11-6 Surface Areas and Volumes of Spheres

Mathematics Florida Standards

MAFS.912.G-GMD.1.3 Use volume formulas for . . . spheres to solve problems.

MAFS.912.G-MG.1.1 Use geometric shapes, their measures, and their properties to describe objects.

MP 1, MP 3, MP 4, MP 6, MP 7, MP 8

Objective To find the surface area and volume of a sphere

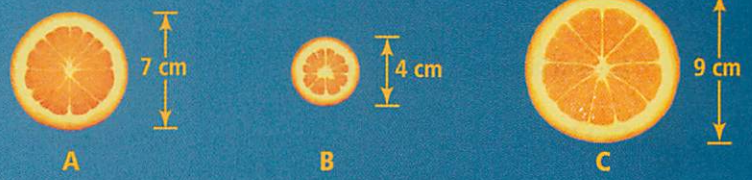
Try drawing a diagram of a sphere to see how different cross sections compare.

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

The three orange slices below were cut from three different oranges. Do you have sufficient information to tell which orange is the largest? If not, explain what information you would need.

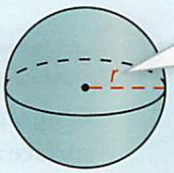


Lesson Vocabulary

- sphere
- center of a sphere
- radius of a sphere
- diameter of a sphere
- circumference of a sphere
- great circle
- hemisphere

In the Solve It, you considered the sizes of objects with circular cross sections.

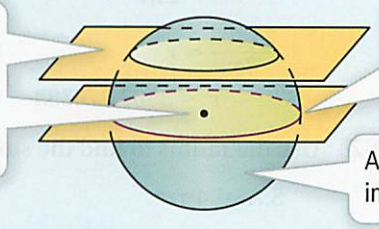
A **sphere** is the set of all points in space equidistant from a given point called the **center.** A **radius** is a segment that has one endpoint at the center and the other endpoint on the sphere. A **diameter** is a segment passing through the center with endpoints on the sphere.



r is the length of the radius of the sphere.

Essential Understanding You can find the surface area and the volume of a sphere when you know its radius.

When a plane and a sphere intersect in more than one point, the intersection is a circle. If the center of the circle is also the center of the sphere, it is called a **great circle.**



The circumference of a great circle is the **circumference** of the sphere.

A great circle divides a sphere into two **hemispheres.**

A baseball can model a sphere. To approximate its surface area, you can take apart its covering. Each of the two pieces suggests a pair of circles with radius *r*, which is approximately the radius of the ball. The area of the four circles, $4\pi r^2$, suggests the surface area of the ball.

The area of each circle is πr^2.

take note

Theorem 11-10 Surface Area of a Sphere

The surface area of a sphere is four times the product of π and the square of the radius of the sphere.

$$\text{S.A.} = 4\pi r^2$$

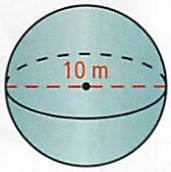
Problem 1 Finding the Surface Area of a Sphere

What is the surface area of the sphere in terms of π?

Plan

What are you given? In sphere problems, make it a habit to note whether you are given the radius or the diameter. In this case, you are given the diameter.

The diameter is 10 m, so the radius is $\frac{10}{2}$ m, or 5 m.



$\text{S.A.} = 4\pi r^2$ Use the formula for surface area of a sphere.

$= 4\pi(5)^2$ Substitute 5 for r.

$= 100\pi$ Simplify.

The surface area is 100π m^2.

Got It? **1.** What is the surface area of a sphere with a diameter of 14 in.? Give your answer in terms of π and rounded to the nearest square inch.

You can use spheres to approximate the surface areas of real-world objects.

Problem 2 Finding Surface Area

Plan

How can you use the length of Earth's equator? Earth's equator is a great circle that divides Earth into two hemispheres. Its length is Earth's circumference. Use it to find Earth's radius.

Geography **Earth's equator is about 24,902 mi long. What is the approximate surface area of Earth? Round to the nearest thousand square miles.**

Step 1 Find the radius of Earth.

$C = 2\pi r$ Use the formula for circumference.

$24{,}902 = 2\pi r$ Substitute 24,902 for C.

$\frac{24{,}902}{2\pi} = r$ Divide each side by 2π.

$r \approx 3963.276393$ Use a calculator.

Step 2 Use the radius to find the surface area of Earth.

$\text{S.A.} = 4\pi r^2$ Use the formula for surface area.

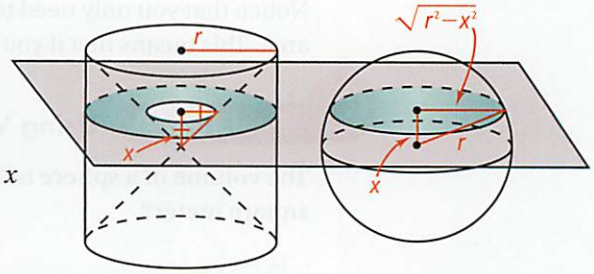
$= 4\pi$ ANS x^2 enter Use a calculator. ANS uses the value of r from Step 1.

≈ 197387017.5

The surface area of Earth is about 197,387,000 mi^2.

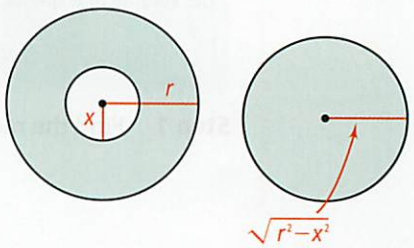
Got It? **2.** What is the surface area of a melon with circumference 18 in.? Round your answer to the nearest ten square inches.

In the previous lesson, you learned that the volume of a cone is $\frac{1}{3}\pi r^3$. You can use this with Cavalieri's Principle to find the formula for the volume of a sphere.



Both figures at the right have a parallel plane x units above their centers that form circular cross sections.

The area of the cross section of the cylinder minus the area of the cross section of the cone is the same as the area of the cross section of the sphere. Every horizontal plane will cut the figures into cross sections of equal area. By Cavalieri's Principle, the volume of the sphere = the volume of the cylinder = the volume of two cones.



$$\begin{aligned}\text{Volume of a sphere} &= \pi r^2(2r) - 2(\tfrac{1}{3}\pi r^3)\\ &= 2\pi r^3 - \tfrac{2}{3}\pi r^3\\ &= \tfrac{4}{3}\pi r^3\end{aligned}$$

take note

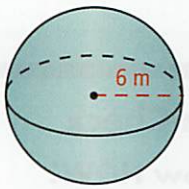
Theorem 11-11 Volume of a Sphere

The volume of a sphere is four thirds the product of π and the cube of the radius of the sphere.

$$V = \frac{4}{3}\pi r^3$$

Problem 3 Finding the Volume of a Sphere

What is the volume of the sphere in terms of π?



$V = \frac{4}{3}\pi r^3$ Use the formula for volume of a sphere.

$= \frac{4}{3}\pi(6)^3$ Substitute.

$= 288\pi$

The volume of the sphere is 288π m^3.

Think

What are the units of the answer?
You are cubing the radius, which is in meters (m), so your answer should be in cubic meters (m^3).

Got It? **3. a.** A sphere has a diameter of 60 in. What is its volume to the nearest cubic inch?

b. Reasoning Suppose the radius of a sphere is halved. How does this affect the volume of the sphere? Explain.

Notice that you only need to know the radius of a sphere to find its volume and surface area. This means that if you know the volume of a sphere, you can find its surface area.

Problem 4 Using Volume to Find Surface Area

The volume of a sphere is 5000 m^3. What is its surface area to the nearest square meter?

Know	Need	Plan
The volume of a sphere	The radius of the sphere	*Work backward* by using the formula for volume and solving for r. Then use the radius to calculate surface area.

Step 1 Find the radius of the sphere.

$V = \frac{4}{3}\pi r^3$ Use the formula for volume of a sphere.

$5000 = \frac{4}{3}\pi r^3$ Substitute.

$5000\left(\frac{3}{4\pi}\right) = r^3$ Solve for r^3.

$\sqrt[3]{5000\left(\frac{3}{4\pi}\right)} = r$ Take the cube root of each side.

$r \approx 10.60784418$ Use a calculator.

Step 2 Find the surface area of the sphere.

S.A. $= 4\pi r^2$ Use the formula for surface area of a sphere.

$= 4\pi$ ANS x^2 enter Use a calculator.

≈ 1414.04792

The surface area of the sphere is about 1414 m^2.

Got It? **4.** The volume of a sphere is 4200 ft^3. What is its surface area to the nearest tenth?

Lesson Check

Do you know HOW?

The diameter of a sphere is 12 ft.

1. What is its surface area in terms of π?
2. What is its volume to the nearest tenth?
3. The volume of a sphere is 80π cm^3. What is its surface area to the nearest whole number?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** What is the ratio of the area of a great circle to the surface area of the sphere?
5. **Error Analysis** Your classmate claims that if you double the radius of a sphere, its surface area and volume will quadruple. What is your classmate's error? Explain.

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Find the surface area of the sphere with the given diameter or radius. Leave your answer in terms of π. See Problem 1.

6. $d = 30$ m **7.** $r = 10$ in. **8.** $d = 32$ mm **9.** $r = 100$ yd

Sports **Find the surface area of each ball. Leave each answer in terms of π.**

10.

$d = 68$ mm

11.

$d = 21$ cm

12.

$d = 2\frac{1}{16}$ in.

Use the given circumference to find the surface area of each spherical object. Round your answer to the nearest whole number. See Problem 2.

13. a grapefruit with $C = 14$ cm

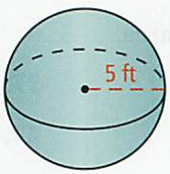
14. a bowling ball with $C = 27$ in.

15. a pincushion with $C = 8$ cm

16. a head of lettuce with $C = 22$ in.

Find the volume of each sphere. Give each answer in terms of π and rounded to the nearest cubic unit. See Problem 3.

17.



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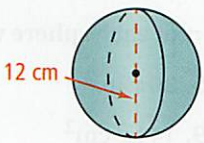

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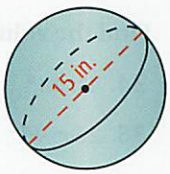

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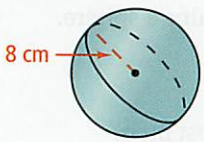

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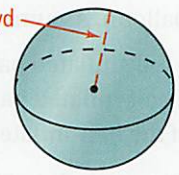

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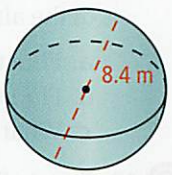


A sphere has the volume given. Find its surface area to the nearest whole number. See Problem 4.

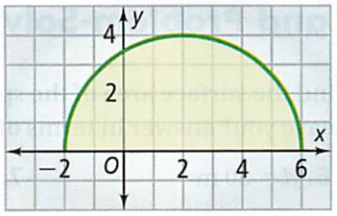
23. $V = 900$ in.^{3} **24.** $V = 3000$ m^3 **25.** $V = 140$ cm^3

B Apply

26. Mental Math Use $\pi \approx 3$ to estimate the surface area and volume of a sphere with radius 3 cm.

27. Open-Ended Give the dimensions of a cylinder and a sphere that have the same volume.

28. Visualization The region enclosed by the semicircle at the right is revolved completely about the x-axis.



a. Describe the solid of revolution that is formed.

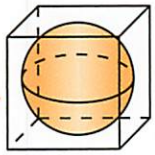
b. Find its volume in terms of π.

c. Find its surface area in terms of π.

29. Think About a Plan A cylindrical tank with diameter 20 in. is half filled with water. How much will the water level in the tank rise if you place a metallic ball with radius 4 in. in the tank? Give your answer to the nearest tenth.

- What causes the water level in the tank to rise?
- Which volume formulas should you use?

30. The sphere at the right fits snugly inside a cube with 6-in. edges. What is the approximate volume of the space between the sphere and cube?



(A) 28.3 in.^{3} (C) 102.9 in.^{3}

(B) 76.5 in.^{3} (D) 113.1 in.^{3}

STEM 31. Meteorology On September 3, 1970, a hailstone with diameter 5.6 in. fell at Coffeyville, Kansas. It weighed about 0.018 lb/in.^{3} compared to the normal 0.033 lb/in.^{3} for ice. About how heavy was this Kansas hailstone?

32. Reasoning Which is greater, the total volume of three spheres, each of which has diameter 3 in., or the volume of one sphere that has diameter 8 in.?

33. Reasoning How many great circles does a sphere have? Explain.

Find the volume in terms of π of each sphere with the given surface area.

34. 4π m^2 **35.** 36π in.^{2} **36.** 9π ft^2 **37.** 100π mm^2

38. 25π yd^2 **39.** 144π cm^2 **40.** 49π m^2 **41.** 225π mi^2

42. Recreation A spherical balloon has a 14-in. diameter when it is fully inflated. Half of the air is let out of the balloon. Assume that the balloon remains a sphere.

a. Find the volume of the fully inflated balloon in terms of π.

b. Find the volume of the half-inflated balloon in terms of π.

c. What is the diameter of the half-inflated balloon to the nearest inch?

43. Sports Equipment The diameter of a golf ball is 1.68 in.

a. Approximate the surface area of the golf ball.

b. Reasoning Do you think that the value you found in part (a) is greater than or less than the actual surface area of the golf ball? Explain.

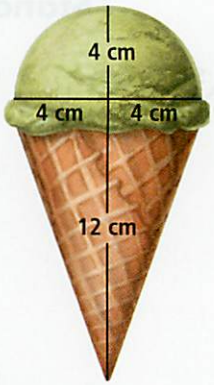
Geometry in 3 Dimensions **A sphere has center (0, 0, 0) and radius 5.**

44. Name the coordinates of six points on the sphere.

45. Tell whether each of the following points is *inside*, *outside*, or *on the sphere*.

a. $A(0, -3, 4)$ **b.** $B(1, -1, -1)$ **c.** $C(4, -6, -10)$

46. Food An ice cream vendor presses a sphere of frozen yogurt into a cone, as shown at the right. If the yogurt melts into the cone, will the cone overflow? Explain.

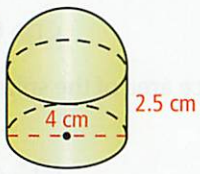


47. The surface area of a sphere is 5541.77 ft^2. What is its volume to the nearest tenth?

48. Geography The circumference of Earth at the equator is approximately 40,075 km. About 71% of Earth is covered by oceans and other bodies of water. To the nearest thousand square kilometers, how much of Earth's surface is land?

Find the surface area and volume of each figure.

49.



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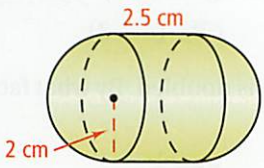

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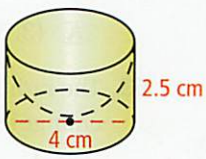


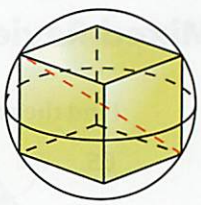
STEM **52. Astronomy** The diameter of Earth is about 7926 mi. The diameter of the moon is about 27% of the diameter of Earth. What percent of the volume of Earth is the volume of the moon? Round your answer to the nearest whole percent.

STEM **53. Science** The density of steel is about 0.28 lb/$in.^3$. Could you lift a solid steel ball with radius 4 in.? With radius 6 in.? Explain.

54. A cube with edges 6 in. long fits snugly inside a sphere as shown at the right. The diagonal of the cube is the diameter of the sphere.

a. Find the length of the diagonal and the radius of the sphere. Leave your answer in simplest radical form.

b. What is the volume of the space between the sphere and the cube to the nearest tenth?



Challenge **Find the radius of a sphere with the given property.**

55. The number of square meters of surface area equals the number of cubic meters of volume.

56. The ratio of surface area in square meters to volume in cubic meters is 1 : 5.

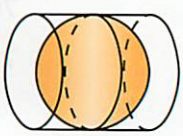
57. Suppose a cube and a sphere have the same volume.

a. Which has the greater surface area? Explain.

b. Writing Explain why spheres are rarely used for packaging.

58. A plane intersects a sphere to form a circular cross section. The radius of the sphere is 17 cm and the plane comes to within 8 cm of the center. Draw a sketch and find the area of the cross section, to the nearest whole number.

59. History At the right, the sphere fits snugly inside the cylinder. Archimedes (about 287–212 B.C.) requested that such a figure be put on his gravestone along with the ratio of their volumes, a finding that he regarded as his greatest discovery. What is that ratio?



Standardized Test Prep

SAT/ACT

60. What is the diameter of a sphere whose surface area is 100π m^2?

(A) 5 m (B) 10 m (C) 5π m (D) 25π m

61. Which of the following statements contradict each other?

I. Opposite sides of $\square ABCD$ are parallel.
II. Diagonals of $\square ABCD$ are perpendicular.
III. $\square ABCD$ is not a rhombus.

(F) I and II (G) II and III (H) I and III (I) none

62. What is the reflection image of (3, 7) across the line $y = 4$?

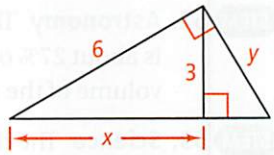
(A) (3, 3) (B) (−7, 3) (C) (3, 1) (D) (3, −7)

63. The radius of a sphere is doubled. By what factor does the surface area of the sphere change?

(F) $\frac{1}{4}$ (G) $\frac{1}{2}$ (H) 2 (I) 4

Short Response

64. Find the values of x and y. Show your work.



Mixed Review

Find the volume of each figure to the nearest cubic unit. See Lesson 11-5.

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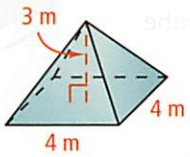

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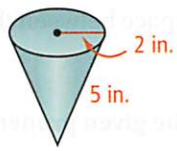

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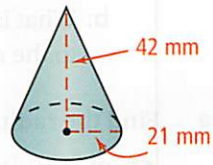

68. A leg of a right triangle has a length of 4 cm and the hypotenuse has a length of 7 cm. Find the measure of each acute angle of the triangle to the nearest degree. See Lessons 6-4 and 8-3.

69. The length of each side of a rhombus is 16. The longer diagonal has length 26. Find the measures of the angles of the rhombus to the nearest degree.

Get Ready! **To prepare for Lesson 11-7, do Exercises 70 and 71.**

Are the figures similar? If so, give the scale factor. See Lesson 7-2.

70. two squares, one with 3-in. sides and the other with 1-in. sides

71. two right isosceles triangles, one with a 3-cm hypotenuse and the other with a 1-cm leg