Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
1	States or implies the formula for differentiation from first principles. $\mathrm{f}(x)=5x^3$ $\mathrm{f}'(x)=\lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$	**B1**	2.1	5th Complete a proof of a derivative function from first principles.
	Correctly applies the formula to the specific formula and expands and simplifies the formula. $\mathrm{f}'(x)=\lim_{h\to 0}\frac{5(x+h)^3-5x^3}{h}$ $\mathrm{f}'(x)=\lim_{h\to 0}\frac{5(x^3+3x^2h+3xh^2+h^3)-5x^3}{h}$ $\mathrm{f}'(x)=\lim_{h\to 0}\frac{15x^2h+15xh^2+5h^3}{h}$	**M1**	1.1b	
	Factorises the '*h*' out of the numerator and then divides by *h* to simplify. $\mathrm{f}'(x)=\lim_{h\to 0}\frac{h(15x^2+15xh+5h^2)}{h}$ $\mathrm{f}'(x)=\lim_{h\to 0}(15x^2+15xh+5h^2)$	**A1**	1.1b	
	States that as $h\to 0$, $15x^2+15xh+5h^2\to 15x^2$ o.e. so derivative = $15x^2$ *	**A1***	2.2a	
				(4 marks)

Notes

Use of δx also acceptable.

Student must show a complete proof (without wrong working) to achieve all 4 marks.

Not all steps need to be present, and additional steps are also acceptable.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
2	Attempts to differentiate.	**M1**	1.1a	5th Use derivatives to determine whether a function is increasing or decreasing in a given interval.
	$f'(x) = 3x^2 - 8x - 35$	**A1**	1.1b	
	States or implies that f(x) is increasing when $f'(x) > 0$	**M1**	1.2	
	Attempts to find the points where the gradient is zero. $(3x + 7)(x - 5) = 0$ (or attempts to solve quadratic inequality)	**M1**	1.1b	
	$x = -\frac{7}{3}$ and $x = 5$, so f(x) is increasing when $\{x : x < -\frac{7}{3}\} \cup \{x : x > 5\}$ (or $x < -\frac{7}{3}$ or $x > 5$)	**A1**	2.2a	
				(5 marks)

Notes

Allow other method to find critical value (e.g. formula or calculator). This may be implied by correct answers.

Correct notation ("or" or "$\cup$") must be seen for final A mark.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
3a	Attempts to differentiate.	M1	1.1a	4th Carry out differentiation of simple functions.
	$\frac{dy}{dx} = 3x^2 - 2x - 1$	A1	1.1b	
		(2)		
3b	Substitutes into equation for C to find y-coordinate. $x = 2, y = 2^3 - 2^2 - 2 + 2 = 4$	M1	1.1b	5th Solve coordinate geometry problems involving tangents and normals using first order derivatives.
	Substitutes $x = 2$ into $f'(x)$ to find gradient of tangent. $\frac{dy}{dx} = 3(4) - 2(2) - 1 = 7$	M1	1.1b	
	Finds equation of tangent using $y - y_1 = m(x - x_1)$ with (2, 4) $y - 4 = 7(x - 2)$	M1	1.1b	
	$y = 7x - 10$ o.e.	A1	1.1b	
		(4)		

3c	States or implies gradient of tangent is 7, so gradient of normal is $-\frac{1}{7}$	**M1**	1.2	5th Solve coordinate geometry problems involving tangents and normals using first order derivatives.
	Finds equation of normal using $y - y_1 = m(x - x_1)$ with (2, 4) $y-4=-\frac{1}{7}(x-2)$	**M1**	1.1a	
	Substitutes $y = 0$ and attempts to solve for x.	**M1**	1.1b	
	$x = 30$, $A(30,0)$	**A1**	1.1b	
		(4)		
				(10 marks)

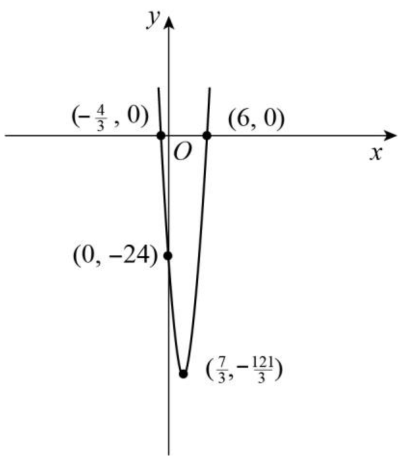
Notes

3b

Using $y = mx + c$ is acceptable. For example $4 = 7 \times 2 + c$, so $c = -10$

3c

Using $y = mx + c$ is acceptable. For example $4=\left(-\frac{1}{7}\right)(2)+c$, so $c=\frac{30}{7}$

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
4	Attempts to differentiate.	**M1**	1.1a	5th Sketch graphs of the gradient function of curves.
	$f'(x) = 3x^2 - 14x - 24$	**A1**	1.1b	
	States or implies that the graph of the gradient function will cut the x-axis when $f'(x) = 0$ $f'(x) = 0 \Rightarrow 3x^2 - 14x - 24 = 0$	**M1**	2.2a	
	Factorises $f'(x)$ to obtain $(3x+4)(x-6) = 0$ $x = -\frac{4}{3}$, $x = 6$	**A1**	1.1b	
	States or implies that the graph of the gradient function will cut the y-axis at $f'(0)$. Substitutes $x = 0$ into $f'(x)$ Gradient function will cut the y-axis at (0, −24).	**M1**	2.2a	
	Attempts to find the turning point of $f'(x)$ by differentiating (i.e. finding $f''(x)$)	**M1**	2.2a	
	$f''(x) = 0 \Rightarrow 6x - 14 = 0 \Rightarrow x = \frac{7}{3}$	**A1**	1.1b	
	Substitutes $x = \frac{7}{3}$ into $f'(x)$ to obtain $y = -\frac{121}{3}$	**A1ft**	1.1b	
	 A parabola with correct orientation with required points correctly labelled.	**A1ft**	2.2a	
				(9 marks)

Notes

A mistake in the earlier part of the question should not count against the students for the last part. If a student sketches a parabola with the correct orientation correctly labelled for their values, award the final mark.

Note that a fully correct sketch without all the working but with all points clearly labelled implies 8 marks in this question.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
5a	States or implies that area of base is x^2.	**M1**	3.3	6th Apply derivatives and the principle of rate of change to real-life contexts.
	States or implies that total surface area of the fish tank is $x^2 + 4xh = 1600$ Use of a letter other than h is acceptable.	**M1**	3.3	
	$h = \frac{400}{x} - \frac{x}{4}$	**M1**	1.1b	
	Substitutes for h in $V = x^2h = x^2\left(\frac{400}{x} - \frac{x}{4}\right)$	**M1**	1.1b	
	Simplifies to obtain $V = 400x - \frac{x^3}{4}$ *	**A1***	1.1b	
		(5)		
5b	Differentiates f(x) $\frac{dV}{dx} = 400 - \frac{3x^2}{4}$	**B1**	3.4	6th Apply derivatives and the principle of rate of change to real-life contexts.
	Attempts to solve $\frac{dV}{dx} = 0$ $400 - \frac{3x^2}{4} = 0$ or $400 = \frac{3x^2}{4}$	**M1**	1.1b	
	$x = \frac{40\sqrt{3}}{3}$ o.e. (NB must be positive)	**A1**	1.1b	
	Substitutes for x in $V = 400x - \frac{x^3}{4}$ $V_{max/min} = \frac{32\,000\sqrt{3}}{9}$ o.e. or awrt 6160	**A1**	1.1b	
		(4)		

5c	Differentiates $f'(x)$ $\frac{d^2V}{dx^2} = -\frac{3x}{2}$ o.e.	**M1**	1.1b	6th Apply derivatives and the principle of rate of change to real-life contexts.
	Substitutes $x = \frac{40\sqrt{3}}{3}$ into $f''(x)$ States $\frac{d^2V}{dx^2} < 0$, so V in part **b** is a maximum value.	**A1**	2.4	
		(2)		
				(11 marks)

Notes

5a

A sketch of a rectangular prism with a base of x by x and a height of h is acceptable for the first method mark.

5c

Other complete methods for demonstrating that V is a maximum are acceptable. For example a sketch of the graph of V against x or calculation of values of V or $\frac{dV}{dx}$ on either side.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
6a	States that the perimeter of the track is $2\pi r + 2x = 300$ The choice of the variable *x* is not important, but there should be a variable other than *r*.	**M1**	3.3	6th Apply derivatives and the principle of rate of change to real-life contexts.
	Correctly solves for *x*. Award method mark if this is seen in a subsequent step. $x = \frac{300 - 2\pi r}{2} = 150 - \pi r$	**A1**	1.1b	
	States that the area of the shape is $A = \pi r^2 + 2rx$	**B1**	3.3	
	Attempts to simplify this by substituting their expression for *x*. $A = \pi r^2 + 2r(150 - \pi r)$ $A = \pi r^2 + 300r - 2\pi r^2$	**M1**	1.1b	
	States that the area is $A = 300r - \pi r^2$ *	**A1***	1.1b	
		(5)		
6b	Attempts to differentiate *A* with respect to *r*	**M1**	1.1a	6th Apply derivatives and the principle of rate of change to real-life contexts.
	Finds $\frac{dA}{dr} = 300 - 2\pi r$	**A1**	3.4	
	Shows or implies that a maximum value will occur when $300 - 2\pi r = 0$	**M1**	1.1a	
	Solves the equation for *r*, stating $r = \frac{150}{\pi}$	**A1**	1.1b	
	Attempts to substitute for *r* in $A = 300r - \pi r^2$, for example writing $A = 300\left(\frac{150}{\pi}\right) - \pi\left(\frac{150}{\pi}\right)^2$	**M1**	1.1b	
	Solves for *A*, stating $A = \frac{22\,500}{\pi}$	**A1**	1.1b	
		(6)		

(11 marks)
Notes **6b** Ignore any attempts at deriving second derivative and related calculations.