

# Mass Balance- in Non-Reactive System Multi unit system



# LEARNING OBJECTIVES



By the end of this topic, you should be able to:

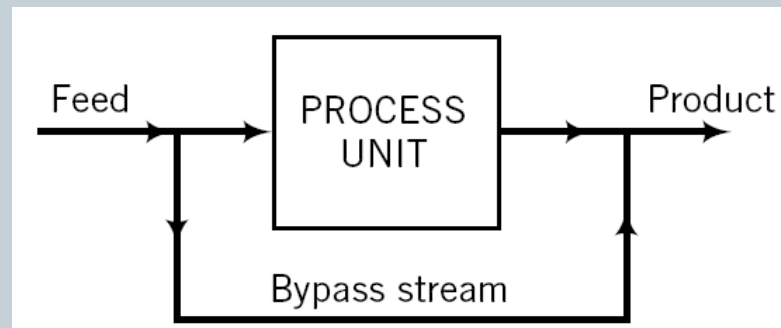
- Performed material balance for system for multiple unit.

# BYPASS AND RECYCLE



## Bypass Stream

- Similar to a recycle, but a fraction of a stream is diverted around a process unit, rather than being returned to it.
- Calculation approach is identical.



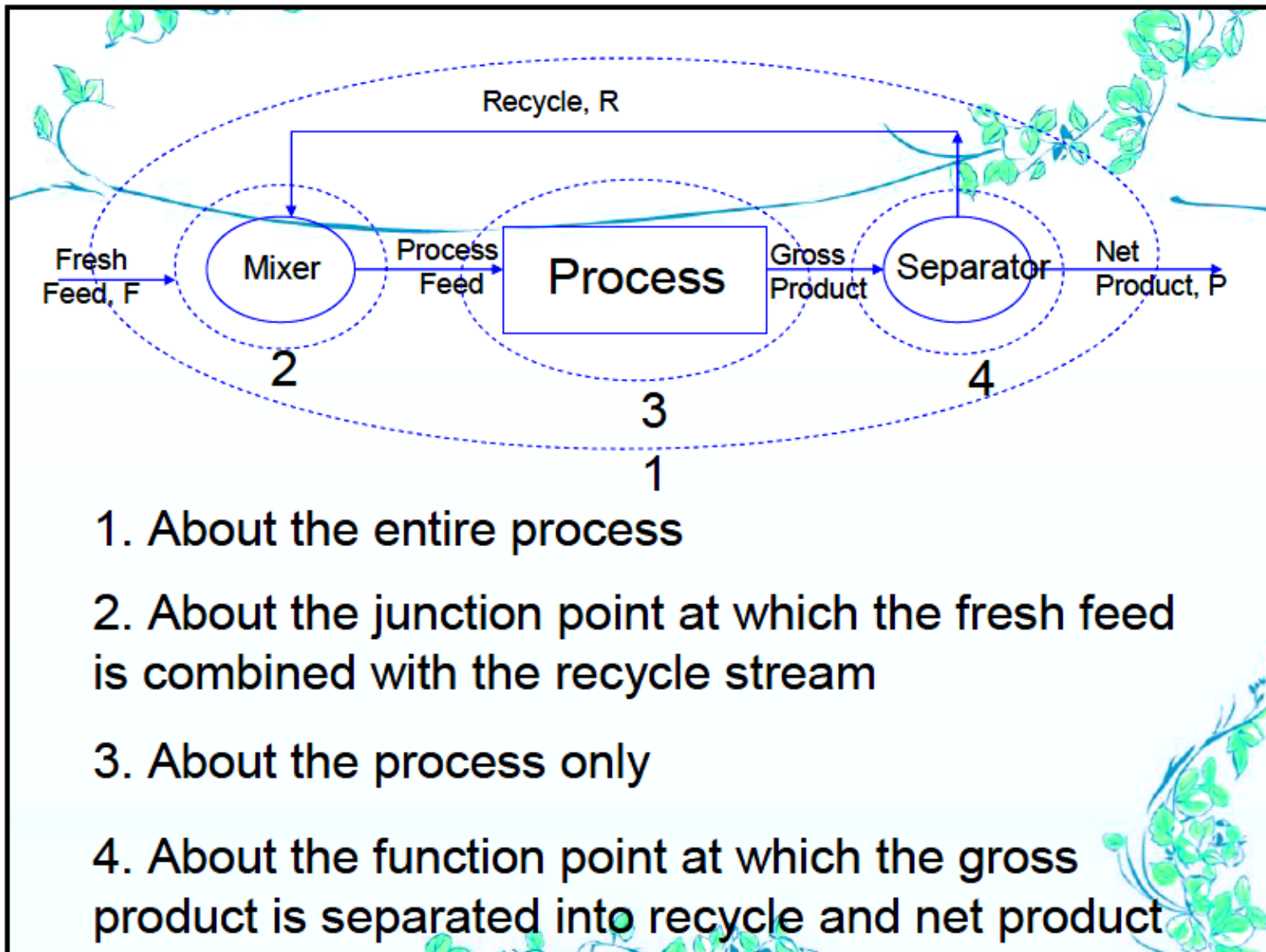


## Recycle, Bypass, and Purge Calculations

A **recycle stream** is a term denoting a process stream that returns material from downstream of a process unit back to the process unit.

A **bypass stream** is the one that skips one or more stages of the process and goes directly to another down stream stage.

A **purge stream** is a stream bled off to remove an accumulation of inert or unwanted material that might otherwise build up in the recycle stream.



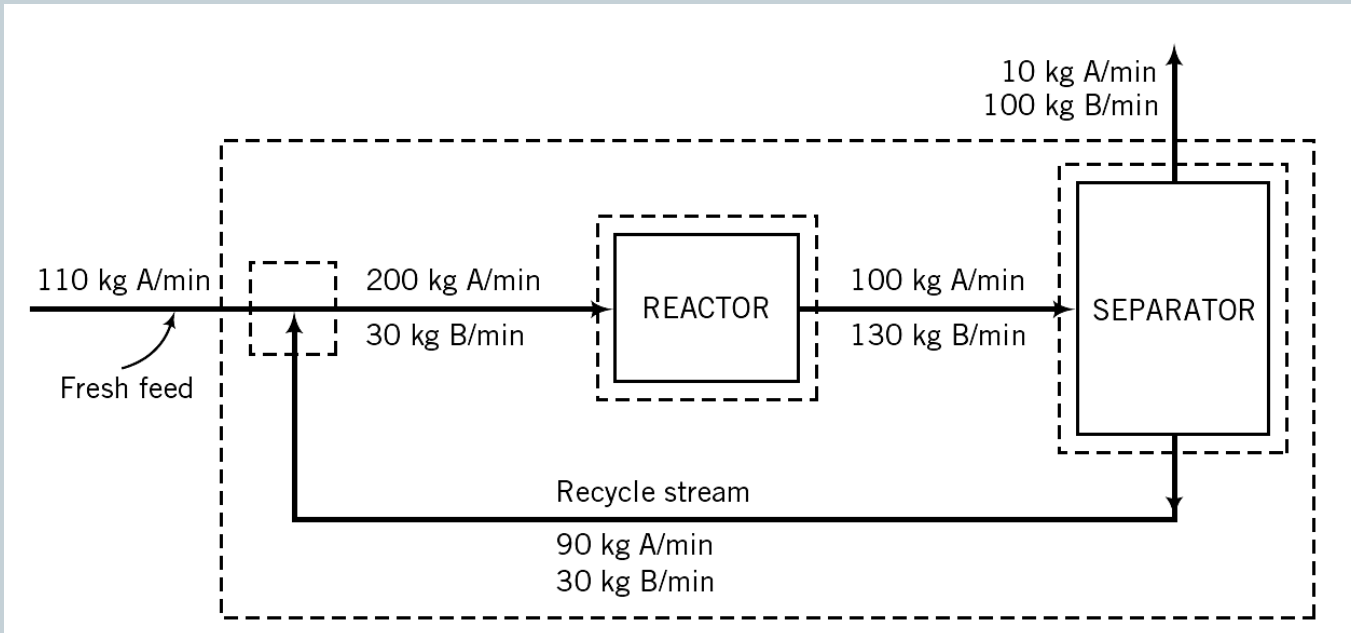
1. About the entire process
2. About the junction point at which the fresh feed is combined with the recycle stream
3. About the process only
4. About the function point at which the gross product is separated into recycle and net product

# BYPASS AND RECYCLE



## Recycle

- It is seldom cost effective to waste reactant fed that does not react to product. More often, this material is separated (recovered), and recycled (returned to its point of origin for reuse).





# Reasons to recycle

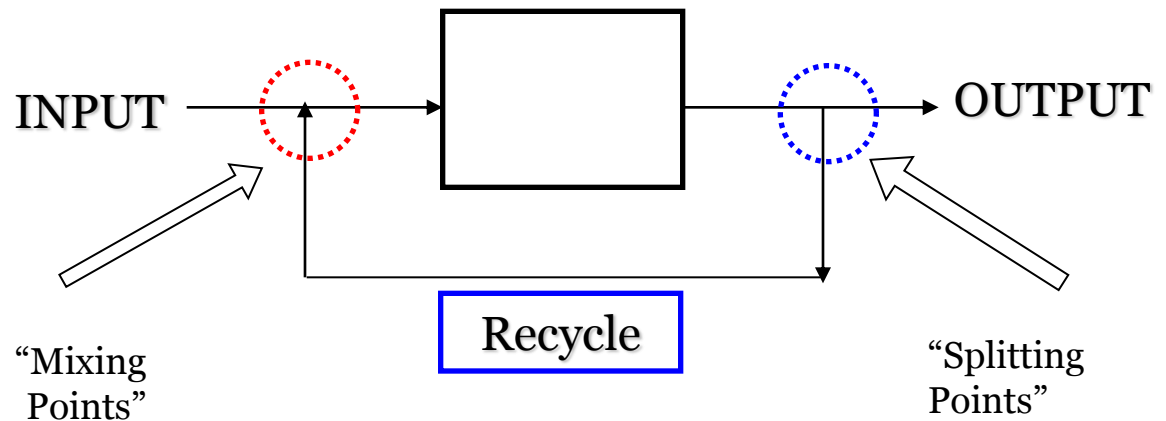
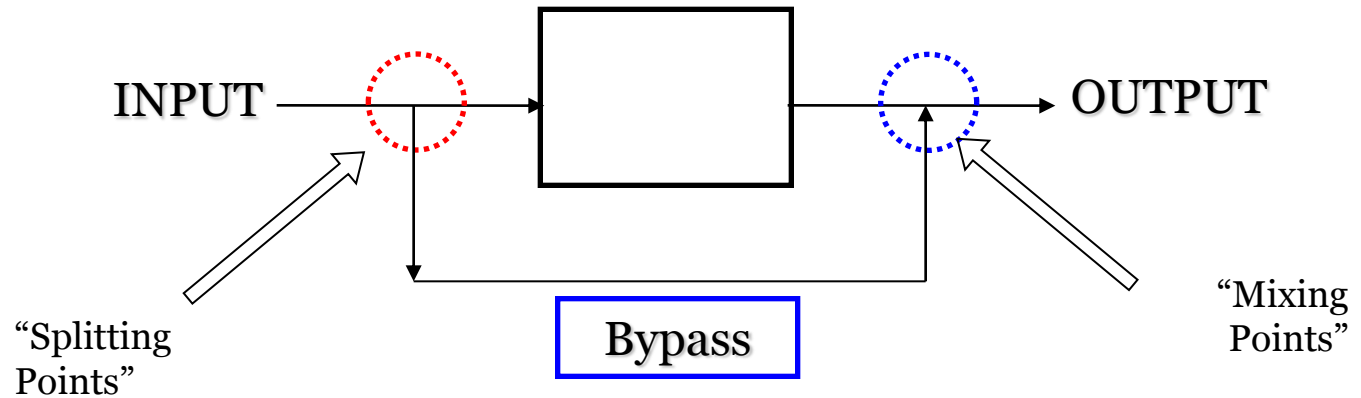


- recover catalyst
  - typically most expensive chemical constituent
- dilute a process stream
  - reduce slurry concentration
- control a process variable
  - control heat produced by highly exothermic reaction
- circulation of a working fluid
  - refrigerant

# BYPASS AND RECYCLE



Note:  
At splitting points, the components mass fractions remain the same. The ONLY difference is the mass flow rate.







## **Example : Recycle without Chemical Reaction**

A distillation column separates 10,000 kg/hr of a 50% benzene-50% toluene mixture. The product D recovered from the condenser at the top of the column contains 95% benzene, and the bottom W from the column contains 96% toluene. The vapor stream V entering the condenser from the top of the column is 8000 kg/hr. A portion of the product from the condenser is returned to the column as reflux, and the rest is withdrawn for use elsewhere. Assume that the compositions of the streams at the top of the column (V), the product withdrawn (D), and the reflux (R) are identical because the V stream is condensed completely. Find the ratio of the amount refluxed R to the product withdrawn (D).



Basis : 1 hr (equal  $F = 10,000$  kg)

Overall Material Balances:

Total material

$$F = D + W$$

$$10,000 = D + W$$

Component (benzene)

$$F\omega_F = D\omega_D + W\omega_W$$

$$10,000(0.50) = D(0.95) + W(0.04)$$

Solving for W and D

$$W = 4950 \text{ kg/hr}$$

$$D = 5050 \text{ kg/hr}$$



## Balance around the condenser

Total material:

$$V = R + D$$

$$8,000 = R + 5,050$$

$$R = 2,950 \text{ kg/hr}$$

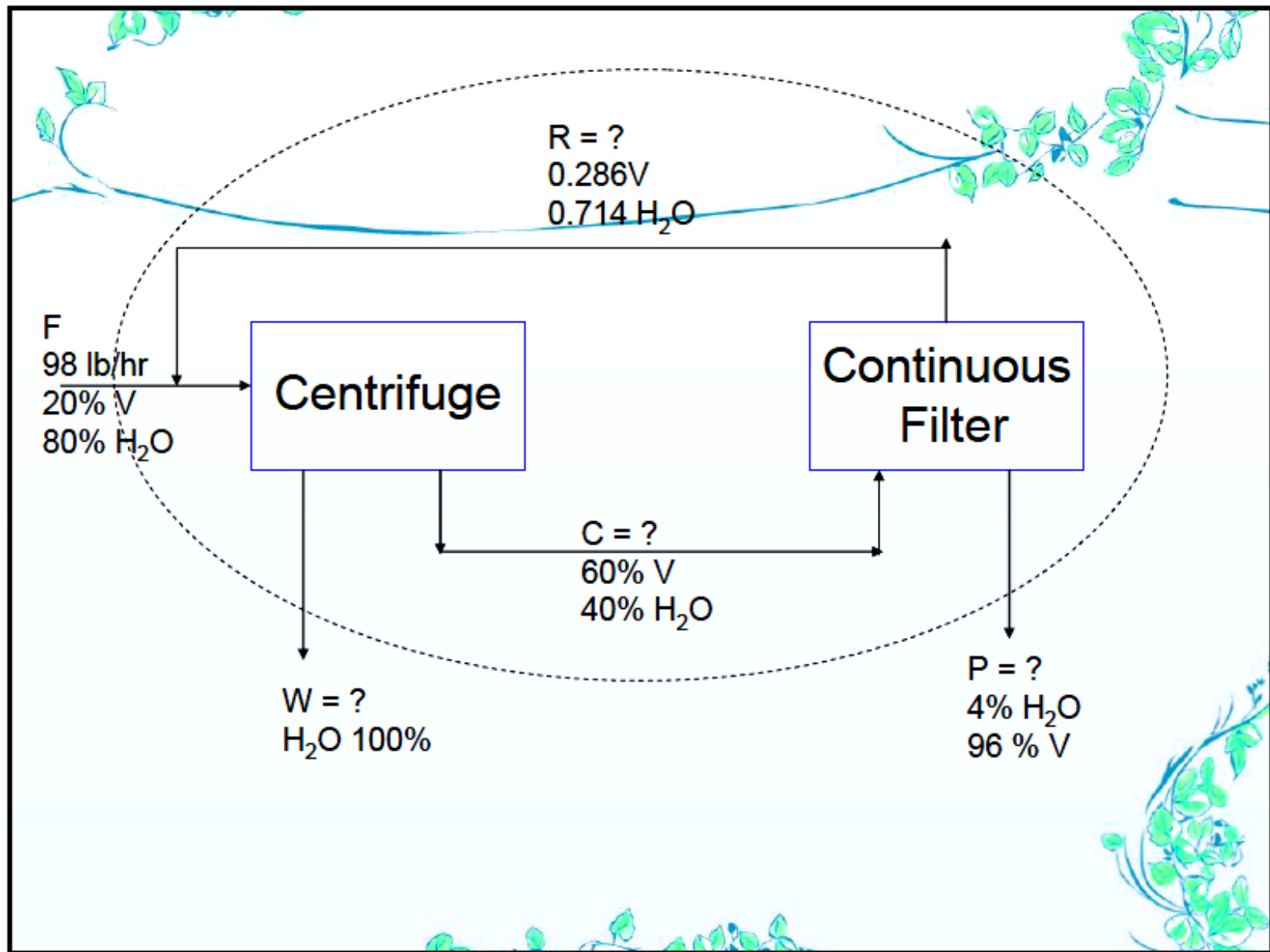
$$R/D = 2950/5050 = 0.58$$



## **Example : Recycle without chemical reaction**

The manufacture of such products as penicillin, tetracycline, vitamins, and other fine organic compounds, usually requires separating the suspended solids from their mother liquor by centrifuging, and then drying the wet cake. What is the lb/hr of the recycle stream R?

This is a steady-state problem without reaction and with recycle.





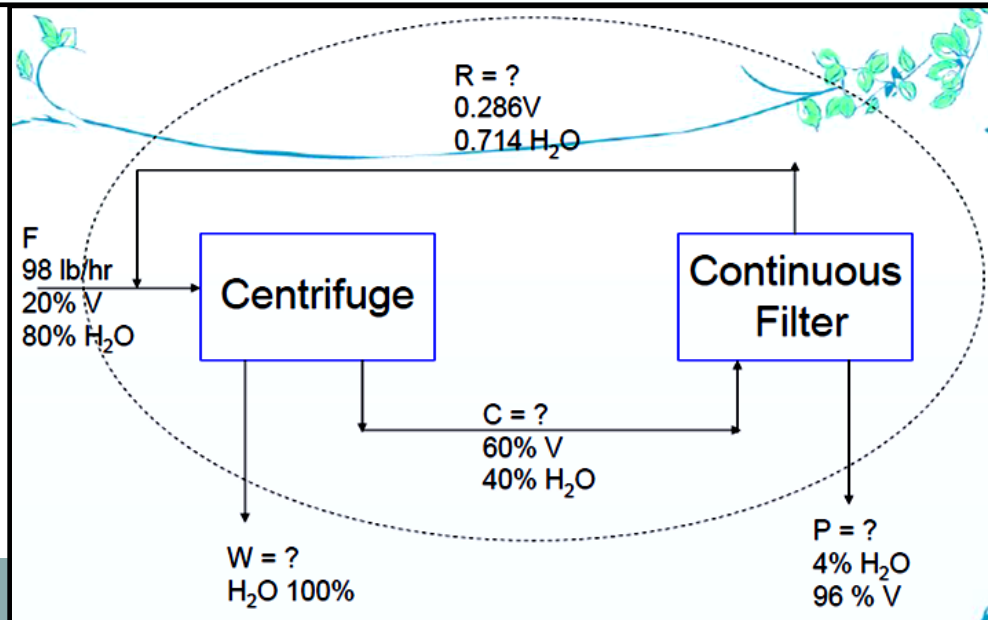
**Basis :** 1 hr ( $F = 98 \text{ lb}$ )

Overall mass balances

	In		Out
V :	$0.20(98)$	$= 0$	$+ 0.96P$
H <sub>2</sub> O :	$0.80(98)$	$= (1.0)W +$	$0.04P$
Total :	98	$= W +$	P

Solving for P and W

$$P = 20.4 \text{ lb} \quad W = 98 - 20.4 = 77.6 \text{ lb}$$





Total balance on filter

$$C = R + P$$

$$C = R + 20.4$$

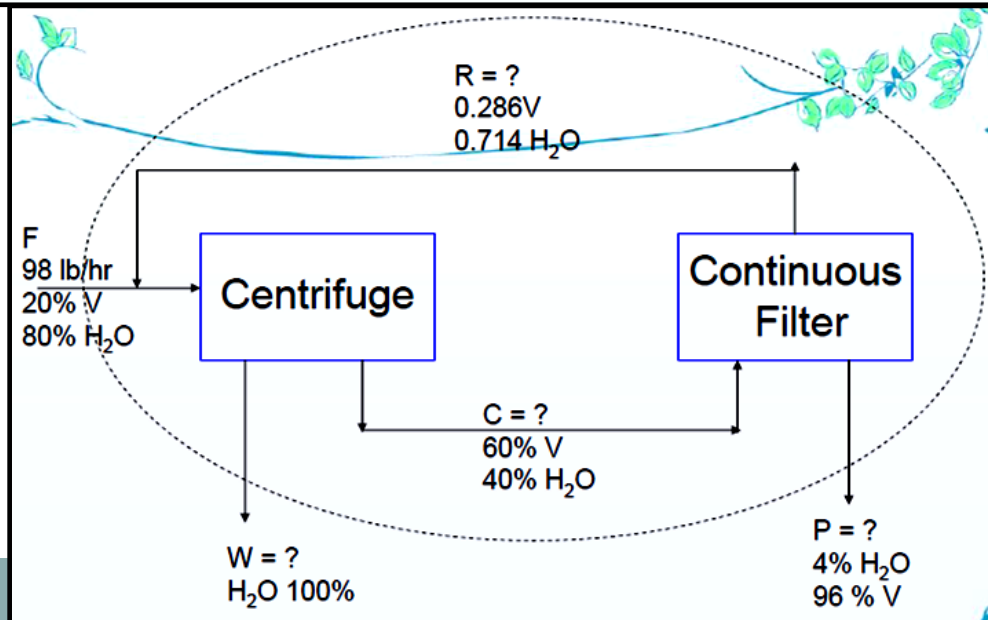
Component V balance on filter

$$C\omega_C = R\omega_R + P\omega_P$$

$$0.6C = 0.286R + 0.96(20.4)$$

Solving for R

$$R = 23.4 \text{ lb/hr}$$

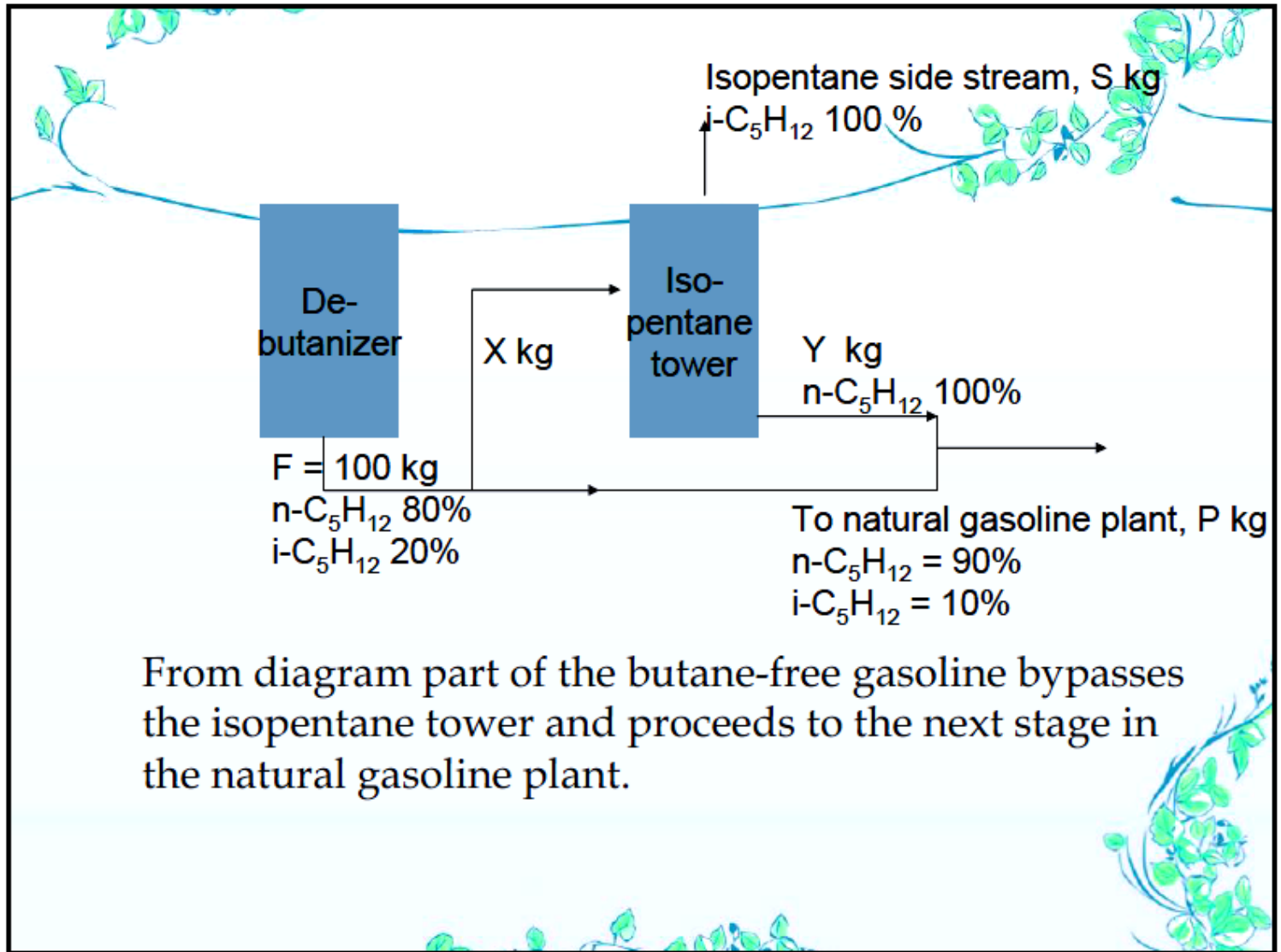




## Example: Bypass Calculations

In the feedstock preparation section of a plant manufacturing natural gasoline, isopentane is removed from butane-free gasoline. Assume for purposes of simplification that the process and components are as shown in figure. What fraction of the butane-free gasoline is passed through the isopentane tower? The process is in the steady state and no reaction occurs.





From diagram part of the butane-free gasoline bypasses the isopentane tower and proceeds to the next stage in the natural gasoline plant.



Basis: 100 kg feed

### Overall balance

Total material balance: In = Out

$$100 = S + P \quad (a)$$

Component balance ( $n\text{-C}_5\text{H}_{12}$ )

$$100(0.8) = S(0) + P(0.9) \quad (b)$$

We get,  $P = 88.9$  kg and  $S = 11.1$  kg

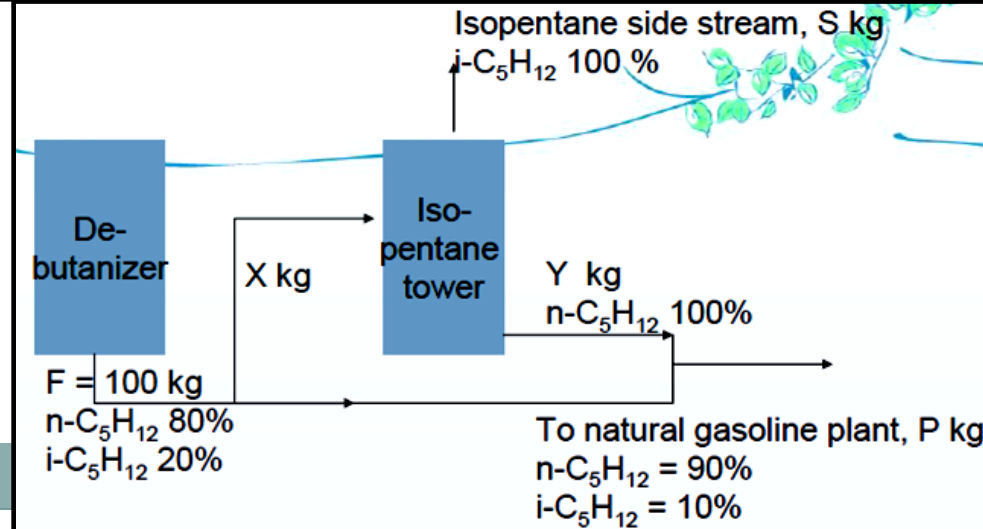
### Balance around isopentane tower

Total material balance:  $x = 11.1 + y$  (c)

Component balance ( $n\text{-C}_5\text{H}_{12}$ )

$$x(0.8) = y(1) \quad (d)$$

We get,  $x = 55.5$  kg,  $y = 44.4$  kg, therefore the fraction of butane free gas passed through isopentane tower is  $55.5/100 = 0.55$





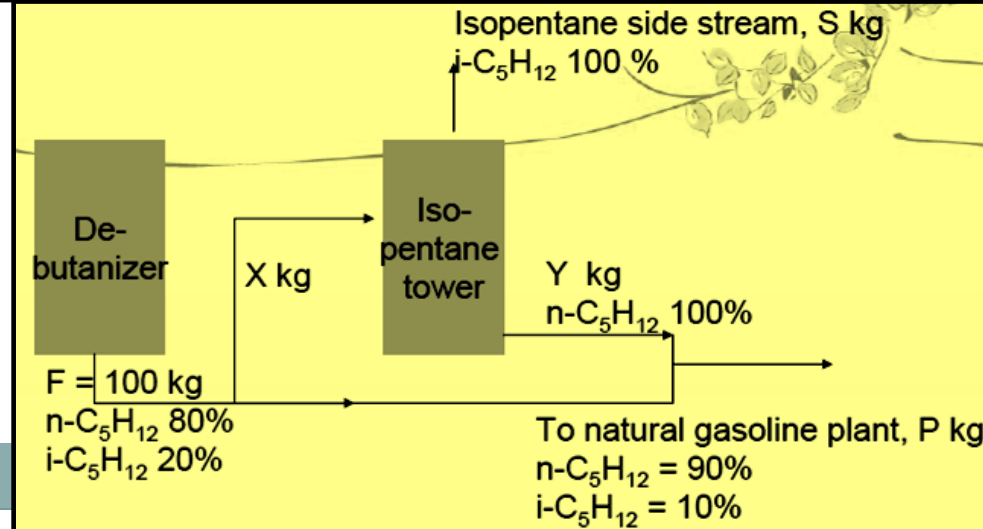
Another approach is to make a balance at mixing points 1 and 2

Balance around mixing point 2

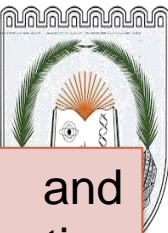
Total material balance:  $(100-x) + y = 88.9$  (e)

Component balance (i-C<sub>5</sub>H<sub>12</sub>):  $(100-x)0.2 = 88.9*0.1$  (f)

We get,  $x = 55.5$  kg

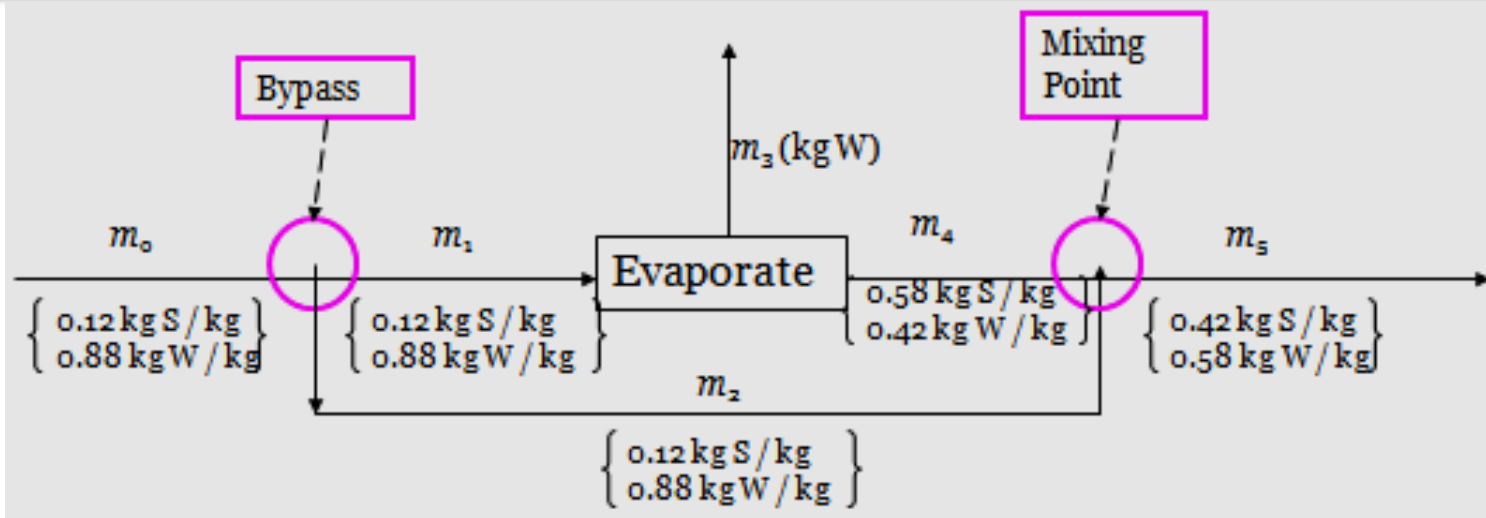


# EXAMPLE



Fresh orange juice contains 12.0 wt% solids and the balance water, and concentrated orange juice contains 42.0 wt% solids. Initially a single evaporation process was used for the concentration, but volatile constituents of the juice escaped with the water, leaving the concentrate with a flat taste. The current process overcomes the problem by bypassing the evaporator with a fraction of fresh juice. The juice that leaves the evaporator is concentrated to 58 wt% solids, and the evaporator product stream is mixed with the bypassed fresh juice to achieve the desired final concentration.

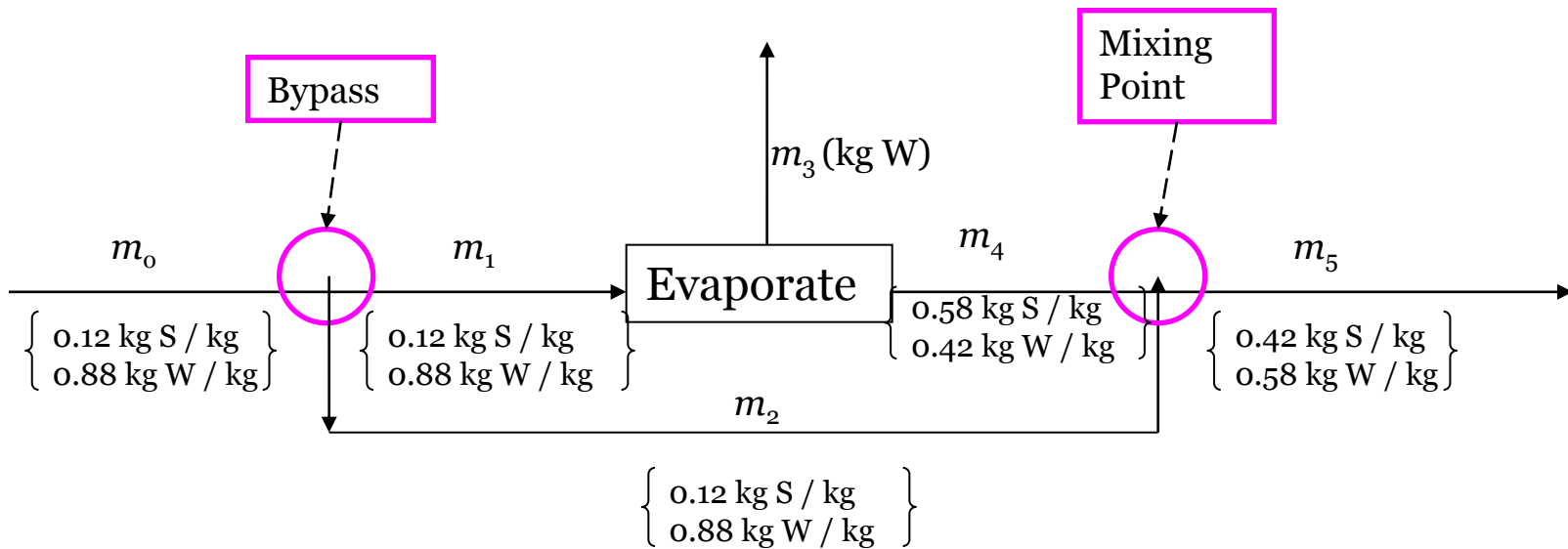
Draw and label the flowchart. Perform the degrees of freedom analyses. Calculate the amount of product (42% concentrate) produced per 100 kg fresh juice fed to the process and the fraction of the feed that bypasses the evaporator.



# EXAMPLE



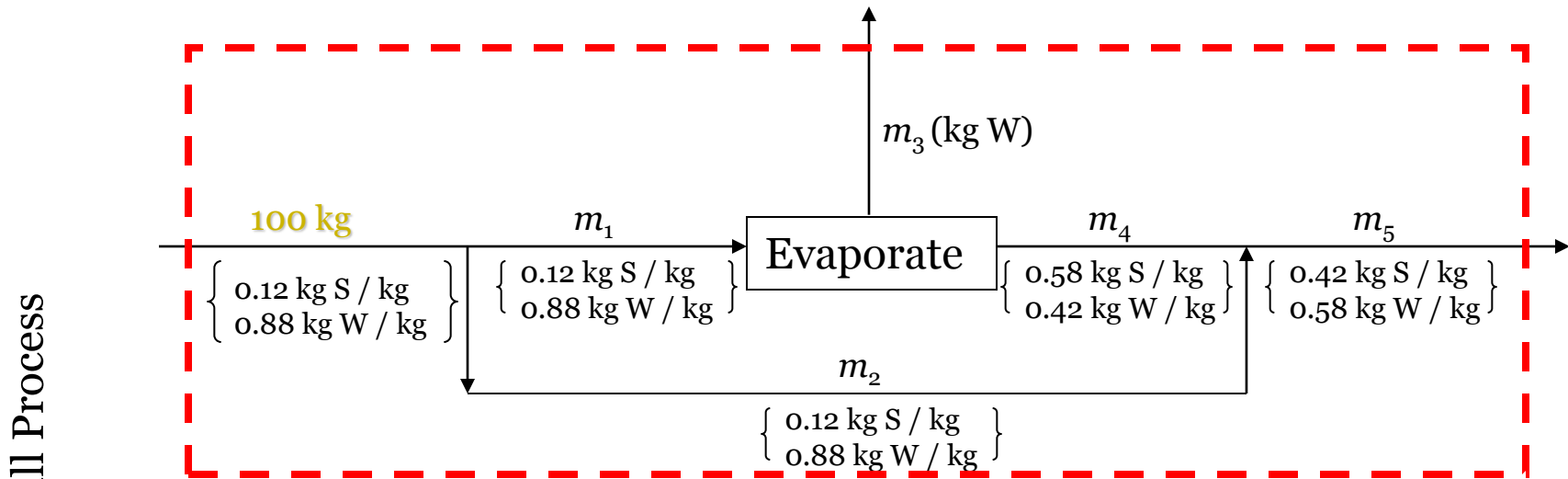
Step 1. Draw and label the flowchart.



# EXAMPLE



- Step 2. Choose a basis of calculation: Given 100 kg fresh juice.  
 Step 3. Perform the DoF analysis.



## Balances

Overall :  $100 = m_3 + m_5$

S :  $(0.12)(100) = 0 + 0.42m_5$

W :  $(0.88)(100) = m_3 + 0.58m_5$

## Degrees of Freedom

2 Unks. ( $m_3, m_5$ )

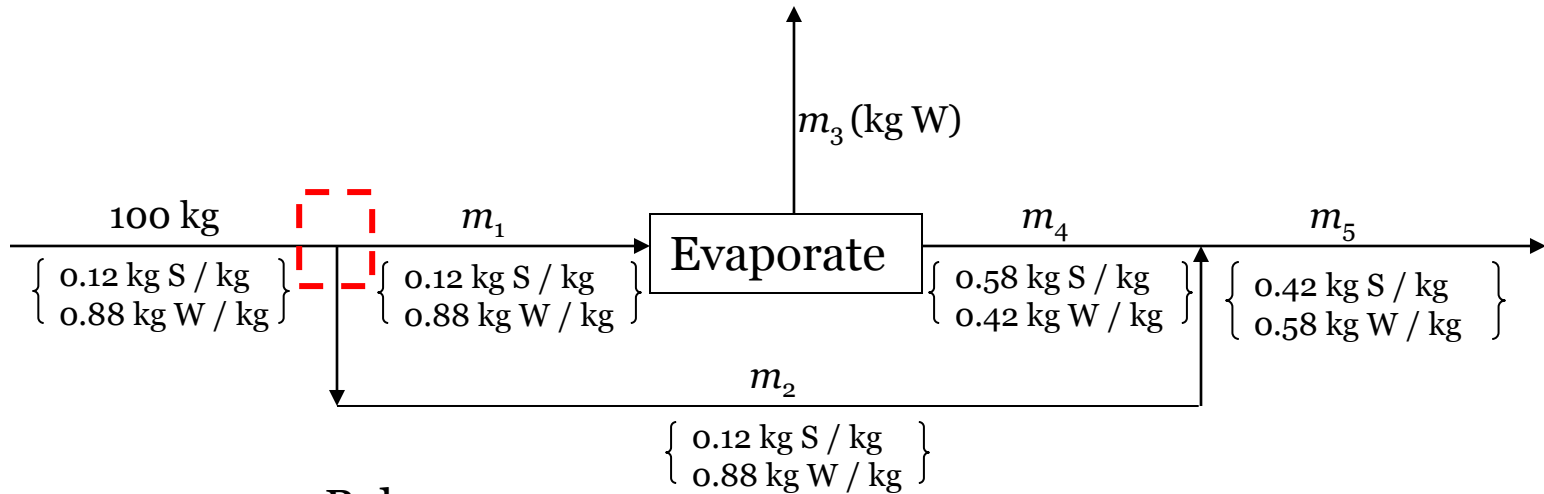
- 2 IE's

0 DoF



# EXAMPLE

Step 3. Perform the DoF analysis (Continuation).



## Balances

Overall :  $100 = m_1 + m_2$

S :  $(0.12)(100) = 0.12m_1 + 0.12m_2$

W :  $(0.88)(100) = 0.88m_1 + 0.88m_2$

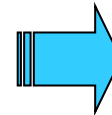
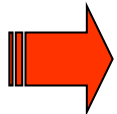
## Degrees of Freedom

2 Unks. ( $m_1, m_2$ )

- 2 IE's

0 DoF

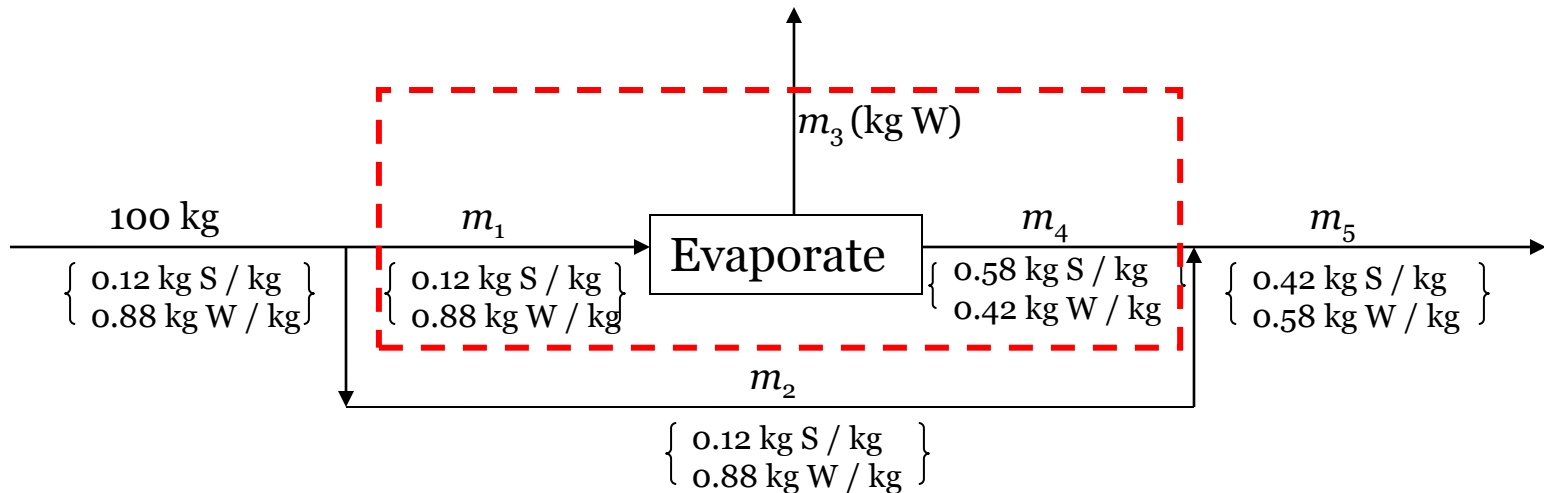
Bypass



# EXAMPLE



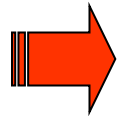
Step 3. Perform the DoF analysis (Continuation).



## Balances

## Degrees of Freedom

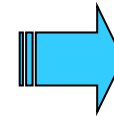
Evaporator



$$\text{Overall : } m_1 = m_3 + m_4$$

$$\text{S : } 0.12m_1 = 0.58m_4$$

$$\text{W : } 0.88m_1 = 0.42m_4 + m_3$$



3 Unks. ( $m_1, m_3, m_4$ ) – 2 IE's

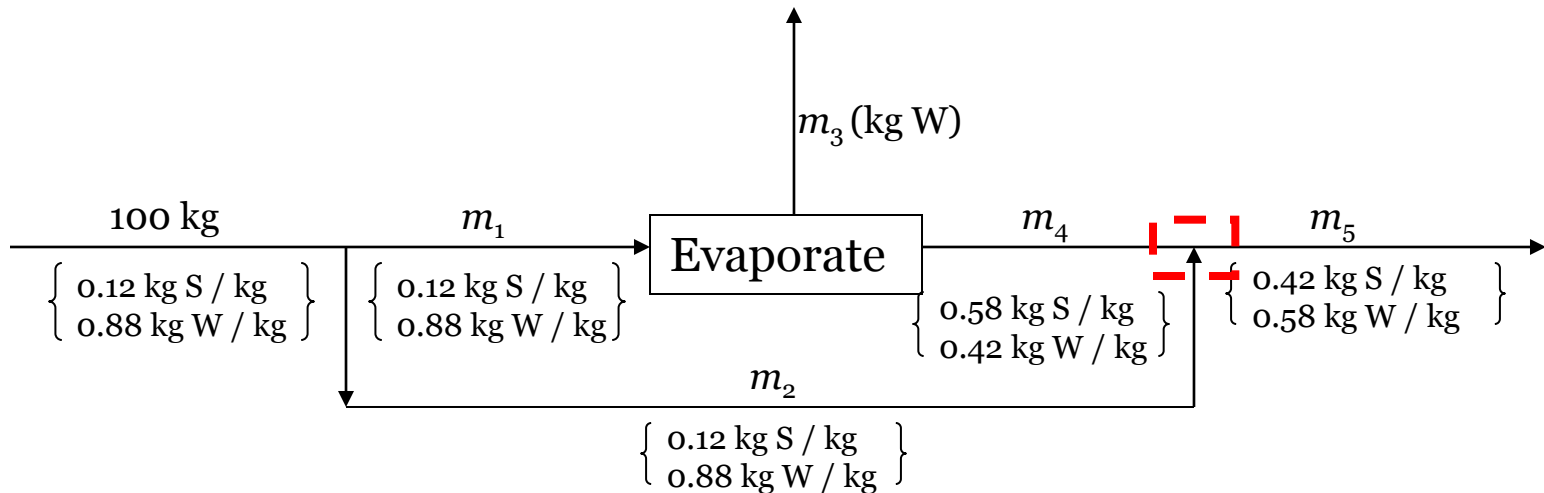
1 DoF



# EXAMPLE



Step 3. Perform the DoF analysis (Continuation).



## Balances

Overall :  $m_4 + m_2 = m_5$

S :  $0.12m_2 + 0.58m_4 = 0.42m_5$

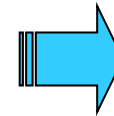
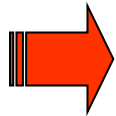
W :  $0.88m_2 = 0.42m_4 + 0.58m_5$

## Degrees of Freedom

3 Unks. ( $m_2, m_4, m_5$ ) – 2 IE's

1 DoF

Mixing Point



# EXAMPLE



Step 4. Do the algebra. Solve the balance equations.  
Calculate the **amount of product** (42% concentrate) produced per 100 kg fresh juice fed to the process and the fraction of the feed that bypasses the evaporator.

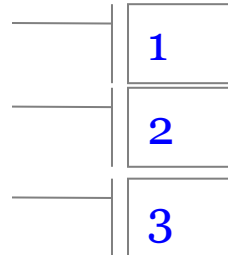
Let's start with the Overall Process because DoF = 0

Recall the material balances for overall process

$$\text{Overall: } 100 = m_2 + m_5$$

$$\text{S: } (0.12)(100) = 0.42m_5$$

$$\text{W: } (0.88)(100) = m_3 + 0.58m_5$$



From (2) :  $m_5 = 28.6 \text{ kg}$

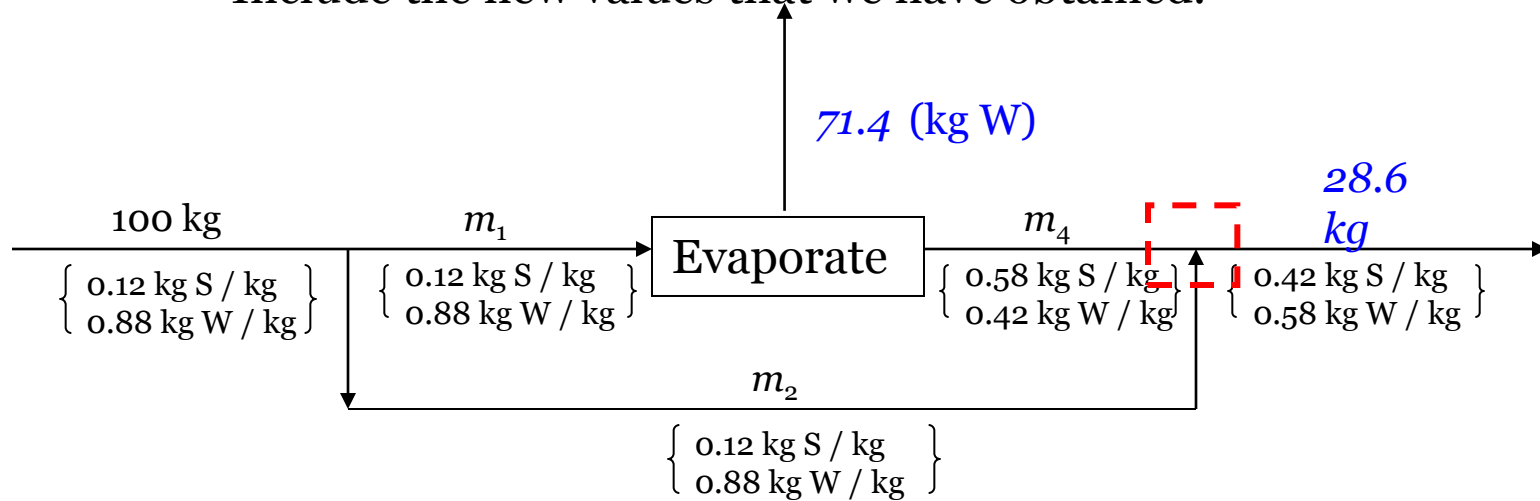
**Amount of product !**

From (1) :  $m_3 = 71.4 \text{ kg}$

# EXAMPLE



Step 4. Do the algebra. Solve the balance equations (Continuation). Include the new values that we have obtained.



DoF (updated)

2 unknowns ( $m_2, m_4$ )  
- 2 IE's

0 DoF

## Balances

$$\text{Overall : } m_4 + m_2 = m_5$$

$$\text{S : } 0.12m_2 + 0.58m_4 = 0.42m_5$$

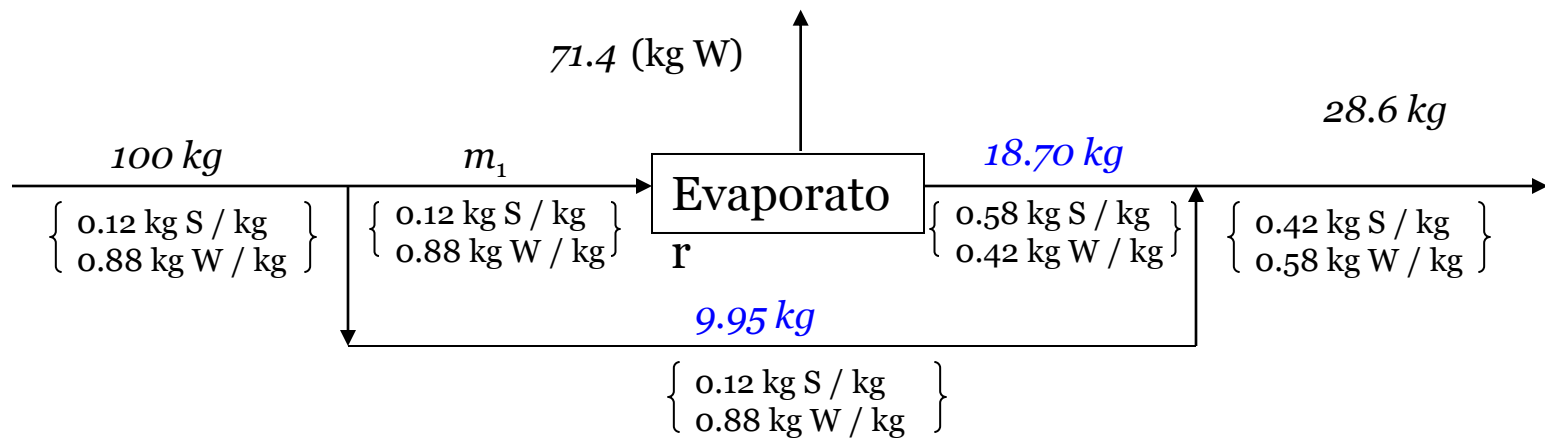
$$\text{W : } 0.88m_2 = 0.42m_4 + 0.58m_5$$

Known  $m_5 = 28.60 \text{ kg}$   
Solve for  $m_4 = 18.70 \text{ kg}$   
 $m_5 = 9.95 \text{ kg}$

# EXAMPLE



Step 4. Do the algebra. Calculate the amount of product (42% concentrate) produced per 100 kg fresh juice fed to the process and **the fraction of the feed that bypasses the evaporator (Continuation).**



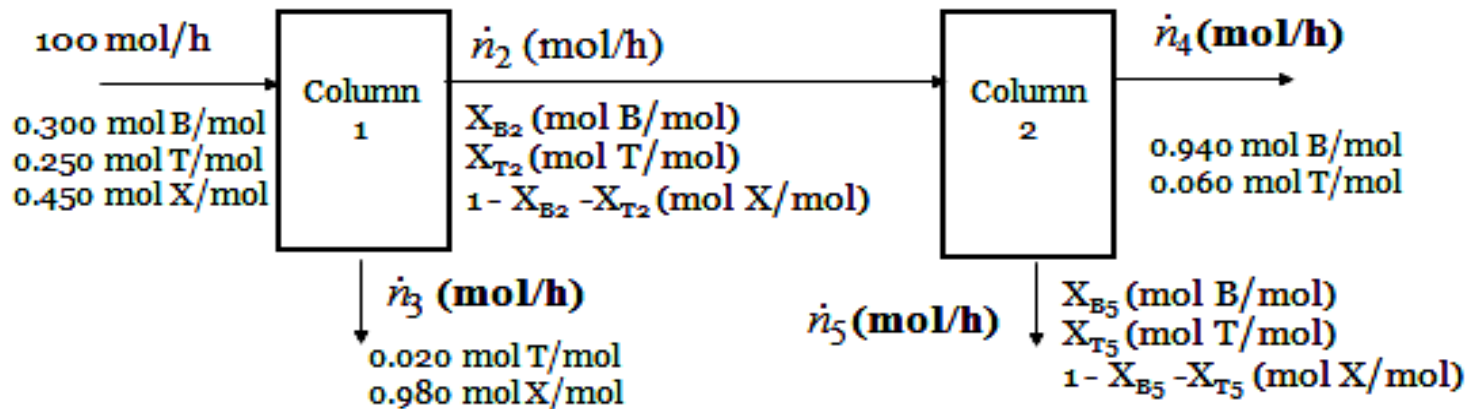
$$\begin{aligned} \text{The bypass fraction} &= \frac{m_2}{100} = \frac{9.95}{100} \\ &= 0.0995 \end{aligned}$$



# EXAMPLE

A liquid mixture containing 30.0 mole% benzene (B), 25.0% toluene (T) and the balance xylene (X) is fed to a distillation column. The bottoms product containing 98.0 mole% X and no B, and 96.0% of the X in the feed is recovered in this stream. The overhead product is fed to a second column. The overhead product from the second column contains 97.0 % of the B in the feed to this column. The composition of this stream is 94.0 mole% of B and the balance T.

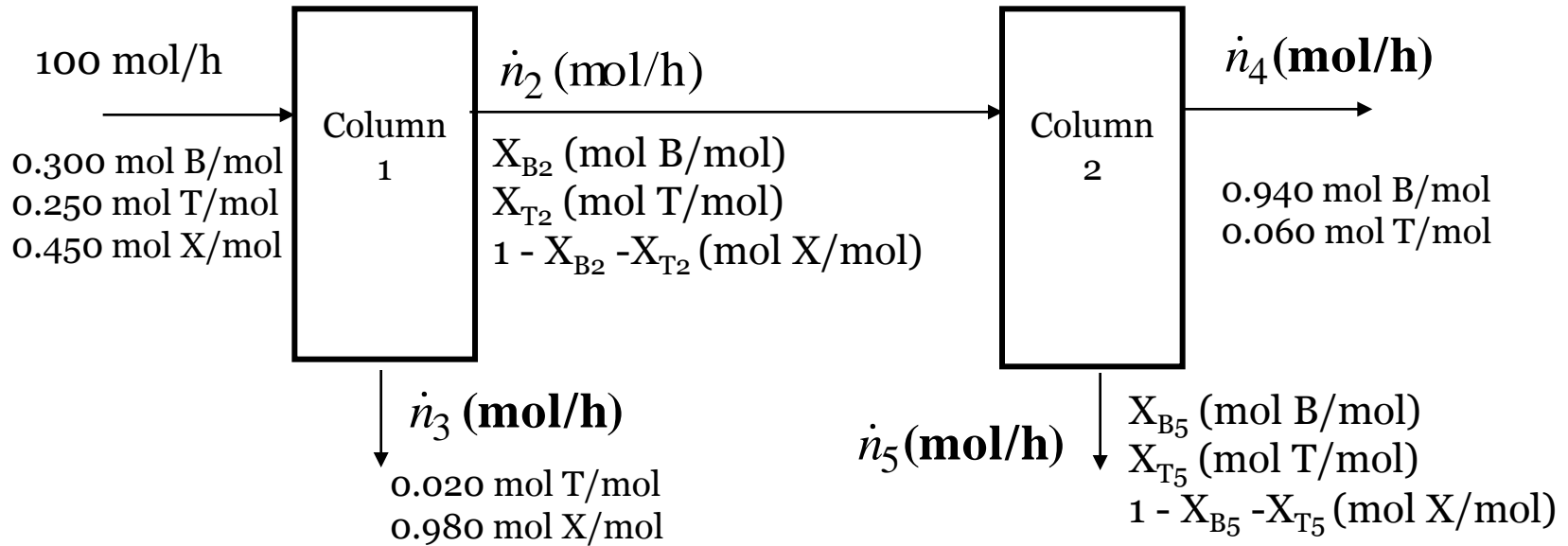
- (a) Draw and label flowchart. Do the degree-of-freedom analysis to prove that for an assumed basis of calculation, molar flow rate and compositions of all process streams can be calculated from the given information.
- (b) Calculate: (i) the percentage of the benzene in the process feed (the feed to the first column) that emerges in the overhead product from the second column and (ii) the percentage of toluene in the process feed that emerges in the bottom product from the second column.



# EXAMPLE

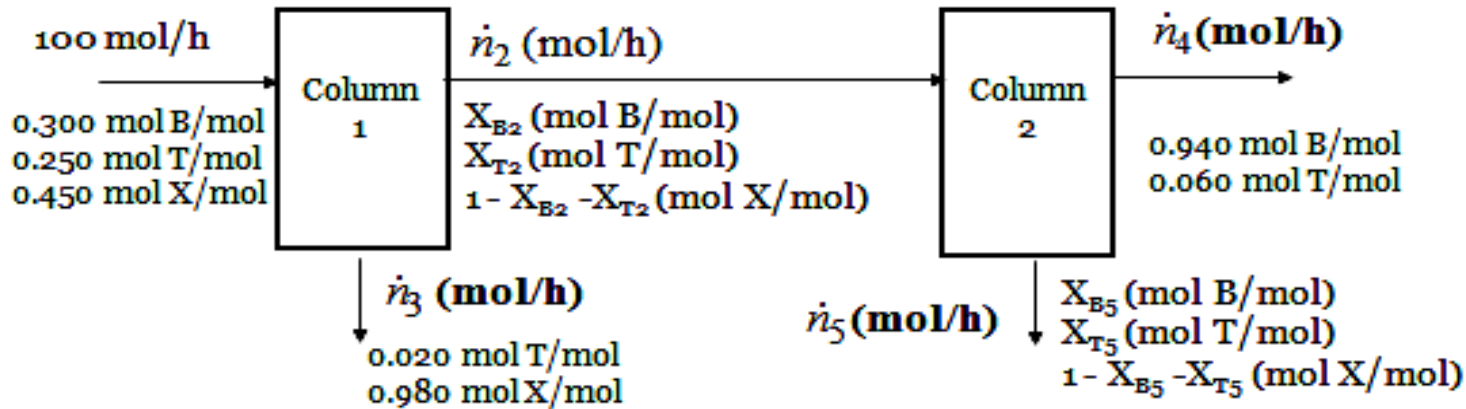


(a) Draw and label flowchart. Do the degree-of-freedom analysis to prove that for an **assumed basis of calculation**, molar flowrate and compositions of all process streams can be calculated from the given information.



# EXAMPLE

(a) D  
basis  
from



## DoF Column 1

- 4 unknowns
- 3 Independent eq<sup>n</sup>
- 1 Recovery of X
- 0 DoF

## Column 1

$$96\% \text{ X Recovery: } 0.960(0.450)(100) = 0.980\dot{n}_3 \quad (1)$$

$$\text{Total mole balance: } 100 = \dot{n}_2 + \dot{n}_3 \quad (2)$$

$$\text{B Balance: } 0.300(100) = x_{B2}\dot{n}_2 \quad (3)$$

$$\text{T Balance: } 0.250(100) = x_{T2}\dot{n}_2 + 0.020\dot{n}_3 \quad (4)$$

## DoF Column 2

- 4 unknowns
- 3 Independent eq<sup>n</sup>
- 1 Recovery of X
- 0 DoF

## Column 2

$$97\% \text{ B Recovery: } 0.970x_{B2}\dot{n}_2 = 0.940\dot{n}_4 \quad (5)$$

$$\text{Total mole balance: } \dot{n}_2 = \dot{n}_4 + \dot{n}_5 \quad (6)$$

$$\text{B Balance: } x_{B2}\dot{n}_2 = 0.940\dot{n}_4 + x_{B5}\dot{n}_5 \quad (7)$$

$$\text{T Balance: } x_{T2}\dot{n}_2 = 0.060\dot{n}_4 + x_{T5}\dot{n}_5 \quad (8)$$

# EXAMPLE



**(b) Calculate:**

- (i) the percentage of the benzene in the process feed (the feed to the first column) that emerges in the overhead product from the second column and
- (ii) the percentage of toluene in the process feed that emerges in the bottom product from the second column.

**Solving all the balances and obtain these results:**

$$\dot{n}_3 = 44.1 \text{ mol/h}$$

$$\dot{n}_4 = 30.95 \text{ mol/h}$$

$$\dot{n}_2 = 55.9 \text{ mol/h}$$

$$\dot{n}_5 = 24.96 \text{ mol/h}$$

$$x_{B2} = 0.536 \text{ molB/h}$$

$$x_{B5} = 0.036 \text{ molB/h}$$

$$x_{T2} = 0.431 \text{ molT/h}$$

$$x_{T5} = 0.892 \text{ molT/h}$$





# EXAMPLE

(b) Calculate:

- (i) the percentage of the benzene in the process feed (the feed to the first column) that emerges in the overhead product from the second column and
- (ii) the percentage of toluene in the process feed that emerges in the bottom product from the second column.

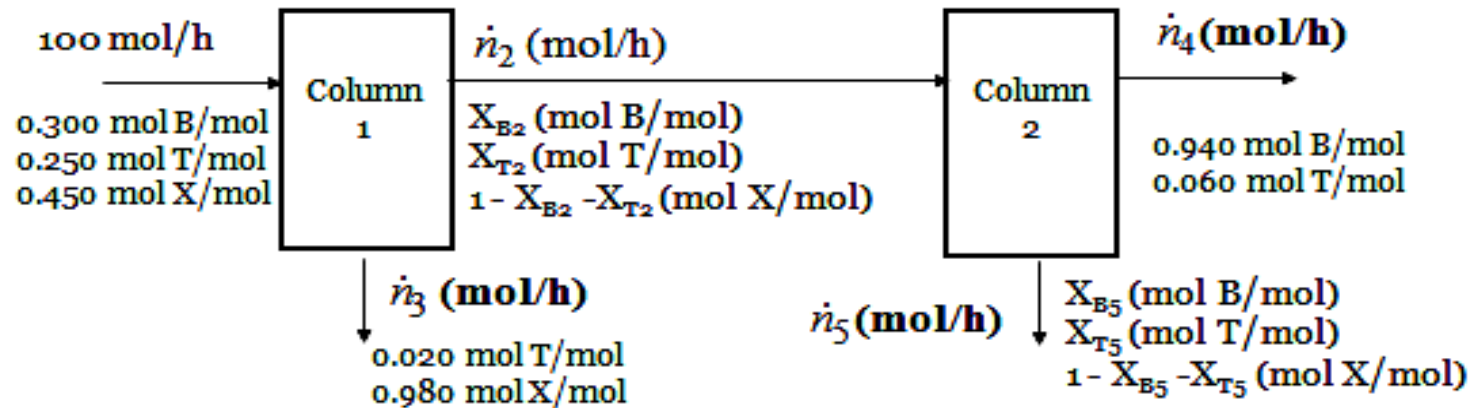
$$\dot{n}_4 = 30.95 \text{ mol/h}$$

Overall Benzene Recovery:  $\frac{0.940(30.95)}{0.300(100)} \times 100\% = 97\%$

$$\dot{n}_5 = 24.96 \text{ mol/h}$$

Overall Toluene Recovery:  $\frac{0.892(24.96)}{0.250(100)} \times 100\% = 89\%$

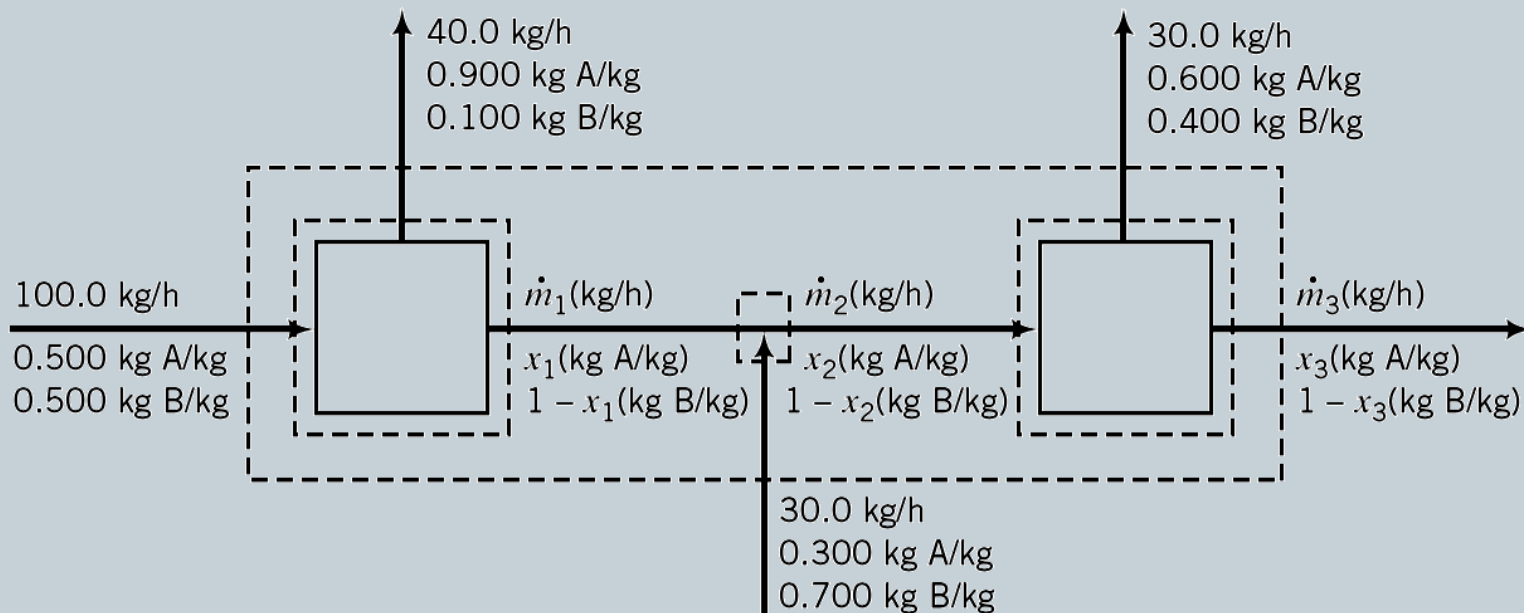
$$x_{T5} = 0.892 \text{ mol T/h}$$



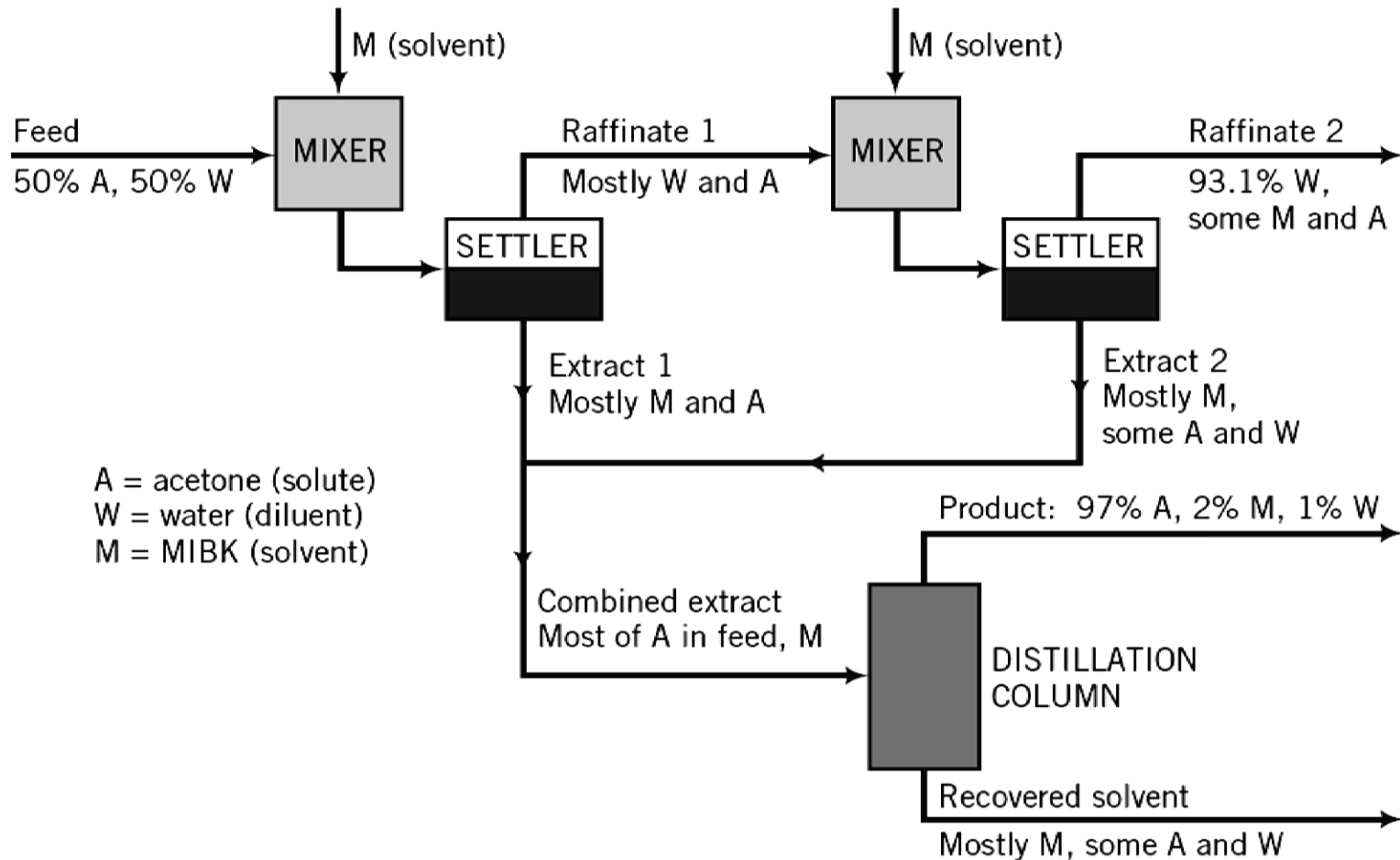


# Two-Unit Process Example

- Degree-of-freedom analysis
  - ❖ overall system: 2 unknowns – 2 balances = 0 (find  $m_3$ ,  $x_3$ )
  - ❖ mixer: 4 unknowns – 2 balances = 2
  - ❖ Unit 1: 2 unknowns – 2 balances = 0 (find  $m_1$ ,  $x_1$ )
  - ❖ mixer: 2 unknowns – 2 balances = 0 (find  $m_2$ ,  $x_2$ )

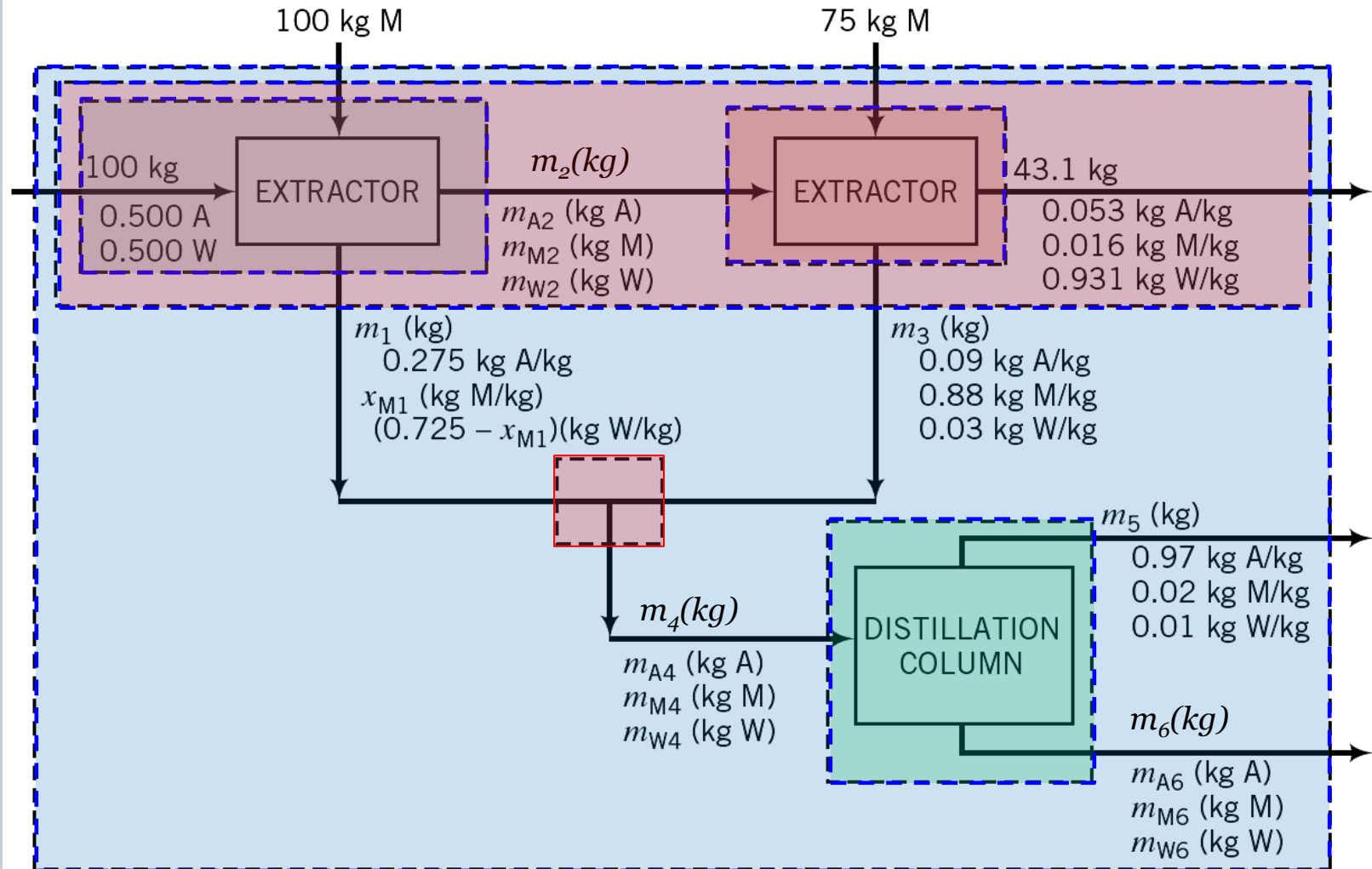


# Extraction-Distillation Process



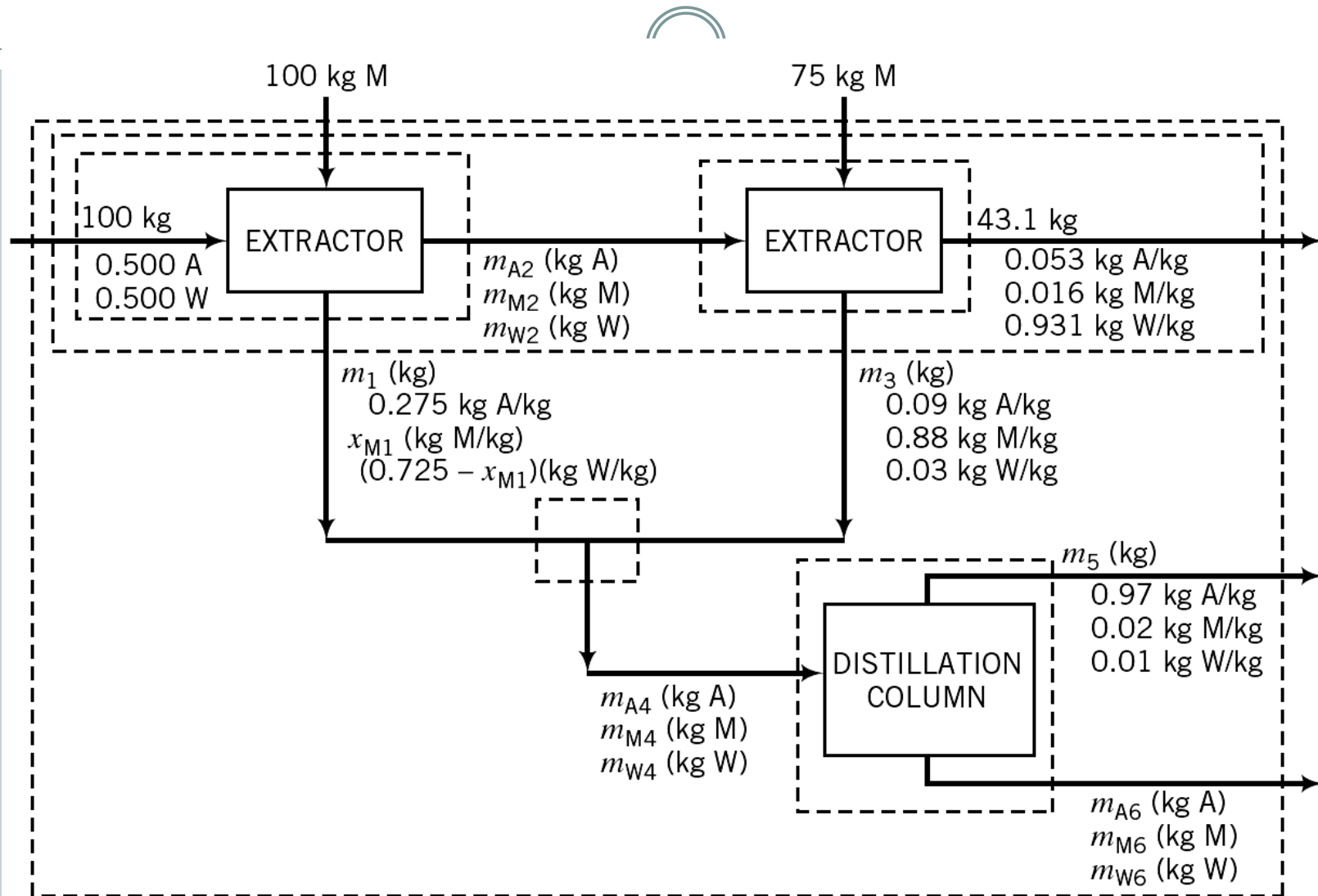
# Class work

## Extraction-Distillation Process



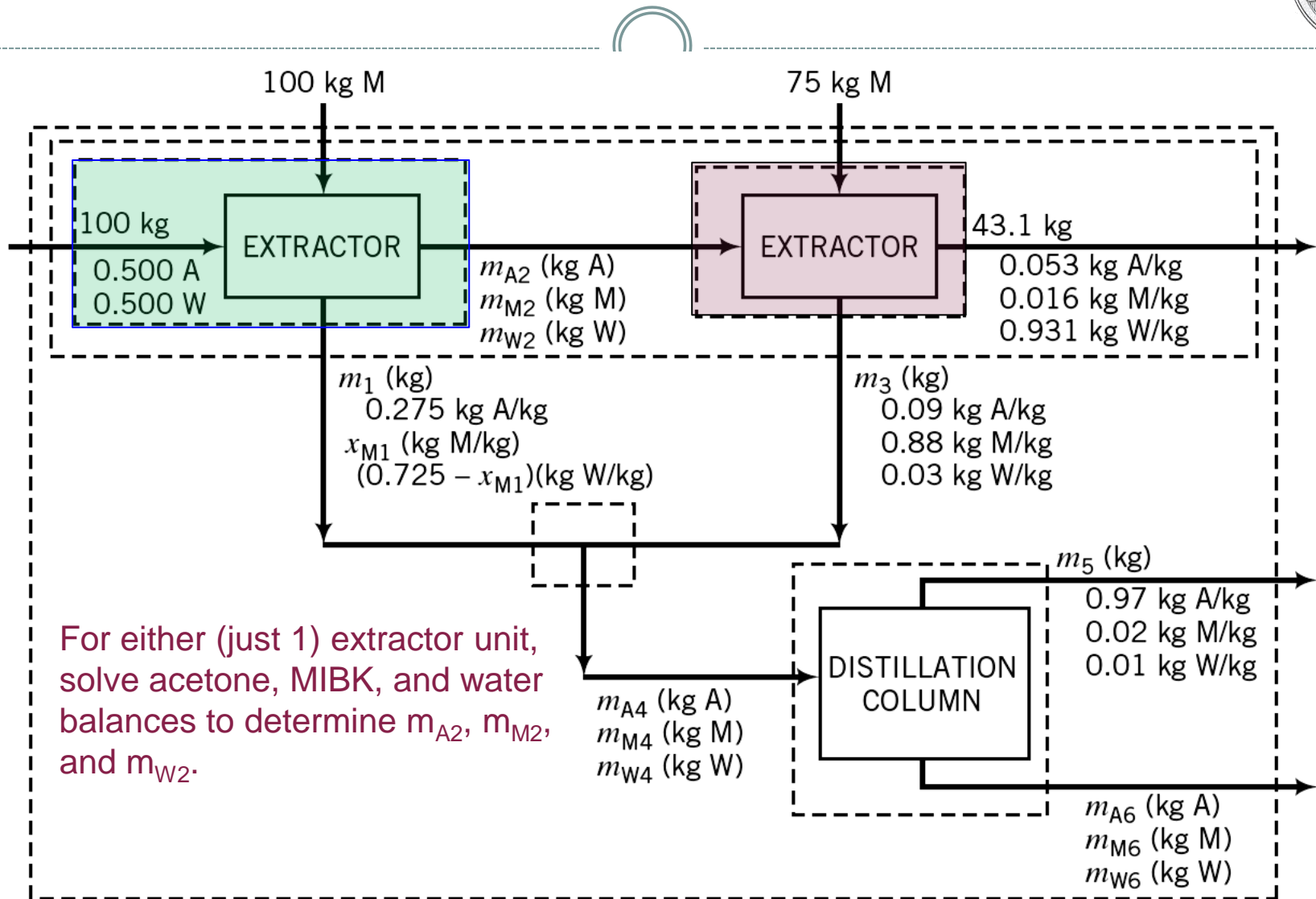


# Extraction-Distillation Process



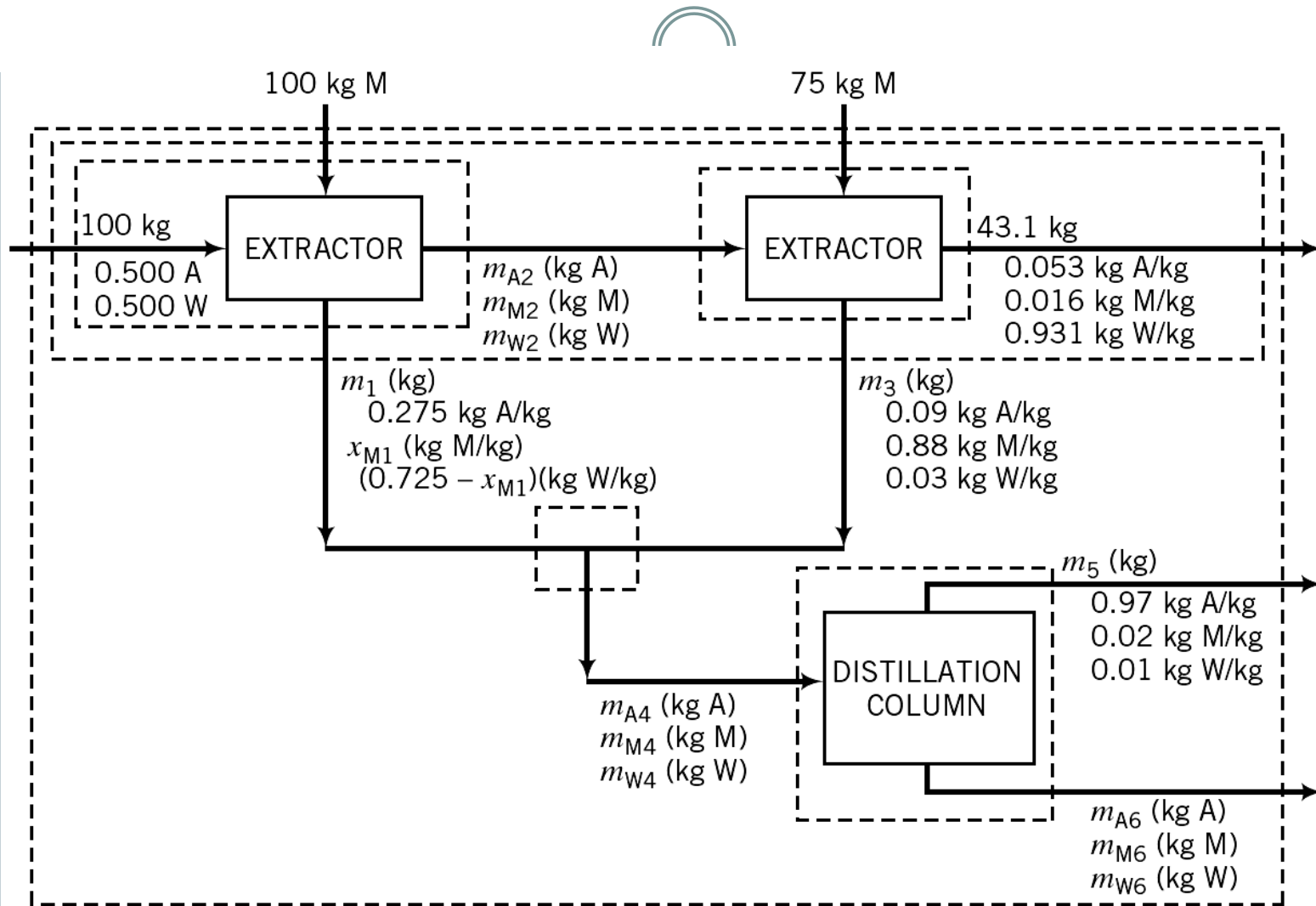


# Extraction-Distillation Process



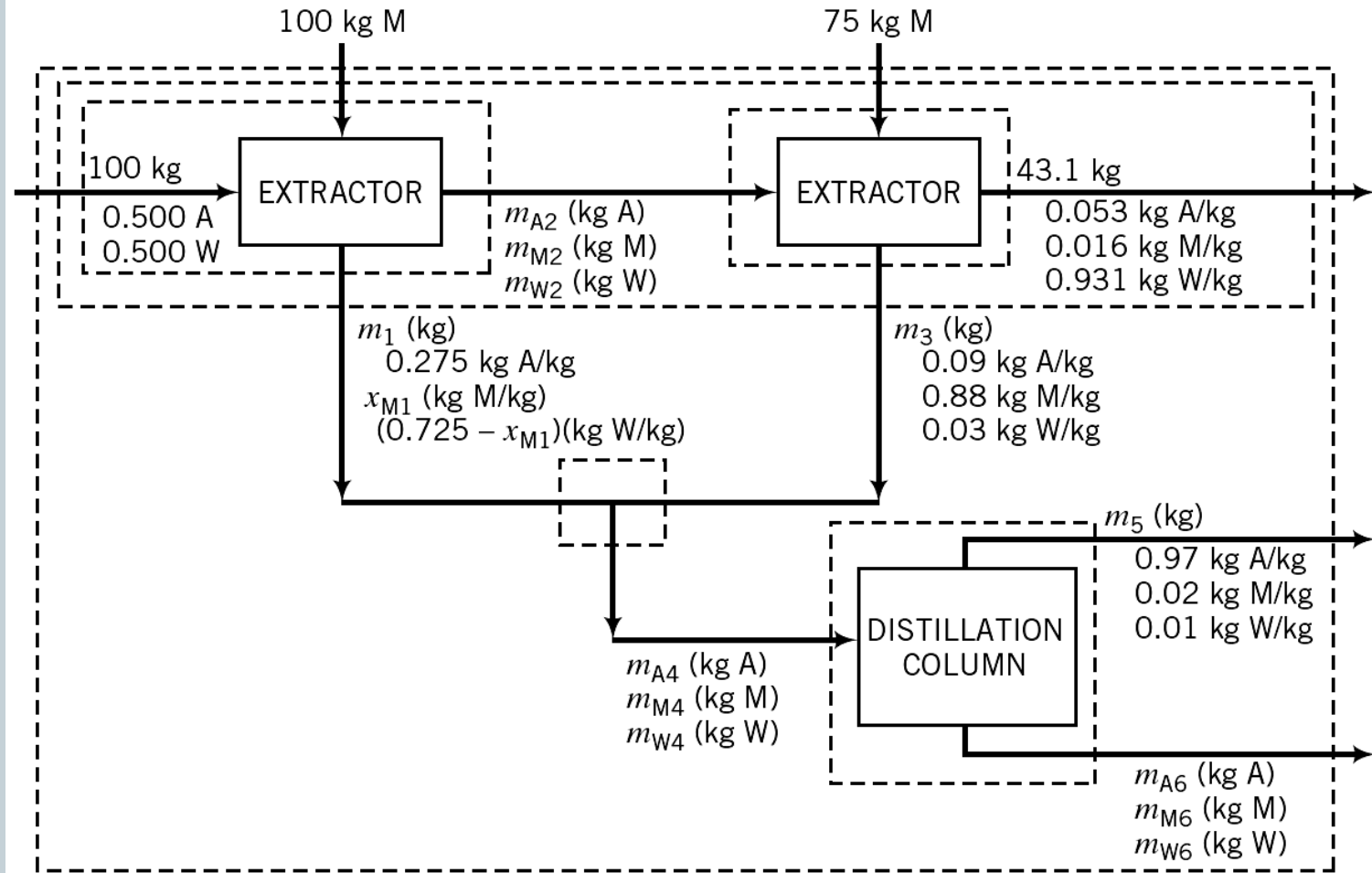


# Extraction-Distillation Process





# Extraction-Distillation Process

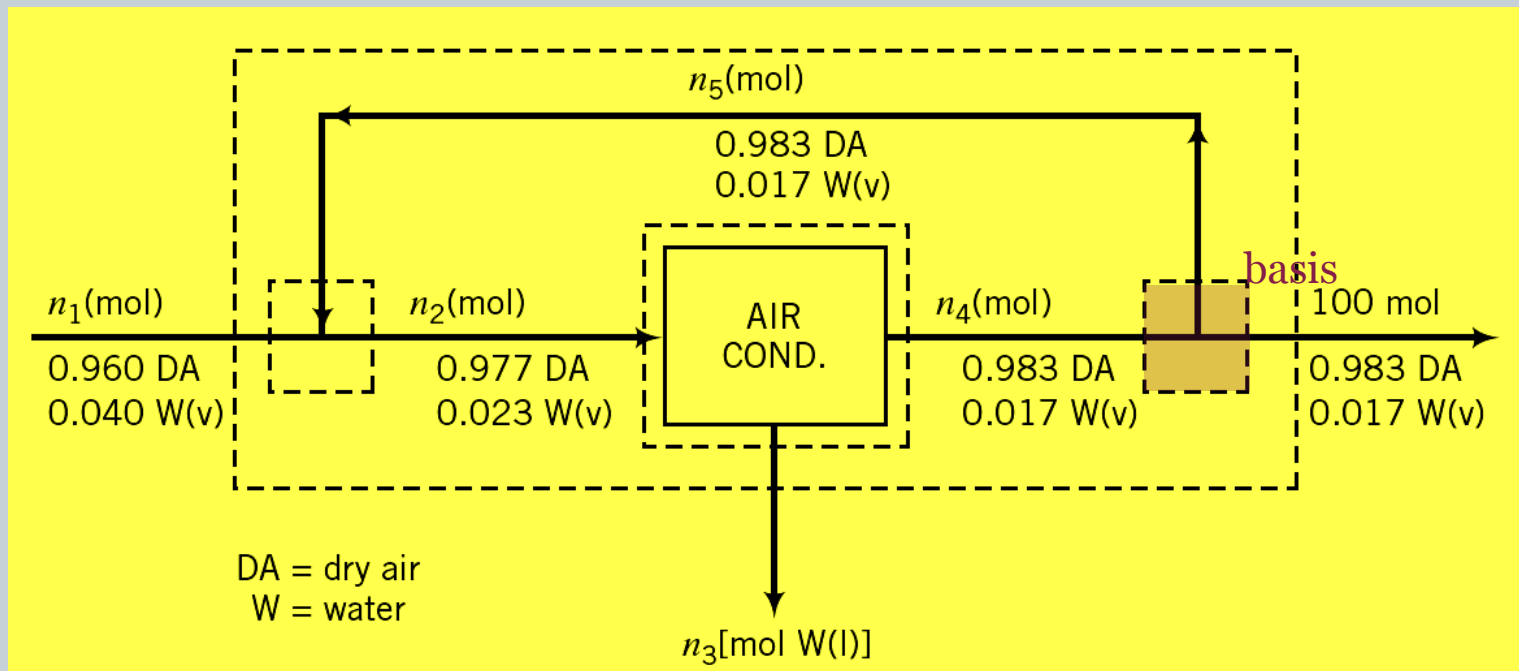






# Balances on an Air Conditioner

- process cools and dehumidifies feed air.
- unknowns:  $n_1$ ,  $n_2$ ,  $n_3$ ,  $n_4$ ,  $n_5$  (requested by problem).
- degree-of-freedom analysis critical to solution.



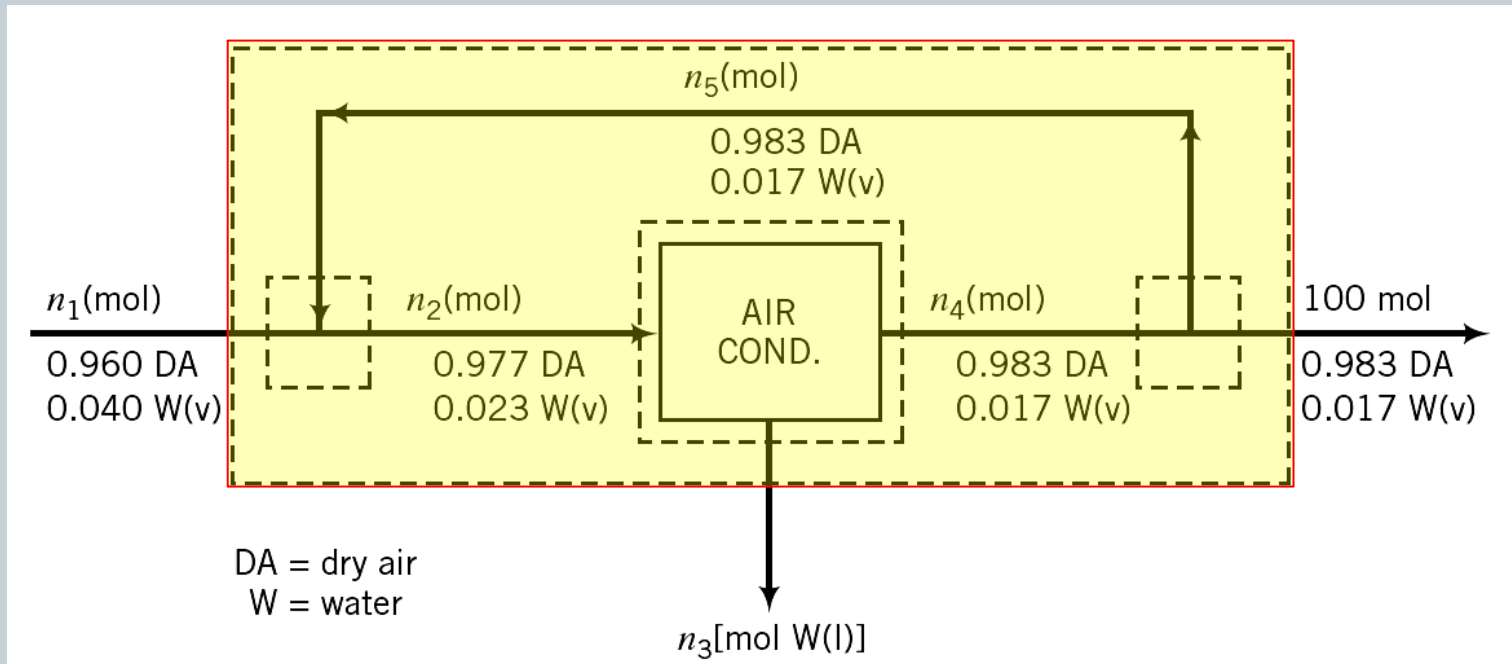


# Balances on an Air Conditioner



## □ Overall system

❖  $n_{df} = 2$  variables ( $n_1, n_3$ ) – 2 balances = 0



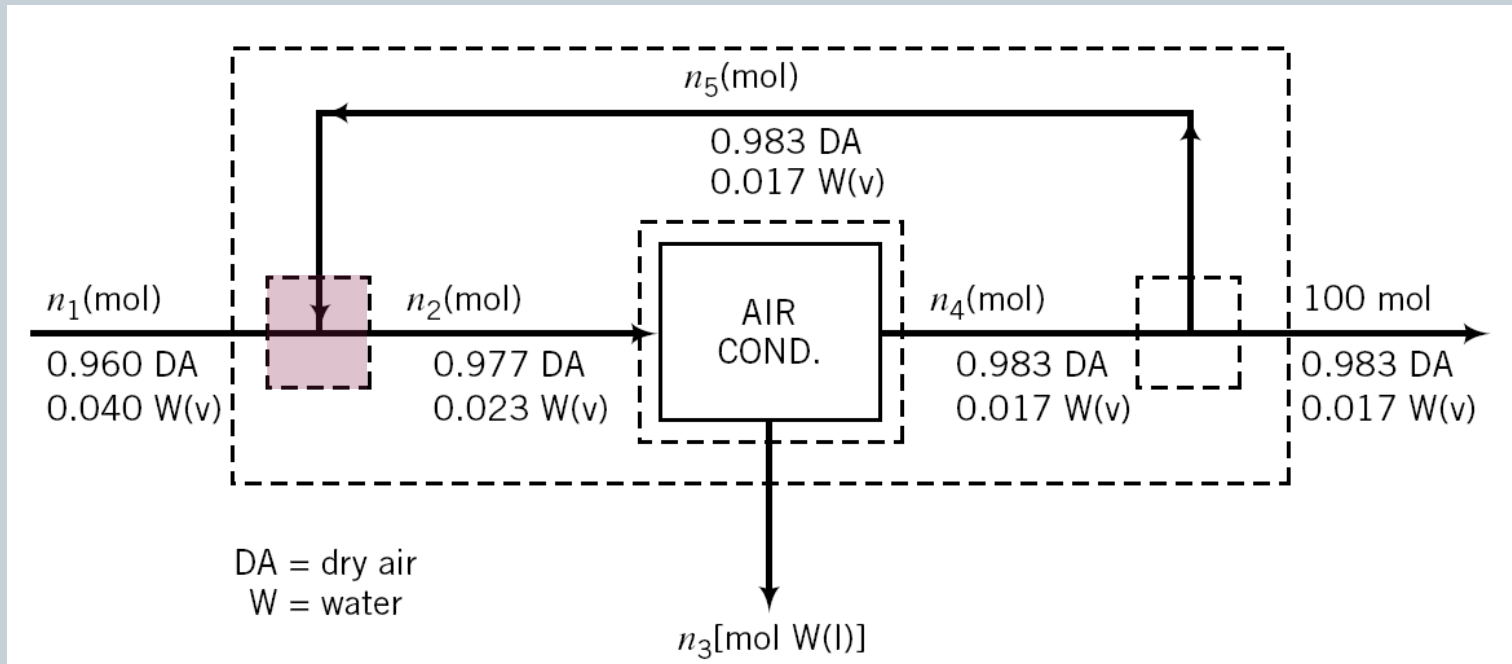


# Balances on an Air Conditioner



## □ Mixer

❖  $n_{df} = 2 \text{ variables } (n_2, n_5) - 2 \text{ balances} = 0$

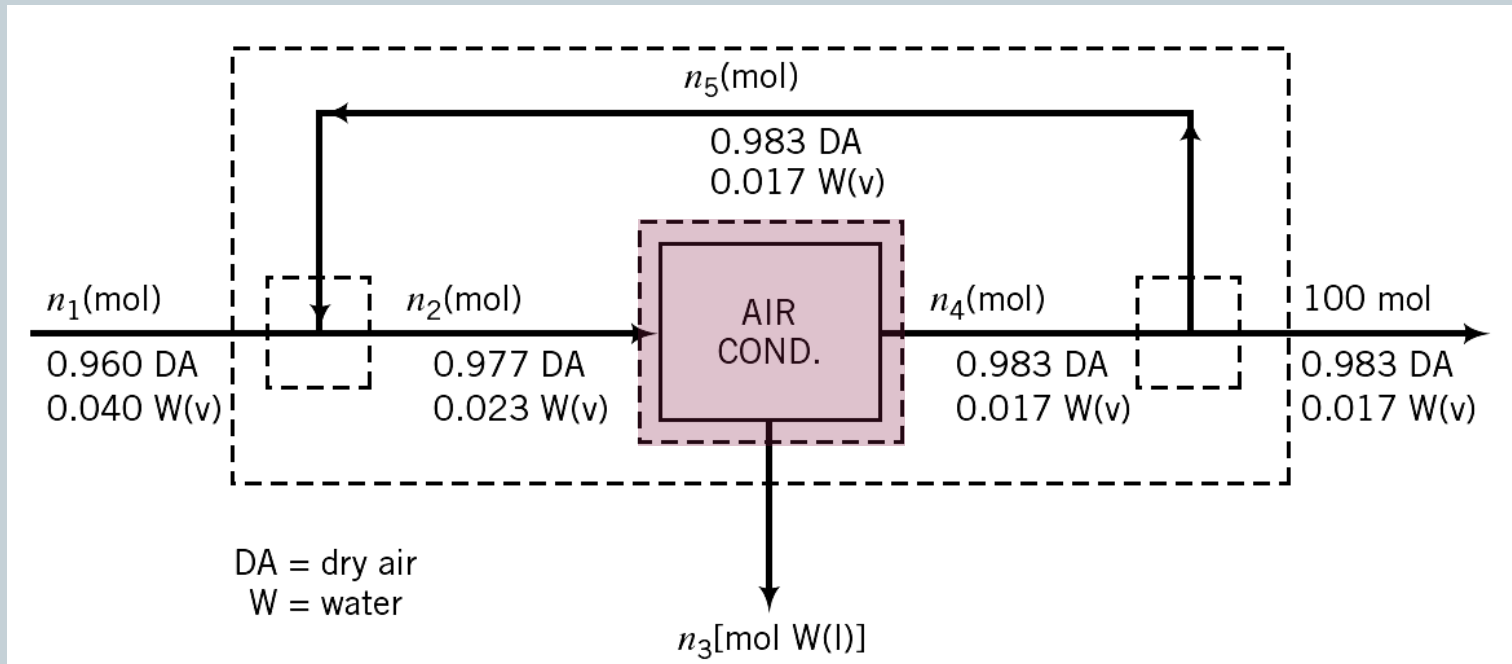




# Balances on an Air Conditioner

## □ Cooler

❖  $n_{df} = 2 \text{ variables } (n_2, n_4) - 2 \text{ balances} = 0$



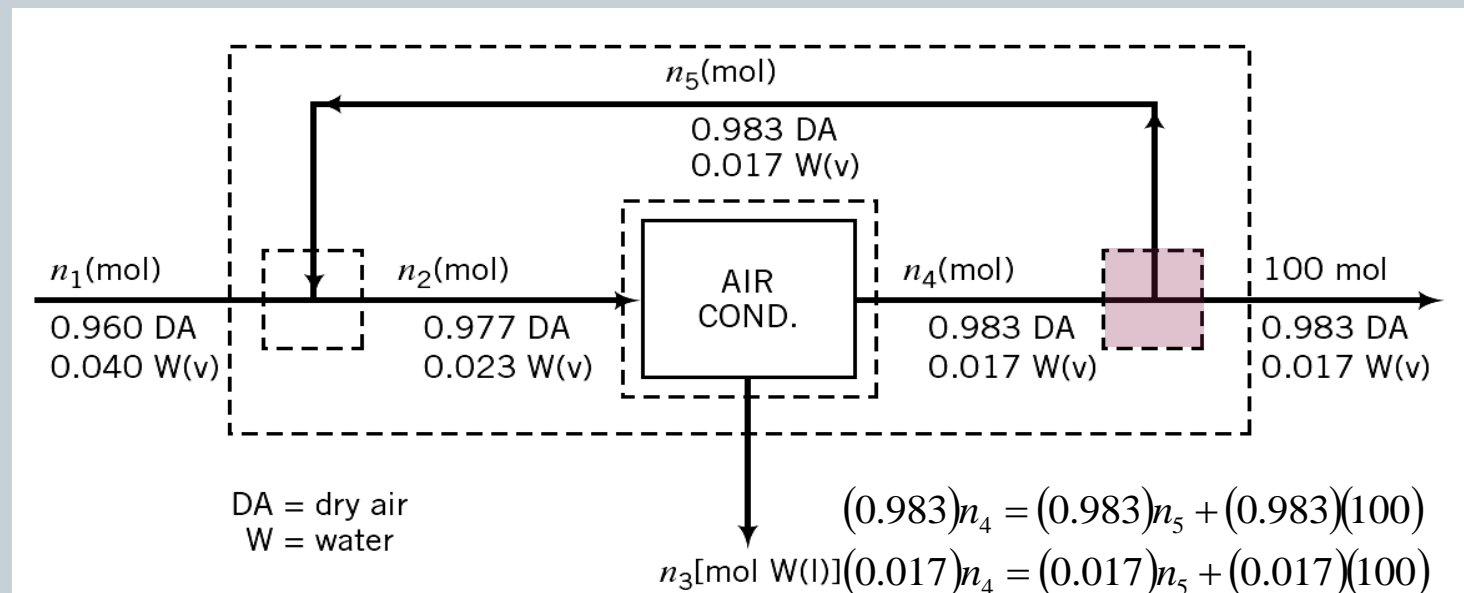


# Balances on an Air Conditioner



## □ Splitter

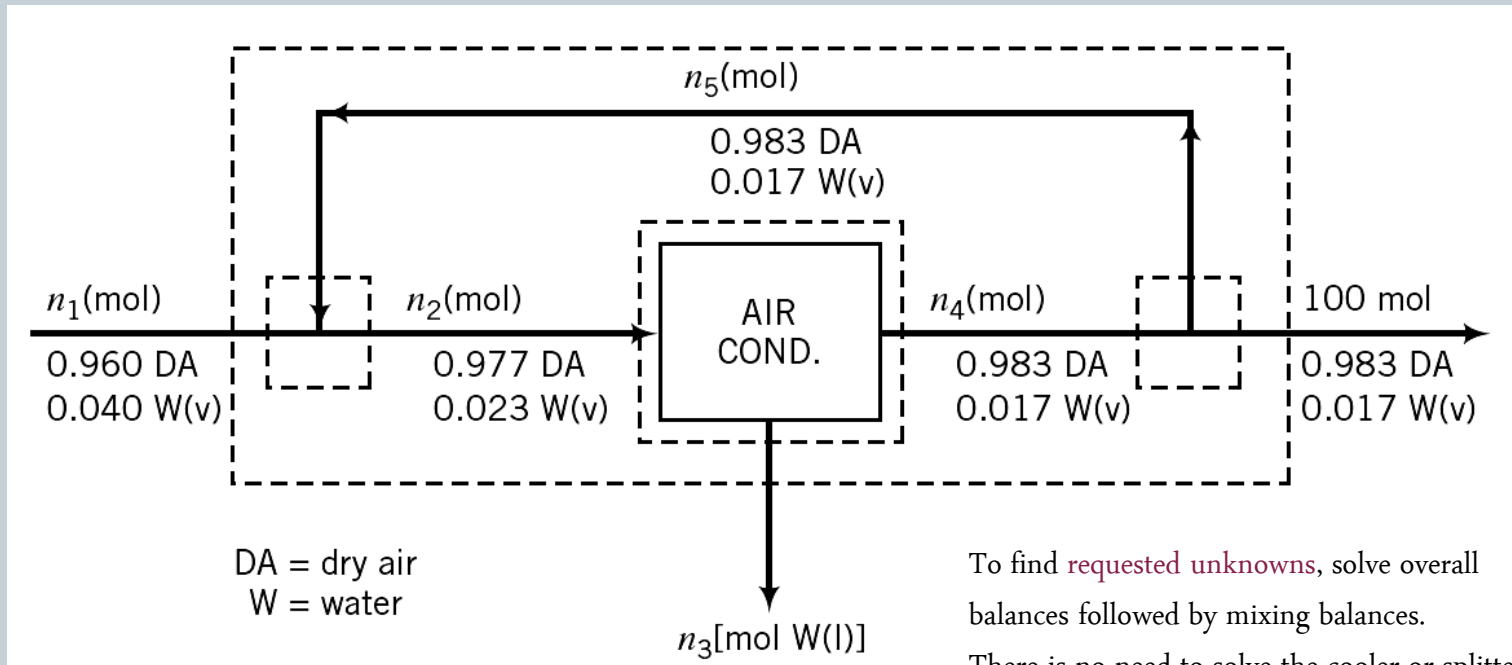
- ❖  $n_{df} = 2$  variables ( $n_4, n_5$ ) – 2 balances = 0
  - ✦ only 1 independent balance can be written on the splitter because the streams entering/leaving have the same composition.





## Balances on an Air Conditioner

- Overall:  $n_{df} = 2$  variables ( $n_1, n_3$ ) – 2 balances = 0
- Mixer:  $n_{df} = 2$  variables ( $n_2, n_5$ ) – 2 balances = 0
- Cooler:  $n_{df} = 2$  variables ( $n_2, n_4$ ) – 2 balances = 0
- Splitter:  $n_{df} = 2$  variables ( $n_4, n_5$ ) – 2 balances = 0

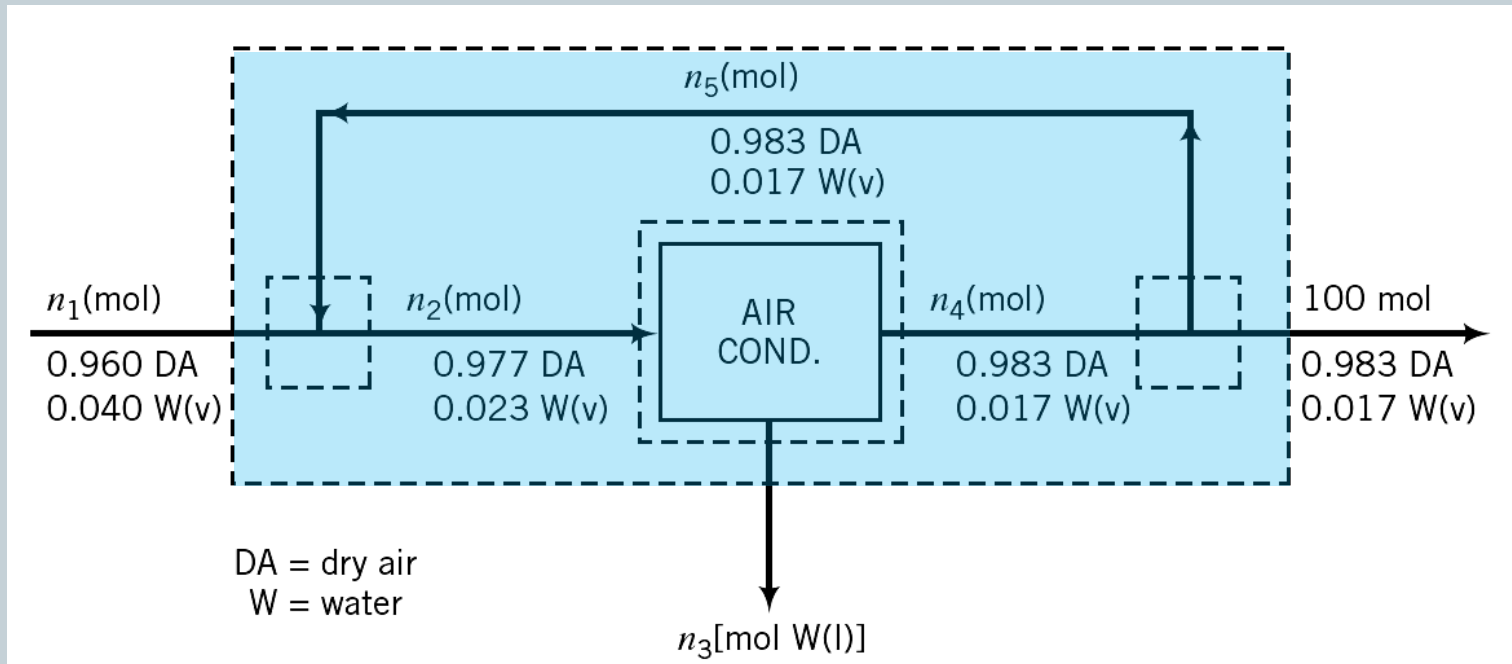




# Balances on an Air Conditioner

- ❑ overall dry air balance
- ❑ overall mole balance

$$(0.960)n_1 = (0.983)(100) \Rightarrow n_1 = 102.4 \text{ mol}$$
$$n_1 = n_3 + (100) \Rightarrow n_3 = 2.4 \text{ mol H}_2\text{O condensed}$$





# Balances on an Air Conditioner

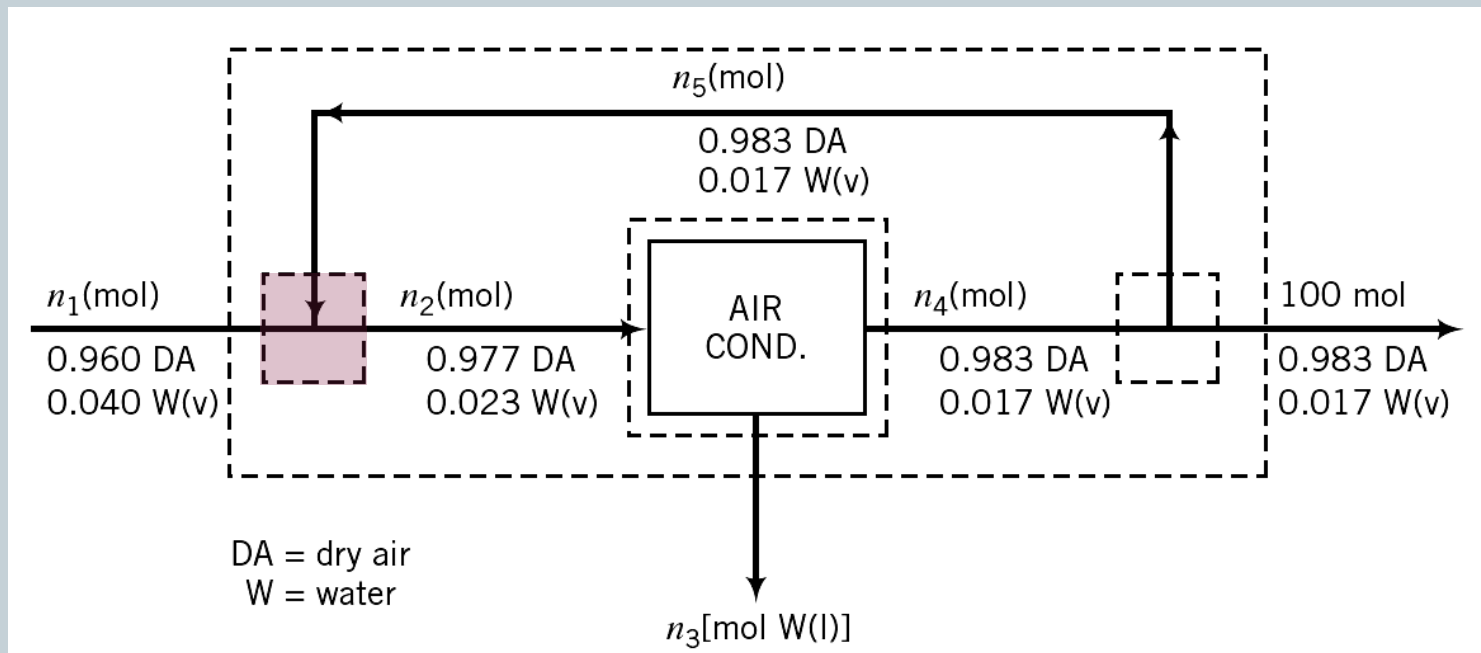


- overall mole balance
- water balance
- solved simultaneously:

$$n_1 + n_5 = n_2$$

$$(0.04)n_1 + (0.017)n_5 = (0.023)n_2$$

$$n_2 = 392.5 \text{ mol}; n_5 = 290 \text{ mol}$$



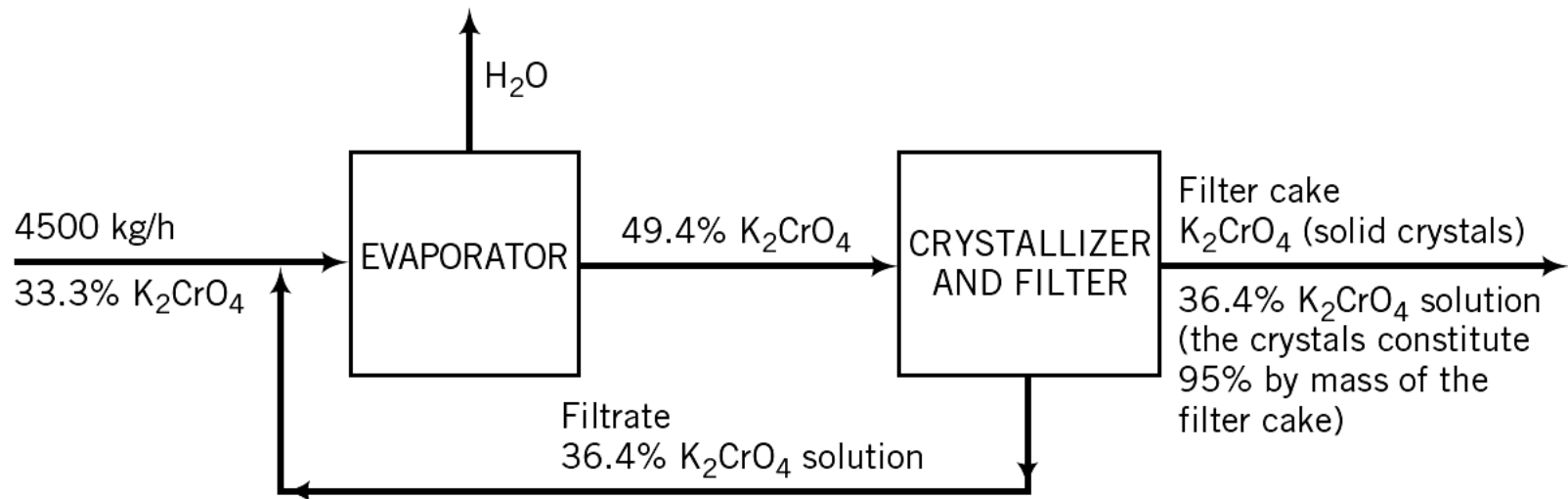




# Evaporative Crystallization Process

## □ Calculate:

- ❖ rate of evaporation
- ❖ rate of production of crystalline  $K_2CrO_4$
- ❖ feed rates to evaporator and crystallizer
- ❖ recycle ratio (mass or recycle/mass of fresh feed)

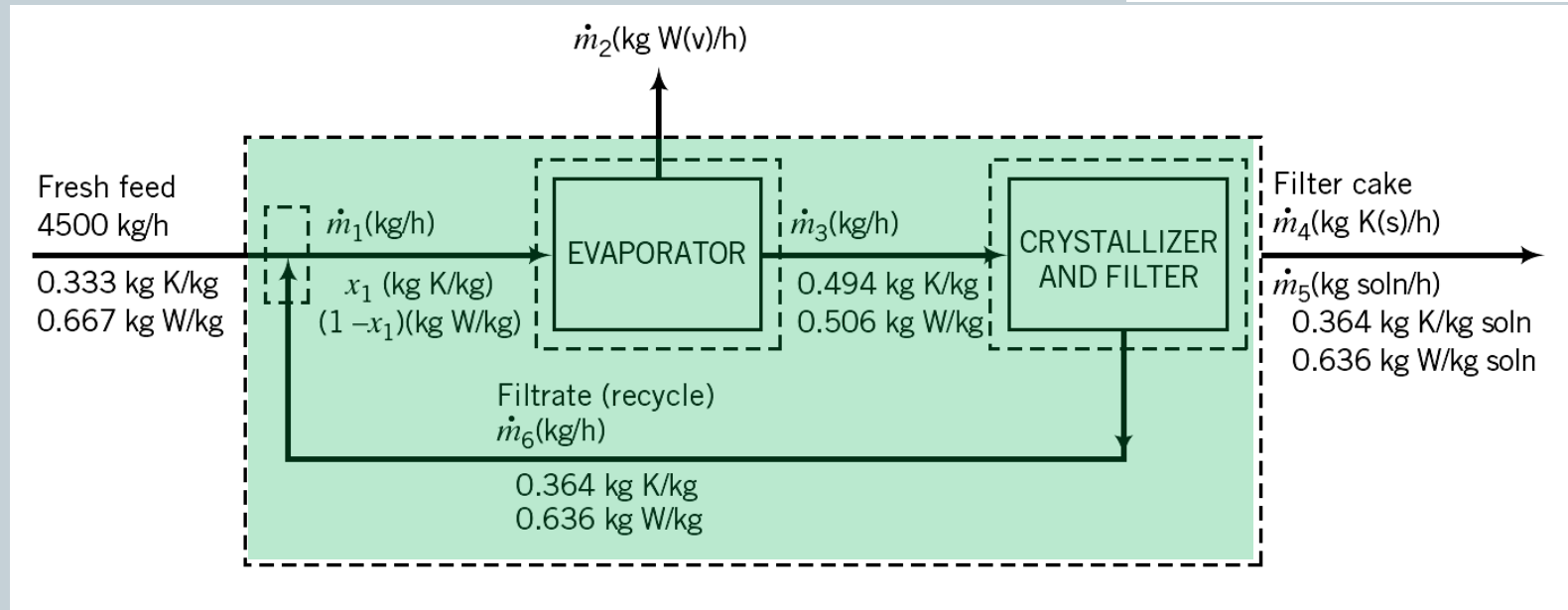




# Evaporative Crystallization Process

- Overall system:
  - $n_{df} = 3$  unknowns ( $m_2, m_4, m_5$ ) – 2 balances – 1 spec = 0
  - specification:  $m_4$  is 95% of total filter cake mass

$$\dot{m}_4 = 0.95(\dot{m}_4 + \dot{m}_5)$$



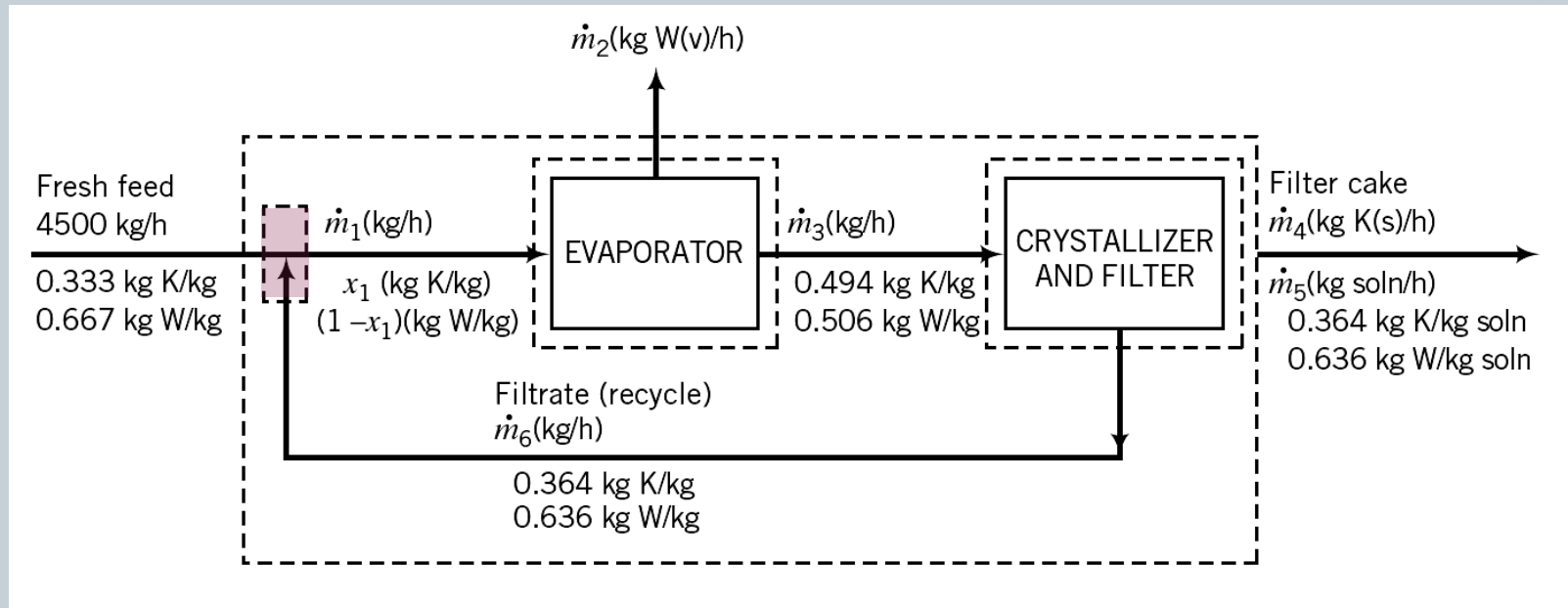


# Evaporative Crystallization Process

## □ Feed/recycle mixer:

- $n_{df} = 3$  unknowns ( $m_6$ ,  $m_1$ ,  $x_1$ ) – 2 balances = 1
- underspecified

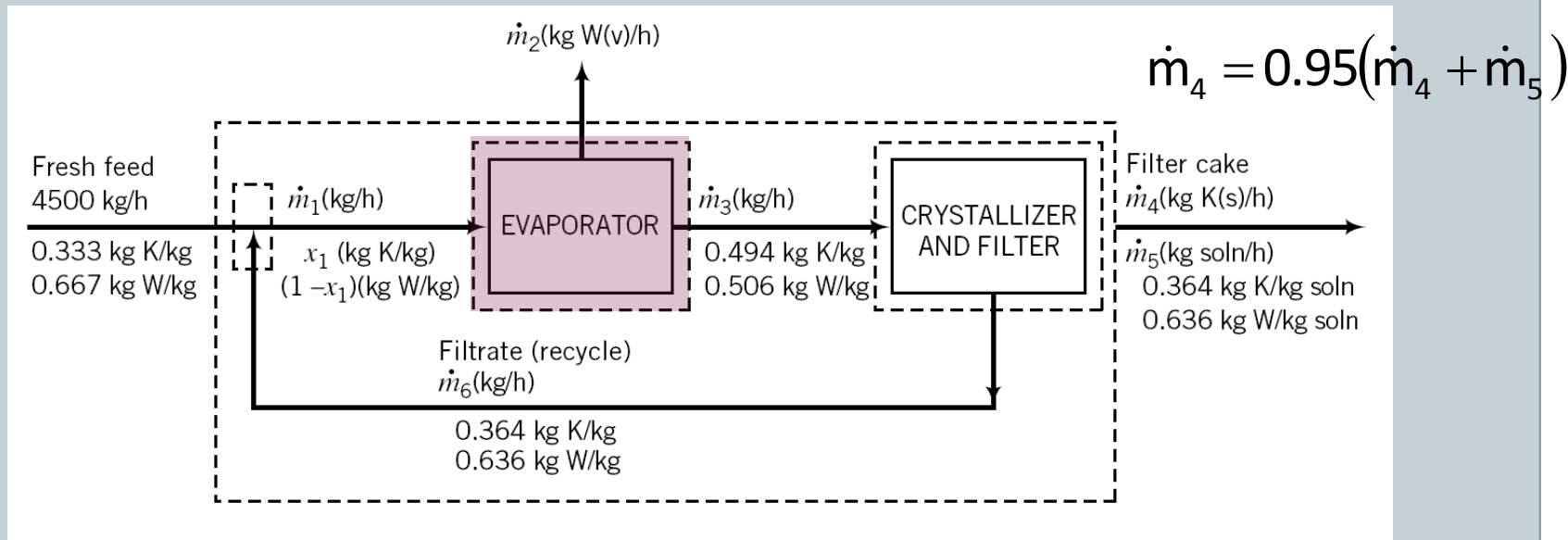
$$\dot{m}_4 = 0.95(\dot{m}_4 + \dot{m}_5)$$





# Evaporative Crystallization Process

- Evaporator:
  - $n_{df} = 3$  unknowns ( $m_3, m_1, x_1$ ) – 2 balances = 1
  - underspecified

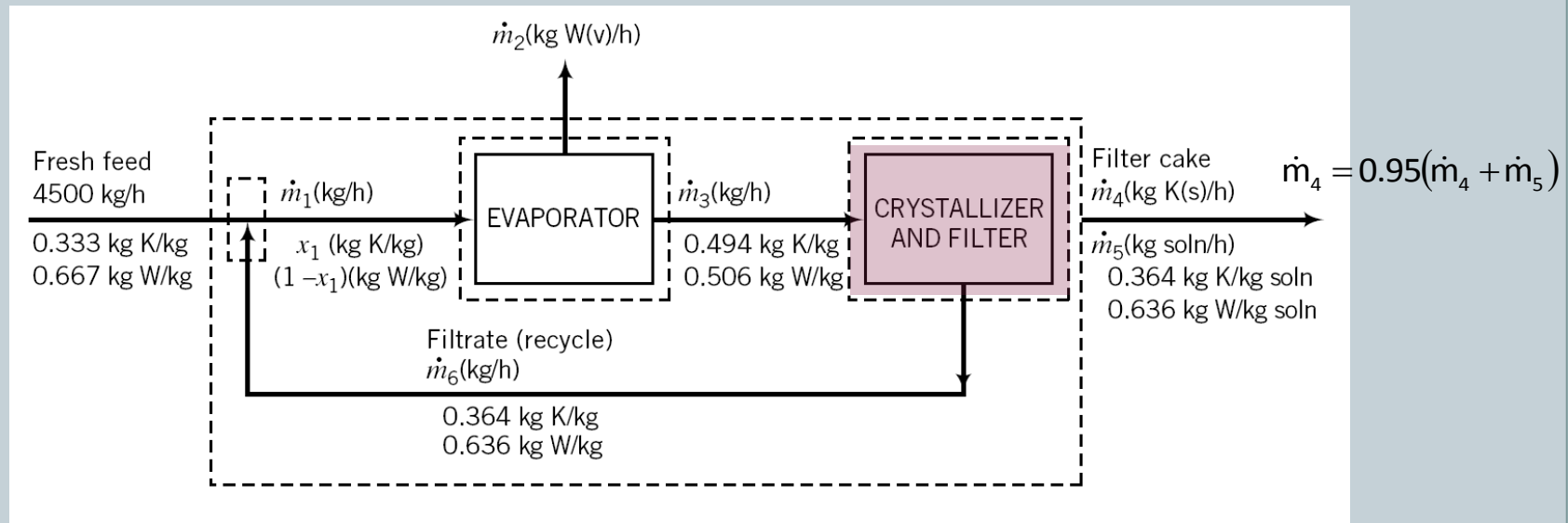




# Evaporative Crystallization Process



- Crystallizer:
  - $n_{df} = 2$  unknowns ( $m_3, m_6$ ) – 2 balances = 0
  - solvable
  - Once  $m_3, m_6$  are known, mixer or evaporator balances can be solved.





# Evaporative Crystallization Process



- Overall system:
  - $K_2CrO_4$  balance
  - water balance
  - total mass balance
  - specification

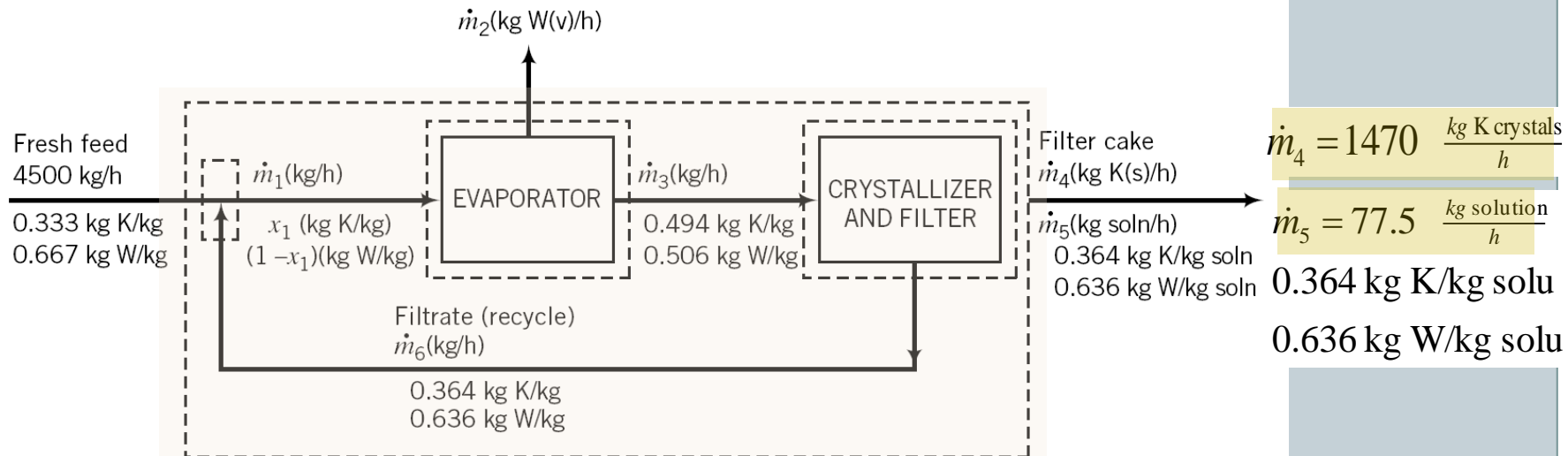
solve simultaneously for  $\dot{m}_4$  and  $\dot{m}_5$

$$(0.333)(4500) \frac{\text{kg K}}{\text{h}} = \dot{m}_4 + (0.364)\dot{m}_5$$

$$(0.667)(4500) \frac{\text{kg K}}{\text{h}} = \dot{m}_2 + (0.636)\dot{m}_5$$

$$4500 \frac{\text{kg}}{\text{h}} = \dot{m}_2 + \dot{m}_4 + \dot{m}_5$$

$$\dot{m}_4 = 0.95(\dot{m}_4 + \dot{m}_5)$$





# Evaporative Crystallization Process

- Overall system:
  - $K_2CrO_4$  balance
  - water balance
  - total mass balance
  - specification

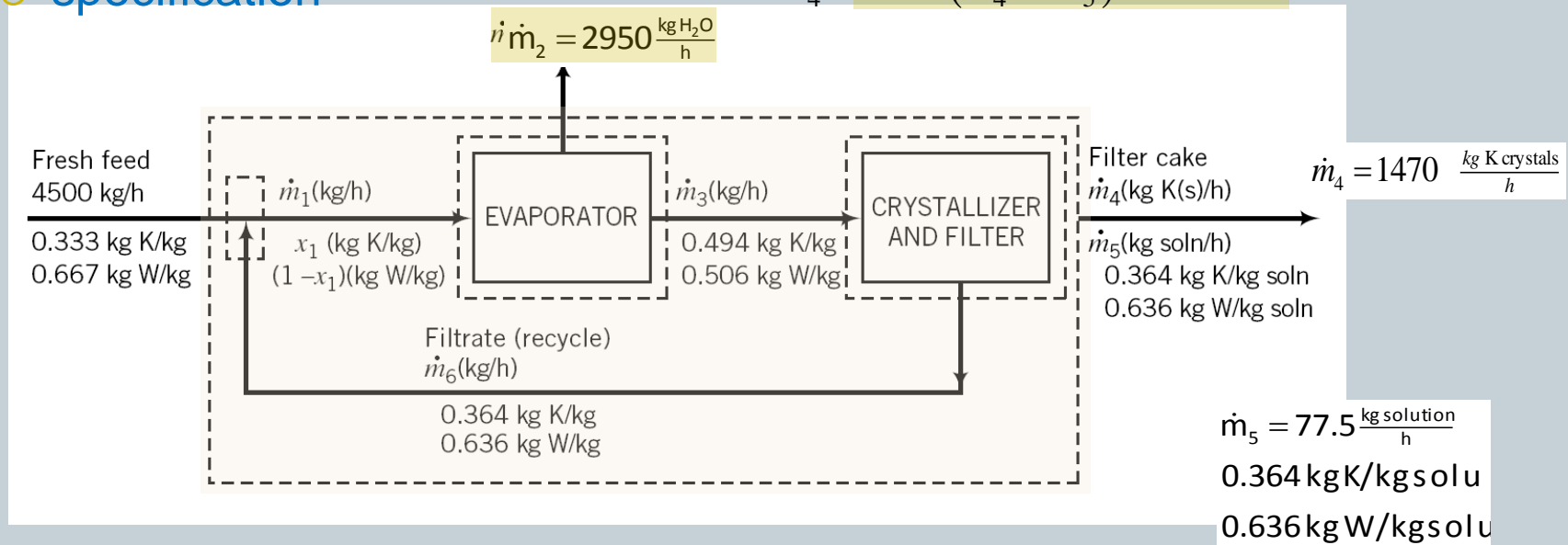
solve for  $m_2$  with knowns  $m_4$  and  $m_5$

$$(0.333)(4500) \frac{kg\ K}{h} = \dot{m}_4 + (0.364)\dot{m}_5$$

$$(0.667)(4500) \frac{kg\ K}{h} = \dot{m}_2 + (0.636)\dot{m}_5$$

$$4500 \frac{kg}{h} = \dot{m}_2 + \dot{m}_4 + \dot{m}_5$$

$$\dot{m}_4 = 0.95(\dot{m}_4 + \dot{m}_5)$$





# Evaporative Crystallization Process



only 3 equations are independent

- Overall system:
  - $K_2CrO_4$  balance
  - water balance
  - total mass balance
  - specification

$$(0.333)(4500) \frac{\text{kg K}}{\text{h}} = \dot{m}_4 + (0.364)\dot{m}_5$$

$$(0.667)(4500) \frac{\text{kg K}}{\text{h}} = \dot{m}_2 + (0.636)\dot{m}_5$$

$$4500 \frac{\text{kg}}{\text{h}} = \dot{m}_2 + \dot{m}_4 + \dot{m}_5$$

$$\dot{m}_4 = 0.95(\dot{m}_4 + \dot{m}_5)$$

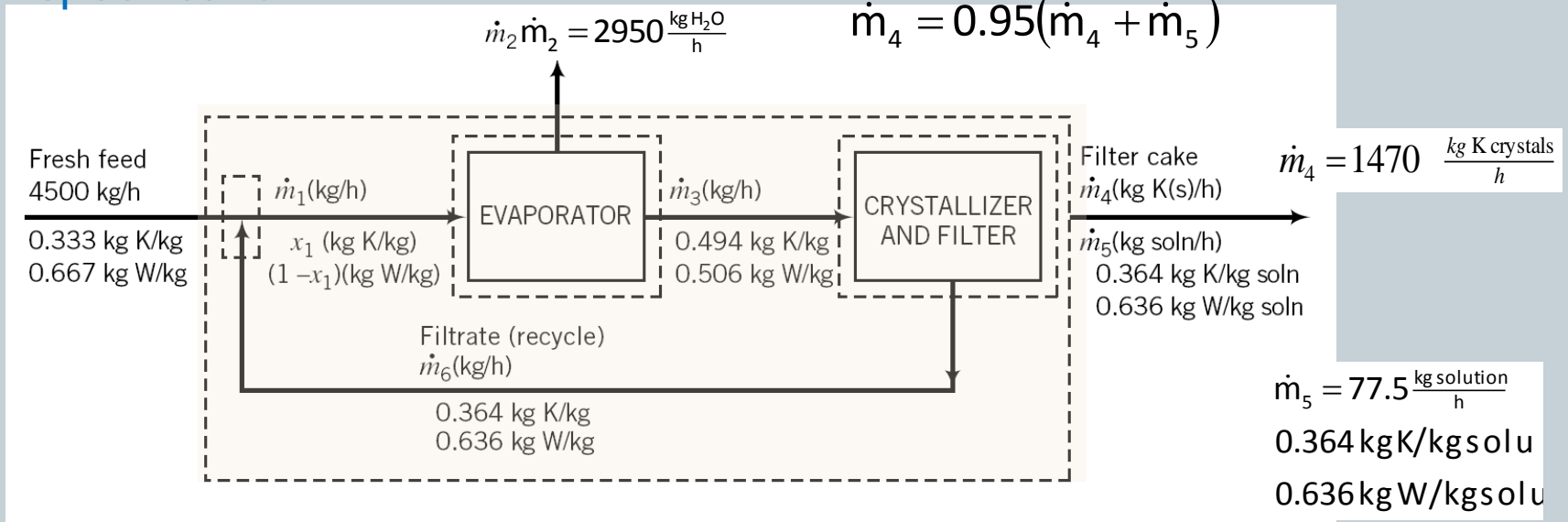
$$\dot{m}_2 \dot{m}_2 = 2950 \frac{\text{kg H}_2\text{O}}{\text{h}}$$

$$\dot{m}_4 = 1470 \frac{\text{kg K crystals}}{\text{h}}$$

$$\dot{m}_5 = 77.5 \frac{\text{kg solution}}{\text{h}}$$

$$0.364 \text{ kg K/kg solu}$$

$$0.636 \text{ kg W/kg solu}$$







# Evaporative Crystallization Process

- Crystallizer:
  - total mass balance
  - water balance

solve simultaneously for  $\dot{m}_3$  and  $\dot{m}_6$

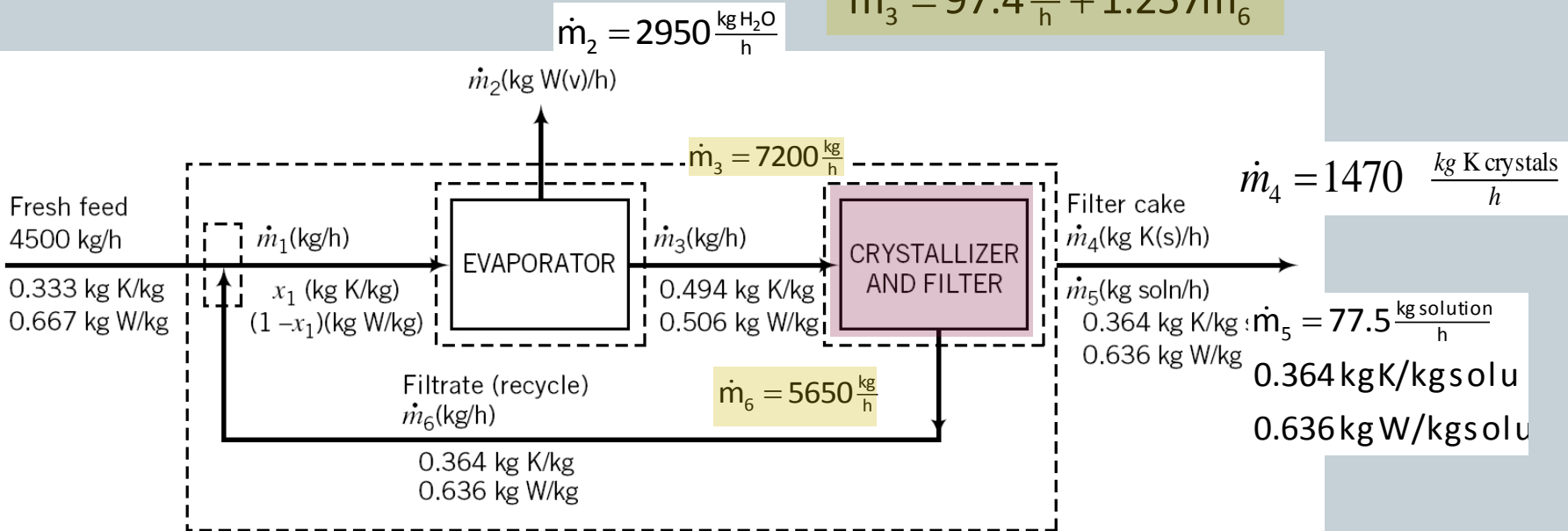
$$\dot{m}_3 = \dot{m}_4 + \dot{m}_5 + \dot{m}_6$$

$$\dot{m}_3 = (1470 + 77.5) \frac{\text{kg}}{\text{h}} + \dot{m}_6$$

$$(0.506)\dot{m}_3 = (0.636)\dot{m}_5 + (0.636)\dot{m}_6$$

$$\dot{m}_3 = 97.4 \frac{\text{kg}}{\text{h}} + 1.257\dot{m}_6$$

$$\dot{m}_2 = 2950 \frac{\text{kg H}_2\text{O}}{\text{h}}$$





# Evaporative Crystallization Process

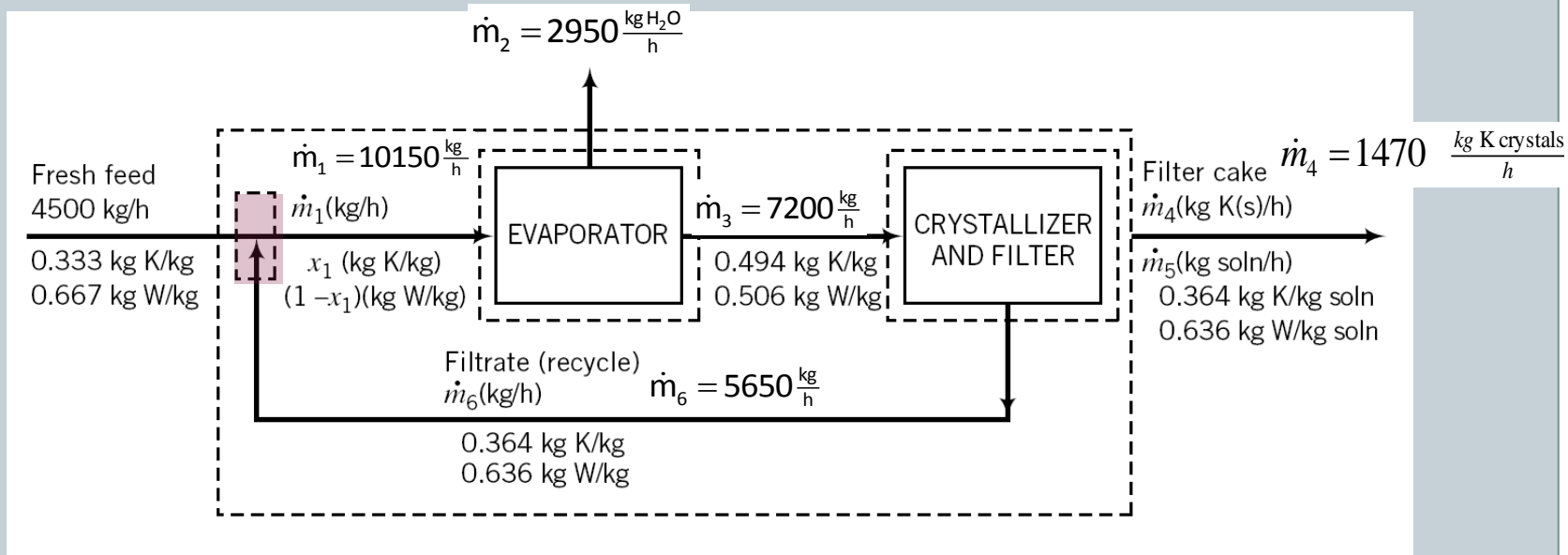


- feed/recycle mixer:

- total mass balance

- water or  $K_2CRO_4$  balance could be used to find  $x_1$  if desired

$$4500 \frac{\text{kg}}{\text{h}} + \dot{m}_6 = \dot{m}_1 \Rightarrow \dot{m}_1 = 10150 \frac{\text{kg}}{\text{h}}$$

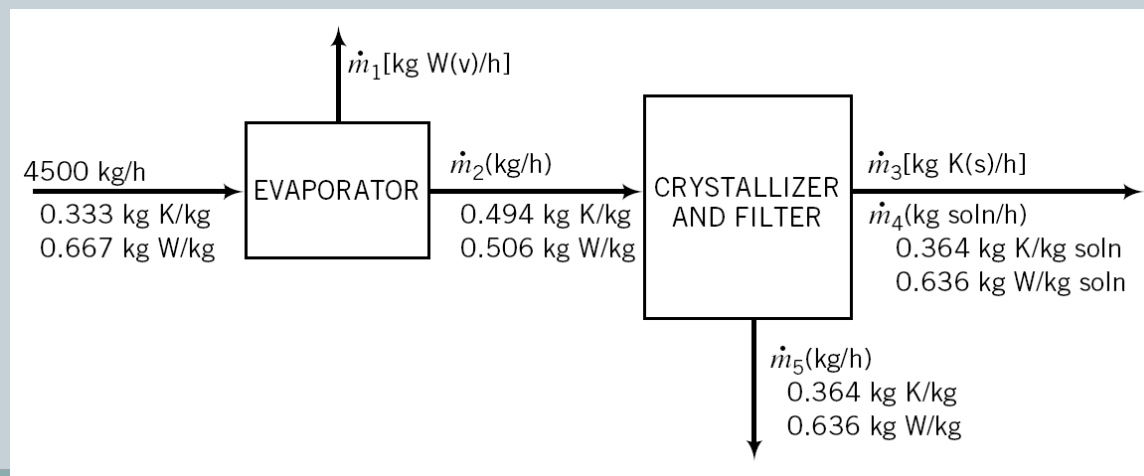




# Evaporative Crystallization Process



- If recycle is not used,
  - crystal production is 622 kg/h vs 1470 kg/h (w/ recycle)
  - discarded filtrate ( $m_4$ ) is 2380 kg/h, representing 866 kg/h of potassium chromate
- What are cost consequences of using recycle vs not?





## Assignment:

Sea water with the below data is entering 10 units evaporators. Roughly each unit, equally amount of water is vaporized then condensed and combined to obtain a fresh water. determine the fractional yield of fresh water from process (kg H<sub>2</sub>O recovered/kg H<sub>2</sub>O in process feed) and the weight percentage of salt in the solution leaving fourth stage.

