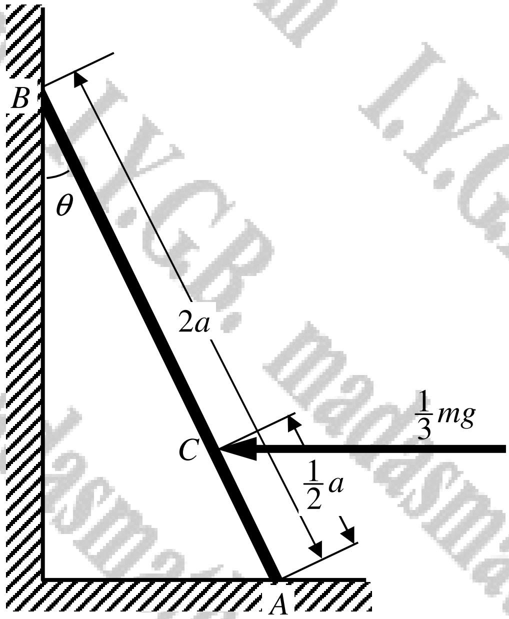
EQUILIBRIUM OF RIGID BODIES

Question 1 (**)



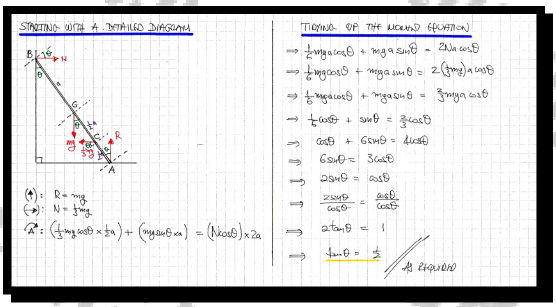
A ladder of length $2a$ and mass m, has one end A on smooth horizontal ground and the other end B against a smooth vertical wall.

The ladder is kept in equilibrium by a horizontal force of magnitude $\frac{1}{3}mg$ acting at a point C on the ladder, where $AC = \frac{1}{2}a$, as shown in the figure above.

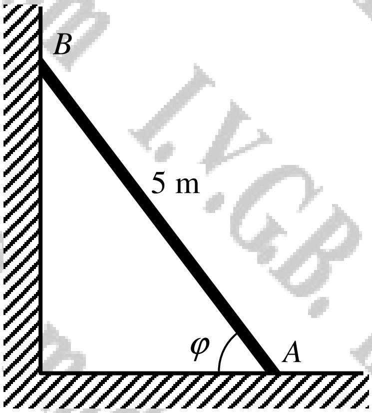
The angle between the ladder and the vertical wall is θ .

By modelling the ladder as a uniform rod show that $\tan\theta = \frac{1}{2}$.

V, M S K, proof



Question 2 (**+)



The figure above shows a ladder AB resting in equilibrium with one end A on rough horizontal ground and the other end B against a smooth vertical wall. The ladder is modelled as a uniform rod of length 5 metres and mass 20 kg, and lies in a vertical plane perpendicular to the wall and the ground, inclined at an angle φ to the horizontal.

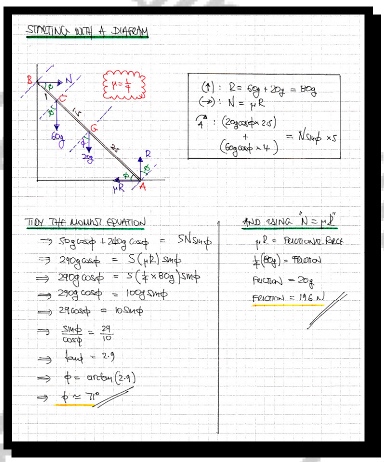
When a person, which is modelled as a particle, of mass 60 kg stands at a point C on the ladder, where $AC = 4$ metres the ladder is at the point of slipping.

Given that the coefficient of friction between the ladder and the ground is $\frac{1}{4}$, find …

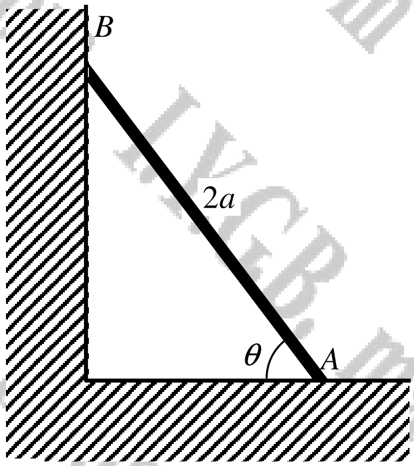
a) … the magnitude of the frictional force of the ground on the ladder.

b) … the value of φ, to the nearest degree.

$\boxed{}$, $\boxed{F = 196\text{N}}$, $\boxed{\varphi \approx 71°}$



Question 3 (**+)



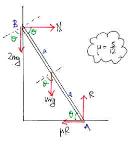
The figure above shows a uniform ladder AB of length $2a$ and of mass m resting with the end A on rough horizontal ground and the end B against a smooth vertical wall. The ladder is inclined at an angle θ to the ground.

When a child of mass $2m$ is standing on the ladder at B, the ladder is about to slip.

Given that the coefficient of friction between the ladder and the ground is $\frac{5}{12}$, find the value of θ.

$\boxed{}$, $\boxed{\theta \approx 63.4°}$

LOOKING AT THE DIAGRAM BELOW



$(\uparrow): R = 3mg$

$(\rightarrow): N = \mu R$

TAKING MOMENTS ABOUT A

$\Rightarrow mg\cos\theta \times a + 2mg\cos\theta \times 2a = N\sin\theta \times 2a$

$\Rightarrow 2N\cancel{a}\sin\theta = 5mg\cancel{a}\cos\theta$

$\Rightarrow 2(\mu R)\sin\theta = 5mg\cos\theta$

$\Rightarrow 2 \times \frac{5}{12} \times 3mg \times \sin\theta = 5mg\cos\theta$

$\Rightarrow \frac{5}{2}\cancel{mg}\sin\theta = 5\cancel{mg}\cos\theta$

$\Rightarrow \sin\theta = 2\cos\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = 2$

$\Rightarrow \tan\theta = 2$

$\Rightarrow \theta \simeq 63.4°$

Question 4 (***)

A uniform ladder AB of mass m and length $2a$ has one of its end A on rough horizontal ground and the other end B against a smooth vertical wall.

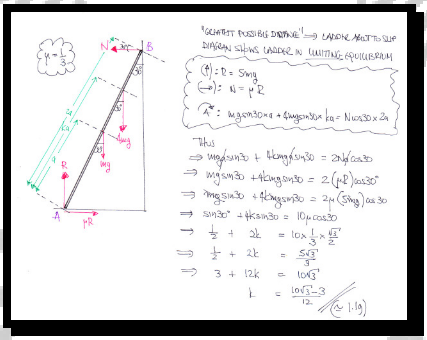
The ladder lies in a vertical plane perpendicular to the wall and the ground, and makes an angle of $30°$ with the vertical wall.

The coefficient of friction between the ladder and the ground is $\frac{1}{3}$.

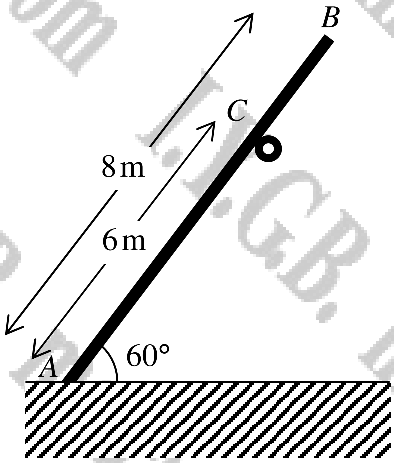
The greatest distance from A that a man of mass $4m$ can walk up this ladder is ka, where k is a positive constant.

By modelling the man as a particle and the ladder as a uniform rod, determine the value of k.

$$\boxed{k = \frac{10\sqrt{3}-3}{12} \approx 1.19}$$



Question 5 (***)



The figure above shows a uniform rod AB of length 8 metres and of mass 15 kg. The rod is resting in equilibrium with the end A on rough horizontal ground and the point C, where $AC = 6$ metres, on a smooth peg. The rod is inclined at $60°$ to the ground.

a) Determine in any order …

i. … the reaction on the rod at the peg.

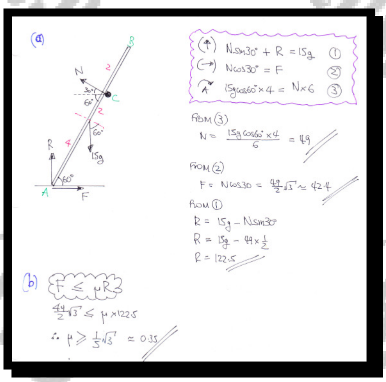
ii. … the normal reaction on the rod at the ground.

iii. … the friction acting on the rod.

The coefficient of friction between the rod and the ground is denoted by μ.

b) Find the range of the possible values of μ.

$\boxed{N_C = 49}$, $\boxed{Fr = \frac{49}{2}\sqrt{3} \approx 42.4}$, $\boxed{R_A = 122.5}$, $\boxed{\mu \geq \frac{1}{5}\sqrt{3}}$



Question 6 (***)

A non uniform ladder of weight 180 N and length 6 metres, rests with its end A on smooth horizontal ground and its end B against a rough vertical wall.

The coefficient of friction between the ladder and the wall is $\frac{1}{4}$.

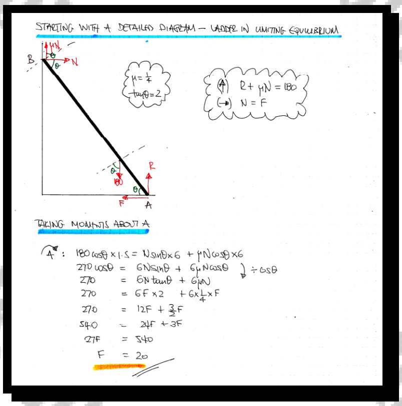
The centre of mass of the ladder is 1.5 metres from A.

The ladder lies in a vertical plane perpendicular to the wall and the ground, and is inclined at an angle θ to the horizontal, where $\tan\theta = 2$.

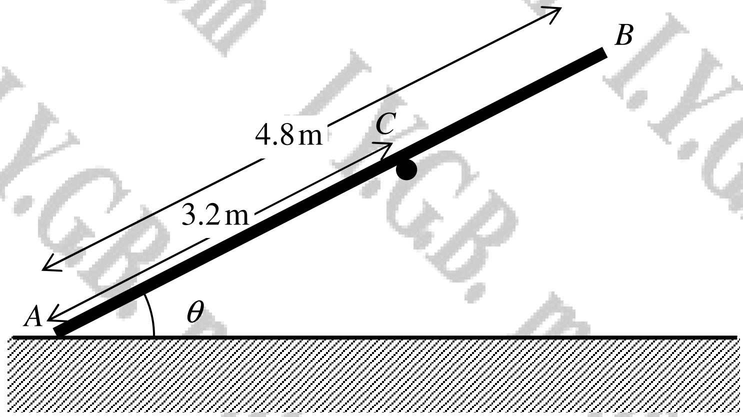
A man can just prevent the ladder from sliding down the wall by pushing the bottom of the ladder with a horizontal force F.

By modelling the ladder as a non uniform rod determine the value of F.

$\boxed{}$, V, $\boxed{F = 20\,\text{N}}$



Question 7 (***)



The figure above shows a plank AB resting on a smooth peg. The plank is modelled a uniform rod of weight W N and of length 4.8 metres, resting on the peg at the point C, where AC is 3.2 metres.

The end A of the plank rests in limiting equilibrium on rough ground, where the coefficient of friction between the plank and the ground is $\frac{9}{13}$.

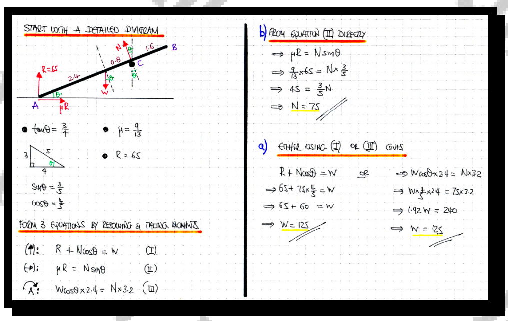
The plank is inclined at angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$. The points A, B and C lie in a vertical plane which is perpendicular to the ground.

Given that the magnitude of the normal reaction of the ground at A is 65 N, find in any order …

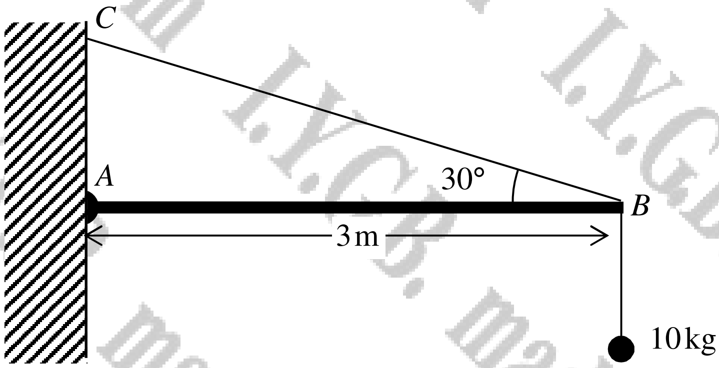
a) … the value of W.

b) … the magnitude of the force between the plank and the peg.

$\boxed{W = 125\text{ N}}$, $\boxed{F = 75\text{ N}}$



Question 8 (***)



The figure above shows a uniform rod AB, of length 3 metres and of mass 20 kg, smoothly hinged at the point A, which lies on a vertical wall.

A particle, of mass 10 kg, is suspended from the end B of the rod. The rod is kept in a horizontal position by a light inextensible string BC, where C lies on the same wall vertically above A.

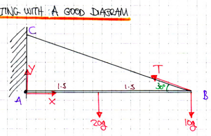
The plane ABC is perpendicular to the wall and the angle ABC is $30°$.

a) Determine the tension in the string.

b) Show that the reaction at the hinge has magnitude $98\sqrt{13}$ N.

V, M, $\boxed{T = 392 \text{ N}}$

a) STARTING WITH A GOOD DIAGRAM



FORMING THREE STANDARD EQUATIONS

$(\uparrow)$: $Y + T\sin30° = 20g + 10g$

$(\rightarrow)$: $X = T\cos30°$

$(\circlearrowright A)$: $(20g \times 1.5) + (10g \times 3) = T\sin30 \times 3$

USING THE "MOMENT" EQUATION

$\Rightarrow 30g + 30g = \frac{3}{2}T$

$\Rightarrow \frac{3}{2}T = 60g$

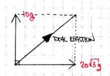
$\Rightarrow T = 40g$

$\Rightarrow T = 392 \text{ N}$ //

b) USING THE "VERTICAL & HORIZONTAL" EQUATIONS WITH $T = 392 = 40g$

- $X = T\cos30$
  $X = 40g \times \frac{\sqrt{3}}{2}$
  $X = 20\sqrt{3}g$
- $Y + T\sin30 = 30g$
  $Y + 40g \times \frac{1}{2} = 30g$
  $Y = 10g$

FINALLY THE "NET REACTION"



TOTAL REACTION $= \sqrt{(10g)^2 + (20\sqrt{3}g)^2}$

$= \sqrt{100g^2 + 400 \times 3 \times g^2}$

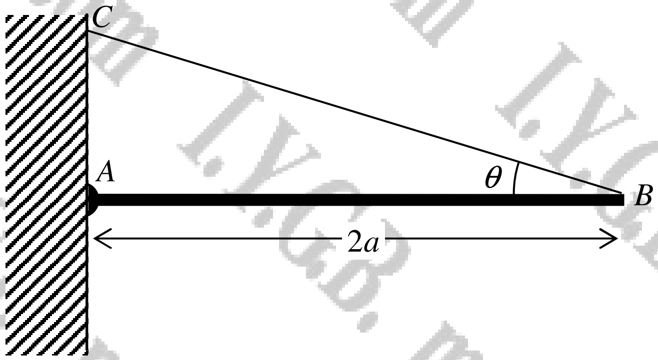
$= \sqrt{1300g^2}$

$= \sqrt{100g^2}\sqrt{13}$

$= 10g\sqrt{13}$

$= 98\sqrt{13}$ // AS REQUIRED

Question 9 (***+)



The figure above shows a uniform rod AB of length $2a$ and of mass m smoothly hinged at the point A, which lies on a vertical wall.

The rod is kept in a horizontal position by a light inextensible string BC, where C lies on the same wall vertically above A.

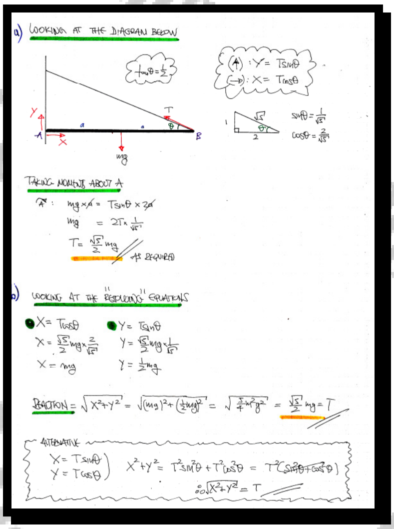
The plane ABC is perpendicular to the wall and the angle ABC is denoted by θ.

Given that $\tan\theta = \frac{1}{2}$, show that …

a) … the tension in the string is $\frac{1}{2}\sqrt{5}mg$.

b) … the magnitude of the reaction at the hinge has the same magnitude as the tension in the string.

$\boxed{}$, $\boxed{\text{proof}}$



a) LOOKING AT THE DIAGRAM BELOW

$\tan\theta = \frac{1}{2}$

$(\uparrow): Y = T\sin\theta$

$(\rightarrow): X = T\cos\theta$

$\sin\theta = \frac{1}{\sqrt{5}}$, $\cos\theta = \frac{2}{\sqrt{5}}$

TAKING MOMENTS ABOUT A

$mg \times \cancel{a} = T\sin\theta \times 2\cancel{a}$

$mg = 2T \times \frac{1}{\sqrt{5}}$

$T = \frac{\sqrt{5}}{2}mg$ AS REQUIRED

b) LOOKING AT THE "RESOLVING" EQUATIONS

$X = T\cos\theta$ | $Y = T\sin\theta$
$X = \frac{\sqrt{5}}{2}mg \times \frac{2}{\sqrt{5}}$ | $Y = \frac{\sqrt{5}}{2}mg \times \frac{1}{\sqrt{5}}$
$X = mg$ | $Y = \frac{1}{2}mg$

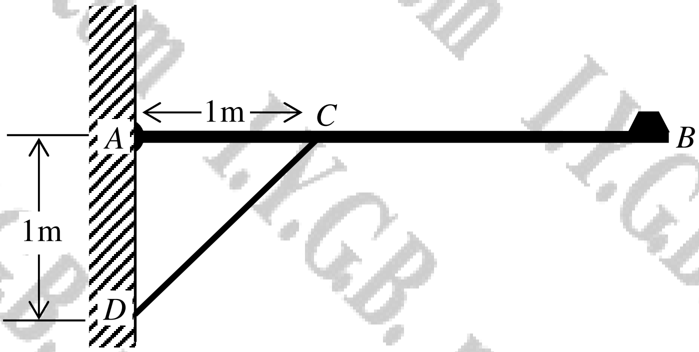
REACTION $= \sqrt{X^2+Y^2} = \sqrt{(mg)^2 + \left(\frac{1}{2}mg\right)^2} = \sqrt{\frac{5}{4}m^2g^2} = \frac{\sqrt{5}}{2}mg = T$

ALTERNATIVE

$X = T\sin\theta$, $Y = T\cos\theta$ $\quad X^2+Y^2 = T^2\sin^2\theta + T^2\cos^2\theta = T^2(\cancel{\sin^2\theta+\cos^2\theta})$

$\therefore \sqrt{X^2+Y^2} = T$

Question 10 (***+)



The figure above shows a uniform rod AB, of length 2.5 metres and mass 10 kg, with one of its ends A smoothly hinged vertical wall.

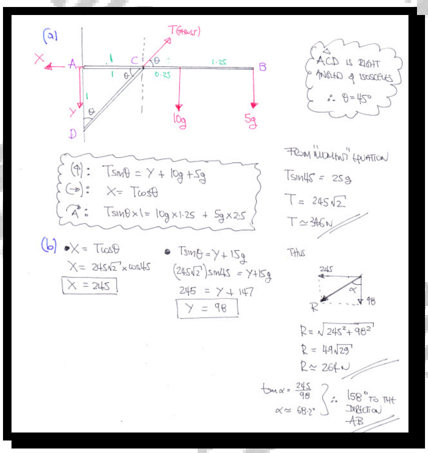
The rod is kept in equilibrium in a horizontal position by a light rigid strut DC, where D lies on the same wall vertically below A and C lies on the rod such that $|AC| = |AD| = 1$ metre.

A particle of mass 5 kg is placed at B. The plane ACD is perpendicular to the wall.

a) Calculate the force exerted by the strut on the rod.

b) Determine the magnitude and direction of the force exerted by the hinge on the rod AB.

$\boxed{\text{Thrust} = 245\sqrt{2} \approx 346 \text{ N}}$, $\boxed{R = 49\sqrt{29} \approx 264 \text{ N}}$, $\boxed{\theta \approx 158°, \text{ to the direction } AB}$



Question 11 (***+)

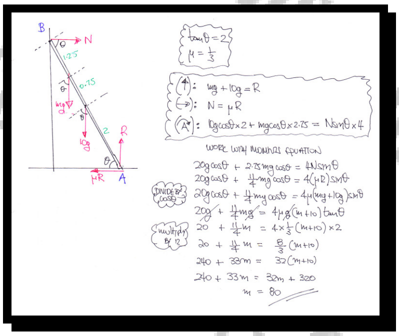
A uniform ladder of mass 10 kg and length 4 metres, rests with its end A on rough horizontal ground and its end B against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and the ground, and is inclined at an angle θ to the horizontal, where $\tan\theta = 2$.

The coefficient of friction between the ladder and the ground is $\frac{1}{3}$.

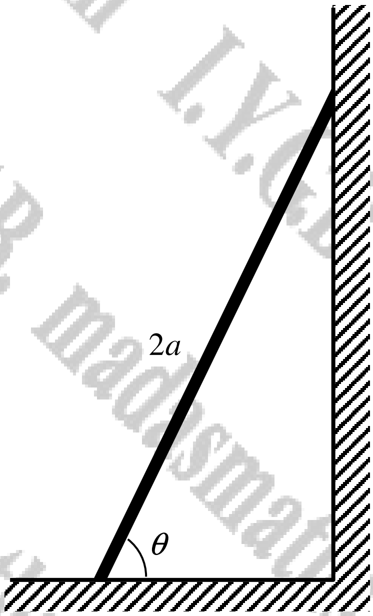
When a man of mass m climbs on the ladder and stands at the point C, where AC is 2.75 metres, the ladder is in limiting equilibrium.

By modelling the man as a particle and the ladder as a uniform rod, determine the value of m.

$\boxed{m = 80}$



Question 12 (***+)

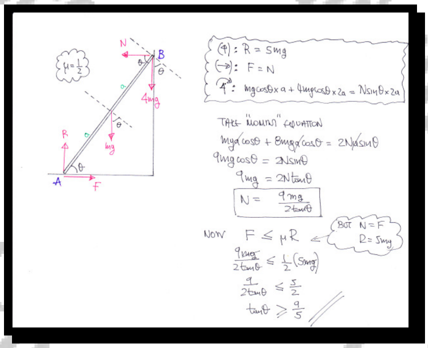


The figure above shows a uniform ladder with one end on rough horizontal ground and the other end against a smooth vertical wall. The ladder is modelled as a uniform rod of length $2a$ and mass m, and lies in a vertical plane perpendicular to the wall and the ground, inclined at an angle θ to the horizontal.

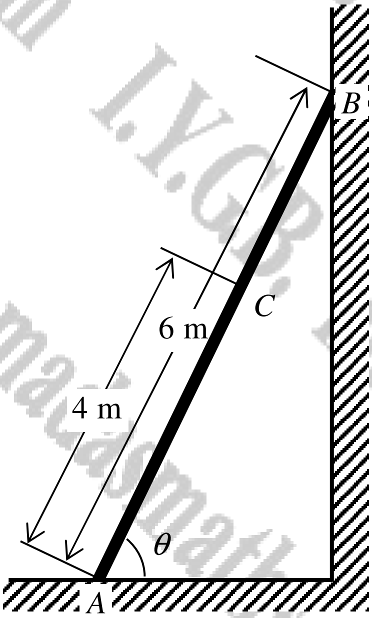
The ladder remains in equilibrium when a man of mass $4m$ stands at the top of the ladder. The coefficient of friction between the ladder and the ground is $\frac{1}{2}$

Show clearly that $\tan\theta \geq \frac{9}{5}$.

proof



Question 13 (***+)

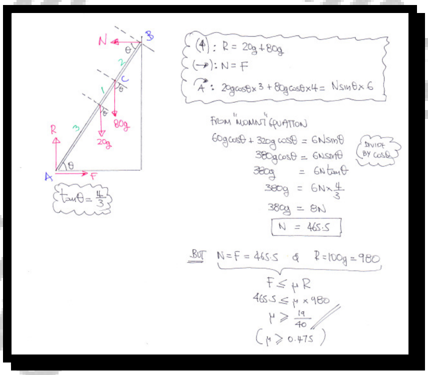


The figure above shows a ladder of length 6 metres and mass 20 kg, with one end A on rough horizontal ground and the other end B against a smooth vertical wall. The ladder is modelled as a uniform rod and lies in a vertical plane perpendicular to the wall and the ground, inclined at an angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$.

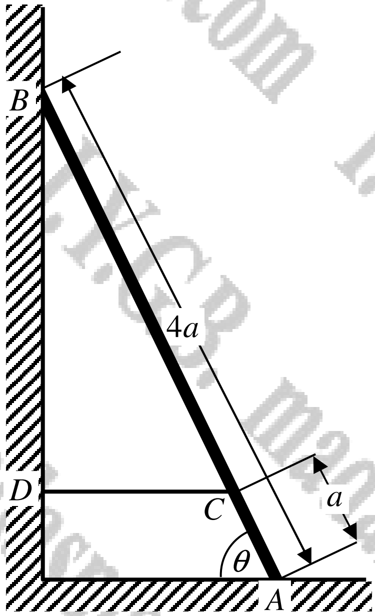
The ladder remains in equilibrium when a man of mass 80 kg stands at a point C on the ladder, where $AC = 4$ metres. The coefficient of friction between the ladder and the ground is denoted by μ.

Find the range of the possible values of μ.

$$\boxed{\mu \geq \frac{19}{40}}$$



Question 14 (***+)



The figure above shows a ladder AB of length $4a$ and weight W, with one end A on smooth horizontal ground and the other end B against a rough vertical wall. A light inextensible string has one of its ends C tied on the ladder, where $AC = a$, and the other end D tied at a point on the wall, so that CD is horizontal. The tension in the string is $3W$ and the coefficient of friction between the ladder and the ground is μ.

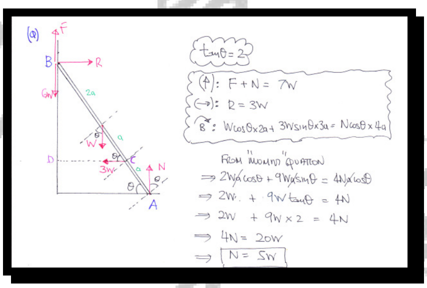
The ladder is in equilibrium when a man of weight $6W$ stands at the point B.

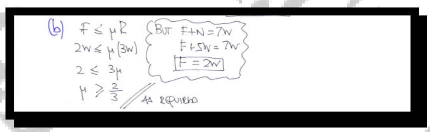
The ladder lies in a vertical plane perpendicular to the wall and the ground and is inclined at an angle θ to the horizontal, where $\tan\theta = 2$. The ladder is modelled as a uniform rod and the man as a particle.

a) Find, in terms of W, the magnitude of the force of the ground on the ladder

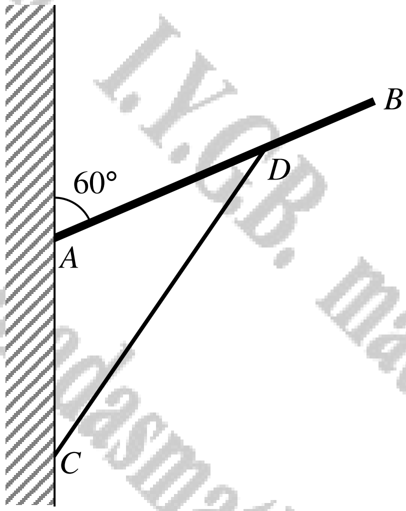
b) Show that $\mu \geq \frac{2}{3}$.

$\boxed{N = 5W}$





Question 15 (***+)



The figure above shows a uniform rod AB, of weight $\sqrt{27}W$ and length $4L$, freely hinged at the end A to a vertical wall.

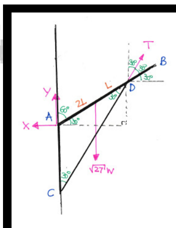
The rod is supported by a light rigid strut CD and rests in equilibrium at an angle of $60°$ to the wall. The strut is freely hinged to the rod at the point D and to the wall at the point C, which is vertically below A. It is further given that $AC = AD = 3L$

The rod and the strut lie in the same vertical plane, which is perpendicular to the wall.

a) Show that the magnitude of the thrust in the strut is $6W$.

b) Find, in terms of W, the magnitude of the force acting on the rod at A.

$\boxed{\text{reaction} = 3W \text{ N}}$



a) LOOKING AT THE DIAGRAM OPPOSITE AND TAKING MOMENTS ABOUT A

A: $\sqrt{27}W \times 2\cancel{L}\cos30 = T\sin30 \times 3\cancel{L}$

$3\sqrt{3}W \times \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}} = T \times \frac{1}{2} \times 3$

$9W = \frac{3}{2}T$

$T = 6W$ // AS REQUIRED

b) RESOLVING WE THEN

(↑) $T\cos30 + Y = \sqrt{27}W$

$6W \times \frac{\sqrt{3}}{2} + Y = 3\sqrt{3}W$

$3\sqrt{3}W + Y = 3\sqrt{3}W$

$Y = 0$

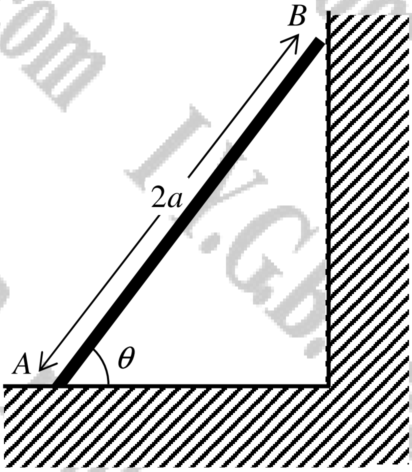
(→) $X = T\sin30$

$X = 6W \times \frac{1}{2}$

$X = 3W$

∴ MAGNITUDE IS $3W$ //

Question 16 (***+)



The figure above shows a uniform ladder AB of length $2a$ and of mass m resting with A on rough horizontal ground, and B against a rough vertical wall.

The ladder is inclined at an angle θ to the ground and the coefficient of friction between the ladder and the ground and between the ladder and the wall is $\frac{2}{3}$.

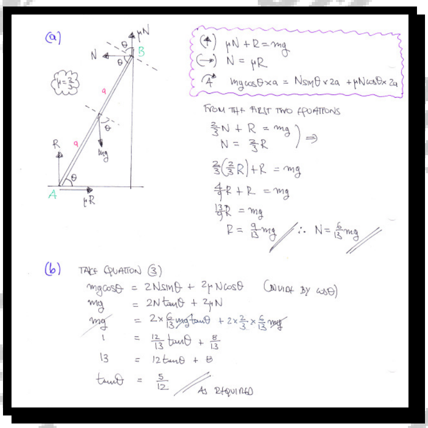
a) Given the ladder is in limiting equilibrium, determine in terms of mg …

i. … the normal reaction on the ladder at A.

ii. … the normal reaction on the ladder at B.

b) Show clearly that $\tan\theta = \frac{5}{12}$.

$\boxed{N_A = \frac{9}{13}mg}$, $\boxed{N_B = \frac{6}{13}mg}$


(a)

$\mu = \frac{2}{3}$

$(\uparrow)$ $\mu N + R = mg$
$(\rightarrow)$ $N = \mu R$
$(\circlearrowleft A)$ $mg\cos\theta \times a = N\sin\theta \times 2a + \mu N\cos\theta \times 2a$

FROM THE FIRST TWO EQUATIONS

$\left.\begin{aligned}\tfrac{2}{3}N + R &= mg\\ N &= \tfrac{2}{3}R\end{aligned}\right\} \Rightarrow$

$\frac{2}{3}\left(\frac{2}{3}R\right) + R = mg$

$\frac{4}{9}R + R = mg$

$\frac{13}{9}R = mg$

$R = \frac{9}{13}mg$ // $\therefore N = \frac{6}{13}mg$ //

(b) TAKE EQUATION (3)

$mg\cos\theta = 2N\sin\theta + 2\mu N\cos\theta$ (DIVIDE BY $\cos\theta$)

$mg = 2N\tan\theta + 2\mu N$

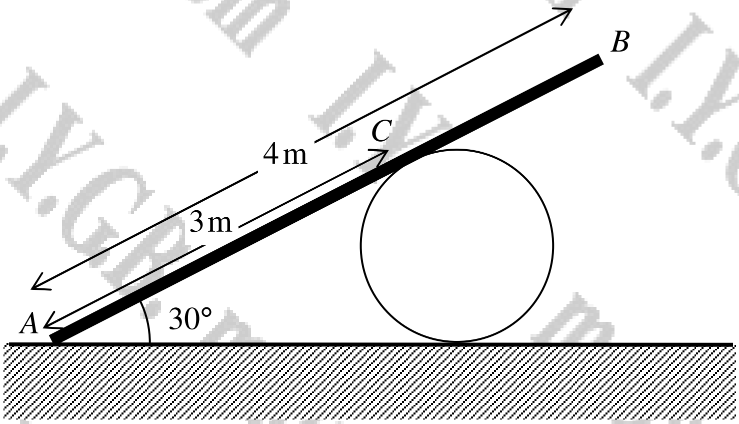
$\cancel{mg} = 2 \times \frac{6}{13}\cancel{mg}\tan\theta + 2 \times \frac{2}{3} \times \frac{6}{13}\cancel{mg}$

$1 = \frac{12}{13}\tan\theta + \frac{8}{13}$

$13 = 12\tan\theta + 8$

$\tan\theta = \frac{5}{12}$ // AS REQUIRED

Question 17 (***+)

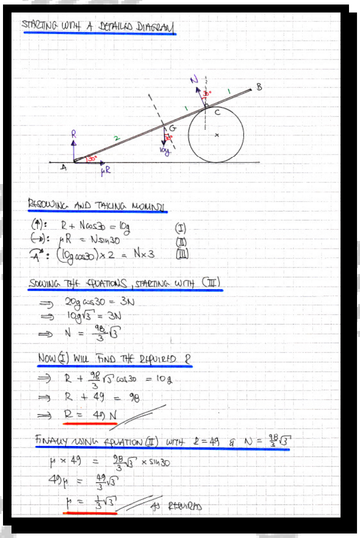


The figure above shows a plank AB resting on a smooth cylinder which is fixed with its axis horizontal to rough horizontal ground. The plank is modelled a uniform rod of mass 10 kg and length 4 metres, resting in equilibrium on the cylinder at the point C, where AC is 3 metres. The end A of the plank rests in limiting equilibrium on the ground, inclined at an angle of $30°$ to the horizontal. The points A, B and C lie in a vertical plane which is perpendicular to the axis of the cylinder.

a) Find the normal reaction between the plank and the ground.

b) Show that the coefficient of friction between the plank and the ground is $\frac{1}{3}\sqrt{3}$.

$\boxed{}$, $\boxed{R = 49\,\text{N}}$



STARTING WITH A DETAILED DIAGRAM

RESOLVING AND TAKING MOMENTS

$(\uparrow)$: $R + N\cos 30 = 10g$ (I)

$(\rightarrow)$: $\mu R = N\sin 30$ (II)

A: $(10g\cos 30) \times 2 = N \times 3$ (III)

SOLVING THE EQUATIONS, STARTING WITH (III)

$\Rightarrow 20g\cos 30 = 3N$

$\Rightarrow 10g\sqrt{3} = 3N$

$\Rightarrow N = \frac{98}{3}\sqrt{3}$

NOW (I) WILL FIND THE REQUIRED R

$\Rightarrow R + \frac{98}{3}\sqrt{3}\cos 30 = 10g$

$\Rightarrow R + 49 = 98$

$\Rightarrow R = 49\text{ N}$

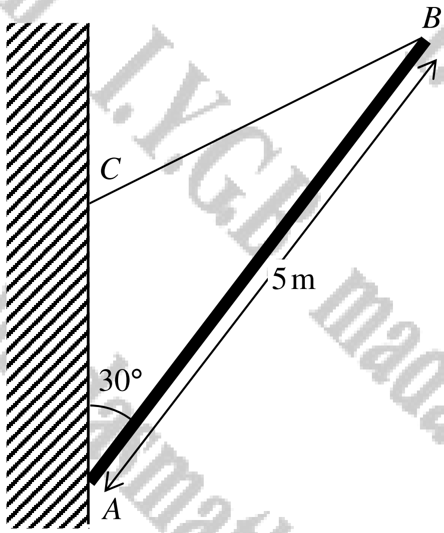
FINALLY USING EQUATION (II) WITH $R = 49$ & $N = \frac{98}{3}\sqrt{3}$

$\mu \times 49 = \frac{98}{3}\sqrt{3} \times \sin 30$

$49\mu = \frac{49}{3}\sqrt{3}$

$\mu = \frac{1}{3}\sqrt{3}$ AS REQUIRED

Question 18 (***+)



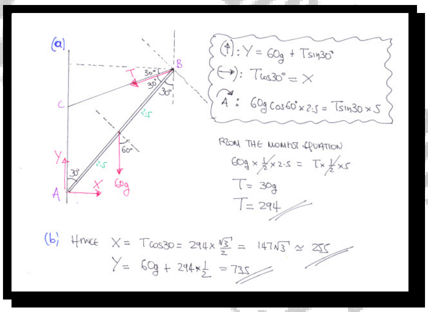
The figure above shows a uniform rod AB of length 5 metres and of mass 60 kg resting in equilibrium with its end A smoothly hinged on a vertical wall. A light string BC is attached to a point C on the wall which lies vertically above A, such that $|AC| = |CB|$.

The plane ABC is perpendicular to the wall and the angle CAB is $30°$.

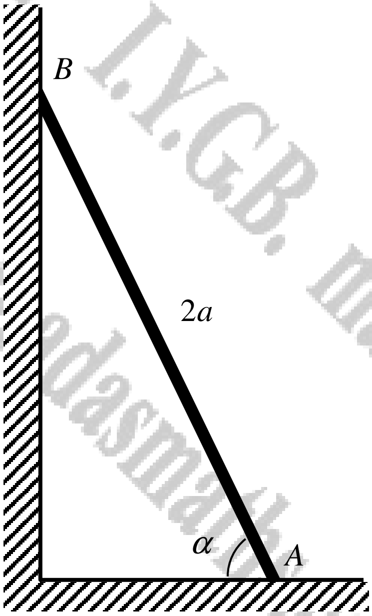
a) Determine the tension in the string.

b) Find the magnitude of the horizontal and vertical components of the force acting on the hinge at A.

$\boxed{T = 294 \text{ N}}$, $\boxed{F_{\uparrow} = 735 \text{ N}}$, $\boxed{F_{\rightarrow} = 147\sqrt{3} \text{ N}}$



Question 19 (***+)

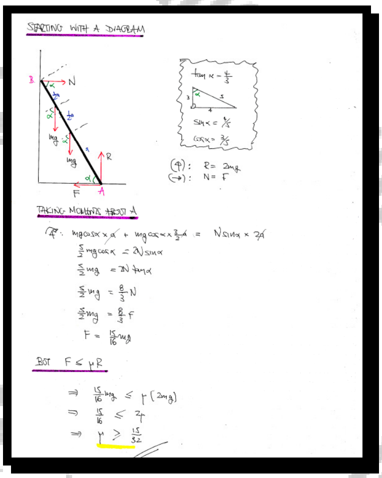


The figure above shows a ladder of length $2a$ and mass m, with one end A on rough horizontal ground and the other end B against a smooth vertical wall.

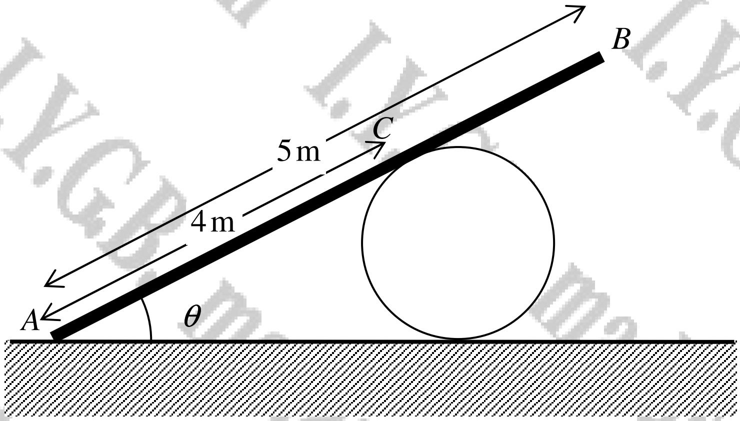
The ladder is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$ and remains in equilibrium when a child of mass m stands at a point C on the ladder, where $AC = \frac{3}{2}a$.

By modelling the ladder as a uniform rod and the child as a particle, find the range of the possible values of the coefficient of friction between the ladder and the ground.

$\boxed{\mu \geq \frac{15}{32}}$



Question 20 (***+)

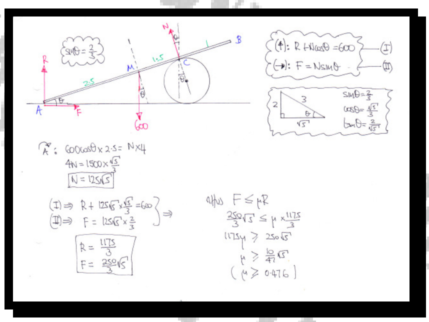


The figure above shows a plank AB of weight 600 N resting in equilibrium with one end A on rough horizontal ground. The coefficient of friction between the ground and the plank is denoted by μ.

The plank rests on a smooth cylindrical drum which fixed to the ground and cannot slide or roll. The point of contact between the plank and the drum is at C, where $AC = 4$ metres. The points A, B and C lie in a vertical plane which is perpendicular to the axis of the drum. The plank is inclined at an angle of θ to the horizontal ground, where $\sin\theta = \frac{2}{3}$.

By modelling the plank as a uniform rod, show that $\mu \geq \frac{10}{47}\sqrt{5}$.

proof



Question 21 (***+)

A ladder AB, of weight W, has one end A resting on rough horizontal ground and the other end B resting against a rough vertical wall. The ladder is modelled as a uniform rod which lies in a vertical plane perpendicular to the wall.

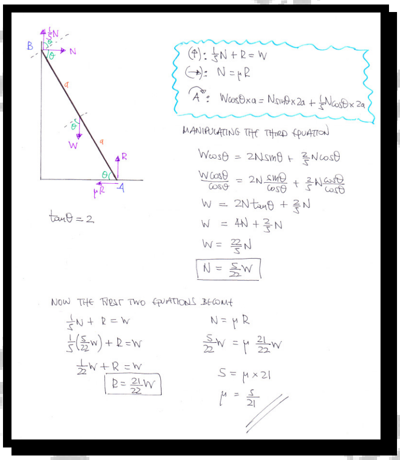
The coefficient of friction between the ladder and the wall is 0.2.

The coefficient of friction between the ladder and the ground is μ.

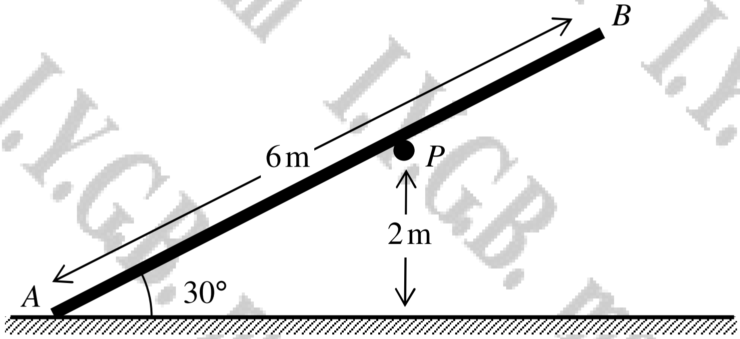
It is further given that the friction is limiting at both A and B, and the ladder is at an angle θ to the ground, where $\tan\theta = 2$.

Find the exact value of μ.

$$\boxed{\mu = \frac{5}{21}}$$



Question 22 (***+)



The figure above shows a uniform rod AB of weight 120 N and length 6 m resting in equilibrium against a rough peg P which is located 2 m above level horizontal ground. The end A of the rod rests on smooth ground, inclined at $30°$ to the ground. The coefficient of friction between the rod and the peg is μ.

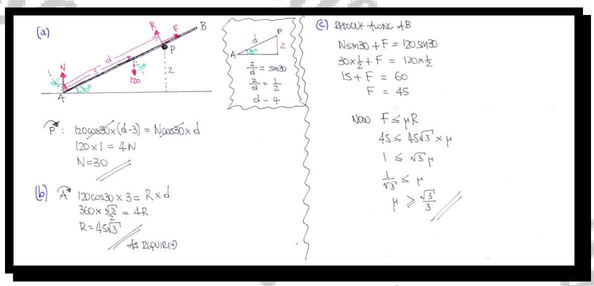
The points A, P and B lie in a vertical plane which is perpendicular to the ground.

a) Calculate the reaction between the rod and the ground.

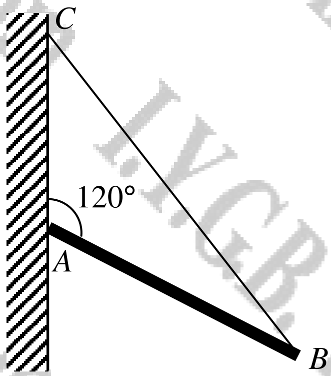
b) Show that the magnitude of the normal reaction between the rod and the peg is $45\sqrt{3}$.

c) Determine the range of the possible values of μ.

$\boxed{N = 30\,\text{N}}$, $\boxed{\mu \geq \frac{1}{3}\sqrt{3}}$



Question 23 (***+)

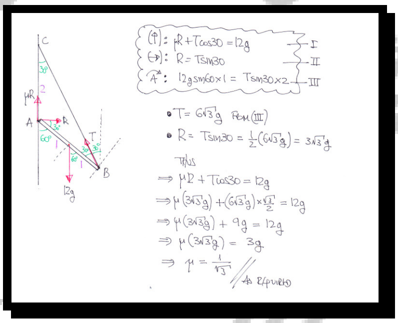


The figure above shows a uniform rod AB of length 2 metres and of mass 12 kg resting in equilibrium with its end against a rough vertical wall. A light string BC is attached to a point C on the wall which lies vertically above A, such that $|AC| = |AB|$.

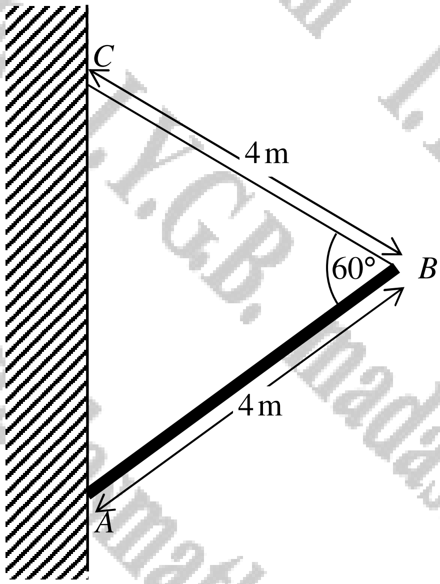
The plane ABC is perpendicular to the wall and the angle CAB is $120°$. The coefficient of friction between the wall and the rod is μ.

Given that the rod is at the point of slipping down the wall, show that $\mu = \dfrac{1}{\sqrt{3}}$.

proof



Question 24 (***+)



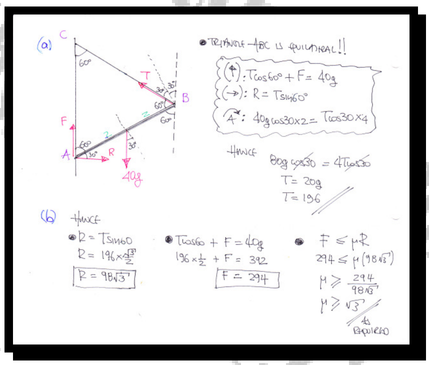
The figure above shows a uniform rod AB of length 4 metres and of mass 40 kg resting in equilibrium with its end A in contact with a rough vertical wall. The coefficient of friction between the rod and the wall is μ. A light string BC of length 4 metres is attached to a point C on the wall which lies vertically above A.

The plane ABC is perpendicular to the wall and the angle ABC is $60°$.

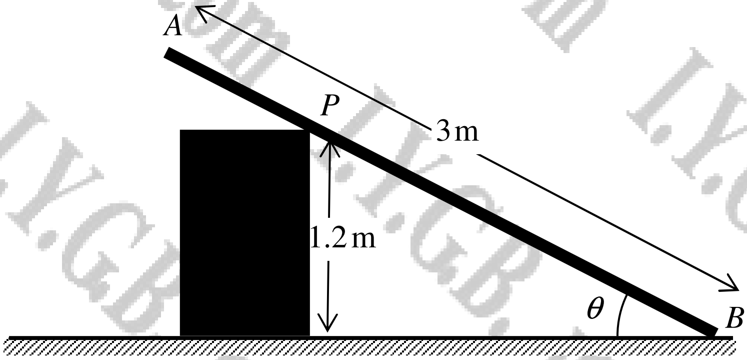
a) Determine the tension in the string.

b) Show clearly that $\mu \geq \sqrt{3}$.

$\boxed{T = 196 \text{ N}}$



Question 25 **(***+)**



The figure above shows a uniform rod AB of weight 120 N and length 3 m rests in limiting equilibrium against a rough box , where the contact point P between the rod and the box located 1.2 m above level horizontal ground.

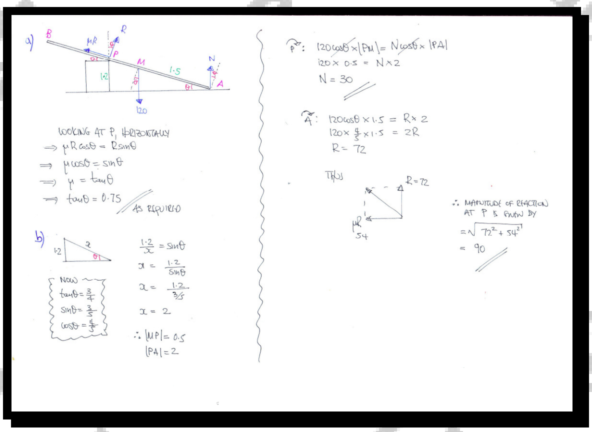
The end A of the rod rests on smooth ground, inclined at π to the ground. The coefficient of friction between the rod and the box is 0.75.

The points A, P and B lie in a vertical plane which is perpendicular to the ground.

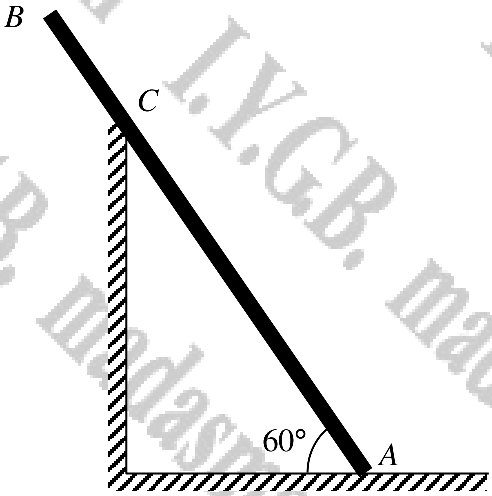
a) Show clearly that $\tan\theta = \dfrac{3}{4}$.

b) Calculate in any order the magnitude of the reaction between the rod and the ground and the **magnitude** of the reaction between the rod and the box.

$\boxed{R_A = 30\,\text{N}}$, $\boxed{R_P = 90\,\text{N}}$



Question 26 (***+)



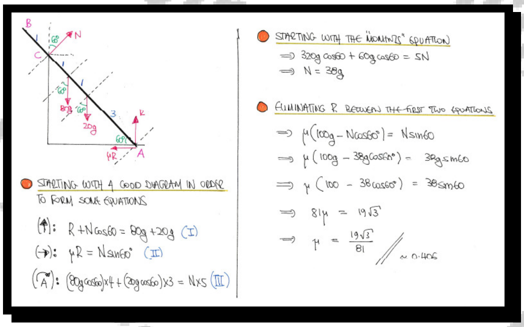
A uniform ladder AB, of length 6 m and mass 20 kg, is placed with its end A on rough horizontal ground and B **over** a smooth vertical wall. The ladder rests on the wall at the point C, so that $|BC| = 1 \text{ m}$.

The ladder is inclined at an angle of $60°$ to the horizontal and the coefficient of friction between the ladder and the ground is μ.

When a man, of mass 80 kg, is standing at the point P on the ladder, so that $|AP| = 4 \text{ m}$, the ladder is in limiting equilibrium.

Determine the exact value of μ.

$\boxed{}$, $\boxed{\mu = \frac{19\sqrt{3}}{81}}$



Question 27 (****)

A uniform ladder AB, of length 12 m and mass $M \text{ kg}$, is placed with its end A on rough horizontal ground and B against a smooth vertical wall.

A light inextensible rope is attached to the ladder at a vertical distance of 3 m **above the ground** and is tied to the wall so that the rope is horizontal. The rope can withstand a maximum tension of 490 N.

The ladder is inclined at $\arctan\frac{4}{3}$ to the horizontal and the coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

Given that the rope breaks when a man of mass 100 kg is standing at the point P on the ladder, where $|AP| = 9 \text{ m}$, find the value of M.

$\boxed{}$, $\boxed{M = 25}$

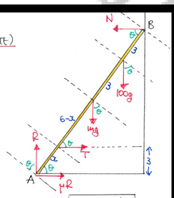
START WITH A DIAGRAM (OPPOSITE)

WHERE $\mu = \frac{1}{4}$, $T = 490$

ALSO $\tan\theta = \frac{4}{3}$

$\cos\theta = \frac{3}{5}$

$\sin\theta = \frac{4}{5}$

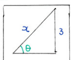


RESOLVING & TAKING MOMENTS

$(\uparrow)$ $R = mg + 100g$

$(\rightarrow)$ $N = T + \mu R$

(A) $(T\sin\theta)x + (mg\cos\theta)\times 6 + (100g\cos\theta)\times 9 = (N\sin\theta)\times 12$



$\frac{3}{x} = \sin\theta$

$x = \frac{3}{\sin\theta}$

$x = \frac{3}{\frac{4}{5}}$

$x = \frac{15}{4}$

TIDYING UP

$\Rightarrow Tx\tan\theta + 6mg + 900g = 12N\tan\theta$

$\Rightarrow \frac{4}{3}Tx + 6mg + 900g = 12N \times \frac{4}{3}$

$\Rightarrow \frac{4}{3}T\left(\frac{15}{4}\right) + 6mg + 900g = 16N$

$\Rightarrow 5T + 6mg + 900g = 16N$

$\Rightarrow (5\times 490) + 6mg + 900g = 16(T + \mu R)$

$\Rightarrow 2450 + 6mg + 900g = 16T + 16\mu R$

$\Rightarrow 2450 + 6mg + 8820 = 16\times 490 + 16\times\frac{1}{4}(mg + 100g)$

$\Rightarrow 11270 + 6mg = 7840 + 4mg + 400g$

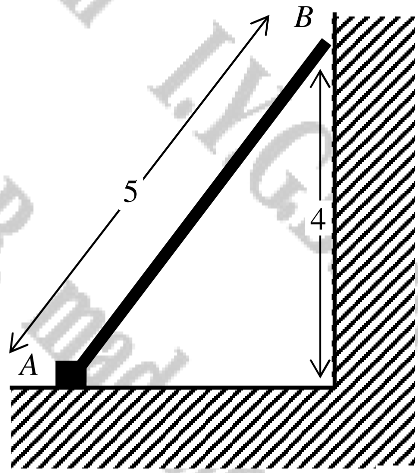
$\Rightarrow 11270 + 6mg = 7840 + 4mg + 3920$

$\Rightarrow 2mg = 490$

$19.6m = 490$

$m = 25 \text{ kg}$

Question 28 (****)



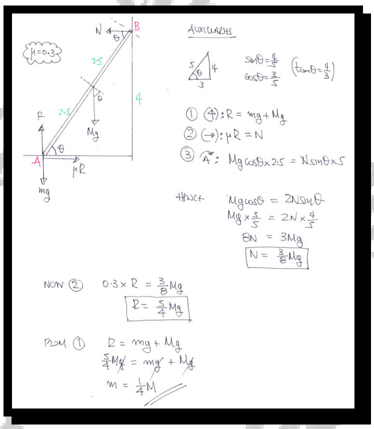
The figure above shows a uniform ladder AB of length 5 metres and of mass M kg resting with the end A on rough horizontal ground and the end B against a smooth vertical wall. The end B is 4 metres from the horizontal ground.

In order for the ladder to stay in equilibrium a block of mass m kg is placed **on the bottom end** of the ladder but **not** in contact with the ground.

The coefficient of friction between the ladder and the ground is $\frac{3}{10}$.

Find, in terms of M, the smallest value of m that will keep the ladder in equilibrium.

$$\boxed{m = \frac{1}{4}M}$$


$\mu = 0.3$

AUXILIARIES

$\sin\theta = \frac{4}{5}$ $\cos\theta = \frac{3}{5}$ $\left(\tan\theta = \frac{4}{3}\right)$

① $(\uparrow)$: $R = mg + Mg$

② $(\rightarrow)$: $\mu R = N$

③ $\overset{\curvearrowleft}{A}$: $Mg\cos\theta \times 2.5 = N\sin\theta \times 5$

HENCE $Mg\cos\theta = 2N\sin\theta$

$Mg \times \frac{3}{5} = 2N \times \frac{4}{5}$

$8N = 3Mg$

$\boxed{N = \frac{3}{8}Mg}$

NOW ② $0.3 \times R = \frac{3}{8}Mg$

$\boxed{R = \frac{5}{4}Mg}$

FROM ① $R = mg + Mg$

$\frac{5}{4}M\cancel{g} = m\cancel{g} + M\cancel{g}$

$m = \frac{1}{4}M$

Question 29 (****)

A uniform ladder rests in limiting equilibrium with its bottom end on rough horizontal ground and its top end against a smooth vertical wall.

The ladder lies in a vertical plane perpendicular to the wall and the ground, and makes an angle of θ with the ground. The coefficient of friction between the ladder and the ground is μ.

a) Show clearly that

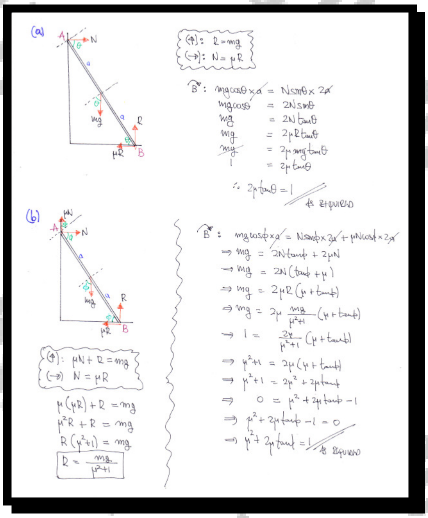
$$2\mu \tan\theta = 1.$$

The uniform ladder is next placed with its bottom end on rough horizontal ground and its top end against a rough vertical wall. The ladder lies in a vertical plane perpendicular to the wall and the ground, and makes an angle of φ with the ground. The coefficient of friction between the ladder and the ground is μ. The coefficient of friction between the ladder and the wall is also μ.

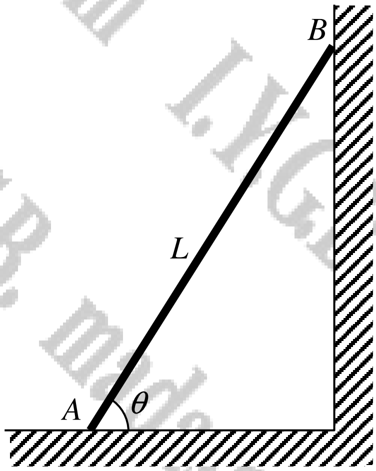
b) Show further that

$$\mu^2 + 2\mu \tan\varphi = 1.$$

$\boxed{\text{proof}}$



Question 30 (****)



The figure above shows a non-uniform rod AB, of mass m and length L, rests in equilibrium with the end A on a rough horizontal floor and the other end B, against a rough vertical wall.

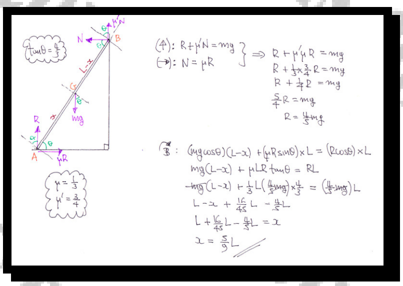
The rod is in a vertical plane perpendicular to the wall and makes an angle θ with the floor, where $\tan\theta = \frac{4}{3}$. The coefficient of friction between the rod and the floor is $\frac{1}{3}$ and the coefficient of friction between the rod and the wall is $\frac{3}{4}$.

The rod is on the point of slipping at both ends.

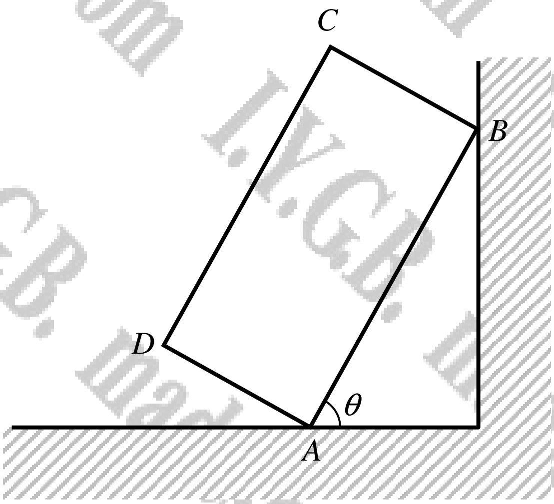
The centre of mass of the rod is at the point G.

Determine, in terms of L, the distance AG.

$$\boxed{|AG| = \frac{5}{9}L}$$



Question 31 (****)

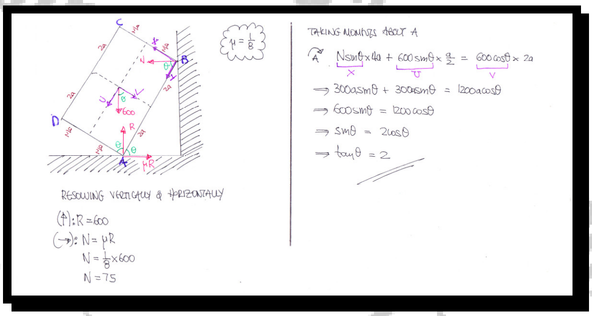


The figure above shows a uniform rectangular lamina $ABCD$, of weight 600 N, resting in equilibrium with the vertex A on rough horizontal ground and the other vertex B, against a smooth vertical wall. It is further given that $|AD| = |BC| = a$ and $|AB| = |CD| = 4a$.

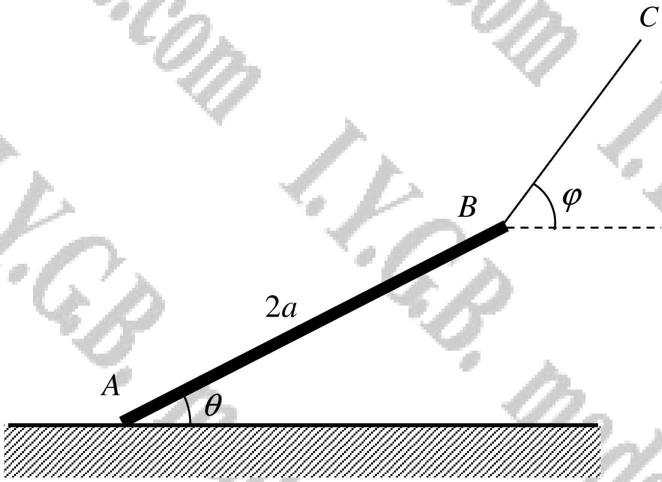
The lamina lies in a vertical plane perpendicular to the wall and makes an angle θ with the ground. The coefficient of friction between the lamina and the ground is $\frac{1}{8}$.

Given that the lamina is in limiting equilibrium, determine the value of $\tan\theta$.

$\boxed{\tan\theta = 2}$



Question 32 (****+)



The figure above shows a straight log AB has weight W and length $2a$.

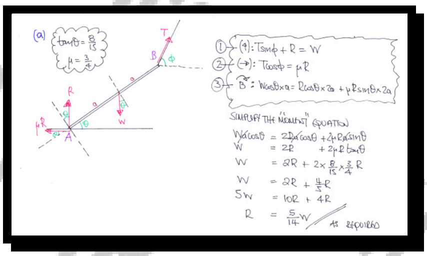
A cable BC is attached to one end of the log B, lifting that end off the ground while the other A end remains in contact with the rough horizontal ground. The cable is making an angle φ to the horizontal while the log is making an angle θ to the ground, where $\tan\theta = \frac{8}{15}$. The tension in the cable is λW. The log is in limiting equilibrium and the coefficient of friction between the log and the ground is $\frac{3}{4}$.

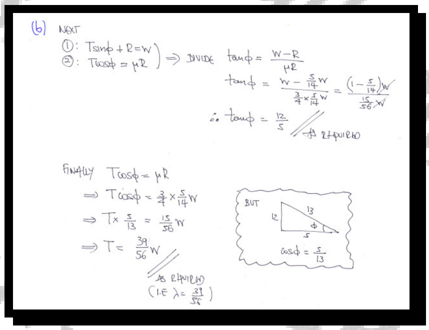
By modelling the log as a uniform rod and the cable as a light inextensible string show clearly that …

a) … the normal reaction between the log and the ground has magnitude $\frac{5}{14}W$.

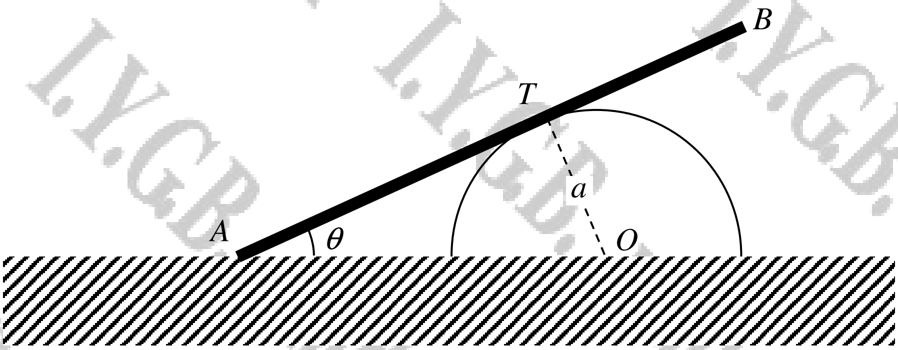
b) … the angle φ satisfies $\tan\varphi = \frac{12}{5}$ and the value of λ is $\frac{39}{56}$.

proof





Question 33 (****+)



A uniform rod AB, of length $2a$, is resting with its end A on rough horizontal ground and a point T on the rod in contact with a rough fixed prism of semicircular cross-section, of radius a. The rod lies in a vertical plane which is perpendicular to the axis of the prism, as shown in the figure above.

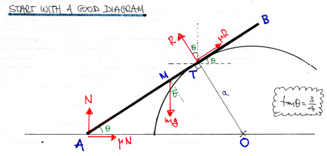
The coefficient of friction between the rod and the ground at A **and** between the rod and the prism at T, is μ, $0 < \mu < 1$.

When the rod is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$, the rod is at the point of slipping.

Determine the value of μ.

$\boxed{\mu \approx 0.425}$

START WITH A GOOD DIAGRAM



FIRSTLY BY SIMPLE TRIGONOMETRY

$\Rightarrow \frac{|OT|}{|AT|} = \tan\theta$

$\Rightarrow \frac{a}{|AT|} = \frac{3}{4}$

$\Rightarrow |AT| = \frac{4}{3}a$

RESOLVING & TAKING MOMENTS YIELDS THE FOLLOWING EQUATIONS

$(\uparrow)$: $N + R\cos\theta + \mu R\sin\theta = mg$

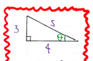
$(\rightarrow)$: $\mu N + \mu R\cos\theta = R\sin\theta$

(A): $mg\cos\theta \times |AM| = R \times |AT|$

FROM THE MOMENT EQUATION WE OBTAIN

$\Rightarrow mg\left(\frac{4}{5}\right) \times \cancel{a} = R \times \frac{4}{3}\cancel{a}$

$\Rightarrow R = \frac{3}{5}mg$



$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

USING $R = \frac{3}{5}mg$ INTO THE OTHER TWO EQUATIONS AND SIMPLIFYING

$$\Rightarrow \left\{ \begin{array}{l} N + \left(\frac{3}{5}mg\right)\left(\frac{4}{5}\right) + \mu\left(\frac{3}{5}mg\right)\left(\frac{3}{5}\right) = mg \\ \mu N + \mu\left(\frac{3}{5}mg\right)\left(\frac{4}{5}\right) = \left(\frac{3}{5}mg\right)\left(\frac{3}{5}\right) \end{array} \right\}$$

$$\Rightarrow \left\{ \begin{array}{l} N + \frac{12}{25}mg + \frac{9}{25}\mu mg = mg \\ \mu N + \frac{12}{25}\mu mg = \frac{9}{25}mg \end{array} \right.$$

ELIMINATE N BETWEEN THE EQUATIONS

$\Rightarrow \mu\left[mg - \frac{12}{25}mg - \frac{9}{25}\mu mg\right] + \frac{12}{25}\mu mg = \frac{9}{25}mg$

$\Rightarrow \mu\left[1 - \frac{12}{25} - \frac{9}{25}\mu\right] + \frac{12}{25}\mu = \frac{9}{25}$

$\Rightarrow \mu\left[25 - 12 - 9\mu\right] + 12\mu = 9$

$\Rightarrow 13\mu - 9\mu^2 + 12\mu = 9$

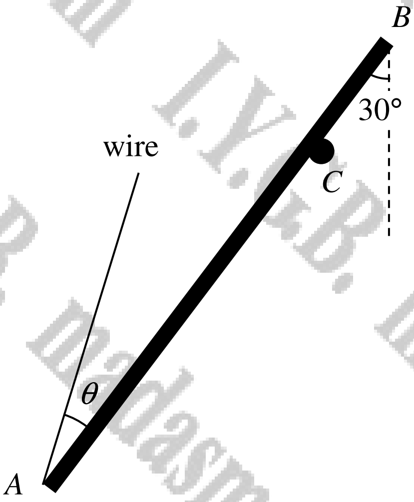
$\Rightarrow 9\mu^2 - 25\mu + 9 = 0$

BY THE QUADRATIC FORMULA

$\Rightarrow \mu = \frac{25 \pm \sqrt{301}}{18}$

$\Rightarrow \mu \approx 0.425$, $\cancel{2.35}$

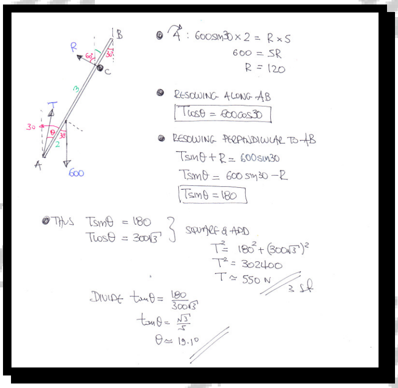
Question 34 (****+)



A non uniform rod AB of length 6 m and weight 600 N rests in equilibrium over a smooth peg at C, where $BC = 1$ m. A light inextensible wire is attached to the rod at A, and is inclined at an angle θ to the rod. The wire and the peg support the rod at $30°$ to the vertical, as shown in the figure above.

Given that the position of the centre of mass of the rod is located 2 m from A, determine the tension in the wire and the value of θ.

$\boxed{T \approx 550 \text{ N}}$, $\boxed{\theta \approx 19.1}$



Question 35 (****+)

A uniform ladder AB of mass m and length $2a$ has one of its end A on rough horizontal ground and the other end B against a smooth vertical wall.

The ladder lies in a vertical plane perpendicular to the wall and the ground, and makes an angle of α with the horizontal ground, where $\tan\alpha = \frac{5}{4}$. The coefficient of friction between the ladder and the ground is $\frac{1}{3}$.

A window cleaner of mass $8m$ stands at the top of the ladder B and his helper pushes the bottom of the ladder A towards the wall with a force of magnitude P N. This force is horizontal and its direction is perpendicular to the wall. The ladder remains in equilibrium.

The window cleaner is modelled as a particle and the ladder as a uniform rod.

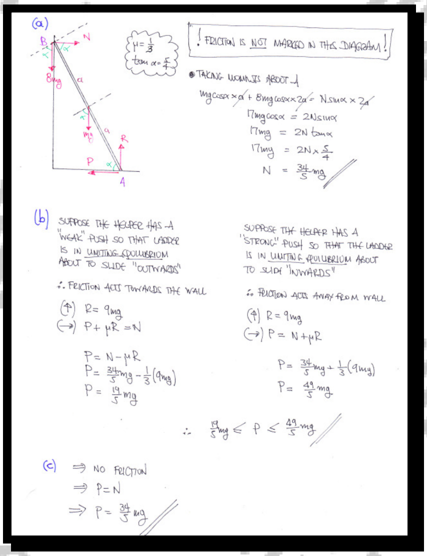
a) Find the magnitude of the force exerted by the wall on the ladder at B.

b) Show clearly that

$$\frac{19}{5}mg \le P \le \frac{49}{5}mg \,.$$

c) State the value of P for which the frictional force on the ladder is zero.

$\boxed{N = \frac{34}{5}mg}$, $\boxed{P = \frac{34}{5}mg}$


(a)

$\mu = \frac{1}{3}$, $\tan\alpha = \frac{5}{4}$

! FRICTION IS NOT MARKED IN THIS DIAGRAM !

• TAKING MOMENTS ABOUT A

$mg\cos\alpha \times a + 8mg\cos\alpha \times 2a = N\sin\alpha \times 2a$

$17mg\cos\alpha = 2N\sin\alpha$

$17mg = 2N\tan\alpha$

$17mg = 2N \times \frac{5}{4}$

$N = \frac{34}{5}mg$

(b) SUPPOSE THE HELPER HAS A "WEAK" PUSH SO THAT LADDER IS IN LIMITING EQUILIBRIUM ABOUT TO SLIDE "OUTWARDS"

∴ FRICTION ACTS TOWARDS THE WALL

$(\uparrow)$ $R = 9mg$

$(\rightarrow)$ $P + \mu R = N$

$P = N - \mu R$

$P = \frac{34}{5}mg - \frac{1}{3}(9mg)$

$P = \frac{19}{5}mg$

SUPPOSE THE HELPER HAS A "STRONG" PUSH SO THAT THE LADDER IS IN LIMITING EQUILIBRIUM ABOUT TO SLIDE "INWARDS"

∴ FRICTION ACTS AWAY FROM WALL

$(\uparrow)$ $R = 9mg$

$(\rightarrow)$ $P = N + \mu R$

$P = \frac{34}{5}mg + \frac{1}{3}(9mg)$

$P = \frac{49}{5}mg$

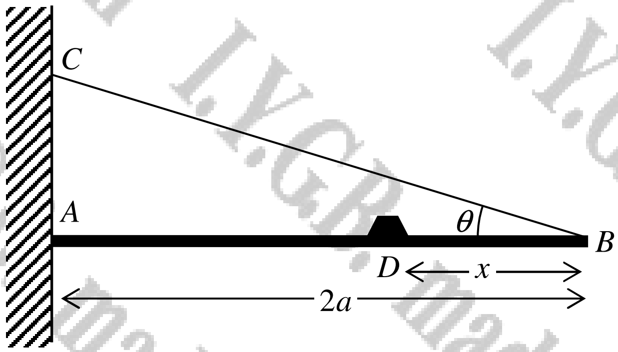
$\therefore \frac{19}{5}mg \le P \le \frac{49}{5}mg$

(c) $\Rightarrow$ NO FRICTION

$\Rightarrow P = N$

$\Rightarrow P = \frac{34}{5}mg$

Question 36 (****+)



The figure above shows a uniform rod AB, of length $2a$ and weight W N, with one of its ends A in contact with a rough vertical wall. The rod is kept in equilibrium in a horizontal position by a light inextensible string BC, where C lies on the same wall vertically above A. A particle of weight 100 N is placed at the point D on the rod, where $BD = x$. The plane ABC is perpendicular to the wall and the angle ABC is θ, where $\sin\theta = \frac{3}{5}$.

a) Show that the tension in the string is

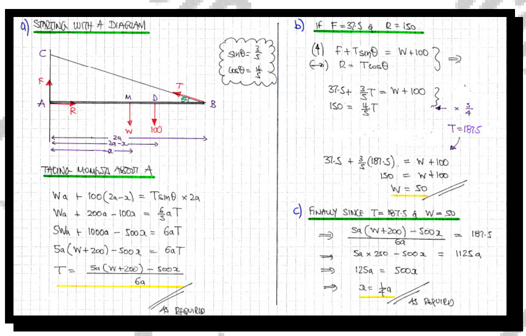
$$\frac{5a(W+200)-500x}{6a}.$$

The normal reaction and the friction of the wall on the rod are 150 N and 37.5 N, respectively.

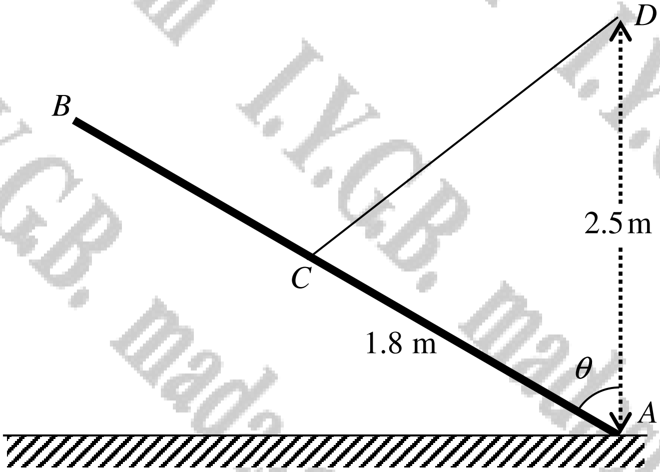
b) Find the value of W.

c) Show that $x = \frac{1}{4}a$.

$\boxed{W = 50 \text{ N}}$



Question 37 (****+)



The figure above shows a uniform rod AB, of mass 60 kg and length 3 m, with the end A resting on rough horizontal ground. The rod is held in equilibrium at an angle θ to the vertical by a light inextensible string. One end of the string is attached to the rod at the point C, where $AC = 1.8$ m. The other end of the string is attached to the point D, which is lies at a height of 2.5 m vertically above A.

a) Determine in terms of g and θ …

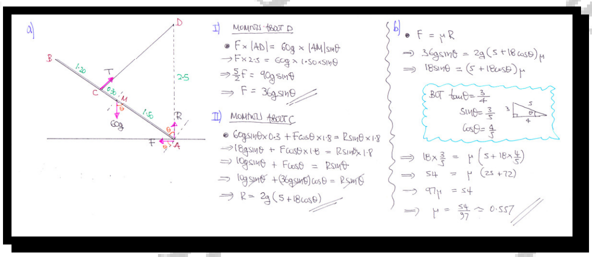
i. … the magnitude of the frictional force at A.

ii. … the magnitude of the normal reaction on the rod at A.

b) The rod is in limiting equilibrium when $\tan\theta = \frac{3}{4}$.

c) Find the coefficient of friction between the rod and the ground.

$\boxed{F = 36g\sin\theta}$, $\boxed{R = 2g(5+18\cos\theta)}$, $\boxed{\mu = \frac{54}{97} \approx 0.557}$



Question 38 (****+)

A non uniform ladder AB has length a, and its centre of mass is located at a distance k from A.

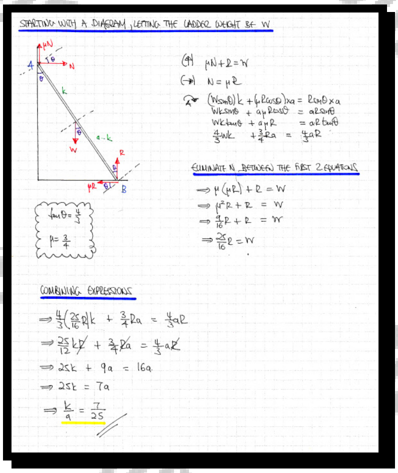
One of the ends of the ladder, A, is resting against a rough vertical wall and the other end, B, is resting on rough horizontal ground.

The coefficient of friction at both A and B is 0.75.

When AB is inclined at $\arctan\left(\frac{4}{3}\right)$ to the wall the equilibrium is limiting at both ends of the ladder.

Determine the value of $\frac{k}{a}$.

$\boxed{}$, $\boxed{\frac{k}{a} = \frac{7}{25}}$



Question 39 (****+)

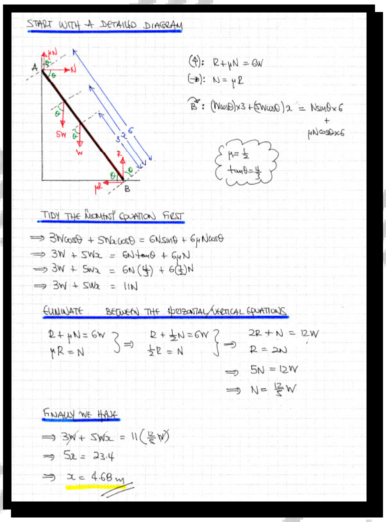
A uniform ladder AB has length 6 m and is inclined at $\arctan\left(\frac{4}{3}\right)$ to the ground.

One of the ends of the ladder, A, is resting against a rough vertical wall and the other end, B, is resting on rough horizontal ground.

The coefficient of friction at both A and B is 0.5.

Calculate the greatest distance from B a man that is **five** times as heavy as the ladder can climb.

$\boxed{}$, $\boxed{d = 4.68 \text{ m}}$



START WITH A DETAILED DIAGRAM

$(\uparrow)$: $R + \mu N = 6W$

$(\rightarrow)$: $N = \mu R$

$\overset{\curvearrowright}{B}$: $(W\cos\theta)\times 3 + (5W\cos\theta)x = N\sin\theta \times 6 + \mu N\cos\theta \times 6$

$\mu = \frac{1}{2}$

$\tan\theta = \frac{4}{3}$

TIDY THE "MOMENT" EQUATION FIRST

$\Rightarrow 3W\cos\theta + 5Wx\cos\theta = 6N\sin\theta + 6\mu N\cos\theta$

$\Rightarrow 3W + 5Wx = 6N\tan\theta + 6\mu N$

$\Rightarrow 3W + 5Wx = 6N\left(\frac{4}{3}\right) + 6\left(\frac{1}{2}\right)N$

$\Rightarrow 3W + 5Wx = 11N$

ELIMINATE BETWEEN THE HORIZONTAL/VERTICAL EQUATIONS

$\left.\begin{array}{l} R + \mu N = 6W \\ \mu R = N \end{array}\right\} \Rightarrow \left.\begin{array}{l} R + \frac{1}{2}N = 6W \\ \frac{1}{2}R = N \end{array}\right\} \Rightarrow \begin{array}{l} 2R + N = 12W \\ R = 2N \end{array}$

$\Rightarrow 5N = 12W$

$\Rightarrow N = \frac{12}{5}W$

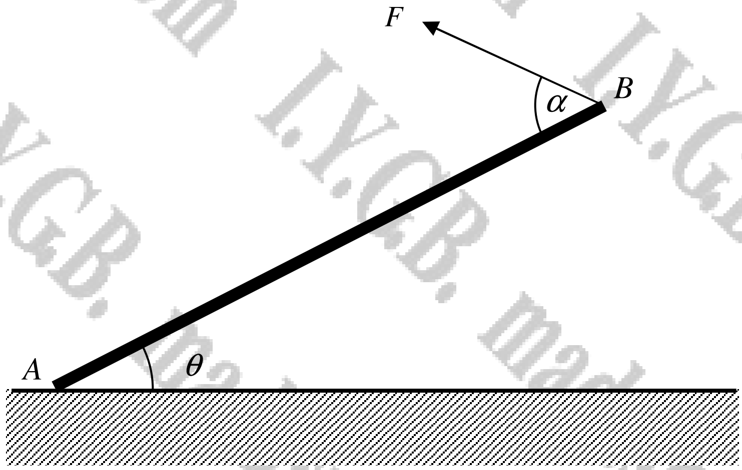
FINALLY WE HAVE

$\Rightarrow 3W + 5Wx = 11\left(\frac{12}{5}W\right)$

$\Rightarrow 5x = 23.4$

$\Rightarrow x = 4.68$ m

Question 40 (****+)



The figure above shows a plank AB modelled a uniform rod of mass m kg.

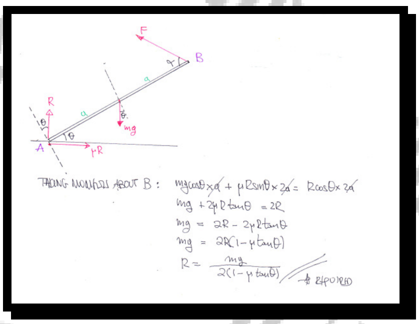
The end of the plank A rests on rough horizontal ground, where the coefficient of friction between the plank and the ground is denoted by μ.

The plank is inclined at an angle θ to the horizontal and is kept in position by a force F N acting at B. The force F is acting at an angle α to the plank, where $a > \theta$.

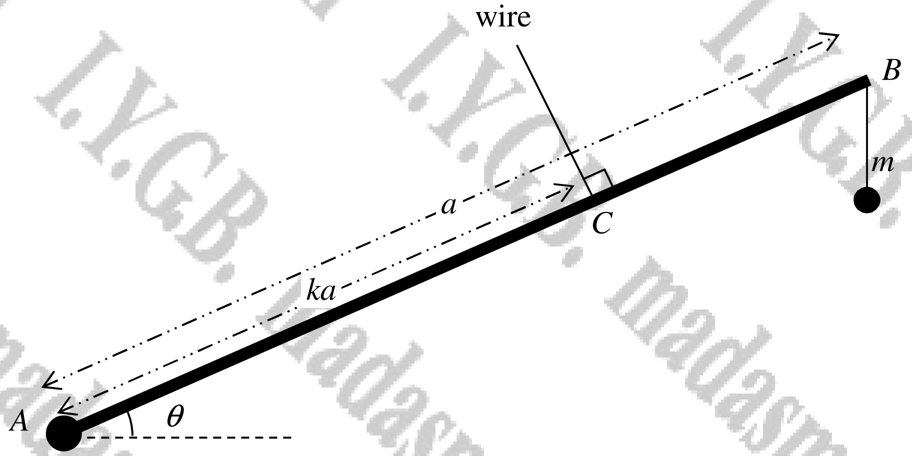
Given the plank is in limiting equilibrium, show that the magnitude of the normal reaction of the ground on the plank at A is

$$\frac{mg}{2(1-\mu\tan\theta)}.$$

proof



Question 41 (****+)



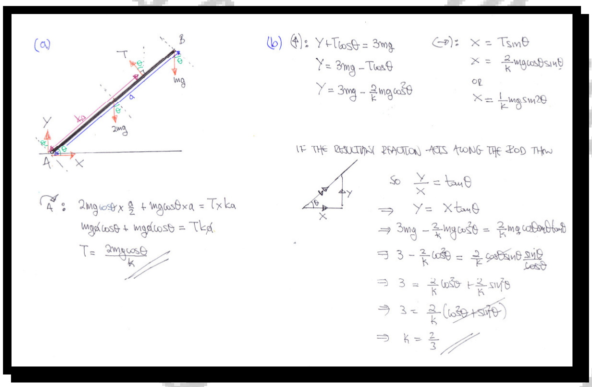
A uniform rod AB, of mass $2m$ and length a, is freely hinged at a fixed point A. A particle of mass m is suspended by a light string at B. The rod is kept in equilibrium, at an angle θ above the horizontal by a light wire attached to the rod at the point C, where $|AC| = ka$, $0 < k < 1$. The wire meets the rod at right angles and lies in the same vertical plane as the rod.

a) Find an expression, in terms of m, g, θ and k, for the tension in the wire.

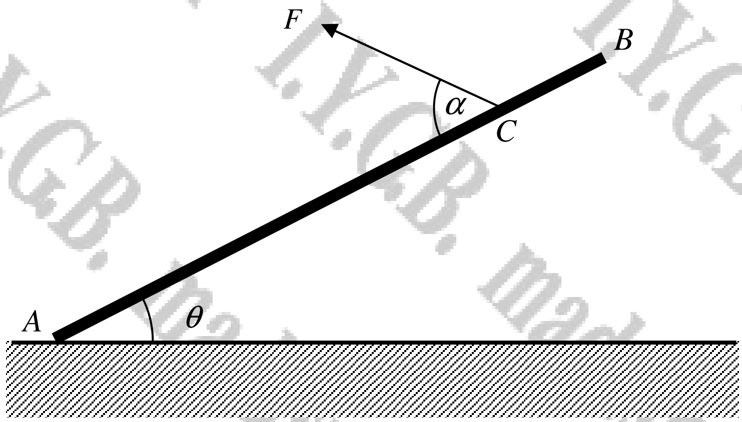
The reaction force of the hinge at A acts in the direction AB.

b) By considering two separate expressions for the horizontal component reaction force of the hinge at A and for the vertical component reaction force of the hinge at A, determine the value of k.

$\boxed{T = \frac{2}{k} mg\cos\theta}$, $\boxed{k = \frac{2}{3}}$



Question 42 (****+)



A metal beam, of length 6 m and mass 70 kg, is modelled a uniform rod.

The end of the beam A rests on rough horizontal ground, where the coefficient of friction between the beam and the ground is $\frac{3}{4}$.

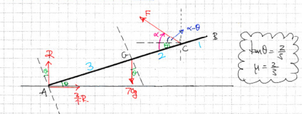
The beam is inclined at an angle θ to the horizontal and is kept in a position of equilibrium by a force F N acting at the point C, on the beam where $|AC| = 5$ m.

The force F is acting at an angle α to the plank, where $a > \theta$.

Given further that the plank is in limiting equilibrium, and $\tan\theta = \frac{2}{5}$, find in exact form the value of $\tan\alpha$ and the magnitude of F.

$\boxed{}$, $\boxed{F = 30\sqrt{2}\,g}$

STARTING WITH A DETAILED DIAGRAM



TAKE MOMENTS ABOUT C TO AVOID THE ANGLE α (UNKNOWN)

$(\text{A}): \frac{3}{4}R\sin\theta \times 5 + 70g\cos\theta \times 2 = R\cos\theta \times 5$

$\Rightarrow \frac{15}{4}R\sin\theta + 140g\cos\theta = 5R\cos\theta$ $\div \cos\theta$

$\Rightarrow \frac{15}{4}R\tan\theta + 140g = 5R$

$\Rightarrow \frac{15}{4}R\left(\frac{2}{5}\right) + 140g = 5R$

$\Rightarrow \frac{3}{2}R + 140g = 5R$ $\times 2$

$\Rightarrow 3R + 280g = 10R$

$\Rightarrow 280g = 7R$

$\Rightarrow R = 40g$

NEXT RESOLVING VERTICALLY & HORIZONTALLY

$(\rightarrow): \frac{3}{4}R = F\cos(\alpha-\theta)$

$\frac{3}{4}(40g) = F\cos(\alpha-\theta)$

$F\cos(\alpha-\theta) = 30g$

$(\uparrow): R + F\sin(\alpha-\theta) = 70g$

$40g + F\sin(\alpha-\theta) = 70g$

$F\sin(\alpha-\theta) = 30g$

DIVIDING THE EQUATIONS SIDE BY SIDE

$\Rightarrow \frac{\cancel{F}\sin(\alpha-\theta)}{\cancel{F}\cos(\alpha-\theta)} = \frac{\cancel{30g}}{\cancel{30g}}$

$\Rightarrow \tan(\alpha-\theta) = 1$

$\Rightarrow \frac{\tan\alpha - \tan\theta}{1 + \tan\alpha\tan\theta} = 1$

$\Rightarrow \frac{\tan\alpha - \frac{2}{5}}{1 + \frac{2}{5}\tan\alpha} = 1$

$\Rightarrow \tan\alpha - \frac{2}{5} = 1 + \frac{2}{5}\tan\alpha$

$\Rightarrow 5\tan\alpha - 2 = 5 + 2\tan\alpha$

$\Rightarrow 3\tan\alpha = 7$

$\Rightarrow \tan\alpha = \frac{7}{3}$ //

SQUARING & ADDING THE EQUATIONS

$\Rightarrow F^2\cos^2(\alpha-\theta) + F^2\sin^2(\alpha-\theta) = (30g)^2 + (30g)^2$

$\Rightarrow F^2\left(\cos^2(\alpha-\theta) + \sin^2(\alpha-\theta)\right) = (30g)^2$

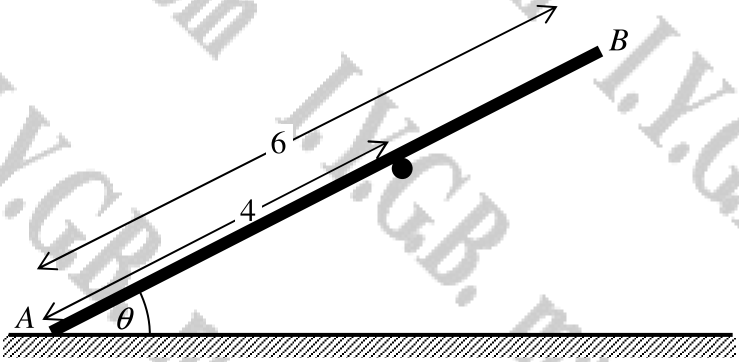
$\Rightarrow F^2 \times 1 = 2(30g)^2$

$\Rightarrow F^2 = 2(30g)^2$

$\Rightarrow F = \sqrt{2}(30g)$

$\Rightarrow F = 30\sqrt{2}\,g$ //

Question 43 (*****)



The figure above shows a uniform rigid rod AB resting on a rough peg. The rod has weight W N and length 6 m and rests on the peg at the point C, where AC is 4 m. The coefficient of friction between the rod and the peg is 0.5.

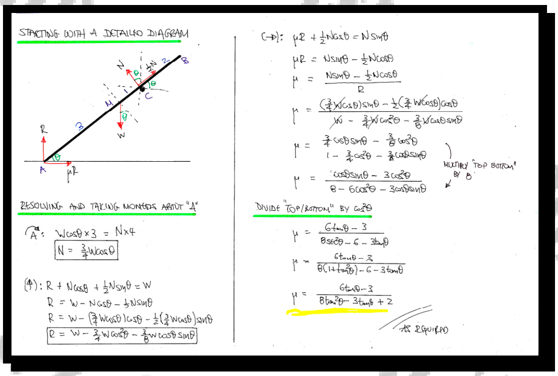
The end A of the rod rests in on rough ground, where the coefficient of friction between the rod and the ground is μ.

The rod is inclined at angle θ to the horizontal and the points A, B and C lie in a vertical plane which is perpendicular to the ground.

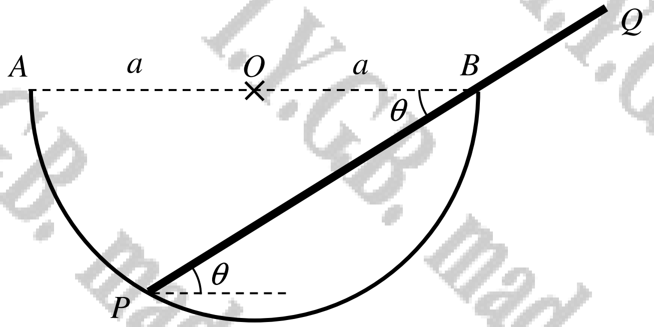
Given the equilibrium is limiting both at A and at C, show clearly that

$$\mu = \frac{6\tan\theta - 3}{8\tan^2\theta - 3\tan\theta + 2}.$$

, proof



Question 44 (*****)



A smooth hollow hemispherical bowl of radius a and centre at O, is fixed so that its circular rim lies in a horizontal plane.

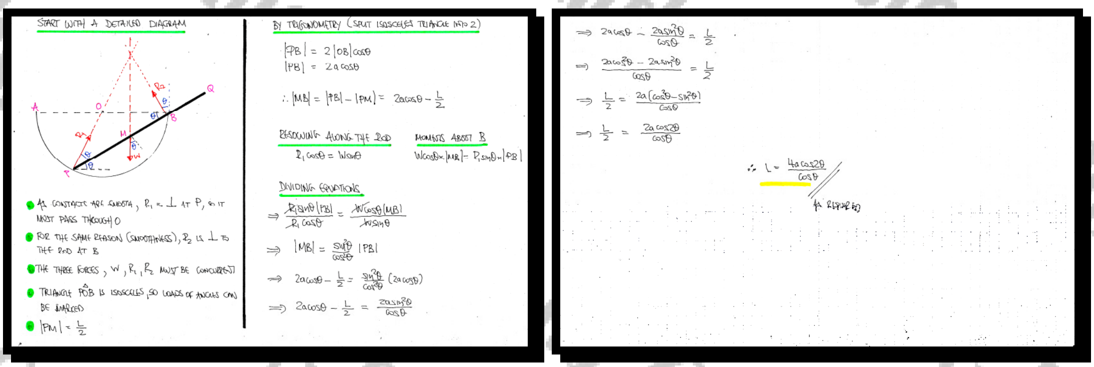
A smooth uniform rod PQ, of length L, rests in equilibrium with its end P at some point inside the bowl, as shown in the figure above.

The rod is in contact with the rim of the hemisphere at some point B, so that $|PB| < L$, and is inclined at an angle θ to the horizontal.

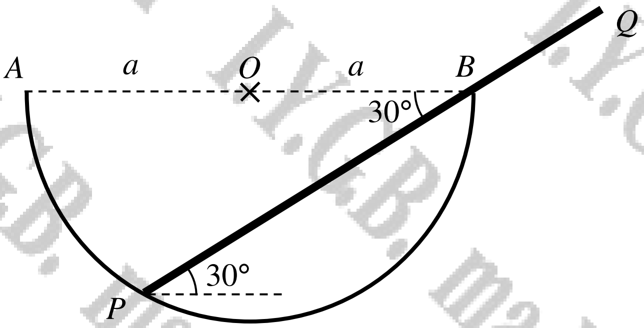
Show that

$$L = \frac{4a\cos 2\theta}{\cos\theta}.$$

[ ], [proof]



Question 45 (*****)



A smooth hollow hemispherical bowl of radius a and centre at O, is fixed so that its circular rim lies in a horizontal plane.

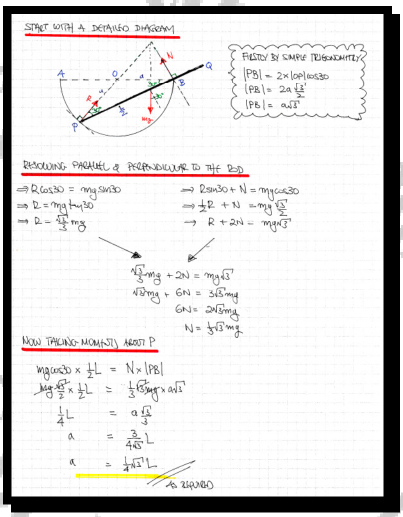
A smooth uniform rod PQ, of length L, rests in equilibrium with its end P at some point inside the bowl, as shown in the figure above.

The rod is in contact with the rim of the hemisphere at some point B, so that $|PB| < L$, and is inclined at an angle $30°$ to the horizontal.

Show that

$$a = \frac{\sqrt{3}}{4} L .$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 46 (*****)

Two spheres of respective radius a and $2a$, are fixed on level horizontal ground.

The spheres are touching each other.

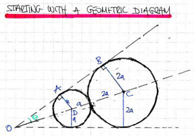
A uniform rod of length $4a$ is gently placed over the two spheres, touching the two spheres in such a way so that the centre of mass of the rod is equidistant from the two contact points of the rod with the spheres.

The coefficient of friction at the contact point with the smaller sphere is μ and at the contact point with the larger sphere is 3μ.

If the rod remains in equilibrium, show that $\mu \geq k\sqrt{2}$, where k is a rational constant.

$\boxed{\text{M456S}}$, $\boxed{k = \frac{4}{7}}$

STARTING WITH A GEOMETRIC DIAGRAM



- LET $OC = d$
- $\sin\theta = \frac{|BC|}{|OC|} = \frac{|AD|}{|OD|}$

$\Longrightarrow \frac{2a}{d} = \frac{a}{d-3a}$

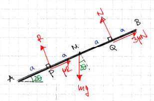
$\frac{2}{d} = \frac{1}{d-3a}$

$2d - 6a = d$

$d = 6a$

- $\sin\theta = \frac{1}{3}$

NOW LOOKING AT THE ROD IN EQUILIBRIUM AND SUPPOSE IT IS LIMITING



$\overset{\curvearrowright}{P}$: $mg\cos2\theta \times a = N \times 2a$

$N = \frac{1}{2}mg\cos2\theta$

$\overset{\curvearrowright}{Q}$: $mg\cos2\theta \times a = R \times 2a$

$R = \frac{1}{2}mg\cos2\theta$

NOW RESOLVING PARALLEL (ALONG) THE ROD

$\mu R + 3\mu N = mg\sin2\theta$

$\mu(R + 3N) = mg\sin2\theta$

$\mu\left(\frac{1}{2}mg\cos2\theta + \frac{3}{2}mg\cos2\theta\right) = mg\sin2\theta$

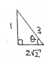
$2\mu\cos2\theta = \sin2\theta$

$\mu = \frac{1}{2}\tan2\theta$

NOW WE NEED THE EXACT VALUE OF $\tan2\theta$

$\tan2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\times\frac{1}{2\sqrt{2}}}{1-\frac{1}{8}} = \frac{\frac{1}{\sqrt{2}}}{\frac{7}{8}} = \frac{8\sqrt{2}}{7}$

$\sin\theta = \frac{1}{3}$



$\cos\theta = \frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{1}{2\sqrt{2}}$

$\therefore \tan\theta = \frac{8}{7}\sqrt{2}$ IF IN LIMITING EQUILIBRIUM

$\therefore \mu \geqslant \frac{1}{2}\tan2\theta$

$\mu \geqslant \frac{4}{7}\sqrt{2}$