VECTOR APPLICATIONS

VECTOR PROBLEMS WITH ACCELERATION

Question 1 (**)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle, of mass 1.5 kg, is moving under the action of a constant force $\mathbf{F}$ N. Initially the particle has velocity $(-2\mathbf{i}+3\mathbf{j})$ ms^{-1} and at time $t=3$ s it has velocity $(10\mathbf{i}-3\mathbf{j})$ ms^{-1}

a) Determine the acceleration of the particle, in vector form.

b) Find the magnitude of $\mathbf{F}$.

c) Calculate the speed of the particle when $t=6$ s.

$\boxed{\mathbf{a}=4\mathbf{i}-2\mathbf{j}}$, $\boxed{|\mathbf{F}|=\sqrt{45}\approx 6.71 \text{ N}}$, $\boxed{|v|=\sqrt{565}\approx 23.8 \text{ ms}^{-1}}$

a) $\underline{v} = \underline{u} + \underline{a}t$
$10\underline{i} - 3\underline{j} = -2\underline{i} + 3\underline{j} + \underline{a} \times 3$
$12\underline{i} - 6\underline{j} = 3\underline{a}$
$\underline{a} = 4\underline{i} - 2\underline{j}$

b) $\underline{F} = m\underline{a}$
$\underline{F} = 1.5(4\underline{i} - 2\underline{j})$
$\underline{F} = 6\underline{i} - 3\underline{j}$
$|\underline{F}| = \sqrt{6^2 + (-3)^2}$
$|\underline{F}| = \sqrt{45} \simeq 6.71$ N

c) $\underline{v} = \underline{u} + \underline{a}t$
$\underline{v} = (-2\underline{i} + 3\underline{j}) + (4\underline{i} - 2\underline{j}) \times 6$
$\underline{v} = 22\underline{i} - 9\underline{i}$
$|\underline{v}| = \sqrt{22^2 + (-9)^2}$
$|\underline{v}| = \sqrt{565} \simeq 23.8$ ms^{-1}

Question 2 (**)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle, of mass 0.5 kg, is moving under the action of a single constant force $\mathbf{F}$ N which produces an acceleration of $(12\mathbf{i} - 16\mathbf{j})$ ms^{-2} .

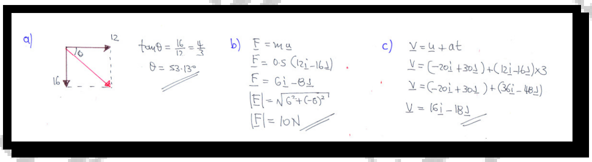
a) Find, in degrees, the angle between the acceleration and the vector $\mathbf{i}$.

b) Determine the magnitude of $\mathbf{F}$.

The velocity of the particle at time t s is $\mathbf{v}$ ms^{-1}.

c) Given further that when $t = 0$ s $\mathbf{v} = (-20\mathbf{i} + 30\mathbf{j})$ ms^{-1}, find the velocity of the particle when $t = 3$ s.

$\boxed{53.13°}$, $\boxed{|\mathbf{F}| = 10 \text{ N}}$, $\boxed{\mathbf{v} = 16\mathbf{i} - 18\mathbf{j}}$



Question 3 (**)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle P, of mass 2 kg, is moving under the action of two constant forces $(2\mathbf{i}+3\mathbf{j}) \text{ N}$ and $(6\mathbf{i}-\mathbf{j}) \text{ N}$.

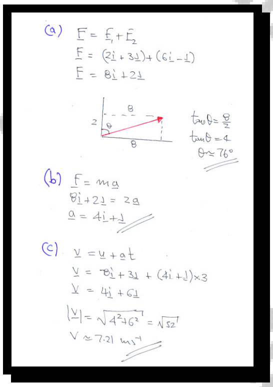
a) Determine, in degrees, the angle between the resultant of these two forces and the vector $\mathbf{j}$.

b) Find the acceleration of P, in vector form.

The initial velocity of P is $-8\mathbf{i}+3\mathbf{j}$.

c) Calculate the speed of P after 3 s.

$\boxed{76°}$, $\boxed{\mathbf{a}=4\mathbf{i}+\mathbf{j}}$, $\boxed{|\mathbf{v}|=\sqrt{52}\approx 7.21 \text{ ms}^{-1}}$



(a) $\underline{F} = \underline{F}_1 + \underline{F}_2$

$\underline{F} = (2\underline{i}+3\underline{j})+(6\underline{i}-\underline{j})$

$\underline{F} = 8\underline{i}+2\underline{j}$

$\tan\theta = \frac{8}{2}$

$\tan\theta = 4$

$\theta \simeq 76°$

(b) $\underline{F} = m\underline{a}$

$8\underline{i}+2\underline{j} = 2\underline{a}$

$\underline{a} = 4\underline{i}+\underline{j}$

(c) $\underline{v} = \underline{u}+\underline{a}t$

$\underline{v} = -8\underline{i}+3\underline{j}+(4\underline{i}+\underline{j})\times 3$

$\underline{v} = 4\underline{i}+6\underline{j}$

$|\underline{v}| = \sqrt{4^2+6^2} = \sqrt{52}$

$v \simeq 7.21 \text{ ms}^{-1}$

Question 4 **(**)**

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle is moving with constant acceleration $(3\mathbf{i}+5\mathbf{j})\ \text{ms}^{-2}$.

At time $t=0$ s the particle has speed u ms^{-1} and at time $t=4$ s the particle has velocity $(4\mathbf{i}+5\mathbf{j})$ ms^{-1}.

Determine the value of u.

$\boxed{u=17}$

USING $\underline{v}=\underline{u}+\underline{a}t$

$4\underline{i}+5\underline{j}=\underline{u}+(3\underline{i}+5\underline{j})\times 4$

$4\underline{i}+5\underline{j}=\underline{u}+12\underline{i}+20\underline{j}$

$-8\underline{i}-15\underline{j}=\underline{u}$

THUS $|\underline{u}|=\sqrt{(-8)^2+(-15)^2}$

$|\underline{u}|=\sqrt{289}$

$|\underline{u}|=17$

$\therefore$ SPEED $u=17$

Question 5 (**)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle P is moving with constant acceleration $(3\mathbf{i}-\mathbf{j})\ \text{ms}^{-2}$.

At time t s the velocity of P is $\mathbf{v}\ \text{ms}^{-1}$.

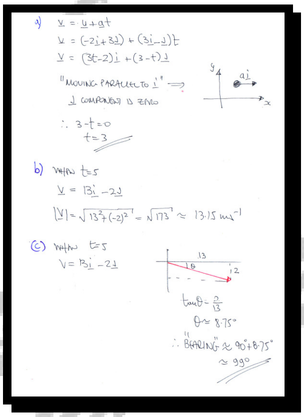
When $t=0$, $\mathbf{v}=(-2\mathbf{i}+3\mathbf{j})$.

a) Find the value of t when P is travelling parallel to the vector $\mathbf{i}$.

b) Determine the speed of P, when $t=5$.

c) Find, in degrees, the angle between the direction of motion of P when $t=5$ and the vector $\mathbf{j}$.

$\boxed{t=3\ \text{s}}$, $\boxed{|\mathbf{v}|=\sqrt{173}\approx 13.15\ \text{ms}^{-1}}$, $\boxed{\approx 99°}$


a) $\underline{v} = \underline{u} + \underline{a}t$

$\underline{v} = (-2\underline{i}+3\underline{j}) + (3\underline{i}-\underline{j})t$

$\underline{v} = (3t-2)\underline{i} + (3-t)\underline{j}$

"MOVING PARALLEL TO $\underline{i}$" $\Longrightarrow$ $\underline{j}$ COMPONENT IS ZERO

$\therefore 3-t=0$

$t=3$

b) WHEN $t=5$

$\underline{v} = 13\underline{i} - 2\underline{j}$

$|\underline{v}| = \sqrt{13^2+(-2)^2} = \sqrt{173} \approx 13.15\ \text{ms}^{-1}$

(c) WHEN $t=5$

$v = 13\underline{i} - 2\underline{j}$

$\tan\theta = \frac{2}{13}$

$\theta \approx 8.75°$

$\therefore$ "BEARING" $\approx 90° + 8.75°$

$\approx 99°$

Question 6 (**+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle P is moving with constant acceleration $(-\mathbf{i}-2\mathbf{j})$ ms^{-2}.

At time t s the velocity of P is $\mathbf{v}$ ms^{-1}. When $t=0$, $\mathbf{v}=(5\mathbf{i}+3\mathbf{j})$.

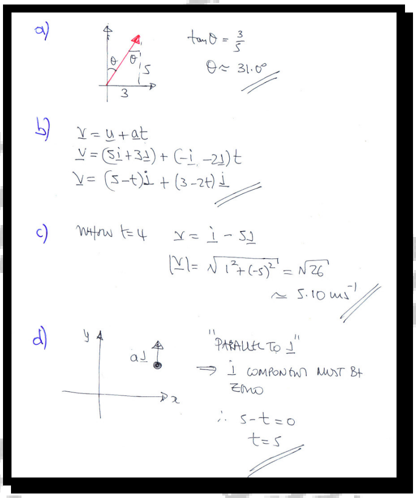
a) Find, in degrees, the angle between the direction of motion of P when $t=0$ and the vector $\mathbf{j}$.

b) Find an expression for $\mathbf{v}$, in terms of t.

c) Determine the speed of P when $t=4$.

d) Calculate the value of t when P is moving parallel to the vector $\mathbf{j}$.

$\boxed{\approx 59.0°}$, $\boxed{\mathbf{v}=(5-t)\mathbf{i}+(3-2t)\mathbf{j}}$, $\boxed{|\mathbf{v}|=\sqrt{26}\approx 5.10 \text{ ms}^{-1}}$, $\boxed{t=5 \text{ s}}$



Question 7 (***)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle P, of mass 2 kg, is moving under the action of a single force $\mathbf{F}$ N.

At time t s the velocity of P is $\mathbf{v}$ ms^{-1}.

When $t = 0$, $\mathbf{v} = (-3\mathbf{i} + \mathbf{j})$ and when $t = 4$, $\mathbf{v} = (5\mathbf{i} + 5\mathbf{j})$.

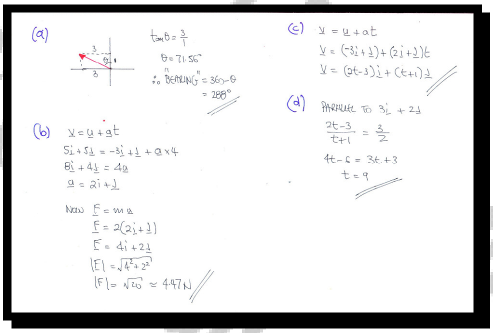
a) Find, in degrees, the bearing of the direction of motion of P, when $t = 0$.

b) Calculate the magnitude of $\mathbf{F}$.

c) Determine, in terms of t, an expression for the velocity of P.

d) Find the time when P is moving parallel to the vector $3\mathbf{i} + 2\mathbf{j}$.

$\boxed{288°}$, $\boxed{|\mathbf{F}| = \sqrt{20} \approx 4.47 \text{ N}}$, $\boxed{\mathbf{v} = (2t-3)\mathbf{i} + (t+1)\mathbf{j}}$, $\boxed{t = 9 \text{ s}}$



(a)

$\tan\theta = \frac{3}{1}$

$\theta = 71.56°$

$\therefore$ "BEARING" $= 360 - \theta$

$= 288°$

(b) $\underline{v} = \underline{u} + \underline{a}t$

$5\underline{i} + 5\underline{j} = -3\underline{i} + \underline{j} + \underline{a} \times 4$

$8\underline{i} + 4\underline{j} = 4\underline{a}$

$\underline{a} = 2\underline{i} + \underline{j}$

Now $\underline{F} = m\underline{a}$

$\underline{F} = 2(2\underline{i} + \underline{j})$

$\underline{F} = 4\underline{i} + 2\underline{j}$

$|\underline{F}| = \sqrt{4^2 + 2^2}$

$|F| = \sqrt{20} \approx 4.47\text{N}$

(c) $\underline{v} = \underline{u} + \underline{a}t$

$\underline{v} = (-3\underline{i} + \underline{j}) + (2\underline{i} + \underline{j})t$

$\underline{v} = (2t-3)\underline{i} + (t+1)\underline{j}$

(d) PARALLEL TO $3\underline{i} + 2\underline{j}$

$\frac{2t-3}{t+1} = \frac{3}{2}$

$4t - 6 = 3t + 3$

$t = 9$

Question 8 (***)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle, of mass 0.5 kg, is moving under the action of a single force $\mathbf{F}$ N.

Initially, the particle is at the point with position vector $(7\mathbf{i} - 2\mathbf{j})$ m and moving with velocity $(3\mathbf{i} - 5\mathbf{j}) \text{ ms}^{-1}$.

Three seconds later the velocity of the particle is $(-3\mathbf{i} + 7\mathbf{j}) \text{ ms}^{-1}$ and its position vector is $(a\mathbf{i} + b\mathbf{j})$ m, where a and b are constants.

a) Determine the magnitude of $\mathbf{F}$.

b) Find the value of a and the value of b.

$\boxed{|\mathbf{F}| = \sqrt{5} \approx 2.24 \text{ N}}$, $\boxed{a = 7, \ b = 1}$

(a) $\underline{v} = \underline{u} + \underline{a}t$

$-3\underline{i} + 7\underline{j} = 3i - 5\underline{j} + \underline{a} \times 3$

$-6\underline{i} + 12\underline{j} = 3\underline{a}$

$a = -2\underline{i} + 4\underline{j}$

Now $\underline{F} = m\underline{a}$

$\underline{F} = 0.5(-2\underline{i} + 4\underline{j})$

$\underline{F} = -\underline{i} + 2\underline{j}$

$|\underline{F}| = \sqrt{(-1)^2 + 2^2} = \sqrt{5} \simeq 2.24 \text{ N}$

(b) $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\underline{r} = (7\underline{i} - 2\underline{j}) + (3\underline{i} - 5\underline{j}) \times 3 + \frac{1}{2}(-2\underline{i} + 4\underline{j}) \times 3^2$

$\underline{r} = (7\underline{i} - 2\underline{j}) + (9\underline{i} - 15\underline{j}) + (-9\underline{i} + 18\underline{j})$

$\underline{r} = 7\underline{i} + \underline{j}$

ie $a = 7$
$b = 1$

Question 9 **(***)**

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle, of mass 0.6 kg, is moving under the action of a single force $(4.5\mathbf{i} - 6\mathbf{j})$ N.

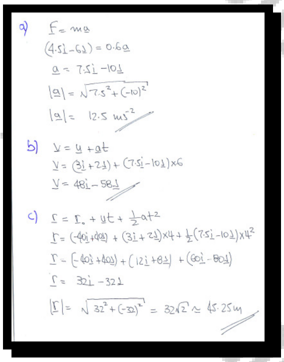
Initially, the particle is at the point with position vector $(-40\mathbf{i} + 40\mathbf{j})$ m and moving with velocity $(3\mathbf{i} + 2\mathbf{j})$ ms^{-1}.

a) Determine the magnitude of the acceleration of the particle.

b) Find the velocity of the particle, after 6 seconds.

c) Calculate the distance of the particle from the origin O, after 4 seconds.

$\boxed{|\mathbf{a}| = 12.5 \text{ ms}^{-2}}$, $\boxed{\mathbf{v} = 48\mathbf{i} - 58\mathbf{j}}$, $\boxed{d = 32\sqrt{2} \approx 45.25 \text{ m}}$



a) $\underline{F} = m\underline{a}$

$(4.5\underline{i} - 6\underline{j}) = 0.6\underline{a}$

$\underline{a} = 7.5\underline{i} - 10\underline{j}$

$|\underline{a}| = \sqrt{7.5^2 + (-10)^2}$

$|\underline{a}| = 12.5 \text{ ms}^{-2}$

b) $\underline{v} = \underline{u} + \underline{a}t$

$\underline{v} = (3\underline{i} + 2\underline{j}) + (7.5\underline{i} - 10\underline{j}) \times 6$

$\underline{v} = 48\underline{i} - 58\underline{j}$

c) $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\underline{r} = (-40\underline{i} + 40\underline{j}) + (3\underline{i} + 2\underline{j}) \times 4 + \frac{1}{2}(7.5\underline{i} - 10\underline{j}) \times 4^2$

$\underline{r} = (-40\underline{i} + 40\underline{j}) + (12\underline{i} + 8\underline{j}) + (60\underline{i} - 80\underline{j})$

$\underline{r} = 32\underline{i} - 32\underline{j}$

$|\underline{r}| = \sqrt{32^2 + (-32)^2} = 32\sqrt{2} \simeq 45.25 \text{ m}$

Question 10 (***+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle P, of mass 1.5 kg, is moving under the action of a single constant force $\mathbf{F}$ N.

When $t = 0$, the velocity of P is $(-6\mathbf{i} + 4\mathbf{j})\text{ ms}^{-1}$.

When $t = 2$, the velocity of P is $(6\mathbf{i} + 20\mathbf{j})\text{ ms}^{-1}$.

a) Find the acceleration of P, in vector form.

b) Determine the magnitude of $\mathbf{F}$.

When $t = 4$, P is at the point with position vector $-6\mathbf{j}$ m. At that instant $\mathbf{F}$ is removed and the particle continues to move freely.

c) Calculate the distance of P from the origin 1 s after $\mathbf{F}$ was removed.

$\boxed{\mathbf{a} = 6\mathbf{i} + 8\mathbf{j}}$, $\boxed{|\mathbf{F}| = 15\text{ N}}$, $\boxed{d \approx 35.0\text{ m}}$

a) $\underline{v} = \underline{u} + \underline{a}t$

$6\underline{i} + 20\underline{j} = -6\underline{i} + 4\underline{j} + \underline{a} \times 2$

$12\underline{i} + 16\underline{j} = 2\underline{a}$

$\underline{a} = 6\underline{i} + 8\underline{j}$

b) $\underline{F} = m\underline{a}$

$\underline{F} = 1.5(6\underline{i} + 8\underline{j})$

$\underline{F} = 9\underline{i} + 12\underline{j}$

$|\underline{F}| = \sqrt{9^2 + 12^2} = 15\text{ N}$

c) when $t = 4$ "$v = u + at$" $\left.\right\}$ $\underline{v} = (-6\underline{i} + 4\underline{j}) + (6\underline{i} + 8\underline{j}) \times 4$

$\underline{v} = 18\underline{i} + 36\underline{j}$

Now $\underline{r} = \underline{r}_0 + \underline{u}t + \cancel{\tfrac{1}{2}\underline{a}t}$

$\underline{r} = (0\underline{i} - 6\underline{j}) + (18\underline{i} + 36\underline{j})$

$\underline{r} = 18\underline{i} + 30\underline{j}$

$d = |18\underline{i} + 30\underline{j}| = \sqrt{18^2 + 30^2} = \sqrt{1224}$

$\approx 35.0\text{ m}$

Question 11 (***)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle P, of mass 2 kg, is moving under the action of a single constant force $\mathbf{F}\text{ N}$. When $t=0\text{ s}$, the velocity of P is $(3\mathbf{i}-5\mathbf{j})\text{ ms}^{-1}$ and when $t=4$ the velocity of P is $(11\mathbf{i}+7\mathbf{j})\text{ ms}^{-1}$.

a) Calculate the speed of the particle when $t=0$.

b) Determine the vector $\mathbf{F}$.

c) Find the value of t when the particle is moving in an eastward direction.

$\boxed{}$, $\boxed{\text{speed}=\sqrt{34}\approx 5.83\text{ ms}^{-1}}$, $\boxed{\mathbf{F}=4\mathbf{i}+6\mathbf{j}}$, $\boxed{t=\frac{5}{3}}$

a) WHEN $t=0$ $\quad \underline{v}=3\underline{i}-5\underline{j}$

SPEED $=|\underline{v}|=|3\underline{i}-5\underline{j}|=\sqrt{3^2+(-5)^2}$

$=\sqrt{9+25}=\sqrt{34}\approx 5.83\text{ ms}^{-1}$

b) USING $\underline{v}=\underline{u}+\underline{a}t$

$\Rightarrow 11\underline{i}+7\underline{j}=3\underline{i}-5\underline{j}+\underline{a}\times 4$

$\Rightarrow 8\underline{i}+12\underline{j}=4\underline{a}$

$\Rightarrow \underline{a}=2\underline{i}+3\underline{j}$

$\underline{F}=m\underline{a}$

$\Rightarrow \underline{F}=2(2\underline{i}+3\underline{j})$

$\Rightarrow \underline{F}=4\underline{i}+6\underline{j}$

c) WRITE A GENERAL EXPRESSION FOR THE VELOCITY VECTOR IN TIME t

$\Rightarrow \underline{v}=\underline{u}+\underline{a}t$

$\Rightarrow \underline{v}=(3\underline{i}-5\underline{j})+(2\underline{i}+3\underline{j})t$

$\Rightarrow \underline{v}=(2t+3)\underline{i}+(3t-5)\underline{j}$

WHEN MOVING DUE EAST, $\underline{v}$ HAS THE FORM $\underline{v}=\lambda\underline{i}$

$\therefore$ NO $\underline{j}$ COMPONENT

$\Rightarrow 3t-5=0$

$\Rightarrow t=5/3$

Question 12 (***)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle P is moving with constant acceleration of $(-\mathbf{i}+\mathbf{j})\ \text{ms}^{-2}$.

It is initially observed passing through the point with position vector $-2\mathbf{j}$ m with velocity of $2\mathbf{i}\ \text{ms}^{-1}$.

a) Find the speed of P, 8 s after it was first observed.

b) Calculate the distance of P from the origin, 8 s after it was first observed.

$\boxed{}$, $\boxed{|\mathbf{v}| = 10\ \text{ms}^{-1}}$, $\boxed{d = 34\ \text{m}}$

a) USING "$\underline{v} = \underline{u} + \underline{a}t$"

$$\underline{v} = \begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}-1\\1\end{pmatrix}t$$

$$\underline{v} = \begin{pmatrix}2-t\\t\end{pmatrix}$$

WHEN $t=8$

$$\underline{v}_8 = \begin{pmatrix}-6\\8\end{pmatrix}$$

SPEED IS |VELOCITY VECTOR|

$$|\underline{v}_8| = \left|\begin{matrix}-6\\8\end{matrix}\right| = \sqrt{(-6)^2+8^2} = \sqrt{36+64} = 10\ \text{ms}^{-1}$$

b) USING $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$$\underline{r} = \begin{pmatrix}0\\-2\end{pmatrix} + \begin{pmatrix}2\\0\end{pmatrix}t + \frac{1}{2}\begin{pmatrix}-1\\1\end{pmatrix}t^2$$

WHEN $t=8$ WE OBTAIN

$$\underline{r}_8 = \begin{pmatrix}0\\-2\end{pmatrix} + 8\begin{pmatrix}2\\0\end{pmatrix} + \frac{1}{2}\times 8^2\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}0+16-32\\-2+0+32\end{pmatrix} = \begin{pmatrix}-16\\30\end{pmatrix}$$

DISTANCE FROM O IS THE MODULUS OF THE ABOVE POSITION VECTOR

$$\text{DISTANCE} = \left|\begin{matrix}-16\\30\end{matrix}\right| = \sqrt{(-16)^2+30^2} = \sqrt{256+900} = \sqrt{1156} = 34\ \text{m}$$

NOTE THAT INTEGRATION & CONDITIONS CAN ALSO BE USED TO SOLVE THIS PROBLEM

Question 13 **(***+)**

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

At time $t = 0$, particle A is at the point with position vector $(7\mathbf{i} - 2\mathbf{j})$ m, moving with velocity $(2\mathbf{i} + 3\mathbf{j})$ ms^{-1}.

At time $t = 0$, particle B is moving with velocity $(3\mathbf{i} + 5\mathbf{j})$ ms^{-1}.

The constant accelerations of the two particles, A and B, are $(0.1\mathbf{i} + 0.2\mathbf{j})$ ms^{-2} and $(-0.2\mathbf{i} + 0.3\mathbf{j})$ ms^{-2}, respectively.

Determine the position vector of B when $t = 0$, given further that A and B collide when $t = 10$.

$\boxed{}$, $\boxed{\mathbf{r}_A = (12\mathbf{i} - 27\mathbf{j}) \text{ m}}$

USING THE EQUATION $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$ FOR EACH PARTICLE

$$\underline{r}_A = \begin{pmatrix} 7 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix}t + \frac{1}{2}\begin{pmatrix} 0.1 \\ 0.2 \end{pmatrix}t^2$$
$$\underline{r}_B = \underline{r}_0 + \begin{pmatrix} 3 \\ 5 \end{pmatrix}t + \frac{1}{2}\begin{pmatrix} -0.2 \\ 0.3 \end{pmatrix}t^2$$

IT IS GIVEN THAT $\underline{r}_A = \underline{r}_B$ WHEN $t = 10$

$$\begin{pmatrix} 7 \\ -2 \end{pmatrix} + 10\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \frac{1}{2}\times 100\begin{pmatrix} 0.1 \\ 0.2 \end{pmatrix} = \underline{r}_0 + 10\begin{pmatrix} 3 \\ 5 \end{pmatrix} + \frac{1}{2}\times 100\begin{pmatrix} -0.2 \\ 0.3 \end{pmatrix}$$
$$\begin{pmatrix} 7 \\ -2 \end{pmatrix} + \begin{pmatrix} 20 \\ 30 \end{pmatrix} + \begin{pmatrix} 5 \\ 10 \end{pmatrix} = \underline{r}_0 + \begin{pmatrix} 30 \\ 50 \end{pmatrix} + \begin{pmatrix} -10 \\ 15 \end{pmatrix}$$
$$\begin{pmatrix} 32 \\ 38 \end{pmatrix} = \underline{r}_0 + \begin{pmatrix} 20 \\ 65 \end{pmatrix}$$
$$\underline{r}_0 = \begin{pmatrix} 12 \\ -27 \end{pmatrix}$$

$\therefore$ IT STARTS FROM $12\underline{i} - 27\underline{j}$

Question 14 (****)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

The velocity of a particle, $\mathbf{v}$ ms^{-1}, at time t s after a given instant is

$$\mathbf{v} = (3-2t)\mathbf{i} + (3t-6)\mathbf{j}.$$

a) Find the speed of the particle when $t = 0$.

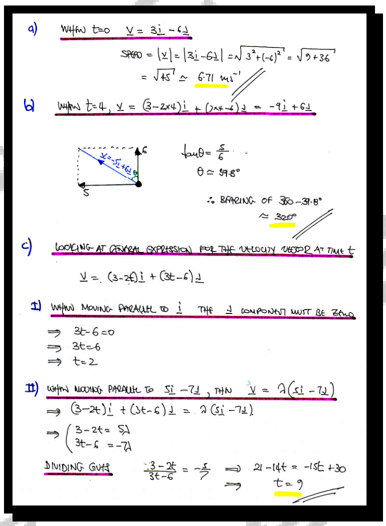
b) Determine the bearing on which the particle is moving when $t = 4$.

c) Calculate the value of t when the particle is moving …

i. … parallel to $\mathbf{i}$.

ii. … parallel to $5\mathbf{i} - 7\mathbf{j}$.

$\boxed{\text{M.S.C.}}$, $\boxed{\text{speed} = \sqrt{45} \approx 6.71 \text{ ms}^{-1}}$, $\boxed{320°}$, $\boxed{t = 2}$, $\boxed{t = 9}$



a) WHEN $t=0$ $\underline{v} = 3\underline{i} - 6\underline{j}$

SPEED $= |\underline{v}| = |3\underline{i} - 6\underline{j}| = \sqrt{3^2 + (-6)^2} = \sqrt{9+36}$

$= \sqrt{45} \simeq 6.71$ ms^{-1}

b) WHEN $t=4$, $\underline{v} = (3-2\times4)\underline{i} + (3\times4-6)\underline{j} = -9\underline{i} + 6\underline{j}$

$\underline{v} = -5\underline{i} + 6\underline{j}$

6

5

$\tan\theta = \frac{5}{6}$

$\theta \simeq 39.8°$

$\therefore$ BEARING OF $360 - 39.8°$

$\approx 320°$

c) LOOKING AT GENERAL EXPRESSION FOR THE VELOCITY VECTOR AT TIME t

$\underline{v} = (3-2t)\underline{i} + (3t-6)\underline{j}$

I) WHEN MOVING PARALLEL TO $\underline{i}$ THE $\underline{j}$ COMPONENT MUST BE ZERO

$\Rightarrow 3t - 6 = 0$

$\Rightarrow 3t = 6$

$\Rightarrow t = 2$

II) WHEN MOVING PARALLEL TO $5\underline{i} - 7\underline{j}$, THEN $\underline{v} = \lambda(5\underline{i} - 7\underline{j})$

$\Rightarrow (3-2t)\underline{i} + (3t-6)\underline{j} = \lambda(5\underline{i} - 7\underline{j})$

$\Rightarrow \begin{cases} 3-2t = 5\lambda \\ 3t-6 = -7\lambda \end{cases}$

DIVIDING GIVES $\frac{3-2t}{3t-6} = -\frac{5}{7} \Rightarrow 21 - 14t = -15t + 30$

$\Rightarrow t = 9$

Question 15 (****)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle P is moving with constant acceleration of $\left(\frac{1}{10}\mathbf{i}-\frac{1}{5}\mathbf{j}\right)$ ms^{-2}.

It is initially observed passing through the point with position vector $\left(-20\mathbf{i}-\frac{15}{2}\mathbf{j}\right)$ m with velocity of $(4\mathbf{i}+2\mathbf{j})$ ms^{-1}.

a) Find an expression for the position vector of P, t s after it was first observed.

b) Calculate the times when P is due east of the origin O.

c) Determine the speed of P when it is travelling in a south-eastern direction.

$\boxed{}$, $\boxed{\mathbf{r}=\left[\left(-20+4t+\frac{1}{20}t^2\right)\mathbf{i}+\left(-\frac{15}{2}+2t-\frac{1}{10}t^2\right)\mathbf{j}\right]\text{m}}$, $\boxed{t=5\text{ s},\ 15\text{ s}}$, $\boxed{v=10\sqrt{2}\approx 14.14\text{ ms}^{-1}}$

a) USING $\underline{r}=\underline{r}_0+\underline{u}t+\frac{1}{2}\underline{a}t^2$

$\underline{r}=\left(-20\underline{i}-\frac{15}{2}\underline{j}\right)+(4\underline{i}+2\underline{j})t+\frac{1}{2}\left(\frac{1}{10}\underline{i}-\frac{1}{5}\underline{j}\right)t^2$

$\underline{r}=\left(-20+4t+\frac{1}{20}t^2\right)\underline{i}+\left(-\frac{15}{2}+2t-\frac{1}{10}t^2\right)\underline{j}$

b) DUE EAST IN POSITION, IMPLIES $\underline{j}$ COMPONENT ZERO & $\underline{i}$ COMPONENT POSITIVE (IN THE POSITION VECTOR)

$-\frac{15}{2}+2t-\frac{1}{10}t^2=0$

$-75+20t-t^2=0$

$t^2-20t+75=0$

$(t-5)(t-15)=0$

$t=\begin{cases}5\\15\end{cases}$

CHECK $\underline{i}$ FOR BEING POSITIVE

$t=5 \Rightarrow -20+4\times5+\frac{1}{20}\times5^2=\frac{5}{4}>0$

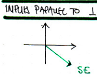
$t=15 \Rightarrow -20+4\times15+\frac{1}{20}\times15^2=\frac{205}{4}>0$

c) USING $\underline{v}=\underline{u}+\underline{a}t$

$\underline{v}=4\underline{i}+2\underline{j}+\left(\frac{1}{10}\underline{i}-\frac{1}{5}\underline{j}\right)t$

$\underline{v}=\left(4+\frac{1}{10}t\right)\underline{i}+\left(2-\frac{1}{5}t\right)\underline{j}$

"SOUTH EAST" DIRECTION OF MOTION, IMPLIES PARALLEL TO $\underline{i}-\underline{j}$



$\Rightarrow \left(4+\frac{1}{10}t\right)\underline{i}+\left(2-\frac{1}{5}t\right)\underline{j}=\lambda(\underline{i}-\underline{j})$

$\Rightarrow \begin{cases}4+\frac{1}{10}t=\lambda\\2-\frac{1}{5}t=-\lambda\end{cases}$

$\Rightarrow 2-\frac{1}{5}t=-4-\frac{1}{10}t$

$\Rightarrow 6=\frac{1}{10}t$

$\Rightarrow t=60$

WHEN $t=60$

$\underline{v}=\left(4+\frac{1}{10}\times60\right)\underline{i}+\left(2-\frac{1}{5}\times60\right)$

$\underline{v}=10\underline{i}-10\underline{j}$

SPEED $=|\underline{v}|=|10\underline{i}-10\underline{j}|=\sqrt{10^2+(-10)^2}$

$=\sqrt{100+100}=\sqrt{200}=10\sqrt{2}\simeq 14.14\text{ ms}^{-1}$

Question 16 (****)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

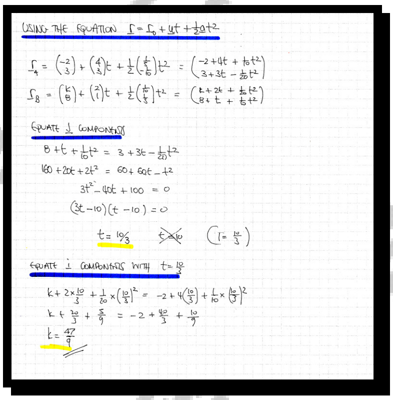
Two particles are moving with constant acceleration on a horizontal surface where O is contained.

At time $t = 0$ s, one of the particles is at the point with position vector $(-2\mathbf{i} + 3\mathbf{j})$ m, moving with velocity $(4\mathbf{i} + 3\mathbf{j})$ ms^{-1} and constant acceleration $\left(\frac{1}{5}\mathbf{i} - \frac{1}{10}\mathbf{j}\right)$ ms^{-2}.

At time $t = 0$ s, the other particle is at the point with position vector $(k\mathbf{i} + 8\mathbf{j})$ m, where k is a scalar constant, moving with velocity $(2\mathbf{i} + \mathbf{j})$ ms^{-1} and constant acceleration $\left(\frac{1}{10}\mathbf{i} + \frac{1}{5}\mathbf{j}\right)$ ms^{-2}.

If the two particles first collide at time $t = T$ s, determine the exact value of k.

$\square$, $\boxed{k = \frac{47}{9}}$



USING THE EQUATION $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$$\underline{r}_A = \begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}4\\3\end{pmatrix}t + \frac{1}{2}\begin{pmatrix}\frac{1}{5}\\-\frac{1}{10}\end{pmatrix}t^2 = \begin{pmatrix}-2+4t+\frac{1}{10}t^2\\3+3t-\frac{1}{20}t^2\end{pmatrix}$$

$$\underline{r}_B = \begin{pmatrix}k\\8\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}t + \frac{1}{2}\begin{pmatrix}\frac{1}{10}\\\frac{1}{5}\end{pmatrix}t^2 = \begin{pmatrix}k+2t+\frac{1}{20}t^2\\8+t+\frac{1}{10}t^2\end{pmatrix}$$

EQUATE $\underline{j}$ COMPONENTS

$$8 + t + \frac{1}{10}t^2 = 3 + 3t - \frac{1}{20}t^2$$
$$160 + 20t + 2t^2 = 60 + 60t - t^2$$
$$3t^2 - 40t + 100 = 0$$
$$(3t-10)(t-10) = 0$$
$$t = 10/3 \qquad \cancel{t = 10} \qquad \left(T = \tfrac{10}{3}\right)$$

EQUATE $\underline{i}$ COMPONENTS WITH $t = \frac{10}{3}$

$$k + 2\times\frac{10}{3} + \frac{1}{20}\times\left(\frac{10}{3}\right)^2 = -2 + 4\left(\frac{10}{3}\right) + \frac{1}{10}\times\left(\frac{10}{3}\right)^2$$
$$k + \frac{20}{3} + \frac{5}{9} = -2 + \frac{40}{3} + \frac{10}{9}$$
$$k = \frac{47}{9}$$

Question 17 (****+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

Two particles are moving with constant acceleration on a horizontal surface where O is contained.

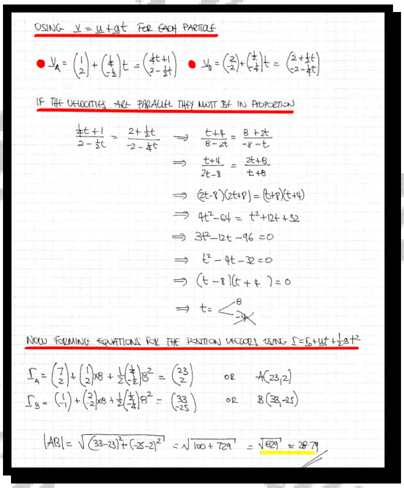
At time $t = 0$ s, one of the particles is at the point with position vector $(7\mathbf{i} + 2\mathbf{j})$ m, moving with velocity $(\mathbf{i} + 2\mathbf{j})$ ms^{-1} and constant acceleration $\left(\frac{1}{4}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$ ms^{-2}.

At time $t = 0$ s, the other particle is at the point with position vector $(\mathbf{i} - \mathbf{j})$ m, moving with velocity $(2\mathbf{i} - 2\mathbf{j})$ ms^{-1} and constant acceleration $\left(\frac{1}{2}\mathbf{i} - \frac{1}{4}\mathbf{j}\right)$ ms^{-2}.

Calculate the distance between the two particles, at the instant when they are moving in parallel directions to one another.

[You may ignore any motion taking place prior to time $t = 0$ s]

$\square$, $\boxed{d = \sqrt{829} \approx 28.79 \text{ m}}$



USING $\underline{v} = \underline{u} + \underline{a}t$ FOR EACH PARTICLE

- $\underline{v}_A = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} \frac{1}{4} \\ -\frac{1}{2} \end{pmatrix} t = \begin{pmatrix} \frac{1}{4}t + 1 \\ 2 - \frac{1}{2}t \end{pmatrix}$
- $\underline{v}_B = \begin{pmatrix} 2 \\ -2 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{4} \end{pmatrix} t = \begin{pmatrix} 2 + \frac{1}{2}t \\ -2 - \frac{1}{4}t \end{pmatrix}$

IF THE VELOCITIES ARE PARALLEL THEY MUST BE IN PROPORTION

$$\frac{\frac{1}{4}t + 1}{2 - \frac{1}{2}t} = \frac{2 + \frac{1}{2}t}{-2 - \frac{1}{4}t} \implies \frac{t+4}{8-2t} = \frac{8+2t}{-8-t}$$
$$\implies \frac{t+4}{2t-8} = \frac{2t+8}{t+8}$$
$$\implies (2t-8)(2t+8) = (t+8)(t+4)$$
$$\implies 4t^2 - 64 = t^2 + 12t + 32$$
$$\implies 3t^2 - 12t - 96 = 0$$
$$\implies t^2 - 4t - 32 = 0$$
$$\implies (t-8)(t+4) = 0$$
$$\implies t = \begin{cases} 8 \\ \cancel{-4} \end{cases}$$

NOW FORMING EQUATIONS FOR THE POSITION VECTORS USING $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$$\underline{r}_A = \begin{pmatrix} 7 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \times 8 + \frac{1}{2}\begin{pmatrix} \frac{1}{4} \\ -\frac{1}{2} \end{pmatrix} 8^2 = \begin{pmatrix} 23 \\ 2 \end{pmatrix} \quad \text{OR} \quad A(23, 2)$$
$$\underline{r}_B = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} \times 8 + \frac{1}{2}\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{4} \end{pmatrix} 8^2 = \begin{pmatrix} 33 \\ -25 \end{pmatrix} \quad \text{OR} \quad B(33, -25)$$

$$|AB| = \sqrt{(33-23)^2 + (-25-2)^2} = \sqrt{100 + 729} = \sqrt{829} \approx 28.79$$

Question 18 (****+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle P is moving from point A to point B, with constant acceleration of $(p\mathbf{i}+q\mathbf{j})\ \text{ms}^{-2}$, where p and q are constants. The velocity of P at A is $(10\mathbf{i}-13\mathbf{j})\ \text{ms}^{-1}$ and the velocity of P at B is $(22\mathbf{i}+22\mathbf{j})\ \text{ms}^{-1}$.

Given that the magnitude of the acceleration is $3.7\ \text{ms}^{-2}$, find the value of p and the value of q.

$\boxed{p=1.2}$, $\boxed{q=3.5}$

USING $\underline{v} = \underline{u} + \underline{a}t$

$22\underline{i}+22\underline{j} = 10\underline{i}-13\underline{j} + (p\underline{i}+q\underline{j})t$

$12\underline{i}+35\underline{j} = pt\underline{i} + qt\underline{j}$

$pt = 12$

$qt = 35$

ACCELERATION MAGNITUDE IS 3.7

$\Rightarrow |\underline{a}| = |p\underline{i}+q\underline{j}| = 3.7$

$\Rightarrow \sqrt{p^2+q^2} = 3.7$

$\Rightarrow p^2+q^2 = 13.69$

SOLVING AS FOLLOWS

$\left.\begin{array}{l} p^2t^2 = 144 \\ q^2t^2 = 1225 \end{array}\right\} \Rightarrow p^2t^2+q^2t^2 = 1369$

$\Rightarrow t^2(p^2+q^2) = 1369$

$\Rightarrow 13.69t^2 = 1369$

$\Rightarrow t^2 = 100$

$\Rightarrow t = 10$

FINALLY

$pt = 12$ & $qt = 35$

$10p = 12$ & $10q = 35$

$p = 1.2$ & $q = 3.5$

Question 19 (****+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle P is moving with constant acceleration of $(0.25\mathbf{i} - 0.5\mathbf{j})\ \text{ms}^{-2}$.

It is initially observed passing through the point with position vector $(2\mathbf{i} - 3\mathbf{j})$ m with velocity of $(\mathbf{i} + 3.5\mathbf{j})\ \text{ms}^{-1}$.

a) Find an expression for the velocity of P, t s after it was first observed.

b) Determine an expression for the position vector of P, t s after it was first observed.

c) Calculate the distance of P from the origin O, when P is moving in a south-eastern direction.

$\boxed{\mathbf{v} = \left[(1+0.25t)\mathbf{i} + (3.5-0.5t)\mathbf{j}\right]\text{ms}^{-1}}$, $\boxed{\mathbf{r} = \left[\left(2+t+\frac{1}{8}t^2\right)\mathbf{i} + \left(-3+3.5t-\frac{1}{4}t^2\right)\mathbf{j}\right]\text{m}}$, $\boxed{d \approx 64.04 \text{ m}}$

a) $\underline{v} = \underline{u} + \underline{a}t$

$\underline{v} = (\underline{i} + 3.5\underline{j}) + (0.25\underline{i} - 0.5\underline{j})t$

$\underline{v} = (1+0.25t)\underline{i} + (3.5-0.5t)\underline{j}$

b) $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\underline{r} = (2\underline{i} - 3\underline{j}) + (\underline{i} + 3.5\underline{j})t + \frac{1}{2}(0.25\underline{i} - 0.5\underline{j})t^2$

$\underline{r} = (2 + t + \frac{1}{8}t^2)\underline{i} + (-3 + 3.5t - \frac{1}{4}t^2)\underline{j}$

c) SOUTH-EAST IS IN THE DIRECTION $\underline{i} - \underline{j}$

$\frac{1+0.25t}{3.5-0.5t} = \frac{1}{-1}$

$-1 - 0.25t = 3.5 - 0.5t$

$0.25t = 4.5$

$\boxed{t = 18}$

$\underline{r} = (2 + 18 + \frac{1}{8} \times 18^2)\underline{i} + (-3 + 3.5 \times 18 - \frac{1}{4} \times 18^2)\underline{j}$

$\underline{r} = 60.5\underline{i} - 21\underline{j}$

HENCE

$|\underline{r}| = \sqrt{60.5^2 + (-21)^2} = 64.0400... \approx 64.0$

Question 20 (****+)

The unit vectors $\mathbf{i}$ *and* $\mathbf{j}$ *are oriented due east and due north, respectively.*

At time $t = 0$ s, a particle of mass 4 kg, is sighted at the point A with position vector $(-17\mathbf{i} - 50\mathbf{j})$ m and moving with constant velocity $(-2\mathbf{i} + 2\mathbf{j})$ ms^{-1}.

At time $t = 10$ s, two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ each of magnitude 50 N, begin to act on the particle until it passes through the point B, after a further period of 10 s.

It is further given that

- $\mathbf{F}_1$ is acting in the direction $(3\mathbf{i} - 4\mathbf{j})$.
- $\mathbf{F}_2$ is acting in the direction $(-7\mathbf{i} + 24\mathbf{j})$.

Determine, correct to the nearest m, the distance between A and B.

$\boxed{}$, $\boxed{d \approx 213 \text{ m}}$

START WITH THE RESULTANT OF THE TWO FORCES

- $|3\underline{i} - 4\underline{j}| = \sqrt{9+16} = \sqrt{25} = 5$

$\therefore F_1 = 10(3\underline{i} - 4\underline{j}) = 30\underline{i} - 40\underline{j}$

- $|-7\underline{i} + 24\underline{j}| = \sqrt{49+576} = \sqrt{625} = 25$

$\therefore \underline{F}_2 = 2(-7\underline{i} + 24\underline{j}) = -14\underline{i} + 48\underline{j}$

$\underline{F} = \underline{F}_1 + \underline{F}_2 = (30\underline{i} - 40\underline{j}) + (-14\underline{i} + 48\underline{j}) = 16\underline{i} + 8\underline{j}$

NEXT FIND THE ACCELERATION, ONCE THE FORCES START ACTING

$\underline{F} = m\underline{a} \implies 16\underline{i} + 8\underline{j} = 4\underline{a}$

$\implies \underline{a} = 4\underline{i} + 2\underline{j}$

NOW TRACE THE JOURNEY

$t=0 \quad \underline{r} = -17\underline{i} - 50\underline{j}, \quad \underline{v} = -2\underline{i} + 2\underline{j}$

$\underline{r} = \underline{r}_0 + \underline{v}t$

$\underline{r} = (-17\underline{i} - 50\underline{j}) + (-2\underline{i} + 2\underline{j}) \times 10$

$\underline{r} = (-17\underline{i} - 50\underline{j}) + (-20\underline{i} + 20\underline{j})$

$\underline{r} = -37\underline{i} - 30\underline{j}$

FINALLY THE LAST 10 SECONDS

$\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\underline{r} = (-37\underline{i} - 30\underline{j}) + (-2\underline{i} + 2\underline{j}) \times 10 + \frac{1}{2}(4\underline{i} + 2\underline{j}) \times 10^2$

$\underline{r} = (-37\underline{i} - 30\underline{j}) + (-20\underline{i} + 20\underline{j}) + (200\underline{i} + 100\underline{j})$

$\underline{r} = 143\underline{i} + 90\underline{j}$

HENCE THE PARTICLE TRAVELLED FROM $-17\underline{i} - 50\underline{j}$ TO $143\underline{i} + 90\underline{j}$
OR IN CO-ORDINATES FROM $(-17, -50)$ TO $(143, 90)$

USING THE DISTANCE FORMULA

$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$d = \sqrt{(90 - (-50))^2 + (143 - (-17))^2}$

$d = \sqrt{140^2 + 160^2}$

$d = 20\sqrt{113} \simeq 213$ m

Question 21 (****+)

The unit vectors $\mathbf{i}$ *and* $\mathbf{j}$ *are oriented due east and due north, respectively.*

A particle moves with constant acceleration between two points A and B.

The particle is passing through A with velocity $(3\mathbf{i}-\mathbf{j})\ \text{ms}^{-1}$ and 10 seconds later is passing through B with velocity $(9\mathbf{i}+11\mathbf{j})\ \text{ms}^{-1}$.

Find the distance AB, and hence determine the **average speed** of the particle as it travels from A to B.

$\boxed{\text{MMS-Z}}$, $\boxed{d \approx 78.10\text{ m}}$, $\boxed{\text{average speed} \approx 7.81\text{ ms}^{-1}}$

START BY FINDING THE CONSTANT ACCELERATION

$\Rightarrow \underline{v} = \underline{u} + \underline{a}t$

$\Rightarrow 9\underline{i} + 11\underline{j} = 3\underline{i} - \underline{j} + \underline{a} \times 10$

$\Rightarrow 6\underline{i} + 12\underline{j} = 10\underline{a}$

$\Rightarrow \underline{a} = \frac{3}{5}\underline{i} + \frac{6}{5}\underline{j}$

NOW USING $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$ WE OBTAIN

$\Rightarrow \underline{r}_B = \underline{r}_A + (3\underline{i} - \underline{j}) \times 10 + \frac{1}{2}\left(\frac{3}{5}\underline{i} + \frac{6}{5}\underline{j}\right) \times 10^2$

$\Rightarrow \underline{r}_B - \underline{r}_A = (30\underline{i} - 10\underline{j}) + (30\underline{i} + 60\underline{j})$

$\Rightarrow \underline{r}_B - \underline{r}_A = 60\underline{i} + 50\underline{j}$ ← POSITION OF B RELATIVE TO A

$\Rightarrow |\underline{r}_B - \underline{r}_A| = \sqrt{60^2 + 50^2}$

$\Rightarrow |\underline{r}_B - \underline{r}_A| = 78.10$ 2 d.p.

FINALLY THE AVERAGE SPEED IS GIVEN BY

$\text{AVERAGE SPEED} = \frac{\text{DISTANCE}}{\text{TIME}} = \frac{78.10}{10} = 7.81\text{ ms}^{-1}$

VECTOR PROBLEMS WITHOUT ACCELERATION

Question 1 (***)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle is moving with constant velocity $(4\mathbf{i} - 2\mathbf{j})\ \text{ms}^{-1}$.

At time $t = 7$ s the particle is at the point with position vector $(19\mathbf{i} + 2\mathbf{j})$ m.

Determine the distance of the particle from the origin O, when $t = 3$.

$$\boxed{d = \sqrt{109} \approx 10.44 \text{ m}}$$

$\underline{r} = \underline{r}_0 + \underline{v}t$

$19\underline{i} + 2\underline{j} = \underline{r}_0 + (4\underline{i} - 2\underline{j}) \times 7$

$19\underline{i} + 2\underline{j} = \underline{r}_0 + 28\underline{i} - 14\underline{j}$

$\underline{r}_0 = -9\underline{i} + 16\underline{j}$

THUS $\underline{r} = -9\underline{i} + 16\underline{j} + (4\underline{i} - 2\underline{j}) \times 3$

$\underline{r} = 3\underline{i} + 10\underline{j}$

$d = |3\underline{i} + 10\underline{j}| = \sqrt{3^2 + 10^2}$

$d = \sqrt{109} \simeq 10.44$ m

Question 2 (***)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

Two ships, P and Q, are sailing in a channel, with constant velocities.

Initially, P is at the point with position vector $(6\mathbf{i} - 2\mathbf{j})$ km and Q is at the point with position vector $-5\mathbf{i}$ km. The velocity P is $12\mathbf{j}$ km h^{-1} and the velocity of Q is $(8\mathbf{i} - 6\mathbf{j})$ km h^{-1}.

a) Determine the speed of Q.

The respective position vectors of P and Q, at time t hours, are $\mathbf{p}$ and $\mathbf{q}$.

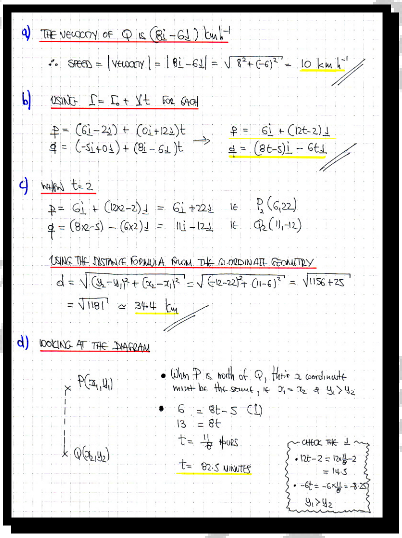
b) Find expressions for $\mathbf{p}$ and $\mathbf{q}$.

c) Calculate the distance between P and Q when $t = 2$.

d) Determine the value of t when P is north of Q.

$\boxed{}$, $\boxed{\text{speed} = 10 \text{ km h}^{-1}}$, $\boxed{\mathbf{p} = \left[6\mathbf{i} + (12t - 2)\mathbf{j}\right] \text{km}, \quad \mathbf{q} = \left[(8t - 5)\mathbf{i} - 6t\mathbf{j}\right] \text{km}}$,

$\boxed{|PQ| = \sqrt{1181} \approx 34.4 \text{ km}}$, $\boxed{t = \frac{11}{8} = 82.5 \text{ minutes}}$


a) THE VELOCITY OF Q IS $(8\underline{i} - 6\underline{j})$ km h^{-1}

$\therefore$ SPEED $= |\text{VELOCITY}| = |8\underline{i} - 6\underline{j}| = \sqrt{8^2 + (-6)^2} = 10$ km h^{-1}

b) USING $\underline{r} = \underline{r}_0 + \underline{v}t$ FOR EACH

$\underline{p} = (6\underline{i} - 2\underline{j}) + (0\underline{i} + 12\underline{j})t$
$\underline{q} = (-5\underline{i} + 0\underline{j}) + (8\underline{i} - 6\underline{j})t$ $\Rightarrow$ $\underline{p} = 6\underline{i} + (12t - 2)\underline{j}$
$\underline{q} = (8t - 5)\underline{i} - 6t\underline{j}$

c) WHEN $t = 2$

$\underline{p} = 6\underline{i} + (12\times2 - 2)\underline{j} = 6\underline{i} + 22\underline{j}$ IE $P_2(6, 22)$
$\underline{q} = (8\times2 - 5) - (6\times2)\underline{j} = 11\underline{i} - 12\underline{j}$ IE $Q_2(11, -12)$

USING THE DISTANCE FORMULA FROM THE CO-ORDINATE GEOMETRY

$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{(-12 - 22)^2 + (11 - 6)^2} = \sqrt{1156 + 25}$
$= \sqrt{1181} \simeq 34.4$ km

d) LOOKING AT THE DIAGRAM

$P(x_1, y_1)$

$Q(x_2, y_2)$

- When P is north of Q, their x coordinate must be the same, IE $x_1 = x_2$ & $y_1 > y_2$
- $6 = 8t - 5$ ($\underline{i}$)
$13 = 8t$
$t = \frac{11}{8}$ HOURS

$t = 82.5$ MINUTES

CHECK THE $\underline{j}$
- $12t - 2 = 12\times\frac{11}{8} - 2$
$= 14.5$
- $-6t = -6\times\frac{11}{8} = -8.25$

$y_1 > y_2$

Question 3 (***)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

At time $t = 0$ s a particle P is observed to be passing through the point with position vector $(-24\mathbf{i} + 17\mathbf{j})$ m and moving with constant velocity $(3\mathbf{i} - 2\mathbf{j})$ ms^{-1}.

a) Calculate the speed of P.

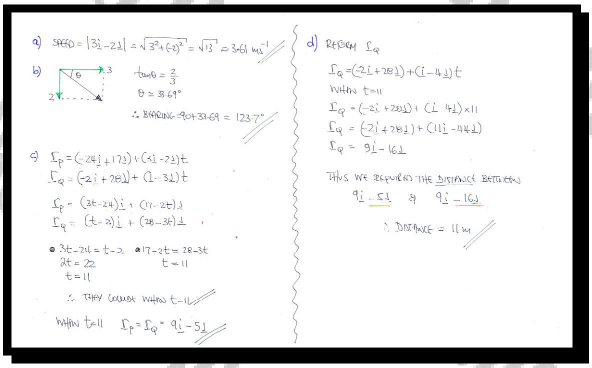
b) Find the direction of motion of P, giving the answer as a bearing.

At time $t = 0$ s, another particle Q, is observed to be passing through the point with position vector $(-2\mathbf{i} + 28\mathbf{j})$ m.

c) Given that Q is moving with constant velocity $(\mathbf{i} - 3\mathbf{j})$ ms^{-1}, show that P and Q will collide at some point A, further determining the position vector of A.

d) Given instead that Q has constant velocity $(\mathbf{i} - 4\mathbf{j})$ ms^{-1}, determine the distance between P and Q, when $t = 11$ s.

$\boxed{\text{speed} = \sqrt{13} \approx 3.61 \text{ ms}^{-1}}$, $\boxed{124°}$, $\boxed{\mathbf{a} = (9\mathbf{i} - 5\mathbf{j})\text{m}}$, $\boxed{d = 11 \text{ m}}$



a) SPEED $= |3\underline{i} - 2\underline{j}| = \sqrt{3^2 + (-2)^2} = \sqrt{13} \approx 3.61$ ms^{-1}

b) $\tan\theta = \frac{2}{3}$

$\theta \simeq 33.69°$

$\therefore$ BEARING $= 90 + 33.69 = 123.7°$

c) $\underline{r}_P = (-24\underline{i} + 17\underline{j}) + (3\underline{i} - 2\underline{j})t$

$\underline{r}_Q = (-2\underline{i} + 28\underline{j}) + (\underline{i} - 3\underline{j})t$

$\underline{r}_P = (3t - 24)\underline{i} + (17 - 2t)\underline{j}$

$\underline{r}_Q = (t - 2)\underline{i} + (28 - 3t)\underline{j}$

• $3t - 24 = t - 2$, $2t = 22$, $t = 11$ • $17 - 2t = 28 - 3t$, $t = 11$

$\therefore$ THEY COLLIDE WHEN $t = 11$

WHEN $t = 11$ $\underline{r}_P = \underline{r}_Q = 9\underline{i} - 5\underline{j}$

d) REFORM $\underline{r}_Q$

$\underline{r}_Q = (-2\underline{i} + 28\underline{j}) + (\underline{i} - 4\underline{j})t$

WHEN $t = 11$

$\underline{r}_Q = (-2\underline{i} + 28\underline{j}) + (\underline{i} - 4\underline{j}) \times 11$

$\underline{r}_Q = (-2\underline{i} + 28\underline{j}) + (11\underline{i} - 44\underline{j})$

$\underline{r}_Q = 9\underline{i} - 16\underline{j}$

THUS WE REQUIRED THE DISTANCE BETWEEN $9\underline{i} - 5\underline{j}$ & $9\underline{i} - 16\underline{j}$

$\therefore$ DISTANCE $= 11$ m

Question 4 (***)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A ship P is sailing with constant velocity $(2\mathbf{i}-6\mathbf{j})\ \text{km h}^{-1}$.

a) Calculate the speed of P.

At noon P is at the point with position vector $(4\mathbf{i}+2\mathbf{j})$ km.

At time t hours after noon the position vector of P is $\mathbf{p}$ km.

b) Determine an expression for $\mathbf{p}$, in terms of t.

The position vector of another ship Q, $\mathbf{q}$ km t hours after noon, is given by

$$\mathbf{q} = 7\mathbf{i} - 4\mathbf{j} + (-4\mathbf{i} - 2\mathbf{j})t.$$

c) Calculate the value of t when Q is west of P.

d) Find the distance between the two ships when Q is west of P.

$\boxed{\text{speed} = \sqrt{40} \approx 6.32\ \text{km h}^{-1}}$, $\boxed{\mathbf{p} = \left[(2t+4)\mathbf{i} + (2-6t)\mathbf{j}\right]\text{km}}$, $\boxed{t = 1.5}$, $\boxed{d = 6\ \text{km}}$

a) SPEED $= |2\underline{i} - 6\underline{j}| = \sqrt{2^2 + (-6)^2} = \sqrt{40} \simeq 6.32\ \text{ms}^{-1}$

b) $\underline{p} = (4\underline{i} + 2\underline{j}) + (2\underline{i} - 6\underline{j})t$

$\underline{p} = (2t+4)\underline{i} + (2-6t)\underline{j}$

c) $\underline{q} = 7\underline{i} - 4\underline{j} + (-4\underline{i} - 2\underline{j})t$

$\underline{q} = (7-4t)\underline{i} + (-4-2t)\underline{j}$

"DUE WEST" $\Rightarrow$ SAME $\underline{j}$ COMPONENT

$\Rightarrow 2 - 6t = -4 - 2t$

$6 = 4t$

$t = \frac{3}{2}$

d) WHEN $t = \frac{3}{2}$ $\quad \underline{p} = 7\underline{i} - 7\underline{j}$

$\underline{q} = \underline{i} - 7\underline{j}$

$\therefore$ DISTANCE IS 6 km

Question 5 (***+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

Two helicopters, P and Q, are flying with constant velocities in the same horizontal plane. At noon, P is at the point with position vector $(20\mathbf{i}+35\mathbf{j})$ km. The position vector of P at time t hours after noon is $\mathbf{p}$.

a) Given that when $t=\frac{1}{2}$ hours, $\mathbf{p}=(65\mathbf{i}+59\mathbf{j})$ km, determine the velocity of P.

b) Find an expression for $\mathbf{p}$, in terms of t.

At noon, Q is at the point with position vector $200\mathbf{j}$ km.

The speed of Q is 125 km h^{-1}, in the direction $(24\mathbf{i}-7\mathbf{j})$.

The position vector of Q, at time t hours after noon, is $\mathbf{q}$.

c) Find an expression for $\mathbf{q}$, in terms of t.

d) Calculate the distance between P and Q when $t=2$.

$\boxed{}$, $\boxed{\mathbf{v}=(90\mathbf{i}+48\mathbf{j})\,\text{km h}^{-1}}$, $\boxed{\mathbf{p}=\left[(90t+20)\mathbf{i}+(48t+35)\mathbf{j}\right]\text{km}}$,

$\boxed{\mathbf{q}=\left[120t\mathbf{i}+(200-35t)\mathbf{j}\right]\text{km}}$, $\boxed{|PQ|=\sqrt{1601}\approx 40.0 \text{ km}}$

a) USING $\underline{r}=\underline{r}_0+\underline{v}t$ WE OBTAIN

$\Rightarrow 65\underline{i}+59\underline{j}=20\underline{i}+35\underline{j}+\frac{1}{2}\underline{v}$

$\Rightarrow 45\underline{i}+24\underline{j}=\frac{1}{2}\underline{v}$

$\Rightarrow \underline{v}=90\underline{i}+48\underline{j}$

b) USING $\underline{p}=\underline{p}_0+\underline{v}t$

$\Rightarrow \underline{p}=(20\underline{i}+35\underline{j})+(90\underline{i}+48\underline{j})t$

$\Rightarrow \underline{p}=(90t+20)\underline{i}+(48t+35)\underline{j}$

c) PROCEED TO FIND VELOCITY VECTOR FOR Q

- IF THE VELOCITY WAS $24\underline{i}-7\underline{j}$, THEN ITS SPEED WOULD HAVE BEEN
$\sqrt{24^2+(-7)^2}=\sqrt{576+49}=25\text{ kmh}^{-1}$
- BUT THE SPEED IS IN FACT 125 kmh^{-1}, IE 5 TIMES LARGER
- $\underline{V}_Q=5(24\underline{i}-7\underline{j})=120\underline{i}-35\underline{j}$

FINALLY WE CAN FIND AN EXPRESSION FOR $\underline{q}$

$\Rightarrow \underline{q}=\underline{q}_0+\underline{V}_Qt$

$\Rightarrow \underline{q}=200\underline{j}+(120\underline{i}-35\underline{j})t$

$\Rightarrow \underline{q}=120t\underline{i}+(200-35t)\underline{j}$

WHEN $t=2$

$\underline{p}=(90\times2+20)\underline{i}+(48\times2+35)\underline{j}=200\underline{i}+131\underline{j}$

$\underline{q}=(120\times2)\underline{i}+(200-35\times2)\underline{j}=240\underline{i}+130\underline{j}$

$\therefore P(200,131)$ & $Q(240,130)$

$|PQ|=\sqrt{(200-240)^2+(131-130)^2}$

$=\sqrt{1600+1}$

$=\sqrt{1601}$

$\approx 40.0\text{ km}$

Question 6 (***+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A coastguard station, located at O, monitors the movement of ships in a sea region. **At noon** the radar of the station picks two ships in danger of collision.

At that instant ship A is at the point with position vector $8\mathbf{i}$ km and moving with constant velocity $(12\mathbf{i}-4\mathbf{j})\ \text{km h}^{-1}$. At the same instant ship B is at the point with position vector $(20\mathbf{i}-12\mathbf{j})$ km and moving with constant velocity $(4\mathbf{i}+4\mathbf{j})\ \text{km h}^{-1}$.

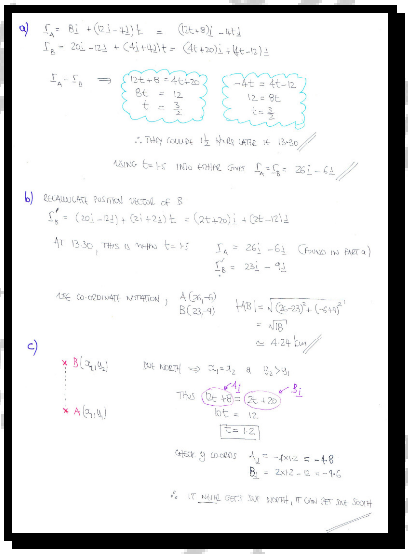
a) Show that if the two ships maintain their velocities, they will eventually collide, further determining the position vector of their collision point.

The coastguard **immediately** orders ship B to change its velocity to $(2\mathbf{i}+2\mathbf{j})\ \text{km h}^{-1}$.

b) Given that ship B obeys the order, calculate the distance between the two ships at 13.30 hours.

c) Determine if there is a time after noon when B is north of A.

$\boxed{(26\mathbf{i}-6\mathbf{j})\,\text{m}}$, $\boxed{|AB|=\sqrt{18}\approx 4.24\ \text{km}}$


a) $\underline{r}_A = 8\underline{i} + (12\underline{i}-4\underline{j})t = (12t+8)\underline{i} - 4t\underline{j}$
$\underline{r}_B = 20\underline{i} - 12\underline{j} + (4\underline{i}+4\underline{j})t = (4t+20)\underline{i} + (4t-12)\underline{j}$

$\underline{r}_A - \underline{r}_B \implies$ $12t+8 = 4t+20$, $8t = 12$, $t = \frac{3}{2}$; $-4t = 4t-12$, $12 = 8t$, $t = \frac{3}{2}$

$\therefore$ THEY COLLIDE $1\frac{1}{2}$ HOURS LATER IE 13:30

USING $t=1.5$ INTO EITHER GIVES $\underline{r}_A = \underline{r}_B = 26\underline{i} - 6\underline{j}$

b) RECALCULATE POSITION VECTOR OF B
$\underline{r}_B' = (20\underline{i}-12\underline{j}) + (2\underline{i}+2\underline{j})t = (2t+20)\underline{i} + (2t-12)\underline{j}$

AT 13.30, THIS IS WHEN $t=1.5$ $\quad \underline{r}_A = 26\underline{i} - 6\underline{j}$ (FOUND IN PART a)
$\underline{r}_B' = 23\underline{i} - 9\underline{j}$

USE CO-ORDINATE NOTATION, $A(26,-6)$, $B(23,-9)$ $\quad |AB| = \sqrt{(26-23)^2+(-6+9)^2} = \sqrt{18} \simeq 4.24$ km

c) $B(x_2,y_2)$, $A(x_1,y_1)$

DUE NORTH $\implies x_1 = x_2$ & $y_2 > y_1$

THUS $\underbrace{12t+8}_{A_i} = \underbrace{2t+20}_{B_i}$
$10t = 12$
$\boxed{t=1.2}$

CHECK y CO-ORDS $\quad A_j = -4\times1.2 = -4.8$
$B_j = 2\times1.2 - 12 = -9.6$

$\therefore$ IT NEVER GETS DUE NORTH, IT CAN GET DUE SOUTH

Question 7 (***+)

A model ship A is moving in a straight line with constant velocity, on the calm water of a pond. Relative to a fixed origin O the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

At time $t = 0$ s its position vector of A is $(-2\mathbf{i} + 3\mathbf{j})$ m and when $t = 5$ s its position vector is $(13\mathbf{i} - 7\mathbf{j})$ m.

a) Find an expression for the position vector of A at time t s.

When $t = 10$ another model ship B passes through the point with position vector $(8\mathbf{i} + 3\mathbf{j})$ m and travels with constant velocity $\mathbf{V}$ ms^{-1}.

b) Given that the two model ships collide when $t = 30$, determine an expression for $\mathbf{V}$ in the form $u\mathbf{i} + v\mathbf{j}$.

$\boxed{}$, $\boxed{\mathbf{r}_A = \left[(3t-2)\mathbf{i} + (3-2t)\mathbf{j}\right]\text{m}}$, $\boxed{\mathbf{V} = 4\mathbf{i} - 3\mathbf{j}}$

a) USING $\underline{r} = \underline{r}_0 + \underline{v}t$

$\Rightarrow (13\underline{i} - 7\underline{j}) = (-2\underline{i} + 3\underline{j}) + \underline{v} \times 5$

$\Rightarrow 15\underline{i} - 10\underline{j} = 5\underline{v}$

$\Rightarrow \underline{v} = 3\underline{i} - 2\underline{j}$

HENCE A GENERAL EXPRESSION WILL BE

$\Rightarrow \underline{r} = (-2\underline{i} + 3\underline{j}) + (3\underline{i} - 2\underline{j})t$

$\Rightarrow \underline{r} = (3t-2)\underline{i} + (3-2t)\underline{j}$

b) FIRSTLY LOOKING AT THE POSITION VECTOR OF SHIP A, WHEN $t=30$

$\underline{r} = (3\times30-2)\underline{i} + (3-2\times30)\underline{j}$

$\underline{r} = 88\underline{i} - 57\underline{j}$

NOW FORMING AN EQUATION FOR THE MOTION OF B

$\Rightarrow$ "$\underline{r} = \underline{r}_0 + \underline{v}t$"

$\Rightarrow \underbrace{(88\underline{i} - 57\underline{j})}_{\text{COLLISION AT THIS POINT WHEN } t=30} = \underbrace{(8\underline{i} + 3\underline{j})}_{\text{POSITION OF B WHEN } t=10} + \underbrace{\underline{V}\times 20}_{\text{MOTION FOR ANOTHER 20s TO REACH } t=30}$

$\Rightarrow 80\underline{i} - 60\underline{j} = 20\underline{V}$

$\Rightarrow \underline{V} = 4\underline{i} - 3\underline{j}$

Question 8 (***+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A ship A is moving with constant velocity.

At noon A is at the point with position vector $(12\mathbf{i}+12\mathbf{j})$ km and at 2 p.m. it has sailed to the point with position vector $(8\mathbf{i}+20\mathbf{j})$ km.

a) Determine the velocity of A.

b) Write down an expression for the position vector of A, t hours after noon.

A ship B is also moving with constant velocity and its position vector, $\mathbf{r}$ km, t hours after noon is given by

$$\mathbf{r} = (2t+k)\mathbf{i} + (2t+33)\mathbf{j},$$

where k is a scalar constant.

c) Given that A intercepts B determine the value of k.

$\boxed{}$, $\boxed{\mathbf{v} = (-2\mathbf{i}+4\mathbf{j})\text{ km h}^{-1}}$, $\boxed{\mathbf{r}_A = [(12-2t)\mathbf{i} + (4t+12)\mathbf{j}]\text{ km}}$, $\boxed{k=-30}$

a) USING $\underline{r} = \underline{r}_0 + \underline{v}t$

$\Rightarrow 8\underline{i} + 20\underline{j} = 12\underline{i} + 12\underline{j} + \underline{v} \times 2$

$\Rightarrow -4\underline{i} + 8\underline{j} = 2\underline{v}$

$\Rightarrow \underline{v} = -2\underline{i} + 4\underline{j}$

b) USING $\underline{r} = \underline{r}_0 + \underline{v}t$ AGAIN WITH VELOCITY FOUND IN (a)

$\Rightarrow \underline{r} = 12\underline{i} + 12\underline{j} + (-2\underline{i} + 4\underline{j})t$

$\Rightarrow \underline{r} = (12-2t)\underline{i} + (12+4t)\underline{j}$

c) INTERCEPTION IMPLIES SAME POSITION VECTOR AT THE SAME TIME

$\underline{r}_A = \underline{r}_B \Rightarrow (12-2t)\underline{i} + (12+4t)\underline{j} = (2t+k)\underline{i} + (2t+33)\underline{j}$

$\Rightarrow \begin{cases} 12-2t = 2t+k \\ 12+4t = 2t+33 \end{cases}$

$\Rightarrow \begin{cases} k = 12-4t \\ 2t = 21 \end{cases}$

$\Rightarrow \begin{cases} k = 12-4t \\ 4t = 42 \end{cases}$

$\Rightarrow k = -30$

Question 9 (***+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A ship P is sailing with constant velocity $(-7\mathbf{i}-5\mathbf{j})\ \text{km h}^{-1}$.

a) Calculate the speed of P.

b) Find the direction in which P is moving, giving the answer as a bearing.

At 12.00 hours, P is observed passing through the point with position vector $(40\mathbf{i}+28\mathbf{j})$ km.

A lighthouse L is located at the point with position vector $(-12\mathbf{i}+\mathbf{j})$ km.

c) Find the distance between P and L at 16.00 hours.

d) Determine the time, using 24 hour clock notation, when P is east of L.

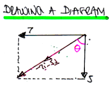
$\boxed{\quad}$, $\boxed{\text{speed} = \sqrt{74} \approx 8.60\ \text{km h}^{-1}}$, $\boxed{234°}$, $\boxed{d = 25\ \text{km}}$, $\boxed{17:24}$

a) SPEED = MAGNITUDE OF VELOCITY VECTOR

$\Rightarrow \text{SPEED} = |\underline{v}| = |-7\underline{i}-5\underline{j}| = \sqrt{49+25} = \sqrt{74}$

$\simeq 8.60\ \text{kmh}^{-1}$

b) DRAWING A DIAGRAM



$\tan\theta = \frac{7}{5}$

$\theta = 54.46...°$

$\therefore$ BEARING $= 180 + 54.46°...$

$\simeq 234°$

c) OBTAIN A GENERAL EXPRESSION FOR THE POSITION VECTOR OF THE SHIP AS IT WILL BE NEEDED IN PART (d) ALSO

$\underline{r} = \underline{r}_0 + \underline{v}t$

$\underline{r} = (40\underline{i}+28\underline{j}) + (-7\underline{i}-5\underline{j})t$

$\underline{r} = (40-7t)\underline{i} + (28-5t)\underline{j}$

$\underline{r}_4 = (40-7\times4)\underline{i} + (28-5\times4)\underline{j}$

$\underline{r}_4 = 12\underline{i} + 8\underline{j}$

DISTANCE BETWEEN $(-12\underline{i}+\underline{j})$ & $(12\underline{i}+8\underline{j})$

$d = \sqrt{(-12-12)^2+(1-8)^2} = \sqrt{576+49} = 25\ \text{km}$

d) EAST OF THE LIGHT-HOUSE $\Rightarrow$ SAME $\underline{j}$ & $\underline{i}$ GREATER THAN 12

$\underline{r} = (40-7t)\underline{i} + (28-5t)\underline{j}$

$\therefore 28-5t = 1$

$5t = 27$

$t = 5.4$

NOW $0.4\times60 = 24$

$\therefore$ 17:24

Question 10 (***+)

Relative to a fixed origin O, the horizontal unit vectors where $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

At time $t = 0$, a ship A moving with constant velocity $(\mathbf{i} + 7\mathbf{j})$ km h^{-1} is observed to be passing through the point with position vector $(8\mathbf{i} + 7\mathbf{j})$ km .

a) Calculate the speed of A and the bearing at which is moving.

b) Find an expression for the position vector of A, at time t hours.

At time $t = 0$, another ship B moving with constant velocity $(2\mathbf{i} + c\mathbf{j})$ km h^{-1} , where c is a constant, is passing through the point with position vector $(4\mathbf{i} + 3\mathbf{j})$ km .

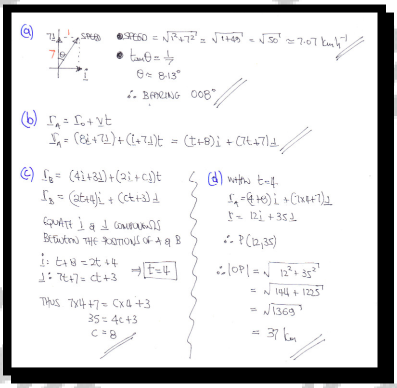
The two ships meet at the point P.

c) Determine the value of c.

d) Calculate the distance OP.

$\boxed{008°}$, $\boxed{\text{speed} = \sqrt{50} \approx 7.07 \text{ km h}^{-1}}$, $\boxed{\mathbf{r}_A = [(t+8)\mathbf{i} + (7t+7)\mathbf{j}] \text{km}}$, $\boxed{c = 8}$,

$\boxed{|\text{OP}| = 37 \text{ km}}$


(a) SPEED = $\sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50} \simeq 7.07$ km h^{-1}

$\tan\theta = \frac{1}{7}$

$\theta \simeq 8.13°$

∴ BEARING 008°

(b) $\underline{r}_A = \underline{r}_0 + \underline{v}t$

$\underline{r}_A = (8\underline{i}+7\underline{j}) + (\underline{i}+7\underline{j})t = (t+8)\underline{i} + (7t+7)\underline{j}$

(c) $\underline{r}_B = (4\underline{i}+3\underline{j}) + (2\underline{i}+c\underline{j})t$

$\underline{r}_B = (2t+4)\underline{i} + (ct+3)\underline{j}$

EQUATE $\underline{i}$ & $\underline{j}$ COMPONENTS BETWEEN THE POSITIONS OF A & B

$\underline{i}$: $t+8 = 2t+4$

$\underline{j}$: $7t+7 = ct+3$ $\Rightarrow \boxed{t=4}$

THUS $7\times4+7 = c\times4+3$

$35 = 4c+3$

$c = 8$

(d) WHEN $t=4$

$\underline{r}_A = (4+8)\underline{i} + (7\times4+7)\underline{j}$

$\underline{r} = 12\underline{i} + 35\underline{j}$

∴ P(12, 35)

∴ $|OP| = \sqrt{12^2+35^2}$

$= \sqrt{144+1225}$

$= \sqrt{1369}$

$= 37$ km

Question 11 (***+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle is moving with constant velocity $(3\mathbf{i}+\mathbf{j})$ ms^{-1}.

a) Calculate the speed of the particle.

b) Find the direction of motion of the particle, giving the answer as a bearing.

At time $t=0$ s, the particle is observed to be passing through the point A with position vector $(-5\mathbf{i}+2\mathbf{j})$ m .

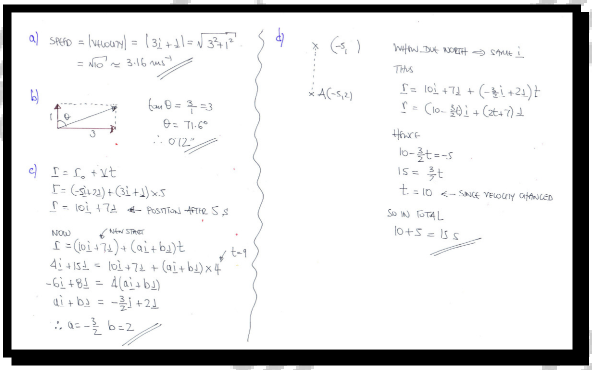
At time $t=5$ s the particle changes its velocity from $(3\mathbf{i}+\mathbf{j})$ ms^{-1} to $(a\mathbf{i}+b\mathbf{j})$ ms^{-1}.

When $t=9$ s the particle is observed to be passing through the point with position vector $(4\mathbf{i}+15\mathbf{j})$ m.

c) Find the value of a and the value of b.

d) Determine the **total** time it took the particle to travel from A to a point which is north from A.

$\boxed{\text{speed} = \sqrt{10} \approx 3.16 \text{ ms}^{-1}}$, $\boxed{072°}$, $\boxed{a=-1.5 \ \ \& \ \ b=2}$, $\boxed{t=15 \text{ s}}$



a) SPEED = |VELOCITY| = $|3\underline{i}+\underline{j}| = \sqrt{3^2+1^2}$
$= \sqrt{10} \approx 3.16$ ms^{-1}

b) $\tan\theta = \frac{3}{1} = 3$
$\theta = 71.6°$
$\therefore$ 072°

c) $\underline{r} = \underline{r}_0 + \underline{v}t$
$\underline{r} = (-5\underline{i}+2\underline{j}) + (3\underline{i}+\underline{j})\times 5$
$\underline{r} = 10\underline{i}+7\underline{j}$ ← POSITION AFTER 5 s

NOW (NEW START)
$\underline{r} = (10\underline{i}+7\underline{j}) + (a\underline{i}+b\underline{j})t$
$4\underline{i}+15\underline{j} = 10\underline{i}+7\underline{j} + (a\underline{i}+b\underline{j})\times 4$ ($t=9$)
$-6\underline{i}+8\underline{j} = 4(a\underline{i}+b\underline{j})$
$a\underline{i}+b\underline{j} = -\frac{3}{2}\underline{i}+2\underline{j}$
$\therefore a = -\frac{3}{2}$ $b=2$

d) × $(-5, \ )$
× $A(-5,2)$

WHEN DUE NORTH ⟹ SAME $\underline{i}$
THUS
$\underline{r} = 10\underline{i}+7\underline{j} + (-\frac{3}{2}\underline{i}+2\underline{j})t$
$\underline{r} = (10-\frac{3}{2}t)\underline{i} + (2t+7)\underline{j}$

HENCE
$10-\frac{3}{2}t = -5$
$15 = \frac{3}{2}t$
$t = 10$ ← SINCE VELOCITY CHANGED

SO IN TOTAL
$10+5 = 15$ s

Question 12 (***+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A ship is moving with constant velocity $(4\mathbf{i}+7.5\mathbf{j})\ \text{km h}^{-1}$.

At noon the ship is at the point with position vector $(-4\mathbf{i}-19\mathbf{j})$ km.

a) Calculate the speed of the ship.

b) Determine the bearing on which the ship is moving.

If the ship continues on this course it will collide with an oil drilling platform at 16.00.

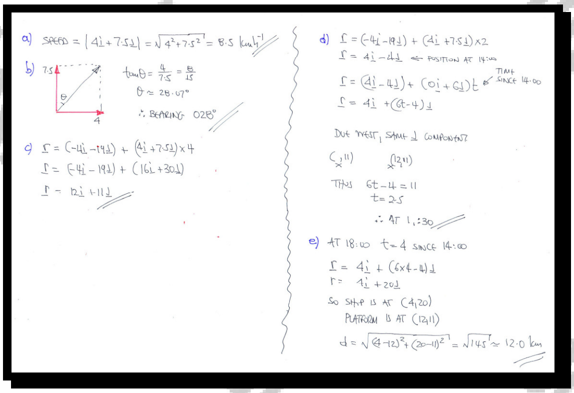
c) Find the position vector of the oil drilling platform.

The ship continues sailing with the same velocity until 14.00 when the ship changes course and begins to sail due north with speed 6 km h^{-1}.

d) Determine the time when the ship will be due west of the oil drilling platform.

e) Calculate the distance of the ship from the platform at 18.00.

$\boxed{\text{speed} = 8.5\ \text{km h}^{-1}}$, $\boxed{028°}$, $\boxed{(12\mathbf{i}+11\mathbf{j})\ \text{km}}$, $\boxed{16:30}$, $\boxed{d=\sqrt{145}\approx 12.0\ \text{km}}$



a) SPEED $= |4\underline{i}+7.5\underline{j}| = \sqrt{4^2+7.5^2} = 8.5\ \text{km h}^{-1}$

b) $\tan\theta = \frac{4}{7.5} = \frac{8}{15}$

$\theta \simeq 28.07°$

$\therefore$ BEARING 028°

c) $\underline{r} = (-4\underline{i}-19\underline{j}) + (4\underline{i}+7.5\underline{j})\times 4$

$\underline{r} = (-4\underline{i}-19\underline{j}) + (16\underline{i}+30\underline{j})$

$\underline{r} = 12\underline{i}+11\underline{j}$

d) $\underline{r} = (-4\underline{i}-19\underline{j}) + (4\underline{i}+7.5\underline{j})\times 2$

$\underline{r} = 4\underline{i}-4\underline{j}$ ⇐ POSITION AT 14:00

$\underline{r} = (4\underline{i}-4\underline{j}) + (0\underline{i}+6\underline{j})t$ ← TIME SINCE 14.00

$\underline{r} = 4\underline{i}+(6t-4)\underline{j}$

DUE WEST, SAME $\underline{j}$ COMPONENT

$(\ ,11)$ $(12,11)$

THUS $6t-4=11$

$t=2.5$

$\therefore$ AT 16:30

e) AT 18:00 $t=4$ SINCE 14:00

$\underline{r} = 4\underline{i}+(6\times4-4)\underline{j}$

$r = 4\underline{i}+20\underline{j}$

SO SHIP IS AT $(4,20)$

PLATFORM IS AT $(12,11)$

$d=\sqrt{(4-12)^2+(20-11)^2}=\sqrt{145}\simeq 12.0\ \text{km}$

Question 13 (***+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

Two cars, A and B, are moving on straight horizontal roads with constant velocities.

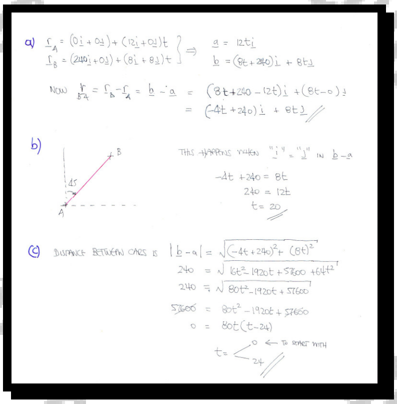
Initially A is at O and B is at the point with position vector $240\mathbf{i}$ m. The velocity of A is 12 ms^{-1} due east and the velocity of B is $(8\mathbf{i}+8\mathbf{j}) \text{ ms}^{-1}$.

a) Find an expression, in terms of t, for the position vector of B relative to A.

b) Hence, calculate the value of t when the bearing of B from A is $045°$.

c) Determine the value of t when the two cars are again 240 m apart.

$\boxed{\left[(240-4t)\mathbf{i}+8t\mathbf{j}\right]\text{m}}$, $\boxed{t=20}$, $\boxed{t=24}$


a) $\underline{r}_A = (0\underline{i}+0\underline{j})+(12\underline{i}+0\underline{j})t$, $\underline{r}_B = (240\underline{i}+0\underline{j})+(8\underline{i}+8\underline{j})t$ $\Rightarrow$ $\underline{a} = 12t\underline{i}$, $\underline{b} = (8t+240)\underline{i} + 8t\underline{j}$

NOW $\underline{r}_{B/A} = \underline{r}_B - \underline{r}_A = \underline{b} - \underline{a} = (8t+240-12t)\underline{i} + (8t-0)\underline{j}$

$= (-4t+240)\underline{i} + 8t\underline{j}$

b)

THIS HAPPENS WHEN "$\underline{i}$" = "$\underline{j}$" IN $\underline{b}-\underline{a}$

$-4t+240 = 8t$

$240 = 12t$

$t = 20$

(c) DISTANCE BETWEEN CARS IS $|\underline{b}-\underline{a}| = \sqrt{(-4t+240)^2+(8t)^2}$

$240 = \sqrt{16t^2-1920t+57600+64t^2}$

$240 = \sqrt{80t^2-1920t+57600}$

$\cancel{57600} = 80t^2-1920t+\cancel{57600}$

$0 = 80t(t-24)$

$t = 0$ ← TO START WITH, $t = 24$

Question 14 (****)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

Two dogs, Archie and Betty, are running in a field.

At time $t = 0$, Archie is at the point with position vector $(7\mathbf{i} + 8\mathbf{j})$ m and running with constant velocity $(2\mathbf{i} - 3\mathbf{j})$ ms^{-1}.

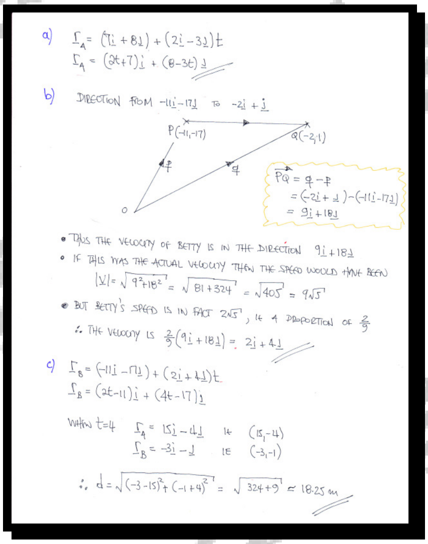
a) Determine the position vector of Archie at time t s, giving the answer in the form $f(t)\mathbf{i} + g(t)\mathbf{j}$ where f and g are functions to be found.

At time $t = 0$, Betty is at the point with position vector $(-11\mathbf{i} - 17\mathbf{j})$ m and running with constant velocity towards the point with position vector $(-2\mathbf{i} + \mathbf{j})$ m.

b) Given that Betty's speed is $2\sqrt{5}$ ms^{-1}, show clearly that Betty's velocity is $(2\mathbf{i} + 4\mathbf{j})$ ms^{-1}.

c) Assuming that the two dogs continue to run with constant velocities, find the distance between them when $t = 4$.

$\boxed{\mathbf{r} = (2t+7)\mathbf{i} + (8-3t)\mathbf{j}}$, $\boxed{d = \sqrt{333} \approx 18.25 \text{ m}}$


a) $\underline{r}_A = (7\underline{i} + 8\underline{j}) + (2\underline{i} - 3\underline{j})t$

$\underline{r}_A = (2t+7)\underline{i} + (8-3t)\underline{j}$

b) DIRECTION FROM $-11\underline{i} - 17\underline{j}$ TO $-2\underline{i} + \underline{j}$

$\vec{PQ} = \underline{q} - \underline{p} = (-2\underline{i} + \underline{j}) - (-11\underline{i} - 17\underline{j}) = 9\underline{i} + 18\underline{j}$

- THUS THE VELOCITY OF BETTY IS IN THE DIRECTION $9\underline{i} + 18\underline{j}$
- IF THIS WAS THE ACTUAL VELOCITY THEN THE SPEED WOULD HAVE BEEN $|\underline{v}| = \sqrt{9^2 + 18^2} = \sqrt{81 + 324} = \sqrt{405} = 9\sqrt{5}$
- BUT BETTY'S SPEED IS IN FACT $2\sqrt{5}$, IE A PROPORTION OF $\frac{2}{9}$

$\therefore$ THE VELOCITY IS $\frac{2}{9}(9\underline{i} + 18\underline{j}) = 2\underline{i} + 4\underline{j}$

c) $\underline{r}_B = (-11\underline{i} - 17\underline{j}) + (2\underline{i} + 4\underline{j})t$

$\underline{r}_B = (2t-11)\underline{i} + (4t-17)\underline{j}$

WHEN $t=4$ $\quad \underline{r}_A = 15\underline{i} - 4\underline{j}$ IE $(15,-4)$

$\qquad \underline{r}_B = -3\underline{i} - \underline{j}$ IE $(-3,-1)$

$\therefore d = \sqrt{(-3-15)^2 + (-1+4)^2} = \sqrt{324+9} \approx 18.25$ m

Question 15 (****)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A small, unmanned boat is drifting in the sea with **constant** velocity.

At 10.00 a.m. it is observed at the point with position vector $(-2\mathbf{i}+3\mathbf{j})$ km and at 10.45 a.m. it has drifted to the point with position vector $(-5\mathbf{i}+3.75\mathbf{j})$ km.

The position vector of the boat, t hours after 10.00 a.m., is $\mathbf{b}$ km.

a) Find an expression for $\mathbf{b}$ in terms of t.

At 11.30 a.m. a patrol boat leaves from the point with position vector $(2\mathbf{i}+\mathbf{j})$ km and intercepts the small unmanned boat at 11.45 a.m. The patrol boat is moving with **constant** velocity, $\mathbf{V}$ km h^{-1}.

b) Find $\mathbf{V}$, giving the answer in the form $a\mathbf{i}+b\mathbf{j}$, where a and b are constants to be found.

$\boxed{}$, $\boxed{\mathbf{b}=(-4t-2)\mathbf{i}+(t+3)\mathbf{j}}$, $\boxed{\mathbf{V}=-44\mathbf{i}+15\mathbf{j}}$

a) USING $\underline{r}=\underline{r}_0+\underline{v}t$ WITH $t=0$ AT 10:00 AM

$\Rightarrow -5\underline{i}+3.75\underline{j} = -2\underline{i}+3\underline{j}+\underline{v}\times\frac{3}{4}$

$\Rightarrow -20\underline{i}+15\underline{j} = -8\underline{i}+12\underline{j}+3\underline{v}$

$\Rightarrow -12\underline{i}+3\underline{j} = 3\underline{v}$

$\Rightarrow \underline{v} = -4\underline{i}+\underline{j}$

USING THE SAME EQUATION AGAIN

$\Rightarrow \underline{b} = \underline{b}_0+\underline{v}t$

$\Rightarrow \underline{b} = -2\underline{i}+3\underline{j}+(-4\underline{i}+\underline{j})\times t$

$\Rightarrow \underline{b} = (-2-4t)\underline{i}+(3+t)\underline{j}$

b) FIND THE POSITION OF THE DRIFTING BOAT AT 11:45, IE $t=1.75$

$\underline{b} = (-2-4\times1.75)\underline{i}+(3+1.75)\underline{j}$

$\underline{b} = -9\underline{i}+4.75\underline{j}$

NOW USING $\underline{r}=\underline{r}_0+\underline{V}T$, WHERE T IS MEASURED FROM 11:30

$-9\underline{i}+4.75\underline{j} = 2\underline{i}+\underline{j}+\underline{V}\times\frac{1}{4}$

$-36\underline{i}+19\underline{j} = 8\underline{i}+4\underline{j}+\underline{V}$

$\underline{V} = -44\underline{i}+15\underline{j}$

Question 16 (****)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

At time $t = 0$ s a footballer A kicks the ball from the point with position vector $(4\mathbf{i} + 2\mathbf{j})$ m giving it a constant speed of 10 ms^{-1} in the direction $(4\mathbf{i} + 3\mathbf{j})$.

The ball is modelled as a particle moving with constant velocity.

a) Show that the position vector of the ball, t s after it was kicked, is given by

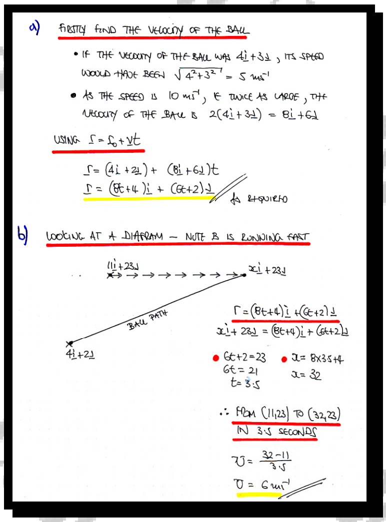
$$(8t + 4)\mathbf{i} + (6t + 2)\mathbf{j}.$$

At time $t = 0$ s, another footballer B is observed to be running through the point with position vector $(11\mathbf{i} + 23\mathbf{j})$ m.

Footballer B is modelled as a particle running due East with constant speed U ms^{-1}.

b) Given that B intercepts the ball determine the value of U.

$\boxed{}$, $\boxed{\mathbf{v} = 8\mathbf{i} + 6\mathbf{j}}$, $\boxed{U = 6}$


a) FIRSTLY FIND THE VELOCITY OF THE BALL

- IF THE VELOCITY OF THE BALL WAS $4\underline{i} + 3\underline{j}$, ITS SPEED WOULD HAVE BEEN $\sqrt{4^2+3^2} = 5 \text{ ms}^{-1}$
- AS THE SPEED IS 10 ms^{-1}, i.e. TWICE AS LARGE, THE VELOCITY OF THE BALL IS $2(4\underline{i} + 3\underline{j}) = 8\underline{i} + 6\underline{j}$

USING $\underline{r} = \underline{r}_0 + \underline{v}t$

$\underline{r} = (4\underline{i} + 2\underline{j}) + (8\underline{i} + 6\underline{j})t$

$\underline{r} = (8t+4)\underline{i} + (6t+2)\underline{j}$ AS REQUIRED

b) LOOKING AT A DIAGRAM – NOTE B IS RUNNING EAST

$\underline{r} = (8t+4)\underline{i} + (6t+2)\underline{j}$

$x\underline{i} + 23\underline{j} = (8t+4)\underline{i} + (6t+2)\underline{j}$

- $6t + 2 = 23$, $6t = 21$, $t = 3.5$
- $x = 8 \times 3.5 + 4$, $x = 32$

$\therefore$ FROM $(11, 23)$ TO $(32, 23)$ IN 3.5 SECONDS

$U = \dfrac{32 - 11}{3.5}$

$U = 6 \text{ ms}^{-1}$

Question 17 (****)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A ship S is sailing with constant velocity.

At 12.00 hours, S is passing through the point with position vector $(18\mathbf{i} - 5\mathbf{j})$ km and half an hour later S is passing through the point with position vector $(16\mathbf{i} - 2\mathbf{j})$ km.

a) Calculate the speed of S.

The position vector of S, $\mathbf{s}$ km, t hours after noon, is given by

$$\mathbf{s} = f(t)\mathbf{i} + g(t)\mathbf{j}.$$

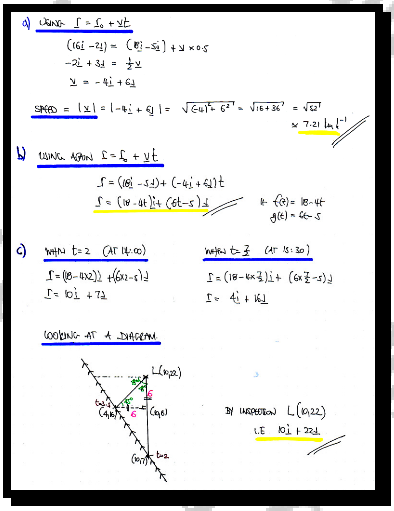
b) Determine an expression for $f(t)$ and an expression for $g(t)$.

At 14.00, to an observer on S, a lighthouse L appears due north of S.

At 15.30, to an observer on S, the same lighthouse L appears north-east of S.

c) Find the position vector of L.

$\boxed{\text{speed} = \sqrt{52} \approx 7.21 \text{ km h}^{-1}}$, $\boxed{\mathbf{r} = \left[(18-4t)\mathbf{i} + (6t-5)\mathbf{j}\right] \text{km}}$, $\boxed{10\mathbf{i} + 22\mathbf{j}}$



Question 18 (****)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A boat is sailing with constant velocity. At time t hours after noon the position vector of the boat is $\mathbf{r}$ km.

When $t=0$, $\mathbf{r}=(-5\mathbf{i}+10\mathbf{j})$ km and when $t=3$, $\mathbf{r}=(4\mathbf{i}-2\mathbf{j})$ km.

a) Calculate the speed of the boat.

b) Find the direction in which the boat is moving, giving the answer as a bearing.

c) Determine an expression for $\mathbf{r}$, in terms of t.

A beacon is located at the point with position vector $(10\mathbf{i}-10\mathbf{j})$ km.

When $t=T$, the boat is at a distance of 15 km from the beacon.

d) Determine the two possible values of T.

$\boxed{}$, $\boxed{\text{speed}=5\ \text{km h}^{-1}}$, $\boxed{143°}$, $\boxed{\mathbf{r}=\left[(3t-5)\mathbf{i}+(10-4t)\mathbf{j}\right]\text{m}}$, $\boxed{T=2,\ 8}$

a) USING $\underline{r}=\underline{r}_0+\underline{v}t$

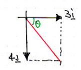
$4\underline{i}-2\underline{j}=(-5\underline{i}+10\underline{j})+\underline{v}\times 3$

$9\underline{i}-12\underline{j}=3\underline{v}$

$\underline{v}=3\underline{i}-4\underline{j}$

$\therefore$ SPEED $=|\underline{v}|=|3\underline{i}-4\underline{j}|=\sqrt{3^2+(-4)^2}=\sqrt{9+16}=5\ \text{ms}^{-1}$

b) USING THE VELOCITY VECTOR



$\tan\theta=\frac{4}{3}$

$\theta=53.13°$

$\therefore$ BEARING IS $90+53.13$ $=143°$

c) USING AGAIN $\underline{r}=\underline{r}_0+\underline{v}t$

$\underline{r}=(-5\underline{i}+10\underline{j})+(3\underline{i}-4\underline{j})t$

$\underline{r}=(3t-5)\underline{i}+(10-4t)\underline{j}$

d) LET $B(10,-10)$ & $R(3t-5,10-4t)$

$\Rightarrow |BR|=\sqrt{[(3t-5)-10]^2+[(10-4t)-(-10)]^2}$

$\Rightarrow 15=\sqrt{(3t-15)^2+(20-4t)^2}$

$\Rightarrow 225=(3t-15)^2+(20-4t)^2$

$\Rightarrow \cancel{225}=9t^2-90t+\cancel{225}+400-160t+16t^2$

$\Rightarrow 0=25t^2-250t+400$

$\Rightarrow t^2-10t+16=0$

$\Rightarrow (t-2)(t-8)=0$

$\Rightarrow t=2,\ 8$

Question 19 (****)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

At noon a ship A is at the point with position vector $(5\mathbf{i}-7\mathbf{j})$ km, travelling with constant velocity $(3\mathbf{i}-4\mathbf{j})$ km h^{-1}.

a) Determine the bearing at which A is travelling.

At noon ship B is at the point with position vector $(3\mathbf{i}+5\mathbf{j})$ km and travelling with constant velocity $(-2\mathbf{i}+8\mathbf{j})$ km h^{-1}.

b) Find the speed of B.

The distance between the two ships, t hours after noon is d km.

c) Show clearly that

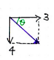
$$d=\sqrt{169t^2+308t+148}\,,\ t\geq 0.$$

d) Show further that the two ships do not collide in the subsequent motion.

e) Calculate the time when the two ships are 25 km apart.

$\boxed{143°}$, $\boxed{\text{speed}=\sqrt{68}\approx 8.25\ \text{km h}^{-1}}$, $\boxed{13.00}$

a) LOOKING AT A DIAGRAM



$\tan\theta = \frac{4}{3}$

$\theta = 53.13°$

$\therefore$ BEARING $= 90° + 53.13$

$= 143°$

b) SPEED IS THE MAGNITUDE OF $-2\underline{i}+8\underline{j}$

SPEED $=\sqrt{(-2)^2+8^2} = \sqrt{4+64} = \sqrt{68} = 8.25\ \text{km h}^{-1}$

c) USING $\underline{r}=\underline{r}_0+\underline{v}t$ FOR EACH SHIP

$\underline{r}_A = (5\underline{i}-7\underline{j})+(3\underline{i}-4\underline{j})t$

$\underline{r}_B = (3\underline{i}+5\underline{j})+(-2\underline{i}+8\underline{j})t$

$\underline{r}_A = (3t+5)\underline{i}+(-4t-7)\underline{j}$

$\underline{r}_B = (3-2t)\underline{i}+(8t+5)\underline{j}$

OR AS "CO-ORDINATES"

$A(3t+5, -4t-7)$

$B(3-2t, 8t+5)$

USING THE DISTANCE FORMULA

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d=\sqrt{[(3-2t)-(3t+5)]^2+[(8t+5)-(-4t-7)]^2}$

$d=\sqrt{(-5t-2)^2+(12t+12)^2}$

$d=\sqrt{25t^2+20t+4+144t^2+288t+144}$

$d=\sqrt{169t^2+308t+148}$

AS REQUIRED

d) FOR COLLISION $d=0$

$\Rightarrow 0=\sqrt{169t^2+308t+148}$

$\Rightarrow 0=169t^2+308t+148$

$b^2-4ac = 308^2-4\times169\times148 = -5184<0$

NO VALUE OF t, GIVES $d=0$, SO THEY NEVER COLLIDE

e) FINALLY $d=25$

$\Rightarrow 25=\sqrt{169t^2+308t+148}$

$\Rightarrow 625 = 169t^2+308t+148$

$\Rightarrow 0 = 169t^2+308t-477$

QUADRATIC FORMULA

$t=\dfrac{-308\pm\sqrt{308^2-4\times169(-477)}}{2\times169} = \begin{cases}1\\ \cancel{-2.82}\end{cases}$

IE AT 13:00

Question 20 (****+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

At midnight a ship P is at the point with position vector $(-11\mathbf{i}-24\mathbf{j})$ km and 4 hours later is at the point with position vector $(9\mathbf{i}+20\mathbf{j})$ km.

a) Find the velocity of P, in vector form.

At midnight another ship Q is at the point with position vector $(5\mathbf{i}-10\mathbf{j})$ km and travelling with constant velocity $8\mathbf{j}\text{ km h}^{-1}$.

The distance between the two ships, t hours after midnight is d km.

b) Show clearly that

$$d^2 = 34t^2 - 244t + 452,\ t \geq 0.$$

An observer on P can only see the lights of Q when d is 10 km or less.

c) Given that an observer on P sees the lights of Q for the first time at 2 a.m. find the time when the lights of Q move out of the sight of the observer on P.

$\boxed{}$, $\boxed{\mathbf{v}=5\mathbf{i}+11\mathbf{j}}$, $\boxed{\mathbf{p}=(5t-11)\mathbf{i}+(11t-24)\mathbf{j}}$, $\boxed{\mathbf{q}=5\mathbf{i}+(8t-10)\mathbf{j}}$, $\boxed{05:11}$

a) USING $\underline{r} = \underline{r}_0 + \underline{v}t$

$9\underline{i}+20\underline{j} = -11\underline{i} - 24\underline{j} + \underline{v}\times 4$

$20\underline{i}+44\underline{j} = 4\underline{v}$

$\underline{v} = 5\underline{i}+11\underline{j}$

b) OBTAIN EXPRESSIONS FOR THE POSITION VECTORS OF EACH SHIP, t HOURS AFTER MIDNIGHT.

$\underline{r}_A = (-11\underline{i}-24\underline{j}) + (5\underline{i}+11\underline{j})t$

$\underline{r}_B = (5\underline{i}-10\underline{j}) + (0\underline{i}+8\underline{j})t$

$\underline{r}_A = (5t-11)\underline{i} + (11t-24)\underline{j}$

$\underline{r}_B = 5\underline{i} + (8t-10)\underline{j}$

OR AS CO-ORDINATES

$A(5t-11,\ 11t-24)$ & $B(5,\ 8t-10)$

USING THE DISTANCE FORMULA $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d = \sqrt{[(5t-11)-5]^2+[(11t-24)-(8t-10)]^2}$

$d = \sqrt{(5t-16)^2+(3t-14)^2}$

$d^2 = 25t^2-160t+256+9t^2-84t+196$

$d^2 = 34t^2-244t+452$ AS REQUIRED

c) LET $d=10$ & NOTE $t=2$ IS A SOLUTION

$\Rightarrow 10^2 = 34t^2-244t^2+452$

$\Rightarrow 0 = 34t^2-244t^2+352$

$\Rightarrow 17t^2-122t+176=0$

$\Rightarrow (t-2)(17t-88)=0$

$t = \begin{cases} 2 & \text{(ALREADY KNOWN)} \\ \frac{88}{17} = 5.1764\ldots \end{cases}$

$= 05:11$

$0.1764\times 60 \simeq 10.58\ldots \simeq 11$

Question 21 (****+)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

At noon a ship A is at the point with position vector $(-2\mathbf{i}+\mathbf{j})$ km and travelling with constant velocity $(4\mathbf{i}-7\mathbf{j})\ \text{km h}^{-1}$.

a) Calculate the speed of A.

At noon ship B is at the point with position vector $(10\mathbf{i}-3\mathbf{j})$ km and travelling with constant velocity $(-2\mathbf{i}+5\mathbf{j})\ \text{km h}^{-1}$.

b) Determine the bearing at which B is travelling.

c) Find an expression for the position vector of A relative to B.

The distance between the two ships, t hours after noon is d km.

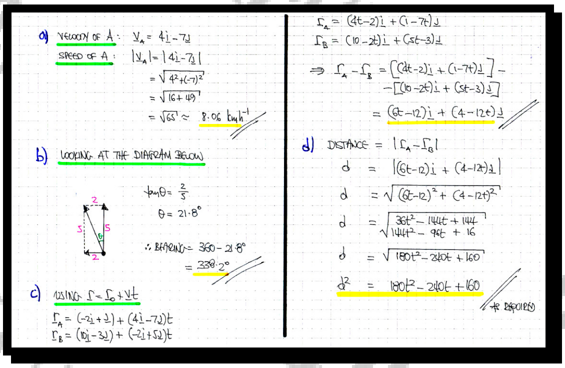
d) Show clearly that

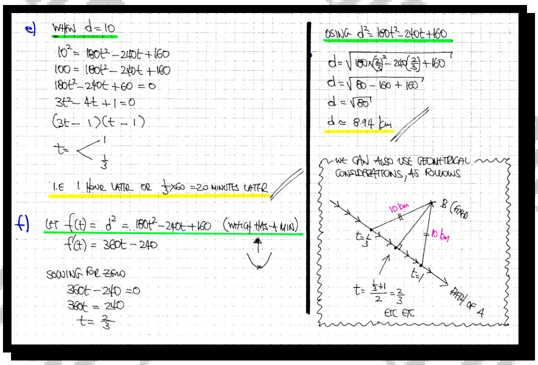
$$d^2 = 180t^2 - 240t + 160,\ t \ge 0.$$

e) Calculate the times when the two ships are 10 km apart.

f) Determine the shortest distance between the two ships.

$\boxed{\text{speed} = \sqrt{65} \approx 8.06\ \text{km h}^{-1}}$, $\boxed{338°}$, $\boxed{{}_A\mathbf{r}_B = (6t-12)\mathbf{i}+(4-12t)\mathbf{j}}$, $\boxed{12.20/13.00}$, $\boxed{}$, $\boxed{d_{\text{min}} = \sqrt{80} \approx 8.94\ \text{km}}$





Question 22 (****+)

The unit vectors $\mathbf{i}$ *and* $\mathbf{j}$ *are oriented due east and due north, respectively.*

Two boats, A and B, are moving in the open sea with constant velocities given in vector form as $(7\mathbf{i}+3\mathbf{j})\ \text{km h}^{-1}$ and $(-3\mathbf{i}+9\mathbf{j})\ \text{km h}^{-1}$, respectively.

At noon, B is on a bearing of $120°$ from A, 12 km away.

a) Determine, correct to the nearest m, the distance between A and B when B is due east of A.

b) Find the time when the two boats are at the closest distance.

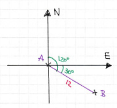
$\boxed{}$, $\boxed{d \approx 392 \text{ m}}$, $\boxed{13:02}$

a) • TAKE THE POSITION OF "A" AT NOON TO BE THE ORIGIN

• THEN AT NOON

$\underline{r}_B = (12\cos 30)\underline{i} - (12\sin 30)\underline{j}$

$\underline{r}_B = 6\sqrt{3}\,\underline{i} - 6\underline{j}$



• THEN THE POSITION VECTORS OF THE TWO SHIPS, t HOURS AFTER NOON, IS GIVEN BY

$\underline{r}_A = (0\underline{i}+0\underline{j}) + (7\underline{i}+3\underline{j})t = 7t\underline{i} + 3t\underline{j}$

$\underline{r}_B = (6\sqrt{3}\,\underline{i}-6\underline{j}) + (-3\underline{i}+9\underline{j})t = (6\sqrt{3}-3t)\underline{i} + (9t-6)\underline{j}$

• $\underline{r}_B - \underline{r}_A = (6\sqrt{3}-10t)\underline{i} + (6t-6)\underline{j}$

• WHEN B IS EAST OF A, $\underline{j}$ COMPONENT MUST BE ZERO

$\Rightarrow 6t-6=0$

$\Rightarrow 6t=6$

$\Rightarrow t=1$

$\therefore$ (i): $6\sqrt{3} - 10\times 1 = -0.3923\ldots$

$\simeq$ 392 m EAST OF A

b) • $|\underline{r}_B - \underline{r}_A|$ = DISTANCE BETWEEN THE SHIPS AT TIME t

$\Rightarrow |\underline{r}_B - \underline{r}_A| = \sqrt{(6\sqrt{3}-10t)^2 + (6t-6)^2}$

$\Rightarrow |\underline{r}_B - \underline{r}_A| = \sqrt{108 - 120\sqrt{3}t + 100t^2 + 36t^2 - 72t + 36}$

$\Rightarrow |\underline{r}_B - \underline{r}_A| = \sqrt{136t^2 - (72+120\sqrt{3})t + 144}$

$\Rightarrow |\underline{r}_B - \underline{r}_A|^2 = 136t^2 - (72+120\sqrt{3})t + 144$

• LET $f(t) = 136t^2 - 24(3+5\sqrt{3})t + 144$

BY COMPLETING THE SQUARE OR CALCULUS

$\Rightarrow f'(t) = 272t - (72+120\sqrt{3})$

• SOLVING FOR ZERO YIELDS

$t = \dfrac{72+120\sqrt{3}}{272} \simeq 1.0288\ldots$

$\simeq$ 1 HOUR – 2 MINUTES

$\simeq$ 13:02

Question 23 (****+)

The unit vectors $\mathbf{i}$ *and* $\mathbf{j}$ *are oriented due east and due north, respectively.*

Two boats, A and B, are moving in the open sea with **constant** velocities.

- At noon, A is at the point with position vector $(\mathbf{i}+2\mathbf{j})$ km and half an hour later is at the point with position vector $(7\mathbf{i}-2\mathbf{j})$ km.
- At noon, B is at the point with position vector $(-\mathbf{i}-\mathbf{j})$ km and half an hour later is at the point with position vector $(9\mathbf{i}+\mathbf{j})$ km.

Calculate the closest distance between A and B, and the time when this closest distance occurs.

$\boxed{d_{\text{min}} \approx 2.846 \text{ km}}$, $\boxed{12:14}$

$A_1(1,2)$ $A_2(7,-2)$
$B_1(-1,-1)$ $B_2(9,1)$

START BY FINDING THEIR RESPECTIVE VELOCITIES USING $\underline{v} = \frac{\underline{r}_2-\underline{r}_1}{t}$

- $\underline{v}_A = \frac{(7\underline{i}-2\underline{j})-(\underline{i}+2\underline{j})}{0.5} = \frac{6\underline{i}-4\underline{j}}{0.5} = 12\underline{i}-8\underline{j}$
- $\underline{v}_B = \frac{(9\underline{i}+\underline{j})-(-\underline{i}-\underline{j})}{0.5} = \frac{10\underline{i}+2\underline{j}}{0.5} = 20\underline{i}+4\underline{j}$

NEXT WE FORM EXPRESSIONS FOR THE POSITION VECTORS

$\underline{r}_A = (\underline{i}+2\underline{j}) + (12\underline{i}-8\underline{j})t$
$\underline{r}_B = (-\underline{i}-\underline{j}) + (9\underline{i}+\underline{j})t$

$\Rightarrow \underline{r}_A-\underline{r}_B = (2\underline{i}+3\underline{j}) + (3\underline{i}-9\underline{j})t$

$\Rightarrow \underline{r}_A-\underline{r}_B = (3t+2)\underline{i} + (3-9t)\underline{j}$

$\Rightarrow |\underline{r}_A-\underline{r}_B|^2 = (3t+2)^2+(3-9t)^2$

$\Rightarrow |\underline{r}_A-\underline{r}_B|^2 = 9t^2+12t+4+9-54t+81t^2$

$\Rightarrow |\underline{r}_A-\underline{r}_B|^2 = 90t^2-42t+13$

$\Rightarrow |\underline{r}_A-\underline{r}_B|^2 = 90\left[t^2-\frac{7}{15}t+\frac{13}{90}\right]$

$\Rightarrow |\underline{r}_A-\underline{r}_B|^2 = 90\left[\left(t-\frac{7}{30}\right)^2-\frac{49}{900}+\frac{13}{90}\right]$

$\Rightarrow |\underline{r}_A-\underline{r}_B|^2 = 90\left[\left(t-\frac{7}{30}\right)^2+\frac{9}{100}\right]$

$\Rightarrow |\underline{r}_A-\underline{r}_B|^2 = 90\left(t-\frac{7}{30}\right)^2+\frac{81}{10}$

THUS THE SHORTEST DISTANCE IS $\sqrt{\frac{81}{10}} = \frac{9}{10}\sqrt{10} \approx 2.846$ km

AND

THE TIME IS $t = \frac{7}{30}$ HOURS $= 14$ MINUTES, IE 12:14

VARIOUS VECTOR PROBLEMS

Question 1 (****)

At time $t = 0$, relative to a fixed origin O, two particles, A and B, have position vectors $(\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$ m and $(-2\mathbf{i} + a\mathbf{j} + 6\mathbf{k})$ m, respectively, where a is a constant.

It is further given that A and B, are travelling with constant velocities of $(2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k})$ ms^{-1} and $(3\mathbf{i} + 12\mathbf{j} + 4\mathbf{k})$ ms^{-1}, respectively.

The distance between A and B is least when $t = 5$ s.

Determine the value of a.

$\boxed{}$, $\boxed{a = -49}$

FORMING EXPRESSIONS FOR EACH PARTICLE, USING $\underline{r} = \underline{r}_0 + \underline{v}t$

$$\underline{r}_A = (1,-2,4) + (2,3,6)t = (2t+1,\ 3t-2,\ 6t+4)$$
$$\underline{r}_B = (-2,a,6) + (3,12,4)t = (3t-2,\ 12t+a,\ 4t+6)$$

$$|AB|^2 = |\underline{r}_B - \underline{r}_A|^2 = (t-3)^2 + (9t+a+2)^2 + (-2t+2)^2$$
$$= (t-3)^2 + (9t+a+2)^2 + 4(t-1)^2$$

USING CALCULUS

LET $f(t) = (t-3)^2 + (9t+a+2)^2 + 4(t-1)^2$

$f'(t) = 2(t-3) + 18(9t+a+2) + 8(t-1)$

NOW WE HAVE $f'(t) = 0$ WHEN $t = 5$

$\Rightarrow 0 = (2 \times 2) + 18(47 + a) + 8 \times 4$

$\Rightarrow 0 = 4 + 18(a+47) + 32$

$\Rightarrow -36 = 18(a+47)$

$\Rightarrow a + 47 = -2$

$\Rightarrow a = -49$

Question 2 (****)

Two forces $\mathbf{F}_1 = (2\mathbf{i} + 7\mathbf{j})$ N and $\mathbf{F}_2 = (4\mathbf{i} + k\mathbf{j})$ N, where k is a constant, act on a particle of mass m kg.

Find the value of m, given that the particle accelerates in the direction $3\mathbf{i} - 2\mathbf{j}$ with magnitude $5\sqrt{13}$ ms^{-2}.

$\boxed{}$, $\boxed{m = \frac{2}{5}}$

PROCEED AS FOLLOWS

$$\Rightarrow \underline{F} = \underline{F}_1 + \underline{F}_2$$

$$\Rightarrow \lambda(3\underline{i} - 2\underline{j}) = (2\underline{i} + 7\underline{j}) + (4\underline{i} + k\underline{j})$$

ACTS PARALLEL TO THIS VECTOR (RESULTANT FORCE ACTS IN THE DIRECTION OF a)

$$\Rightarrow 3\lambda\underline{i} - 2\lambda\underline{j} = 6\underline{i} + (k+7)\underline{j}$$

- $3\lambda = 6$, $\lambda = 2$
- $-2\lambda = k + 7$, $-4 = k + 7$, $k = -11$

NOW THE EQUATION OF MOTION GIVES IF $k = -11$

$$\Rightarrow \underline{F} = m\underline{a}$$

$$\Rightarrow (2\underline{i} + 7\underline{j}) + (4\underline{i} - 11\underline{j}) = m\underline{a}$$

$$\Rightarrow 6\underline{i} - 4\underline{j} = m\underline{a}$$

$$\Rightarrow |6\underline{i} - 4\underline{j}| = m|\underline{a}|$$

$$\Rightarrow 2|3\underline{i} - 2\underline{j}| = m \times 5\sqrt{13}$$

$$\Rightarrow 2\sqrt{3^2 + (-2)^2} = 5m\sqrt{13}$$

$$\Rightarrow 2\cancel{\sqrt{13}} = 5m\cancel{\sqrt{13}}$$

$$\Rightarrow m = \frac{2}{5}$$

Question 3 (****)

The vectors $\mathbf{i}$ and $\mathbf{j}$ are mutually perpendicular unit vectors.

The resultant of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ is given by $\mathbf{R} = (-\mathbf{i} + 6\mathbf{j})$ N.

a) Calculate the magnitude of $\mathbf{R}$.

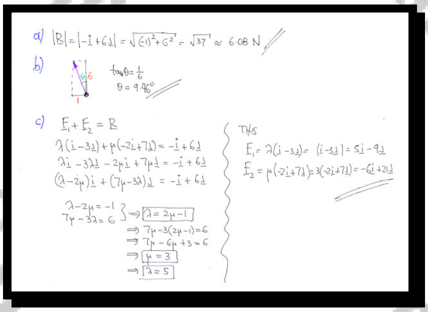
b) Find the size of the angle between the direction of $\mathbf{R}$ and the vector $\mathbf{j}$.

The line of action of $\mathbf{F}_1$ is parallel to the vector $(\mathbf{i} - 3\mathbf{j})$.

The line of action of $\mathbf{F}_2$ is parallel to the vector $(-2\mathbf{i} + 7\mathbf{j})$.

c) Determine $\mathbf{F}_1$ and $\mathbf{F}_2$, in terms of $\mathbf{i}$ and $\mathbf{j}$.

$\boxed{|\mathbf{R}| = \sqrt{37} \approx 6.08 \text{ N}}$, $\boxed{\approx 9.46°}$, $\boxed{\mathbf{F}_1 = (5\mathbf{i} - 9\mathbf{j}) \text{ N}}$, $\boxed{\mathbf{F}_2 = (-6\mathbf{i} + 21\mathbf{j}) \text{ N}}$



a) $|\underline{R}| = |-\underline{i} + 6\underline{j}| = \sqrt{(-1)^2 + 6^2} = \sqrt{37} \approx 6.08$ N

b) $\tan\theta = \frac{1}{6}$

$\theta = 9.46°$

c) $\underline{F}_1 + \underline{F}_2 = \underline{R}$

$\lambda(\underline{i} - 3\underline{j}) + \mu(-2\underline{i} + 7\underline{j}) = -\underline{i} + 6\underline{j}$

$\lambda\underline{i} - 3\lambda\underline{j} - 2\mu\underline{i} + 7\mu\underline{j} = -\underline{i} + 6\underline{j}$

$(\lambda - 2\mu)\underline{i} + (7\mu - 3\lambda)\underline{j} = -\underline{i} + 6\underline{j}$

$\left.\begin{array}{l}\lambda - 2\mu = -1\\ 7\mu - 3\lambda = 6\end{array}\right\} \Rightarrow \boxed{\lambda = 2\mu - 1}$

$\Rightarrow 7\mu - 3(2\mu - 1) = 6$

$\Rightarrow 7\mu - 6\mu + 3 = 6$

$\Rightarrow \boxed{\mu = 3}$

$\Rightarrow \boxed{\lambda = 5}$

THUS

$\underline{F}_1 = \lambda(\underline{i} - 3\underline{j}) = 5(\underline{i} - 3\underline{j}) = 5\underline{i} - 9\underline{j}$

$\underline{F}_2 = \mu(-2\underline{i} + 7\underline{j}) = 3(-2\underline{i} + 7\underline{j}) = -6\underline{i} + 21\underline{j}$

Question 4 (****)

Two forces $\mathbf{F}_1 = (3\mathbf{i} + 4\mathbf{j})$ N and $\mathbf{F}_2 = (a\mathbf{i} + b\mathbf{j})$ N, where a and b are constants, act on a particle of mass 1.5 kg.

Find the value of a and the value of b, given that the particle accelerates in the direction $\mathbf{i} - \mathbf{j}$ with magnitude $4\sqrt{2}$ ms^{-2}.

$\boxed{}$, $\boxed{a = 3}$, $\boxed{b = -10}$

SOLVING AS FOLLOWS

$$\Rightarrow \underline{F} = m\underline{a}$$

$$\Rightarrow \underline{F}_1 + \underline{F}_2 = \frac{3}{2}\lambda(\underline{i} - \underline{j})$$

IN THE DIRECTION $\underline{i} - \underline{j}$

$$\Rightarrow (3\underline{i} + 4\underline{j}) + (a\underline{i} + b\underline{j}) = \frac{3}{2}\lambda(\underline{i} - \underline{j})$$

$$\Rightarrow (a+3)\underline{i} + (b+4)\underline{j} = \frac{3}{2}\lambda\underline{i} - \frac{3}{2}\lambda\underline{j}$$

HENCE WE HAVE

$$\boxed{\begin{array}{l} a + 3 = \frac{3}{2}\lambda \\ b + 4 = -\frac{3}{2}\lambda \end{array}}$$

ALSO WE HAVE THE MAGNITUDE OF THE ACCELERATION

$$|\lambda(\underline{i} - \underline{j})| = 4\sqrt{2}$$

$$|\lambda\underline{i} - \lambda\underline{j}| = 4\sqrt{2}$$

$$\sqrt{\lambda^2 + \lambda^2} = 4\sqrt{2}$$

$$\sqrt{2\lambda^2} = 4\sqrt{2}$$

$$\sqrt{2}\lambda = 4\sqrt{2}$$

$$\lambda = 4$$

FINALLY WE HAVE

$$a = \frac{3}{2} \times 4 - 3$$

$$b = -\frac{3}{2} \times 4 - 4$$

$$\therefore \quad a = 3, \quad b = -10$$