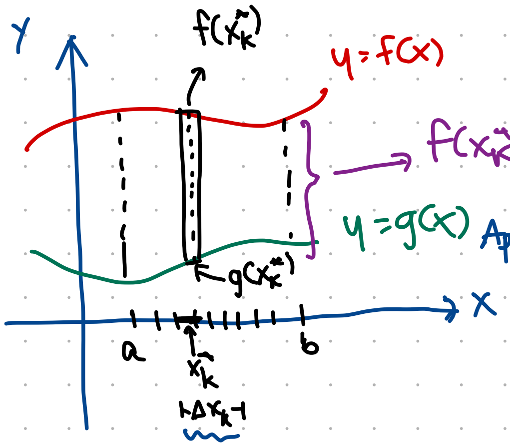
[illegible] Area between curves

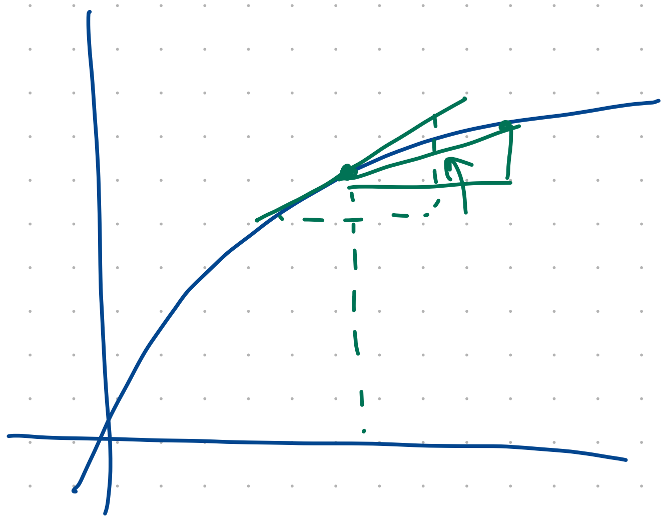


⊛ $f(x) \geq g(x) \quad \forall x \in [a,b]$

$$\text{Approx Area} = \sum_{i=1}^{n} \left(f(x_k^*) - g(x_k^*)\right) \Delta x_k$$

$$A = \lim_{\max \Delta x_k \to 0} \sum_{i=1}^{n} \left(f(x_k^*) - g(x_k^*)\right) \Delta x_k$$

$$= \int_a^b [f(x) - g(x)]\, dx$$



Chapter 8

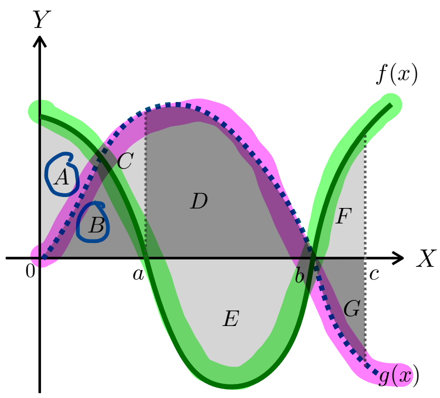
Applications of the Definite Integral in Geometry and Improper Integrals
การประยุกต์ปริพันธ์ในทางเรขาคณิต และปริพันธ์ไม่ตรงแบบ

8.1 Area between Curves
พื้นที่ระหว่างโค้ง

ข้อ 107 From the given figure, find the following integrals in terms of A, B, C, D, E, F, where each of them represents the area of each region.

จากภาพต่อไปนี้ จงหาค่าอินทิกรัลต่อไปนี้ในรูปของ A, B, C, D, E, F, G เมื่อ A, B, C, D, E, F, G แทนพื้นที่แต่ละส่วน

↓ ในรูป integral เราจะมองหา net-signed area



107.1 $\int_0^a f(x)\,dx$ = $A+B$

$\int_a^b f(x)dx = -E \Rightarrow \int_b^a f(x)dx = -\int_a^b f(x)dx$

107.2 $\int_b^a f(x)\,dx$ = E

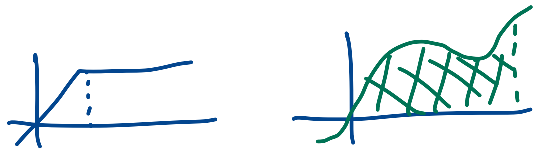
107.3 $\int_a^b [f(x)+g(x)]\,dx$ = $\int_a^b f(x)dx + \int_a^b g(x)dx$

$= -E + D$

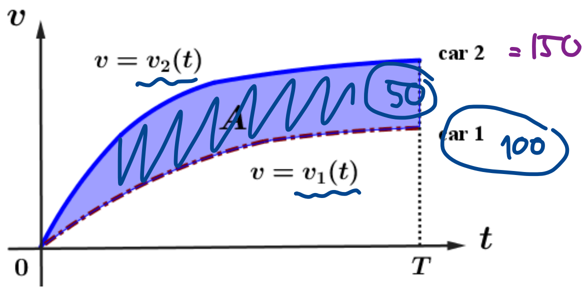
107.4 $\int_a^c [f(x)-g(x)]\,dx$ = $\int_a^b [f(x)-g(x)]dx + \int_b^c [f(x)-g(x)]dx$

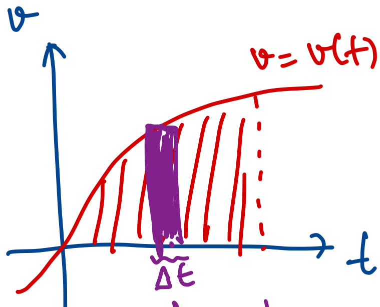
$= \int_a^b -[g(x)-f(x)]dx + \int_b^c [f(x)-g(x)]dx$

$= -(D+E) + (F+G)$



Example 8.3 Figure shows velocity versus time curves for two race cars that move along a straight track, starting from rest at the same time. Give a physical interpretation of the area A between the curves over the interval $0 \le t \le T$.





ความหมายของ พ.ท.ใต้กราฟ $v=v(t)$

ระยะทาง = ความเร็ว×เวลา

$S = vt$

∴ พ.ท.ใต้กราฟ $v=v(t)$ คือ ระยะทางที่เคลื่อนที่ได้

$A =$ พ.ท.ใต้กราฟของ $v_2(t)$ จาก 0 ถึง T − พ.ท.ใต้กราฟ $v_1(t)$ จาก 0 ถึง T

$$= \int_0^T v_2(t)\,dt - \int_0^T v_1(t)\,dt = \int_0^T [v_2(t) - v_1(t)]\,dt$$

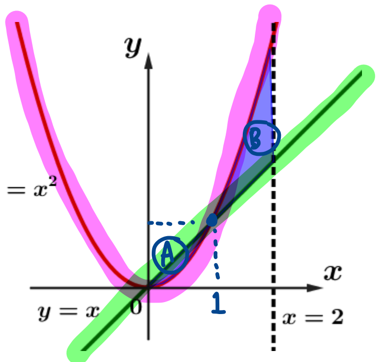
จาก $v_2(t) \geq v_1(t)$ ทุก $t \in [0,T]$ และ

พ.ท.ใต้กราฟ จาก 0 ถึง T ของ $v_2(t)$ คือ ระยะทางที่ car 2 วิ่งได้

ทำนองเดียวกัน พ.ท.ใต้กราฟ จาก 0 ถึง T ของ $v_1(t)$ คือระยะทางที่ car 1 วิ่งได้

∴ พ.ท. A คือ ระยะทางที่รถ car 2 วิ่งนำหน้ารถคันที่ 1

Example 8.4 Find the area of the region enclosed by $y = x$, $y = x^2$, $x = 0$ and $x = 2$.



หาจุดแบ่งช่วง แก้ระบบสมการ

$y = x^2$, $y = x$

$\therefore x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

$$A = \int_0^1 [x - x^2]\,dx + \int_1^2 [x^2 - x]\,dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 + \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^2$$

$$= \left[\frac{1}{2} - \frac{1}{3} - 0 + 0\right] + \left[\frac{8}{3} - \frac{4}{2} - \frac{1}{3} + \frac{1}{2}\right]$$

$$= 1 + \frac{8}{3} - \frac{2}{3} - 2$$

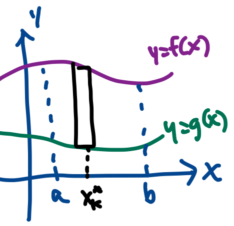
$$= 1 + 2 - 2 = 1$$

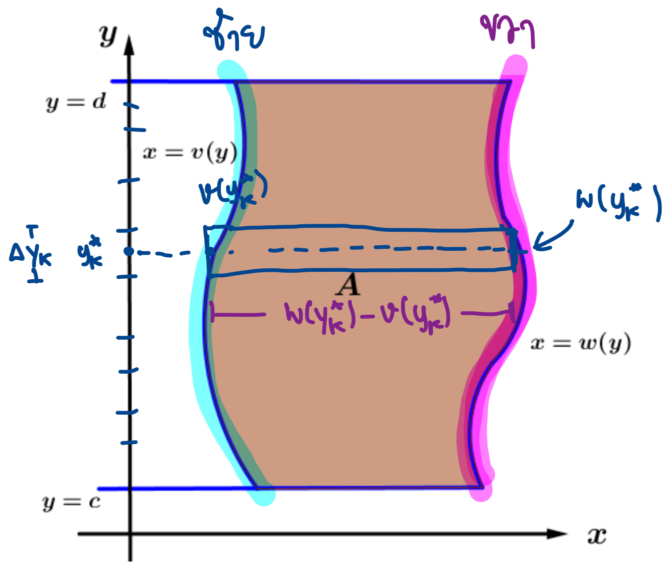
#

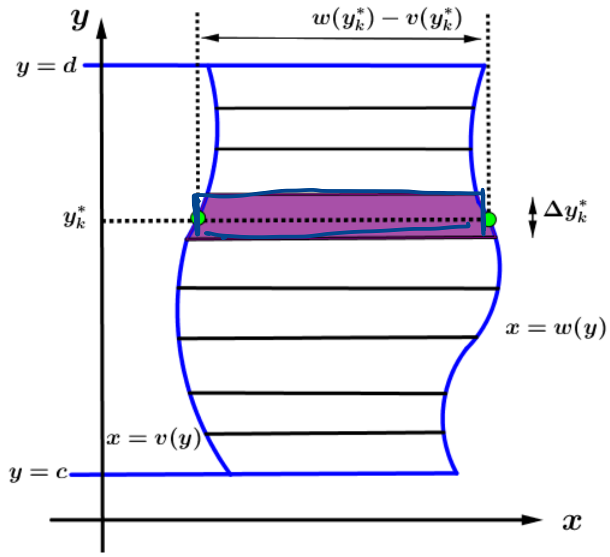
Reversing the Roles of x and y

Theorem 8.2 If w and v are continuous functions on the interval [c, d] and $w(y) \geq v(y)$ for all y in [c, d]. Then the area of the region bounded on the right by $x = w(y)$, on the left by $x = v(y)$, below by the line $y = c$, and above by the line $y = d$ is

$$A = \int_c^d [w(y) - v(y)]dy \tag{8.2}$$



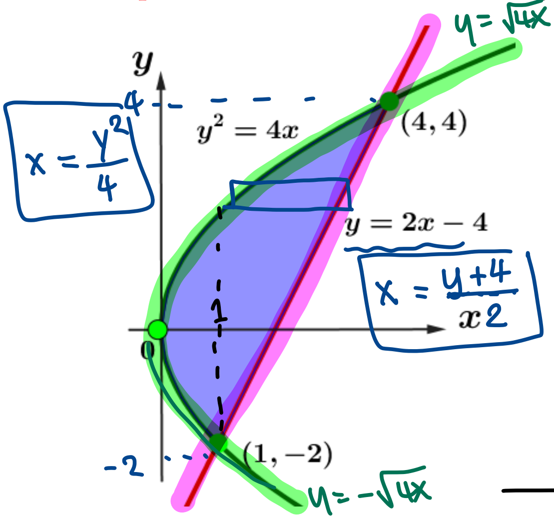




ขวา ซ้าย ขวา ซ้าย

$$A = \lim_{max \Delta y_k \to 0} \sum_{k=1}^{n} [w(y_k^*) - v(y_k^*)]\, \Delta y_k = \int_c^d [w(y) - v(y)]dy$$

Example 8.5 Find the area of the region enclosed by $y^2 = 4x$ and $y = 2x - 4$.



พ.ท.ตามแนวแกน y [อินทิเกรตเทียบ y]

$$A = \int_{-2}^{4} \left[\frac{y+4}{2} - \frac{y^2}{4}\right] dy$$

$$= \int_{-2}^{4} \left[\frac{y}{2} + 2 - \frac{y^2}{4}\right] dy = \left[\frac{y^2}{4} + 2y - \frac{y^3}{12}\right]_{-2}^{4}$$

$$= \left(\frac{16}{4} + 8 - \frac{64}{12}\right) - \left(\frac{4}{4} - 4 + \frac{8}{12}\right) = 9 \text{ ตร.หน่วย}$$

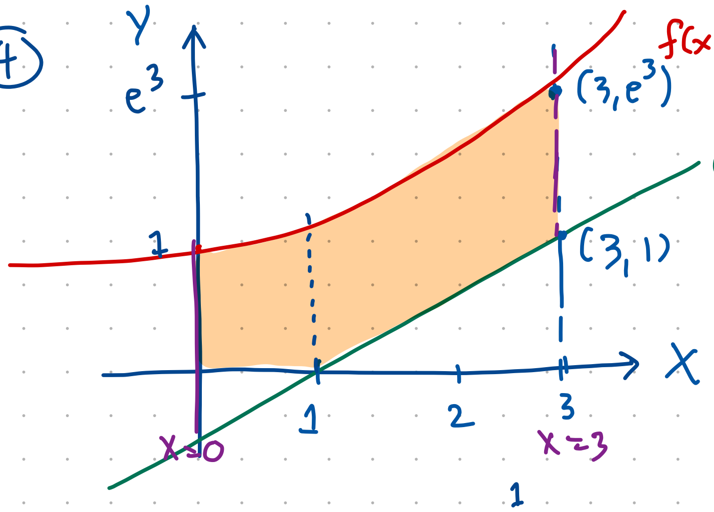
อินทิเกรตเทียบ x :

$$A = \int_0^1 [\sqrt{4x} - (-\sqrt{4x})]dx + \int_1^4 [\sqrt{4x} - (2x-4)]dx$$

$$\vdots$$

$= 9$ ตร.หน่วย

④



$f(x) = e^x \Rightarrow y = e^x$

$x = \ln y$

$g(x) = \frac{x-1}{2} \Rightarrow y = \frac{x-1}{2}$

$\therefore x = 2y+1$

<u>อินทิเกรตเทียบ x</u>

$$A = \int_0^1 \left[e^x - 0 \right] dx + \int_1^3 \left[e^x - \frac{x-1}{2} \right] dx$$

<u>อินทิเกรตเทียบ y</u>

$$A = \int_0^1 \left[(2y+1) - 0 \right] dy + \int_1^{e^3} \left[3 - \ln y \right] dy$$

7. $y = x^2, \quad y = \sqrt{x}, \quad x = \frac{1}{4}, \quad x = 1.$

8. $y = x^3 - 4x, \quad y = 0, \quad x = 0, \quad x = 2.$

9. $y = \cos 2x, \quad y = 0, \quad x = \pi/4, \quad x = \pi/2.$

10. $y = \sec^2 x, \quad y = 2, \quad x = -\pi/4, \quad x = \pi/4.$

11. $y = \sin y, \quad x = 0, \quad y = \pi/4, \quad y = 3\pi/4.$

12. $x^2 = y, \quad x = y - 2.$

13. $y = e^x, \quad y = e^{2x}, \quad x = 0, \quad x = \ln 2.$

14. $x = 1/y, \quad x = 0, \quad y = 1, \quad y = e.$

15. $y = 2/(1 + x^2), \quad y = |x|.$

16. $y = 1/\sqrt{1 - x^2}, \quad y = 2.$

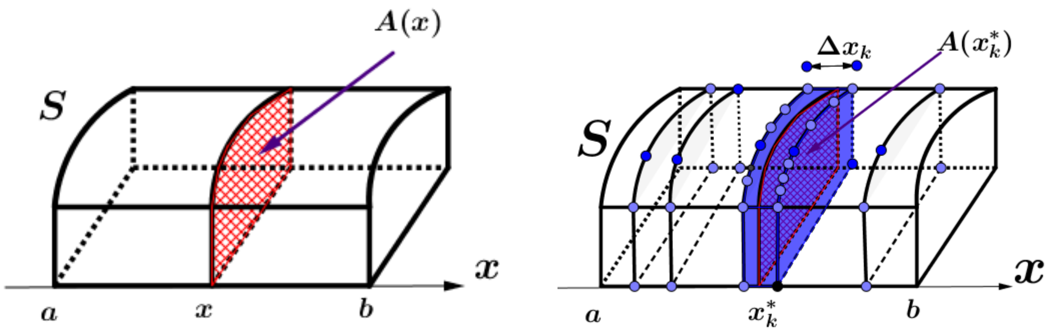
17. $y = x, \quad y = 4x, \quad y = -x + 2.$

8.2 Volumes by Slicing; Disks and Washers

การหาปริมาตรของทรงตัน

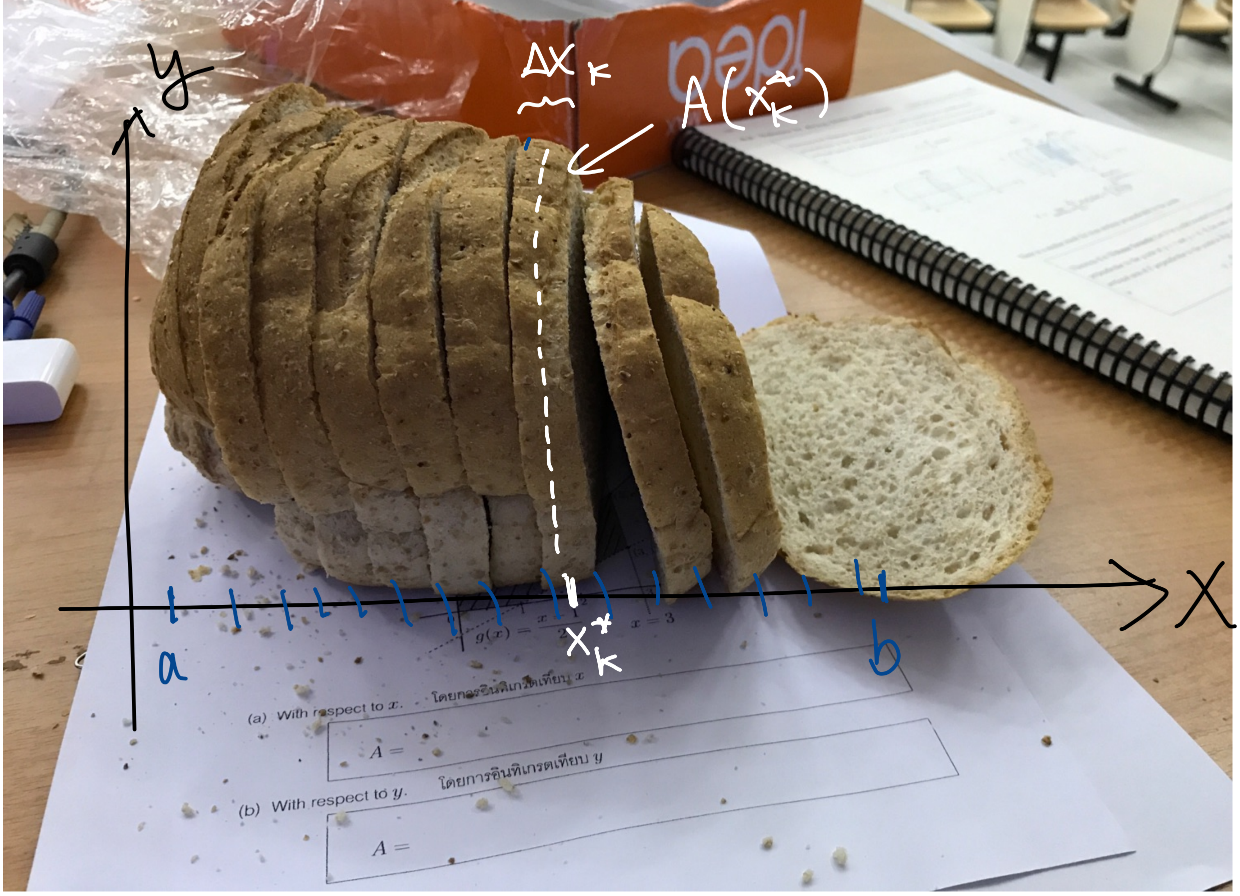
Theorem 8.3 (**Volume formula**) Let S be a solid bounded by two parallel planes perpendicular to the x-axis at $x = a$ and $x = b$. If, for each x in [a, b], the cross-sectional area of S perpendicular to the x-axis is $A(x)$, then the volume of the solid is

$$V = \int_a^b A(x)dx. \tag{8.3}$$



$$V = \lim_{max \Delta x_k \to 0} \sum_{k=1}^{n} A(x_k^*) \Delta x_k = \int_a^b A(x)dx$$

There is a similar result for cross sections perpendicular to the y-axis.



V ของขนมปัง 1 แผ่น $= A(x_k^*)\Delta x_k$

V ของขนมปัง n แผ่น $= \sum_{k=1}^{n} A(x_k^*)\Delta x_k \quad \therefore V = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} A(x_k^*)\Delta x_k$

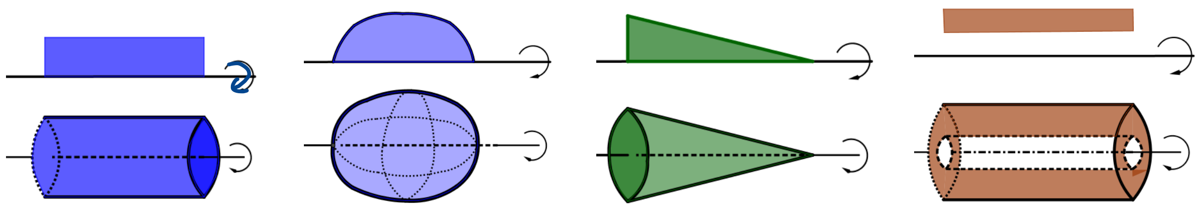
$= \int_{a} A(x)\,dx$

Theorem 8.4 (**Volume formula**) Let S be a solid bounded by two parallel planes perpendicular to the y-axis at $y = c$ and $y = d$. If, for each y in [c, d], the cross-sectional area of S perpendicular to the y-axis is $A(y)$, then the volume of the solid is

$$V = \int_c^d A(y)dy. \tag{8.4}$$

Solid of Revolution

ทรงตันที่เกิดจากการหมุนรอบแกน.

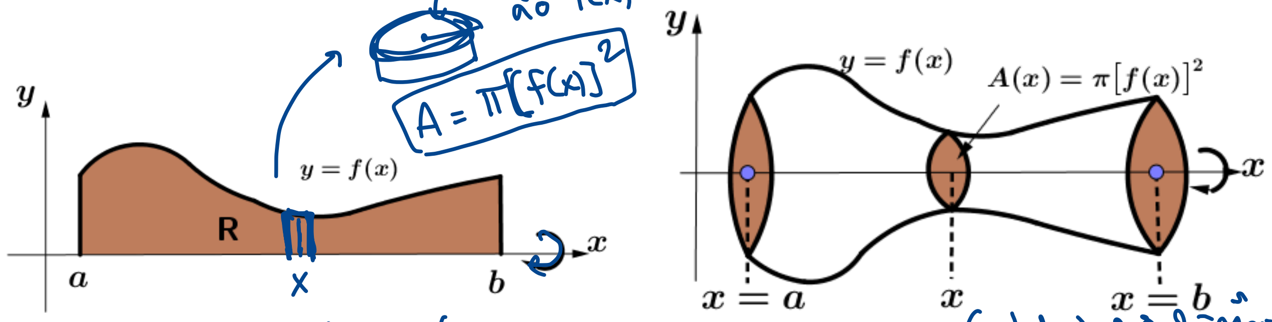


Volume by Disks perpendicular to the X-axis

Problem: Let f be continuous and nonnegative on [a, b], and let R be the region that is bounded above by $y = f(x)$, below by the x-axis, and on the sides by the lines $x = a$ and $x = b$. Find the volume of the solid of revolution that is generated by revolving the region R about the X-axis.

รัศมีของทรงกระบอกชิ้นเล็ก คือ $f(x)$

$\boxed{A = \pi[f(x)]^2}$



⊛ disk method (การหมุนแบบตัน) ⇒ สร้างแถบ (strip) สร้างให้ตั้งฉากกับแกนหมุน

We can solve this problem by slicing. For this purpose, observe that the cross section of the solid taken perpendicular to the X-axis at the point x is a circular disk of radius $f(x)$. The area of this region is

$$V = \int_a^b A(x)\,dx$$

$$A(x) = \pi[f(x)]^2.$$

Thus, from (8.3) the volume of the solid is

$$A(x) = \pi[f(x)]^2$$

$$V = \int_a^b \pi[f(x)]^2 dx. \quad \therefore \boxed{V = \int_a^b \pi[f(x)]^2 dx} \tag{8.5}$$