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Reinsurance credit risk arises whenever a direct insurer is exposed to loss if a reinsurer fails to pay reinsurance recovery. This special type of credit risk is discussed and a method for calibrating limits of exposure to reinsurance credit risk is proposed in this paper. The proposed method is based on risk optimisation and considered as an integral part of insurer's active risk management.

Keywords: reinsurance credit risk, DFA simulation model, non-linear optimisation.

CONTENTS

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In practice general[1] insurers usually hedge against their risk of having extremely large losses through reinsurance market. By purchasing reinsurance cover they transfer a part of un-tolerated risk to smooth insurance results and protect against insolvency. This type of risk hedging in insurance is an important risk management tool and often used in the maximisation of shareholders value, but it also creates another, rather new type of risk - *reinsurance credit risk*.

The reinsurance credit risk is often defined as *"the risk of loss arising from failure to collect reinsurance recoveries from a reinsurer due to either unwilingness or inability to fulfill its contractual obligations"*. In its nature it can be seen as a special type of credit risk in general, as it resonates with increased:

- *single name concentration* - the number of reinsurers is small (when compared to the number of bond issuers) and so a typical insurer - however prudent - is likely to have a concentrated exposure to individual names;
- *industry sector concentration* - by definition reinsurer exposure is specific to one industry sector (insurance) so correlations are likely higher than in a more diversified portfolio;
- *tail dependence* - the ceding insurer is in the same industry - catastrophe events will weaken the balance sheet of the reinsurers at the same time (potentially) as the ceding insurers portfolio is stressed.

Consequently, without an effective management of such risk the very reinsurance function aimed at the maximisation of shareholders value would be seriously compromised. There are two obvious choices here: either **neutralise** the reinsurance credit risk via hedging it in the CDS[2] derivative market, or **retain** it, and thus model it, hold the capital to support it and actively manage exposure to it.

CHOICE 1: HEDGE THE RISK. At first glance, the first choice seems to be more sensible and somewhat attractive. Indeed, the insurance company should neutralise the reinsurance credit risk and not retain it. This is because the insurer's main specialty is insurance risk and not making money by retaining reinsurance credit risk. Shareholders investing in a general insurance company are cognisant of the fact that large natural peril losses is the significant portion of insurance risk, and thus expect insurance risk managers to optimise that risk by transfering a certain amount of it to reinsurers, but do not expect any part of the transferred risk to bounce back to 'base'. Put another way, shareholders perceive the reinsurance credit risk rather as a part of insurer's specific risk and, hence, do not require to be rewarded for it.

Now, if the insurer can use the CDS derivative market to hedge the reinsurance credit risk, the only thing that is left to risk managers to be sorted out is to optimise the total cost of reinsurance and hedging the reinsurance credit risk. Such a cost optimisation problem has been recently proposed and studied in Bodoff [2].

The attractiveness of this option, however, comes at the expense of some significant shortcomings that should be taken into account. Firstly, the market for hedging reinsurance credit risk through CDS is not always available,

1 In Australia the non-life type of insurance is called 'general' insurance.
2 Credit Default Swaps.

and/or in some jurisdictions insurance regulator may even discourage insurers from using CDS. Secondly, using CDS will not completely neutrilise the reinsurance credit risk, as CDS does not cover reinsurer's unwillingness to pay and taking CDS cover gives a rise to a further residual risk - risk of CDS counterparty failing to fulfill its obligations.

CHOICE 2: RETAIN THE RISK. The second choice - retaining reinsurance credit risk - implies, first of all, modelling it, then pricing it and holding extra amount of capital to support it, and finally mitigating it through active management of its exposure.
Reinsurance credit risk modelling. In order to accurately model the reinsurance risk a great deal of care needs to be taken. This is mainly because the reinsurance credit risk significantly differs from credit risk of a conventional bond. This means that traditional methods often used to assess bond issuer default cannot be easily transferred to assessment of reinsurance default. Differences arise in relation to probability of default, exposure, and recoveries. In regards to default event, the reinsurer default is usually defined as an asset impairment event, whereas the credit markets consider an issuer default as having occurred when there is a shortfall in interest and/or principal payments on its obligation. Under financial distress reinsurers often go into runoff and usually enter into a commutation agreement with cedants, and therefore it is often unclear if the default defined in financial sense has taken place or not.

The exposure profile of an insurer (particularly the modelled exposure within a DFA model) is also different. The insolvency of a reinsurer will lead to costs on the part of the insurer:

- any amounts owing as a result of claims settled with the reinsurer but not yet paid will be impaired;
- amounts potentially owing in respect of claims incurred but not yet advised to the ceding insurer are potentially not recoverable;
- any potential recoveries in respect of policies in force are unlikely to be met in full;
- the ceding company may be required to purchase replacement cover to remain protected against events from the time of reinsurance default to the time the contract would have expired.

Within the constructs of a DFA model there is a further difficulty, in that exposure may arise in respect of contracts not in force at the calculation date, but expected to be written in respect of future years. All of those modelling issues were addressed in the DFA approach to modelling reinsurance credit risk proposed in Britt and Krvavych [3].
Active management of risk exposure. The key role of active management is to, first of all, determine an optimal limit (i.e. acceptable maximum amount) of credit risk exposure to a single reinsurance counterparty for each credit rating. Here, the exposure limits are optimised such that the insurer's marginal risk attributable to reinsurance default is minimised under the constraints of budget cost of underlying reinsurance program. Once it has been done it then monitors and manages the reinsurance counterparty exposure through:

- *Replacement of defaulting reinsurer* by another one of the same credit quality;
- *Commutation/collaterisation* in the case of adverse exposure movements due to increased realised claims, and/or deterioration of reinsurance counterparty credit rating;

- *Imposing restrictions on credit quality* of reinsurers providing cover, say the minimum acceptable credit rating is BBB+; and

- *Mitigating the risk of 'unwillingness to pay'* via, for example, using a clientele of reinsurers whom the primary insurer has been maintaining a strong business relationship with.

This paper is mainly focusing on choice 2 - 'retain the risk', and, in particular, proposes a quantitative approach to modelling optimal exposure limits under which the retained reinsurance credit risk is minimised.

The structure of this paper is as follows. In section 2, the general concepts of quantifying reinsurance credit risk in a DFA environment are discussed and then the problem of minimising that risk through deriving optimal limits of exposure to a single counterparty for each credit rating is formulated. In section 3, we provide a numerical example of solving proposed risk minimisation problem based on simulation. Finally, brief conclusions are given in section 4.

2 MINIMISATION OF RETAINED REINSURANCE CREDIT RISK

Before we start minimising the reinsurance credit risk we first outline the foundation of its modelling. This is done in subsection 2.1 Then, in subsection 2.2, the formal setup of risk optimisation problem is provided.

2.1 *Modelling the reinsurance credit risk - background*

In the modelling approach proposed in Britt and Krvavych [3] authors consider quantification of reinsurance credit risk for a generic general insurance company that uses a DFA model in determining capital requirements that would particularly allow for reinsurance credit risk emerging over a one year time horizon of writing business as well as the run-off of this risk to extinction.

In high level terms, the model setup uses the following key modelling assumptions to calculate the cost arising from the reinsurance default:

- *Exposure* - a small number of representative 'proxy reinsurers' is created to capture the company's exposure to reinsurers default. The company's exposure to a proxy reinsurer varies by exposure type, e.g. by catastrophe (cat) and non-cat, small and large cat events. In the event of default, the defaulted proxy reinsurer will be replaced by a new proxy reinsurer of the same quality. We model proxy reinsurers to by pass the issue of future exposure. In effect we model the credit policy, not its realisation at any one point in time;

- *Dependent default events* - the default of any proxy is assumed to occur at the beginning of any projection time period, that is assumed here to be a quarter, and is modelled as a binary event using Bernoulli random variable with the default rate dependent on the state ('normal' or 'stressed') of the global reinsurance market;

- *Tail dependency* between large natural peril losses sustained by a primary insurer and the global reinsurance market transiting to 'stressed' state;

- *Loss Given Default* calculated by applying a recovery rate to the exposure to proxy reinsurers (i.e. reinsurance recoveries) at the end of previous period and the replacement cost of unexpired reinsurance cover.

Exposure limits

It is assumed that the primary insurer defines the set of m plausible credit ratings. If, say, lowest plausible credit quality is BBB+, then we are dealing with eight credit ratings, i.e. $m = 8$. For each plausible credit rating we use a single representative *proxy reinsurer* to capture the company's exposure to reinsurer default. The proxy reinsurers are chosen so as to capture the credit exposure and concentration levels typical of company's reinsurer exposure. These representative exposures will remain static over the life of the projection. It is expected that two sets of exposure limits will be maintained: one for catastrophe exposure in *main catastrophe tower*[3] (Upper Layers) and the one for both catastrophe exposure below main cat retention and long-tail (non-catastrophe) exposure (Lower Layers).

Thus exposure limits for upper(lower) layers are defined as positive weights $w_i^{U(L)} \in [0,1]$, $i = 1, \dots, m$ - a maximum share of exposure-at-default (i.e. potential reinsurance recoverables in the event of default) the primary insurer is willing to assign to a single proxy reinsurer of credit quality i. For each set of exposure limits those weights will add up to one, i.e. $\sum_{i=1}^m w_i^{U(L)} = 1$.

Modelling default using normal and stressed default rates

According to the model setup the default event of each proxy reinsurer in force is modelled at the beginning of every quarter by Bernoulli random variable rate of which is dependent on the pre-generated global market state ('normal' or 'stressed'). If a particular proxy reinsurer defaults it is replaced by another one of the same credit quality. It is assumed that once the global reinsurance market transits into stressed state, it remains stressed for some period of time. The reasonable duration of 'stress' state is considered to be between 1 to 4 years[4]. The following formula formalises the default event of reinsurance proxy i in a quarter period j of year one:

$$\left\{\mathcal{D}_{ij}(Z)|Z = z\right\} \sim Be\left[q_n^{I(z,j)} q_s^{1-I(z,j)}\right], \tag{1}$$

where q_n and q_s are quarterly normal and stressed default rates respectively,

$$I(z,j) = \begin{cases} 1, & z > j; \\ 0, & z \le j \end{cases} \tag{2}$$

and $Z \sim TruncGeom[p]$ is the Truncated Geometric random variable over the period of four quarters with the quarterly transition rate p and distribution

$$\mathbb{P}[Z = z] = \begin{cases} (1-p)^{z-1}p, & z = 1, \dots, 4; \\ (1-p)^4, & z = 5 \end{cases} \tag{3}$$

where $z = 1, \dots, 4$ is the ordering number of quarter when for the first time the market transits into 'stressed' state, $z = 5$ indicates that the market remains in normal state during the first four quarters. For example, if the market transits into stressed state at the beginning of the second quarter (i.e. $z = 2$), then the proxy reinsurer either

1) defaults with normal default rate during the first quarter; or

2) survives the default till the beginning of the second quarter in the market being in unstressed environment, and then either defaults with stressed default rate in the period from quarter two to quarter four or survives the default.

3 The tower of layers of catastrophe reinsurance programme above the main cat retention.

4 For example, industry research shows that following Hurricane Katrina, there were several downgrades of reinsurers for a period of approximately 18 months whilst those reinsurers rebuilt their balance sheets.

The use of a 'stress' scenario that affects all reinsurers is a useful method by which co-dependency between reinsurer defaults can be allowed for.

In addition to this, it is assumed that the global reinsurance market transits into 'stressed' state in a particular period j if the quarterly gross natural peril loss X_j exceeds its $(1-p)$-th percentile, i.e. when $X_j > \mathrm{VaR}_{1-p}(X_j)$. It is also assumed that the quarterly transition of the market is independent of the cumulative catastrophe losses incurred in the periods prior to market transition, as it is believed that 'unstressed' reinsurers sustaining losses in a quarter would quickly re-capitalise during the same period. Those additional assumptions allow for tail dependencies between the primary insurer and reinsurers providing cover to it.

Loss given default assumptions

The default recoveries from outstanding potential reinsurance recoverables in the event of default are modelled as a default recovery rate ρ times exposure-at-default. The recovery rate can be modelled either deterministically or stochastically with *Beta* distribution on interval from zero to one. The recovery rate ρ is credit rating specific. The ultimate cost of default is then calculated as exposure-at-default net of default recoveries plus the cost of replacing residual cover lost due to default. More specifically, we define the ultimate default cost on i-th reinsurer proxy providing cover on both upper and lower layers, and defaulting in quarter j as

$$\begin{aligned} &(1-\rho_i) \times \left[w_i^U \times R_U(X_j) + w_i^L \times \left(R_L(X_j) + R_{LT}(j)\right)\right] \\ &+\frac{4-j}{4}\left[w_i^U \times \pi_U(i) + w_i^L \times \pi_L(i)\right], \end{aligned} \tag{4}$$

where

- $R_U(X_j)$ and $R_L(X_j) + R_{LT}(j)$ is the gross exposure-at-default for upper and lower layers respectively, with $R_{U(L)}(X_j)$ being the total recoveries from upper(lower) layers of the reinsurance program on catastrophe losses X_j incurred in period j , and $R_{LT}(j)$ being the total recoverables on long-tail (liability) losses at the end of period j. It is assumed here that cat losses incurred in the quarter period are fully paid in the same period, which implies that a reinsurance proxy defaulting at the beginning of period gives rise to gross exposure-at-default equal to the total recoveries on cat losses incurred only during the period. On the other hand, the total recoverables $R_{LT}(j)$ on long-tail losses at the end of the period would also include outstanding recoveries in relation to losses incurred prior to default event.

- $\frac{4-j}{4}\left(w_i^U \times \pi_U(i) + w_i^L \times \pi_L(i)\right)$ is, in essence, a portion of Deferred Reinsurance Expense of reinsurance program shared by reinsurer proxy i, which, in the event of default, indicates (an 'optimistic' estimate of) the total cost of replacing residual cover lost due to default of the proxy. Here, $\pi_{U(L)}(i)$ is the cost of the part of company's reinsurance program providing cover in upper(lower) layers assuming the cover is 100% placed with reinsurance proxy i.

Ultimate total default cost as a 'carrier' of retained reinsurance credit risk

The ultimate total default cost specific to upper(lower) layers of the reinsurance program are then defined as a risk function (carrier) linear in weights (i.e. exposure limits) $\mathbf{w}^U = \left(w_1^U, \ldots, w_m^U\right)$ and $\mathbf{w}^L = \left(w_1^L, \ldots, w_m^L\right)$

$$\begin{aligned} C_L\left(\mathbf{w}^L\right) &= \\ &= \sum_{i=1}^{m}\sum_{j=1}^{4}\Big\{(1-\rho_i)\times w_i^L \times \left(R_L(X_j) + R_{LT}(j)\right) \\ &\quad + \frac{4-j}{4} w_i^L \times \pi_L(i)\Big\} \times \mathcal{D}_{ij}(Z), \end{aligned} \tag{5}$$

and

$$\begin{aligned} C_U\left(\mathbf{w}^U\right) &= \\ &= \sum_{i=1}^{m}\sum_{j=1}^{4}\Big\{(1-\rho_i)\times w_i^U \times R_U(X_j) \\ &\quad + \frac{4-j}{4} w_i^U \times \pi_U(i)\Big\} \times \mathcal{D}_{ij}(Z), \end{aligned}$$

Then the total ultimate cost default across the reinsurance program is

$$C\left(\mathbf{w}^L, \mathbf{w}^U\right) = C_L\left(\mathbf{w}^L\right) + C_U\left(\mathbf{w}^U\right).$$

2.2 *Risk minimisation and derivation of optimal exposure limits*

As was already mentioned in the beginning of this section, the minimisation of risk is based on the following four pillars:

1) *definition/perception of risk*;
2) *risk carrier*, which in turn depends on the way the risk realisation is modelled;
3) *risk appetite/tollerance*; and
4) *risk measure* is driven by 1) and 3), and applied to risk carrier to measure the risk.

Those four pillars will determine the objective function of our risk minimisation problem. To find out how it looks and can be minimised, we first need to get a good understanding of each of the four pillars listed above.

Perception of risk

The reinsurance credit risk is the risk of the reinsurance counterparty failing to pay reinsurance recoveries to the ceding insurer in a timely manner, or even not paying them at all. It emerges mainly because the ceding insurer pays insurance claims to policyholders before reclaiming reinsurer's part. Default by a reinsurer will - potentially - lead to losses to the ceding insurer distressing insurance results in Profit and Loss statement and capital position in the Balance Sheet.

Risk carrier

The reinsurance credit risk carrier is defined as the total ultimate cost of default over the period of one year, $C\left(\mathbf{w}^U, \mathbf{w}^L\right)$. The approach to modelling the risk carrier was provided above in subsection 2.1.

Risk appetite and incentives to minimise/mitigate specific risk

The primary goal of underlying reinsurance program is to protect the insurer from insolvency and provide stability of insurance results. It is usually designed in such a way that the cover in upper layers, i.e. in the 'main cat tower', would protect the insurer from imminent insolvency in the events when it sustains initial impact from large catastrophe losses, whereas the cover in lower layers would be purchased to enhance stability of insurance results once solvency protection from upper layers is ensured. From shareholders point of view the use of reinsurance by insurance risk managers to hedge against large (un-tolerated) risks is usually welcomed as it increases the return on capital, however the reinsurance credit risk that emerges as a by-product of reinsurance purchasing is not expected to bounce back to the ceding company. Therefore, the reinsurance credit risk is perceived (by shareholders) as a company's specific risk, which, when retained, must be minimised. The insurance risk managers will minimise:

- any reduction, due to reinsurance credit risk, in the benefits of lowering insurance results volatility anticipated from reinsurance program; and
- capital consumption attributable to reinsurance credit risk.

Risk measure

The choice of risk measure is mainly dependent on how the retained reinsurance credit risk impacts the benefits anticipated from the reinsurance program. When considering volatility of insurance results one would traditionally opt for standard deviation or even one-sided standard deviation of insurance results falling below their expected value. On the other hand, when looking at solvency protection, tail-sensitive risk measures are common choice in risk management.

In general, the use of variance-based risk measures in risk minimisation can be justified mathematically by either/both of assumptions: 1) shareholders' (risk managers') utility function is quadratic; 2) the distribution of risk carrier is normal, implying that the standard deviation is fully characterises the distribution. Unfortunately, neither of those two assumptions are realistic in insurance, as insurance losses are extremely skewed and shareholders' absolute risk averseness decreases with wealth[5] in contrast to investors with quadratic utility. While the use of variance-based risk measures in insurance, in particular when pricing underwriting risk (attritional losses), could still be justified through diversification of large risk portfolio (asymptotically converging to 'normality'), it is not appropriate for measuring risks with highly skewed and/or heavy tailed distribution, e.g. catastrophe losses or losses from reinsurance default. Therefore, risk measures capturing most, if not all, of the characteristics of distribution are more desirable. When measuring and minimising risk one could, for example, 'leapfrog' the variance and consider higher moments, measuring skewness and tail heaviness of risk distribution (as it was done in Powers [8]), or even a quantile-based risk measure, like a TVaR measure averaging risks in the distribution tail above their central estimate.

When measuring risk insolvency(financial distress) both insurers and regulators often employ tail-sensitive risk measures like VaR or TVaR. In particular, when deriving risk capital of an insurer under solvency restriction VaR is preferable candidate. This can been explained from the following

5 As was shown in Krvavych [7] the maximization of shareholder value under solvency restrictions is approximately equivalent to the maximization of shareholder value using utility approach with a specific isoelastic utility function.

two points of view. From the regulator's point of view, when considering simultaneous minimisation of policyholders deficit and cost of capital, VaR comes up as an optimal solution to this problem (e.g. see pp. 73-75 in Denuit et al. [5]). From insurer's point of view, risk managers are not interested in 'how bad is bad' in the event of insolvency, as at that instance they will be replaced and/or control of the company will be taken over by regulator. Hence, VaR is used again.

Now, returning back to the problem of assessing the impact of retained reinsurance credit risk on the benefits anticipated from underlying reinsurance program we consider the following risk measures:

- $\mathbf{RM}_L$ - (preferably) a TVaR risk measure averaging risk carrier above its central estimate, or a variance-based measure (e.g. standard deviation or semi-standard deviation). The risk measure $\mathbf{RM}_L$ is used to measure the reduction in the benefits of the reinsurance program in lower layers through the difference

$$\begin{aligned}&\mathbf{RM}_L\left[(X-R)+(Y-R_{LT})+C_L\left(\mathbf{w}^L\right)\right]\\&-\mathbf{RM}_L\left[(X-R)+(Y-R_{LT})\right],\end{aligned}$$

  where $X=\sum_{j=1}^{4}X_j$ is the total catastrophe losses incurred over the period of one year, $R=\sum_{j=1}^{4}R\left(X_j\right)$ is the total cat recoveries, Y and R_{LT} - is the total attritional claims (i.e. small working plus large liability claims) incurred over the period of one year and their non-cat reinsurance recoveries respectively, and $C_L\left(\mathbf{w}^L\right)$ is the total ultimate cost of reinsurance default over one year specific to lower layers of reinsurance program; and

- $\mathbf{RM}_U$ - a TVaR risk measure averaging risk carrier in the tail of its distribution above a certain percentile. The total cost of reinsurance default, $C\left(\mathbf{w}^U,\mathbf{w}^L\right)$, plays the role of risk carrier here. This risk measure is used to measure capital consumptions due to retained reinsurance credit risk. It should be noted that while the main risk based capital of the company is likely to be calculated using a VaR risk measure, it is more sensible to calculate marginal capital consumption attributable to additional amount of risk using a TVaR. Indeed, in this situation a TVaR measure indicates how much of the main capital is eroded on average due to reinsurance default when the cost of reinsurance default exceeds a certain tolerance level. The choice of tolerance level, i.e. the percentile of the TVaR measure, is customary and depends on the risk managers' tolerance to reinsurance credit risk.

2.2.1 *Convex optimisation of exposure limits - numerical approach*

We consider the following problem of minimising retained reinsurance credit risk

$$\begin{cases}\min\limits_{\mathbf{w}^L,\mathbf{w}^U} & \left\{a\times\mathbf{RM}_L\left[Z+C_L\left(\mathbf{w}^L\right)\right]+\mathbf{RM}_U\left[C\left(\mathbf{w}^U,\mathbf{w}^L\right)\right]\right\}\\[2ex] \text{s.t.} & \sum_{i=1}^{m}w_i^L=1;\;\;\sum_{i=1}^{m}w_i^U=1;\\ & w_i^L\geq 0;\;\;w_i^U\geq 0;\\ & \sum_{i=1}^{m}\left(w_i^L\times\pi_L(i)+w_i^U\times\pi_U(i)\right)=c>0\end{cases}\tag{6}$$

where $Z=(X-R)+(Y-R_{LT})$ is the total annual net(of reinsurance) claims expense excluding cost of default, and the last constraint equality is budget constraint of insurer's annual reinsurance expense. The target function of the minimisation problem (6) is the mixture of two risk measures $\mathbf{RM}_L$ and

$\mathbf{RM}_U$ measuring impact of retained reinsurance credit risk on effectiveness of underlying reinsurance program in lower and upper layers respectively. The coefficient a is the positive weight assigned to $\mathbf{RM}_L$ to indicate the level of significance of this risk measure relative to $\mathbf{RM}_U$ in the minimisation of target function.

The optimisation problem (6) can be reduced to convex programming and then solved numerically via simulation. The convexity of (6) is mainly due to convexity of chosen risk measures. Indeed, variance-based measures (like standard deviation or semi-standard deviation) are convex, and TVaR is coherent which implies it is convex too.

Despite those nice properties of the risk measures used in (6), there is one inconvenience related to calculation of TVaR. Direct use of TVaR in its canonical form would require calculation of VaR, which could be problematic when applying it to non-continuous random variables (e.g. random mixture of Bernouili random variables). This inconvenience can be avoided by replacing TVaR with the limiting function from its 'representation form'. In general, for a TVaR of a random variable V its corresponding representation form is:

$$\begin{aligned} \mathrm{TVaR}_\alpha[V] &= \inf_{x\in\mathbb{R}} \Upsilon_\alpha[V,x] \\ &= \inf_{x\in\mathbb{R}} \left\{ x + \frac{1}{1-\alpha}\mathbb{E}\left[(V-x)_+\right] \right\}. \end{aligned} \tag{7}$$

The TVaR representation form is induced by the following regulator's problem of finding an optimal solvency capital requirements $r[V]$ (from the set of risk measures) that minimises the total of policyholder deficit $\mathbb{E}\left[(V-r[V])_+\right]$ and the cost of capital $\varepsilon \times r[V]$ with $\varepsilon = 1-\alpha,\ \varepsilon \in (0,1)$:

$$\min_{r[V]} \left\{ \varepsilon \times r[V] + \mathbb{E}\left[(V - r[V])_+\right] \right\}, \tag{8}$$

which gives $\mathrm{VaR}_{1-\varepsilon}[V]$ as the optimal solution[6].

Furthermore, when the random variable V is the linear combination of different random variables with coefficients $\mathbf{w}$ the function $\Upsilon_\alpha[V(\mathbf{w}),x]$ is convex in $(\mathbf{w},x)$, and, as it was shown in Rockafeller and Uryasev [9], the convexity holds for general random variables admitting discreteness.

Now, by replacing the TVaR risk measure with the limiting function Υ we restate the target function in (6) as:

$$\begin{aligned} f &\left(\mathbf{w}^U, \mathbf{w}^L, x, y\right) \\ = &\ \Upsilon_\alpha\left[C\left(\mathbf{w}^U,\mathbf{w}^L\right), x\right] \\ &+ \begin{cases} a \times \Upsilon_{0.5}\left[Z + C_L\left(\mathbf{w}^L\right), y\right], & \text{(when using TVaR)} \\ a \times \mathbf{RM}_L\left[Z + C_L\left(\mathbf{w}^L\right)\right], & \text{(when using variance-based)} \end{cases} \end{aligned} \tag{9}$$

which is convex in $\left(\mathbf{w}^U, \mathbf{w}^L, x, y\right)$, and apply numerical convex programming to minimise $f\left(\mathbf{w}^U, \mathbf{w}^L, x, y\right)$ with respect to $\left(\mathbf{w}^U, \mathbf{w}^L, x, y\right)$.

3 NUMERICAL EXAMPLE

3.1 *Setup*

In this section we provide a numerical example of solving optimisation problem (6) for optimal limits of exposure to reinsurance credit risk. We consider a general insurer that writes insurance business with the following profile of loss exposure:

6 Please refer to Denuit et al. [5]) for an elegant geometric proof of this result.

- aggregate annual attritional loss Y that follows a log-normal distribution with mean 2.5Bn, CoV of 20%, and no reinsurance cover for large liability losses, i.e. $R_{LT} = 0$;
- aggregate quarterly catastrophe loss X_j following a Pareto distribution with mean 70M and CoV of 300%;
- the insurer buys an annual reinsurance cat XOL cover of 100M xs 1,900M for the main cat tower (upper layers), and a combined (XOL and Stop-Loss) cover for losses between 25M and 100M (lower layers), and all the covers can be purchased from reinsurers of credit quality not worse than BBB+;
- aggregate quarterly recoveries on cat losses from lower and upper layers are respectively
  - $R_L(X_j)$ following a Pareto distribution with mean 50M and CoV of 125%; and
  - $R_U(X_j)$ following a Pareto distribution with mean 70M and CoV of 400%.

It is also assumed that the cost of reinsurance program for both lower and upper layers varies by credit rating of reinsurer as follows (premiums in M) and the total reinsurance expense is budgeted at 255M:

	AAA	AA+	AA	AA-	A+	A	A-	BBB+
π_U	160	153	147	140	135	130	124	118
π_L	130	120	115	110	107	105	102	100

The assumptions listed above would have been available as outputs from a DFA model that is used for capital modelling. Furthermore, it is assumed that both risk measures specific to upper(lower) layers of the reinsurance program are defined by a TVaR with 50-th percentile used for the risk measure specific to lower layers and 90-th percentile for the one specific to upper layers.
In addition to this, we adopt from Britt and Krvavych [3] the following exogenous assumptions in relation to reinsurance default frequency and severity:

Default rates - annual forward rates in 'normal' and 'stressed' states

Calibrated conditional 'normal' annual forward rates of default (in %)

YEAR	AAA	AA+	AA	AA-	A+	A	A-	BBB+
1	$6.9{\times}10^{-5}$	0.038	0.089	0.148	0.218	0.277	0.350	0.744

Calibrated conditional 'stressed' annual forward rates of default (in %)

YEAR	AAA	AA+	AA	AA-	A+	A	A-	BBB+
1	0.987	2.466	3.688	4.839	6.016	6.926	7.951	12.589

Those conditional default rates were calibrated in Britt and Krvavych [3] using reinsurance default modelling approach outlined in subsection 2.1 to recover unconditional default rates from the AM Best research study[7] [1] titled "Securitisation of Reinsurance Recoverables" published in August 2007.

7 The study defines the reinsurer default as an asset impairment event.

Global reinsurance market transition rate

It is assumed that the global reinsurance market enters 'stressed' state with the transition rate of 10% p.a., and that the duration of 'stress' state equals two years.

Average loss rates given default

The default recoveries from outstanding potential reinsurance recoverables can be modelled either deterministically or stochastically via introducing a random recovery rate with *Beta* distribution on interval from zero to one. In this setup we elect deterministic approach to modelling recoveries. The following average loss rates given default were derived from the industry study of reinsurance default recovery rates conducted by GIRO Working Group of the UK Institute of Actuaries (see Bulmer et al. [4])

AAA	AA+	AA	AA-	A+	A	A-	BBB+
0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6

3.2 *Result of numerical optimisation*

The convex programming procedure was run with 50,000 simulation trials using `NMinimize` library function in MATHEMATICA™ programming suite, and the following optimal exposure limits (in %) were obtained:

	AAA	AA+	AA	AA-	A+	A	A-	BBB+
w^U	19.7	17.0	16.4	15.7	13.1	9.8	6.6	1.6
w^L	17.2	16.7	16.4	16.2	13.9	10.6	7.2	1.8

4 CONCLUSIONS

In this paper we considered an approach to managing exposure to reinsurance credit risk. While comparing different options for active management of such type of risk, we distinguished the one based on the idea of retaining and actively managing the risk as the most realistic option for insurance companies. Under such circumstances risk, perceived as insurer's specific risk, must be minimised in order to enhance shareholders value. The proposed approach outlines key concepts of the modelling of reinsurance credit risk and minimises it trough setting optimal exposure limits per credit rating represented by a single proxy reinsurer. The convex programming is used to find the set of optimal reinsurance credit risk exposure limits that minimise the negative impact of reinsurance default on the benefits anticipated from underlying reinsurance cover. This was done numerically via stochastic simulation and illustrated with the numerical example for a generic non-life insurer.

REFERENCES

[1] AM Best. Securitisation of Reinsurance Recoverables. *Best's Rating Methodology*, pages 1–7, August 2007. http://www.ambest.com/ratings/methodology.

[2] Neil M. Bodoff. Discarding Risk Avoidance and Embracing Risk Optimization: Managing Reinsurance Credit Risk. *ERM Symposium, April 12-15, 2010, Chicago, IL*, 2010. Web link to PDF file.

[3] Stephen P. Britt and Yuriy Krvavych. Reinsurance Credit Risk Modelling - DFA Approach. *ASTIN Colloquium, June 1-5, 2009, Helsinki*, 2009. Web link to PDF file.

[4] Richard Bulmer et al. Reinsurance Bad Debt Provisions for General Insurance Companies. *GIRO Working Group, The Institute of Actuaries (UK)*, 2005. Web link to PDF file.

[5] M. Denuit et al. *Actuarial Theory for Dependent Risks: Measures, Orders and Models*. Wiley, UK, 2005.

[6] Emanuel Derman, Paul Wilmott. The Financial Modelers' Manifesto. Web puplication, 2009. http://www.wilmott.com.

[7] Yuriy Krvavych. Enhancing insurer value through reinsurance, dividends and capital optimization: an expected utility approach. *ASTIN Colloquium, June 19-22, 2007, Orlando, FL*, 2007. Web link to PDF file.

[8] Michael R. Powers. 'Leapfrogging' the Variance: The Financial Management of Extreme-Event Risk. *Journal of Risk Finance*, (4):26–39, 2003.

[9] Tyrrell Rockafeller and Stanislav Uryasev. Conditional value-at-risk for general loss distributions. *Journal of Banking and Finance*, (26): 1443–1471, 2002.

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