Tab 4: Assessment

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Tab 4: Assessment
Master Materials List

Blank paper
Chart paper or white board space
Current textbook(s) or local curriculum (optional)

Copies of all the PowerPoints with space for note taking
What's Your Problem? Handouts

The following materials are not in the notebook. They can be accessed on the CD through the links below.

PowerPoint: Opener

PowerPoint: *What's Your Problem?*

PowerPoint: The Power of Creating

PowerPoint: Closer

Activity: Opening Activity – Assessment Pyramid

Overview: Assessment happens in many forms of classroom interaction: questioning, homework, quizzes, tests, projects, classroom tasks. All of these forms of assessment have their own important place in the mathematics classroom. They all have in common the idea of ascertaining what students know and what they do not know. These assessments can be thought of in three dimensions: level of reasoning required, level of difficulty, and degree of skill or conceptual understanding required. The Assessment pyramid can be helpful in providing language and a perspective when looking at specific items, at a complete test, and at an entire course. Over time, all classroom assessments should generally fill the pyramid. The pyramid is not a rectangular prism, suggesting equal amounts of low and high level questions because it takes fewer high level reasoning questions to assess mathematical understanding. It takes more low level reasoning questions to assess mathematical understanding.

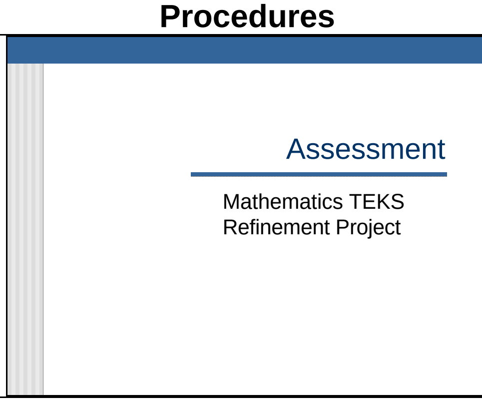
Participants will examine and discuss the Assessment Pyramid. They will consider several assessment items and their approximate positions in the pyramid. Participants will reflect on their own assessment practices and consider several guiding questions, including how to change their existing questions. This activity should prime them for the next section – how to change questions.

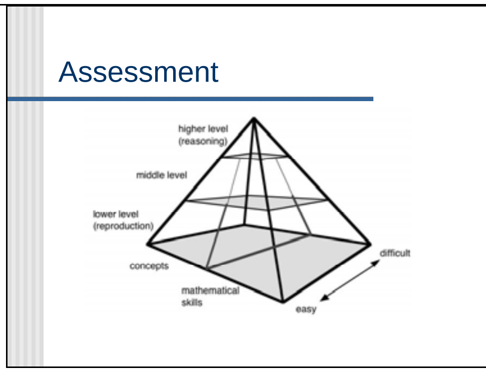
Materials: PowerPoint: Opener
Copies of the PowerPoint with space for note taking

Grouping: Tables of 4

Time: 30 minutes

Lesson: Distribute the PowerPoint copies to participants to help them focus on the important ideas from the PowerPoint presentation as they take notes. Show the PowerPoint presentation *Opener*. Use the following note pages to elaborate on the content of each slide.

Procedures		Notes
Slide 1		

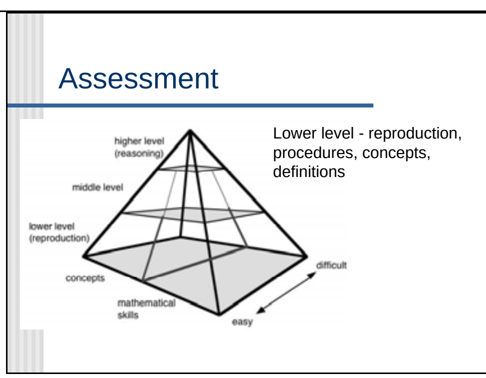
<table>
<tr><th colspan="2">Procedures</th><th>Notes</th></tr>
<tr><td>Slide 2</td><td>


</td><td>
Introduce the Assessment Pyramid (adapted from de Lange's Assessment Pyramid.) Note that the pyramid is a graphic that helps identify qualities or properties or dimensions of assessment items. Point out the two primary sections:

- concepts
- mathematical skills

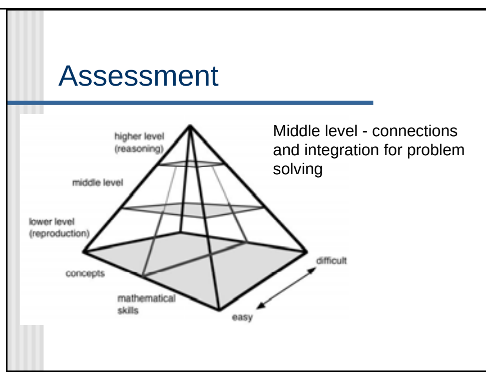
Level of difficulty runs from front (easy) to back (difficult). Levels of reasoning go from bottom (reproduction) to top (higher level). The line separating concepts and skills gets thinner as you move up the pyramid, suggesting that the lines blur between concepts and skills as you raise the level of reasoning required to solve problems.

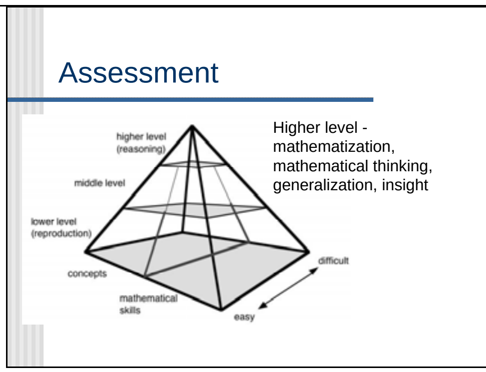
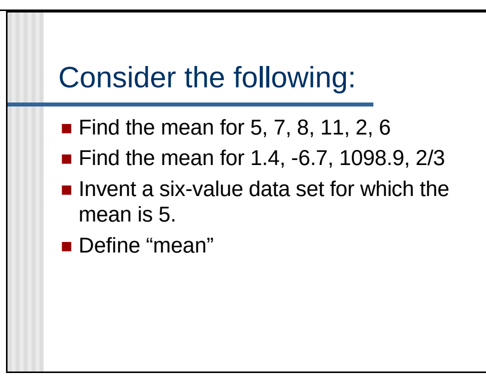
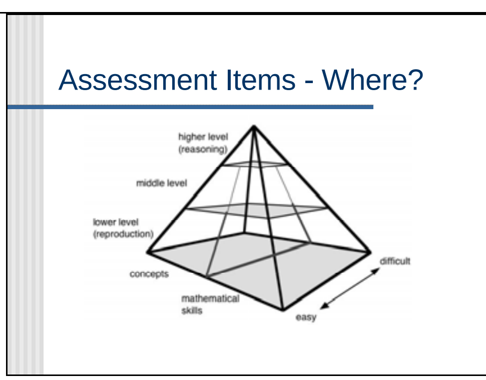
The next few slides give the opportunity to discuss the levels of reasoning in more depth.

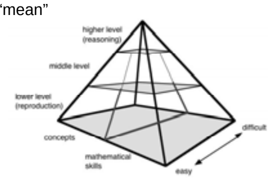
Be very clear that participants should not get bogged down with the pyramid. It can be useful and helpful but it should not cause arguments through the rest of the training about just how high or deep an item should be placed, etc. Its purpose is to get the conversation going and provide some vocabulary and perspective.
</td></tr>
<tr><td>Slide 3</td><td>


</td><td>
Throughout the discussion, be sure to juxtapose the ideas of low level to high level reasoning versus easy to difficult. Many teachers talk past each other by using the same words to discuss different qualities of assessment items.

Some teachers say, “I do not understand why my students are not successful. I ask them really hard questions. I do not know how to better prepare them for the hard/difficult questions they see on TAKS or other tests.”
</td></tr>
</table>

Procedures	Notes
	But is there a difference between hard or difficult questions and questions that require a higher level of reasoning? Now look at each level of reasoning separately. Lower level reasoning items “deal with knowing facts, representing, recognizing equivalents, recalling mathematical objects and properties, performing routine procedures, applying standard algorithms, and developing technical skills, as well as dealing and operating with statements and expressions that contain symbols and formulas in ‘standard’ form. Test items at this level are often similar to those on standardized tests and on chapter tests related to conventional curricula. These are familiar tasks for teachers and tend to be the types of tasks they are able to create.” (Romberg, 17)
Slide 4 	“At this level, students start making connections within and between the different domains of mathematics, integrate information in order to solve simple problems, have a choice of strategies, and have a choice in the mathematical tools. At this level, students also are expected to handle representations according to situations and purpose and need to be able to distinguish and relate a variety of statements (eg., definitions, claims, examples, conditioned assertion, proofs). Items at this level often are placed within a context and engage students in mathematical decision making. These tasks tend to be open and similar to instructional activities.” (Romberg, 18) Note that at this level, the lines blur between concepts and skills. It is more about connections. Also, it takes fewer middle level reasoning items to assess mathematical achievement. A student’s

Procedures	Notes
	answers to items higher up the pyramid give a more complete picture of that student's understanding and skill. Therefore teachers need less higher level reasoning questions to ascertain a student's mathematical achievement. Also, those higher level reasoning items generally take more time and involve more work.
Slide 5 	"At this level, students are asked to mathematize situations: recognize and extract the mathematics embedded in the situation and use mathematics to solve the problem; analyze; interpret; develop models and strategies; and make mathematical arguments, proofs, and generalizations. Items at this level involve extended-response questions with multiple answers." (Romberg, 18) Over time, a complete assessment program should "fill" the pyramid.
Slide 6 	Ask participants to consider the four items on Slide 6. How would they rate the items? Easy, hard? What other kinds of words would they use to describe the items? Have them discuss in their groups.
Slide 7 	Where might the items fit in the pyramid?

Procedures		Notes
Slide 8	Assessment Items - Where? ▪ Find the mean for 5, 7, 8, 11, 2, 6 ▪ Find the mean for 1.4, -6.7, 1098.9, 2/3 ▪ Invent a six-value data set for which the mean is 5. ▪ Define “mean”  	Ask participants to consider where each of the four items might sit in the pyramid. Have them discuss this with their group. Then ask groups to share their thinking with the whole group. #1 - Finding the mean is a fairly easy skill question with lower level thinking needed. #2 – This question is still a skill question with lower-level thinking, but it is difficult because of the crazy numbers. Do teachers sometimes confuse “difficult” with higher-level thinking? Do teachers sometimes create difficult questions because of the computation and fail to create higher level thinking questions? #3 – This is an un-do kind of question. Students have to understand how to find the mean in order to invent a data set. This requires a higher level of thinking and a conceptual knowledge of what a mean is. If a student has a good understanding of what a mean is, this is actually a fairly easy question. To change it up, ask for a 5-value data set. To make it more difficult, ask for 2 different data sets. #4 – This could be considered a concept question but one that only requires memory and therefore easy and low level. An objective here is to get participants to talk about the difference between computationally more difficult questions and higher level reasoning questions.

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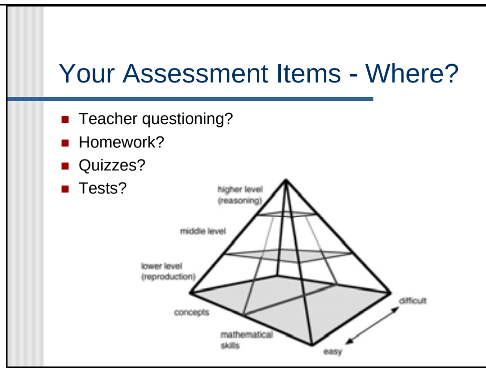
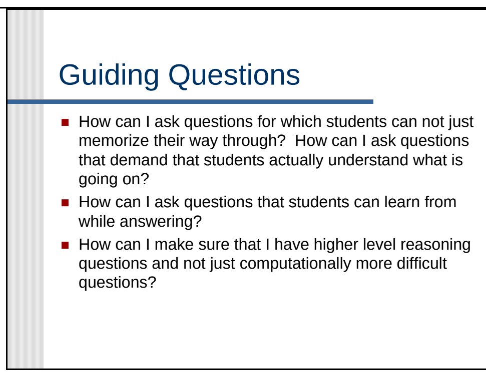
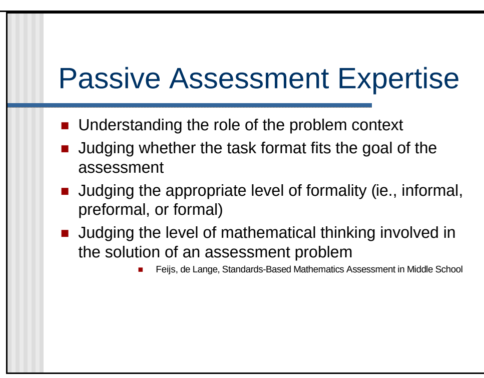
<tr><th colspan="2">Procedures</th><th>Notes</th></tr>
<tr><td>Slide 9</td><td>
</td><td>Assessment is a broad term that for many has different implications. Is it all about grades? Is it a continual process that informs instructional decisions?

At this point, ask participants to consider what they deem “assessment” and where their assessments might fit in the pyramid.

While assessment includes teacher questioning, homework, quizzes, tests, and more, the discussion that follows will focus on individual assessment items - specific questions, tasks, problems - and how teachers can differentiate where the items are on the pyramid so that teachers can make better assessment decisions.</td></tr>
<tr><td>Slide 10</td><td>
</td><td>Ask participants to consider these guiding questions as we continue.</td></tr>
<tr><td>Slide 11</td><td>
</td><td>The goal of this assessment discussion is to help teachers to select the assessment items that fit their needs and purposes, as they consider their entire assessment program. It is not to teach teachers to create such items, but to judge the appropriateness of those items from which they are selecting.</td></tr>
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<table>
<tr><th colspan="2">Procedures</th><th>Notes</th></tr>
<tr><td>Slide 12</td><td>The Assessment Principle

Assessment should become a routine part of the ongoing classroom activity rather than an interruption.
NCTM's Principles and Standards for School Mathematics (2000)</td><td>Before participants try to answer the guiding questions together, look at the Assessment Principle from the NCTM standards. Assessment should be a routine part, not an interruption. How can participants do that better? The next activities will give participants some ideas.</td></tr>
<tr><td>Slide 13</td><td>TAKS Item 7th Grade 2004

Terri collected data on the number of cans donated by each homeroom in her grade for a food drive. The table below shows the results of the food drive.

Which number could be added to the set of data in order for the median and mode of the set to be equal?
a. 54
b. 63
c. 80
d. 88

Canned-Food Drive<table><tr><th>Homeroom Teacher</th><th>Number of Cans</th></tr><tr><td>Mr. Campbell</td><td>45</td></tr><tr><td>Mrs. Padilla</td><td>63</td></tr><tr><td>Ms. Pogue</td><td>92</td></tr><tr><td>Mrs. Malmgren</td><td>27</td></tr><tr><td>Mr. Dawson</td><td>115</td></tr><tr><td>Ms. Morgan</td><td></td></tr></table></td><td>Here is a TAKS released item from 2004 - Seventh grade.

Ask participants: Which of the items that we looked at would better prepare students for this item? Specifically - would the computationally more difficult question better prepare students for the curve in this question? Or would the more open ended, higher level thinking question better prepare students for this question?

It is not possible to predict all of the ways that a TEKS will be assessed.

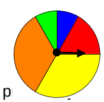
Would higher level thinking questions open the door for students to at least be thinking in that direction?</td></tr>
<tr><td>Slide 14</td><td>Our focus

▪ Think about current classroom assessments
▪ How can they improve?</td><td>Again, focusing on classroom assessment items, raise the question, “How can they improve?”

Don’t answer this yet.</td></tr>
</table>

Procedures		Notes
Slide 15	**Take one typical assessment** ▪ What is the purpose of the assessment? ▪ Where are the items in the pyramid? ▪ Are you satisfied with the balance?	Here Trainers could have participants choose one of their classrooms' assessments (assignments, quizzes, tests) and answer these questions. This could be a group activity where the group looks at a common assessment, perhaps a textbook assessment or shared exam. Trainers could also have each teacher individually select a quiz he/she has given and then share with others an example of an item from 3 different places in the pyramid.
Slide 16	**Changing existing questions** ▪ to higher leveling reasoning ▪ to concept questions ▪ maintain balance between concept and skill questions ▪ shift focus from what students do not know to what they do know	This is a transition slide to get participants primed for the next activity which is changing and improving items.
Slide 17	**Targeted Content** ▪ 6(10) Probability and statistics. The student uses statistical representations to analyze data. The student is expected to: (A) **select and use an appropriate representation for presenting and displaying** different graphical representations of the same data **including line plot, line graph, bar graph, and stem and leaf plot;** (B) **identify mean (using concrete objects and pictorial models)** , median, mode, and range of a set of data.	Note that the items and discussion will be based on targeted content - the TEKS listed in the following slides.

Procedures		Notes
Slide 18	**Targeted Content** ■ 7(10) Probability and Statistics. The student recognizes that a physical or mathematical model can be used to describe the experimental and theoretical probability of real-life events. The student is expected to: **(A) construct sample spaces for simple or composite experiments.**	
Slide 19	**Targeted Content** ■ 8(12) Probability and statistics. The student is expected to: (C) **select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots**, circle graphs, bar graphs, **box and whisker plots**, histograms, and **Venn diagrams**, with and without **the use of** technology.	
Slide 20	**Additional Areas to Highlight** ■ 6(4) Patterns relationships, and algebraic thinking. The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes. The student is expected to: (A) use tables and symbols to represent and describe proportional and other relationships such as those involving conversions, **arithmetic sequences (with a constant rate of change)** , perimeter and area.	
Slide 21	**Additional Areas to Highlight** ■ 7(5) Patterns, relationships, and algebraic thinking. The student uses equations to solve problems. The student is expected to: (B) formulate problem **situations** when given a simple equation **and formulate an equation when given a problem situation**.	

Procedures		Notes
Slide 22	So, let’s look at some ways to improve …	
Slide 23	Consider the following: ▪ What is the probability of rolling a 1 or a 6 on a dot cube? ▪ What is the probability of landing on blue?  ▪ Create 3 different spinners so that the p landing on red or blue is 3/4. ▪ Explain what each number in the ratio means if the probability of an event is 3/4.	This is the alternate set of questions - more probability than statistics.

Resources: Romberg, Thomas A. ed., *Standards-Based Assessment in Middle School: Rethinking Classroom Practice*. New York: Teachers College Press. 2004

Verhage, H., & de Lange, J. (1997, April). Mathematics education and assessment. *Pythagoras*, 42, 14-20.

Activity: What's Your Problem?

Overview: Examination, discussion, and writing of three types of assessment items: snapshot problems, un-doing problems, and error analysis problems.

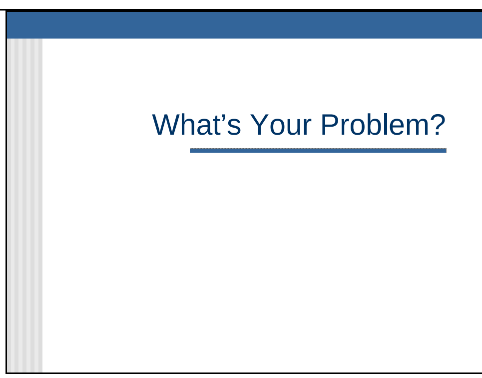
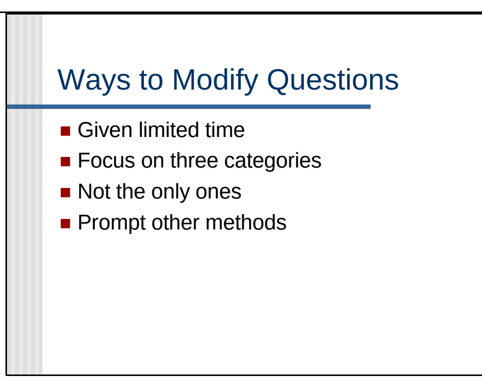
Materials: Handout 1-Sample Assessment Items-*What's the Difference?*
(pages 4-25 – 4-26)
Handout 2-Three Problem Types Labels, 1 for large group
(pages 4-27 – 4-29)
Handout 3-*What's Your Problem?* Items (pages 4-30 – 4-65)
Handout 4-Three Problem Types – How to Write (page 4-66)
Blank paper
PowerPoint: *What's Your Problem?*

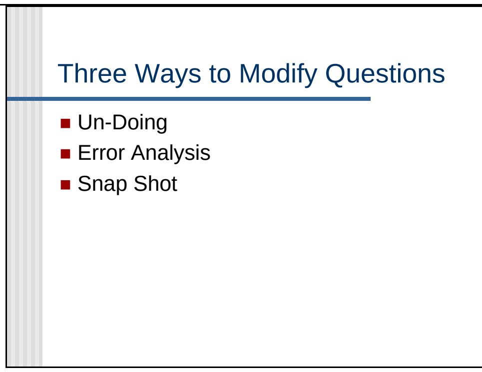
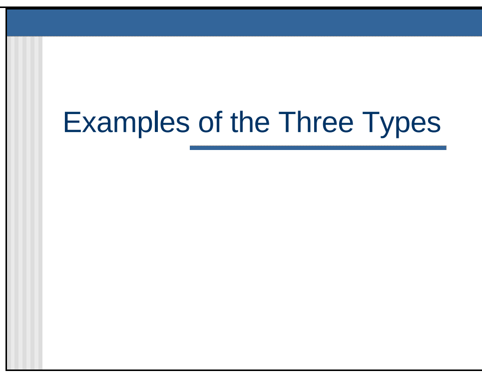
Copies of the PowerPoint with space for note taking

Grouping: Groups of 3

Time: 1.5 hours

Lesson: Distribute the PowerPoint copies to participants to help them focus on the important ideas from the PowerPoint presentation as they take notes. Show the PowerPoint presentation *What's Your Problem*? Use the following notes pages to elaborate on the content of each slide.

Procedures		Notes
Slide 1		The purpose of this PowerPoint is to give participants examples and experience with alternate problem types.
Slide 2		Discuss the disclaimers: We only have a limited amount of time. Therefore we are going to focus on three categories. They are certainly not the only ways to turn lower level thinking, closed questions into higher level thinking questions – there are other ways for sure, but these are helpful

<table>
<tr><th>Procedures</th><th>Notes</th></tr>
<tr><td></td><td>generalities to look at. They are a place to start.
This discussion will certainly prompt other methods, and that is also one of the goals – to get participants to consider alternatives.</td></tr>
<tr><td>Slide 3

</td><td>So, there are three possibilities that will be referred to loosely as “Un-doing,” “Error Analysis,” and “Snap Shot.” These are not technical names, just general descriptors of broad categories, again with the intent of giving participants alternatives.</td></tr>
<tr><td>Slide 4

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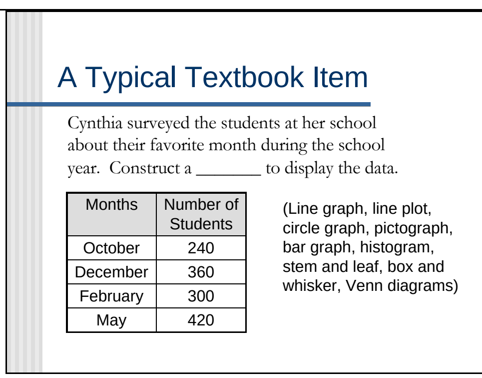
</td><td>A typical item - students are given data and asked to represent it with a particular graph. Do students need to be able to do this? Of course. What other kinds of questions could be asked so that students would learn more about representing data?</td></tr>
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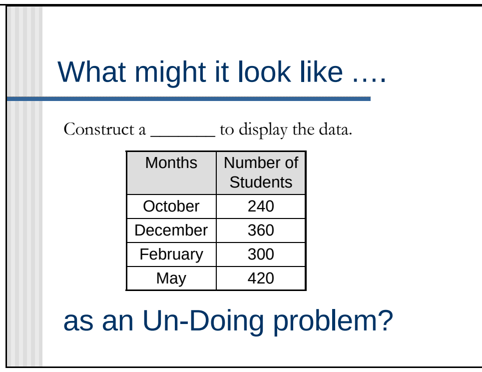
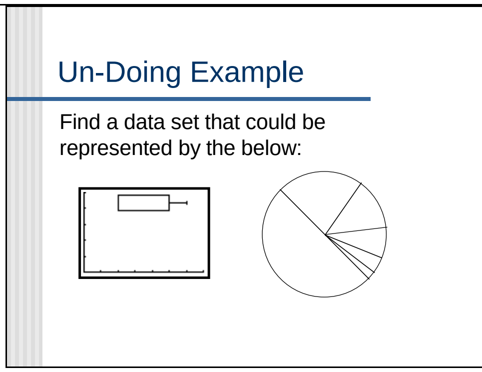
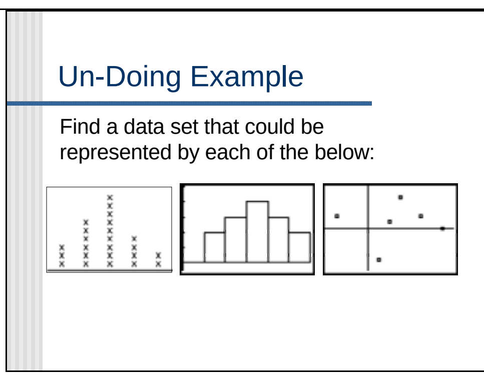
<tr><th colspan="2">Procedures</th><th>Notes</th></tr>
<tr><td>Slide 6</td><td>


</td><td></td></tr>
<tr><td>Slide 7</td><td>


</td><td>Have participants briefly discuss this problem with a partner or group.

In mathematics, we often do something and then un-do it. We multiply, we factor. We add, we subtract. Here, instead of giving students data and having them represent it with a particular graph type, we give students the graph and ask them to come up with the data needed to produce such a graph.

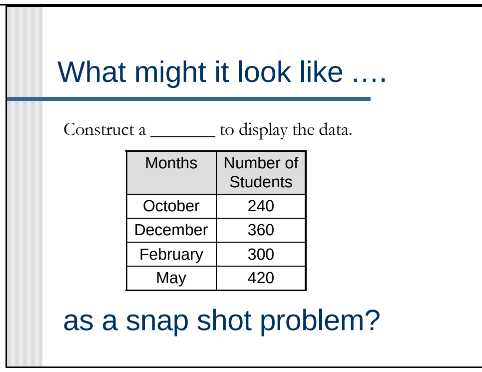
What kind of thinking and understanding is required to be able to do this?</td></tr>
<tr><td>Slide 8</td><td>


</td><td>Here are some more examples of the kinds of graphs students could be given and asked to create the data set. (Note that each graph does not necessarily represent the same data set.)

Have participants briefly discuss. Ask them to consider how a student might think differently to solve these problems. What extra or additional parts about graphs might be embedded in this problem, beyond what students had to think about to do the original “make a graph” problem?

Did this and the previous example seem more open? Might this possibly allow the teacher to see a greater variety of solutions and strategies that when discussed can build strength in connecting the different approaches? How many different answers</td></tr>
</table>

<table>
<tr><th colspan="2">Procedures</th><th>Notes</th></tr>
<tr><td colspan="2"></td><td>are possible for each?

If participants do not mention it, ask about the labels on the graphs. How does leaving off the labels open the question up even more?</td></tr>
<tr><td>Slide 9</td><td>
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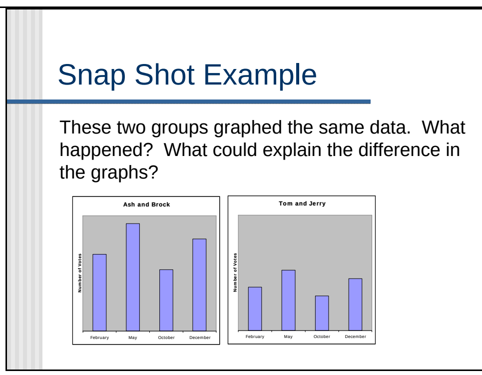
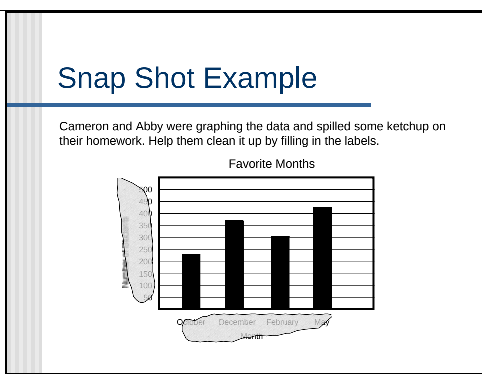
<tr><td>Slide 10</td><td>
</td><td>An important issue in graphing is scale.

Set the stage: In class the day before, two groups had graphed the same data but their graphs looked different because of scale. This came out in the class discussion.

Did all students make sense of the change of scale and its effect on the graph? Here is one way to assess.</td></tr>
<tr><td>Slide 11</td><td>
</td><td>Another way to open up the discussion is to take a procedure or process and hide carefully selected parts of it. In this way students have to think about someone else's strategies. This is a way to see if students really know what is going on, the reasons for the steps, or if they are stuck in one way of "doing" it.</td></tr>
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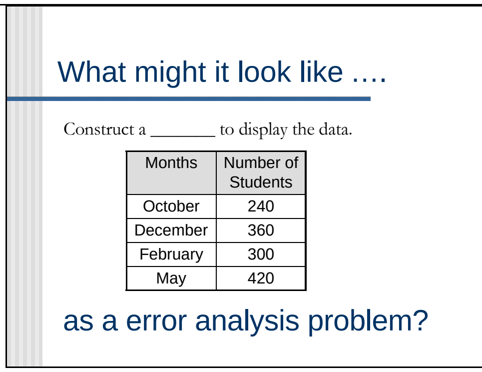
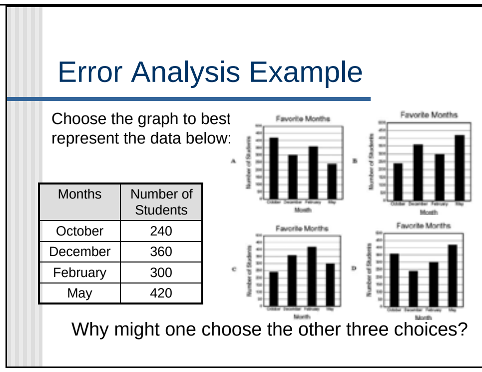
Procedures	Notes

Slide 12



Months	Number of Students
October	240
December	360
February	300
May	420

Slide 13



Months	Number of Students
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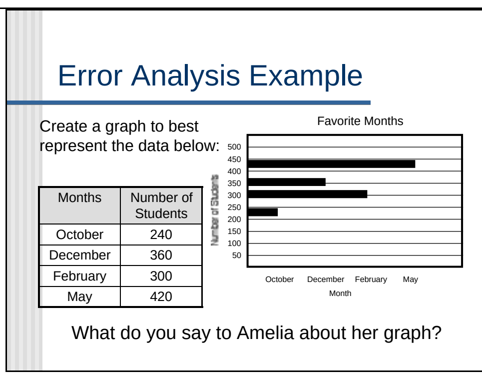
A technique to help students is to determine why others might choose an incorrect answer.

Slide 14



Months	Number of Students
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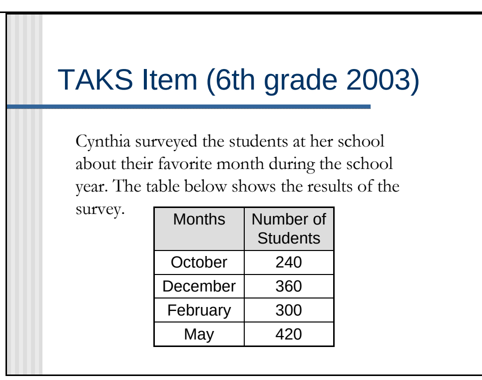
May	420

Students are asked to examine erroneous solutions and find the error(s). Ask teachers to consider the common errors of their own students. Suggest that instead of only re-teaching the correct method, they might also consider asking students to analyze the common errors made in their own classrooms.

Here we see that Amelia put a bar at the value in the table for each month and extended the bar out to the month. The bar does not represent a numerical amount.

Slide 15



Months	Number of Students
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February	300
May	420

Here is a released item from the 2003 6th grade TAKS. Ask participants to consider the various problems at which they have just looked and how these problems might have prepared students for this item.

Compare the original, typical question's place on the pyramid with the locations of the other questions.

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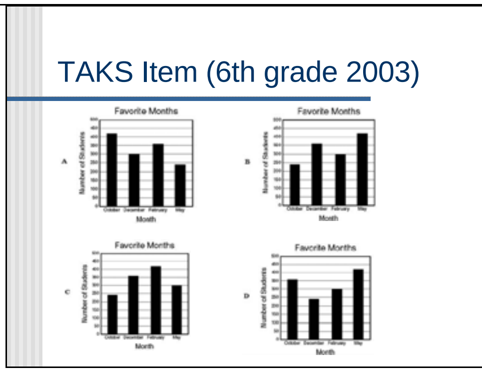
<tr><td>Slide 16</td><td>
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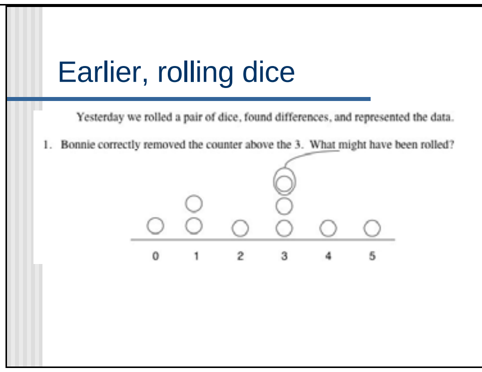
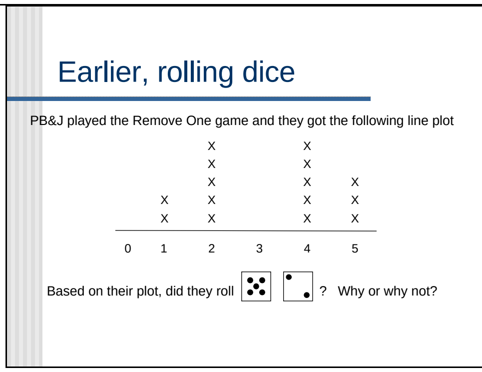
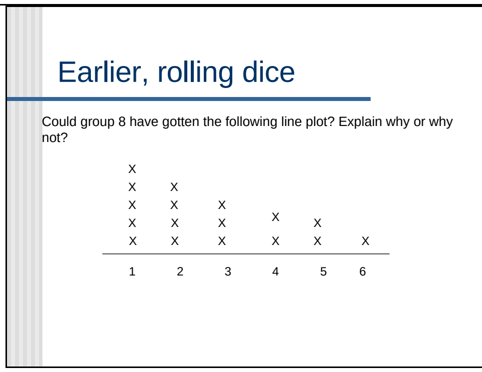
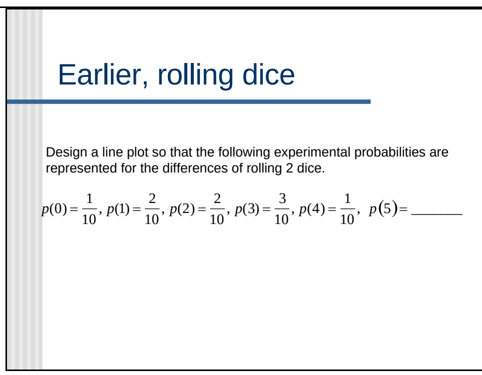
<tr><td>Slide 17</td><td>
</td><td>A more effective discussion of assessment begins with a common experience to discuss. After participants have done the What's the Difference? activity (pages 3-3 – 3-12), they can now have a rich discussion about how to assess it. Ask participants to brainstorm how they might assess the What's the Difference? activity.

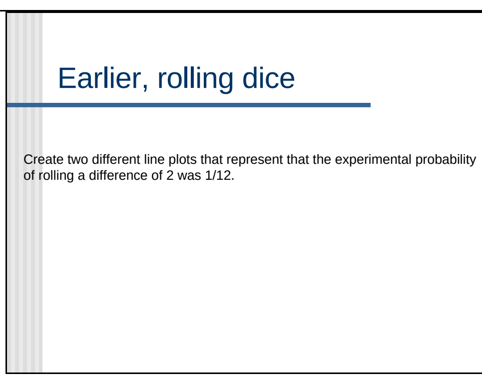
The following are several examples of possibilities, ranging from low level to mid level reasoning questions. Ask participants to generally categorize the problem as Un-Do, Error Analysis, Snap Shot or other. (These examples are provided in Handout 1-Sample Assessment Items-What's the Difference?, pages 4-25 – 4-26)

This problem can be considered a Snap Shot with an un-doing feel - taking a snap shot of the activity and asking students to un-do, or reverse the procedures they did in What's the Difference?. In the game, they rolled and took away a counter. Here they need to determine what was rolled if a counter was correctly removed.</td></tr>
</table>

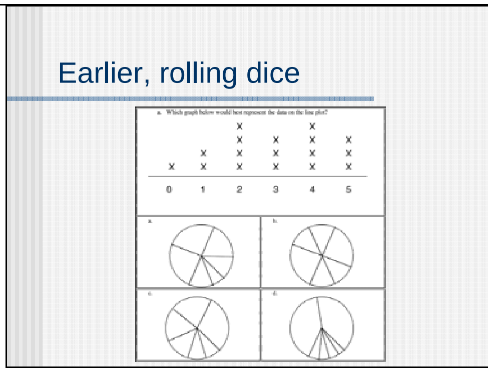
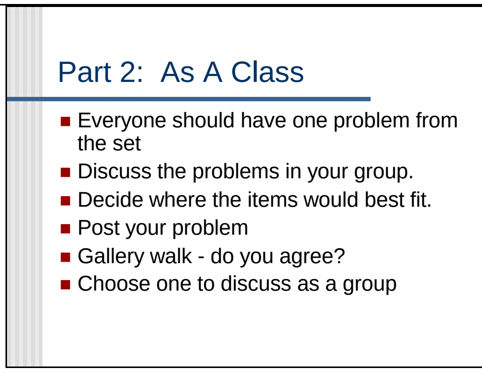
<table>
<tr><th colspan="2">Procedures</th><th>Notes</th></tr>
<tr><td>Slide 18</td><td>
</td><td>Snap Shot with an un-do twist.

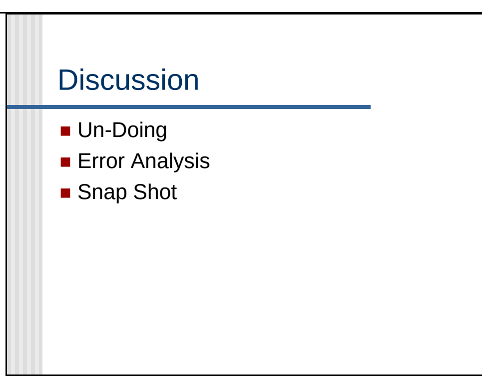
Were they paying attention in class? Did they understand the rules? Can they go from the graph to the roll?</td></tr>
<tr><td>Slide 19</td><td>
</td><td>Error analysis.

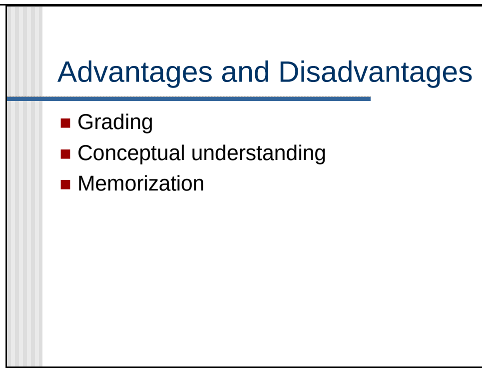
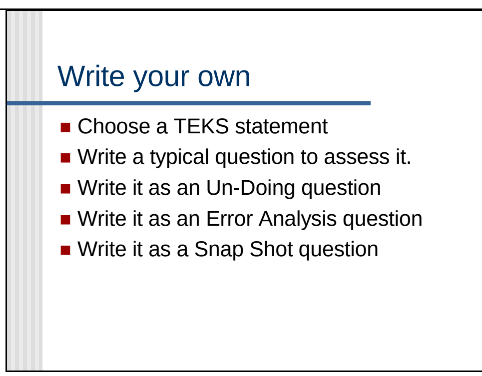
What do the numbers on the line plot represent?</td></tr>
<tr><td>Slide 20</td><td>
</td><td>Un-Do.

In the activity, students rolled dice, removed counters and recorded experimental probabilities. Can students go backward? Do they understand what those experimental probabilities represent? Can they go from one representation to another?</td></tr>
<tr><td>Slide 21</td><td>
</td><td>Un-Do.

This is an open ended item where students are asked to create two different representations, going backward from the experimental probability to the plot. If they understood where the experimental probabilities came from, they should be able to create the scenarios and the plots</td></tr>
</table>

Procedures		Notes
Slide 22		Error analysis - moving between representations.
Slide 23		Have participants look at some more items to continue to get a better sense of ways to alter and adjust classroom assessment for better student learning. Use Handout 2 (pages 4-27 – 4-29) to label sections of the room as Un-Do, Snap Shot, and Error Analysis. Distribute the problem items (Handout 3 pages 4-30 – 4-65), one per person if possible. Have participants decide as a group and post the problem items under previously labeled sections of the room (Un-Do, Snap Shot, and Error Analysis). After items are posted, participants consider if they agree as they take a gallery walk. For items they think are posted incorrectly, participants could flag them with a red flag. The purpose here is not so much the three problem types, it is more to expose participants to alternative ways to assess. As a whole group, discuss the groupings. Bring out the following points. **Un-Doing:** Much of mathematics is doing something and un-doing it. Many times a great question to assess if students “got it” when doing something is to ask students to un-do it, to back up from the answer, to come at it from a different direction or

Procedures	Notes
	representation. Some of the Un-Doing questions are a specific type – a creating type. This is a fine time to discuss this type that, for our purposes, is included in the Un-Doing group. Note: In the creating type of Un-Doing questions, students are asked to create or generate different answers. Posing questions where the answer becomes the question opens up the social “space” in the classroom to allow all students the opportunity to participate and makes them accountable for the content they are learning. “Generative design centers on taking tasks that typically converge to one outcome and turning them into tasks where students can create a space of responses.” Stroup, Ares, Hurford, 2005 **Error analysis:** Taking common misconceptions and mistakes and putting them up front for students to consider and explain. **Snapshots:** Taking a snapshot out of the middle of a process or solution or activity and asking students about it.
Slide 24 	Have groups share out the one or two problems they found most interesting or compelling. Use the red flags, if any, to generate more conversation about the problems. It is less important if everyone agrees on the type of item. It is more important to discuss how the items assess student thinking.

Procedures		Notes
Slide 25	 	If they have not come up already, briefly discuss the advantages and disadvantages of these kinds of assessment items (Un-Do, Snap Shot, and Error Analysis). **One disadvantage:** • Grading: Many of the problems are more open ended. This may be a barrier for participants who have little experience or few resources to deal with more open-ended questions. In the *Closer* section, there are resources to help. **Two advantages:** • Conceptual understanding: These assessments demand more conceptual understanding than many typical textbook bare problems. • Memorization: Students cannot just memorize their way through these problems. They actually have to know what is going on.
Slide 26	 	Distribute Handout 4-Three Problem Types: How to Write (page 4-66) and blank paper. With participants in groups, have them create some assessment items on the blank paper using the How to Write handout. Participants can consult their textbooks for typical questions. They can also do a “search and rescue” as they search the textbook for examples of the three types of items. Have each group choose one or two and share out with the whole group. Collect, copy, and then hand them out so that everyone in the group will have more examples. Ask participants to label the items in some way as to suggest where they might use each item (on which assignment, test,

Procedures		Notes
		project, what time of the year, etc.)
Slide 27	**Snap Shot Problems** ▪ What are two ideas, processes, or representations that students mix up? Juxtapose them and ask which is which. ▪ What part of a large activity can you grab to assess if students got the gist of the large activity?	Trainers might show this slide while participants are sharing Snap Shot problems they created.
Slide 28	**Un-Doing Problems** ▪ Can you start with the answer? ▪ Can you start in the middle? ▪ Can you change one constraint? ▪ Can you start with a different representation? ▪ Ask students to create or invent the beginning of a problem.	Trainers might show this slide while participants are sharing Un-Doing problems they created.
Slide 29	**Error Analysis** ▪ What are the typical errors that students make? ▪ Pose an incorrect solution ▪ Ask students to explain what went wrong. ▪ Sometimes show the incorrect process, sometimes just show the incorrect answer	Trainers might show this slide while participants are sharing Error Analysis problems they created.
Slide 30	**The Assessment Principle** Assessment should become a routine part of the ongoing classroom activity rather than an interruption. NCTM's *Principles and Standards for School Mathematics* (2000)	Close by discussing the Assessment principle. How can these different problem types help teachers with on-going assessment?

Procedures		Notes
Slide 31	**Another Example** The following slides begin with a different stem problem based on percents	Trainers can use the rest of these slides as more examples of the three problem types based on a different TAKS stem. The stem is based on percents.
Slide 32	**A Typical Textbook Item** The cost of Matt and Natalie's dinner was $27.35. They want to leave a 20% tip. What should they leave for a tip?	A typical item. Students are asked to find percents. Do students need to be able to do this? Of course. What other kinds of questions could we ask so that students learn more about percents?
Slide 33	**What might it look like ….** The cost of Matt and Natalie's dinner was $27.35. They want to leave a 20% tip. What should they leave for a tip? as an un-doing problem?	
Slide 34	**Un-Doing Example** Some folks choose to pay different tips - some choose 10%, others 15%, others 20%, and some may even pay 30% for excellent service. If the tip paid was $5.00, what might the price of the meal have been?	In mathematics, we often do something and then un-do it. We multiply, we factor. We add, we subtract. Here, instead of giving students an amount and asking them to find the percent to find the tip, students are given the tip and the percents and asked to find the starting amount.

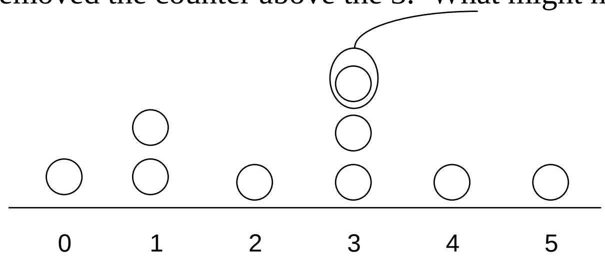
	Procedures	Notes
Slide 35	**Un-Doing Example** Miquel had the following written on his quiz. 10% of 27.35 is 2.74 2 x 2.74 = 5.48 $5.48 What might have been the question he answered?	Did this and the previous example seem more open? Might this possibly allow the teacher to see a greater variety of solutions and strategies that when discussed can build strength in connecting the different approaches? How many different answers are possible for each? If participants do not mention it, pose the question, “Does the context have to be tips? Could students use and compare several different contexts?” (Taxes, how much of a computer file you have downloaded, sale prices)
Slide 36	**What might it look like ….** The cost of Matt and Natalie’s dinner was $27.35. They want to leave a 20% tip. What should they leave for a tip? as a snap shot problem?	
Slide 37	**Snap Shot Example** The meal cost $27.35. Bethany wanted to pay a 25% tip. She knows that 10% of 27.35 is about 2.70. She took half of 10% to find that 5% is about 1.35. How can she use these to find the 25% tip? What would the tip be?	Another way to open up the discussion is to take a procedure or process and ask about selected parts of it. In this way students have to think about someone else’s strategies. This is a way to see if students really know what is going on, the reasons for the steps, or if they are stuck in one way of “doing” it.

	Procedures	Notes
Slide 38	What might it look like …. The cost of Matt and Natalie's dinner was $27.35. They want to leave a 20% tip. What should they leave for a tip? as a error analysis problem?	
Slide 39	Error Analysis Example Abigail: 27.35 rounds to 20. 10% of 20 is 2 20% of 20 is 4 $4.00 tip of 20% Craig: 27.35 rounds to 30. 10% of 30 is 3 5% of 30 is 1.5 $1.50 tip of 15% Cameron: 10% of 27.35 is about 0.25 $0.25 tip of 10%	A technique to help students is to determine why others might choose an incorrect answer. Students are asked to examine erroneous solutions and find the error(s). Ask participants to consider the common errors of their own students. Suggest that instead of only re-teaching the correct method, they might also consider asking students to analyze the common errors made in their own classrooms.
Slide 40	TAKS Item (7th grade 2003) The cost of Matt and Natalie 's dinner was $27.35. They want to leave a 20% tip. Which of the following is closest to the amount of the tip they want to leave? A. $4.00 B. $4.50 C. $5.00 D. $5.50	Here is a released item from the 2003 7th grade TAKS. Ask participants to consider the various problems at which they have just looked, and how these problems might have prepared students for this item. Trainers might also compare the original, typical question's place on the pyramid with the locations of the other questions.

Sample Assessment Items – What's the Difference?

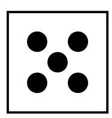
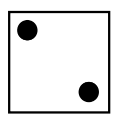
Yesterday we rolled a pair of dice, found differences, and represented the data.

1. Bonnie correctly removed the counter above the 3. What might have been rolled?



2. PB & J played the Remove One game and they got the following line plot.

		X		X	
		X		X	
		X		X	X
	X	X		X	X
X	X	X		X	X
0	1	2	3	4	5

Based on their plot, did they roll   ? Why or why not?

3. Could Group 8 have gotten the following line plot? Explain why or why not?

X					
X	X				
X	X	X			
X	X	X	X	X	
X	X	X	X	X	X
1	2	3	4	5	6

4. Design a line plot so that the following experimental probabilities are represented for the differences of rolling 2 dice.

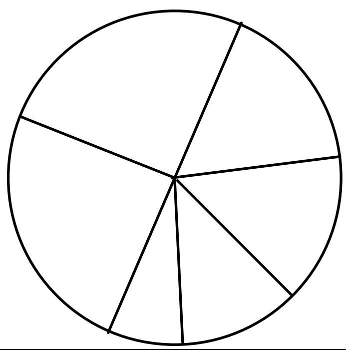
$$p(0)=\frac{1}{10},\ p(1)=\frac{2}{10},\ p(2)=\frac{2}{10},\ p(3)=\frac{3}{10},\ p(4)=\frac{1}{10},\ p(5)=\underline{\qquad}$$

5. Create two different line plots that represent that the experimental probability of rolling a difference of 2 was 1/12.

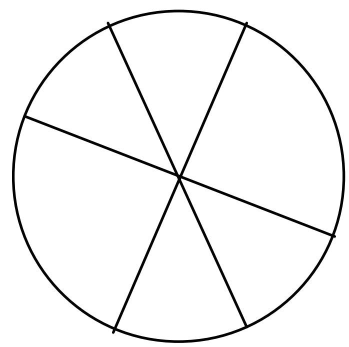
6. Which graph below would best represent the data on the line plot?

		X		X	
		X	X	X	X
	X	X	X	X	X
X	X	X	X	X	X
0	1	2	3	4	5

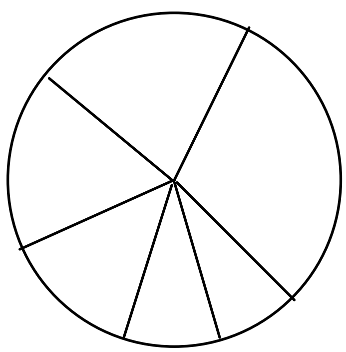
a.



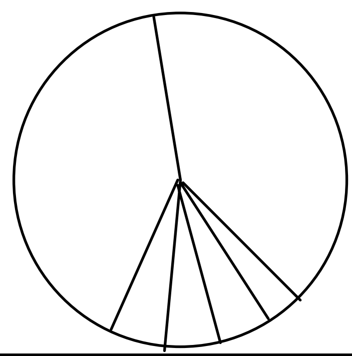
b.



c.



d.

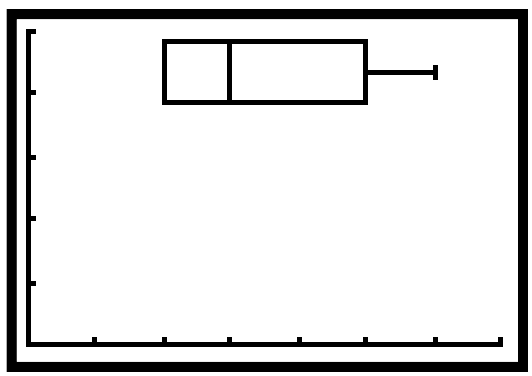


Un-Doing

Snap Shot

Error Analysis

Create a six-value data set that would produce the following graph:



Find a pair of numbers, if possible, whose lcm is

a. the larger of the pair of numbers
b. the smaller of the pair of numbers
c. the product of the pair of numbers
d. a number between the larger of the pair and the product of the pair.

Generate two 5-value data sets that have a mean of 12.

The mean number of children in 6 families is 5 children.

a. What is the total number of children in the six families?
b. Other than the six families of 5 children, create a set of families that fits this information.
c. Would another classmate's set of families for question *b* have to be the same as yours?

Connected Mathematics Project

a. Give three pairs of numbers, each consisting of a positive and a negative number, with a difference of 100.

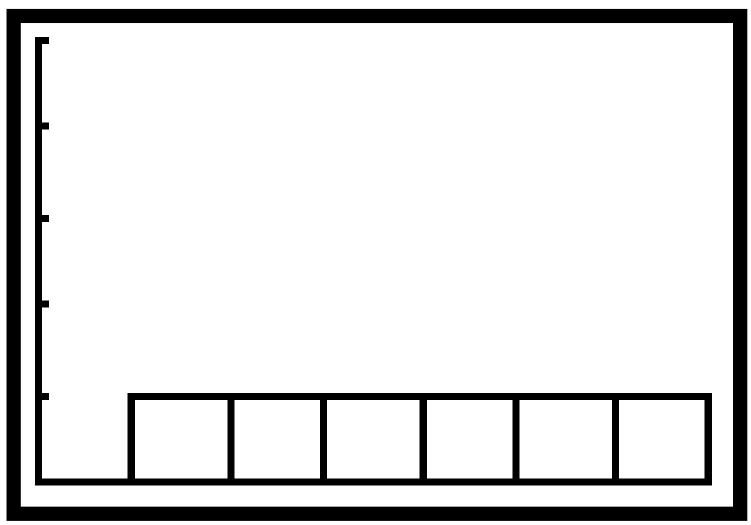
b. Give three pairs of negative numbers with a difference of 50.

Mathematics in Context

Make up a context problem that fits with $10\frac{1}{2} \div \frac{3}{4}$

Mathematics in Context

Find a six-value data set that would produce the following graph:



Create a context and a 6-value data set where the mean is a better *average* of the data than the median or the mode. Discuss why.

Jana wants to be able to have 30 completely different outfits consisting of pants, a shirt, and shoes. Create 2 different possible wardrobes for her.

Create 3 different spinners so that the probability of landing on blue is 1/2.

Create a context and a 6-value data set where the mode is a better *average* of the data than the median or the mean. Discuss why.

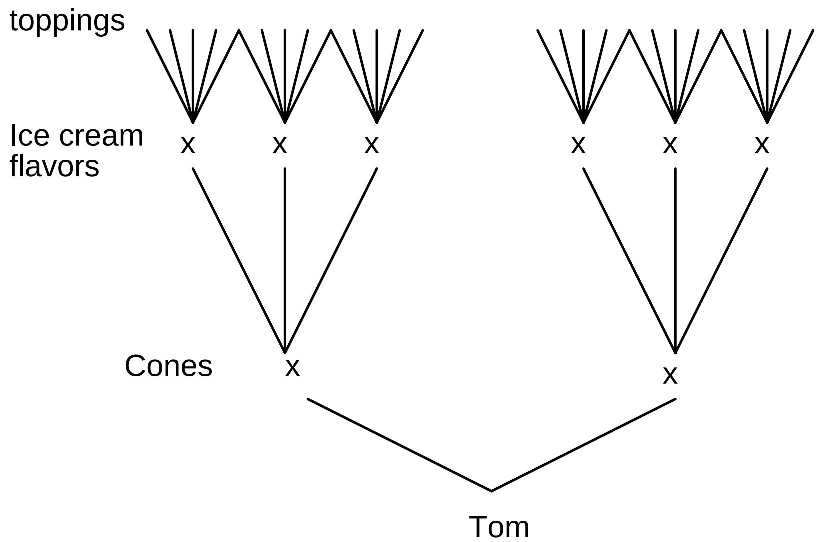
Write three new "frog problems" – one that you think is easy, one that is more difficult, and one that is very difficult. Describe how to solve each problem.

Mathematics in Context

If the probability of Abby winning the drawing at the school carnival is 1/30, name 3 possible combinations of tickets she bought and total tickets sold.

Create a spinner so that the chance for getting hot dogs is 12.5%, the chance for pizza is 37.5%, the chance for hamburgers is 25%. The last choice is ham sandwiches. What is the chance of ham sandwiches?

Adapted from *Connected Mathematics Project*



Tom is choosing an ice-cream cone.

1. How many kinds of cones does he have to choose from?
2. How many ice-cream flavors does he have to choose from?
3. How many toppings does he have to choose from?

Design a line plot so that the following probabilities are represented for the differences of rolling 2 dice.

$$p(0)=\frac{1}{10},\ p(1)=\frac{2}{10},\ p(2)=\frac{2}{10},\ p(3)=\frac{3}{10},\ p(4)=\frac{1}{10},\ p(5)=\_\_\_$$

A bag contains several marbles. Some are red, some are white, and some are blue. You count the marbles and find the theoretical probability of choosing a red marble is 1/5. You also find the theoretical probability of choosing a white marble is 3/10.

a. What is the least number of marbles that can be in the bag?
b. Can the bag contain 60 marbles? If so how many of each color does it contain?
c. If the bag contains 4 red marbles and 6 white marbles, how many blue marbles does it contain?
d. How can you find the probability of choosing a blue marble?

Connected Mathematics Project

About how many feet of fencing are needed to enclose a rectangular garden with a 6 ft long side and a 10 ft long diagonal?

Abby wrote:

$$6^2 + 10^2 = 136$$

$$\sqrt{136} = \sqrt{100} + \sqrt{36} = 16$$

$$2(16) + 2(6) = 32 + 12 = 44$$

44 ft. of fencing

If you have a rectangle that is 2 cm by 3 cm and you dilate it by a scale factor of 4, what is the area of the new figure?

Joanne showed the following work:

$$2 \times 3 = 6$$

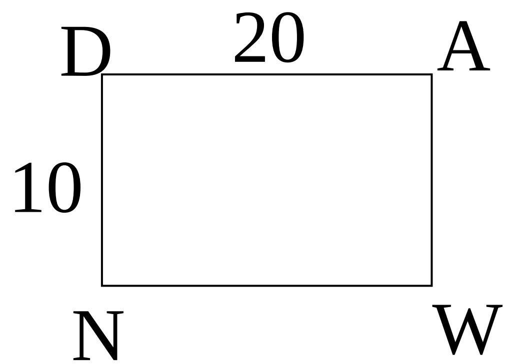
$$6(4) = 24$$

$$24 \text{ cm}^2$$

What do you say to Joanne?

A hat had two blue cubes, four yellow cubes, and six red cubes. Ralph says that the probability the cube is blue is 12/4. Eleanor says that 12/4 is impossible. Who is correct? Explain.

The rectangle DAWN was enlarged by a scale factor of 2:3 to form a new similar rectangle COLD. What is the perimeter of COLD?



Justin’s work is below. What do you say to Justin?

10 to 15, 20 to 30

so (15) (30) = 450

450 cm

Aran knows that if you roll a number cube once, there is a 50% chance of getting an even number. He says that if you roll a number cube twice, the chance of getting at least one even number is doubled. Is he correct? Explain.

Connected Mathematics Project

Carrie wonders what would happen to the figure she made if she multiplied the coordinates by –3. This is what some of her classmates think.

John says, “It would be upside down and three times as big.”

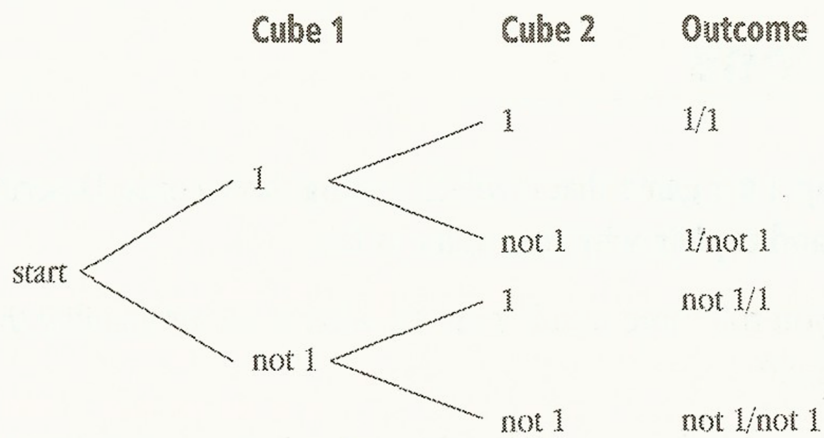
Mauri says, “I guess it would be nine times as big.”

Emily says, “The coordinates of the top point which were (2,3) would be (9,8).”

Reflect: Comment on the thinking of each of Carrie’s three classmates.

Adapted from *Mathematics in Context*

9. Tricia wants to determine the probability of getting two 1s when two number cubes are rolled. She made a counting tree and used it to list the possible outcomes.



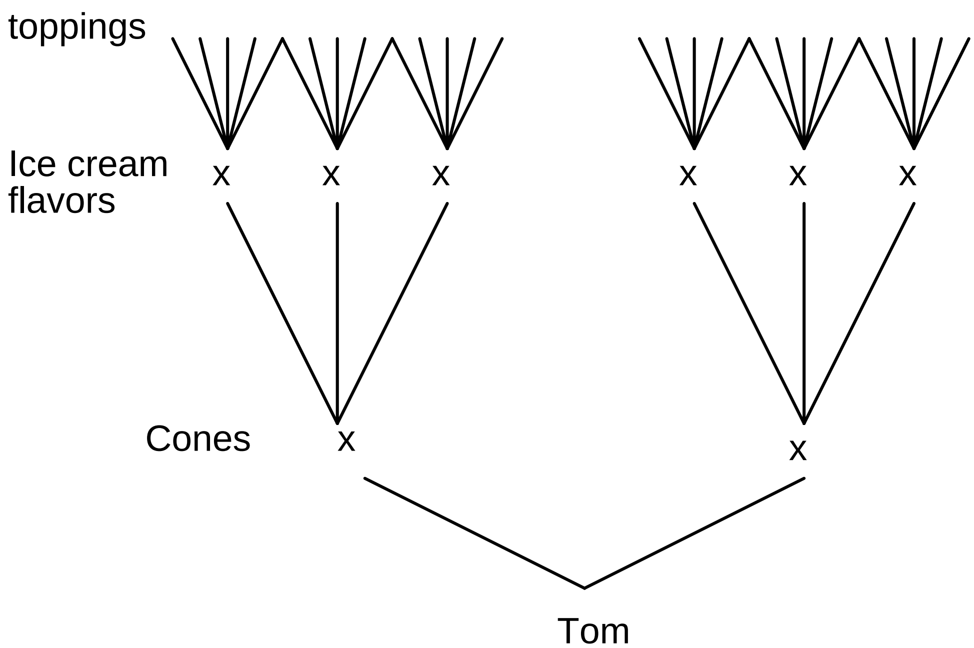
She says that, since there are four possible outcomes, the probability of getting 1 on both number cubes is $\frac{1}{4}$. Is Tricia right? Why or why not?

Connected Mathematics Project

Michael said that the mean, median, and the mode of the following data is 7. What do you think?

3, 5, 6, 8, 9, 11

How many different ice-cream cones are possible?



Tom wrote:

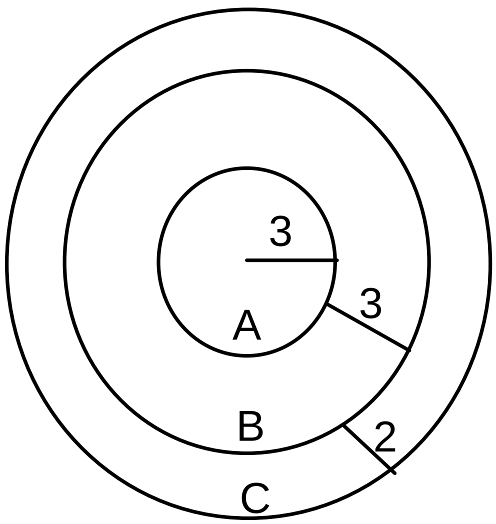
$2 + 3 + 4 = 7$

What do you tell him?

Gil, Lashonda, and Greg are discussing how they might shrink a triangle.
Gil says, "You could multiply the coordinates by – 2,"
Lashonda says, "That is not right. You would have to multiply the coordinates by ½."
Greg says, "Why not multiply by – ½?"
Which of these statements do you think is/are correct?

Adapted from *Mathematics in Context*

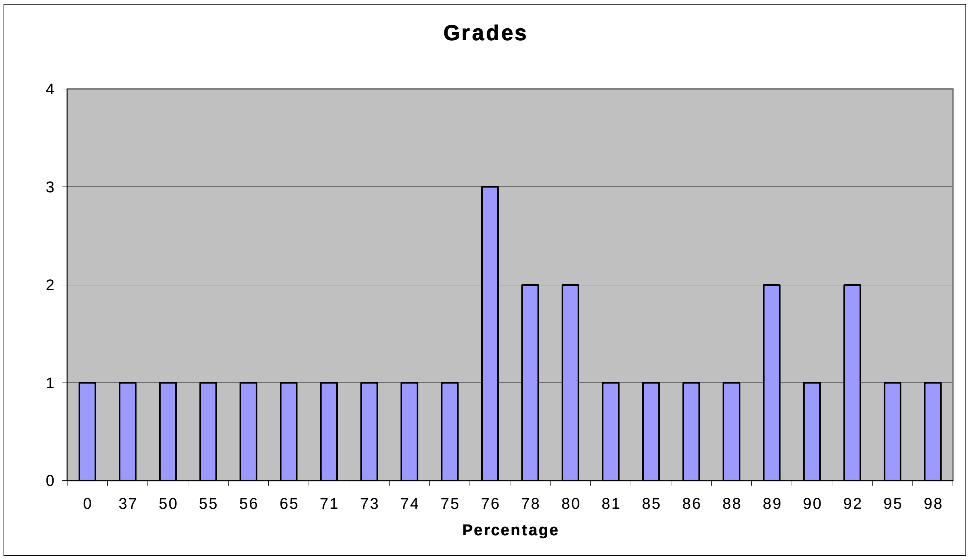
Robin found the probability of hitting section A in the dart game below. Is she correct?

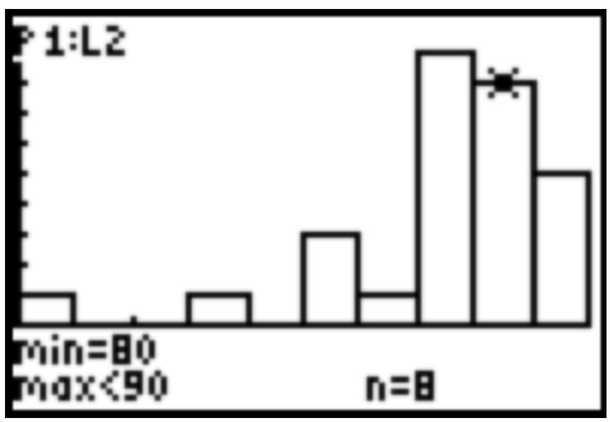


$\frac{3^2\pi}{4^2\pi} = \frac{9\pi}{16\pi} = \frac{9}{16}$, so the probability of hitting the A section is 9/16.

In the Clue[R] board game, players try to solve a murder mystery. To win, a player must identify the murderer, the murder weapon, and the room in which the murder was committed. Amadi claims that there are 118 possible solutions to the game. His sister Ayana, who has never played the game, says she can't believe this is true. Why does she say this?

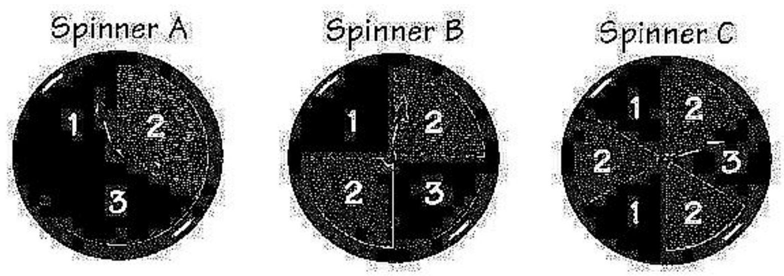
Connected Mathematics Project





Which of the graphs is more helpful to the teacher to see the grade layout quickly?

Molly designs a game for a class project. She makes the three spinners shown. She tests to see which one she likes best for her game. She spins each pointer 20 times and writes down her results, but she forgets to record which spinner gives which set of data. Match each spinner with one of the data sets. Explain your answer.



First data set: 1, 2, 3, 2, 1, 1, 2, 1, 2, 2, 2, 3, 2, 1, 2, 2, 2, 3, 2, 2

Second data set: 2, 3, 1, 1, 3, 3, 3, 1, 1, 2, 3, 2, 2, 2, 1, 1, 1, 3, 3, 3

Third data set: 1, 2, 3, 3, 1, 2, 2, 2, 3, 2, 1, 2, 2, 2, 3, 2, 2, 3, 2, 1

Connected Mathematics Project

	50	.....
.....	1500	60
7	.....	14

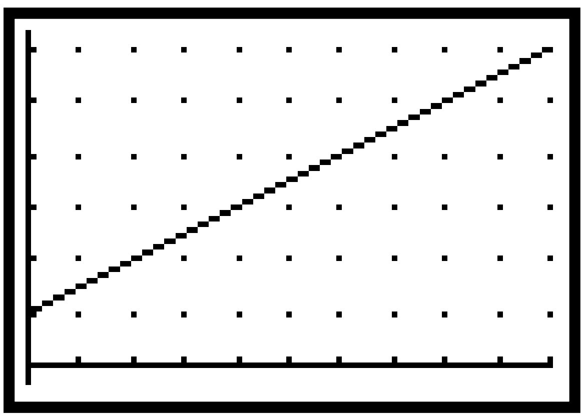
a. Copy and complete the area model in your notebook and fill in the missing numbers.

b. What multiplication problem fits problem a? Use the area model to find the answer.

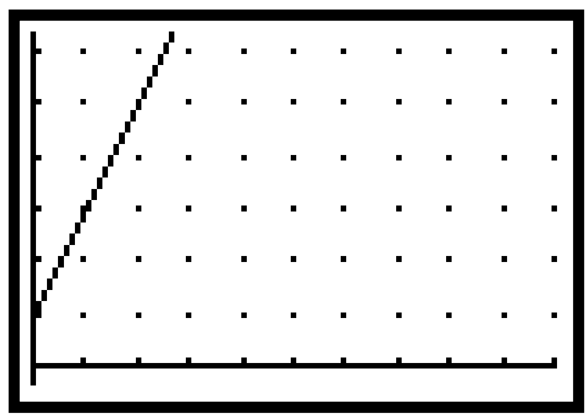
Mathematics in Context

Yesterday we walked around in front of the motion detector. Which person below was walking faster? Why?

Amanda



Jessica



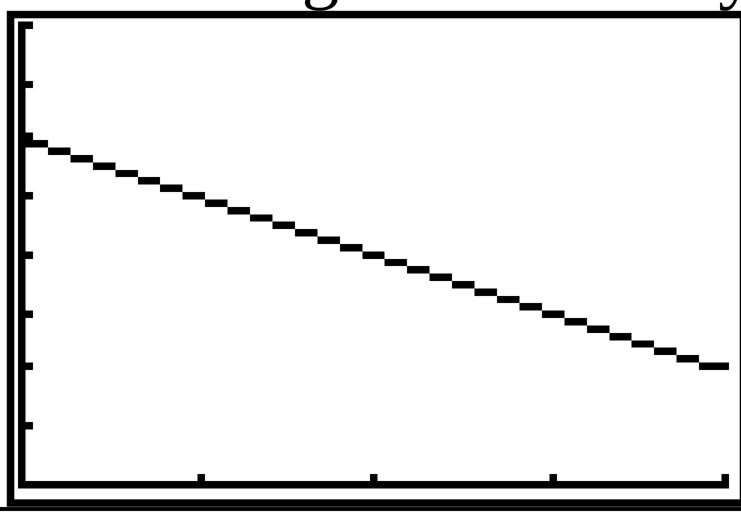
A math teacher at Springfield Middle School would like to have calculators for her class. The school store offers calculators for $7 each. She asked her sixth-grade students to calculate the total price for 32 calculators. Here is the strategy of one of her students. Describe the steps Sondra used for her ratio table.

Sondra:

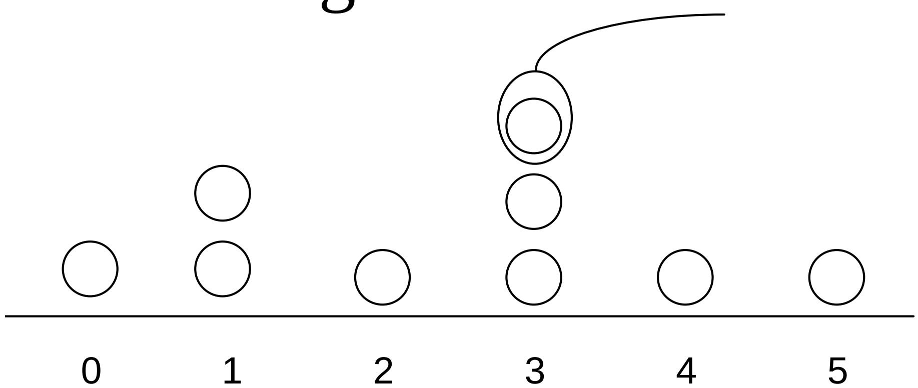
Number of Calculators	1	10	20	30	2	32
Price (in dollars)	7	70	140	210	14	224

Adapted from *Mathematics in Context*

Yesterday we walked around in front of the motion detector. What direction was the person walking below? Why?



In the *What's the Difference* game, Bonnie removed the counter above the 3. What might have been rolled?



Three Problem Types – How to Write

Snap Shot Problems:

What are two ideas, processes, or representations that students mix up?
Juxtapose them and ask which is which.
What part of a large activity can you grab to assess if students got the gist of the large activity?

Un-Doing Problems:

Can you start with the answer?
Can you start in the middle?
Can you change one constraint?
Can you start with a different representation?
Ask students to create or invent the beginning of a problem.

Error Analysis

What are the typical errors that students make?
Pose an incorrect solution.
Ask students to explain what went wrong.
Sometimes show the incorrect process, sometimes just show the incorrect answer.

Activity: The Power of Creating

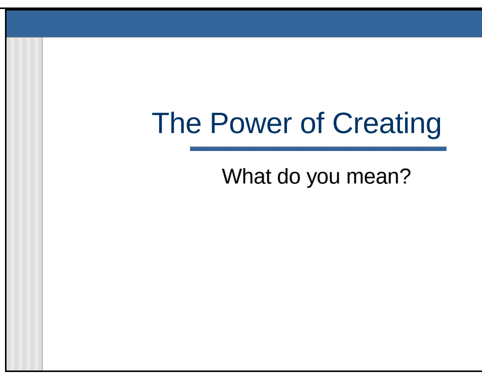
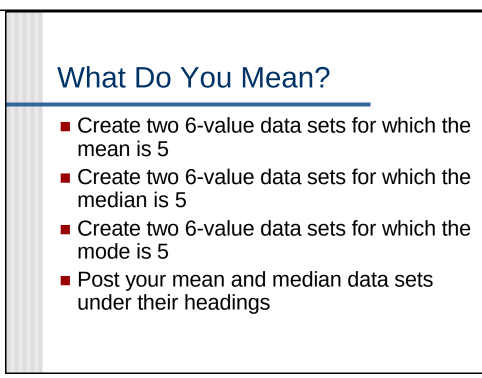
Overview: Participants explore the power of “creating” problems.

Materials: Three places to record answers (chart paper or white board space)
[PowerPoint: The Power of Creating]

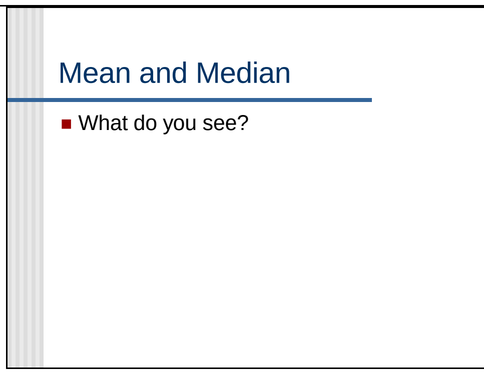
Grouping: Partners

Time: 30 minutes

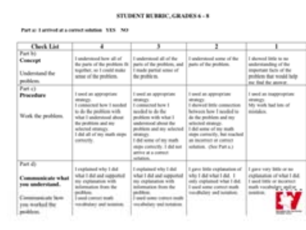
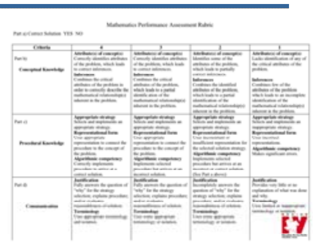
Lesson: Distribute the PowerPoint copies to participants to help them focus on the important ideas from the PowerPoint presentation as they take notes. Show the PowerPoint presentation *Power of Creating*. Use the following notes pages to elaborate on the content of each slide.

<table>
<tr><th colspan="2">Procedures</th><th>Notes</th></tr>
<tr><td>Slide 1</td><td>
</td><td>In the set of problems that the participants classified as Un-Doing problems, there was a subset of those assessment items that are uniquely designed to teach while assessing. This subset is referred to here as “Creating Problems” because the students are asked to create or generate a response that contributes to a space of responses that as a whole increase students’ depth of understanding.</td></tr>
<tr><td>Slide 2</td><td>
</td><td>Explain the 4 steps. Participants should work together in partners to understand the problem and find the 3 data sets. Then have one partner post the “mean” data set and the other partner post the “median” data set in the appropriate places. (Do not yet post the “mode” data sets.)

As Trainers circulate, they should encourage some groups to be clever in their choices. For instance, ask groups if they can make a data set where the mean is really obvious. Ask some to create a data set where the mean is not obvious at all. Ask some to use “complicated” numbers (fractions, decimals, really large, really small, negative, etc.)</td></tr>
</table>

Procedures	Notes
	For a different twist, ask participants to find a five-value data set, instead of a six-value data set. On the board or on chart paper, have two locations, one labeled “Mean” and the other “Median”. Since “Mode” lists will be posted later, also have that location planned.
Slide 3  	**Focus on the “mean” data set.** Ask: How did you find your numbers? Where did you start? What were you thinking? Did anyone have a specific strategy that you think will work every time? Did anyone guess and check? Would your strategy work if I had asked for a five-value data set, instead of a six-value data set? Have participants share their strategies. Ask: What do these sets of numbers have in common?” (They all have a mean of 5, they all sum to 30, …) **Focus on the “median” data set.** Ask: What do these sets of numbers have in common?” (They all have a middle number that is 5, they do not all have the same sum, …) **Focus on the “median” data set.** Ask: What do these sets of numbers have in common?” (They all have a middle number that is 5, they do not all have the same sum, …) **Focus on the “mode” data set.** Ask: What do you think we will see when we post all of your “mode” data sets? Talk with your partner about what you think you will see.”

Procedures	Notes
	After the partner discussion, have 2 participants share what they discussed.
Slide 4 **Mode** ▪ What do you think you will see?	Have the participants post their "mode' data sets. Ask how their predictions compare to the results. Note that the TEKS ask students to identify measures of central tendency from a set of data. Here students are asked to un-do this process. They are asked to find a set of data given the mean, median, or mode. Why? This process helps students clarify in their minds the relationship between the data and the measure of central tendency. It also helps them solidify how to find those measures from a set of data because they have to do that in the midst of a larger activity. If students can "un-do," they will certainly be able to "do" even better.
Slide 5 **Summative** ▪ Find one data set for which the mean, median, and mode are 5. You may choose from the class lists or write your own.	Ask participants to find one data set for which the mean, median, and mode are 5. They can choose one from the lists or write their own. This is a summative kind of task.
Slide 6 **The Power of Creating** ▪ Powerful by themselves ▪ More powerful when put together to look at commonalities.	Discussion: Creating problems are powerful by themselves, but even more powerful when generated answers are compared and commonalities are found. This is a participant discussion – get out of the content and broaden the discussion to creating problems in general.

Procedures		Notes
Slide 7	**The Power of Creating** Generative design centers on taking tasks that typically converge to one outcome and turning them into tasks where students can create a space of responses. **Stroup, Ares & Hurford, 2005**	Ask participants to comment on this description of Creating Problems. What is a “space of responses”? How does this “space of responses” enhance learning?
Slide 8	**Open Ended Questions**  http://books.heinemann.com/math/	If participants need support in using open ended questions, this site has excellent resources that are free for teachers.
Slide 9	**Rubrics**  http://www.mathbenchmarks.org/rubric.htm	Grading help can be found at the Region 4 site: http://www.mathbenchmarks.org/rubric.htm. This is the student rubric for grades 6-8.
Slide 10	**Rubrics**  http://www.mathbenchmarks.org/rubric.htm	This is the more detailed teacher rubric.

Procedures		Notes
Slide 11	Other Examples: ▪ Create 3 different spinners so that the probability of landing on blue is 1/2 ▪ Jana wants to be able to have 30 completely different outfits consisting of pants, a shirt, and shoes. Create 2 different possible wardrobes for her.	These are just two examples of creating problems. Ask participants to brainstorm others and share.

Activity: Closing Activity – Assessment Should Drive Instruction

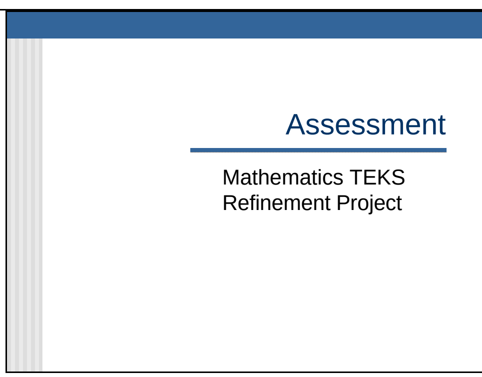
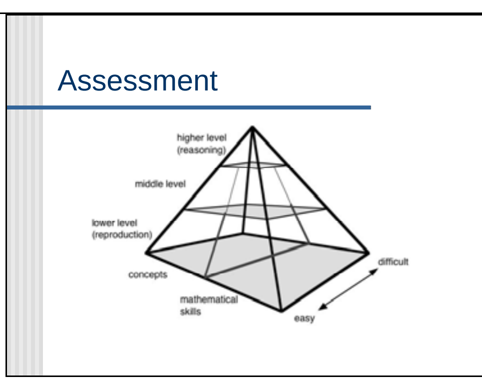
Overview: Participants will discuss common definitions of Diagnostic, Formative, and Summative assessment. Formative assessment, being defined as assessment that informs instruction, should dominate and be an integral part of mathematics classrooms. Several definitions are provided for discussion. Participants end the section by participating in an assessment scavenger hunt, looking for assessment items that appear all over the Assessment Pyramid.

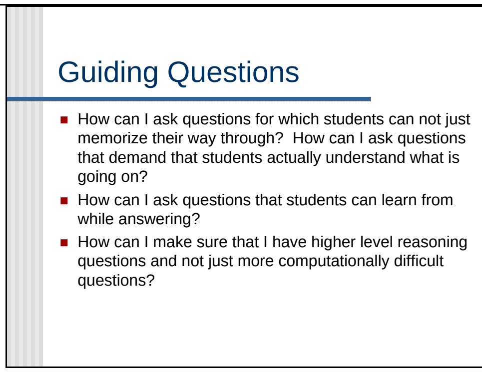
Materials: PowerPoint: Closer
Current textbook(s) or local curriculum, optional

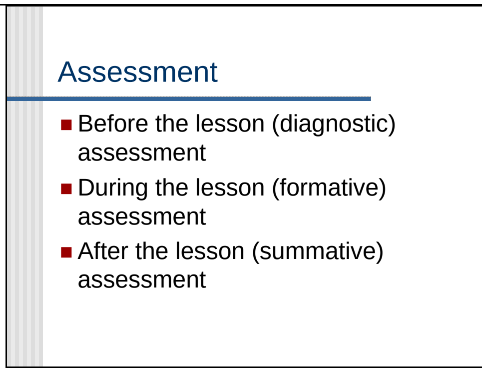
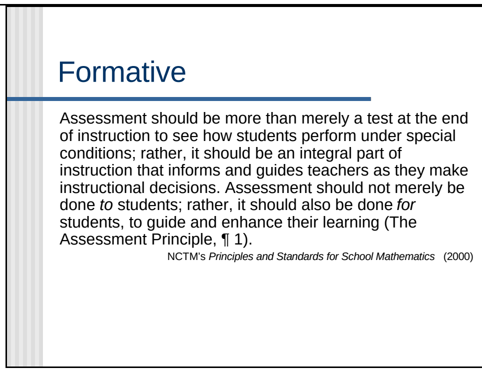
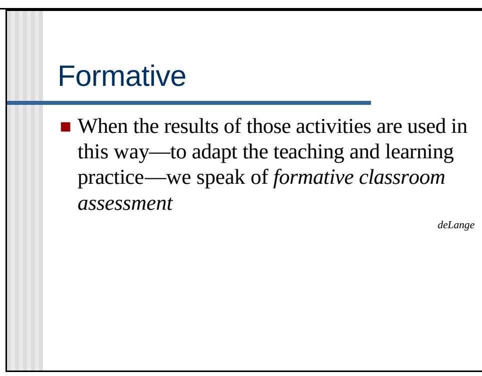
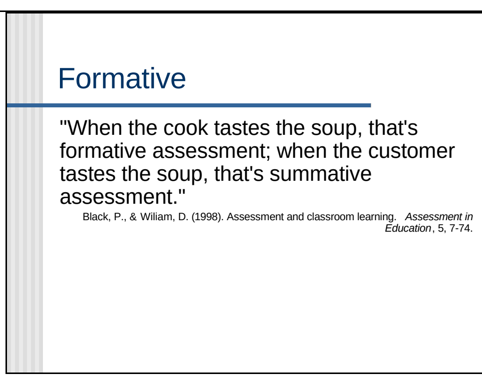
Grouping: Tables of 4

Time: 30 minutes

Lesson: Distribute the PowerPoint copies to participants to help them focus on the important ideas from the PowerPoint presentation as they take notes. Show the PowerPoint presentation Closer. Use the following notes pages to elaborate on the content of each slide.

<table>
<tr><th colspan="2">Procedures</th><th>Notes</th></tr>
<tr><td>Slide 1</td><td>
</td><td>Briefly recall the Assessment Pyramid and the guiding questions. Ask participants to reflect on their current assessment practices. How do they use assessment to inform instruction?</td></tr>
<tr><td>Slide 2</td><td>
</td><td>Ask participants to reflect on the assessment items they have seen in the presentation and to reflect on these guiding questions.
Do you have a clearer picture of the difference between levels of reasoning and levels of difficulty and how they interact?
What items exemplify the kind of question for which students</td></tr>
</table>

Procedures	Notes
	cannot just memorize their way through? • What items exemplify the kind of question from which students can learn while answering?
Slide 3 	Discuss the descriptions of diagnostic, formative, and summative assessment. Whichever it is called, assessment should be a routine part of the classroom, not an interruption. Formative – assessment that helps teachers make decisions about the content or form of instruction. Summative – used to judge students' attainment. (Principles and Standards, p.24). "Some identify classroom assessment with formative assessment. We agree with Biggs (1998) that formative assessment and summative assessment are not mutually exclusive, as suggested by Black and Wiliam (1998). Their argument is that feedback concerning the gap between what *is* and what *should be* is regarded as formative only when comparison of actual and reference levels yields information that is then used to alter the gap. But if the information cannot lead to appropriate action, then it is not formative. Summative assessment in the form of end-of-year tests gives teachers the proof of how well they handled the formative assessment, assuming that the underlying philosophy is coherent and consequent. The differences in formative and summative assessment within the classroom are more related to timing and the amount of cumulation than anything else. Needed for both, of course, is that the assessment is criterion-referenced, incorporating the

Procedures		Notes
		curriculum and resulting in aligned assessment." (DeLange "Framework", p. 4)
Slide 4		Use the rest of the slides to clarify any terms as they come up in the discussion.
Slide 5		
Slide 6		
Slide 7		

Procedures		Notes
Slide 8	**Summative** ■ Judging students' progress/ attainment	
Slide 9	**Degrees of Openess** ■ Closed Task - one correct answer, one route to arriving at that answer ■ Open-Middled Task - one correct answer but many routes to arriving at that answer. ■ Open-Ended Task - several correct answers and many routes to arriving at those answers.	
Slide 10	**Assessing Mathematical Skills** "An assessment task that focus primarily on mathematical skills gives students a chance to apply a well-practiced and important procedure or algorithm." Mathematics Assessment: A Practical Handbook, NCTM, 2000	
Slide 11	**Assessing Mathematical Skills** "These tasks are usually- ■ routine; ■ short; ■ based upon recalling a well-known procedure; ■ cast in a simple context or no context at all; ■ focused on a single correct answer."	

	Procedures	Notes
Slide 12	**Assessing Conceptual Understanding** "Assessment tasks that focus primarily on mathematical concepts give students a chance to apply a concept in a new situation, to reformulate it, and to express it in their own terms. These tasks probe the understanding of an idea." Mathematics Assessment: A Practical Handbook, NCTM, 2000	
Slide 13	**Assessing Conceptual Understanding** "They are usually- ▪ non-routine; ▪ short; ▪ based upon reconstruction, rather than memorization ▪ cast in a context; ▪ focused on representation and explanation of the solution."	
Slide 14	**Assessing Problem Solving** "An assessment task that focuses primarily on mathematical problem solving gives students a chance to select and use problem-solving strategies." Mathematics Assessment: A Practical Handbook, NCTM, 2000	
Slide 15	**Assessing Problem Solving** "Problem solving tasks are usually- ▪ non-routine; ▪ long; ▪ Predicated on the high-level use of facts, concepts, and skills ▪ cast in a context; ▪ focused on the students' abilities to develop and use strategies to solve."	

Procedures		Notes
Slide 16	**The Assessment Principle** Assessment should become a routine part of the ongoing classroom activity rather than an interruption. NCTM's *Principles and Standards for School Mathematics* (2000)	How has this presentation affected your view of assessment?
Slide 17	**Scavenger Hunt** Using your current text and assessments ▪find examples all over the pyramid, ▪justify your examples, ▪share with the group.	

Resources

deLange, Jan. FRAMEWORK FOR CLASSROOM ASSESSMENT IN MATHEMATICS, Freudenthal Institute & National Center for Improving Student Learning and Achievement in Mathematics and Science, September 1999.

Lappan, Glenda, James Fey, Susan Friel, William Fitzgerald, and Elizabeth Phillips. *The Connected Mathematics Project*. Neeham, Massachusetts. Pearson Prentice Hall, 2004.

National Center for Research in Mathematical Sciences and Freudenthal Institute. *Mathematics in Context*. Chicago, Ill.: Encyclopaedia Britannica Educational Corp., 2006.

National Council of Teachers of Mathematics (NCTM). *Mathematics Assessment: A Practical handbook For Grades 6-8.* Reston, Va.: NCTM, 2000.

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National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM, 1989.

National Council of Teachers of Mathematics (NCTM).. *Principles and Standards for School Mathematics*. Reston, Va.: NCTM, 2000.

Romberg, Thomas, A. ed., *Standards-Based Mathematics Assessment in Middle School: Rethinking Classroom Practice*; New York: Teachers College Press. 2004.

Stroup, W., Ares, N., & Hurford, A., (2005). A Dialectic Analysis of Generativity: Issues of Network Supported Design in Mathematics and Science. *Journal of Mathematical Thinking and Learning*,7(3), 181–206.