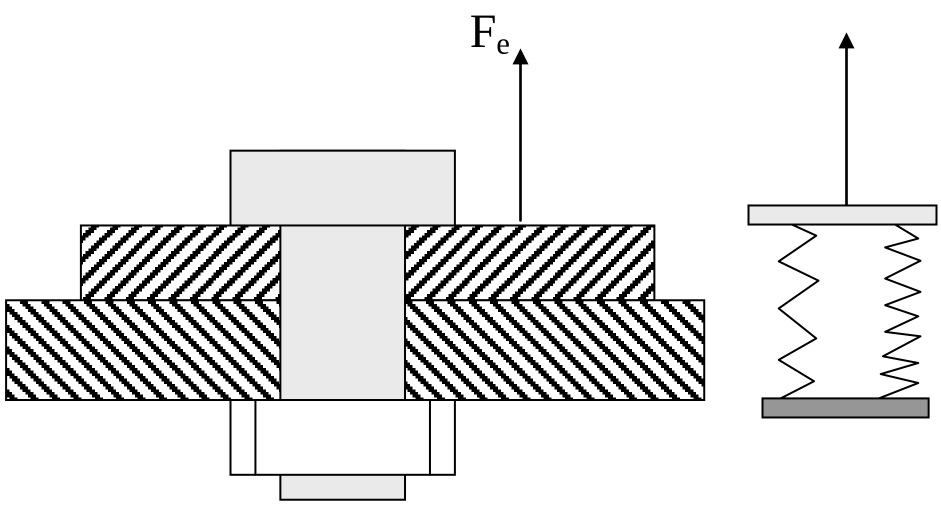
Machine Design

Bolt Selections and Design

- Dimensions of standard threads (UNF/UNC)
- Strength specifications (grades) of bolts.

Clamping forces



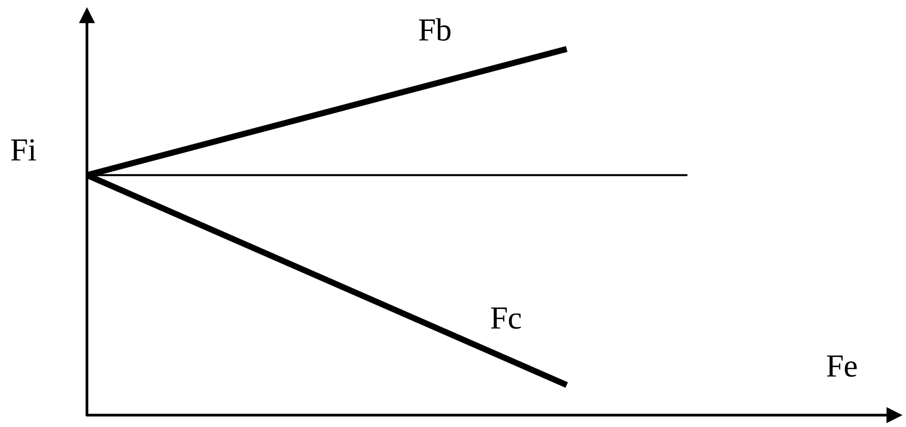
The bolt force is

$$F_b = F_i + \frac{k_b}{k_b + k_c} F_e$$

Where K_b and K_C are the bolt and the clamping material stiffness and F_i is the initial bolt tensioning. Calculating K_b and K_c are relatively difficult and exam problems often give you theses stiffnesses or their ratio.

The clamping force is

$$F_c = F_i - \frac{k_c}{k_b + k_c} F_e$$



Recommended initial tension (for reusable bolts)

F_i = (0.75 to 0.90) S_pA_t

Where S_p is the proof strength and A_t is the tensile area of the bolt.

Recommended tightening torque (based on power screw formulas):

T = 0.20 F_id

Where *d* is the nominal bolt size.

Design of bolts in tension

$F_b = A_t S_p$

Where A_t is the tensile area.

Example M_{1a}

Given: Two plates are bolted with initial clamping force of 2250 lbs. The bolt stiffness is twice the clamping material stiffness.

Find: External separating load that would reduce the clamping force to 225 lbs. Find the bolt force at this external load.

Solution

$$F_c = F_i - \frac{k_c}{k_b + k_c} F_e$$

$$225 = 2250 - \frac{1}{1+2} F_e$$

$$F_e = 6075\ lbs$$

$$F_b = F_i + \frac{k_b}{k_b + k_c} F_e = 2250 + (\frac{2}{3})(6075) = 6300\ lbs$$

Example M_{1b}

Select a bolt that would withstand 6300 lbs load in direct tension. Apply a factor of safety of 2.5. Use a bolt with SAE strength grade of 2 (which has a proof strength of 55 ksi).

$$F_{b,design} = 6300(2.5) = 15750$$

$$F_{b,design} = S_p A_t$$

$$A_t = \frac{(15750)}{55000} ===> A_t = 0.286\, in^2$$

A ¾" 10-UNC bolt has a tensile area of 0.336 square inches.

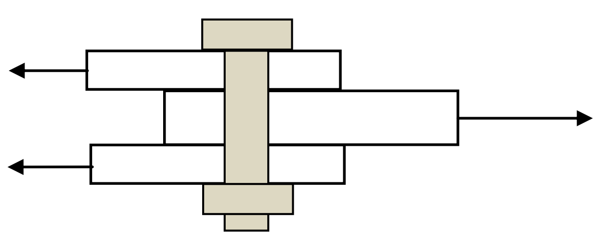
Bolts under shear loading

Example M_{1c}

A 1"-12 UNF steel bolt of SAE grade 5 is under direct double shear loading. The coefficient of friction between mating surfaces is 0.4. The bolt is tightened to its full proof strength. Tensile area is 0.663 in^2. Proof strength is 85 kpsi, and yield strength is 92 kpsi

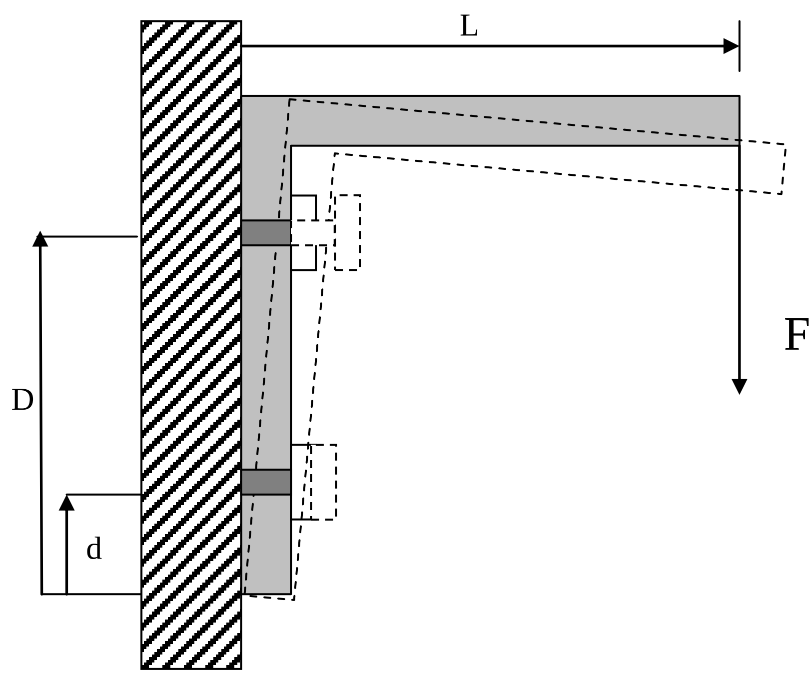
a) What shear force would the friction carry?
b) What shear load can the bolt withstand w/o yielding if the friction between clamped members is completely lost? Base the calculation on the thread root area.

Approximate Answers: a) 22500 lbs, b) 70740 lbs



Design of Bolt Groups in Bending

- Assume bolted frame is rigid.
- Use geometry to determine bolt elongations.
- Assume load distribution proportional to elongations.
- Assume shear loads carried by friction.



Example M_3

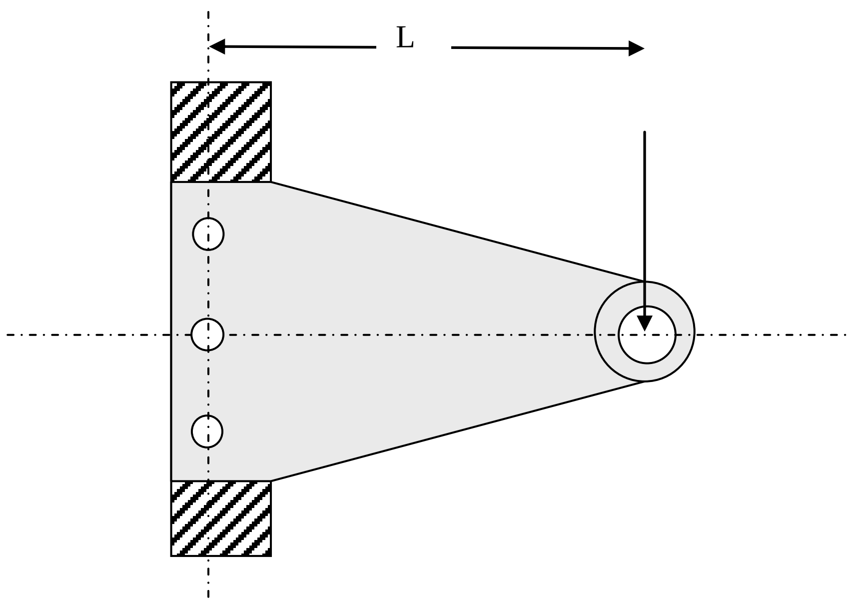
Consider the bracket shown above. Assume the bracket is rigid and the shear loads are carried by friction. The bracket is bolted by four bolts. The following is known: F=5400 lbs, L=40 inches, D=12 inches, d=4 inches. Find appropriate UNC bolt specifications for bolts of 120 Ksi proof strength using a factor of safety of 4.

Answer: ¾”- 10UNC

Design of Bolt Groups in Torsion

- Assume bolted frame is rigid.
- Use geometry to determine bolt distortion.
- Assume torque distribution proportional to distortions.
- Assume bracket rotates around the bolt group C.G.
- Ignore direct shear stress if its magnitude is small.
- Assume friction is lost
- Usually the bolt shank areas are used for analysis of stresses.

Example M_4



The bolts are ½"-13UNC. The distance between bolts is 1.25". The load is 2700 lbs and L=8". Find the shear stress on each bolt.

Answer: 44250 psi

Design of Bolts in Fatigue Loading

The factor of safety against fatigue failure of a bolt or screw is:

$$n = \frac{S_a}{\sigma_a}$$

Where $S_a = \frac{S_e S_u - \sigma_i S_e}{S_e + S_u}$ and σ_i is the stress due to initial tension

Example: A M16*2 SAE grade 8.8 bolt is subject to a cyclic stress. The minimum nominal stress in the tensile area is calculated to be 400 Mpa (for initial tension with no external load) and maximum nominal stress is 500 Mpa (for maximum external load). Determine the factor of safety guarding against eventual fatigue failure for this bolt. Fully corrected endurance limit, including thread effects, is 129 Mpa. The ultimate strength of the bolt material is 830 Mpa.

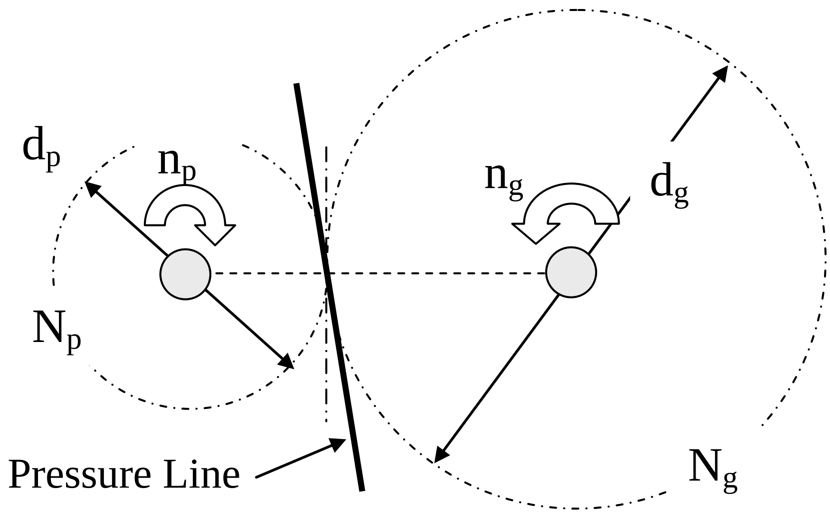
$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{500 - 400}{2} = 50 \quad MPa$$

$$S_a = \frac{129(830) - 400(129)}{129 + 830} = 57.8$$

$$n = \frac{57.8}{50} = 1.15$$

Gear Geometry

Kinematic model of a gear set



Terminology

Diametral pitch (or just pitch) P : determines the size of the tooth. All standard pairs of meshing gears have the same pitch.

$$P = \frac{N}{d} \qquad \textit{Teeth per inch}$$

$$p = \frac{\pi d}{N} \qquad \textit{Inches per tooth}$$

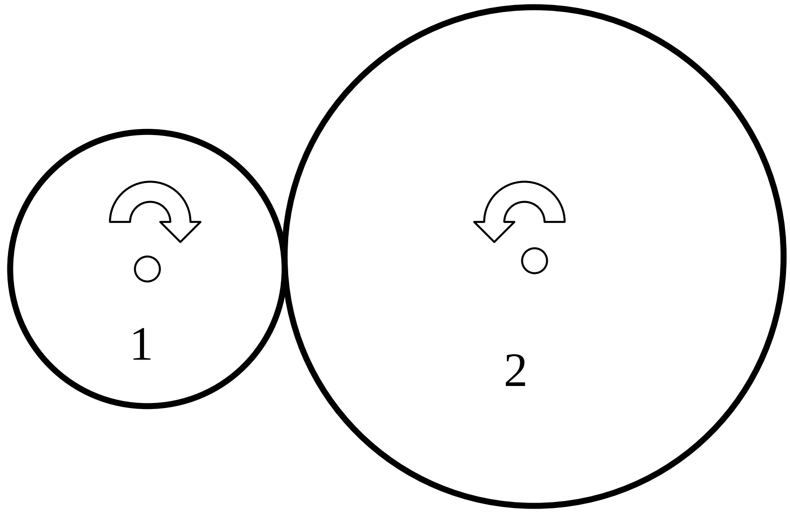
$$m = \frac{d}{N} \qquad \textit{mm per tooth}$$

$$Pp = \pi$$

P is pitch, p is circular pitch and *m* is the module.

I) Regular Gear Trains (**External gears**)

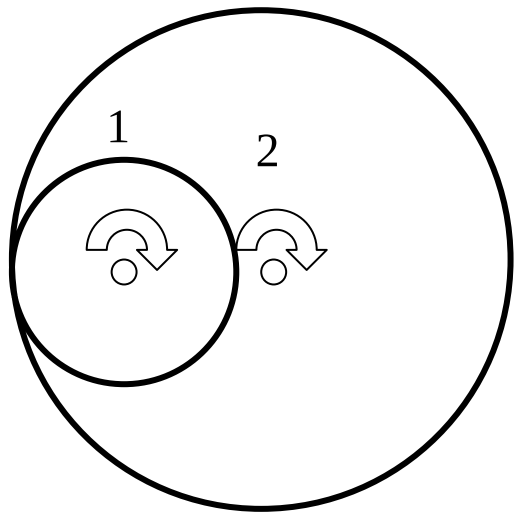
$$\frac{n_1}{n_2} = -\frac{N_2}{N_1}$$



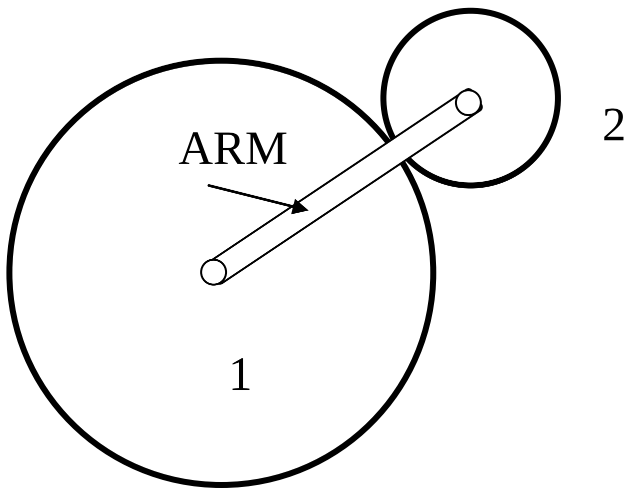
N_1 and N_2 are the number of teeth in each gear, and n_1 and n_2 are the gear speed in rpm or similar units.

Internal gears

$$\frac{n_1}{n_2} = \frac{N_2}{N_1}$$



II) Epicyclic (Planetary) Gear Trains

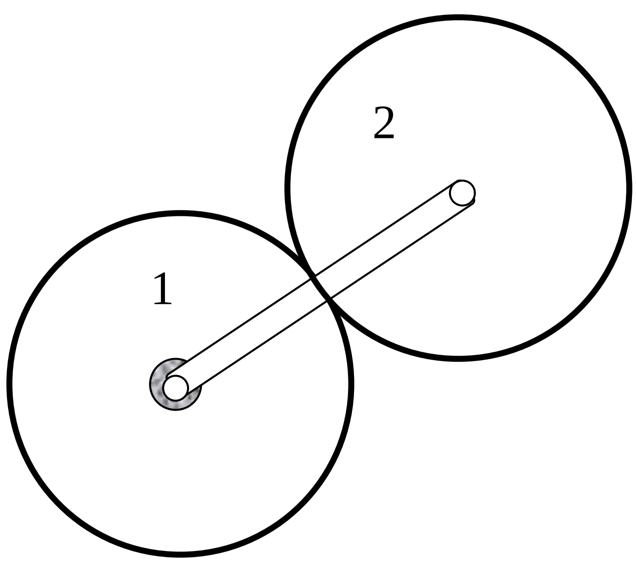


Planetary gear trains have two degrees of freedom – They require two inputs.

Note: When Arm is held stationary, or with respect to the Arm, the gears behave like regular gear trains:

$n_{2/A}$: the rpm of 2 with respect to Arm

$n_{1/A}$: the rpm of 1 seen standing on Arm



Planetary gear trains can be solved by the following two relationships. (two equations in three unknowns)

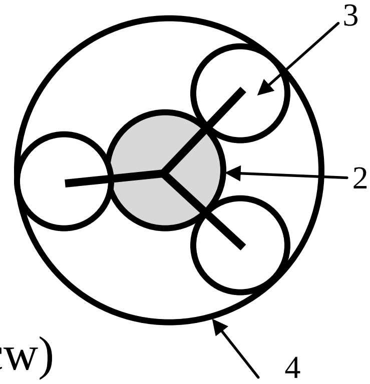
1) Relative angular velocity formula:

$$\frac{n_2 - n_A}{n_1 - n_A} = \frac{n_{2/A}}{n_{1/A}}$$

2) Regular gear train formula with Arm stationary

$$\frac{n_{2/A}}{n_{1/A}} = -\frac{N_1}{N_2}$$

The Toy Gearbox



- Sun gear N2=24
- Planet gear N3=18
- Ring Gear = N2 + 2 N3 = 60

Find Arm speed (assume n_2=100 cw)

$$\frac{n_{2/A}}{n_{4/A}} = \frac{n_2 - n_A}{n_4 - n_A} = \frac{100 - n_A}{0 - n_A}$$

$$\frac{n_{2/A}}{n_{4/A}} = \frac{n_{2/A}}{n_{3/A}}\frac{n_{3/A}}{n_{4/A}} = -\frac{N_3}{N_2}(+\frac{N_4}{N_3}) = -2.5$$

$$100 - n_A = -2.5(-n_A) \Rightarrow n_A = 28.5\,cw$$

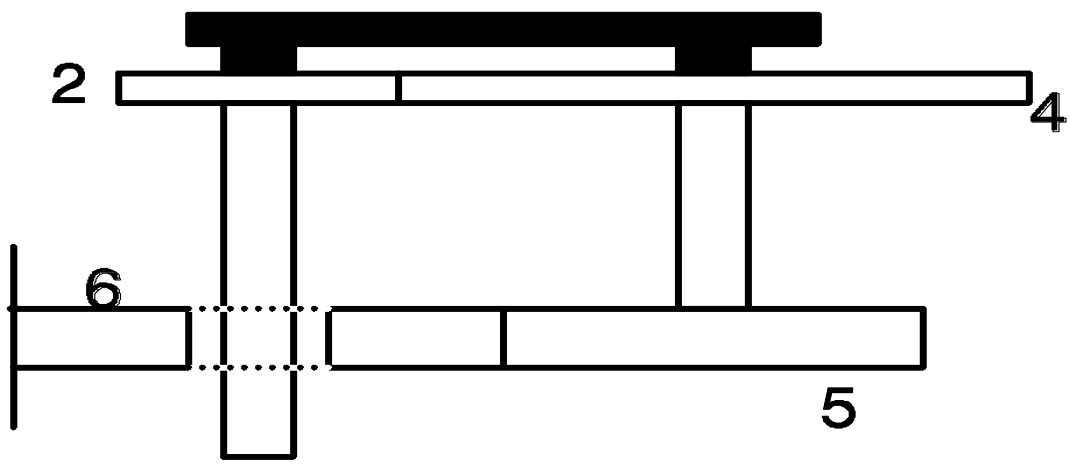
Problem #M5: Gear kinematics

The figure shows an planetary gear train. The number of teeth on each gear is as follows:

N_2=20 N_5=16

N_4=30

The input is Gear 2 and its speed is 250 rpm clockwise. Gear 6 is fixed. Determine the speed of the arm and the speed of Gear 4. The drawing is not to scale.



$d_5 + d_6 = d_2 + d_4$ and assuming all P are the same we get $N_5 + N_6 = N_2 + N_4$ and N_6 = 34 teeth

$$\frac{n_{2/A}}{n_{6/A}} = \frac{n_2 - n_A}{n_6 - n_A} = \frac{250 - n_A}{0 - n_A}$$

$$\frac{n_{2/A}}{n_{6/A}} = \frac{n_{2/A}}{n_{4/A}} \frac{n_{4/A}}{n_{5/A}} \frac{n_{5/A}}{n_{6/A}} = -\frac{N_4}{N_2}(1)(-\frac{N_6}{N_5})$$

Substituting for the number of teeth on each gear

$$\frac{n_{2/A}}{n_{6/A}} = (-\frac{30}{20})(-\frac{34}{16}) = 3.187$$

$$n_A = -114.3$$

Also

$$\frac{n_{4/A}}{n_{2/A}} = \frac{n_4 - n_A}{n_2 - n_A} = \frac{n_4 - (-114.3)}{250 - (-114.3)}$$

$$\frac{n_{4/A}}{n_{2/A}} = -(\frac{N_2}{N_4}) = -\frac{20}{30}$$

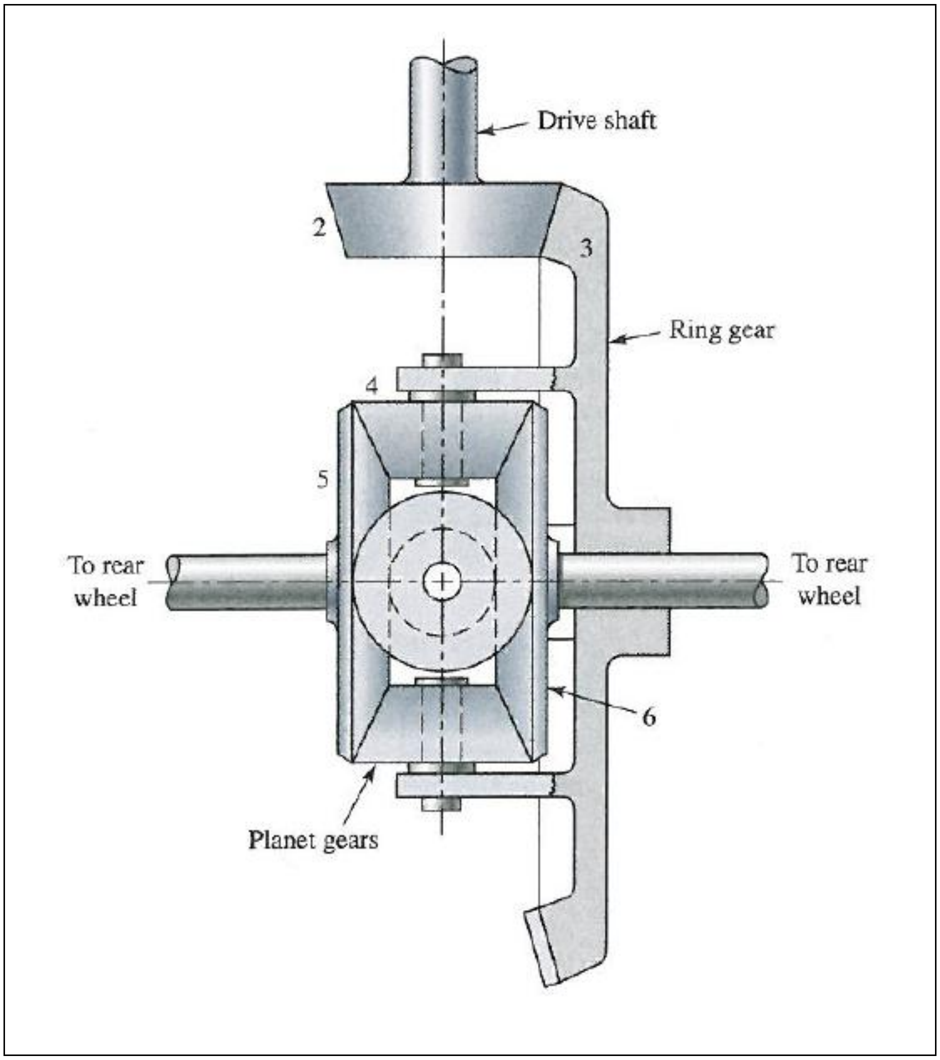
From above:

n_4=-357.1 rpm

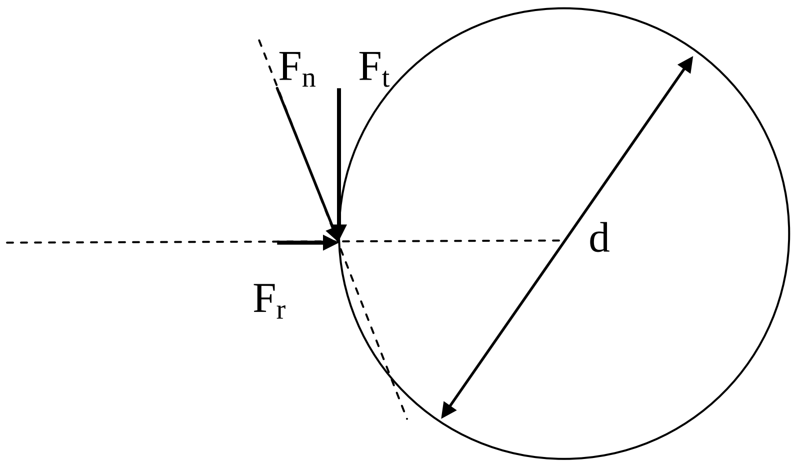
Kinematics of Automobile Differential

Considering the Right Wheel, Left Wheel, the Ring Gear and the Drive Shaft.

$$n_{RW} + n_{LW} = 2n_{RG}$$



Gear Force Analysis



F_n : Normal force
F_t : Torque-producing tangential force
F_r : Radial force.

When *n* is in rpm and *d* is in inches:

$$F_t = \frac{33000(hp)}{V} \quad and \quad V = \frac{\pi n d}{12}$$

and

$$F_r = F_t \tan\phi$$

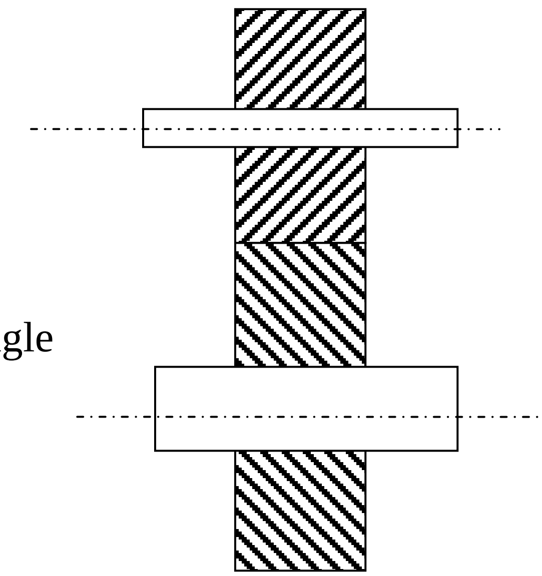
In SI units:

$$Watts = T\omega \quad and \quad T = F_t\left(\frac{d}{2}\right)$$

Helical gears

Geometric parameters

P_n : Normal pitch
P : Plane of rotation pitch
ψ : Helix angle
ϕ_n: Normal pressure angle
ϕ : Plane of rotation pressure angle
N : Number of teeth
d: pitch diameter
p_n and p : circular pitches
p_a : axial pitch



Geometric relationships:

$$P = \frac{N}{d} \quad and \quad P = P_n \cos(\psi)$$

$$\tan(\phi) = \frac{\tan(\phi_n)}{\cos(\psi)}$$

$$Pp = P_n p_n = \pi \qquad and \qquad p_a = \frac{p}{\tan(\psi)}$$

Helical gear forces

$$F_t = From\ Power$$

$$F_r = F_t \tan(\phi)$$

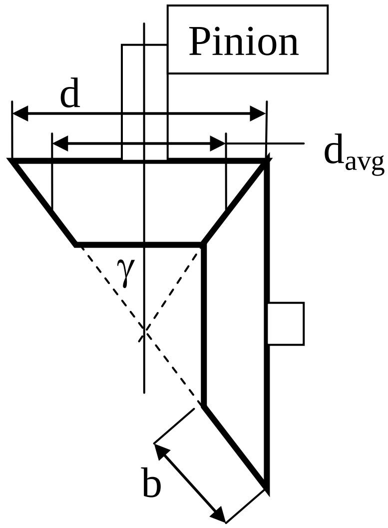
$$F_a = F_t \tan(\psi)$$

When shaft axes are parallel, the helix hands of the two gears must be opposite of each other.

Straight Bevel gears

Geometric Parameters (**Pinion**)

d_p: pitch diameter
$d_{avg,p}$: average diameter
b: Face width
γ_p: Pitch cone angle



Bevel gear forces

$$F_t = \frac{33000(hp)}{V_{avg}} \quad where \quad V_{avg} = \frac{\pi n d_{avg}}{12} \quad and$$

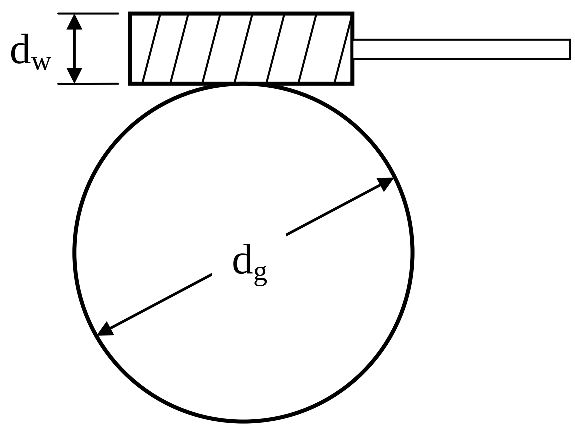
$$d_{avg} = d - b\sin(\gamma)$$

$$F_r = F_t \tan(\phi)\cos(\gamma)$$

$$F_a = F_t \tan(\phi)\sin(\gamma)$$

These forces are for pinion and act through the tooth midpoint. Forces acting on the gear are the same but act on opposite directions.

Worm Gear Kinematics



The velocity ratio of a worm gear set is determined by the number of teeth in gear and the number of worm threads (not the ratio of the pitch diameters).

$$\frac{\omega_g}{\omega_w} = \frac{N_w}{N_g}$$

N_w = Number of threads (single thread =1, double thread =2, etc)

The worm's lead is

$$L = p_a N_W$$

The worm's axial pitch p_a must be the same as the gear's plane of rotation circular pitch p.

The worm's lead angle λ is the same as the gear's helix angle ψ. The gear and worm must have the same hand.

Example

For a speed reduction of 30 fold and a double threaded worm, what should be the number of teeth on a matching worm gear.

$N_g = (2)\,(30) = 60$ teeth

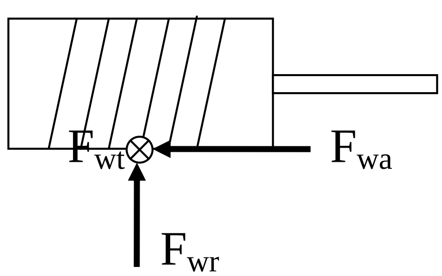
The geometric relation for finding worm lead angle

$$\tan(\lambda) = \frac{L}{\pi d_w}$$

Worm Gear Forces

The forces in a worm gearset when the worm is driving is

$F_{gr} = F_{wr}$ $F_{gt} = F_{wa}$ $F_{ga} = F_{wt}$



The F_{wt} is obtained from the motor *hp* and *rpm* as before. The other forces are:

$$F_{wa} = \frac{\cos(\phi_n)\cos(\lambda) - f\sin(\lambda)}{\cos(\phi_n)\sin(\lambda) + f\cos(\lambda)} F_{wt}$$

The worm and gear radial forces are:

$$F_{wr} = \frac{\sin(\phi_n)}{\cos(\phi_n)\sin(\lambda) + f\cos(\lambda)} F_{wt}$$

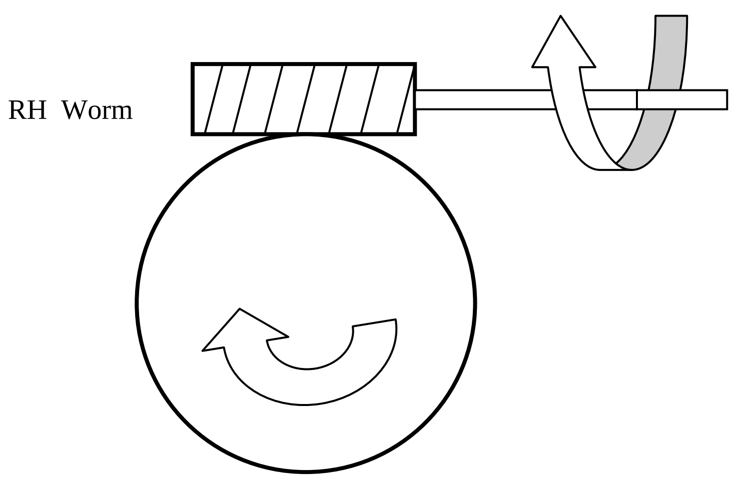
The worm gearset efficiency is:

$$e = \frac{\cos(\phi_n) - f\tan(\lambda)}{\cos(\phi_n) + f\cot(\lambda)}$$

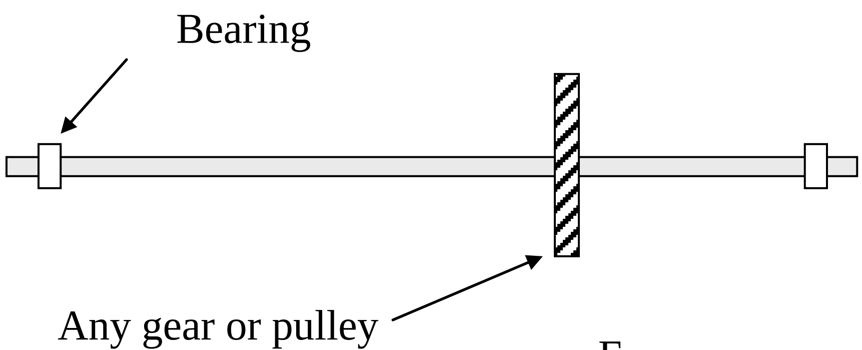
Where *f* is the coefficient of friction. Condition for self-locking when worm is the driver

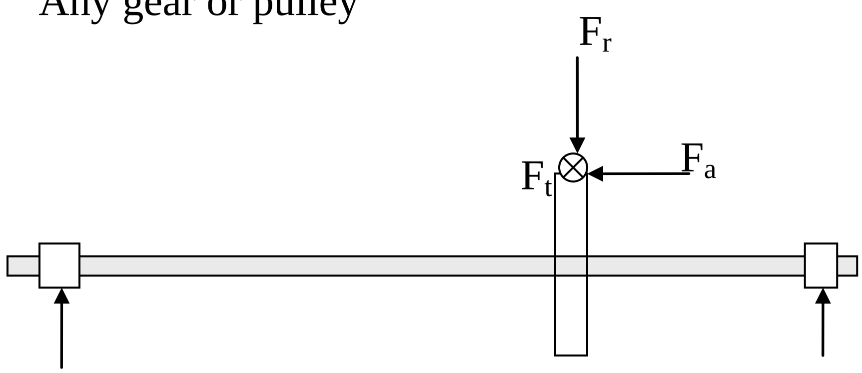
$$f > \cos(\phi_n)\tan(\lambda)$$

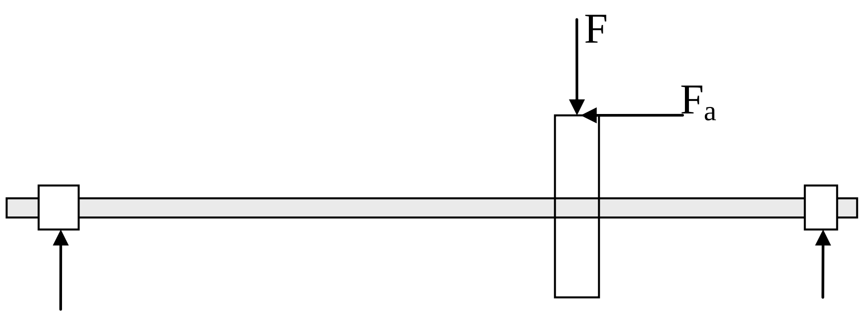
Note: In a RH worm, the teeth spiral away as they turn in a CW direction when observed along the worm axis. When the worm in turning in CW direction, the teeth sweep toward the observer seen along the axis of the worm (imagine a regular bolt and nut).



Bearing Reaction Forces







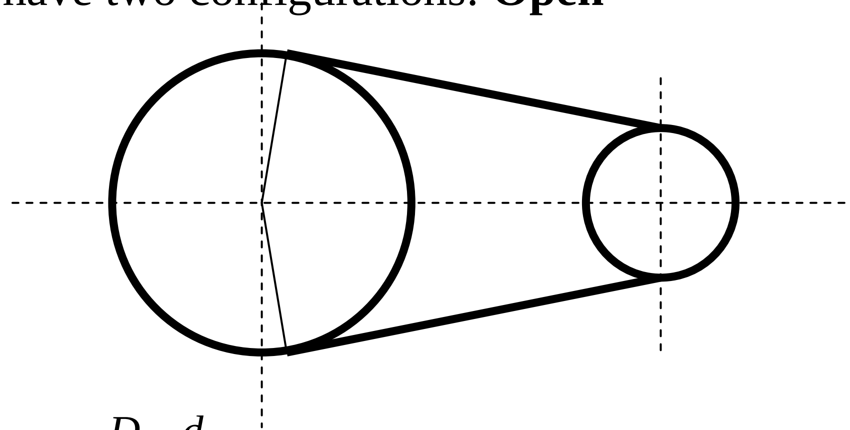
Total thrust load on bearings is F_a

For the radial reaction forces for spur gears (no axial forces) combine the radial and tangential forces into F:

$$F = \sqrt{F_r^2 + F_t^2}$$

Flat Belts

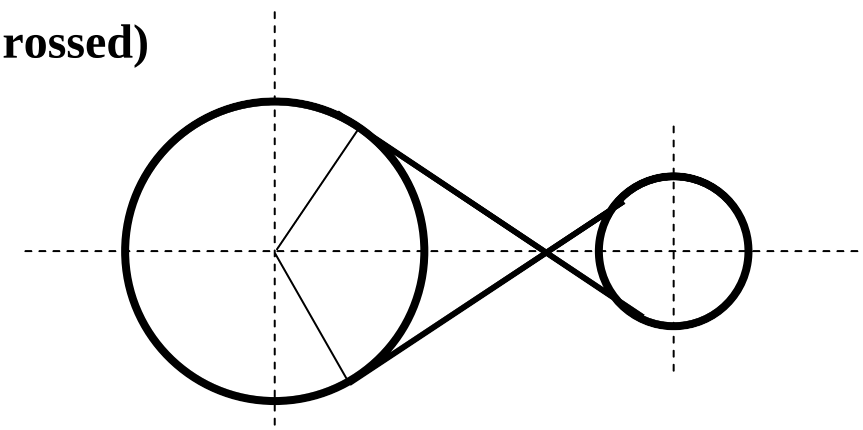
Flat belts have two configurations: **Open**



$$\theta_d = \pi - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

$$\theta_D = \pi + 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

Closed (Crossed)



$$\theta_d = \theta_D = \pi + 2\sin^{-1}\left(\frac{D+d}{2C}\right)$$

Where

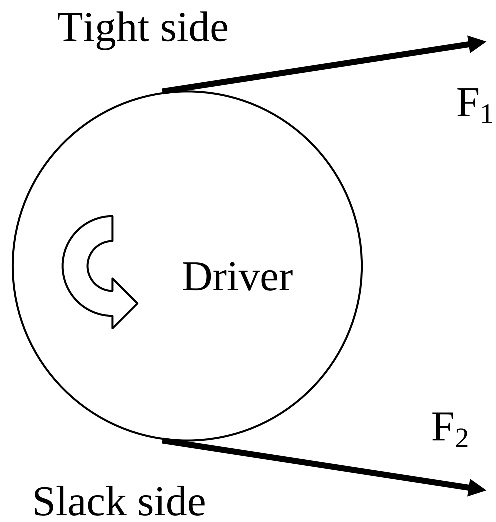
C: Center-to-center distance

D,d: Diameters of larger and smaller rims

θ : Angle of wrap around pulley

Slippage Relationship

(True only at the verge of slippage)



$$\frac{F_1}{F_2} = e^{\mu\theta}$$

θ is in radians.

Transmitted Hp is

$$H_p = \frac{(F_1 - F_2)V}{33000}$$

Where F_1 and F_2 are in lbs and V is in ft/min.

Initial Tension

Belts are tensioned to a specified value of F_i. When the belt is not transmitting torque:

$F_1 = F_2 = F_i$

As the belt start transmitting power,

$F_1 = F_i + \Delta F$

$F_2 = F_i - \Delta F$

The force imbalance continues until the slippage limit is reached.

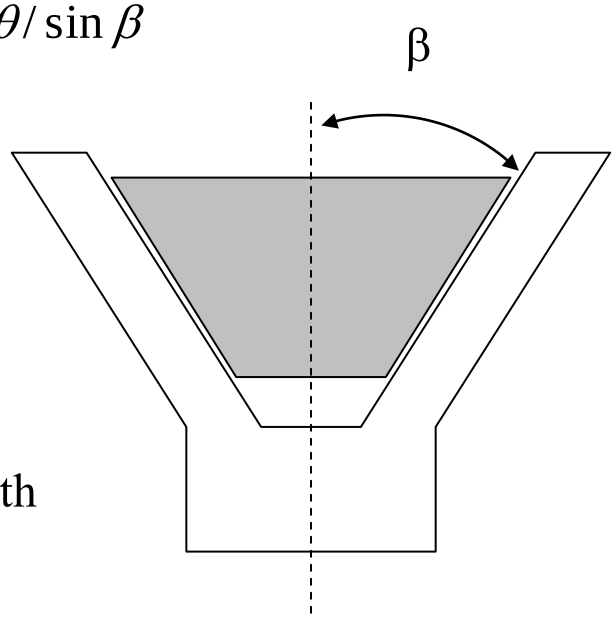
Problem M7

A 10”-wide flat belt is used with a driving pulley of diameter 16” and a driven pulley of rim diameter 36” in an open configuration. The center distance between the two pulleys is 15 feet. The friction coefficient between the belt and the pulley is 0.80. The belt speed is required to be 3600 ft/min. The belts are initially tensioned to 544 lbs. Determine the following. (answers are in parentheses)

a) Belt engagement angle on the smaller pulley (3.03 radians).
b) Force in belt in the tight side just before slippage. (1000 lbs).
c) Maximum transmitted Hp. (99.4 hp)

Formula for V-belts

$$\frac{P_1 - P_c}{P_2 - P_c} = e^{\mu\theta/\sin\beta}$$

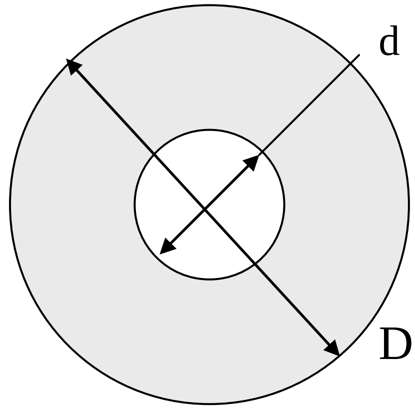


where

$$P_c = m'\omega^2 r^2$$

m’=Mass per unit length
r=Pulley radius

Disk Brakes and Clutches



Torque capacity under "Uniform Wear" condition per friction surface (when brake pads are not new)

$$T = \frac{\pi f p_a d}{8}(D^2 - d^2)$$

Where

f: Coefficient of friction

p_a: Maximum pressure on brake pad

d,D: Inner and outer pad diameters

Torque capacity under "Uniform Pressure" conditions per friction surface (when brake pads are new)

$$T = \frac{\pi f p_a}{12}(D^3 - d^3)$$

Maximum clamping forces to develop full torque

For Uniform Wear

$$F = \frac{\pi p_a d}{2}(D - d)$$

For Uniform Pressure

$$F = \frac{\pi p_a}{4}(D^2 - d^2)$$

Example M_8

Given: A multi-plate disk clutch

d=0.5”

D=6”

P_{max}=100 psi

Coefficient of friction=0.1

Power transmitted= 15 hp at 1500 rpm

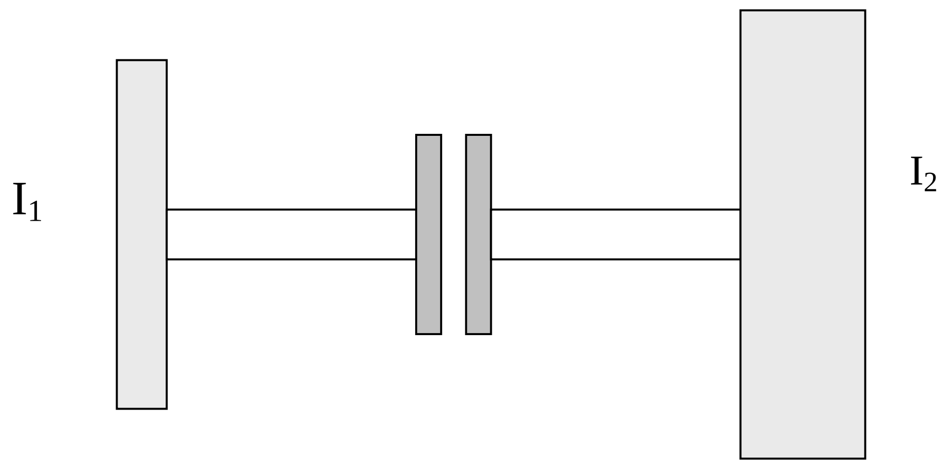
Find: Minimum number of friction surfaces required

Answer: N=2 (uniform pressure)

N=9 (uniform wear)

Energy Dissipation in Clutches and Brakes

The time it takes for two rotational inertia to reach the same speed after engagement through a clutch is:



$$t = \frac{I_1 I_2 (\omega_1 - \omega_2)}{T(I_1 + I_2)}$$

where

T: Common transmitted torque

ω: angular speed in rad/sec

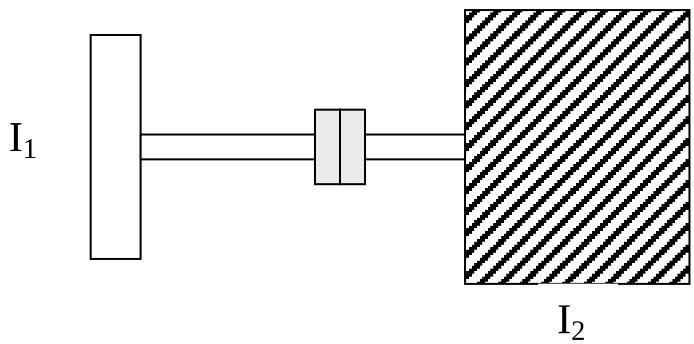
The total energy dissipated during clutching (braking) is:

$$E = \frac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$$

If the answer is needed in BTU, divide the energy in in-lb by 9336.

Problem M_9

A brake with braking torque capacity of 230 ft-lb brings a rotational inertia I_1 to rest from 1800 rpm in 8 seconds. Determine the rotational inertia. Also, determine the energy dissipated by the brake.



Solution hints:

Convert rpm to rad/sec: ω_1 = 188 rad/sec
Note that ω_2=0
Find the ratio (I_1I_2/I_1+I_2) using time and torque=>9.79
Note that I_2 is infinitely large => I_1=9.79 slugs-ft
Find energy from equation=>173000 ft-lb

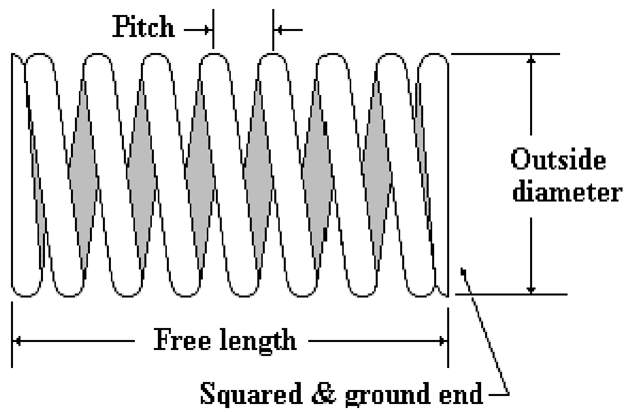
Springs

Coverage:

- Helical compression springs in static loading

Terminology:

- d: Wire diameter
- D: Mean coil diameter
- C: Spring index (D/d)
- N_t: Total # of coils
- N: Num. of active coils
- p: Coil pitch
- L_f: Free length = N*p
- L_s: Solid length



End detail and number of active coils:

	Plain	Plain & Ground	Square	Square & Ground
L_s	$(N_t+1)d$	$N_t d$	$(N_t+1)d$	$N_t d$
N_t	N	N+1	N+2	N+2
L_f	pN+d	p(N+1)	pN+3d	pN+2

Note: Spring geometry, especially the end-condition relationships, are not exact. Other books may have slightly different relationships.

Spring Rate of Helical Springs (compression/extension)

$$k = \frac{d^4 G}{8D^3 N} = \frac{dG}{8NC^3}$$

where : N is the number of active coils

G: shear modulus = E/2(1+ν)

G=11.5*10^6 psi for steels

Shear stress in helical springs for static loading

$$\tau_{\max} = K_s \frac{8FD}{\pi d^3}$$

where $K_s = 1 + \frac{1}{2C}$ and C is the spring index.

Shear strength in springs

$\tau \le 0.45 S_u$ Ferrous without presetting

$\tau \le 0.65 S_u$ Ferrous with presetting

Note: it is common in practice (but not academia) to specify strength as "Allowable Stress". Allowable stress is defined as the strength (yield or shear strength) divided by the factor of safety.

Spring Surge Frequency

$$f_n = \frac{1}{2}\sqrt{\frac{kg}{W_a}}$$

Where g is the gravitational acceleration and W_a is the weight of the active coils:

$$W_a = \frac{1}{4}\pi^2 d^2 DN\gamma$$

with γ being the specific gravity of spring material. For steel springs when d and D are in inches:

$$f_n = \frac{13900d}{ND^2} \quad HZ$$

Example M_{10}

Consider a helical compression spring with the following information (not all are necessarily needed):

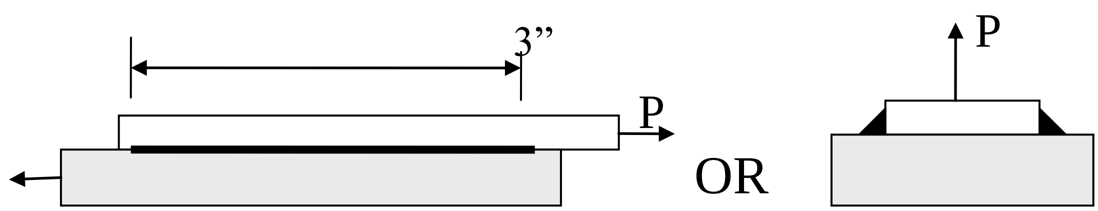
Ends: Squared and ground
Spring is not preset
Material: Music wire (steel) with S_{ut}=283 ksi
d=.055 inches and D=0.48 inches
L_f=1.36 inches and N_t=10

Find the following. Answers are given in parentheses.

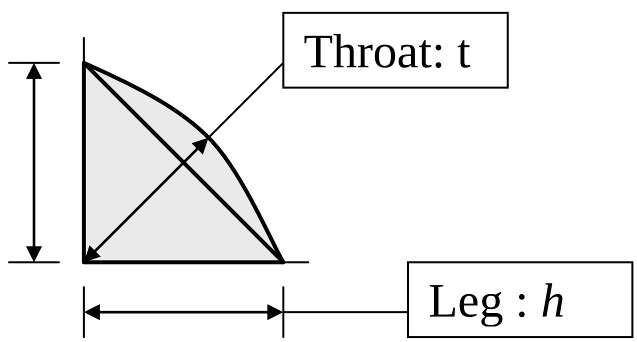
Spring constant, K (14.87 lb/in)
Length at minimum working load of 5 lbs (1.02”)
Length at maximum load of 10 lbs (0.69”)
Solid length (0.55”)
Load corresponding to solid length (12.04 lbs)
Clash allowance ($L_{Fmax} - L_S$) (0.137”)
Shear stress at solid length (93496 psi)
Surge frequency of the spring (415 Hz)

Design of Welds

Welds in parallel loading and transverse loading



Weld Geometry



Analysis Convention

- Critical stresses are due to shear stresses in throat area of the weld in both parallel and transverse loading.
- For convex welds, t=0.707*h* is used.
- The shear strength in weld analysis is taken as 30% of the weld ultimate strength.

Analysis Methodology

- Under combined loading, different stresses are calculated and combined as vectors.

Stresses based on weld leg (h)

Direct tension/compression:

$$\tau = \frac{F}{Lt} = \frac{1.41}{Lh}$$

Direct shear:

$$\tau = \frac{1.41V}{Lh}$$

Bending:

$$\tau = \frac{Mc}{I} = \frac{Mc}{I_u t}$$

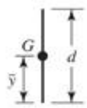
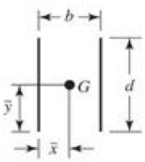
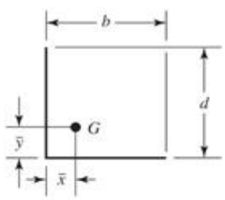
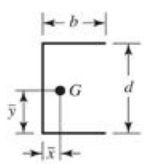
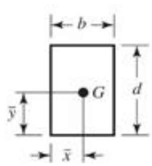
Torsion:

$$\tau = \frac{Tr}{J_u t}$$

Formulas for I_u, and J_u are attached for different weld shapes.

Table 9–1

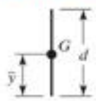
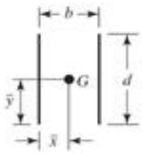
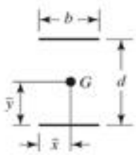
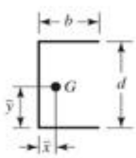
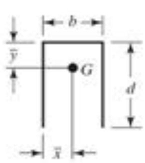
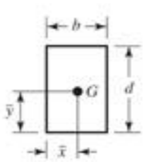
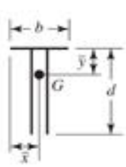
Torsional Properties of Fillet Welds*

Weld	Throat Area	Location of G	Unit Second Polar Moment of Area
	$A = 0.70\,hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$J_u = d^3/12$
	$A = 1.41\,hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$J_u = \dfrac{d(3b^2 + d^2)}{6}$
	$A = 0.707h(2b + d)$	$\bar{x} = \dfrac{b^2}{2(b+d)}$ $\bar{y} = \dfrac{d^2}{2(b+d)}$	$J_u = \dfrac{(b+d)^4 - 6b^2d^2}{12(b+d)}$
	$A = 0.707h(2b + d)$	$\bar{x} = \dfrac{b^2}{2b+d}$ $\bar{y} = d/2$	$J_u = \dfrac{8b^3 + 6bd^2 + d^3}{12} - \dfrac{b^4}{2b+d}$
	$A = 1.414h(b + d)$	$\bar{x} = b/2$ $\bar{y} = d/2$	$J_u = \dfrac{(b+d)^3}{6}$
	$A = 1.414\,\pi hr$		$J_u = 2\pi r^3$

*G is centroid of weld group; h is weld size; plane of torque couple is in the plane of the paper; all welds are of unit width.

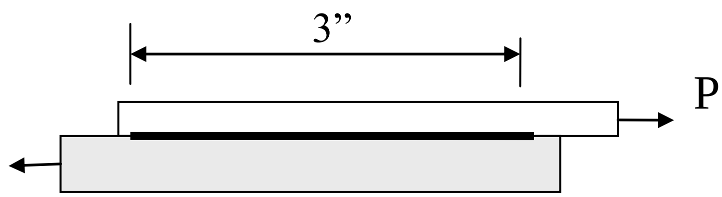
Table 9–2

Bending Properties of Fillet Welds*

Weld	Throat Area	Location of G	Unit Second Moment of Area
	$A = 0.707hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$I_u = \dfrac{d^3}{12}$
	$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{d^3}{6}$
	$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{bd^2}{2}$
	$A = 0.707h(2b + d)$	$\bar{x} = \dfrac{b^2}{2b+d}$ $\bar{y} = d/2$	$I_u = \dfrac{d^2}{12}(6b+d)$
	$A = 0.707h(b + 2d)$	$\bar{x} = b/2$ $\bar{y} = \dfrac{d^2}{b+2d}$	$I_u = \dfrac{2d^3}{3} - 2d^2\bar{y} + (b+2d)\bar{y}^2$
	$A = 1.414h(b + d)$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{d^2}{6}(3b+d)$
	$A = 0.707h(b + 2d)$	$\bar{x} = b/2$ $\bar{y} = \dfrac{d^2}{b+2d}$	$I_u = \dfrac{2d^3}{3} - 2d^2\bar{y} + (b+2d)\bar{y}^2$

Problem M_{11a} -Welds subject to direct shear

Two steel plates welded and are under a direct shear load P. The weld length is 3 inches on each side of the plate and the weld leg is 0.375 inches. What maximum load can be applied if the factor of safety is 2 against yielding? The weld material is E60.



Solution (of M_{11a})

L = 2d = 6

$$\tau = \frac{1.41V}{Lh} = \frac{1.41P}{6(.375)} = 0.6284P \quad psi$$

The design strength of the weld material in shear is:

S_{ys}=0.3 S_{ut} = 0.3(60) = 18 ksi

Using a factor of safety of 2, the allowable shear stress is:

τ_{all} = 18/2 = 9 ksi

Equating stress and strength

.6284P = 9000 ➔ P=14322 lbs

Problem M_{11b} – Welds subject to torsion

A round steel bar is welded to a rigid surface with a ¼ " fillet weld all around. The bar's outer diameter is 4.5". Determine the critical shear stresses in the weld when the bar is subjected to a 20,000 lb-in pure torque.

$$J_u = 2\pi(r^3) = 2\pi(4.5/2)^3 = 71.57 \quad in^3$$

$$J_t = (71.57)(\frac{1}{4})(0.707) = 12.6 \quad in^4$$

$$\tau = \frac{Tc}{J_t} = \frac{(20000)(2.25)}{12.6} = 3571\ psi$$

Problem M_{11c} – Welds subject to bending

Solve the previous problem with a bending moment of 35000 lb-in acting on the welds instead of the torsion load.

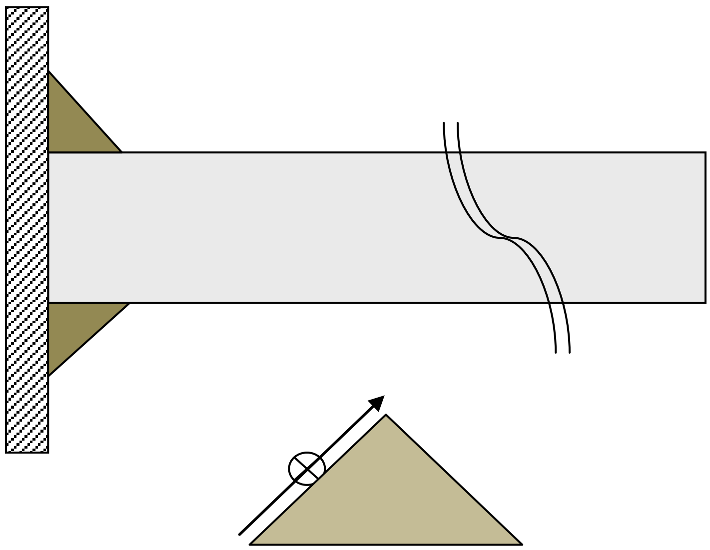
$$I_u = \pi r^3 = \pi(2.25)^3 = 35.78\, in^3$$

$$I_t = 35.78(0.25)(0.707) = 6.32 \quad in^4$$

$$\tau = \frac{Mc}{I_t} = \frac{(35000)(2.25)}{6.32} = 12460\ psi$$

Problem M_{11d} – Welds subject to combined loads

If the shear strength (S_{ys}) in the weld is 27800 psi, what is the factor of safety against yielding when both stresses in previous two problems are acting on the bar.



$$\tau = \sqrt{\tau_1^2 + \tau_2^2} = \sqrt{3571^2 + 12460^2} = 12961\ psi$$

FS = 27800/12961=2.14

Ball and Roller Bearings

Terminology

- Rated or catalog Capacity, C_{10} : Radial load for a life of 1000,000 cycles (or other L_{10}) and 90% reliability.
- Application or design radial load F_r.
- Application or design life L

Load/Life relationships (Palmgren formula)

$$F_1 L_1^{0.333} = F_2 L_2^{0.333}$$

This means if we double the load, the life of the ball bearing would be reduced by a factor of 10. This formula is for ball bearings. For roller bearings use 0.3 as the exponent.

Example: By what factor the radial load capacity of a roller bearing has to be increased if the bearing is to last twice as long as its catalog rating.

$$\left(\frac{F_1}{F_2}\right) = \left(\frac{L_2}{L_1}\right)^{0.3} = \left(\frac{2}{1}\right)^{0.3} = 1.23 \quad \Rightarrow F_2 = \frac{F_1}{1.23}$$

Example: Given:

02 series Deep Groove ball bearing,
Radial load is 4 KN,
Application factor $K_D = 1.2$
Design life 540 million cycles
95% reliability

Find: Suitable bearing catalog rating based on 10^6 cycle L_{10} life.

Solution:

Life multiplier due to reliability

x_1= 0.65 (at 95%) - See reliability multiplier below

Adjusted design life:

L_D = 540/0.65 = 830.77 million cycles

Force multiplier due to life being different from 10^6 cycles

$K_1=(830.77)^{.333} = 9.38$

Adjusted Design Load

F_{DA} = 4 (1.2) (9.38) = 45 KN

Selection: 60 mm bore with 47.5 KN capacity.

11-2

...ons and Load Ratings for Single-Row 02-Series Deep-Groove and Angular-Contact Ball Bearings

OD, mm	Width, mm	Fillet Radius, mm	Shoulder Diameter, mm d_S	d_H	Load Ratings, kN Deep Groove C_{10}	C_0	Angular Contact C_{10}	C_0
30	9	0.6	12.5	27	5.07	2.24	4.94	2.12
32	10	0.6	14.5	28	6.89	3.10	7.02	3.05
35	11	0.6	17.5	31	7.80	3.55	8.06	3.65
40	12	0.6	19.5	34	9.56	4.50	9.95	4.75
47	14	1.0	25	41	12.7	6.20	13.3	6.55
52	15	1.0	30	47	14.0	6.95	14.8	7.65
62	16	1.0	35	55	19.5	10.0	20.3	11.0
72	17	1.0	41	65	25.5	13.7	27.0	15.0
80	18	1.0	46	72	30.7	16.6	31.9	18.6
85	19	1.0	52	77	33.2	18.6	35.8	21.2
90	20	1.0	56	82	35.1	19.6	37.7	22.8
100	21	1.5	63	90	43.6	25.0	46.2	28.5
110	22	1.5	70	99	47.5	28.0	55.9	35.5
120	23	1.5	74	109	55.9	34.0	63.7	41.5
125	24	1.5	79	114	61.8	37.5	68.9	45.5
130	25	1.5	86	119	66.3	40.5	71.5	49.0
140	26	2.0	93	127	70.2	45.0	80.6	55.0
150	28	2.0	99	136	83.2	53.0	90.4	63.0
160	30	2.0	104	146	95.6	62.0	106	73.5
					108	69.5	121	85.0

Problem M_{12} – Bearings

An angular contact 02-series ball bearing is required to run for 50000 hours at 480 rpm. The design radial load is 610 lbs and the application factor (load multiplier) is 1.4. For a reliability of 90%, what is the required capacity of this bearing? Answer: 42.9 KN (capacities are in SI units)

- If the required reliability is different than 90%, apply a reliability factor to the life. (See Juvinall).

  K_r = Reliability factor

90% $K_r = 1$	50% : $K_r = 5$
95% $K_r = 0.65$	99% : $K_r = 0.20$

- If there is substantial thrust or axial loading, then an equivalent radial load should be used. For radial ball bearings (F_t is thrust load):

  Ignore F_t and use $F_e = F_r$ if $F_t < 35\% F_r$

  Use $F_e = 1.18 F_t$ if $F_t > 10 F_r$

  If $F_t > 35\% F_r$ but less than 10 F_r use

$$F_e = F_r\left(1+1.115\left(\frac{F_t}{F_r}-.35\right)\right)$$

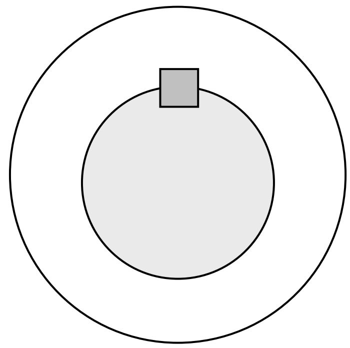
- When a bearing system includes several bearings (*n* bearings), and the reliability of the entire system is given (R_{sys}), then the reliability of each bearing must be:

$$R_D = \sqrt[n]{R_{sys}}$$

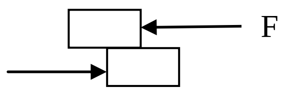
Keys

Square keys

w = d/4   (d is shaft diameter)
Length = L



Shear stresses at torque T



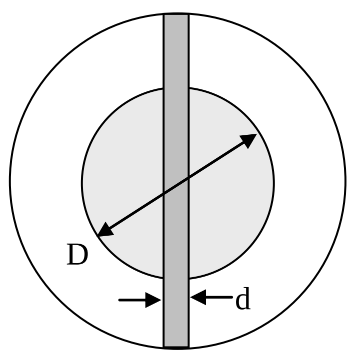
Setting F = 2T/d and balancing the force and stress

$$\frac{2T}{d} = \frac{d}{4}(L)\tau \quad \Rightarrow \quad \tau = \frac{8T}{d^2 L}$$

Torque capacity for this key in shear is obtained by setting stress to its yielding limiting value:

$$T_{\max} = \frac{Ld^2}{8}(0.58\, S_y)$$

For round pins in double shear



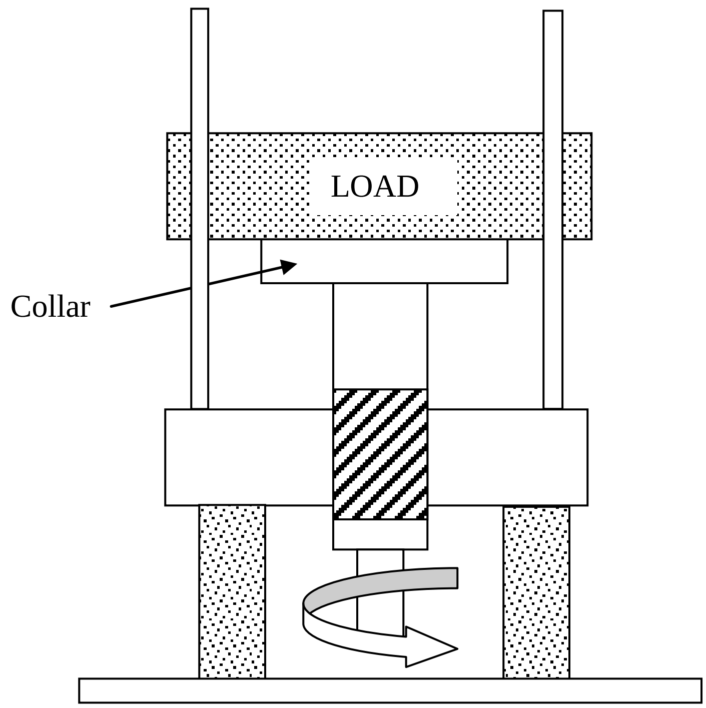
$$\tau = \frac{4T}{\pi D d^2}$$

The torque capacity is

$$T = \frac{\pi D d^2}{4}(0.58 S_y)$$

Other key types and splines are all treated similarly: Equating the shear area to the force created by the transmitted torque.

Power Screws



Torque required to raise a load

$$T_R = \frac{Fd_m}{2}\left(\frac{L + \pi f d_m}{\pi d_m - fL}\right) + \frac{F f_c d_c}{2}$$

F: Load , d_m=Screw mean diameter
L: Screw lead = N_W * p
f: Thread coefficient of friction
f_c : Collar coefficient of friction
d_c: Mean collar diameter

Torque required to lower the load

$$T_L = \frac{Fd_m}{2}\left(\frac{-L + \pi f d_m}{\pi d_m + fL}\right) + \frac{F f_c d_c}{2}$$

Note: All formulas are for power screws with square threads which are the most common type.

Condition for self locking:

$$f > \tan(\lambda) = \frac{L}{\pi d_m}$$

Efficiency of power screws (includes collar losses)

$$e = \frac{FL}{2\pi T_R}$$

Problem M13 – power screws

A single thread power screw is carrying a load of 12500 lbs. The mean screw diameter is 1 inch and the screw pitch is 0.25 (4 threads per inch). The mean collar diameter is 1.5 inch. The coefficient of friction of both threads and collar are 0.1. The thread shape is square. Find a) Major diameter of the screw, b) Torque required to lift the load, c) Minimum f to make the screw self locking if f_c=0, d) power screw efficiency

Answers a)1.125 in , b)2070 lb-in, c) 0.08, d)24%