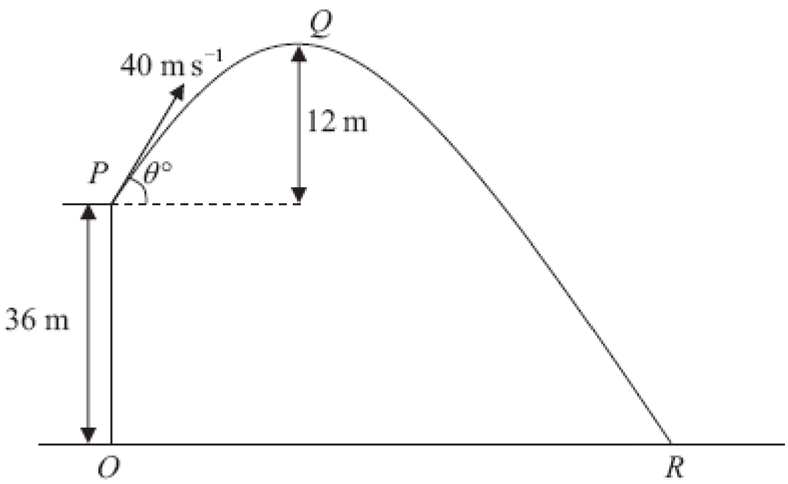
M2 MECHANICS. PAST-PAPER QUESTIONS ON KINEMATICS

1. A particle P moves on the x-axis. The acceleration of P at time t seconds, $t \geq 0$, is $(3t + 5)$ m s^{-2} in the positive x-direction. When $t = 0$, the velocity of P is 2 ms^{-1} in the positive x-direction. When $t = T$, the velocity of P is 6 m s^{-1} in the positive x-direction. Find the value of T.

(Total 6 marks)

2.



A ball is projected with speed 40 m s^{-1} from a point P on a cliff above horizontal ground. The point O on the ground is vertically below P and OP is 36 m. The ball is projected at an angle θ° to the horizontal. The point Q is the highest point of the path of the ball and is 12 m above the level of P. The ball moves freely under gravity and hits the ground at the point R, as shown in the diagram above. Find

(a) the value of θ,

(3)

(b) the distance OR,

(6)

(c) the speed of the ball as it hits the ground at R.

(3)

(Total 12 marks)

3. A particle P moves along the x-axis. At time t seconds the velocity of P is v m s^{-1} in the positive x-direction, where $v = 3t^2 - 4t + 3$. When $t = 0$, P is at the origin O. Find the distance of P from O when P is moving with minimum velocity.

(Total 8 marks)

4. [*In this question* **i** *and* **j** *are unit vectors in a horizontal and upward vertical direction respectively*]

A particle P is projected from a fixed point O on horizontal ground with velocity $u(\mathbf{i} + c\mathbf{j})$ ms^{-1}, where c and u are positive constants. The particle moves freely under gravity until it strikes the ground at A, where it immediately comes to rest. Relative to O, the position vector of a point on the path of P is $(x\mathbf{i} + y\mathbf{j})$ m.

(a) Show that

$$y = cx - \frac{4.9x^2}{u^2}.$$

(5)

Given that $u = 7$, $OA = R$ m and the maximum vertical height of P above the ground is H m,

(b) using the result in part (a), or otherwise, find, in terms of c,

(i) R

(ii) H.

(6)

Given also that when P is at the point Q, the velocity of P is at right angles to its initial velocity,

(c) find, in terms of c, the value of x at Q.

(6)
(Total 17 marks)

5. At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds the velocity of P is v m s^{-1}, where

$$v = 8t - t^2.$$

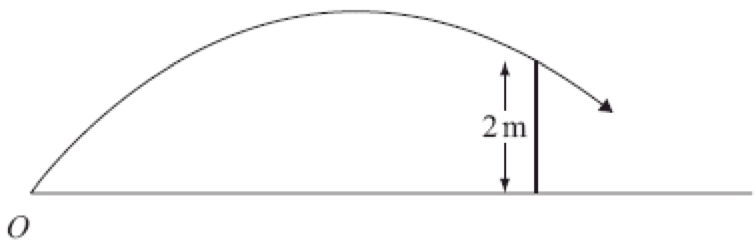
(a) Find the maximum value of v.

(4)

(b) Find the time taken for P to return to O.

(5)
(Total 9 marks)

6.



A child playing cricket on horizontal ground hits the ball towards a fence 10 m away. The ball moves in a vertical plane which is perpendicular to the fence. The ball just passes over the top of the fence, which is 2 m above the ground, as shown in the diagram above.

The ball is modelled as a particle projected with initial speed u m s^{-1} from point O on the ground at an angle α to the ground.

(a) By writing down expressions for the horizontal and vertical distances, from O of the ball t seconds after it was hit, show that

$$2 = 10 \tan \alpha - \frac{50g}{u^2 \cos^2 \alpha}.$$

(6)

Given that $\alpha = 45°$,

(b) find the speed of the ball as it passes over the fence.

(6)
(Total 12 marks)

7. A particle P moves along the x-axis in a straight line so that, at time t seconds, the velocity of P is v m s^{-1}, where

$$v = \begin{cases} 10t - 2t^2, & 0 \le t \le 6, \\ \dfrac{-432}{t^2}, & t > 6. \end{cases}$$

At $t = 0$, P is at the origin O. Find the displacement of P from O when

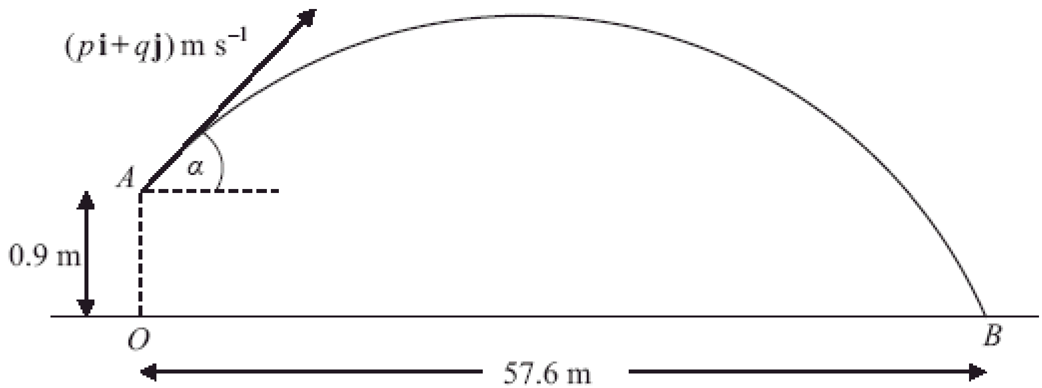
(a) $t = 6$,

(3)

(b) $t = 10$.

(5)
(Total 8 marks)

8.



A cricket ball is hit from a point A with velocity of $(p\mathbf{i} + q\mathbf{j})$ m s^{-1}, at an angle α above the horizontal. The unit vectors i and j are respectively horizontal and vertically upwards. The point A is 0.9 m vertically above the point O, which is on horizontal ground.

The ball takes 3 seconds to travel from A to B, where B is on the ground and OB = 57.6 m, as shown in the diagram above. By modelling the motion of the cricket ball as that of a particle moving freely under gravity,

(a) find the value of p,

(2)

(b) show that $q = 14.4$,

(3)

(c) find the initial speed of the cricket ball,

(2)

(d) find the exact value of $\tan \alpha$.

(1)

(e) Find the length of time for which the cricket ball is at least 4 m above the ground.

(6)

(f) State an additional physical factor which may be taken into account in a refinement of the above model to make it more realistic.

(1)
(Total 15 marks)

9. A particle P of mass 0.5 kg is moving under the action of a single force **F** newtons. At time t seconds,

$$\mathbf{F} = (6t - 5)\,\mathbf{i} + (t^2 - 2t)\,\mathbf{j}.$$

The velocity of P at time t seconds is **v** m s–1. When $t = 0$, $\mathbf{v} = \mathbf{i} - 4\mathbf{j}$.

(a) Find **v** at time t seconds.

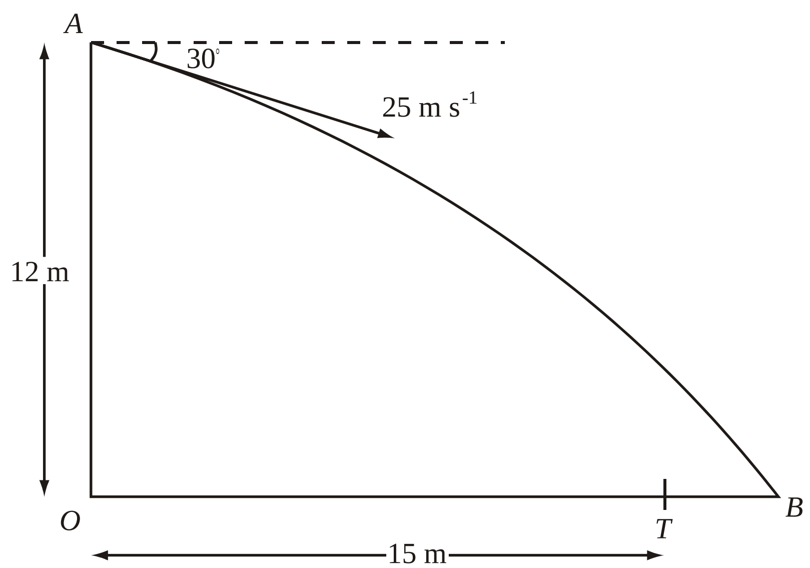
(6)

When $t = 3$, the particle P receives an impulse $(-5\mathbf{i} + 12\mathbf{j})$ N s.

(b) Find the speed of P immediately after it receives the impulse.

(6)
(Total 12 marks)

10.



A ball is thrown from a point A at a target, which is on horizontal ground. The point A is 12 m above the point O on the ground. The ball is thrown from A with speed 25 m s^{-1} at an angle of 30° below the horizontal. The ball is modelled as a particle and the target as a point T. The distance OT is 15 m. The ball misses the target and hits the ground at the point B, where OTB is a straight line, as shown in the diagram above. Find

(a) the time taken by the ball to travel from A to B,

(5)

(b) the distance TB.

(4)

The point X is on the path of the ball vertically above T.

(c) Find the speed of the ball at X.

(5)
(Total 14 marks)

11. At time t seconds ($t \geq 0$), a particle P has position vector **p** metres, with respect to a fixed origin O, where

$$\mathbf{p} = (3t^2 - 6t + 4)\mathbf{i} + (3t^3 - 4t)\mathbf{j}.$$

Find

(a) the velocity of P at time t seconds,

(2)

(b) the value of t when P is moving parallel to the vector **i**.

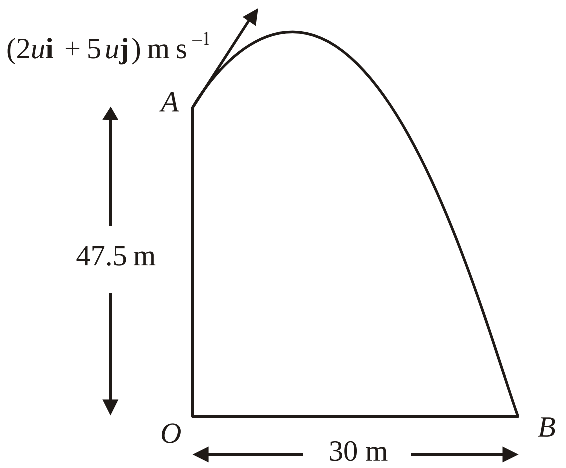
(3)

When $t = 1$, the particle P receives an impulse of $(2\mathbf{i} - 6\mathbf{j})$ N s. Given that the mass of P is 0.5 kg,

(c) find the velocity of P immediately after the impulse.

(4)
(Total 9 marks)

12.



[*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertical.*]

A particle P is projected from the point A which has position vector $47.5\mathbf{j}$ metres with respect to a fixed origin O. The velocity of projection of P is $(2u\mathbf{i} + 5u\mathbf{j})$ m s^{-1}. The particle moves freely under gravity passing through the point B with position vector $30\mathbf{i}$ metres, as shown in the diagram above.

(a) Show that the time taken for P to move from A to B is 5 s.

(6)

(b) Find the value of u.

(2)

(c) Find the speed of P at B.

(5)
(Total 13 marks)

13. A particle P of mass 0.5 kg moves under the action of a single force $\mathbf{F}$ newtons. At time t seconds, the velocity $\mathbf{v}$ m s^{-1} of P is given by

$$\mathbf{v} = 3t^2\mathbf{i} + (1 - 4t)\mathbf{j}.$$

Find

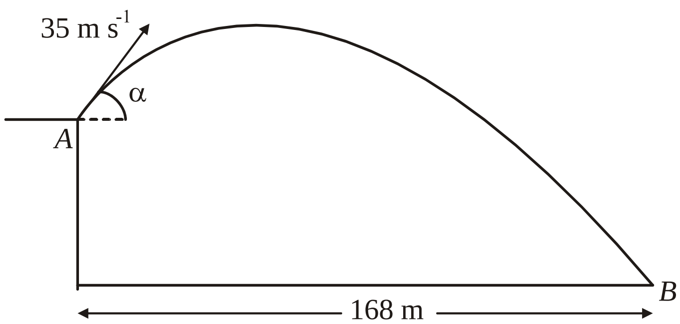
(a) the acceleration of P at time t seconds,

(2)

(b) the magnitude of $\mathbf{F}$ when $t = 2$.

(4)
(Total 6 marks)

14.



A golf ball P is projected with speed 35 m s^{-1} from a point A on a cliff above horizontal ground. The angle of projection is α to the horizontal, where $\tan \alpha = \frac{4}{3}$. The ball moves freely under gravity and hits the ground at the point B, as shown in the diagram above.

(a) Find the greatest height of P above the level of A.

(3)

The horizontal distance from A to B is 168 m.

(b) Find the height of A above the ground.

(6)

By considering energy, or otherwise,

(c) find the speed of P as it hits the ground at B.

(3)
(Total 12 marks)

15. A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where v is given by

$$v = \begin{cases} 8t - \frac{3}{2}t^2, & 0 \leq t \leq 4, \\ 16 - 2t, & t > 4. \end{cases}$$

When $t = 0$, P is at the origin O.

Find

(a) the greatest speed of P in the interval $0 \leq t \leq 4$,

(4)

(b) the distance of P from O when $t = 4$,

(3)

(c) the time at which P is instantaneously at rest for $t > 4$,

(1)

(d) the total distance travelled by P in the first 10 s of its motion.

(8)
(Total 16 marks)

16. A particle P of mass 0.5 kg is moving under the action of a single force **F** newtons. At time t seconds, $\mathbf{F} = (1.5t^2 - 3)\mathbf{i} + 2t\mathbf{j}$. When $t = 2$, the velocity of P is $(-4\mathbf{i} + 5\mathbf{j})$ m s^{-1}.

(a) Find the acceleration of P at time t seconds.

(2)

(b) Show that, when $t = 3$, the velocity of P is $(9\mathbf{i} + 15\mathbf{j})$ m s^{-1}.

(5)

When $t = 3$, the particle P receives an impulse **Q** N s. Immediately after the impulse the velocity of P is $(-3\mathbf{i} + 20\mathbf{j})$ m s^{-1}. Find

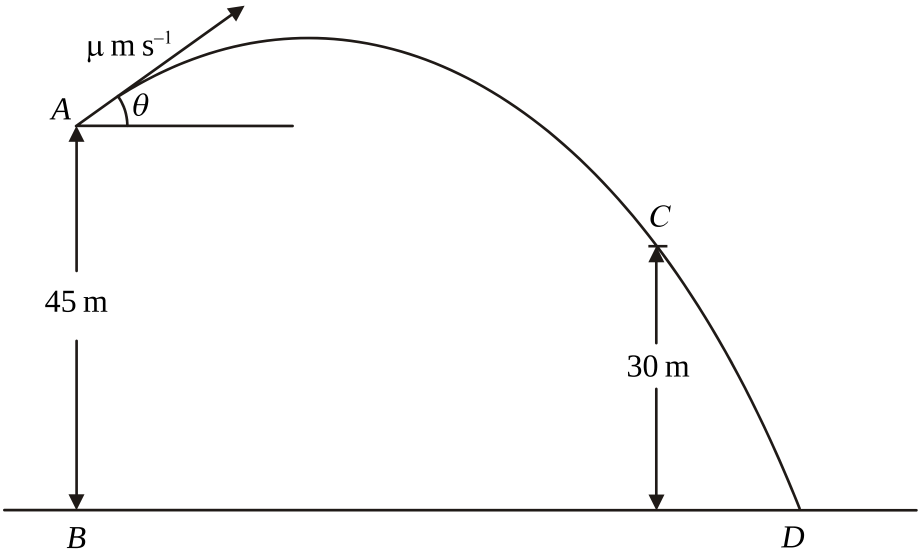
(c) the magnitude of **Q**,

(3)

(d) the angle between **Q** and **i**.

(3)
(Total 13 marks)

17.



A particle P is projected from a point A with speed u m s^{-1} at an angle of elevation θ, where $\cos\theta = \frac{4}{5}$. The point B, on horizontal ground, is vertically below A and AB = 45 m.
After projection, P moves freely under gravity passing through a point C, 30 m above the ground, before striking the ground at the point D, as shown in Figure 3.

Given that P passes through C with speed 24.5 m s^{-1},

(a) using conservation of energy, or otherwise, show that u = 17.5,

(4)

(b) find the size of the angle which the velocity of P makes with the horizontal as P passes through C,

(3)

(c) find the distance BD.

(7)
(Total 14 marks)

18. A particle P moves on the x-axis. At time t seconds, its acceleration is $(5 - 2t)$m s^{-2}, measured in the direction of x increasing. When t = 0, its velocity is 6 m s^{-1} measured in the direction of x increasing. Find the time when P is instantaneously at rest in the subsequent motion.

(Total 6 marks)

19. A cricket ball of mass 0.5 kg is struck by a bat. Immediately before being struck, the velocity of the ball is $(-30\mathbf{i})$ m s^{-1}. Immediately after being struck, the velocity of the ball is $(16\mathbf{i} + 20\mathbf{j})$ m s^{-1}.

(a) Find the magnitude of the impulse exerted on the ball by the bat.

(4)

In the subsequent motion, the position vector of the ball is $\mathbf{r}$ metres at time t seconds. In a model of the situation, it is assumed that $\mathbf{r} = [16t\mathbf{i} + (20t - 5t^2)\mathbf{j}]$. Using this model,

(b) find the speed of the ball when t = 3. **(4) (Total 8 marks)**

20. A vertical cliff is 73.5 m high. Two stones A and B are projected simultaneously. Stone A is projected horizontally from the top of the cliff with speed 28 m s^{-1}. Stone B is projected from the bottom of the cliff with speed 35 m s^{-1} at an angle α above the horizontal. The stones move freely under gravity in the same vertical plane and collide in mid-air. By considering the horizontal motion of each stone,

(a) prove that $\cos\alpha = \frac{4}{5}$.

(4)

(b) Find the time which elapses between the instant when the stones are projected and the instant when they collide.

(4)
(Total 8 marks)

21. A particle P of mass 0.4 kg is moving so that its position vector $\mathbf{r}$ metres at time t seconds is given by

$$\mathbf{r} = (t^2 + 4t)\mathbf{i} + (3t - t^3)\mathbf{j}.$$

(a) Calculate the speed of P when $t = 3$.

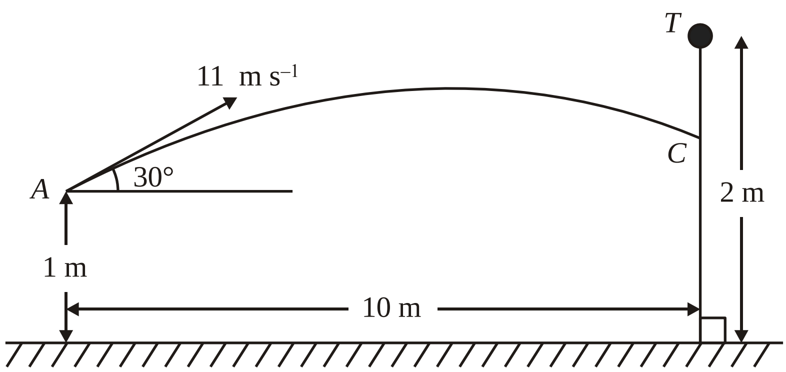
(5)

When $t = 3$, the particle P is given an impulse $(8\mathbf{i} - 12\mathbf{j})$ N s.

(b) Find the velocity of P immediately after the impulse.

(3)
(Total 8 marks)

22.



The object of a game is to throw a ball B from a point A to hit a target T which is placed at the top of a vertical pole, as shown in the figure above. The point A is 1 m above horizontal ground and the height of the pole is 2 m. The pole is at a horizontal distance of 10 m from A. The ball B is projected from A with a speed of 11 m s^{-1} at an angle of elevation of 30°. The ball hits the pole at the point C. The ball B and the target T are modelled as particles.

(a) Calculate, to 2 decimal places, the time taken for B to move from A to C.

(3)

(b) Show that C is approximately 0.63 m below T.

(4)

The ball is thrown again from A. The speed of projection of B is increased to V m s^{-1}, the angle of elevation remaining 30°. This time B hits T.

(c) Calculate the value of V.

(6)

(d) Explain why, in practice, a range of values of V would result in B hitting the target.

(1)
(Total 14 marks)

23. A particle P moves in a horizontal plane. At time t seconds, the position vector of P is $\mathbf{r}$ metres relative to a fixed origin O, and $\mathbf{r}$ is given by

$$\mathbf{r} = (18t - 4t^3)\mathbf{i} + ct^2\mathbf{j},$$

where c is a positive constant. When $t = 1.5$, the speed of P is 15 m s^{-1}. Find

(a) the value of c,

(6)

(b) the acceleration of P when $t = 1.5$.

(3)
(Total 9 marks)

24. A darts player throws darts at a dart board which hangs vertically. The motion of a dart is modelled as that of a particle moving freely under gravity. The darts move in a vertical plane which is perpendicular to the plane of the dart board. A dart is thrown horizontally with speed 12.6 m s^{-1}. It hits the board at a point which is 10 cm below the level from which it was thrown.

(a) Find the horizontal distance from the point where the dart was thrown to the dart board.

(4)

The darts player moves his position. He now throws a dart from a point which is at a horizontal distance of 2.5 m from the board. He throws the dart at an angle of elevation α to the horizontal, where $\tan \alpha = \frac{7}{24}$. This dart hits the board at a point which is at the same level as the point from which it was thrown.

(b) Find the speed with which the dart is thrown.

(6)
(Total 10 marks)

25. A particle P of mass 0.4 kg is moving under the action of a single force $\mathbf{F}$ newtons. At time t seconds, the velocity of P, $\mathbf{v}$ m s^{-1}, is given by

$$\mathbf{v} = (6t + 4)\mathbf{i} + (t^2 + 3t)\mathbf{j}.$$

When $t = 0$, P is at the point with position vector $(-3\mathbf{i} + 4\mathbf{j})$ m. When $t = 4$, P is at the point S.

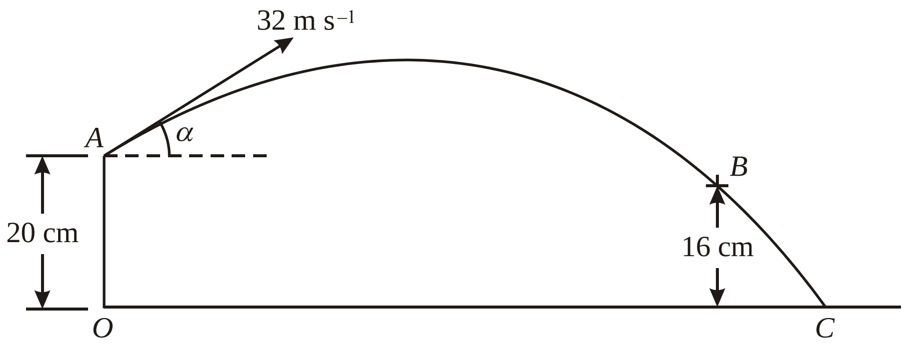
(a) Calculate the magnitude of $\mathbf{F}$ when $t = 4$.

(4)

(b) Calculate the distance OS.

(5)
(Total 9 marks)

26.



A particle P is projected from a point A with speed 32 m s^{-1} at an angle of elevation α, where sin α = $\frac{3}{5}$. The point O is on horizontal ground, with O vertically below A and OA = 20 m. The particle P moves freely under gravity and passes through a point B, which is 16 m above the ground, before reaching the ground at the point C, as shown in the diagram.

Calculate

(a) the time of the flight from A to C,

(5)

(b) the distance OC,

(3)

(c) the speed of P at B,

(4)

(d) the angle that the velocity of P at B makes with the horizontal.

(3)
(Total 15 marks)

27. At time t seconds, the velocity of a particle P is $[(4t - 7)\mathbf{i} - 5\mathbf{j}]$ m s^{-1}. When $t = 0$, P is at the point with position vector $(3\mathbf{i} + 5\mathbf{j})$ m relative to a fixed origin O.

(a) Find an expression for the position vector of P after t seconds, giving your answer in the form $(a\mathbf{i} + b\mathbf{j})$ m.

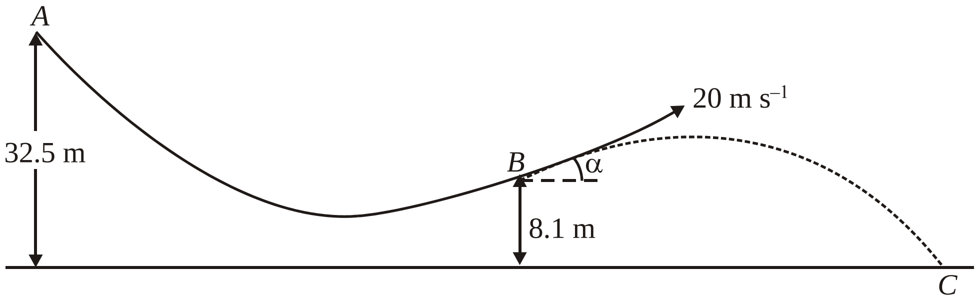
(4)

A second particle Q moves with constant velocity $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1}. When $t = 0$, the position vector of Q is $(-7\mathbf{i})$ m.

(b) Prove that P and Q collide.

(6)
(Total 10 marks)

28.



In a ski-jump competition, a skier of mass 80 kg moves from rest at a point A on a ski-slope. The skier's path is an arc AB. The starting point A of the slope is 32.5 m above horizontal ground. The end B of the slope is 8.1 m above the ground. When the skier reaches B, she is travelling at 20 m s^{-1}, and moving upwards at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$, as shown in the diagram above. The distance along the slope from A to B is 60 m. The resistance to motion while she is on the slope is modelled as a force of constant magnitude R newtons. By using the work-energy principle,

(a) find the value of R.

(5)

On reaching B, the skier then moves through the air and reaches the ground at the point C. The motion of the skier in moving from B to C is modelled as that of a particle moving freely under gravity.

(b) Find the time for the skier to move from B to C.

(5)

(c) Find the horizontal distance from B to C.

(2)

(d) Find the speed of the skier immediately before she reaches C.

(5)
(Total 17 marks)

29. A particle P of mass 0.75 kg is moving under the action of a single force $\mathbf{F}$ newtons. At time t seconds, the velocity $\mathbf{v}$ m s^{-1} of P is given by

$$\mathbf{v} = (t^2 + 2)\,\mathbf{i} - 6t\mathbf{j}.$$

(a) Find the magnitude of $\mathbf{F}$ when $t = 4$.

(5)

When $t = 5$, the particle P receives an impulse of magnitude $9\sqrt{2}$ Ns in the direction of the vector $\mathbf{i} - \mathbf{j}$.

(b) Find the velocity of P immediately after the impulse.

(4)
(Total 9 marks)

30. A particle P is projected with velocity $(2u\mathbf{i} + 3u\mathbf{j})$ m s^{-1} from a point O on a horizontal plane, where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical unit vectors respectively. The particle P strikes the plane at the point A which is 735 m from O.

(a) Show that $u = 24.5$.

(6)

(b) Find the time of flight from O to A.

(2)

The particle P passes through a point B with speed 65 m s^{-1}.

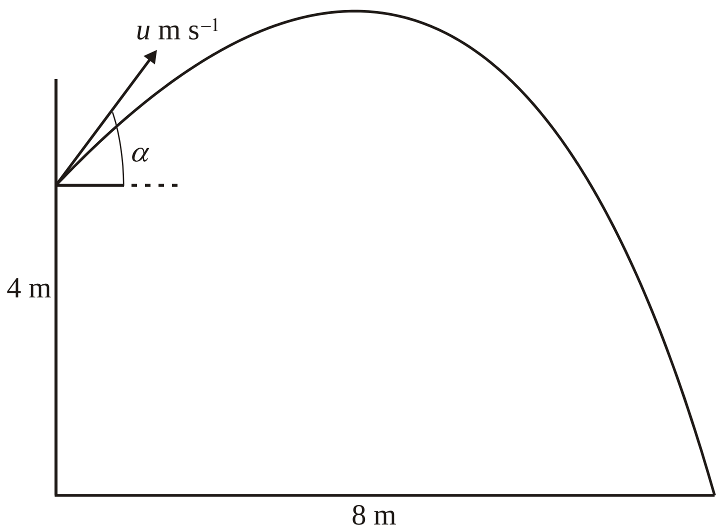
(c) Find the height of B above the horizontal plane.

(4)
(Total 12 marks)

31. A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where $v = 6t - 2t^2$. When $t = 0$, P is at the origin O. Find the distance of P from O when P comes to instantaneous rest after leaving O.

(Total 5 marks)

32.



A ball is thrown from a point 4 m above horizontal ground. The ball is projected at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$. The ball hits the ground at a point which is a horizontal distance 8 m from its point of projection, as shown in the diagram above. The initial speed of the ball is u m s^{-1} and the time of flight is T seconds.

(a) Prove that $uT = 10$.

(2)

(b) Find the value of u.

(5)

As the ball hits the ground, its direction of motion makes an angle ϕ with the horizontal. Find $\tan\phi$. **(5) (Total 12 marks)**

33. A particle P moves on the x-axis. The acceleration of P at time t seconds is $(4t - 8)$ m s^{-2}, measured in the direction of x increasing. The velocity of P at time t seconds is v m s^{-1}. Given that $v = 6$ when $t = 0$, find

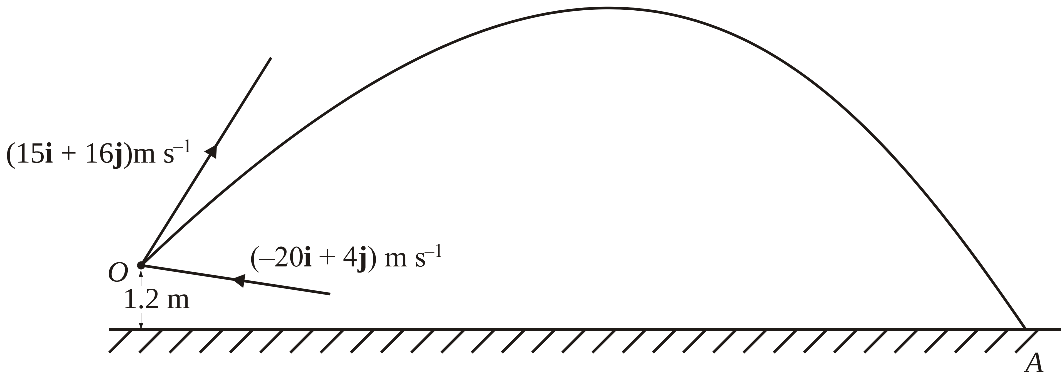
(a) v in terms of t,

(4)

(b) the distance between the two points where P is instantaneously at rest.

(7)
(Total 11 marks)

34.



A ball B of mass 0.4 kg is struck by a bat at a point O which is 1.2 m above horizontal ground. The unit vectors **i** and **j** are respectively horizontal and vertical. Immediately before being struck, B has velocity $(-20\mathbf{i} + 4\mathbf{j})$ m s^{-1}. Immediately after being struck it has velocity $(15\mathbf{i} + 16\mathbf{j})$ m s^{-1}.

After B has been struck, it moves freely under gravity and strikes the ground at the point A, as shown in the diagram above. The ball is modelled as a particle.

(a) Calculate the magnitude of the impulse exerted by the bat on B.

(4)

(b) By using the principle of conservation of energy, or otherwise, find the speed of B when it reaches A.

(6)

(c) Calculate the angle which the velocity of B makes with the ground when B reaches A.

(4)

(d) State two additional physical factors which could be taken into account in a refinement of the model of the situation which would make it more realistic.

(2)
(Total 16 marks)

35. At time t seconds the acceleration, $\mathbf{a}$ m s^{-2}, of a particle P relative to a fixed origin O, is given by $\mathbf{a} = 2\mathbf{i} + 6t\mathbf{j}$. Initially the velocity of P is $(2\mathbf{i} - 4\mathbf{j})$ m s^{-1}.

(a) Find the velocity of P at time t seconds.

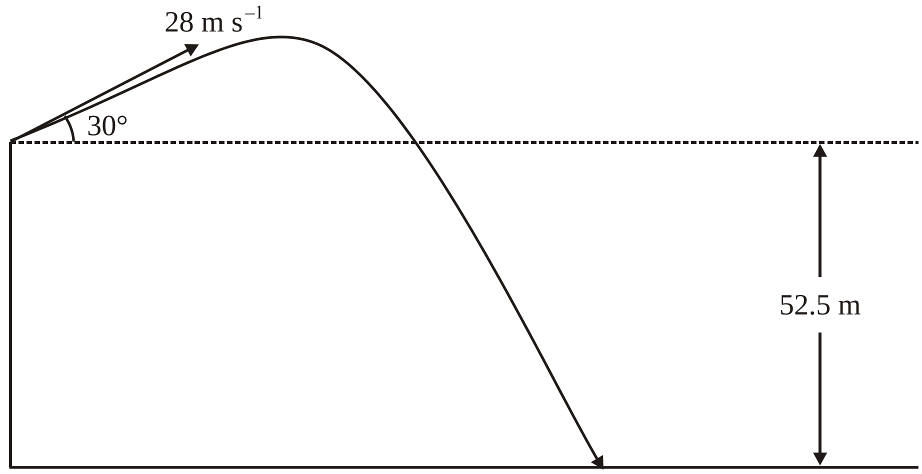
(3)

At time $t = 2$ seconds the particle P is given an impulse $(3\mathbf{i} - 1.5\mathbf{j})$ Ns. Given that the particle P has mass 0.5 kg,

(b) find the speed of P immediately after the impulse has been applied.

(6)
(Total 9 marks)

36.



A shot is projected upwards from the top of a cliff with a velocity of 28 m s^{-1} at an angle of 30° above the horizontal. It strikes the ground 52.5 m vertically below the level of the point of projection, as shown in the diagram above. The motion of the shot is modelled as that of a particle moving freely under gravity.

Find, to 3 significant figures,

(a) the horizontal distance from the point of projection at which the shot strikes the ground,

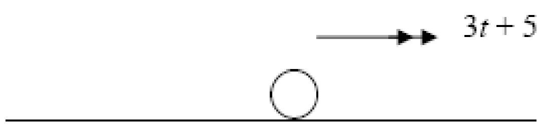
(8)

(b) the speed of the shot as it strikes the ground.

(5)
(Total 13 marks)

MARK SCHEMES

1.



$$\frac{dv}{dt} = 3t + 5$$

$v = \int (3t+5)\,dt$	M1 *
$v = \frac{3}{2}t^2 + 5t \quad (+c)$	A1
$t = 0 \quad v = 2 \quad \Rightarrow \quad c = 2$	B1
$v = \frac{3}{2}t^2 + 5t + 2$	

$t = T \qquad 6 = \frac{3}{2}T^2 + 5T + 2$ DM1 *

$12 = 3T^2 + 10T + 4$

$3T^2 + 10T - 8 = 0$

$(3T - 2)(T + 4) = 0$ M1

$T = \frac{2}{3} \quad (T = -4)$

$\therefore T = \frac{2}{3}$ (or 0.67) A1

[6]

2. (a) Vertical motion: $v^2 = u^2 + 2as$ M1

$(40 \sin \theta^2) = 2 \times g \times 12$ A1

$(\sin \theta)^2 = \frac{2 \times g \times 12}{40^2}$

$\theta = 22.54 = 22.5°$ (accept 23) A13

(b) Vert motion $P \to R$: $s = ut + \frac{1}{2}at^2$

$-36 = 40 \sin \theta t - \frac{g}{2}t^2$ M1

$\frac{g}{2}t^2 - 40 \sin \theta t - 36 = 0$ A1 A1

$$t = \frac{40 \sin 22.54 + \sqrt{(40 \sin 22.54)^2 + 4 \times 4.9 \times 36}}{9.8}$$

$t = 4.694...$ A1

Horizontal P to R: $s = 40 \cos \theta t$ M1

= 173 m (or 170 m) A16

(c) Using Energy:

$\frac{1}{2}mv^2 - \frac{1}{2}m \times 40^2 = m \times g \times 36$ M1 A1

$v^2 = 2\left(9.8 \times 36 + \frac{1}{2} \times 40^2\right)$

$v = 48.0\ldots..$

$v = 48$ m s^{-1} (accept 48.0) A13

[12]

3. $\frac{dv}{dt} = 6t - 4$ M1 A1

$6t - 4 = 0 \Rightarrow t = \frac{2}{3}$ M1 A1

$s = \int 3t^2 - 4t + 3 \, dt = t^3 - 2t^2 + 3t \ (+ c)$ M1 A1

$t = \frac{2}{3} \Rightarrow s = -\frac{16}{27} + 2$ so distance is $\frac{38}{27}$ m M1 A1

[8]

4. (a) $x = ut$ B1

$y = cut - 4.9t^2$ M1 A1

eliminating t and simplifying to give $y = cx - \frac{4.9x^2}{u^2}$ ** DM1 A15

(b) (i) $0 = cx - \frac{4.9x^2}{u^2}$ M1

$0 = x\left(c - \frac{4.9x}{u^2}\right) \Rightarrow R = \frac{u^2 c}{4.9} = 10c$ M1 A1

(ii) When $x = 5c$, $y = H$ M1

$= 5c^2 - \frac{(5c)^2}{10} = 2.5c^2$ M1 A16

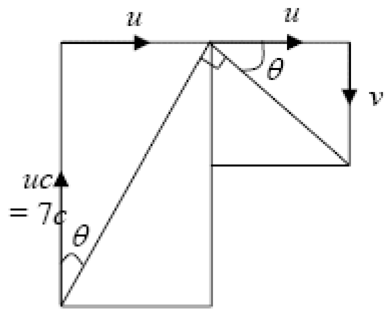
(c) $\frac{dy}{dx} = c - \frac{9.8x}{u^2} = c - \frac{x}{5}$ M1 A1

When $x = 0$, $\frac{dy}{dx} = c$ B1

So, $c - \frac{x}{5} = \frac{-1}{c}$ DM1 A1

$x = 5\left(c + \frac{1}{c}\right)$ A16

<u>Alternative</u>



$\tan\theta = \dfrac{u}{cu} = \dfrac{1}{c} = \dfrac{v}{u}$ B1

$\Rightarrow v = \dfrac{u}{c} = \dfrac{7}{c}$ M1 A1

$v = u + at$; $-\dfrac{7}{c} = 7c - 9.8t$ M1

$t = \dfrac{7}{9.8}\left(c + \dfrac{1}{c}\right)$ A1

$x = ut = 7t$; $x = 5\left(c + \dfrac{1}{c}\right)$ A1

[17]

5. (a) $\dfrac{dv}{dt} = 8 - 2t$ M1

$8 - 2t = 0$ M1

Max $v = 8 \times 4 - 4^2 = 16$ (ms^{-1}) M1A14

(b) $\int 8t - t^2 dt = 4t^2 - \dfrac{1}{3}t^3 (+C)$ M1A1

($t = 0$, displacement = 0 $\Rightarrow$ c=0)

$4T^2 - \dfrac{1}{3}T^3 = 0$ DM1

$T^2\left(4 - \dfrac{T}{3}\right) = 0 \Rightarrow T = 0{,}12$ DM1

$T = 12$ (seconds) A15

[9]

6. (a) $\rightarrow x = u \cos \alpha t = 10$ M1A1

$\uparrow \quad y = u\sin\alpha t - \frac{1}{2}gt^2 = 2$ M1A1

$\Rightarrow t = \frac{10}{u\cos\alpha}$

$2 = u\sin\alpha \times \frac{10}{u\cos\alpha} - \frac{g}{2} \times \frac{100}{u^2\cos^2\alpha}$ M1

$= 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha}$ (given answer) A16

(b) $2 = 10 \times 1 - \frac{100g \times 2}{2u^2 \times 1}$ M1A1

$u^2 = \frac{100g}{8}, u = \sqrt{\frac{100g}{8}} = 11.1\left(\mathrm{m\,s^{-1}}\right)$ A1

$\frac{1}{2}mu^2 = m \times 9.8 \times 2 + \frac{1}{2}mv^2$ M1A1

$v = 9.1ms^{-1}$ A16

[12]

7. (a) $v = 10t - 2t^2$, s = $\int vdt$ M1

$= 5t^2 - \frac{2t^3}{3}(+C)$ A1

$t = 6 \Rightarrow s = 180 - 144 = \underline{36}$ (m) A13

(b) $\underline{s} = \int vdt = \underline{\frac{-432t^{-1}}{-1}}(+k) = \underline{\frac{432}{t}}(+k)$ $\underline{\text{B1}}$

t = 6, s = "36" $\Rightarrow 36 = \frac{432}{6} + K$ M1 *

$\Rightarrow K = -36$ A1

At $t = 10$, $s = \frac{432}{10} - 36 = \underline{7.2}$(m) d * M1

$\underline{\text{A1}}$5

[8]

8. (a) Horizontal distance: $57.6 = p \times 3$ M1

$p = 19.2$ A12

(b) Use $s = ut + \frac{1}{2}at^2$ for vertical displacement. M1

$-0.9 = q \times 3 - \frac{1}{2}g \times 3^2$ A1

$-0.9 = 3q - \frac{9g}{2} = 3q - 44.1$

$q = \frac{43.2}{3} = 14.4$ * **AG** * A1 cso3

(c) initial speed $\sqrt{p^2 + 14.4^2}$ (with their p) M1

$= \sqrt{576} = \underline{24}\,(\text{m s}^{-1})$ A1 cao2

(d) $\tan\alpha = \frac{14.4}{p} \left(= \frac{3}{4}\right)$ (with their p) B11

(e) When the ball is 4 m above ground:

$3.1 = ut + \frac{1}{2}at^2$ used M1

3.1 = 14.4t $-\frac{1}{2}gt^2$ o.e $(4.9t^2 - 14.4t + 3.1 = 0)$ A1

$\Rightarrow t = \frac{14.4 \pm \sqrt{(14.4)^2 - 4(4.9)(3.1)}}{2(4.9)}$ seen or implied M1

$t = \frac{14.4 \pm \sqrt{146.6}}{9.8} = 0.023389..$ .or 2.70488... awrt 0.23 and 2.7 A1

duration = 2.70488... – 0.23389... M1
= 2.47 or 2.5 (seconds) A16

or

M1A1M1 as above

$t = \frac{14.4 \pm \sqrt{146.6}}{9.8}$ A1

Duration $2 \times \frac{\sqrt{146.6}}{9.8}$ o.e. M1

$= 2.47$ or 2.5 (seconds) A1 6

(f) Eg. : Variable 'g', Air resistance, Speed of wind, Swing of ball, The ball is not a particle. B1 1

[15]

9. (a) N2L $(6t - 5)\mathbf{i} + (t^2 - 2t)\mathbf{j} = 0.5\mathbf{a}$ M1

$\mathbf{a} = (12t - 10)\mathbf{i} + (2t^2 - 4t)\mathbf{j}$ A1

$\mathbf{v} = (6t^2 - 10t)\mathbf{i} + \left(\frac{2}{3}t^3 - 2t^2\right)\mathbf{j}\ (+\mathbf{C})$ ft their **a** M1A1ft+A1ft

$\mathbf{v} = (6t^2 - 10t + 1)\mathbf{i} + \left(\frac{2}{3}t^3 - 2t^2 - 4\right)\mathbf{j}$ A1 6

(b) When $t = 3$, $\mathbf{v}_3 = 25\mathbf{i} - 4\mathbf{j}$ M1

$-5\mathbf{i} + 12\mathbf{j} = 0.5(\mathbf{v} - (25\mathbf{i} - 4\mathbf{j}))$ ft their $\mathbf{v}_3$ M1A1ft

$\mathbf{v} = 15\mathbf{i} + 20\mathbf{j}$ A1

$|\mathbf{v}| = \sqrt{(15^2 + 20^2)} = 25$ (m s^{-1}) cso M1A1 6

[12]

10. (a) $(\downarrow)\ u_y = 25 \sin 30° \ (= 12.5)$ B1

$12 = 12.5t + 4.9t^2$ – 1 each error M1A2(1,0)

Leading to $t = 0.743, 0.74$ A1 5

(b) $(\rightarrow)\ u_x = 25 \cos 30° \left(= \frac{25\sqrt{3}}{2} \approx 21.65\right)$ B1

$OB = 25 \cos 30° \times t \ (\approx 16.094\ 58)$ ft their (a) M1A1ft

$TB \approx 1.1\ (m)$ awrt 1.09 A1 4

(c) $(\rightarrow)\ 15 = u_x \times t \Rightarrow t = \frac{15}{u_x} \left(= \frac{2\sqrt{3}}{5} \approx 0.693 \text{ or } 0.69\right)$ M1A1

either $(\downarrow)\ v_y = 12.5 + 9.8t\ (\approx 19.2896)$ M1

$V^2 = u_x^{\,2} + v_y^{\,2}\ (\approx 840.840)$

$V \approx 29$ (m s^{-1}), 29.0 M1A1 5

or $(\downarrow)\ s_y = 12.5t + 4.9t^2\ (\approx 11.0)$ M1

$\frac{1}{2}m \times 25^2 + mg \times s_y = \frac{1}{2}mv^2$

$V \approx 29$ (m s^{-1}), 29.0 M1A1

[14]

11. (a) $\dot{\mathbf{p}} = (6t - 6)\mathbf{i} + (9t^2 - 4)\mathbf{j}$ (m s^{-1}) M1A1 2

	(b)	$9t^2 - 4 = 0$		M1
		$t = \frac{2}{3}$		DM1A1
	(c)	$t = 1 \Rightarrow \dot{\mathbf{p}} = 5\mathbf{j}$	ft their $\dot{\mathbf{p}}$	B1ft
		(+/–) $2\mathbf{i} - 6\mathbf{j} = 0.5(\mathbf{v} - 5\mathbf{j})$		M1
		$\mathbf{v} = 4\mathbf{i} - 7\mathbf{j}$ (m s^{-1})		M1 A1 4
				[9]
12.	(a)	$\rightarrow 30 = 2ut$		B1
		$\uparrow -47.5 = 5ut - 4.9t^2$		M1 A1
		$-47.5 = 75 - 4.9\,t^2$	eliminating u or t	DM1
		$t^2 = \frac{75+47.5}{4.9}$ (= 25)		DM1
		$t = 5$ *	cso	A1 6
	(b)	$30 = 2ut \Rightarrow 30 = 10u \Rightarrow u = 3$		M1A1 2
	(c)	$\uparrow\ \dot{y} = 5u - 9.8t = -34$	M1 requires both	M1A1
		$\rightarrow\ \dot{x} = 2u = 6$	$\dot{x}$ and $\dot{y}$	A1
		$v^2 = 6^2 + (-34)^2$		DM1
		$v \approx 34.5$ (m s^{-1})	accept 35	A1 5
		Alternative		
		$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = m \times g \times 47.5$ with $v_A^2 = 6^2 + 15^2 = 261$		M1A1(2,1,0)
		v_B^2 =261 + 2 × 9.8 × 47.5 (= 1192)		DM1
		$v_B \approx 34.5$ (m s^{-1})	accept 35	A1 5
		BEWARE : Watch out for incorrect use of $v^2 = u^2 + 2as$		
				[13]
13.	(a)	$\mathbf{a} = \mathrm{d}\mathbf{v}/\mathrm{d}t = 6t\mathbf{i} - 4\mathbf{j}$		M1A12
	(b)	Using $\mathbf{F} = ½\mathbf{a}$, sub $t = 2$, finding modulus		M1, M1, M1
		e.g. at $t = 2$, $\mathbf{a} = 12\mathbf{i} - 4\mathbf{j}$		
		$\mathbf{F} = 6\mathbf{i} - 2\mathbf{j}$		
		$\lvert\mathbf{F}\rvert = \sqrt{(6^2 + 2^2)} \approx$ <u>6.32 N</u>		A1(CSO)4

M1 Clear attempt to differentiate. Condone **i** or **j** missing.

A1 both terms correct (column vectors are OK)

The 3 method marks can be tackled in any order, but for consistency on epen grid please enter as:

M1 **F** = m**a** (their **a**, (correct **a** or following from (a)), not **v**, **F** = $\frac{1}{2}$ **a**).

Condone **a** not a vector for this mark.

M1 subst $t = 2$ into candidate's vector **F** or **a** (**a** correct or following from (a), not **v**)

M1 Modulus of candidate's **F** or **a** (not **v**)

A1 CSO All correct (beware fortuitous answers e.g. from 6t**i** + 4**j**))

Accept 6.3, awrt 6.32, any exact equivalent e.g. 2 □ 10, □ 40, $\frac{\sqrt{160}}{2}$

[6]

14. (a) $0 = (35 \sin \alpha)^2 - 2gh$ M1A1

$h =$ <u>40 m</u> A13

(b) $x = 168 \Rightarrow 168 = 35 \cos \alpha . t$ ($\Rightarrow t = 8$s) M1A1

At $t = 8$, $y = 35\sin\alpha \times t - \frac{1}{2}gt^2$ (= 28.8 – ½.g.8^2 = –89.6 m) M1A1

Hence height of A = <u>89.6 m</u> or 90 m DM1A16

(c) $\frac{1}{2}mv^2 = \frac{1}{2}.m.35^2 + mg.89.6$ M1A1

$\Rightarrow v =$ <u>54.6 or 55 m s^{-1}</u> A13

M1 Use of $v^2 = u^2 + 2as$, or possibly a 2 stage method using $v = u + at$ and $s = ut + \frac{1}{2}at^2$

A1 Correct expression. Alternatives need a complete method leading to an equation in h only.

A1 40(m) No more than 2sf due to use of g.

M1 Use of $x = u\cos \alpha . t$ to find t.

A1 $168 = 35 \times$ their $\cos \alpha \times t$

M1 Use of $s = ut + \frac{1}{2}at^2$ to find vertical distance for their t. (AB or top to B)

A1 $y = 35 \sin \alpha \times \ t - \frac{1}{2} g t^2$ (u, t consistent)

DM1 This mark dependent of the previous 2 M marks. Complete method for *AB*.
Eliminate *t* and solve for *s*.

A1 cso.

(NB some candidates will make heavy weather of this, working from *A* to max height (40m) and then down again to *B* (129.6m))

OR: Using $y = x \tan \alpha - \dfrac{gx^2 \sec^2 \alpha}{2u^2}$

M1 formula used (condone sign error)
A1 x, u substituted correctly
M1 α terms substituted correctly.
A1 fully correct formula
M1, A1 as above

M1 Conservation of energy: change in KE = change in GPE.
All terms present. One side correct (follow their *h*).
(will probably work *A* to *B*, but could work top to *B*).

A1 Correct expression (follow their *h*)

A1 54.6 or 55 (m/s)

OR: M1 horizontal and vertical components found and combined using Pythagoras
$v_x = 21$
$v_y = 28 - 9.8 \times 8$ (–50.4)
A1 v_x and v_y expressions correct (as above). Follow their *h*, *t*.
A1 54.6 or 55

NB Penalty for inappropriate rounding after use of g only applies once per question.

[12]

15. (a) $0 \le t \le 4$: $a = 8 - 3t$ M1
$a = 0 \Rightarrow t = 8/3$ s DM1
$\rightarrow v = 8.\frac{8}{3} - \frac{3}{2}.\left(\frac{8}{3}\right)^2 = \frac{32}{3}$ (m/s) DM1A14

second M1 dependent on the first, and third dependent on the second.

(b) $s = 4t^2 - r^3/2$ M1
$t = 4$: $s = 64 - 64/2 = \underline{32 \text{ m}}$ M1A13

(c) $t > 4$: $v = 0 \Rightarrow t = \underline{8 \text{ s}}$ B11

(d) *Either*

$t > 4$ $s = 16t - t^2$ (+C) M1

$t = 4$, $s = 32 \to C = -16 \Rightarrow s = 16t - t^2 - 16$ M1A1

$t = 10 \to s = 44$ m M1A1

But direction changed, so: $t = 8$, $s = 48$ M1

Hence total dist travelled = 48 + 4 = <u>52 m</u> DM1A18

Or (probably accompanied by a sketch?)

$t = 4$ $v = 8$, $t = 8$ $v = 0$, so area under line = $\frac{1}{2}\times(8-4)\times 8$ M1A1A1

$t = 8$ $v = 0$, $t = 10$ $v = -4$, so area above line = $\frac{1}{2}\times(10-8)\times 4$ M1A1A1

$\therefore$ total distance = 32 (from B) + 16 + 4 = <u>52 m</u> M1A18

Or M1, A1 for $t > 4$ $\frac{dv}{dt} = -2$, = constant

$t = 4$, $v = 8$; $t = 8$, $v = 0$; $t = 10$, $v = -4$

M1, A1 $s = \frac{u+v}{2}t = \frac{32}{2}t, = 16$ working for $t = 4$ to $t = 8$

M1, A1 $s = \frac{u+v}{2}t = \frac{-4}{2}t, = 4$ working for $t = 8$ to $t = 10$

M1, A1 total = 32 + 14 + 4, = 52

M1 Differentiate to obtain acceleration

DM1 set acceleration = 0 and solve for t

DM1 use their t to find the value of v

A1 32/3, 10.7 or better

OR using trial and improvement:

M1 Iterative method that goes beyond integer values

M1 Establish maximum occurs for t in an interval no bigger than $2.5 < t < 3.5$

M1 Establish maximum occurs for t in an interval no bigger than $2.6 < t < 2.8$

OR M1 Find/state the coordinates of both points where the curve cuts the x axis.

DM1 Find the midpoint of these two values

M1A1 as above.

OR M1 Convincing attempt to complete the square:

DM1 substantially correct $8t - \frac{3t^2}{2} = -\frac{3}{2}(t - \frac{8}{3})^2 + \frac{3}{2} \times \frac{64}{9}$

DM1 Max value = constant term

A1 CSO

M1 Integrate the correct expression

DM1 Substitute $t = 4$ to find distance ($s = 0$ when $t = 0$ – condone omission/ignoring of constant of integration)

A1 32(m) only

B1 $t = 8$ (s) only

M1 Integrate $16 - 2t$

M1 Use $t = 4$, s = their value from (b) to find the value of the constant of integration.
or 32 + integral with a lower limit of 4 (in which case you probably see these two marks occurring with the next two. First A1 will be for 4 correctly substituted.)

A1 $s = 16t - t^2 - 16$ or equivalent

M1 substitute $t = 10$

A1 44

M1 Substitute $t = 8$ (their value from (c))

DM1 Calculate total distance (M mark dependent on the previous M mark.)

A1 52 (m)

OR the candidate who recognizes $v = 16 - 2t$ as a straight line can divide the shape into two triangles:

M1 distance for $t = 4$ to t = candidate's 8 = ½ × change in time × change in speed.

A1 8 – 4

A1 8 – 0

M1 distance for t = their 8 to $t = 10$ = ½ × change in time × change in speed.

A1 10 – 8

A1 0 – (–4)

M1 Total distance = their (b) plus the two triangles (=32 + 16 + 4)

A1 52(m)

NB: This order on epen grid (the A's and M's will not match up.)

[16]

16.	(a)	N2L $(1.5t^2 - 3)\mathbf{i} + 2t\mathbf{j} = 0.5\mathbf{a}$		Ml
		$\mathbf{a} = (3t^2 - 6)\mathbf{i} + 4t\mathbf{j}$		Al2
	(b)	$\mathbf{v} = (t^3 - 6t)\mathbf{i} + 2t^2\mathbf{j}$ (+c)		Ml Al
		$t = 2$ $-4\mathbf{i} + 5\mathbf{j} = -4\mathbf{i} + 8\mathbf{j} + \mathbf{c}$ (c = –3j)		Ml
		$\mathbf{v} = (t^3 - 6t)\mathbf{i} - (2t^2 - 3)\mathbf{j}$ (m s^{-1})		Al
		$t = 3$ $\mathbf{v} = 9\mathbf{i} + 15\mathbf{j}$ (m s^{-1}) *	cso	Al5
	(c)	$\mathbf{Q} = 0.5(-3\mathbf{i} + 20\mathbf{j} - (9\mathbf{i} + 15\mathrm{j}))$ $(=0.5(-12\mathbf{i} + 5\mathbf{j}))$		Ml
		$\lvert\mathbf{Q}\rvert = 0.5\sqrt{(5^2 + 12^2)} = 6.5$		Ml Al3
	(d)	acute angle is $\arctan\frac{5}{12} \approx 23°$		M1A1
		or required angle is $\arctan\frac{-5}{12}$		
		or acute angle is $\arccos\frac{12}{13} \approx 23°$		
		or required angle is $\arccos\frac{-12}{13}$		
		required angle is 157°	awrt 157°, 203°	Al3
				[13]
17.	(a)	Energy $\frac{1}{2}m(24.5^2 - u^2) = mg \times 15$		M1A1=A1
		$u^2 = 24.5^2 - 30g = 306.25$		
		$u = \sqrt{306.25} = 17.5$ *	cso	Al4
		Alternative		
		$\rightarrow u_x = u\cos\theta = 0.8u$, $\uparrow u_y = u\sin\theta = 0.6u$		
		$v_y^2 = 0.36u^2 + 2 \times 9.8 \times 15 = 0.36u^2 + 294$		
		$24.5^2 = u_x^2 + v_y^2 = 0.64u^2, +0.36u^2 + 294$		Ml A1,A1
		$u^2 = 306.25 \Rightarrow u = 17.5$ *	cso	Al4
	(b)	$\rightarrow u_x = u\cos\theta = 17.5 \times 0.8 = 14$		Bl
		$\psi = \arccos\frac{14}{24.5} \approx 55°$	accept 55.2°	M1A13
			(0.96 rads. or 0.963 rads)	
		Alternative		
		$\rightarrow u_x = u\cos\theta = 17.5 \times 0.8 = 14$		Bl

	$\uparrow\ v_y^2 = u^2 \sin^2\theta + 2 \times 9.8 \times 15 = 404.25$		
	$\psi = \arctan \dfrac{\sqrt{404.25}}{14} \approx 55°$	accept 55.2°	M1A13
(c)	$\uparrow\ u_y = u \sin\theta = 17.5 \times 0.6 = 10.5$		Bl
	$s = ut + \dfrac{1}{2}at^2 \Rightarrow -45 = 10.5t - 4.9t^2$		Ml Al
	leading to $t = 4.3$. awrt $t = 4.3$ or $t = 4\dfrac{2}{7}$		Al
	$\rightarrow BD = 14 \times 4\dfrac{2}{7}$ $(14 \times t)$	ft their t	Ml Alft
	= 60 (m) only		Al7
	Alternative		
	Use of $y = x\tan\theta - \dfrac{g\sec^2\vartheta}{2u^2}x^2$		M1
	$-45 = \dfrac{3}{4}x,\ -\dfrac{g}{2\times 17.5^2} \times \dfrac{25}{16}x^2$		B1,A1
	$x^2 - 30x - 1800 = 0$ o.e.		Al
	Factors or quadratic formula		Ml Alft
	BD = 60 (m)		Al
			[14]

18.	$a = 5 - 2t \Rightarrow v = 5t - t^2, + 6$	M1 A1 A1
	$v = 0 \Rightarrow t^2 - 5t - 6 = 0$	indep M1
	$(t - 6)(t + 1) = 0$	dep M1
	$t = 6$ s	A1
		[6]

19.	(a)	$I = \pm 0.5(16\mathbf{i} + 20\mathbf{j} - (-30\mathbf{i}))$	M1
		$= \pm(23\mathbf{i} + 10\text{j})$	Indep M1
		magn = $\sqrt{(23^2 + 10^2)} \approx \underline{25.1 \text{ Ns}}$	Indep M1 A1
	(b)	$v = 16\mathbf{i} + (20 - 10\text{t})\mathbf{j}$	M1
		$t = 3 \Rightarrow \mathbf{v} = 16\mathbf{i} - 10\mathbf{j}$	indep M1
		$v = \sqrt{(16^2 + 10^2)} \quad \approx 18.9 \text{ ms}^{-1}$	indep M1 A1
			[8]

20. (a) $x_A = 28t$ $\quad x_B = 35 \cos \alpha\, t$ — B1 B1

Meet $\Rightarrow 28t = 35 \cos \alpha\, t \Rightarrow \cos \alpha = 28 / 35 = 4 / 5$ * — M1 A1 4

(b) $y_A = 73.5 - ½\, gt^2$ $\quad y_B = 2\text{l}t - ½\, gt^2$ — B1 B1

Meet $\Rightarrow 73.5 = 21t \Rightarrow t = \underline{3.5 \text{ s}}$ — M1 A1 4

[8]

21. (a) $\dot{\mathbf{r}} = (2t + 4)\mathbf{i} + (3 - 3t^2)\mathbf{j}$ — M1 A1

$\dot{\mathbf{r}}_3 = 10\mathbf{i} - 24\mathbf{j}$ substituting $t = 3$ — M1

$|\dot{\mathbf{r}}_3| = \sqrt{(10^2 + 24^2)} = 26$ (m s^{-1}) — M1 A1 5

(b) $0.4\,(\mathbf{v} - (10\mathbf{i} - 0\ 24\mathbf{j}) = 8\mathbf{i} - 12\mathbf{j}$ ft their $\dot{\mathbf{r}}_3$ — M1 A1ft

$\mathbf{v} = 30\,\mathbf{I} - 54\mathbf{j}$ (m s^{-1}) — A1 3

[8]

22. (a) $u_x = 11 \cos 30°$ — B1

$\rightarrow 11 \cos 30° \times t = 10 \quad \Rightarrow \quad t = 1.05$ (s) cao — M1 A1 3

(b) $s = \underline{11 \cos 30°} \times t - 4.9t^2 \approx 0.37$ — B1 M1 A1

$(2 - 1) - 0.37 = 0.63$ (m) — A1 4

(c) $V \cos 30° \times t = 10$ $\quad \left(t = \dfrac{10}{V \cos 30°}\right)$ — M1 A1

$s = V \sin 30° \times \dfrac{10}{V \cos 30°} - \dfrac{4.9 \times 100}{V^2 \cos^2 \theta} = 1$ — M1 A1

$V^2 = 136.86$ — M1

$V \approx 12$ accept 11.7 — A1 6

(d) *B* and/or *T* are not particles — B1 1

(They have extension giving a range of answers)

[14]

23. (a) $\mathbf{v} = (18 - 12t^2)\mathbf{I} + 2ct\mathbf{j}$ — M1 A1 A1

$t = \dfrac{3}{2}$: $\mathbf{v} = -9\mathbf{i} + 3c\mathbf{j}$ — M1

$|\mathbf{v}| = 15 \Rightarrow 9^2 + (3c)^2 = 15^2$ — M1

$\Rightarrow (3c)^2 = 144 \Rightarrow c = 4$ — A1 6

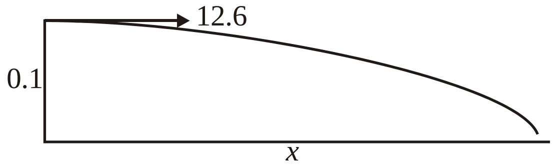
(b) $\mathbf{a} = -24t\mathbf{i} + 8\mathbf{j}$ M1

$t = \frac{3}{2}$ $\quad \mathbf{a} = -36\mathbf{i} + 8\mathbf{j}$ M1

A1ft 3

[9]

24. (a)



$\rightarrow 12.6t = x$ B1

$\downarrow 0.1 = 4.9t^2$ B1

$\Rightarrow 0.1 = 4.9 \times \frac{x^2}{12.6}$ M1

$\Rightarrow x = 1.8$ m A1 4

(b)

$\rightarrow u \cos \alpha.t = 2.5$ M1 A1

$\uparrow u \sin\alpha.t = \frac{1}{2}gt^2$ M1 A1

$u.\frac{24}{25}\,t = 2.5$

$u.\frac{7}{25} = 4.9.\frac{2.5.25}{24u}$

$u^2 = \frac{4.9 \times 2.5 \times 25^2}{7 \times 24}$

$\Rightarrow u \approx 6.75$ or 6.8 m s^{-1} M1 A1 6

[10]

25. (a) $\ddot{\mathbf{r}} = 6\mathbf{i} + (2t + 3)\mathbf{j}$ B1

$\mathbf{F} = 0.4(6\mathbf{i} + 11\mathbf{j})$ M1

0.4× something obtained by differentiation, with t = 4

$|\mathbf{F}| = \sqrt{(2.4^2 + 4.4^2)}$ M1

modulus of a vector

≈ 5.0 A1 4

accept more accurate answers

(b) $\mathbf{r} = (3t^2 + 4t)\mathbf{i} + (\frac{1}{3}t^3 + \frac{3}{2}t^2)\mathbf{j}(+C)$ M1

Using boundary values, $\mathbf{r} = (3t^2 + 4t - 3)\mathbf{i} + (\frac{1}{3}t^3 + \frac{3}{2}t^2 + 4)\mathbf{j}$ A1

$t = 4$, $\quad \mathbf{r} = 61\mathbf{i} + 49\frac{1}{3}\mathbf{j}$ A1

$OS = \sqrt{(61^2 + 49\frac{1}{3}^2)} \approx 78$ (m) M1 A1 5

accept more accurate answers

[9]

26. (a) $\uparrow \quad u_y = 32 \times \frac{3}{5}$ (= 19.2) B1

$-20 = 19.2t - 4.9t^2$ M1 A2 (1, 0)

–1 each error

$t \approx 4.8$ or 4.77 (s) A1 5

(b) $\rightarrow \quad u_x = 32 \times \frac{4}{5}$ (= 25.6) B1

$d = 25.6 \times 4.77\ldots$ M1

≈ 120 or 122 (m) A1 3

(c) $\uparrow \quad v_y^2 = 19.2^2 + 2 \times 9.8 \times 4$ [$v_y^2 = 447.04$, $v_y \approx 21.14$] M1

$V^2 = 447.04 + 25.6^2$ M1 A1

$V = 33$ or 33.2 (ms^{-1}) A1 4

(d) $\tan\theta = \dfrac{21.14}{25.6} \quad \left(\text{or } \cos\theta = \dfrac{25.6}{33.2}, \ldots\right)$ M1 A1ft

ft their components
or resultant

$\theta \approx 40°$ or 39.6° A1 3

[15]

Alternative for (c)

$\frac{1}{2}m(V^2 - 32^2) = mg \times 4$ M1 A1

$V^2 = 1102.4$ M1

$V = 33$ or 33.2 (ms^{-1}) A1 4

There is a maximum penalty of one mark per question for not rounding to appropriate accuracy.

27.	(a)	$\mathbf{p} = (2t^2 - 7t)\mathbf{I} - 5t\mathbf{j}, + 3\mathbf{i} + 5\mathbf{j}$	M1, M1
		$= (2t^2 - 7t + 3)\mathbf{I} + (5 - 5t)\mathbf{j}$	A1+A1 4
	(b)	$\mathbf{q} = (2\mathbf{i} - 3\mathbf{j})t - 7\mathbf{i}$	M1 A1
		$\mathbf{j} : 5 - 5t = -3t \Rightarrow t = 2.5$ equating and solving	M1 A1
		At $t = 2.5$ $\mathbf{i} : p_x = 2 \times 2.5^2 - 7 \times 2.5 + 3 = -2$	
		$q_x = 2 \times 2.5 - 7 = -2$	both M1
		$p_x = q_x \Rightarrow$ collision	cso A1 6
			[10]

Alternative in (b)

$\mathbf{i} : 2t^2 - 7t + 3 = 2t - 7 \Rightarrow 2t^2 - 9t + 10 = 0$	
$t = 2, 2.5$ equating and solving	M1 A1
At $t = 2.5$ $\mathbf{j} : p_y = 5 - 5 \times 2.5 = -7.5$	
$q_y = -3 \times 2.5 = -7.5$	both M1
$p_y = q_y \Rightarrow$ collision	cso A1

In alternative, ignore any working associated with $t = 2$

28.	(a)	Work-Energy $R \times 60 = 80 \times 9.8 \times 24.4 - \frac{1}{2} \times 80 \times 20^2$	M1 A2(1, 0)
		$(= 19129.6 - 16000 = 3129.6)$	
		$R = 52$ (N)	M1 A1 5
		accept 52.2	
	(b)	$-8.1 = 20 \sin \alpha \times t - \frac{1}{2} gt^2$	M1 A2(1, 0)
		$4.9t^2 - 12t - 8.1 = 0$	M1
		$t = 3$ (s)	A1 5
	(c)	$20\cos\alpha \times 3 = 16 \times 3 = 48$ (m)	M1 A1ft
		ft their t	
	(d)	Energy $\frac{1}{2} mv^2 - \frac{1}{2} m \times 20^2 = m \times 9.8 \times 8.1$	M1 A2(1, 0)
		$v = \sqrt{(558.56)} \approx 24$ (ms^{-1})	M1 A1 5

accept 23.6

[17]

Alternative to (d)

$\uparrow v_y = 12 - 3g = -17.4$	M1 A1
$\rightarrow v_x = 16$	A1
$v = \sqrt{(17.4^2 + 16^2)} \approx 24$ (ms^{-1}) *accept 23.6*	M1 A1 5

29.	(a)	$\mathbf{a} = 2t\mathbf{i} - 6\mathbf{j}$	M1
		$t = 4$: $\mathbf{a} = 8\mathbf{i} - 6\mathbf{j}$	**dep**. M1
		$\lvert\mathbf{F}\rvert = 0.75\sqrt{(8^2 + 6^2)} = 7.5$N	M1 M1 A1
	(b)	$\mathbf{I} = 9\mathbf{i} - 9\mathbf{j}$	B1
		$9\mathbf{i} - 9\mathbf{j} = \frac{3}{4}(\mathbf{v} - (27\mathbf{i} - 30\mathbf{j}))$	M1 A1 f.t.
		$\mathbf{v} = 39\mathbf{i} - 42\mathbf{j}$ m s^{-1}	A1 4
			[9]
30.	(a)	$2ut = 735$	M1 A1
		$0 = 3ut - \frac{1}{2}gt^2$	M1 A1
		eliminating t	**dep.** M1
		$u = 24.5$ *	A1 6
	(b)	$t = \frac{735}{49} = 15$	M1 A1 2
	(c)	Initially: $v^2 = (2u)^2 + (3u)^2$ (7803.25)	M1
		$\frac{1}{2}mv^2 - \frac{1}{2}m\,65^2 = mgh$	M1 A1
		$h = 180$ m (183 m)	A1 4
	OR	$v_y^2 = 65^2 - (2u)^2$ (1824)	M1
		$v_y^2 = (3u)^2 - 2gh$	M1 A1
		$h = 180$ m (183 m)	A1 (4)
			[12]
31.		$x = \int 6t - 2t^2\,dt$	M1
		$= 3t^2 - \frac{2}{3}t^3$ (+ C)	A1

$v = 0 \Rightarrow 6t - 2t^2 = 0 \Rightarrow t = 3$ (or 0) M1

$t = 3$: $x = (3 \times 9) - (\frac{2}{3} \times 27) = 9$ m M1 A1

[5]

32. (a) $(\rightarrow)$: $u \cos \alpha \times T = 8$

$u \times \frac{4}{5} \times T = 8$ M1

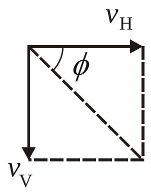
$uT = 10$ (*) A1 2

(b) $(\uparrow)$: $-4 = u \sin \alpha\, T - \frac{1}{2} gT^2$ M1 A1

$-4 = u \times \frac{3}{5}\left(\frac{10}{u}\right) - \frac{1}{2} \times 9.8\left(\frac{10}{u}\right)^2$ M1

$u = 7$ M1 A1 7

(c)



$v_H = u \cos \alpha = \frac{28}{5}$ B1 ft

$v_V^2 = (-u \sin \alpha)^2 + 2g \times 4$ M1

$\Rightarrow v_V = 9.8$ (= $\frac{49}{5}$) A1 ft

$\tan \phi = \dfrac{49/5}{28/5} = \dfrac{7}{4}$ M1 A1 cao

[12]

33. (a) $v = \int a\, dt = 2t^2 - 8t\ (+c)$ M1 A1

Using $v = 6$, $t = 0$; $v = 2t^2 - 8t + 6$ M1 A1 4

(b) $v = 0 \Rightarrow 2t^2 - 8t + 6 = 0, \Rightarrow t = 1,3$ M1 A1

$S = \int (2t^2 - 8t + 6)\mathrm{d}t = \left[\frac{2}{3}t^3 - 4t^2 + 6t\right]$ M1 A2, 1, 0

$= 0 - 2\frac{2}{3}$ M1

Distance is $(\pm)2\frac{2}{3}$ m A1 7

[11]

34. (a) $\mathbf{I} = 0.4(15\mathbf{i} + 16\mathbf{j} + 20\mathbf{i} - 4\mathbf{j})$ $(= 0.4(35\mathbf{i} + 12\mathbf{j}) = 14\mathbf{i} + 4.8\mathbf{j})$ M1

$|\mathbf{I}| = \sqrt{(14^2 + 4.8^2)}$ *or* $0.4\sqrt{(35^2 + 12^2)}$ M1 A1

M1 for any magnitude

= 14.8 (Ns) A1 4

(b) Initial K.E. = $\frac{1}{2} m(15^2 + 16^2)$ (= $240.5m$ = 96.2 J) M1

$\frac{1}{2}mv^2 = \frac{1}{2}m(15^2 + 16^2) = m \times 9.8 \times 1.2$ M1 A2, 1,0

–1 each incorrect term

$v^2 = 504.52$ M1

$v = 22$ (m s^{-1})

accept 22.5 A1 6

(c) $\arccos \dfrac{15}{22.5} = 48°$ M1 A1 A1 A1

accept 48.1 °

(d) Air resistance B1, B1 2

Wind (problem not 2 dimensional)

Rotation of ball (ball is not a particle)

any 2

[16]

Alt (b)

Resolve ↑ with 16 and 9.8 M1

(↑) $v_y^2 = 16^2 + 2 \times (-9.8) \times (-1.2)$ M1 A1

($v_y^2 = 279.52$, $v_y \approx 16.7$…..)

$v^2 = 15^2 + 279.52$ M1 A1

$v = 22$ (m s^{-1}) A1 6

accept 22.5

Alt (c)

$\arctan \dfrac{16.7}{15} = 48°$ M1 A1 A1 A1

[10]

35. (a) $\mathbf{v} = \int 2\mathbf{i} + 6t\mathbf{j}\,\mathrm{d}t = 2t\mathbf{i} + 3t^2\,\mathbf{j}\ (+\,\mathbf{c})$ M1 A1

c = 2i **– 4**j A13

$\mathbf{v} = (2t + 2)\mathbf{i} + (3t^2 - 4)\mathbf{j}$

(b) $t = 2$: $\mathbf{v} = 6\mathbf{i} + 8\mathbf{j}$ B1

$3\mathbf{i} - 1.5\mathbf{j} = 0.5(\mathbf{v} - (6\mathbf{i} + 8\mathbf{j}))$ M1 A1

$\Rightarrow \mathbf{v} = 12\mathbf{i} + 5\mathbf{j}$ A1

$\Rightarrow |\mathbf{v}| = \sqrt{(12^2 + 5^2)} = 13 \text{ m s}^{-1}$ M1 A16

[9]

36. (a) $(\uparrow)$: $-52.5 = 14t - \frac{1}{2} \times 9.8t^2$ M1 A2

$7t^2 - 20t - 75 = 0$

$(7t + 15)(t - 5) = 0$

$t = 5$ (or $t = -\frac{15}{7}$) M1 A1 A1

$(\rightarrow)$: $S = 28 \cos 30° \times 5$ M1

$= 70\sqrt{3} = 121$ m (3 s.f.) A18

(b) $v_{\text{horizontal}}$: $28 \cos 30° = 14\sqrt{3}$

v_{vertical}: $28 \sin 30° - 5g = -35$ M1 A1

$\therefore$ speed $= \sqrt{((14\sqrt{3})^2 + 35^2)} = \sqrt{1813} = 42.6 \text{ m s}^{-1}$ M1 A15

OR

KE gain = PE loss

$\frac{1}{2} m (v^2 - 28^2) = mg \times 52.5$ M1 A2

$\Rightarrow v = \sqrt{1813} = 42.6 \text{ m s}^{-1}$ M1 A15

[13]

EXAMINERS' REPORTS

1. This question proved very accessible and gave most candidates a confident start to the paper. There were very few incorrect answers, with the overwhelming majority integrating the given acceleration correctly. Any errors in the integration were mostly when $3t^2$ was not divided by 2. There was some confusion about the constant of integration in a few cases, often taken in error to be zero. Nearly all candidates set their velocity expressions equal to 6 and attempted to solve the resulting quadratic equation. There were some basic algebraic or arithmetical slips resulting in incorrect equations. A method was not always shown in the solution of a quadratic. This should be discouraged as credit can be given for correct working if it is seen.

There were a small number of candidates who tried to apply "*suvat*" to the motion, losing 5 out of the 6 marks available.

2. There were many confident solutions to this question, but over specification of answers following the use of $g = 9.8$ was a common problem, causing many candidates to throw away a mark.

In part (a) many students used correct methods for calculating the angle, although 22.54 rather than 22.5 was common. Very few went wrong here, though some took longer routes than necessary, failing to spot that they could use $v^2 = u^2 + 2as$ to obtain the angle in one step, and there were a few who attempted to use distances to find the angle.

In part (b) the quickest method here was to use a displacement of –36 in an equation to find time and then use this time in a horizontal equation to find displacement. Some took this in two stages: time to the highest point and then time to the bottom. It was pleasing to find very few resolution and both methods were used correctly in many solutions. The most common error was to consider only part of the flight and then use an incorrect time to find the horizontal displacement. The over-specified answer 173.4 rather than 173 was common.

In part (c) those candidates who used conservation of energy were usually successful but the most common method was to find horizontal and vertical components of velocity and hence find the speed using Pythagoras' Theorem. Unfortunately, those who used this method often found only the vertical component of the velocity and lost all the marks here.

3. The majority of candidates offered confident responses to this opening question. Most of them successfully integrated the given velocity to find out the displacement of the particle. When it came to finding out the time of minimum velocity, most candidates used calculus again to find acceleration and made it equal to zero but some preferred to complete the square or use the expression for the turning point of a parabola. A few candidates attempted to find the time when the velocity of the particle was zero, believing this to be the minimum. A common error was find the minimum velocity and substitute this, rather than the time, into the displacement equation.

4. (a) Many correct solutions were seen, with the majority of candidates clearly familiar with the method for deriving the equation of the parabolic path. However some candidates substituted into all the suvat equations and were clearly struggling to find a way forward.

(b) Those candidates who were not able to complete (a) started afresh at this point, and were often successful in earning marks in this part. Most candidates used the given formula to find R having identified this as the value of x when y = 0. Some candidates used the fact that maximum height occurs when $x = \frac{R}{2}$, but many preferred to work with the initial information and to use $v^2 = u^2 + 2as$ to find H.

(c) Only a few candidates made a concerted attempt at this part with a correct solution a rarity. Most attempts used a vector approach rather than calculus. Several candidates demonstrated an understanding of perpendicular vectors, and the partially correct velocity $\begin{pmatrix} cu \\ -u \end{pmatrix}$ in place of the correct answer $\begin{pmatrix} u \\ \frac{-u}{c} \end{pmatrix}$ was quite common. Diagrams were rarely seen, possibly accounting for the common incorrect answer $\begin{pmatrix} u \\ -cu \end{pmatrix}$. Confusion between distances and velocities often marred any work beyond this stage.

5. This question provided the opportunity for candidates to show that they could both differentiate the velocity function to find the acceleration and integrate it to find the displacement. In general both were done successfully, although as usual there were candidates who incorrectly attempted to solve the problem using constant acceleration formulae.

Although the majority of candidates used differentiation in part (a), there was also a large number who treated it by completing the square, and they were often successful in this approach. A number of candidates produced a table of discrete time values and corresponding speeds of the particle. Unfortunately they rarely scored full marks for their effort as the supporting statement about the symmetry of a quadratic function was usually missing. The most common error among candidates using differentiation was to stop when they had found the time and not go on to find the speed. In part (b) it would have been reassuring to have seen more candidates - even the successful ones - giving a more rigorous treatment of the constant of integration. Algebraic errors in solving the equation $4T^2 - \frac{1}{3}T^3 = 0$ were surprisingly common.

6. Part (a) was answered well by many candidates. The structure of the question, explicitly requiring the horizontal and vertical components as a starting point, led to many more successful answers than might otherwise have been the case. Most achieved the method marks successfully for both components but several had a sign error for the vertical component or some sin/cos confusion. As the answer was given, most candidates managed to recover sufficiently from initial errors to gain full marks. There were a few cases when a candidate had an incorrect expression but then wrote down the correct result! Some lost the final mark because another step was needed to show the given result.

In part (b) many candidates managed to use the given formula, allowing them to find u correctly, even if they had made an error in part (a). However, a significant number of candidates made errors in manipulating the given expression arriving at an incorrect value for u. A few candidates did not see that parts (a) and (b) were connected and started again with horizontal and vertical components. Others did not realise they could find u by substituting into the given equation. It was common for candidates to lose the last three marks. In finding u some thought they had already found the speed as it passed over the fence and proceeded no further.

A few candidates chose to use the more simple method of work-energy for the final part. These candidates usually scored full marks. The most common approach was to try to combine horizontal and vertical components of the final velocity. Some found only one component of the speed (usually the vertical one) and gave this as their answer. Others used u unresolved in either the vertical or horizontal direction and then combined to find v incorrectly. There were a significant number of responses making inappropriate use of $v^2 = u^2 + 2as$ to gain 9.13 incorrectly.

A few candidates scored no marks in part (b) because they made no attempt to find u.

7. A few candidates were clearly confused by velocity being defined in terms of two separate functions. Nevertheless, virtually all candidates knew they had to integrate the relevant expression for velocity in order to find the displacement and they did this correctly in part (a). As the constant was zero in this part of the question, candidates who had overlooked it were not penalised. There were occasional mistakes such as differentiating instead of integrating, and some candidates who tried to use the equations for constant acceleration.

In part (b), although most correctly integrated the expression, for those that went along the indefinite integral route, the constant of integration was often just assumed to be zero because the displacement was zero at the start. Several candidates even demonstrated that the constant of integration was zero, apparently having no problem with equating 432/0 to zero! These candidates clearly did not realise that the expression was not relevant at the start. Those who found the definite integral were generally more successful. Other errors in part (b) included using $t = 4$, using $t = 7$ as a lower limit for the second integral (apparently not recognising the continuous nature of time), or reaching the correct solution but then adding the answer from (a) a second time.

8. Many candidates scored well in this question, with parts (a) to (d) generally answered correctly. However, a few candidates were confused right at the start with the vector form of the initial velocity and tried to bring resolution into the problem and so failed to find p and q.

Some candidates were initially confused about the direction of p and q – it was common to see attempts at parts (a) and (b) relabelled when a candidate discovered their error. In (b) there was often insufficient evidence that the given answer had been reached correctly – essential steps in the working were omitted. Some candidates used long drawn out trigonometric methods to find tan α in part (d), often finding cosα and sin α before finally reaching $\frac{3}{4}$.

In part (e) most candidates used $s = ut + \frac{1}{2}at^2$ from the point of projection but there were a number of other possible methods which were also successful. It was evident that some candidates are relying on the use of calculators to solve quadratic equations. When the initial quadratic equation was incorrect, marks were often lost as a result of failing to show sufficient evidence of use of an appropriate method. It was common to see 4 used in place of 3.1 in the initial equation. Candidates using alternative approaches often got lost in the complexities of the logic of what they were trying to do.

The responses in part (f) showed that virtually all candidates could find (at least) one physical factor which could also be taken into account although a few mis-worded their answers to imply the opposite. For example, many suggested “air resistance”, but it was also common to see “no air resistance”.

9. This was generally well answered. The mechanical principles were well understood by nearly all candidates.

(a) Many fully correct answers were seen. The most frequent errors were due either to problems in dividing by 0.5, or because candidates integrated ***F*** rather than ***a***.
The use of $v = u + at$ or differentiation instead of integration were occasionally seen.
Another, less common, error was to substitute $t = 0$ first. Also the constant of integration was very occasionally written with ‘–1’ instead of ‘+1’ in the **i** term.
The great majority of candidates remembered the initial conditions although a few lost the final A mark for omission of **i** – 4**j**.

(b) In many cases correct use of the impulse equation resulting in a velocity of 15**i** + 20**j** was seen but then too many candidates did not go on to find the speed of the particle. Use of the initial velocity as **i** – 4**j** was a common error.
Less common, but costly, was to start by finding the magnitude of the impulse and attempt to use this.
Also, some candidates attempted an impulse equation but ignored the mass of the particle. Candidates with errors early in the question were often able to gain a mark at the end for finding the speed from their velocity.

10. This was a well answered question with almost all candidates working in appropriate vertical and horizontal directions. However, some candidates were confused by the fact that the ball was projected below the horizontal – many of these assuming that it was projected horizontally and then followed the path illustrated.

(a) Most candidates used $s = ut + ½at^2$ to produce a quadratic equation albeit often with a sign error. When marks were lost this was usually because of difficulties with positive and negative vertical terms. The alternative, longer method, of finding v then t was not infrequent.

(b) The method was clearly understood but the numbers involved were very sensitive to rounding errors. Too many candidates over-rounded their value for t and obtained the inaccurate value of 1.02.

(c) It was pleasing to see that most candidates adopted the correct approach of finding t then vy but some failed to then combine v_x and v_y to obtain the speed. Again, sign errors with the acceleration resulted in errors in v_y being less than the original vertical component, which one would have hoped might ring an alarm bell.
Few candidates used the energy method and those that did often used the incorrect vertical displacement.

11. The candidates did well in this question compared to similar questions on previous papers.

(a) This was usually well answered with most candidates confident in using the **i**, **j** notation in the differentiation. A very small minority of candidates chose to integrate. The differentiation was well done but there was an occasional misread of $3t^3$ as $3t^2$.

(b) The majority of candidates used the **j** component of their velocity to find the value of *t* but some used the **i** component in error. A very small number used the **j** component of ***p***. Candidates starting with the correct equation, $9t^2 - 4 = 0$, often made errors in their attempt to solve for *t*; common incorrect answers included 2/9, 4/3 and 3/2. Some candidates demonstrated little understanding of what the question was asking for.

(c) Impulse was well understood but there were still some candidates confused between the initial and the final velocity. There were also some elegant solutions to provide the velocity in terms of *t* and going no further. Here too some candidates lost the final mark due to algebraic or sign errors.

12. This question proved to be very challenging for many candidates. The best candidates worked through swiftly and efficiently scoring full marks in a relatively short solution. A number eliminated *t* rather than *u* in part (a), thus finding *u* first but this did not cause any great difficulty.

Unfortunately, several candidates were completely unprepared to deal with the initial velocity in vector form. Many of these went on to recombine the components and find a speed and angle of projection, then faithfully worked on with sin/cos and $\sqrt{29}$ without realising that it came back to 2 and 5! The candidates who made the least progress were those who tried to use the equations of motion with the velocity in complete vector form and the displacement and acceleration as scalars.

Although it created considerable extra work, and invariably went wrong, a small proportion of candidates tried to break the task down by working from the point of projection to the highest point and then from the highest point to *B*.
Several candidates who arrived at a correct equation in *u* and *t* then went on to apply the quadratic formula inappropriately to obtain an expression for *t* in terms of *u*. A number also got the wrong answer for *t* in part (a) but did not then pick up the given answer of $t = 5$; they persevered with their incorrect result and hence lost many more marks.

In part (c) we encountered all the usual errors, although it was good to find far fewer candidates making inappropriate use of $v^2 = u + 2as$. Most students understood the need to find both the horizontal and the vertical component of the velocity at *B*. Several candidates were not sufficiently clear about the direction of motion, and made errors due to confusion over signs. A minority of candidates used conservation of energy without being required to do so, and were usually successful. Too many candidates lost the final mark due to an inappropriate level of accuracy in their final answers.

13. There were many correct solutions to this question. Only a small minority of candidates failed to differentiate **v** to find **a** in part (a), and most candidates obtained the correct value for the magnitude of the force in part (b). Incorrect answers were usually due to arithmetic errors, or originated from the sign error $\mathbf{a} = 6t\mathbf{i} + 4\mathbf{j}$ in part (a). Other common differentiation errors gave $\mathbf{a} = 6t\mathbf{i} - 4t\mathbf{j}$ or $\mathbf{a} = 6t\mathbf{i} + (1 - 4)\mathbf{j}$

14. (a) There were several instances of a possible misread of $\tan\alpha = \frac{3}{4}$ in this question, although it was not always possible to tell whether the error was a misread or use of an incorrect expression $\tan\alpha = \frac{\cos\alpha}{\sin\alpha}$. Most candidates used the formula *as* $v^2 = u^2 + 2as$ to find the maximum height. This was often found correctly, but a common error was to forget to square their value for u. Some candidates made the task more difficult than necessary by adopting a method with two, or more, stages.

(b) Having found the time to travel a horizontal distance of 168m it is possible to find the vertical distance in one step, but many candidates elected either to find the time from the highest point to the ground, or to find the time from when the ball returns to the level of A until it reaches B. Candidates choosing one of these longer alternatives did not always match up their value for time taken with an appropriate value for the initial vertical speed. Having reached a value for vertical distance there was then some confusion about whether or not to add or subtract their answer from part (a).

(c) Whether using energy or an alternative approach, the most common error in this part of the question was to concentrate on the vertical speed and to ignore or omit the horizontal component. This resulted in many candidates scoring no marks here.

15. Completely correct solutions to this question were rare, with parts (b) and (c) proving to be a better source of marks than parts (a) or (d).

(a) There are several possible methods for finding the maximum speed in this interval. The expected method was to differentiate, find the value of t for which the acceleration is equal to zero, and use this to find the corresponding value of *v*. Candidates using this approach sometimes got as far as the value for *t* and then stopped as if they thought they had answered the question. As an alternative, candidates who recognised this as part of a parabola, either went on to complete the square (with considerable success despite the nature of the algebra involved), or found the average of the two times when the speed is zero to locate the time for maximum speed and hence the speed, or simply quoted formulae for the location of the turning point. Many candidates simply substituted integer values of *t* in to the formula for *v* and stated their largest answer. This alone was not sufficient. Although it is possible to arrive at the correct answer using trial and improvement, most candidates who embarked on this route failed to demonstrated that their answer was indeed a maximum – they usually offered a sequence of increasing values, but did not demonstrate that they had located the turning point in an interval of appropriate width.

(b) Many candidates answered this correctly – even those who did not differentiate in part (a) did choose to integrate here. There is a false method, assuming constant speed throughout the interval, which gives the answer 32 incorrectly by finding the speed when $t = 4$ and multiplying the result by 4 – many candidates used this without considering the possibility of variable speed and acceleration.

(c) This was usually answered correctly, but some candidates appeared to think that they were being asked to find out when $8t - \frac{3}{2}t^2 = 0$ or when $8t - \frac{3}{2}t^2 = 16 - 2t$.

(d) Those candidates who realised that the particle was now moving with uniform acceleration had the simple task of finding the area of two triangles, assuming that they appreciated the significance of $v < 0$ for $t > 8$. Alternatively they could use the equations for motion under uniform acceleration, with the same proviso. For the great majority of candidates, this was about integration and choosing appropriate limits. The integration itself was usually correct, but common errors included ignoring the lower limit of the interval, or not using $s = 32$ when $t = 4$, and stopping after using the upper limit of $t = 10$. Some candidates thought that the limits for t should be from $t = 0$ to $t = 6$, and a large number thought that they should be starting from $t = 5$. Very few of the candidates who found the integral went on to consider what happened between $t = 8$ and $t = 10$.

16. Candidates were confident in their solutions to this question with many scoring all but the last mark. It was pleasing to see much competent use of calculus and vector mechanics. Only a few failed to include a constant in working towards the given solution for (b). It was disappointing to find candidates with an incorrect answer in (b) attempting to fudge the given answer rather than look for the error in their working. Attempts to find the impulse $\boldsymbol{Q}$ were usually correct, but several candidates with a correct $\boldsymbol{Q}$ did not go on to find the magnitude. Errors in $\boldsymbol{Q}$ were often due to arithmetic errors in the subtraction, but some candidates did omit the mass. In (d), many candidates failed to score the final mark. There appeared to be little appreciation of the actual angle required, even from some candidates who had drawn a correct diagram.

17. This proved to be the most challenging question. Although it looked as though some candidates had run out of time to complete this question satisfactorily, others had sufficient time to make multiple attempts. Some candidates failed to appreciate the nature of velocity direction and tried to use equations for constant acceleration without taking direction into consideration. The most successful way to answer the first part was to use the method suggested – conservation of energy. In (b) some failed to appreciate that the velocity at C was at an angle (not necessarily equal to � and that the horizontal velocity at A was constant throughout. Few candidates answered this concisely. The greatest difficulties were encountered in (c). A significant number of candidates failed to appreciate the use of displacement in constant acceleration equations and broke up the problem, quite unnecessarily, into sections where the particle was travelling up and then travelling down, making the solution of the problem much more difficult than it needed to be. There are several possible approaches to this question, some of them producing pleasingly concise solutions.

18. This proved to be an easy starter and was generally very well answered with the vast majority of candidates scoring 5 or 6 marks. There were some errors in integration, with some candidates failing to include a constant and some unable to solve the required quadratic equation. Of those that could, some failed to reject the negative solution. A few candidates assumed constant acceleration and scored little.

19. There were few errors on this question Only a few candidates failed to use vectors to calculate the impulse in part (a) but some forgot to calculate the magnitude of their vector. The second part was mostly completely correct. Candidates need to read the question carefully and ensure that they answer the question asked. Those who forgot to calculate magnitudes lost four marks in this question!

20. The first part was reasonably well done although some candidates failed to appreciate that it was a 4 mark proof and therefore required a full explanation – those that simply equated the horizontal velocity components only scored two of the four marks. Part (b) was more discriminating and many didn't appreciate that they needed two vertical distance-time equations and of those that did, a significant number were unable to combine them correctly.

21. Most candidates realised that they needed to differentiate, although there was the odd integration. Having found a velocity vector some then failed to find the modulus to obtain the speed. In part (b), a few used $\mathbf{I} = m(\mathbf{u} - \mathbf{v})$ and a very small number worked with scalars, but generally candidates reached the correct vector solution. Some candidates thought they then had to find the magnitude of their vector – they were not penalised for this.

22. Parts (a) and (b) were usually correct although some candidates made things more difficult for themselves by splitting the motion into "up and down" etc but they still usually got there in the end. The third part was more challenging but most candidates were able to pick up some marks although the algebraic manipulation required to solve their equations for V defeated some of the weaker candidates. Part (d) proved to be a good discriminator, with most candidates simply trotting out the "usual" air resistance without really thinking about the situation.

23. There were many fully correct solutions but there was some sloppy use of vector notation and also evidence of poor algebra. Most knew that they needed to differentiate but some lost the **i**'s and **j**'s. Others, in (a), were unable to deal with the magnitude correctly and simply put
$15 = -9 + c$. In the second part many unnecessarily went on to find the magnitude of the acceleration.

24. Part (a) was generally well done although a significant minority used 10 cm instead of 0.1 m and were not put off by the fact that this meant that the darts player was throwing darts at a board which was 18 m away! The second part proved to be more challenging. Most tried to work from 1st principles and set up two equations and eliminate t but a few successfully used quoted formulae for the range or the equation of the path. Candidates are reminded that in this case if they misquote formulae they will lose all of the marks.

25. This was an excellent source of marks for the great majority of candidates, although there were a few who differentiated where they should have integrated and vice versa. In part (a), a few candidates stopped when they had found **F**. In part (b), the use of the inappropriate formula $\mathbf{r} = \mathbf{r}_0 + \mathbf{V}t$ was less frequently seen than in some recent examinations. Some candidates, having found the constant of integration, added it again, or subtracted it. This usually arose from not recognising the convention that *O* referred to the origin.

26. As has been noted in previous examinations, those who attempted part (a) in one go, obtaining an equation equivalent to $4.9t^2 - 19.2t - 20 = 0$ were usually successful and, although there were some slips in sign, these were not common. Those who broke the time up into sections, for example, from *A* to the maximum height and from the maximum height to *C*, produced completely correct solutions less frequently. More stages in a calculation give more scope for error. Such methods are also, not infrequently, given in an incomplete form. Part (b) was well done and an incorrect answer in part (a) lost only one mark here. The most popular method in part (c) was to consider the components of the velocity at *B*. The alternative method using conservation of energy was rarely seen. It was not uncommon for candidates to find the vertical component at *B* and stop. This often did seem to be an error in understanding what was asked for in this part of the question, as such candidates often interpreted their answer correctly and completed part (d) successfully. An inefficient method of solution to part (c), not infrequently seen, was to find the time of flight from *A* to *B* and then to find the vertical component of the velocity. This calculation is awkward and carries a greater risk of sign errors than using $v^2 = u^2 + 2as$. Part (d) was well done although, as noted above, errors due to premature approximation were often seen.

27. Most recognised that integration was needed in part (a), although a few used an inappropriate formula such as $\mathbf{r} = \mathbf{r}_0 + \mathbf{V}t$. Almost all candidates knew how to incorporate the initial position but errors in manipulation were seen and an error in bracketing frequently led to the incorrect $\left(2t^2 - 7t + 3\right)\mathbf{i} - \left(5 + 5t\right)\mathbf{j}$. This lost the last mark in (a) and from this result it was impossible to complete part (b) correctly. However nearly all candidates were able to demonstrate the method needed in (b) and the question was a substantial source of marks for the great majority of candidates.

28. Part (a) of this question proved to be the most testing and the work-energy principle was not understood by many candidates. It was not unusual to see kinetic energy omitted entirely and, when it was included, it often had the wrong sign. Some candidates did not know that work was force multiplied by distance and force multiplied by velocity was not infrequently seen. Those who attempted part (b) in one go, obtaining an equation equivalent to $4.9t^2 - 12t - 8.1 = 0$ were usually successful, although there were, as expected, some slips in sign. Those who broke the time up into sections, for example, from *B* to the maximum height and from the maximum height to *C*, produced completely correct solutions much less frequently although it is not clear if this is because of the intrinsic difficulty of the method or if it was because this method was favoured by weaker candidates. Any answer in part (b) was followed through in part (c) and the majority of the candidates gained the two marks here. The most popular method in part (d) was to consider the components of the velocity at *C*. A few having found the vertical component at *C* stopped but this part of the question was often completed successfully. The alternative method using conservation of energy was rarely seen but, when seen, it was usually correct.

29. The majority of candidates were successful in part (a). A few candidates wrongly tried integrating or using constant acceleration formulae. Part (b) was far less successful with only a small minority of candidates producing a correct solution. A further few had the correct idea but either made numerical slips (particularly with the negative coefficient of the $\underline{\mathbf{j}}$ component of the impulse) or by having the impulse as $k(\underline{\mathbf{i}} - \underline{\mathbf{j}})$ where $k = 9$. About half of the candidates failed to realise that impulse is a vector quantity and either equated the given magnitude of the impulse to a vector velocity or to the speed.

30. Many candidates coped well with parts (a) and (b). Those who provided correct solutions to part (c) usually did so using vertical components of velocity rather than energy. A common error in (c) was to treat 65 as the vertical component of velocity, and use it in $v^2 = u^2 + 2as$. This gave the correct answer but received no credit. A number of candidates interchanged components and then proceeded well. A major source of error came from candidates used to using *u* and an angle for initial conditions who often spent a lot of time and space converting vector form to non-vector form, rarely with success.

Some quoted a formula for the range and confused *u* in the formula with *u* in question. Comparatively few completed part (c) and it was therefore very disappointing that some who managed it lost the final mark by giving the answer to 4 or 5 sig figs. They are generally getting better at this but still need to be reminded.

31. This proved to be an easy starter and there were very few errors. A few differentiated instead of integrating and there was the odd algebraic or arithmetical slip.

32. (a) There were many correct solutions to this part although the weaker candidates struggled generally with the whole of this question.

(b) This was more demanding and discriminated well at the top end. Common errors were signs and failing to resolve *u*. Those who tried to split the motion were rarely successful.

(c) The method here was generally well known with few candidates using distances in their tan expression.

33. Full marks were common for this question. A few candidates omitted the constant from the first integration or tried to use $v = u + at$ to find the velocity. Candidates needed to show, in part (a), how the +6 was obtained in the velocity equation and, in part (b), it was necessary to be clear that the distance travelled *between* the two times found had been calculated. In part (b), it was encouraging to note how many candidates, on obtaining an answer of $-\frac{2}{3}$, re-wrote the answer without the negative sign and with a

unit, making some suitable comment on the nature of "distance".

34. As is appropriate for the last question, this proved the most difficult on the paper. Those who were confident with vector notation often produced accurate and concise solutions. Many lost marks in part (a) by not seeing that the question asked for the *magnitude* of the impulse. Others found $m(|v|-|u|)$. Part (b) caused a lot of confusion among candidates. They were often uncertain whether they should be using vectors, scalars, component velocities or the resultant and often ended up using unacceptable combinations of these. It was common to see $v^2 = u^2 - 2gs$ being used with $u^2 = 15^2 + 16^2$ and, also, for the final vertical velocity component to be given as the required speed. Part (c) caused less confusion and most candidates were aware that they had to draw a triangle and use trigonometry to find the required angle. At the relatively low speeds on this question, it was thought that sensible answers to part (d) were air resistance and the possibility of cross winds. A number of candidates mentioned that the initial impact between the bat and ball would probably impart some spin or rotation to the ball and that a full analysis of the motion would take this into account and not consider the ball as a particle. This was accepted.

35. No Report available for this question.

36. No Report available for this question.