Maharashtra State Board
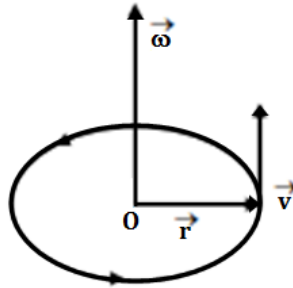## Class XII Physics
Board Paper –2016 Solution

SECTION I

1.

(i) **Analytical method:**

Consider a particle revolving in the anticlockwise sense along the circumference of a circle of radius r with centre O as shown.

Let $\vec{\omega}$ = angular velocity of the particle

$\quad\vec{v}$ = linear velocity of the particle

$\quad\vec{r}$ = radius vector of the particle

In vector form, the linear displacement is

$$\vec{\delta s} = \vec{\delta\theta} \times \vec{r}$$

Dividing both side by δt, we get

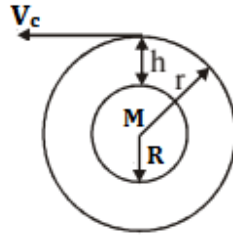$$\frac{\vec{\delta s}}{\delta t} = \frac{\vec{\delta\theta}}{\delta t} \times \vec{r}$$

$$\lim_{\delta t \to 0} \frac{\vec{\delta s}}{\delta t} = \lim_{\delta t \to 0} \frac{\vec{\delta\theta}}{\delta t} \times \vec{r}$$

$$\therefore \frac{\vec{ds}}{dt} = \frac{\vec{d\theta}}{\delta t} \times \vec{r}$$

But, $\dfrac{\vec{ds}}{dt} = \vec{v}$ = Linear velocity and $\dfrac{\vec{d\theta}}{\delta t} = \omega$ = Angular velocity

$$\therefore \vec{v} = \omega \times \vec{r}$$

(ii)

Consider a satellite of mass m revolving round the Earth at a height 'h' above the surface of the Earth.

Let M be the mass and R be the radius of the Earth.

The satellite is moving with velocity V_c and the radius of the circular orbit is r = R + h.

Centripetal force = Gravitational force

$$\therefore \frac{mV_c^2}{r} = \frac{GMm}{r^2}$$

$$\therefore V_c^2 = \frac{GM}{r}$$

$$\therefore V_c = \sqrt{\frac{GM}{R+h}} \quad \ldots\ldots (Equation\ 1)$$

This is the expression for critical velocity of a satellite moving in a circular orbit around the Earth.

We know that,

$$g_h = \frac{GM}{(R+h)^2}$$

$$\therefore GM = g_h (R+h)^2$$

Substituting in equation 1, we get

$$\therefore V_c = \sqrt{\frac{g_h (R+h)^2}{R+h}}$$

$$\therefore V_c = \sqrt{g_h (R+h)}$$

where g_h is the acceleration due to gravity at a height h above the surface of the Earth.

(iii) Let M and R be the mass and radius of the body, V is the translation speed, ω is the angular speed and I is the moment of inertia of the body about an axis passing through the centre of mass.

Kinetic energy of rotation, $E_R = \dfrac{1}{2}MV^2$

Kinetic energy of translation, $E_T = \dfrac{1}{2}I\omega^2$

Thus, the total kinetic energy 'E' of the rolling body is

$E = E_R + E_T$

$\quad = \dfrac{1}{2}MV^2 + \dfrac{1}{2}I\omega^2$

$\quad = \dfrac{1}{2}MV^2 + \dfrac{1}{2}MK^2\omega^2$ .... $(I = MK^2$ and K is the radius of gyration)

$\quad = \dfrac{1}{2}MR^2\omega^2 + \dfrac{1}{2}MK^2\omega^2$ .... $(\because V = R\omega)$

$\therefore E = \dfrac{1}{2}M\omega^2\left(R^2 + K^2\right)$

$\therefore E = \dfrac{1}{2}M\dfrac{V^2}{R^2}\left(R^2 + K^2\right)$

$\therefore E = \dfrac{1}{2}MV^2\left(1 + \dfrac{K^2}{R^2}\right)$

Hence proved.

(iv) Emissive power of a body at a given temperature is the quantity of radiant energy emitted by the body per unit time per unit surface area of the body at that temperature.

If 'Q' is the amount of radiant energy emitted, 'A' is the surface area of the body and 't' is the time for which body radiates energy, then the emissive power is

$E = \dfrac{Q}{At}$

Coefficient of emission of a body is the ratio of the emissive power of the body at a given temperature to the emissive power of a perfectly black body at the same temperature.

Coefficient of emission, $e = \dfrac{E}{E_b}$

(v) Given that r = 5 cm = 0.05 m,

$\qquad$ n = 90 r.p.m. = 1.5 Hz

Limiting force of static friction = Centrifugal force

$$\mu_s mg = mr\omega^2$$

$$\therefore \mu_s = \frac{r\omega^2}{g} = \frac{r(2\pi n)^2}{g}$$

$$\therefore \mu_s = \frac{4r\pi^2 n^2}{g}$$

$$\therefore \mu_s = \frac{4 \times 0.05 \times (3.14)^2 \times (1.5)^2}{9.8}$$

$$\therefore \mu_s = 0.4527$$

(vi) Given that $n_3 = n_o$

where n_3 = frequency of the third overtone of the open pipe

n_o = fundamental frequency of the closed pipe

Third overtone of open pipe is

$$n_3 = 4\left(\frac{V}{2L_3}\right)$$

Fundamental frequency of closed pipe at one end is

$$n_o = \frac{V}{4L_o}$$

$$\frac{V}{4L_o} = 4\left(\frac{V}{2L_3}\right) \quad \text{........} \quad (\because n_o = n_3)$$

$$\therefore \frac{L_o}{L_3} = \frac{1}{8}$$

$$\therefore L_o : L_3 = 1 : 8$$

(vii)

Here, T = 6.28 sec,

Pathlength = 20 cm $\rightarrow$ a = 10 cm = 10×10^{-2}m

$x = 6 \times 10^{-2}$m

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6.28s} = 1 \text{ rad/s}$$

$$v = \pm\omega\sqrt{(100 - 36) \times 10^{-4}}$$

$$= 1 \times 8 \times 10^{-2}$$

$$\therefore v = 8 \times 10^{-2} \text{m/s}$$

(viii) Given that E = 5πT

Surface energy E = T $\times$ dA -------- (Equation 1)

dA = $4\pi r^2$ -------- (where r is the radius of the liquid drop)

Substituting in Equation 1, we get

E = T $\times$ $4\pi r^2$

5πT = T $\times$ $4\pi r^2$ -------- (since E = 5πT)

$$\therefore r^2 = \frac{5}{4}$$

$$\therefore r = \frac{\sqrt{5}}{2}$$

Diameter, $d = 2r = 2 \times \dfrac{\sqrt{5}}{2}$

$$\therefore d = \sqrt{5} = 2.23 \text{ cm}$$

2.

(i) **(d)** $\dfrac{2vt}{D}$

Velocity $= v$; Diameter $= D \rightarrow$ Radius $= \dfrac{D}{2}$

$\omega = \dfrac{\theta}{t}$ and $v = r \times \omega$

$$\therefore v = \frac{D}{2} \times \frac{\theta}{t}$$

$$\therefore \theta = \frac{2vt}{D}$$

(ii) **(b)** $W_1 < W_2$

$F_1 = K_1 x_1$ and $F_2 = K_2 x_2$

Force is same

$$\therefore K_1 x_1 = K_2 x_2$$

$\therefore x_1 < x_2$ .... $(\because K_1 > K_2)$

Work done, $W_1 = \dfrac{1}{2} K_1 x_1^2$ and $W_2 = \dfrac{1}{2} K_2 x_1^2$

$\therefore W_1 < W_2$........ $(\because K_1 > K_2$ and $x_1 < x_2)$

(iii) **(a)** four times that of A.

$$\frac{\text{Stress on B}}{\text{Stress on A}} = \frac{r_A^2}{r_B^2}$$

$$\rightarrow \frac{\text{Stress on B}}{\text{Stress on A}} = \frac{(2r_B)^2}{r_B^2} \quad \text{........} \quad (\because r_A = 2r_B)$$

$$\therefore \frac{\text{Stress on B}}{\text{Stress on A}} = 4$$

$\rightarrow$ Stress on B $= 4 \times$ Stress on A

(iv) **(d)** π rad

There is a phase change of 180°, i.e. the phase of the wave changes by π radians.

(v) **(b)** $n_1 \sqrt{\dfrac{\sigma - 1}{\sigma}}$

Specific gravity or relative density is

$\dfrac{n_1}{n_2} = \sqrt{\dfrac{\sigma}{\sigma - 1}}$

$\therefore n_2 = n_1 \sqrt{\dfrac{\sigma - 1}{\sigma}}$

(vi) **(c)** $\dfrac{4 + f}{3 + f}$

By the law of equipartition of energy, for one mole of polyatomic gas

$C_P = (4 + f)R$ and $C_V = (3 + f)R$

$\therefore \dfrac{C_P}{C_V} = \dfrac{(4 + f)R}{(3 + f)R} = \dfrac{(4 + f)}{(3 + f)}$

(vii) **(c)** 3 m/s

Given that $\dfrac{1}{2} I \omega^2 = \dfrac{1}{2} mv^2$

$\therefore \dfrac{1}{2} \times 5 \ kgm^2 \times 6^2 \ rad/s = \dfrac{1}{2} \times 20 \ kg \times v^2$

$\therefore v^2 = 9$

$\therefore v = 3 \, m/s$

3. Linear S.H.M. is defined as the linear periodic motion of a body in which the restoring force (or acceleration) is always directed towards its mean position and its magnitude is directly proportional to the displacement from the mean position.
Consider a particle 'P' moving along the circumference of a circle of radius 'a' and centre O, with uniform angular speed of 'ω' in anticlockwise direction as shown.
Particle P along circumference of the circle has its projection particle on diameter AB at point M.
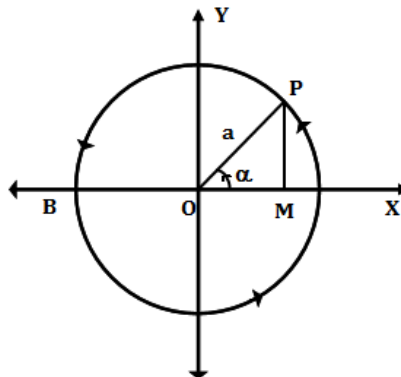
Suppose that particle P starts from the initial position with initial phase α (angle between radius OP and the x-axis at the time t = 0).

In time t, the angle between OP and x-axis is $(\omega t + \alpha)$ as particle P moving with constant angular velocity (ω) as shown.
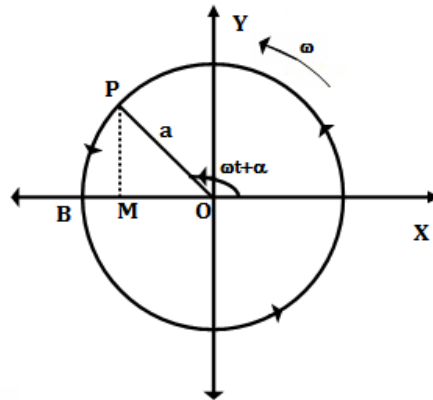
$$\cos(\omega t + \alpha) = \frac{x}{a}$$

$\therefore x = a\cos(\omega t + \alpha)$ .... (Equation 1)

This is the expression for displacement of particle M at time t.

As velocity of the particle is the time rate of change of displacement then we have

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[a\cos(\omega t + \alpha)\right]$$

$\therefore v = -a\omega\sin(\omega t + \alpha)$ .... (Equation 2)

As acceleration of particle is the time rate of change of velocity, we have

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[-a\omega\sin(\omega t + \alpha)\right]$$

$\therefore a = -a\omega^2\cos(\omega t + \alpha)$

$\therefore a = -\omega^2 x$

Hence, the projection of a uniform circular motion on a diameter of a circle is simple harmonic motion.

Numerical:

Given that for the metal sphere

$$\left(\frac{d\theta}{dt}\right)_1 = 4°C \, / \, min,$$

$\theta_1 = 50°C$, $\theta_2 = 45°C$ and $\theta_o = 25°C$

By Newton's law of cooling,

$$\left(\frac{d\theta}{dt}\right) = k\left(\theta - \theta_o\right)$$

$$\therefore \frac{\left(d\theta/dt\right)_1}{\left(d\theta/dt\right)_2} = \frac{\left(\theta_1 - \theta_o\right)}{\left(\theta_2 - \theta_o\right)}$$

$$\therefore \frac{\left(d\theta/dt\right)_1}{\left(d\theta/dt\right)_2} = \frac{\left(50° - 25°\right)}{\left(45° - 25°\right)}$$

$$\therefore \left(\frac{d\theta}{dt}\right)_2 = \frac{20°}{25°} \times \left(\frac{d\theta}{dt}\right)_1 = \frac{20°}{25°} \times 4 = 3.2$$

$$\therefore \left(\frac{d\theta}{dt}\right)_2 = 3.2°C \, / \, min$$

OR

Consider two simple harmonic progressive waves of equal amplitude and frequency propagating on a long uniform string in opposite directions.

If wave of frequency 'n' and wavelength 'l' is travelling along the positive X – axis, then

$$Y_1 = a \sin 2\pi \left(nt - \frac{x}{\lambda} \right) \dots\dots \text{ (Equation 1)}$$

If wave of frequency 'n' and wavelength 'l' is travelling along the negative X – axis, then

$$Y_2 = a \sin 2\pi \left(nt + \frac{x}{\lambda} \right) \dots\dots \text{ (Equation 2)}$$

These waves interfere to produce stationary waves. The resultant displacement of stationary waves is given by the principle of superposition of waves.

$$Y = Y_2 + Y_1$$

$$= a \sin 2\pi \left(nt + \frac{x}{\lambda} \right) + a \sin 2\pi \left(nt - \frac{x}{\lambda} \right)$$

Uisng $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$
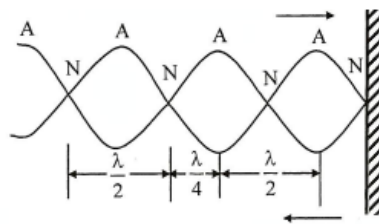
and $\cos(-\theta) = \cos \theta$, we get

$$\therefore Y = 2a \left[\sin(2\pi nt) \cos \left(\frac{2\pi x}{\lambda} \right) \right]$$

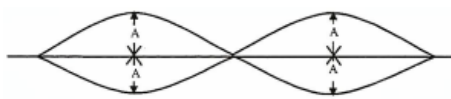$$\therefore Y = \left(2a \cos \frac{2\pi x}{\lambda} \right) \sin(2\pi nt)$$

or $Y = A \sin(2\pi nt)$

where A = Amplitude of resultant wave = $2a \cos \dfrac{2\pi x}{\lambda}$

or $Y = A \sin \omega t$ ....(where $\omega = 2\pi n$)

Position of nodes and antinodes on staionary wave

Position of antinodes

Amplitude of anitodes is maximum, $A = \pm 2a$

$$A = 2a\cos\frac{2\pi x}{\lambda}$$

$$\therefore \pm 2a = 2a\cos\frac{2\pi x}{\lambda}$$

$$\therefore \cos\frac{2\pi x}{\lambda} = \pm 1$$

$$\therefore \frac{2\pi x}{\lambda} = 0,\ \pi,\ 2\pi....$$

or $\dfrac{2\pi x}{\lambda} = P\pi$

$$\therefore x = \frac{P\lambda}{2} = P\left(\frac{\lambda}{2}\right)\ldots\ldots(\text{where } P = 0, 1, 2...)$$

For $x = 0, \dfrac{\lambda}{2}, \lambda, \dfrac{3\lambda}{2}, ...$ antinodes are produced.

Thus, distance between any two successive antinodes is $\dfrac{\lambda}{2}$.

Amplitude of nodes is zero, $A = 0$

$$\therefore A = 2a\cos\frac{2\pi x}{\lambda}$$

$$\therefore 0 = 2a\cos\frac{2\pi x}{\lambda}$$

$$\therefore \cos\frac{2\pi x}{\lambda} = 0$$

$$\therefore \frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}....$$

$$\therefore x = (2P - 1)\frac{\lambda}{4}\ \ldots\ldots\ldots(\text{where } P = 1, 2...)$$

For $x = \dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}....$ nodes are produced.

Thus, distance between any two successive nodes is $\dfrac{\lambda}{2}$.

The distance between node and adjacent anitnodes is $\dfrac{\lambda}{4}$.

Numerical:

Given that $n_1 = 1.5\, n_{48}$ and beat frequency = 5 Hz

The set of tuning forks are arranged in decreasing order of frequencies.

$\therefore n_2 = n_1 - 4$

$n_3 = n_2 - 4 = n_1 - 2 \times 4$

$n_{48} = n_{47} - 4 = n_1 - 47 \times 4$

$\therefore n_{48} = n_1 - 188$

$\therefore n_{48} = 1.5 n_{48} - 188 \,\,........\left(n_1 = 1.5 n_{48} \right)$

$\therefore 0.5\, n_{48} = 188$

$\therefore n_{48} = 376$

$\rightarrow n_1 = 1.5 n_{48} = 1.5 \times 376 = 564$

$n_{42} = n_{41} - 4 = n_1 - 4 \times 41$

$\therefore n_{42} = n_1 - 160 = 564 - 164$

$\therefore n_{42} = 400$ Hz

4.

(i) Given that m = 600 kg, d = 5000 m,

R = 6400 km = 6.4×10^6 m

Weight of the body on the surface of the Earth = $600 \times 9.8 = 5880$ N

At depth d, gravitation acceleration is

$$g_d = g\left[1 - \frac{d}{R} \right]$$

$$\therefore g_d = g\left[1 - \frac{5}{6400} \right] = 9.8 \times 0.999$$

$\therefore g_d = 9.7902$ m/s^2

Weight on surface = mg

$$= 600 \times 9.8$$

$\therefore$ Weight on surface = 5880 N

Weight of the body at depth = mg_d

$$= 600 \times 9.7902$$

$$= 5874\text{ N}$$

$\therefore$ Decrease in weight = mg - mg_d

$$= 5880\text{ N} - 5874\text{ N}$$

$\therefore$ Decrease in weight = 6N

(ii) **Theorem of parallel axes:** The moment of inertia of a body about any axis is equal to the sums of its moment of inertia about a parallel axis passing through its centre of mass and the product of its mass and the square of the perpendicular distance between the two parallel axes.
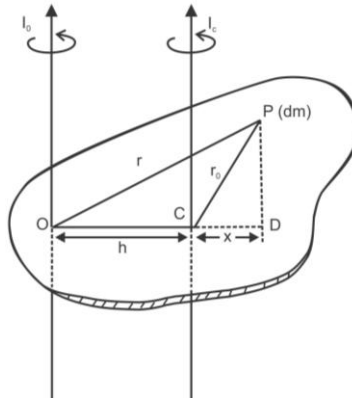
Consider a rigid of mass 'M' rotating about an axis passing through a point 'O' and perpendicular to the plane of the figure.
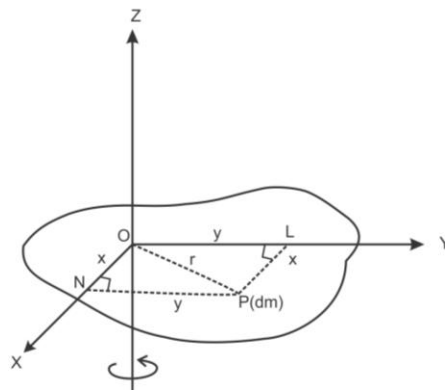
Let 'I_o' be the moment of inertia of the body about an axis passing through point 'O'. Take another parallel axis of rotation passing through the centre of mass of the body.

Let 'I_c' be the moment of inertia of the body about point 'C'.

Let the distance between the two parallel axes be OC = h.

OP = r and CP = r_o

Take a small element of body of mass 'dm' situated at a point P. Join OP and CP, then

$$I_o = \int OP^2 dm = \int r^2 dm$$

$$I_C = \int CP^2 dm = \int r_0^2 dm$$

From point P draw a perpendicular to OC produced.

Let CD = x

From the figure,

$$OP^2 = OD^2 + PD^2$$

$$\therefore OP^2 = (h + CD)^2 + PD^2$$

$$= h^2 + CD^2 + 2hCD + PD^2$$

$$\therefore OP^2 = CP^2 + h^2 + 2hCD \ldots\ldots\ldots \left(\because CD^2 + PD^2 = CP^2\right)$$

$$\therefore r^2 = r_0^2 + h^2 + 2hx$$

Multiplying the above equation with 'dm' on both the sides and integrating, we get

$$\int r^2 dm = \int r_0^2 dm + \int h^2 dm + \int 2hx dm$$

$$\therefore \int r^2 dm = \int r_0^2 dm + \int h^2 dm + 2h \int x dm$$

$\int x dm = 0$ as 'C' is the centre of mass and algebriac sum of moments of all the particles about the centre of mass is always zero, for body in equilibrium.

$$\therefore \int r^2 dm = \int r_0^2 dm + h^2 \int dm + 0 \ldots\ldots\ldots \text{(Equation 1)}$$
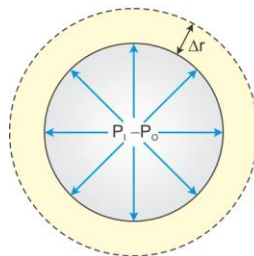
But $\int dm = M = \text{Mass of the body}$,

$\int r^2 dm = I_o$ and $\int r_0^2 dm = I_c$

Substituing in equation 1, we get

$$I_o = I_c + Mh^2$$

This proves the theorm of parallel axes about moment of inertia.

(iii) Consider a spherical liquid drop and let the outside pressure be P_o and inside pressure be P_i, such that the excess pressure is $P_i - P_o$.

Let the radius of the drop increase from r to Δr, where Δr is very small, so that the pressure inside the drop remains almost constant.

Initial surface area $(A_1) = 4\pi r^2$

Final surface area $(A_2) = 4\pi(r + \Delta r)^2$

$$= 4\pi(r^2 + 2r\Delta r + \Delta r^2)$$

$$= 4\pi r^2 + 8\pi r\Delta r + 4\pi\Delta r^2$$

As Δr is very small, Δr^2 is neglected (i.e. $4\pi\Delta r^2 \cong 0$)

Increase in surface area (dA) = $A_2 - A_1 = 4\pi r^2 + 8\pi r\Delta r - 4\pi r^2$

Increase in surface area (dA) = $8\pi r\Delta r$

Work done to increase the surface area $8\pi r\Delta r$ is extra energy.

$\therefore$ dW = TdA

$\therefore$ dW = T $\times$ $8\pi r\Delta r$ --------- (Equation 1)

This work done is equal to the product of the force and the distance Δr.

dF = $(P_i - P_o)$ $4\pi r^2$

The increase in the radius of the bubble is Δr.

dW = dFΔr = $(P_i - P_o)$ $4\pi r^2$ $\times$ Δr --------- (Equation 2)

Comparing Equations 1 and 2, we get

$(P_i - P_o)$ $4\pi r^2$ $\times$ Δr = T $\times$ $8\pi r\Delta r$

$\therefore (P_i - P_o) = 2T/r$

This is called the Laplace's law of spherical membrane.

(iv) Given that A = 1.5 mm^2, lateral strain = 1.5×10^{-5},

$\qquad\qquad$ Y_{steel} = 2×10^{11} N/m^2, σ = 0.291 and g = 9.8 m/s^2

Poisson's ratio, $\sigma = \dfrac{\text{Lateral strain}}{\text{Longitudinal strain}}$

$0.291 = \dfrac{1.5 \times 10^{-5}}{\text{Longitudinal strain}}$

$\therefore$ Longitudinal strain $= \dfrac{1.5 \times 10^{-5}}{0.291} = 5.14 \times 10^{-5}$

Longitudinal stress $= Y \times$ Longitudinal strain

$\qquad\qquad\qquad = 2 \times 10^{11} \times 5.14 \times 10^{-5}$

$\therefore$ Longitudinal stress $= 10.28 \times 10^6$

But longitudinal stress $= \dfrac{Mg}{A}$

$\rightarrow 10.28 \times 10^6 = \dfrac{M \times 9.8}{1.5 \times 10^{-6}}$

$\therefore M = \dfrac{10.28 \times 10^6 \times 1.5 \times 10^{-6}}{9.8} = \dfrac{15.42}{9.8}$

$\therefore M = 1.58$ kg

5.

(i) Bending of light near the edges of an obstacle or slit and spreading into the region of geometrical shadow is known as diffraction of light.

The diffraction phenomenon is classified into two types:

1. **Fraunhofer diffraction:** The source of light and the screen on which the diffraction pattern is obtained are effectively at infinite distance from the diffracting system. In this case, we consider plane wavefront. The diffraction pattern is obtained by using a convex lens.

2. **Fresnel diffraction:** The source of light and the screen are kept at finite distance from the diffracting system. In this case, we consider a cylindrical or spherical wavefront.

(ii)
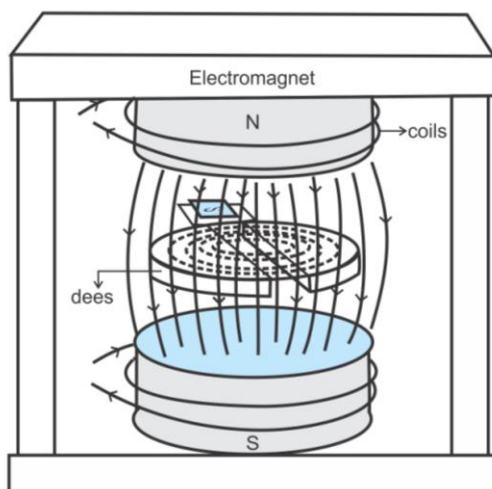
(iii)

Paramagnetic substance	Ferromagnetic substance
Substances which are weakly attracted by a magnet are called paramagnetic substances.	Substances which are strongly attracted by a magnet are called ferromagnetic substances.
Paramagnetic materials lose their magnetism on removal of the external field and hence cannot be used to make permanent magnets.	Ferromagnetic materials retain some magnetism on removal of external field and hence can be used to make permanent magnets.
The susceptibility is positive but small.	The susceptibility is positive but very high.
In the absence of electric field, the net dipole moment is zero.	In the absence of electric field, the net dipole moment is non-zero.
Aluminium, manganese, chromium and platinum are some examples of paramagnetic substances.	Iron, nickel, cobalt, gadolinium, dysprosium and their alloys are some examples of ferromagnetic substances.

(iv)
- When the radio waves from the transmitting antenna propagate along the surface of the Earth to reach the receiving antenna, the wave propagation is called ground wave propagation or surface wave propagation.
- Electromagnetic waves which are vertically polarised can travel in this mode.
- The horizontal component of electric field in contact with the Earth is short circuited. The radio waves induce current in the ground through which they pass.
- There is loss of power in a signal during its propagation on the surface of the Earth due to partial absorption of energy by the ground. Loss of energy is also due to the diffraction effect. The absorption of energy is high for high frequency. Hence, groundwave propagation is suitable for low frequency and medium frequency.
- It is used for local broadcasting, e.g. ship communication and radio navigation. For TV and FM signals (HF), groundwave propagation is not used.

(v)   Here, $G = 500\Omega$ and $R_{eq} = 21\Omega$

$$\frac{1}{R_{eq}} = \frac{1}{G} + \frac{1}{S}$$

$$\therefore \frac{1}{S} = \frac{1}{R_{eq}} - \frac{1}{G} = \frac{G - R_{eq}}{R_{eq}G}$$

$$\therefore S = \frac{R_{eq}G}{G - R_{eq}}$$

$$\therefore S = \frac{500 \times 21}{500 - 21} = \frac{10500}{479}$$

$$\therefore S = 21.92 \ \Omega$$

(vi)   Given, $T = 200K$, $\chi_1 = 1.8 \times 10^{-5}$

$$\chi_1 - \chi_2 = 6 \times 10^{-6} = 0.6 \times 10^{-5}$$

$$\rightarrow \chi_2 = \chi_1 - 0.6 \times 10^{-5}$$

$$= 1.8 \times 10^{-5} - 0.6 \times 10^{-5}$$

$$\therefore \chi_2 = 1.2 \times 10^{-5}$$

$$\because \chi T = cons\tan t$$

$$\because \chi_1 T_1 = \chi_2 T_2$$

$$\rightarrow T_1 = \frac{\chi_2 T_2}{\chi_1}$$

$$\therefore T_2 = \left(\frac{1.8 \times 10^{-5}}{1.2 \times 10^{-5}}\right) \times 200 \ K$$

$$\therefore T_2 = 300 \ K$$

(vii) Here, the co-efficient of mutual induction, M = 2H

4A is cut-off in 2.5×10^{-4} seconds.

Induced e.m.f., $E = M \dfrac{di}{dt}$

$\therefore E = 2H \times \dfrac{4A}{2.5 \times 10^{-4}}$

$\therefore E = 3.2 \times 10^{-4} V$

(viii) Here, $\lambda = 4.33 \times 10^{-4}$ per year

$t_{1/2} = \dfrac{0.6931}{\lambda}$

$\therefore t_{1/2} = \dfrac{0.6931}{\lambda} = \dfrac{0.6931}{4.33 \times 10^{-4}}$

$\therefore t_{1/2} = 1600.69$ years

or $t_{1/2} = 0.16 \times 10^{4} \times 365$ days

$\therefore t_{1/2} = 584000$ days

6.

(i) **(d)** $\sqrt{3}$

Refractive index, $\mu = \tan i_p$

$\rightarrow \mu = \tan 60°$

$\therefore \mu = \sqrt{3}$

(ii) **(c)** diameter of an objective

R.P of telescope $= \dfrac{1}{d\theta} = \dfrac{a}{1.22\lambda}$

Thus, it is clear that a telescope with a large diameter of the objective has higher resolving power. Thus, the resolving power of a telescope depends on the diameter of an objective.

(iii) **(b)** increase in dielectric constant

Electric feild intensity at a point outside a charged conducting sphere is given as

$E = \dfrac{1}{4\pi\varepsilon} \dfrac{q}{r^2}$

(iv) **(a)** $I_1 = I_2 \left(\dfrac{R + r}{R} \right)$

The internal resistance of a cell is given as

$$r = \left(\frac{I_1 - I_2}{I_2} \right) R$$

$$\rightarrow \frac{r}{R} = \frac{I_1 - I_2}{I_2}$$

$$\therefore rI_2 = RI_1 - RI_2$$

$$\therefore I_2 (R + r) = RI_1$$

$$\therefore I_1 = I_2 \frac{(R + r)}{R}$$

(v) **(d)** $\dfrac{hc}{\lambda}$

Energy of a photon $E = h\upsilon = \dfrac{hc}{\lambda}$

(vi) **(b)** NOR

(vii) **(c)** modulation
The process of superimposing a low frequency signal on a high frequency wave is known as modulation.

7. **Principle of working of a transformer:**
A transformer works on the principle that whenever the magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.

Construction:
It consists of two coils, primary (P) and secondary (S), insulated from each other and wound on a soft iron core as shown in the figure below.
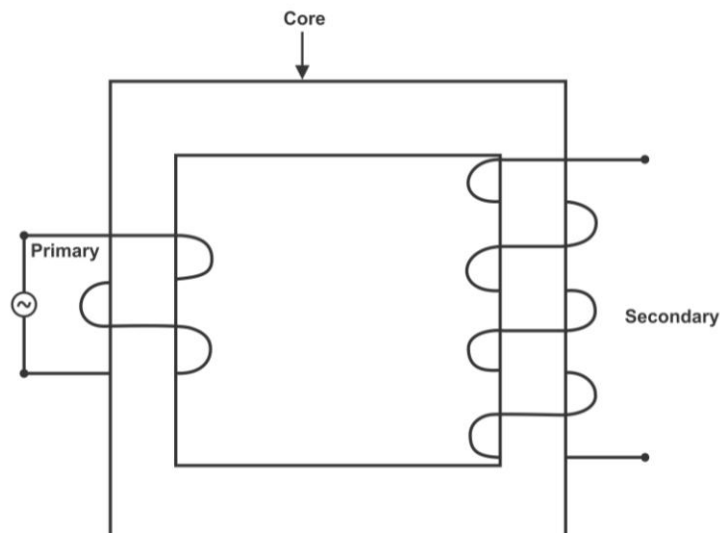
The primary coil is called the input coil and the secondary coil is called the output coil.

Working:
When an alternating voltage is applied to the primary coil, the current through the coil goes on changing. Hence, the magnetic flux through the core also changes. As this changing magnetic flux is linked with both coils, an emf is induced in each of them.
The amount of magnetic flux linked with the coil depends on the number of turns of the coil.

Derivation:
Let 'ϕ' be the magnetic flux linked per turn with both coils at a certain instant of time 't'.
Let the number of turns of the primary and secondary coils be 'N_p' and 'N_s', respectively.
Therefore, the total magnetic flux linked with the primary coil at certain instant of time 't' is $N_p\phi$. Similarly, the total magnetic flux linked with the secondary coil at certain instant of time 't' is $N_s\phi$.
Now, the induced emf in a coil is
$$e = -\frac{d\phi}{dt}$$
Therefore, the induced emf in the primary coil is
$$e_p = -\frac{d\phi_p}{dt} = -\frac{dN_p\phi}{dt} = -N_p\frac{d\phi}{dt} \qquad \dots\dots (1)$$
Similarly, the induced emf in the secondary coil is
$$e_s = -\frac{d\phi_s}{dt} = -\frac{dN_s\phi}{dt} = -N_s\frac{d\phi}{dt} \qquad \dots\dots (2)$$
Dividing equations (1) and (2), we get
$$\frac{e_s}{e_p} = \frac{-N_s\frac{d\phi}{dt}}{-N_p\frac{d\phi}{dt}} = \frac{N_s}{N_p} \qquad \dots\dots (3)$$

The above equation is called the equation of the transformer and the ratio $\frac{N_s}{N_p}$ is known as the turns ratio of the transformer.

Now, for an ideal transformer, we know that the input power is equal to the output power.

$\therefore P_p = P_s$

$\therefore e_p i_p = e_s i_s$

$\therefore \dfrac{e_s}{e_p} = \dfrac{i_p}{i_s}$

From equation (3), we have

$\dfrac{e_s}{e_p} = \dfrac{N_s}{N_p}$

$\therefore \dfrac{e_s}{e_p} = \dfrac{N_s}{N_p} = \dfrac{i_p}{i_s}$

Numerical:

Given: $Q = 0.2 \ \mu C = 0.2 \times 10^{-6} \ C$

$A = 40 \ cm^2 = 40 \times 10^{-4} \ m^2$

$\varepsilon_0 = 8.85 \times 10^{-12}$ SI units

The electric field intensity just outside the surface of a charged conductor of any shape is

$E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$

$\therefore E = \dfrac{0.2 \times 10^{-6}}{40 \times 10^{-4} \times 8.85 \times 10^{-12}}$

$\therefore E = 5.65 \times 10^6 \ N/C$
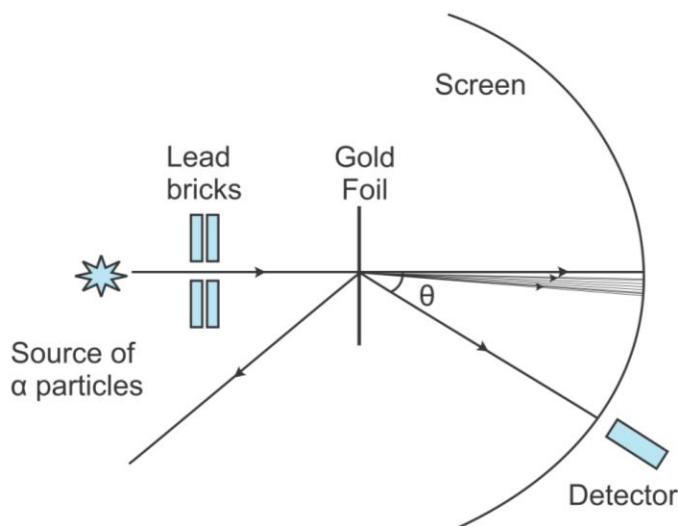
Now, the mechanical force per unit area of a conductor is

$f = \dfrac{1}{2} \varepsilon_0 E^2 = \dfrac{1}{2} \times 8.85 \times 10^{-12} \times \left(5.65 \times 10^6\right)^2$

$\therefore f = 141.25 \ N/m^2$

OR

Geiger–Marsden experiment:

The setup of the Geiger–Marsden experiment is as shown below.

In this experiment, a narrow beam of α-particles from a radioactive source was incident on a gold foil. The scattered α-particles were detected by the detector fixed on a rotating stand. The detector used had a zinc sulphide screen and a microscope.

The α-particles produced scintillations on the screen which could be observed through a microscope. This entire setup is enclosed in an evacuated chamber.

They observed the number of α-particles as a function of scattering angle. Now, the scattering angle is the deviation (θ) of α-particles from its original direction.

They observed that most α-particles passed undeviated and only a few (~0.14%) scattered by more than 1°. Few were deflected slightly and only a few (1 in 8000) deflected by more than 90°. Some particles even bounced back with 180°.

Mass defect:

It is observed that the mass of a nucleus is smaller than the sum of the masses of the constituent nucleons in the free state. The difference between the actual mass of the nucleus and the sum of the masses of constituent nucleons is called mass defect.

The mass defect is

$$\Delta m = \left[Zm_p + (A-Z)m_n \right] - M$$

where Z is the atomic number (number of protons), A is the mass number, (A – Z) is the mass of neutrons, m_p is the mass of a proton, m_n is the mass of a neutron and M is the measured mass of a nucleus.

Problem:

Given: $\phi_0 = 2.3$ eV $= 2.3 \times 1.6 \times 10^{-19}$ J $= 3.68 \times 10^{-19}$ J

$\lambda = 6800 \ \overset{o}{A} = 6800 \times 10^{-10}$ m; $c = 3 \times 10^8$ m/s;

$h = 6.63 \times 10^{-34}$ Js

We know that the incident frequency is given as

$$v = \frac{c}{\lambda}$$

$$\therefore v = \frac{3 \times 10^8}{6800 \times 10^{-10}} = 4.41 \times 10^{14} \text{ Hz}$$

Now, if the incident frequency is greater than the threshold frequency, then photoelectrons will be emitted from the metal surface. The threshold frequency is given from work function as

$$v_0 = \frac{\phi_0}{h}$$

$$\therefore v_0 = \frac{3.68 \times 10^{-19}}{6.63 \times 10^{-34}} = 5.55 \times 10^{14} \text{ Hz}$$

Since, $v < v_0$ photoelectrons will not be emitted.

8.

(i)

Given: $\mu_g = 1.5$; $n = 4 \times 10^{14}$ Hz; $c = 3 \times 10^8$ m/s

The wavelength of light incident on glass from air is

$$\lambda_a = \frac{c}{n} = \frac{3 \times 10^8}{4 \times 10^{14}} = 7.5 \times 10^{-7} \text{ m} = 7500 \times 10^{-10} \text{ m} = 7500 \ \overset{o}{A}$$

Now, the velocity of light in glass is given from its refractive index as

$$\mu_g = \frac{c}{v_g}$$

We also know that velocity is product of frequency and wavelength.

$$\therefore \mu_g = \frac{c}{v_g} = \frac{n\lambda_a}{n\lambda_g} = \frac{\lambda_a}{\lambda_g}$$

$$\therefore \lambda_g = \frac{\lambda_a}{\mu_g} = \frac{7500}{1.5} = 5000 \ \overset{o}{A}$$

Therefore, the difference in wavelength is

$$\lambda_a - \lambda_g = 7500 - 5000 = 2500 \ \overset{o}{A}$$

The wave number is the reciprocal of the wavelength.

Therefore, the wave number in glass is

$$\overline{\lambda_g} = \frac{1}{\lambda_g}$$

$$\therefore \overline{\lambda_g} = \frac{1}{5 \times 10^{-7}} = 2 \times 10^6 \ m^{-1}$$

(ii) In the biprism experiment, the 10th dark band is observed.

The distance between the mth dark band with the central bright band is

$$x_m = (2m - 1)\frac{\lambda D}{2d}$$

Therefore, the distance for the 10th dark band is

$$x_{10} = ((2 \times 10) - 1)\frac{\lambda D}{2d} = \frac{19\lambda D}{2d}$$

Now, when red light is used, we have

$$(x_{10})_r = \frac{19\lambda_r D}{2d} \qquad \qquad \text{...... (1)}$$

Similarly, for blue light, we have

$$(x_{10})_b = \frac{19\lambda_b D}{2d} \qquad \qquad \text{...... (2)}$$

Now, the fringe width is

$$X = \frac{\lambda D}{d}$$

$$\therefore X_r = \frac{\lambda_r D}{d} \qquad \qquad \text{...... (3)}$$

$$\therefore X_b = \frac{\lambda_b D}{d} \qquad \qquad \text{...... (4)}$$

From equations (1) and (3), we get

$$(x_{10})_r = \frac{19X_r}{2} = 2.09 \ mm$$

$$\therefore X_r = \frac{2 \times 2.09}{19} = 0.22 \ mm$$

Dividing equations (1) and (2), we get

$$\frac{(x_{10})_r}{(x_{10})_b} = \frac{\frac{19\lambda_r D}{2d}}{\frac{19\lambda_b D}{2d}} = \frac{\lambda_r}{\lambda_b}$$

$$\therefore (x_{10})_b = \frac{\lambda_b \times (x_{10})_r}{\lambda_r} = \frac{4800 \times 2.09}{6400} = 1.57 \ mm$$

Now, from equations (2) and (4), we get

$$\left(x_{10}\right)_b = \frac{19X_b}{2} = 1.57 \text{ mm}$$

$$\therefore X_b = \frac{2 \times 1.57}{19} = 0.165 \text{ mm}$$

Therefore, the change in fringe width when blue light is used instead of red is

$$X_r - X_b = 0.22 - 0.165 = 0.055 \text{ mm}$$

(iii) Kelvin's method to determine the resistance of the galvanometer by using a meter bridge:

A galvanometer whose resistance 'G' is to be determined is connected in one gap (left gap) of a Wheatstone's meter bridge and a resistance box is connected in the other gap (right gap).
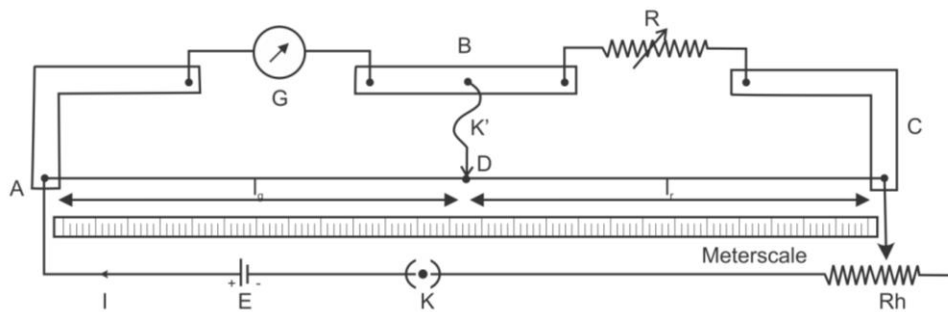
G: Galvanometer

R: Resistance from the resistance box

AC: Metal wire one metre long

R_h: Rheostat

E: Cell

K: Plug key

K': Jockey

- A cell of emf 'E', key K and rheostat R_h are connected in series with the bridge wire AC. The junction 'B' of the galvanometer and the resistance box is connected to the jockey which can slide along wire AC.
- A suitable resistance 'R' is taken in the resistance box and a current 'J' is sent round the circuit. Without touching the jockey to any point of AC, note the deflection in the galvanometer.
- A rheostat is adjusted to get a suitable deflection (e.g. 0.15, 20 divisions) in the galvanometer.
- Place the jockey at points A and C, and see the deflection on the galvanometer. It should be on opposite sides.
- By touching the jockey to different points of wire AC, find (obtain) the point of contact 'D' for which the galvanometer shows the same deflection as before, i.e. points B and D are equipotential (i.e. the point gives the same deflection in the galvanometer with or without the contact of the jockey.)

- In this method, the null point is not obtained. Thus, Kelvin's method is a deflection method. The point 'D' is called the balanced point.
- Let l_g and l_r be the distances of point 'D' from ends 'A' and 'C' of wire AC, respectively. The resistance per unit length of wire AC is 'σ'. Here also G, R and resistances of wire of lengths l_g and l_r form four arms of a balanced Wheatstone's network.

$$\frac{G}{R} = \frac{\text{Resistance of wire AD of length } l_g}{\text{Resistance of wire CD of length} l_r}$$

$$\therefore \frac{G}{R} = \frac{\sigma l_g}{\sigma l_r} = \left(\frac{l_g}{l_r}\right)$$

-

$$\therefore G = X = R\left(\frac{l_g}{l_r}\right)$$

$$\therefore G = R\left(\frac{l_g}{100 - l_g}\right)$$

- Thus, the resistance of galvanometer 'G' can be calculated by knowing the values of R and l_g in the above equation.

(iv) **Oscillator:**

An oscillator is an electronic device which generates an AC signal from a DC source. Oscillators are used in radio/TV receiver sets, radio/TV transmitters, RADAR, smartphones and microwave ovens. No input is applied to the oscillator, yet an output is obtained from it.

An oscillator requires an amplifier and a feedback network with frequency-determining components. When a part of the output of an amplifier is coupled to the input of the amplifier, it is called feedback. When the feedback sample is out of phase with the input, it is called negative feedback. When the feedback sample is in phase with the input, it is called positive feedback. For an oscillator, a positive feedback is required. The block diagram of an oscillator is shown below.
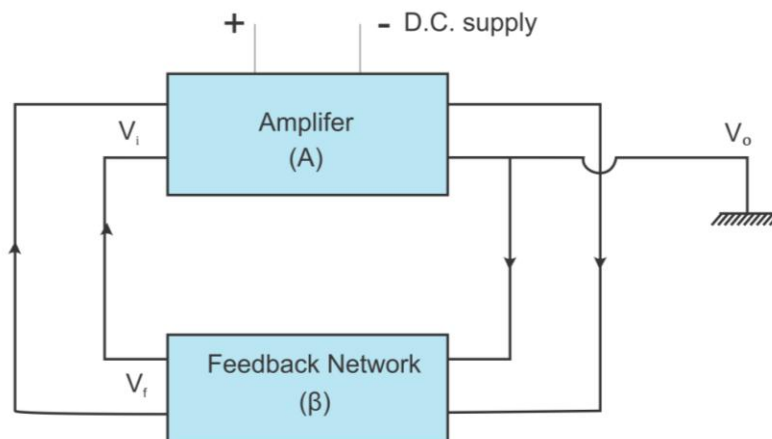
The voltage gain of a complete system is given by

$$A_f = \frac{A}{1 - A\beta}$$

where A_f is the voltage gain with feedback, A is voltage gain without feedback and β is the feedback factor.

If for some frequency $A\beta = 1$, then the system gain becomes ∞ and the circuit begins to oscillate at that frequency. This condition ($A\beta = 1$) is called the Barkhausen criterion for sustained oscillations.
The frequency of oscillations depends on the LC or RC combinations used in a feedback network. When the power supply connected to the oscillator is turned ON, electrical noise of a wide range of frequencies is generated in the circuit, but the condition $A\beta = 1$ is satisfied only for a particular frequency and the oscillator oscillates at that frequency.