9-6 Dilations

Mathematics Florida Standards

MAFS.912.G-SRT.1.1a A dilation takes a line not passing through the center of the dilation to a parallel line, . . . **Also MAFS.912.G-SRT.1.1b, MAFS.912.G-CO.1.2, MAFS.912.G-SRT.1.2**

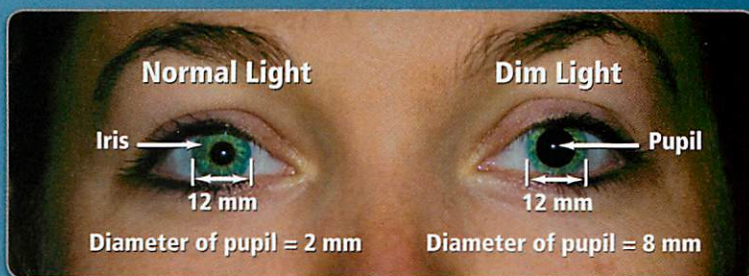
MP 1, MP 3, MP 4, MP 7

Objective To understand dilation images of figures

MATHEMATICAL PRACTICES

SOLVE IT! **Getting Ready!**

The pupil is the opening in the iris that lets light into the eye. Depending on the amount of light available, the size of the pupil changes.



Observe the size and shape of the iris in normal light and in dim light. What characteristics stay the same and what characteristics change? How do these observations compare to transformation of figures you have learned about earlier in the chapter?

Lesson Vocabulary
- dilation
- center of dilation
- scale factor of a dilation
- enlargement
- reduction

In the Solve It, you looked at how the pupil of an eye changes in size, or *dilates*. In this lesson, you will learn how to dilate geometric figures.

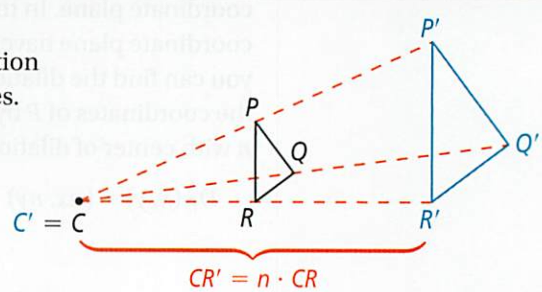
Essential Understanding You can use a scale factor to make a larger or smaller copy of a figure that is also similar to the original figure.

take note

Key Concept Dilation

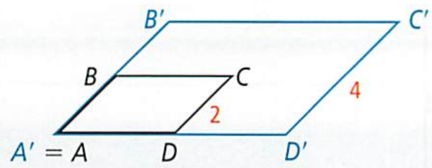
A **dilation** with **center of dilation** C and **scale factor** n, $n > 0$, can be written as $D_{(n,\, C)}$. A dilation is a transformation with the following properties.

- The image of C is itself (that is, $C' = C$).
- For any other point R, R' is on $\overrightarrow{CR}$ and $CR' = n \cdot CR$, or $n = \frac{CR'}{CR}$.
- Dilations preserve angle measure.

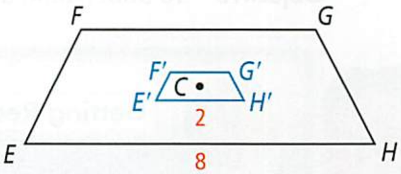


The scale factor n of a dilation is the ratio of a length of the image to the corresponding length in the preimage, with the image length always in the numerator. For the figure shown on page 587, $n = \frac{CR'}{CR} = \frac{R'P'}{RP} = \frac{P'Q'}{PQ} = \frac{Q'R'}{QR}$.

A dilation is an **enlargement** if the scale factor n is greater than 1. The dilation is a **reduction** if the scale factor n is between 0 and 1.



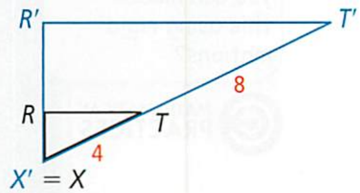
Enlargement
center A, scale factor 2



Reduction
center C, scale factor $\frac{1}{4}$

Problem 1 Finding a Scale Factor

Multiple Choice Is $D_{(n, X)}(\triangle XTR) = \triangle X'T'R'$ an enlargement or a reduction? What is the scale factor n of the dilation?



Ⓐ enlargement; $n = 2$

Ⓑ enlargement; $n = 3$

Ⓒ reduction; $n = \frac{1}{3}$

Ⓓ reduction; $n = 3$

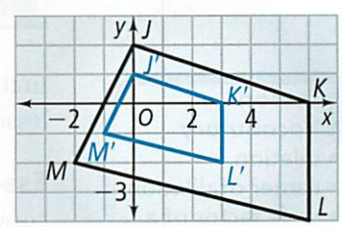
Think
Why is the scale factor not $\frac{4}{12}$, or $\frac{1}{3}$?
The scale factor of a dilation always has the image length (or the distance between a point on the image and the center of dilation) in the numerator.

The image is larger than the preimage, so the dilation is an enlargement.
Use the ratio of the lengths of corresponding sides to find the scale factor.

$$n = \frac{X'T'}{XT} = \frac{4+8}{4} = \frac{12}{4} = 3$$

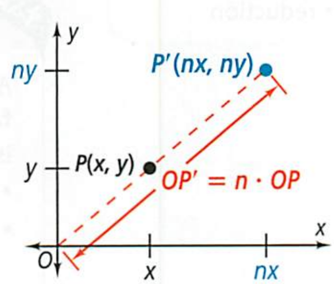
$\triangle X'T'R'$ is an enlargement of $\triangle XTR$, with a scale factor of 3. The correct answer is B.

Got It? 1. Is $D_{(n, O)}(JKLM) = J'K'L'M'$ an enlargement or a reduction? What is the scale factor n of the dilation?



In Got It 1, you looked at a dilation of a figure drawn in the coordinate plane. In this book, all dilations of figures in the coordinate plane have the origin as the center of dilation. So you can find the dilation image of a point $P(x, y)$ by multiplying the coordinates of P by the scale factor n. A dilation of scale factor n with center of dilation at the origin can be written as

$$D_n(x, y) = (nx, ny)$$

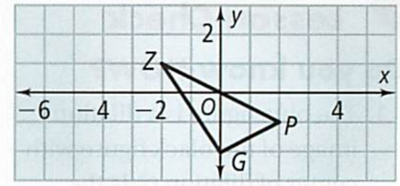


Think

Will the vertices of the triangle move closer to (0, 0) or farther from (0, 0)?
The scale factor is 2, so the dilation is an enlargement. The vertices will move farther from (0, 0).

Problem 2 Finding a Dilation Image

What are the coordinates of the vertices of $D_2(\triangle PZG)$? Graph the image of $\triangle PZG$.



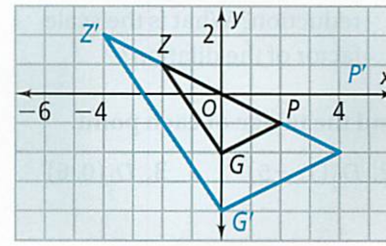
Identify the coordinates of each vertex. The center of dilation is the origin and the scale factor is 2, so use the dilation rule $D_2(x, y) = (2x, 2y)$.

$D_2(P) = (2 \cdot 2, 2 \cdot (-1))$, or $P'(4, -2)$.

$D_2(Z) = (2 \cdot (-2), 2 \cdot 1)$, or $Z'(-4, 2)$.

$D_2(G) = (2 \cdot 0, 2 \cdot (-2))$, or $G'(0, -4)$.

To graph the image of $\triangle PZG$, graph P', Z', and G'. Then draw $\triangle P'Z'G'$.



Got It? 2. a. What are the coordinates of the vertices of $D_{\frac{1}{2}}(\triangle PZG)$?

b. **Reasoning** How are $\overline{PZ}$ and $\overline{P'Z'}$ related? How are $\overline{PG}$ and $\overline{P'G'}$, and $\overline{GZ}$ and $\overline{G'Z'}$ related? Use these relationships to make a conjecture about the effects of dilations on lines.

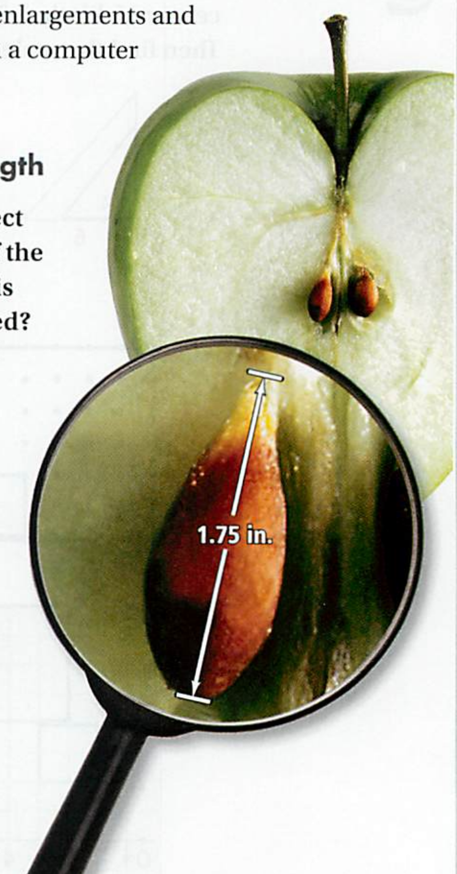
Dilations and scale factors help you understand real-world enlargements and reductions, such as images seen through a microscope or on a computer screen.

Think

What does a scale factor of 7 tell you?
A scale factor of 7 tells you that the ratio of the image length to the actual length is 7, or $\frac{\text{image length}}{\text{actual length}} = 7$.

Problem 3 Using a Scale Factor to Find a Length

Biology **A magnifying glass shows you an image of an object that is 7 times the object's actual size. So the scale factor of the enlargement is 7. The photo shows an apple seed under this magnifying glass. What is the actual length of the apple seed?**



$1.75 = 7 \cdot p$	image length = scale factor • actual length
$0.25 = p$	Divide each side by 7.

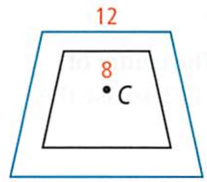
The actual length of the apple seed is 0.25 in.

Got It? 3. The height of a document on your computer screen is 20.4 cm. When you change the zoom setting on your screen from 100% to 25%, the new image of your document is a dilation of the previous image with scale factor 0.25. What is the height of the new image?

Lesson Check

Do you know HOW?

1. The blue figure is a dilation image of the black figure with center of dilation C. Is the dilation an enlargement or a reduction? What is the scale factor of the dilation?



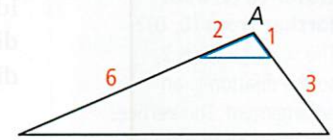
Find the image of each point.

2. $D_2(1, -5)$
3. $D_{\frac{1}{2}}(0, 6)$
4. $D_{10}(0, 0)$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** Describe the scale factor of a reduction.

6. **Error Analysis** The blue figure is a dilation image of the black figure for a dilation with center A.



Two students made errors when asked to find the scale factor. Explain and correct their errors.

A. $\cancel{n = \frac{2}{6} = \frac{1}{3}}$

B. $\cancel{n = \frac{4}{1} = 4}$

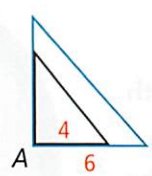
Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

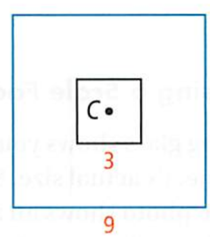
The blue figure is a dilation image of the black figure. The labeled point is the center of dilation. Tell whether the dilation is an enlargement or a reduction. Then find the scale factor of the dilation.

See Problem 1.

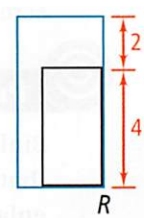
7.



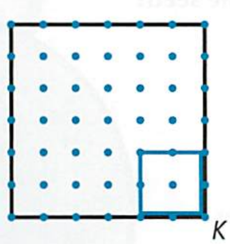
8.



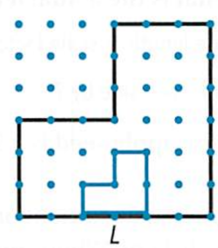
9.



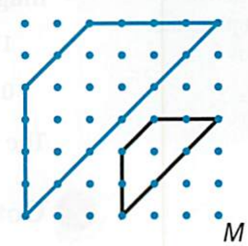
10.



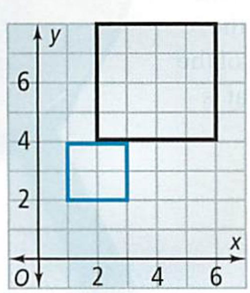
11.



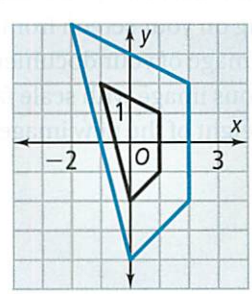
12.



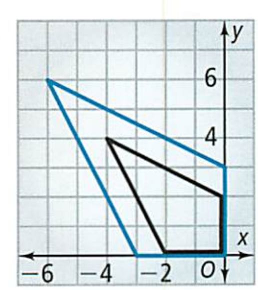
13.



14.

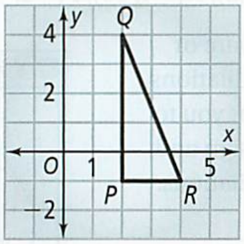


15.

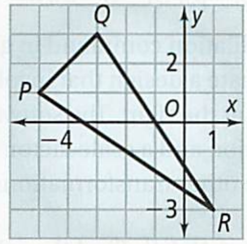


Find the images of the vertices of $\triangle PQR$ for each dilation. Graph the image. See Problem 2.

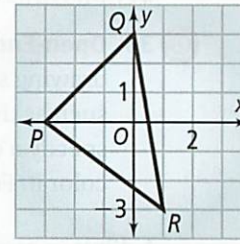
16. $D_3(\triangle PQR)$



17. $D_{10}(\triangle PQR)$



18. $D_{\frac{3}{4}}(\triangle PQR)$



Magnification **You look at each object described in Exercises 19–22 under a magnifying glass. Find the actual dimension of each object.** See Problem 3.

19. The image of a button is 5 times the button's actual size and has a diameter of 6 cm.

20. The image of a pinhead is 8 times the pinhead's actual size and has a width of 1.36 cm.

21. The image of an ant is 7 times the ant's actual size and has a length of 1.4 cm.

22. The image of a capital letter N is 6 times the letter's actual size and has a height of 1.68 cm.

B Apply

Find the image of each point for the given scale factor.

23. $L(-3, 0)$; $D_5(L)$

24. $N(-4, 7)$; $D_{0.2}(N)$

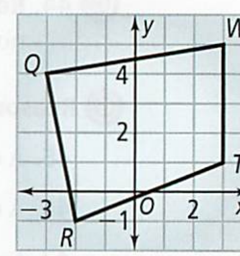
25. $A(-6, 2)$; $D_{1.5}(A)$

26. $F(3, -2)$; $D_{\frac{1}{3}}(F)$

27. $B\left(\frac{5}{4}, -\frac{3}{2}\right)$; $D_{\frac{1}{10}}(B)$

28. $Q\left(6, \frac{\sqrt{3}}{2}\right)$; $D_{\sqrt{6}}(Q)$

Use the graph at the right. Find the vertices of the image of $QRTW$ for a dilation with center (0, 0) and the given scale factor.



29. $\frac{1}{4}$

30. 0.6

31. 0.9

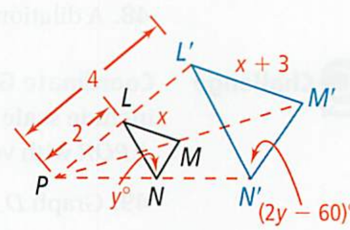
32. 10

33. 100

34. Compare and Contrast Compare the definition of scale factor of a dilation to the definition of scale factor of two similar polygons. How are they alike? How are they different?

35. Think About a Plan The diagram at the right shows $\triangle LMN$ and its image $\triangle L'M'N'$ for a dilation with center P. Find the values of x and y. Explain your reasoning.

- What is the relationship between $\triangle LMN$ and $\triangle L'M'N'$?
- What is the scale factor of the dilation?
- Which variable can you find using the scale factor?



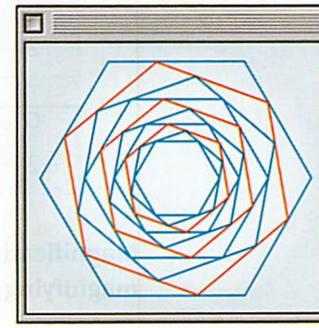
36. Writing An equilateral triangle has 4-in. sides. Describe its image for a dilation with center at one of the triangle's vertices and scale factor 2.5.

Coordinate Geometry **Graph $MNPQ$ and its image $M'N'P'Q'$ for a dilation with center (0, 0) and the given scale factor.**

37. $M(1, 3), N(-3, 3), P(-5, -3), Q(-1, -3); 3$

38. $M(2, 6), N(-4, 10), P(-4, -8), Q(-2, -12); \frac{1}{4}$

39. Open-Ended Use the dilation command in geometry software or drawing software to create a design that involves repeated dilations, such as the one shown at the right. The software will prompt you to specify a center of dilation and a scale factor. Print your design and color it. Feel free to use other transformations along with dilations.



A dilation maps $\triangle HIJ$ onto $\triangle H'I'J'$. Find the missing values.

40. $HI = 8$ in. $H'I' = 16$ in.
$IJ = 5$ in. $I'J' = $ ■ in.
$HJ = 6$ in. $H'J' = $ ■ in.

41. $HI = $ ■ ft $H'I' = 8$ ft
$IJ = 30$ ft $I'J' = $ ■ ft
$HJ = 24$ ft $H'J' = 6$ ft

42. Let ℓ be a line through the origin. Show that $D_k(\ell) = \ell$ by showing that if $C = (c_1, c_2)$ is on ℓ, then $D_k(C)$ is also on ℓ.

43. Let $A = (a_1, a_2)$ and $B = (b_1, b_2)$, let $A' = D_k(A)$ and $B' = D_k(B)$ with $k \neq 1$, and suppose that $\overleftrightarrow{AB}$ does not pass through the origin.

a. Show that $\overleftrightarrow{AB} \neq \overleftrightarrow{A'B'}$ (*Hint:* What happens to the x- and y-intercepts of $\overleftrightarrow{AB}$ under the dilation D_k?)

b. Suppose that $a_1 \neq b_1$. Show that $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{A'B'}$ by showing that they have the same slope.

c. Show that $\overleftrightarrow{AB} \,||\, \overleftrightarrow{A'B'}$ if $a_1 = b_1$.

44. Reasoning You are given $\overline{AB}$ and its dilation image $\overline{A'B'}$ with A, B, A', and B' noncollinear. Explain how to find the center of dilation and scale factor.

Reasoning **Write *true* or *false* for Exercises 45–48. Explain your answers.**

45. A dilation is an isometry.

46. A dilation with a scale factor greater than 1 is a reduction.

47. For a dilation, corresponding angles of the image and preimage are congruent.

48. A dilation image cannot have any points in common with its preimage.

Challenge

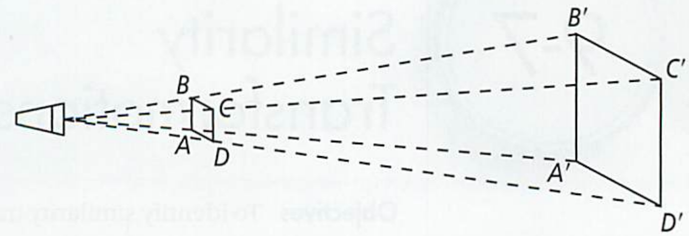
Coordinate Geometry **In the coordinate plane, you can extend dilations to include scale factors that are negative numbers. For Exercises 49 and 50, use $\triangle PQR$ with vertices $P(1, 2)$, $Q(3, 4)$, and $R(4, 1)$.**

49. Graph $D_{-3}(\triangle PQR)$.

50. a. Graph $D_{-1}(\triangle PQR)$.

b. Explain why the dilation in part (a) may be called a *reflection through a point.* Extend your explanation to a new definition of point symmetry.

51. Shadows A flashlight projects an image of rectangle $ABCD$ on a wall so that each vertex of $ABCD$ is 3 ft away from the corresponding vertex of $A'B'C'D'$. The length of $\overline{AB}$ is 3 in. The length of $\overline{A'B'}$ is 1 ft. How far from each vertex of $ABCD$ is the light?



Standardized Test Prep

SAT/ACT

52. A dilation maps $\triangle CDE$ onto $\triangle C'D'E'$. If $CD = 7.5$ ft, $CE = 15$ ft, $D'E' = 3.25$ ft, and $C'D' = 2.5$ ft, what is DE?

(A) 1.08 ft (B) 5 ft (C) 9.75 ft (D) 19 ft

53. You want to prove indirectly that the diagonals of a rectangle are congruent. As the first step of your proof, what should you assume?

(F) A quadrilateral is not a rectangle.

(G) The diagonals of a rectangle are not congruent.

(H) A quadrilateral has no diagonals.

(I) The diagonals of a rectangle are congruent.

54. Which word can describe a kite?

(A) equilateral (B) equiangular (C) convex (D) scalene

Short Response

55. Use the figure at the right to answer the questions below.

a. Does the figure have rotational symmetry? If so, identify the angle of rotation.

b. Does the figure have reflectional symmetry? If so, how many lines of symmetry does it have?

Mixed Review

56. $\triangle JKL$ has vertices $J(23, 2)$, $K(4, 1)$, and $L(1, 23)$. What are the coordinates of J', K', and L' if $(R_{x\text{-axis}} \circ T_{<2, -3>})(\triangle JKL) = \triangle J'K'L'$?

See Lesson 9-5.

Get Ready! **To prepare for Lesson 9-7, do Exercises 55–57.**

Algebra $TRSU \sim NMYZ$. Find the value of each variable.

See Lesson 7-2.

57. a **58.** b **59.** c