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Paper : **TERRITORIES, BOUNDARIES, AND CONNECTIONS**

Topic: Architecture

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Abstract:
Reading Frei Otto's very stimulating book [1], I was incited to pursue investigations I exposed at GA2005 [2] about distance maps and other ways of generating territories. Frei Otto is well know for his work on lightweight structures and constructions inspired by nature, but in this book he explores more fundamental topics about space, how it is occupied and how places are connected. His main preoccupation is about town planning, but his experiments and reflections go far beyond that field.
Frei Otto does not use computers and his book is illustrated by rough sketches and photos of simple experiments involving magnetic needles and polystyrene chips floating on water, soap bubbles, flowing sand, and so on. But those processes may be simulated by algorithms, such as the ones I experimented in my previous paper [2].
This paper will explore dynamic processes leading to the formation of territories, and of boundaries between them, and to connections between points. Those topics involve once again the notion of dimension, and the relationship between dimensions. They imply also questions of duality and reversibility between centres and junctions, and between boundaries. The question of whether and how those processes are generative will also be at the core of my theoretical reflection.
This paper will be illustrated by works of the author, and also by some works of her students.

References:
[1] Frei Otto, "*Occupying and Connecting.* ", Edition Axel Menges, Stuttgart/London, 2009
[2] Marie-Pascale Corcuff, "*Generativity and the Question of Space*", GA2005, www.generativeart.com

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Territories, Boundaries, and Connections

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Abstract

Reading Frei Otto's very stimulating book [1], I was incited to pursue investigations I exposed at GA2005 [2] about distance maps and other ways of generating territories. Frei Otto is well know for his work on lightweight structures and constructions inspired by nature, but in this book he explores more fundamental topics about space, how it is occupied and how places are connected. His main preoccupation is about town planning, but his experiments and reflections go far beyond that field.
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1. Groundings: Frei Otto's research

1.1 An architect inspired by nature

Frei Ottto, born in 1925, is a German architect and engineer, well known for his lightweight tensile and membrane structures, like for instance the cable net of the Expo '67 German Pavillon (Montréal, Canada), or the roof of the Munich Olympic Arena (1972, with Günter Behnisch). As such, he is in the tradition of great constructors like Felix Candela or Pier Luigi Nervi, or architects like Buckminster Fuller with whom he bears some similarities. His architectural vision has left his mark on the 20th century, and was a strong influence on younger architects, like Shigeru

Ban, with whom Otto collaborated on the Japanese Pavillon of Expo 2000 (Hanover, Germany), which has got a roof made entirely of paper.

Frei Otto «is a technician, artist and philosopher in one, and his central concern is for a new and all-embracing link with nature in building» [1]. One must say also that he is a true architectural «researcher». He founded in 1961 the research team *Biologie und Bauen*, and in 1964 the *Institut für Flächentrageweke* at the *Tescnische Hochschule in Stuttgart*, where he and his team experimented with models inspired by natural structures, in collaboration with biologists like Johann-Gerhard Helmcke. The result of this research was for instance the self-standing bell-tower of a church in Berlin-Schönow which was referred to the skeleton of a diatom. His exploration of natural forms does not only concern biological structures, but also phenomena like the formation of bubbles.

Otto's concern in nature was surely grounded, at least in part, in his reading D'arcy Thompson's famous book [2], which has been so influencial on so many biologists, mathematicians, artists, and architects (Louis I. Kahn, for instance). It is remarkable that the only reference, outside of self-references (including his Institute), in Otto's book [1] is precisely Thompson's book, though with no precisions on how Thompson's ideas are exploited. Thompson insisted on physical laws and mechanics as determinant on the form and structure of living organisms. It is then not surprising that architects may be interested by his work.

In *Occupying and Connecting. Thoughts on Territories and Spheres of Influence with Particular Reference to Human Settlement* [1] (Fig. 1), Frei Otto does not particularly look for ideas for constructive structures, but, as the very explicit title of his book says it, he explores very fundamental topics about space, how it is *occupied* and how places are *connected*.

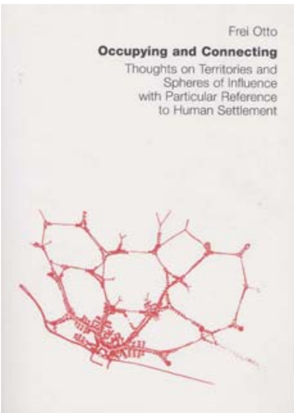

Fig. 1: Frei Otto, Occupying and Connecting [1]

1.2 Frei Otto's method

Otto's method consists in observation, speculation, and experimentation. He observes how phenomena happen in nature, either as the result of the behaviour of living organisms, or with inert materials responding to physical laws, and also how such phenomena occur when human beings intervene. Those phenomena are described with words, and also with rough sketches. Even by those means, Otto speculates on the interpretation of those phenomena. This speculation is continued by the setting of experiments that are supposed to simulate the observed phenomena. In [1], Otto examines essentially three types of phenomena: there is what he calls «occupation», but which concerns actually two topics: the *distribution* of points, and the definition or formation of *territories*; and there is the issue of *connections*, or paths. I shall explain his method on some of these topics which have particularly retained my attention, and on which I have worked later on.

Distributions

Frei Otto begins his book by observing and describing (with words an sketches) how «objects», considered as «points», occupy lines, surfaces, or 3D spaces. Those «objects» may be birds on a wire, fog dew on spider webs, trees in a wood, birds in a flock, water droplets in a cloud, and so on. He distinguishes first between random occupations, which he does define much, and planned occupations.

But more accurately he observes that two «forces» are at stake in any process of occupation: he qualifies some occupations as «distancing» (which could have been called «repulsive»), others as «attractive», and remarks that many occupation mechanisms are both attractive and distancing. Those types of «occupations» (i. e. distributions) are illustrated with sketches.

Attraction and repulsion are present in two physical forces: magnetism and static electricity. Those are the forces that Otto uses in his experiments.

In order to obtain «distancing» distributions of points, Otto's experimental apparatus is a basin of water in which «small rod magnets float (...), each with the same pole uppermost. The magnets repel each other and move away from each other. (...) They adopt a form of occupation which can be described as distancing. (...) Beneath the water, a template marks the surface which can be occupied.» ([1], p. 16). Otto experiments with different shapes and different numbers of needles (Fig. 2).

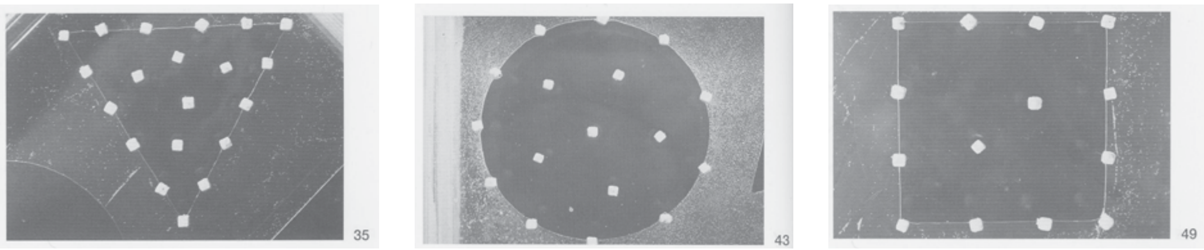

Fig. 2: [1] pp. 21, 22, 23

Attractive occupations are experimented with small soap bubbles. But when trying to experiment both attractive and distancing occupations (with magnetized needles and bubbles) Otto realized that the attractive force between the bubbles was too strong and pulled the needles with their rafts of bubbles. He then used magnetized needles and polystyrene chips (Fig. 3). He also used iron dust with magnetised needles.

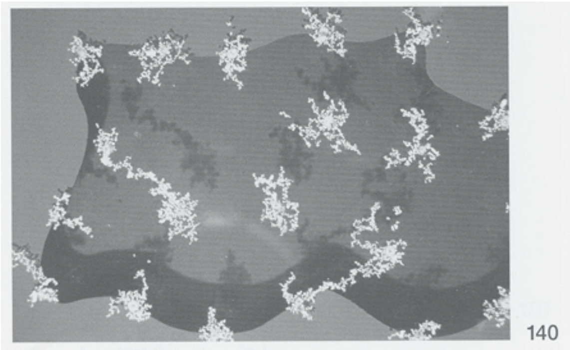

Fig. 3: [1], p. 45

1.2.2 Territories

Very soon in his book, Frei Otto relates the distributions of points to «territories», whose formation is described in these words: «one demarcates the territory by the perpendicular bisectors of the nearest points» ([1], p. 10), and a sketch (Fig. 4).

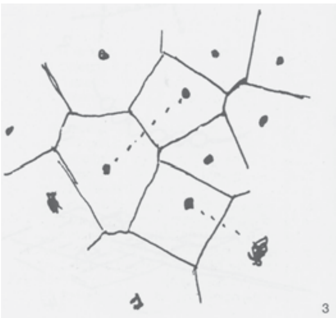

Fig. 4: [1], p. 10

Concerning the formation of territories, Otto describes a particular process in those words: «(...) seeds fall on five points. The occupied areas soon come into contact with each other, until the available surface is completely filled.» [1, p.32]. Even if it is not impossible to actually observe such a phenomenon, it is improbable that this observation was an actual one. Otto supposes that the «territories», which are the areas where some hypothetical plant has expanded from one seed, grow as concentric circles. Even more, he supposes that each circle he draws corresponds to the growth in one year. Otto draws three sketches: in the first one, 5 points (A-E) are marked inside a shape, each being the centre of three concentric circles. Territories issued from A, C and D, are already touching, and Otto draws a dotted line at the frontier. In the second sketch, all five territories have reached each other, and also the edge of the shape. The last sketch shows the end of the process, until the shape is completely covered, and even the border eventually crossed over (Fig. 5).

Fig. 5: [1], p. 32

Otto draws also a sketch (Fig. 6) of an experimental apparatus used at his Institute to study the expansion of territories, the «so-called sand flow apparatus»: «A flat box is filled with sand. It has holes in its bottom arranged in a pattern. The sand trickling out leaves craters and forms growing cones of debris.» [1, p.32]

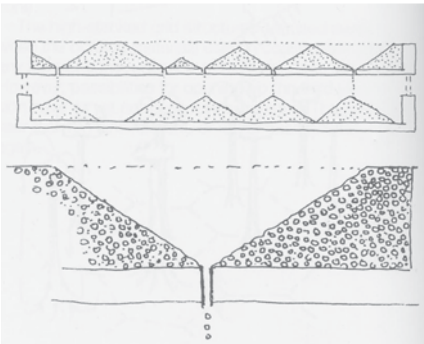

Fig. 6: [1] Ill. 89-90, p.32

1.2.3 Connections

Regarding connections, or paths, which I did not analyse so much as territories and distributions, one may remark first, that boundaries of territories are themselves possible connections (between junction points of those boundaries): in the examples given p. 51 in [1], Otto displays a section of dragonfly's wing, a maple leaf, a crack pattern, a soap bubble raft (which are all rather examples of boundaries) together with a thread model, a road network, or a minimal path network. Working on territories is then a way of working on connections, at least when connections generate closed units.

It is, for that matter, in this part, that Otto experiments with inkblots, or other drippings, which we would have awaited in the part about the formation of territories. The experiments dedicated to connections involve thread (dipped in water), and the «soap bubble skin apparatus».

1.3 A generative method: from material to digital experimentation

Observing natural processes and trying to reproduce them with some device is the starting point of a generative method. Frei Otto may not be qualified as a «generative» architect, but his observation of nature and his experiments may inspire some generative processes.

In his book, Otto ignores absolutely computer simulations. It is obvious, though, that many of his observations and experiments have their digital counterpart. Reading his book, I recognized some of my previous attempts, and I was induced to pursue them, or to imagine some new ones. In the next part, I shall display those experiments, which follow rather closely Frei Otto's exploration.

2. Experiments

1.1 Distance maps

Territories issued from sites, or «centres», are easily obtained by calculating a *distance map*. Let's remind the principle of a distance map: given a bitmap, in which some points, called centres, have been chosen, one gives to each pixel of the bitmap a level of grey according to its distance from the nearest centre. If we translate this level of grey into an altitude in a mesh, we get the result of the sand-flow apparatus. If we want to give a representation more alike Otto's sketches (Fig. 5), we can apply to pixels a level of grey which is calculated as a function based upon the sine of the distance from the nearest centre. In the next figures, the distribution chosen is practically the same as the one in Otto's sketches.

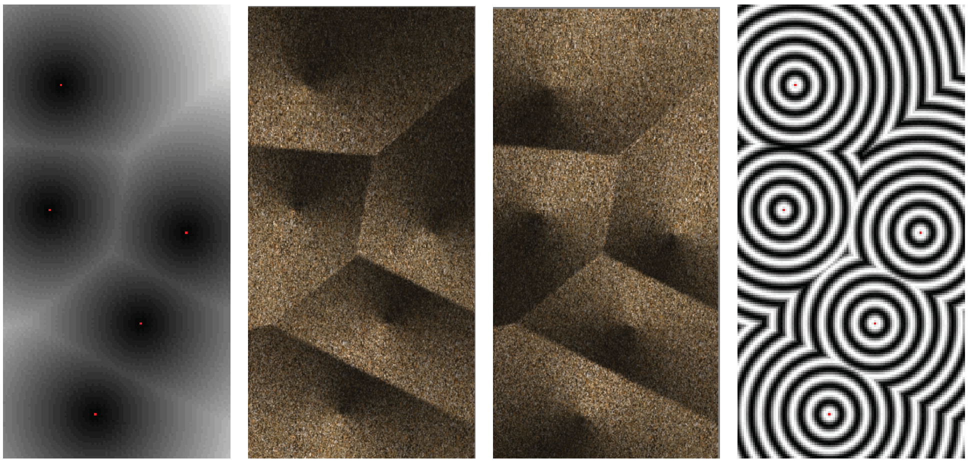

Fig. 7: distance map for 5 centres; sand craters and heaps; sinusoidal distance map

Distance maps may be related to medial axes (*cf* [2]), but also to Voronoi diagrams, anyway when centres are scattered, and not grouped. Otto does not mention this reference, though one of his first sketches (Fig. 4) and the description he gives («one demarcates the territory of an object by the perpendicular bisectors of the nearest points») are roughly the definition of Voronoi diagrams.

Though Voronoi cells are discernible in distance maps by the whitish lines that delineate them, and by the crest lines in the «sand flow» representaiton, one can get them more clearly by slightly changing the way of calculating the map: instead of affecting each pixel with a level of grey, one affects it with a colour corresponding to the nearest centre.

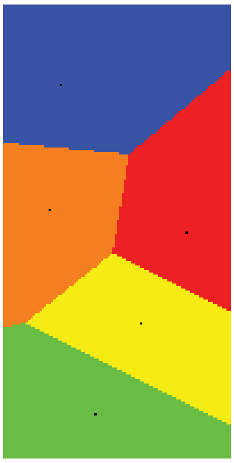

Fig. 8: Voronoi diagram for the same 5 centres

All of these experiments I had already done for my previous paper [2]. But Otto suggests two time-related variants, which I had not thought of. Distance maps are not processes evolving in time, like cellular automata for instance. In a way, time is translated into distance, and as distances are fixed at the start, distance maps only show a state of things. However, one can simulate a time-related effect by changing the way distance is calculated, as we shall see.

The first variant Otto suggests is to delay the start of growth for some centres: «If point A initially occupies its own territory, B follows, time-displaced, in year 3, C in year 6 and D is added in year 10.» ([1], p. 33) (Fig. 9)

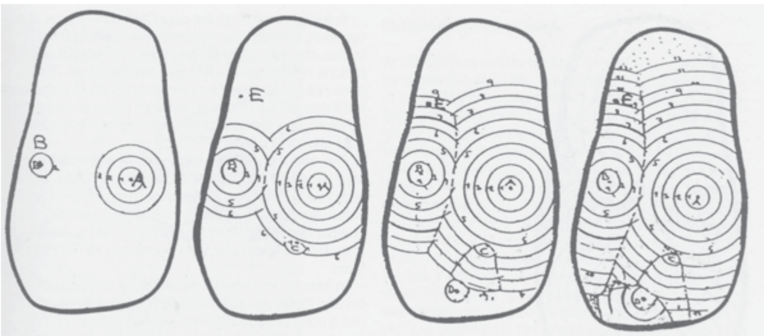

Fig. 9: [1], p. 33

Here we acknowledge that the concentric circles correspond for Otto to years of expansion. This can be simulated in the sand-flow apparatus: « A time lag in the flow is created by inserting small tubes of different heights into each hole.» In the same way, in the computing of the distance map, one can affect to each centre a number which will be *added* to the distance to this centre, when determining which centre is the nearest (Fig. 10):

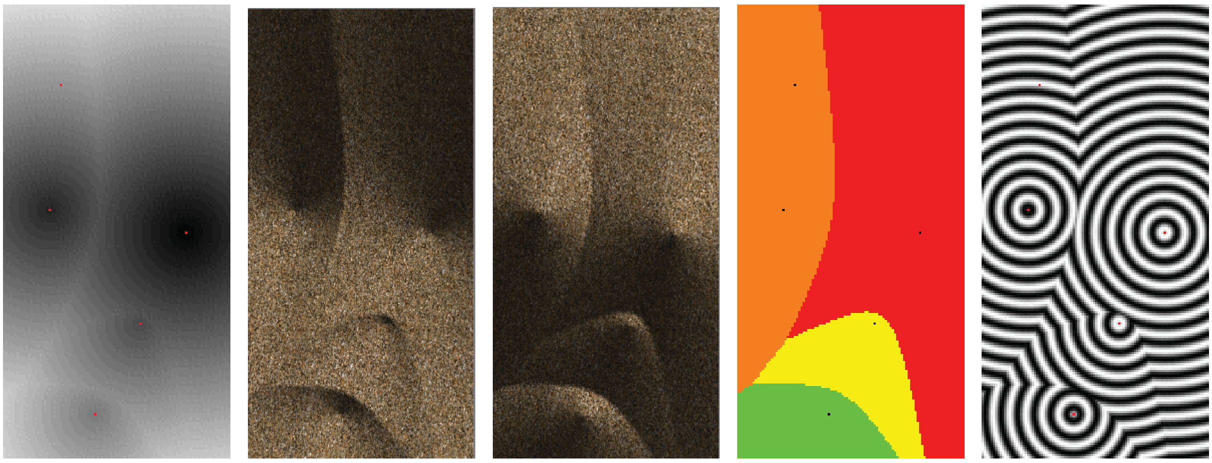

Fig. 10: distance maps, sand craters and heaps, and Voronoi diagram with time-lags

The second variant considers different speeds for the different centres. Otto sketches concentric circles more or less close to each other.

Fig. 11: [1], p. 34

He claims first that «speed of distribution can be adjusted by variations in the diameter of hole or tube» ([1], p. 34]. But then he admits that «this form of territory expansion can not be simulated with the sandbox, but diagrammatically and by means of computation.» (id). Indeed, it is easy to simulate different speeds by affecting each centre with a number, with which we *multiply* the distance from this centre, when determining which centre is the nearest (Fig. 12):

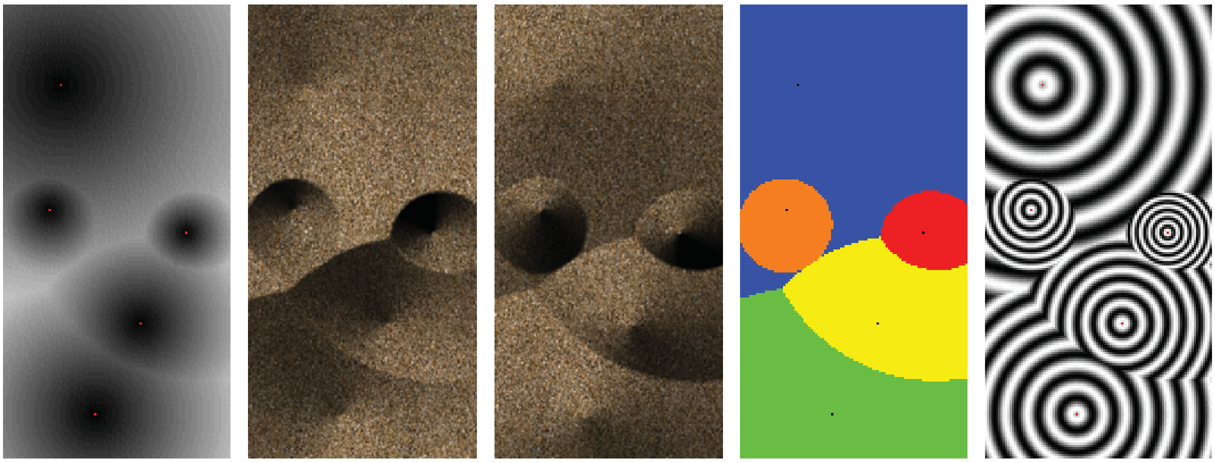

Fig.12: distance maps, sand craters and heaps, and Voronoï diagram for 5 centres with different speeds

Returning to distance maps with no time related differences, we can remark that boundaries of Voronoï diagrams provide us with an interesting net of connections between points, those points being the junctions between cells (Fig. 13):

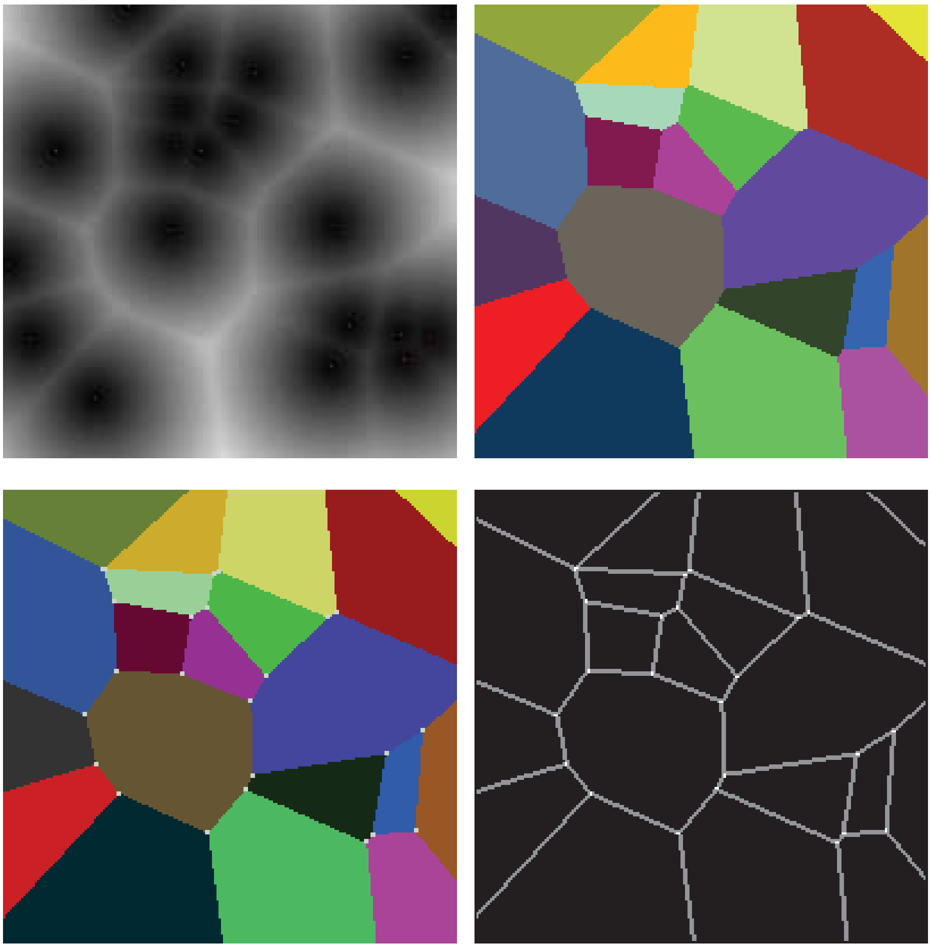

Fig. 13: distance map for a given set of 20 points, Voronoi diagram, junctions and connections

If the junctions of the Voronoi diagram of a set of centres was the dual of this set of centres, i. e. if the junctions of this first Voronoi diagram generated a new Voronoi diagram whose junctions were the first set of centres, then we would have a good way to generate connections between centres. It is the case only with regular configurations corresponding to regular tessellations of the plane (see [2]). But, unfortunately, in common cases, the junctions of the diagram of Voronoi generate a new diagram whose junctions are not the first set of centres.

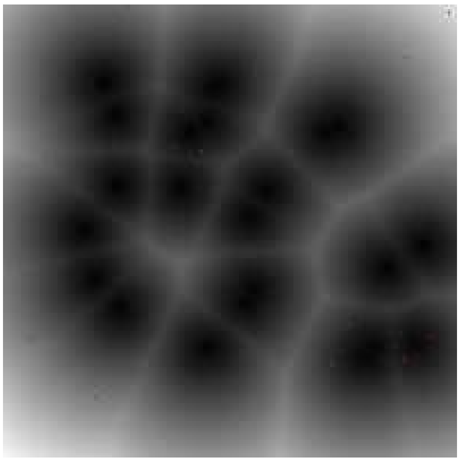

Fig. 14: distance map of the junctions found in Fig. 13 (in red, the original centres)

2.2 Minmax algorithm

The experiment described by Otto for distancing occupations (see 1.2.1 and Fig. 2) is a dynamic rearrangement. We don't know if the magnets are all put at the same time in the water, or if they are put one by one, but in any case it is obvious that they all move till they attain their definitive disposition. I preferred to simulate a more static as well as progressive one: my «magnets» are put one by one, each of them trying to be as far as possible from the ones already there. The simulation of this distancing mechanism is made by finding the pixel for which the minimal distance from «centres» already there is maximal (minmax algorithm). Fig. 15 shows the result of the algorithm with 10 centres and Fig. 16 with 50 centres. Distance maps are calculated, in order to better compare the distributions.

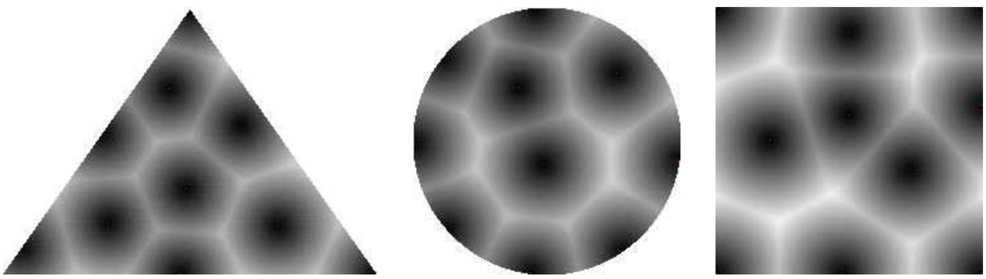

Fig. 15: minmax algorithm for 10 points

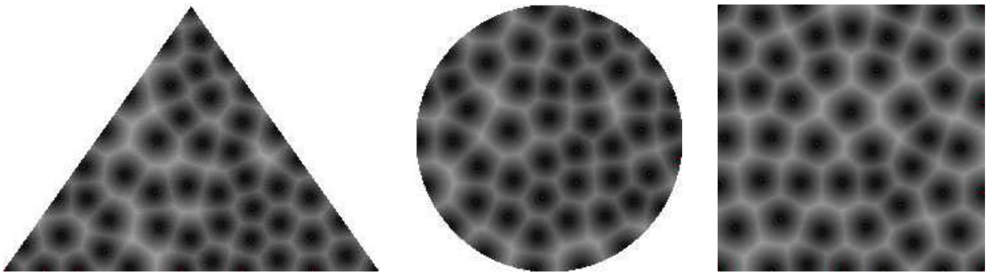

Fig. 16: minmax algorithm for 50 points

We see that, though not absolutely regular, the distributions we obtain are very near from a grid of equilateral triangles; the territories, as shown by the distance maps, form an hexagonal tiling. The more centres are added, the more equilibrated the distributions are. In any case, the distributions are very different from random ones (Fig. 17)

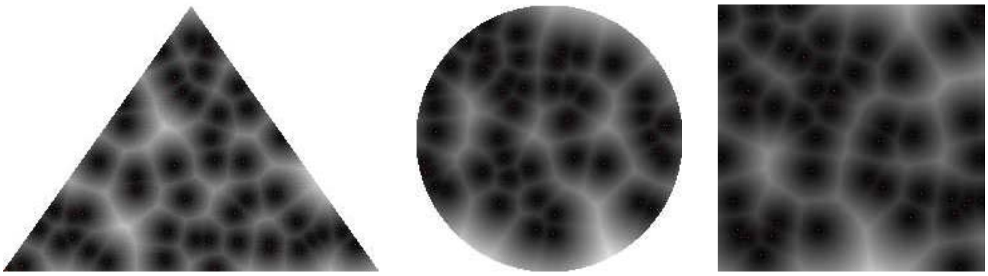

Fig. 17: random distributions of 50 points

2.3 Aggregations

In order to experiment attractive occupations, Frei Otto uses polystyrene chips attracted to each other by static electricity. The images of his experiments (Fig. 3) remind us of the various occurrences of forms which can be related to the well known diffusion-limited aggregation model, and its variants.

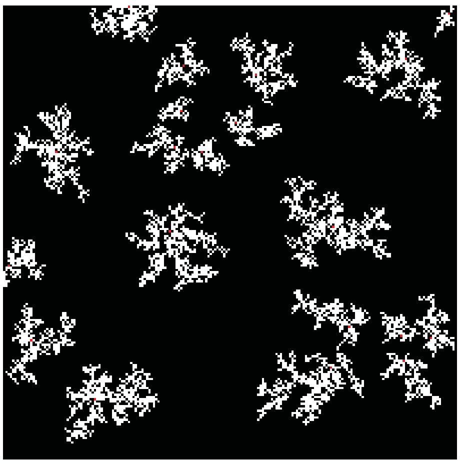

Fig. 18: DLA-like aggregation for a given set of 20 points

In this variant, «cells» are created at random and «walk» randomly till they attain an occupied cell. They then aggregate themselves to this cell (in their previous position in the random walk). One result based upon the same distribution of 20 points than in Fig. 13 is shown Fig. 18.

Returning to Otto's initial example, using this way of generating territories seems more adequate to his description of plants growing from five seeds (Fig. 19):

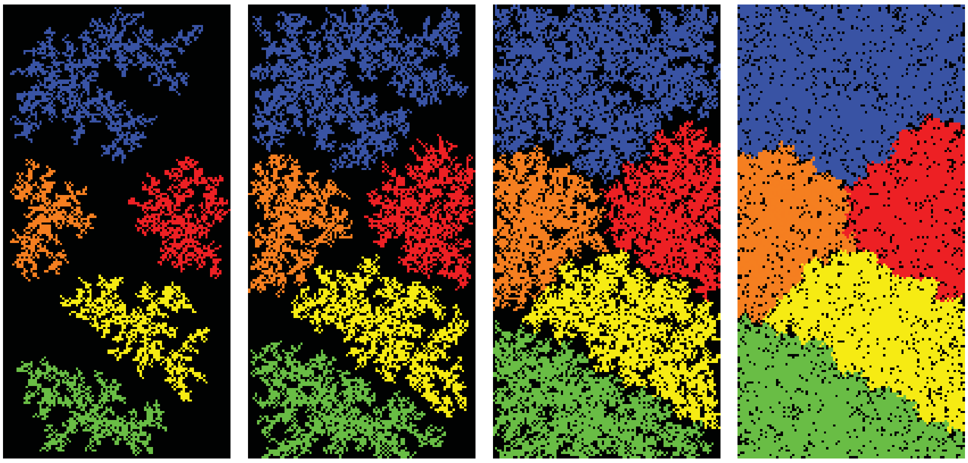

Fig. 19: DLA-like aggregation for a set of 5 points

And territories get to fill completely the space, with boundaries not very different from those obtained by distance maps.

3. Further developments and reflections

3.1 More about distance

Euclidean distances versus other distances

Most of the previous experiments are based upon distance: distance maps determine the «nearest» centre, the minmax algorithm the maximum of minimal distances. The Euclidean distance has been used. But it is possible to imagine other distances: the so-called Manhattan distance, for instance, which is very simple (distman (p1,p2)=abs(x1-x2)+abs(y1-y2)), gives a different distance map (Fig. 20) for the same set of points than in Fig. 13:

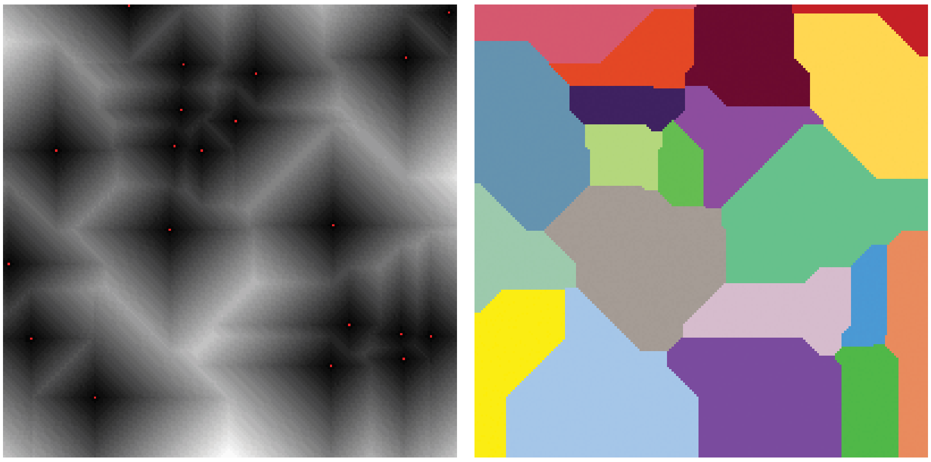

Fig. 20: distance map for a given set of 20 points (Manhattan distance)

Instead of cones, craters or heaps, it forms square based pyramids (Fig. 21):

Fig. 21: interpretation in sand of the distance map of Fig. 20

Minimal triangulation

Another way of using distance (Euclidean distance) was imagined by one of my students, Yann Huet. He explored the connections between nearest points of a set. One of his options was to link each point to the two nearest points, and also to link those three vertices of a triangle to its geometric barycentre.

What is interesting in this very simple algorithm, based only upon distance, is that for a random set of points, we obtain a net that is connected, or not connected. If we add points to the set, sometimes this addition connects parts that were disconnected, in other cases, it disconnects connected parts. Here are an example in 2D (Fig. 22) and an example in 3D (Fig. 23):

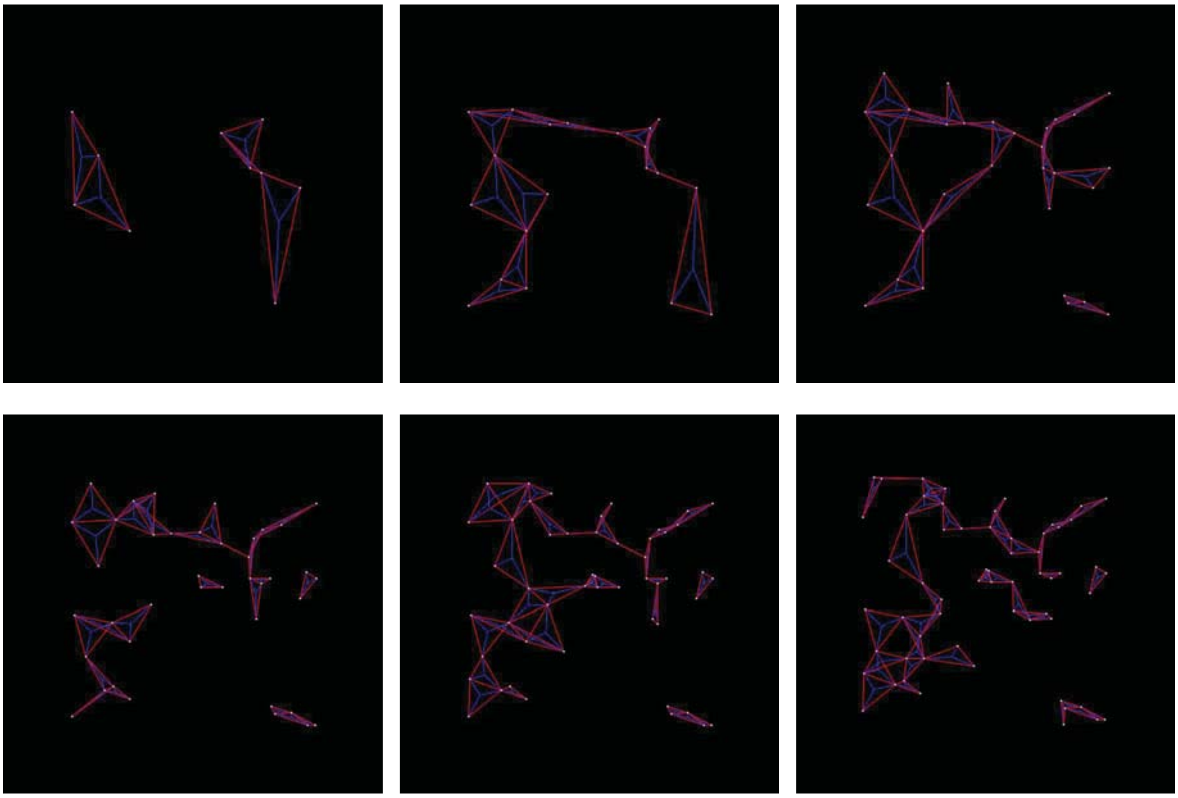

Fig. 22: minimal triangulation of a random set of points (10, 20, 30, 40, 50, 60 points)

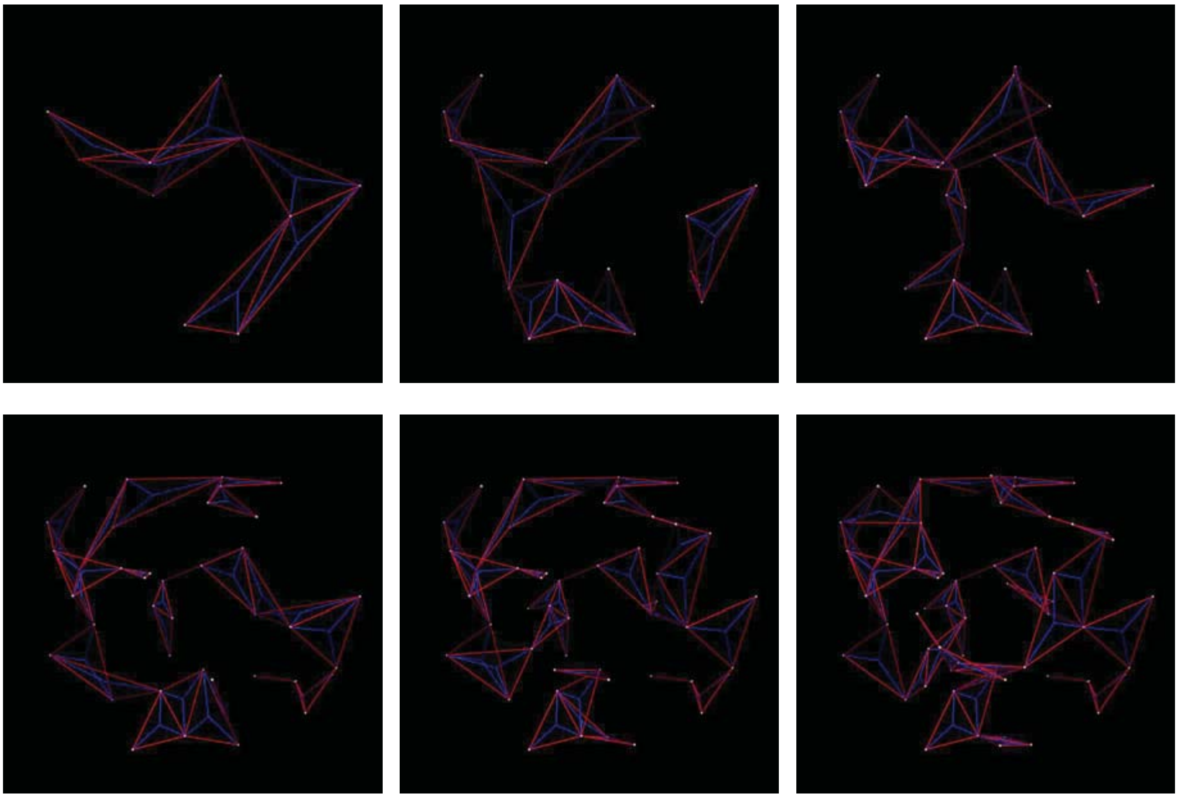

Fig. 23: minimal triangulation of a random set of points in 3D (10, 20, 30, 40, 50, 60 points)

It must be emphasized that this «minimal» triangulation is not the Delaunay triangulation, which is the dual of the Voronoi diagram.

3.2 Discrete spaces

A Voronoi diagram [4] (or Voronoi decomposition, or yet Voronoi tessellation) is a decomposition of any *metric* space (which means that you must have some *distance* to apply) and, given a number of objects («sites», or «generators», or «seeds», or yet «centres»), it is defined as the set of «cells» which are constituted, for each centre, by the points of the space whose distance to this centre is not greater than the distance to the other objects.

Calculating Voronoi diagrams may be difficult. Distance maps are an easy way to obtain a Voronoi diagram in a discrete space. A bitmap is such a space; the same computing may be done in 3D providing one can work in a 3D discrete space, whose elements may be called «voxels» (Fig. 24):

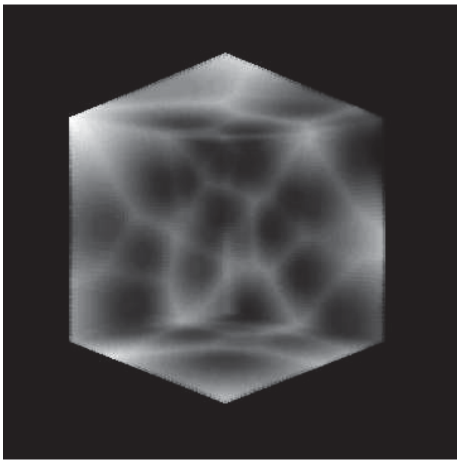

Fig. 24: 3D distance map

3.3 Dimensions

Locations, territories, boundaries, and connections, these topics imply once again the primordial notion of *dimension*. One can say that, in any n-space, a territory is a connected subset of the same dimension, its boundary is a closed manifold of dimension n-1. Locations are points (0-dim), and connections are lines (1-dim). For instance, on a line, territories are segments, their boundaries are points: there is then identity between locations and boundaries; but the idea of connection is not very gratifying, as there is only one way to connect two points, which is by the space itself. On a surface, on the plane for instance, territories are bounded surfaces, their boundaries are closed lines. Those lines may be interpreted as connections. And those lines may have vertices, which are points, and there may be a duality between locations and vertices of the boundaries. In 3D space, territories are volumes, their boundaries are closed surfaces. But those surfaces may have edges, which are lines (and then be interpreted as connections), and themselves have vertices which can be interpreted as locations.

Working in discrete spaces makes things easy, as we saw before, but algorithms that actually calculate Voronoi diagrams in continuous spaces are not so simple. In the Euclidean plane, the boundaries of the cells are defined by bisectors of centres taken two by two, which poses at least two problems: finding the equations of these bisectors is not too difficult, but cutting them to determine into edges of the polygon that encloses a given cell may be tricky; and finding which pairs of centres one must consider is also tedious...

But, if one works with a software that permits to «slice» or «cut» a given object by a plane, or if one implements that function, then one can easily produce the Voronoi diagram for a portion of a plane (or space), for instance a square (or a cube). One determines, for each centre, the planes that are bisectors of all pairs of centres that comport that particular centre, and one cuts the whole square (or cube) by those planes. Some cuts are not necessary, but it does not matter if you are not especially interested in time efficiency. Fig. 25 shows the result of this operation for the same 20 points as before; and Fig. 26 shows the same process in 3D:

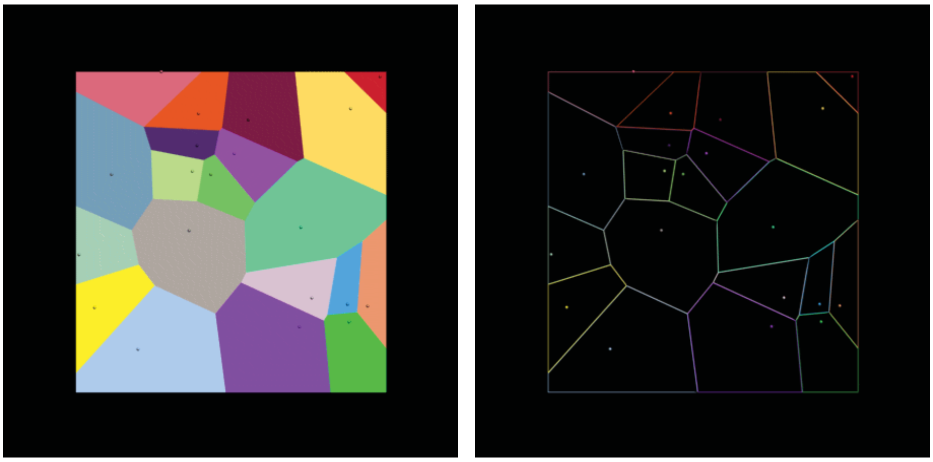

Fig. 25: Voronoi diagram «by cutting» for a given set of 20 points

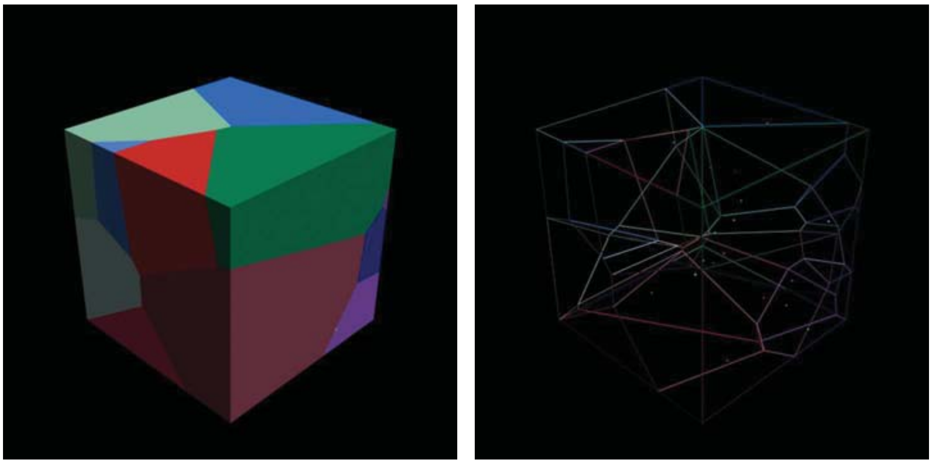

Fig. 26:Voronoi diagram «by cutting» for 20 points in 3D

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