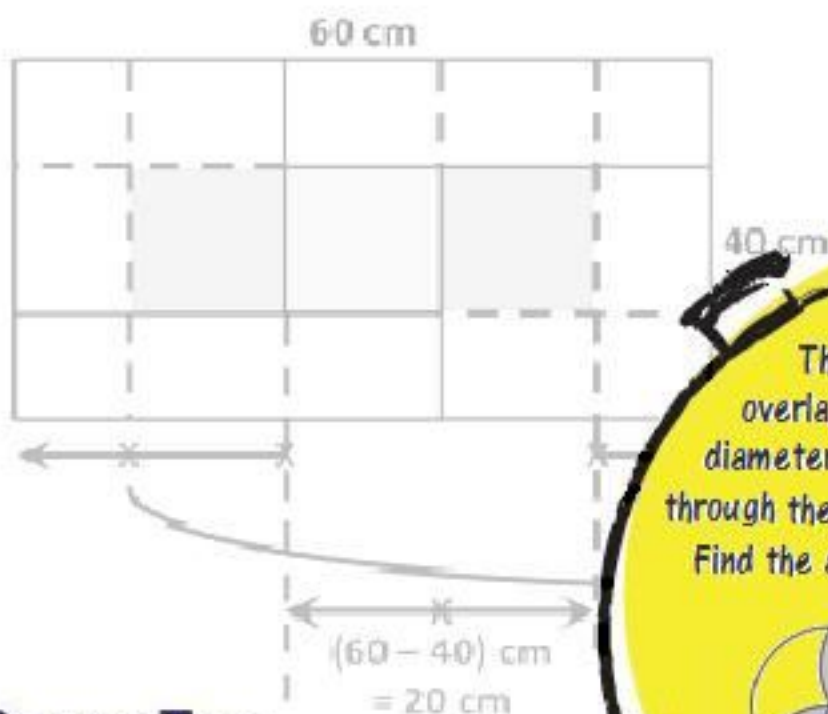


Spatial Visualisation

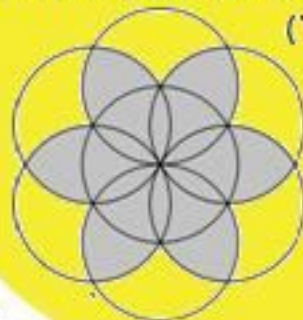
A Problem-solving Tool

for Challenging Problems
in Upper Primary Mathematics
(Primary 5 & 6)



The figure is formed by overlapping 7 circles of 14 cm diameter. The inner circle passes through the centres of the 6 outer circles. Find the area of the shaded regions.

(Take $\pi = \frac{22}{7}$)



Page 137, Qn 9

Solve in 5 minutes!

Sunny Tan

Mastering Heuristics Series

Handbook for discerning parents

Spatial Visualisation

A Problem-solving Tool
for Challenging Problems
in Area and Perimeter
(Primary 5 & 6)

Sunny Tan

Maths Heuristics Private Limited

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* **Chapter 2** Completely involves Circles Scenarios, hence entire chapter is for Primary 6 only.

** **Chapters 3, 5, 6 and 7** Problems in Examples and Let's Apply sections are grouped into Non-Circles and Circles Scenarios.

PREFACE

Heuristics in Primary Maths Syllabus

Heuristics is a specialised mathematical problem-solving concept. Mastering it facilitates efficiency in solving regular as well as challenging mathematical problems. The Ministry of Education in Singapore has incorporated 11 Problem-solving Heuristics into all primary-level mathematics syllabuses.

Learning Heuristics Effectively

However, the 11 Problem-solving Heuristics are not taught systematically; they have been dispersed into the regular curriculum. This not only makes it difficult for students to pick up Heuristics skills, but can also make mathematics confusing for them. For us parents, it is difficult to put aside the regular-syllabus mathematical concepts we were brought up on to re-learn Heuristics, much less teach our own children this new concept.

Take the Heuristic technique of Algebraic Equations, for instance. Parents and educators may attempt to teach their children to solve complex mathematical questions using Algebraic Equations. This will only confuse their children as many are too young to grasp the abstract concept. Instead, other Heuristics techniques should be used, according to current primary-level mathematics syllabus.

These and other challenges were what I observed firsthand during my years as a mathematics teacher, and provided me the impetus for my post-graduate studies, mathsHeuristics™ programmes and the Mastering Heuristics Series of guidebooks.

About Mastering Heuristics Series

This series of books is a culmination of my systematic thinking and experience, supported by professional instructional writing and editing, to facilitate understanding and mastery of Heuristics. I have neatly packaged Heuristics into main techniques (series of guidebooks) and mathematical scenarios (chapters within each guidebook). For each mathematical scenario, I offer several examples, showing how a particular heuristics technique may be applied, and then explaining the application in easy-to-follow steps and illustrations – without skipping a beat.

This particular guidebook in the series deals specifically with *Spatial Visualisation Techniques for Area and Perimeter* – the use of the mind's eye to manipulate given shapes to solve problems in Area and Perimeter. Mastery of this technique is necessary for solving especially-complex problems involving composite shapes. Spatial Visualisation is a very powerful problem-solving technique because it helps students literally see the solution to the problem posed.

The Mastering Heuristics Series provides a comprehensive guide to Heuristics. While each guidebook introduces parents to how Heuristics works, students have the opportunity to see the technique applied in different scenarios and to get in some practice. For students enrolled in mathsHeuristics™ programmes, each guidebook serves as a great companion, while keeping parents well-informed of what their children are learning.

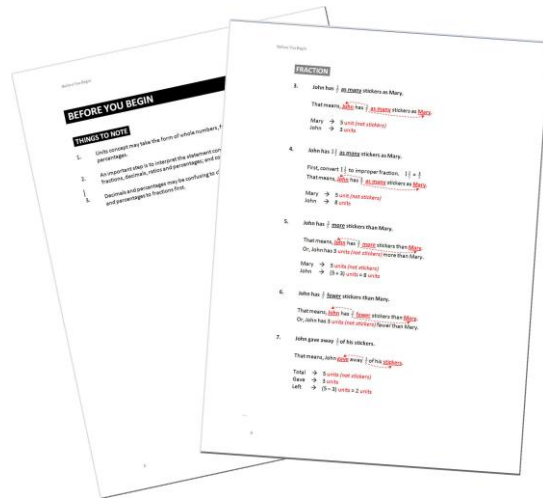
Sunny Tan
February 2014

HOW TO USE THIS BOOK

BEFORE YOU BEGIN

This chapter instills basic but important steps and truths in the heuristics technique that must be applied across every question in the guidebook. This helps to standardise the given information for easy application of the technique being taught.

In this guidebook on Spatial Visualisation, the steps include being familiar with parts and formulae of basic and derivative shapes.



CHAPTERS AND SECTIONS

Various scenarios are neatly separated into different chapters and sections. This allows the heuristics technique to be learnt and applied in a focused manner.

EXAMPLES

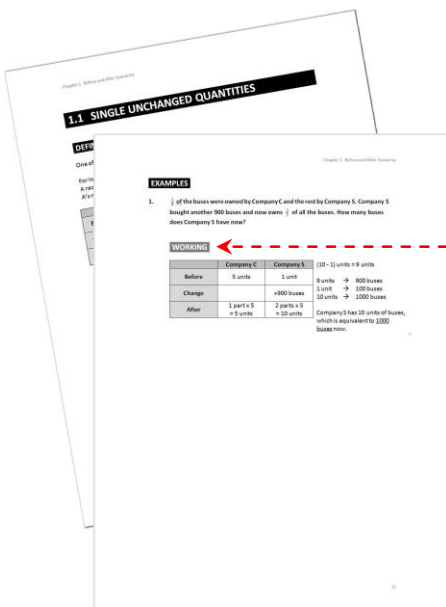
Each example of heuristics application comes with “Working” and “Explanation”, and includes “Confusion Alert” and “Alternative” boxes.

In some chapters, the examples are grouped as follows:

- Non-circles Scenarios (for Primary 5 and Primary 6)
- Circles Scenarios (for Primary 6 only).

WORKING

“Working” shows heuristics application in action (how quick it is to solve a question).



Chapter 6 Before and After Heuristics

EXPLANATION

List all given before-change, change and after-change information. No conversion is needed since the information is already in units and parts.

	Company C	Company S
Before	8 units	2 units
Change		+900 buses
After	2 units	2 units

CONFUSION ALERT

The 2 fractions are for different situations (before change and after-change). That means 1 measure in first fraction is different from 1 measure in second fraction. Hence, we differentiate with units and parts.

We know that Company C's after-change number of buses remains unchanged. So, we make Company C's after-change 1 part equal to before-change 8 units. We do this by multiplying Company C's after-change 1 part by 8.

Whatever we do to a number, we must also do to the other numbers in the same way to maintain the ratio.

Therefore, we must multiply Company S's after-change 2 parts by 8.

These actions will convert the after-change row's parts to units.

	Company C	Company S
Before	8 units	1 units
Change		+900 buses
After	1 part = 8 2 parts = 16	2 parts = 16

The difference between Company C's before-change and after-change is $(8 - 16)$ units, that is 8 units, which is equivalent to 800 buses.

2 parts = 900 buses = 450 buses

16 parts = 450×16 buses = 7200 buses

Company S has 16 units of buses, which is equivalent to 7200 buses.

EXPLANATION

“Explanation” shows the thought process (the detailed steps) behind the heuristics application. It takes readers through the solution in the following manner:

- step-by-step method so readers can follow what happens at every stage.
- systematic approach so readers begin to see a pattern in applying the technique.
- easy-to-follow steps so readers can quickly understand the technique minus the frustration.

For Spatial Visualisation, readers will see that its application always involves:

- identifying basic shapes, derivative shapes and parts of shapes,
- analysing how these are inter-related, and how these may be manipulated to arrive at the answer, and
- the application of formulae (as listed in the “Before You Begin” Chapter) to carry out the actual solution.

This quickly helps readers see and understand how to approach each question, including picking out hints often provided in the questions.

“Confusion Alert” boxes highlight areas where students are likely to falter or make mistakes in. It also gives the rationale to help clarify their doubts.

“Alternative” boxes show other approaches to the solution process. This acknowledges the different views that students may have to the problem.

LET'S APPLY

Learning is only effective with practice. Hence, at the end of each chapter/section is a list of questions to hone readers’ skills in the heuristics technique.

Where applicable, questions are grouped as follows:

- Non-circles Scenarios (for Primary 5 and Primary 6)
- Circles Scenarios (for Primary 6 only).

ADDITIONAL TIPS

For on-going sharing and discussions on the use of Spatial Visualisation, visit:

www.facebook.com/mathsheuristics

For detailed workings to all the Spatial Visualisation “Let’s Apply” sections, visit:

www.mathsheuristics.com/?page_id=472

Chapter 6 Before and After Heuristics

LET'S APPLY Problems Involving Case 1: Case 2

1. A number of chocolate is shared among Andy, Ben and Christopher. If Ben gives 5 chocolate to Andy, Andy will have twice as many sweets as Ben. If Andy gives 5 chocolate to Ben, both of them will have the same number of chocolate. Christopher's share is the difference of the other two boys' share. What is the total number of chocolate they shared?
2. In a year's time, Harry will be twice her sister's age. Five years ago, the ratio of James' age to her sister's age was 2:1. How old is Harry now?
3. When 30 boys leave, the ratio of the remaining boys to girls in a classroom becomes 1:2. On the other hand, when 10 girls leave the classroom, there will be 80% as many girls as boys. How many pupils are there in the class?
4. Elaine and Fran each had some money. If Elaine spends \$20, the ratio of the amount of money Elaine has to the amount of money Fran has is 2:3. If Fran spends \$80, she will have three as much money as Fran. How much money does each girl have?
5. Gerry and Holly received some money each. If Gerry spends \$20 per week and Holly spends \$15 per week, Gerry will have \$100 left while Holly will have spent all her money. If Gerry spends \$15 per week and Holly spends \$20 per week, Gerry will have \$10 left while Holly will have spent all her money.
 - a) How much money did Gerry receive?
 - b) How much money did Holly receive?

BEFORE YOU BEGIN

THINGS TO NOTE

1. Where applicable, problems are grouped into Non-circles and Circles Scenarios.
 - **Non-circles Scenarios** are for **Primary 5 and Primary 6**.
 - **Circles Scenarios** are for **Primary 6 only**.

Chapter 2	Completely involves Circles Scenarios, hence entire chapter is for Primary 6 only.
------------------	--

Chapters 3, 5, 6 and 7	Problems in Examples and Let's Apply sections are grouped into Non-Circles and Circles Scenarios.
-------------------------------	---

2. You need to be familiar with the various formulae for different **basic shapes** – rectangle, square, triangle, circle. However, the topic on circles is only covered at Primary 6.
3. These formulae will also be needed when working with **derivative shapes** – semi-circle (half circle), quadrant (quarter circle), etc.
4. Challenging higher-order questions involve a **combination of shapes**. The combination sometimes forms a larger **composite shape**.

Examples of **combination of shapes**.

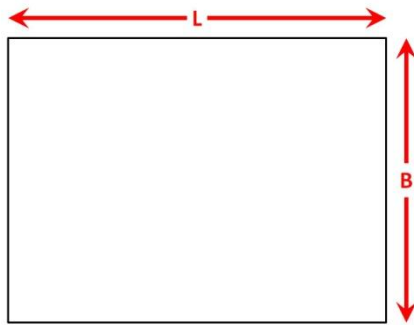
- A semi-circle, a $\frac{1}{4}$ circle and a triangle within a square.
- A square within a circle.

Examples of **composite shape**.

- 2 triangles, forming a square.
- Various squares and rectangles of different sizes, forming a square.

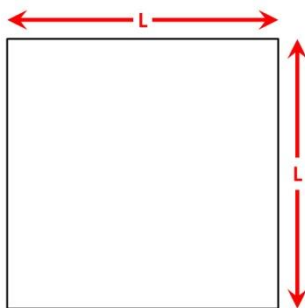
4. **Basic shapes and their formulae.**

a) **Rectangle**



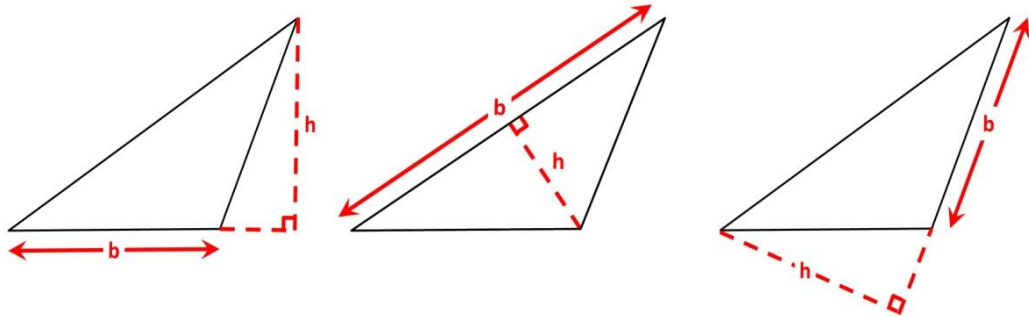
Area	= Length x Breadth
	= L x B
Perimeter	= 2 Length + 2 Breadth
	= 2L + 2B
	= 2(L + B)

b) **Square**



Length	= Breadth
Area	= Length x Length
	= L x L
Perimeter	= 4 x Length
	= 4 x L

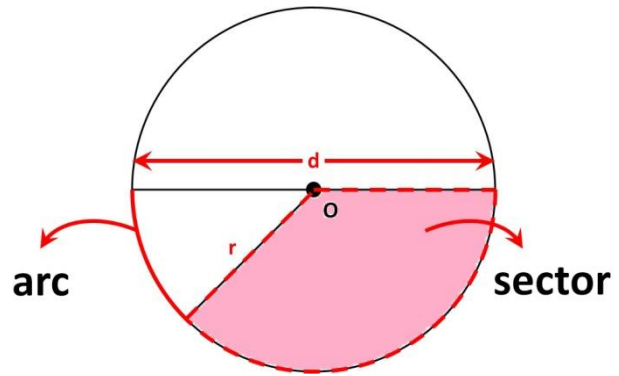
c) **Triangle**



Area	= $\frac{1}{2}$ x Base x Height
	= $\frac{1}{2}$ x b x h
Perimeter	= Sum of 3 sides

$(\frac{1}{2} bh)$

d) Circle

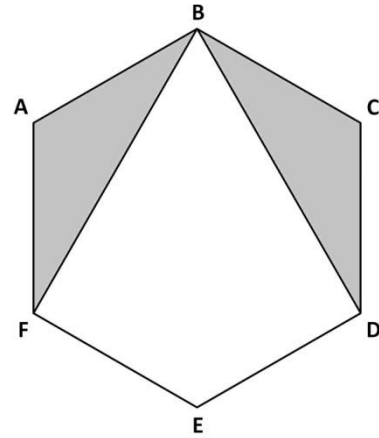


Diameter	= 2 x radius	(2r)
Circumference	= π x diameter	(πd)
	= 2 x π x radius	($2\pi r$)
Area	= π x radius x radius	(πr^2)
π can assume various forms. Read the question carefully to determine which form of π to apply.		
▪ $\pi = 3.14$		
▪ $\pi = \frac{22}{7}$		
▪ Calculator π		

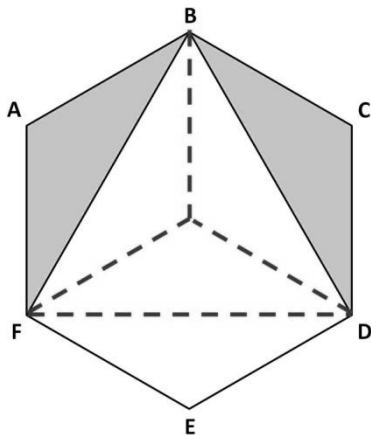
CHAPTER 1 REARRANGING PARTS: NON-CIRCLES

EXAMPLES

1. In the figure below, $AB = BC = CD = DE = EF = FA$.
The area of the whole figure is 120 cm^2 .
Find the area of the shaded part.



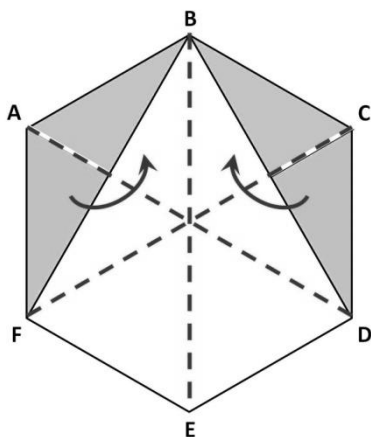
WORKING #1



$$\begin{aligned} 6 \text{ parts} &= 120 \\ 2 \text{ parts shaded} &= \frac{2}{6} \times 120 \\ &= \frac{1}{3} \times 120 \\ &= 40 \end{aligned}$$

Area of the shaded part is 40 cm^2 .

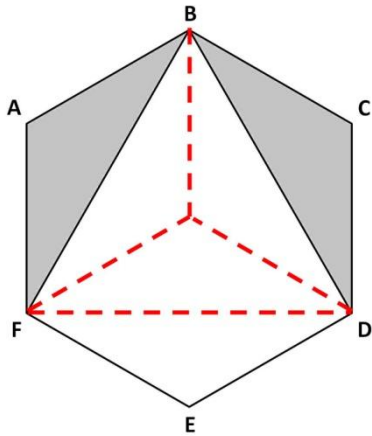
WORKING #2



$$\begin{aligned} 6 \text{ parts} &= 120 \\ 2 \text{ parts shaded} &= \frac{2}{6} \times 120 \\ &= \frac{1}{3} \times 120 \\ &= 40 \end{aligned}$$

Area of the shaded part is 40 cm^2 .

EXPLANATION #1

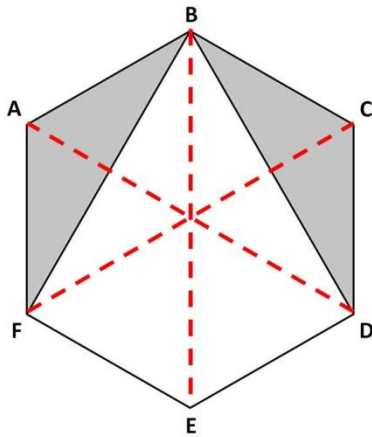


Draw lines as shown.
6 identical triangles are formed.

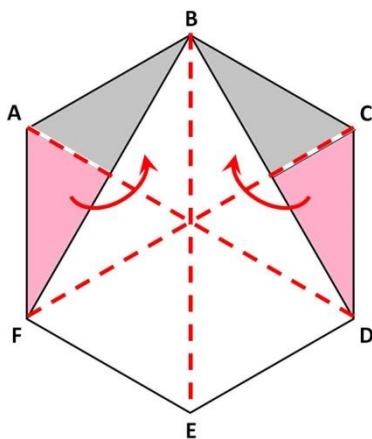
$$\begin{aligned}
 6 \text{ identical triangles} &= 120 \\
 2 \text{ identical triangles shaded} &= \frac{2}{6} \times 120 \\
 &= \frac{1}{3} \times 120 \\
 &= 40
 \end{aligned}$$

Area of the shaded part is 40 cm².

EXPLANATION #2

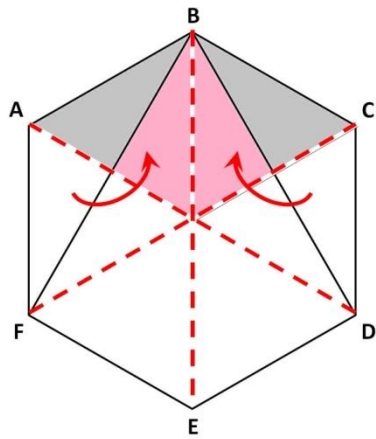


Draw lines as shown.
Looking at the newly-drawn lines,
6 identical segment triangles are formed.



Pay attention to the 2 parts shaded pink.

Each pink-shaded part has
a matching unshaded part.
Switch the corresponding parts.

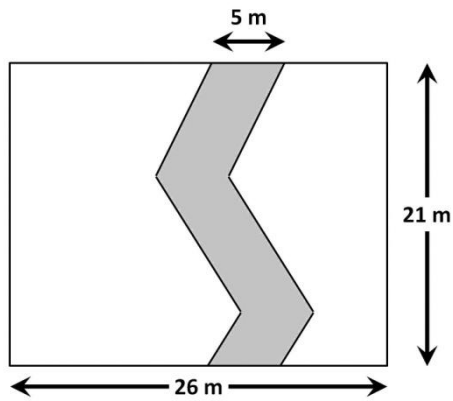


2 of the 6 identical segment triangles are shaded.

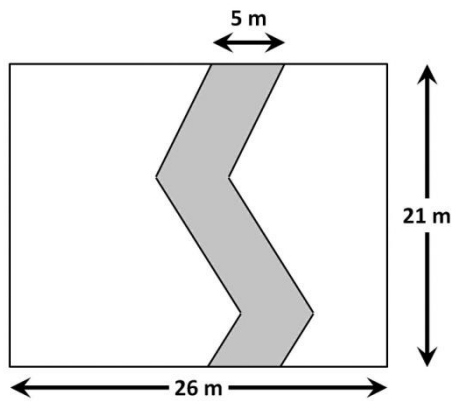
$$\begin{aligned}
 6 \text{ identical triangles} &= 120 \\
 2 \text{ identical triangles shaded} &= \frac{2}{6} \times 120 \\
 &= \frac{1}{3} \times 120 \\
 &= 40
 \end{aligned}$$

Area of the shaded part is 40 cm².

2. A rectangular garden measuring 21 m by 26 m has a 5-m walking path running through it. Find the area of the walking path.



WORKING



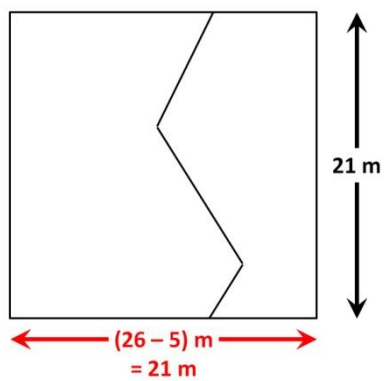
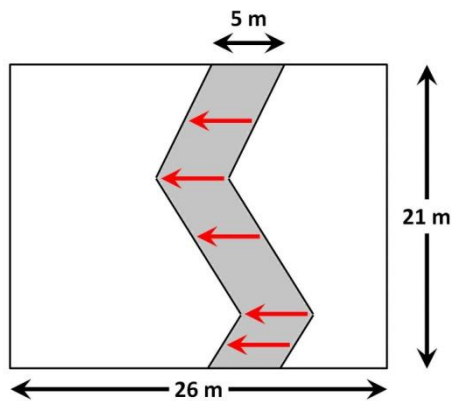
$$\begin{aligned}\text{Garden and path} &= L \times B \\ &= 26 \times 21 \\ &= 546\end{aligned}$$

$$\begin{aligned}\text{Garden only} &= L \times B \\ &= (26 - 5) \times 21 \\ &= 21 \times 21 \\ &= 441\end{aligned}$$

$$\begin{aligned}\text{Path} &= 546 - 441 \\ &= 105\end{aligned}$$

Area of the walking path is 105 m²

EXPLANATION



Remove the path and join the 2 garden parts.

This forms a shorter rectangle measuring 21 m by (26 - 5) m, that is 21 m by 21 m.

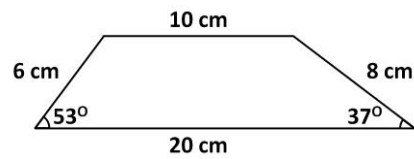
$$\begin{aligned}\text{Longer rectangle} &= L \times B \\ &= 26 \times 21 \\ &= 546\end{aligned}$$

$$\begin{aligned}\text{Shorter rectangle} &= L \times B \\ &= 21 \times 21 \\ &= 441\end{aligned}$$

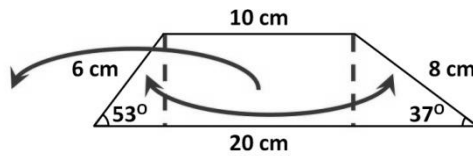
$$\begin{aligned}\text{Path} &= \text{Longer rectangle} - \text{Shorter rectangle} \\ &= 546 - 441 \\ &= 105\end{aligned}$$

Area of the walking path is 105 m².

3. Find the area of the trapezium shown in the figure below.



WORKING



The 2 triangles

$$\begin{aligned} &= \frac{1}{2} \times B \times H \\ &= \frac{1}{2} \times 6 \times 8 \\ &= 24 \end{aligned}$$

Base of the

$$\begin{aligned} &2 \text{ triangles together} \\ &= 20 - 10 \\ &= 10 \\ &= \text{Base of rectangle} \end{aligned}$$

Trapezium

$$\begin{aligned} &= \text{The 2 triangles} \\ &\quad + \text{Rectangle} \\ &= 24 + 48 \\ &= 72 \end{aligned}$$

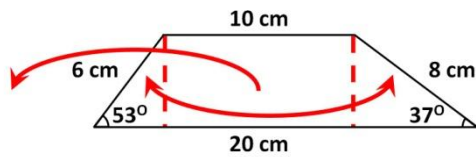
Rectangle

$$\begin{aligned} &= 2 \times \text{The 2 triangle} \\ &= 2 \times 24 \\ &= 48 \end{aligned}$$

Area of the

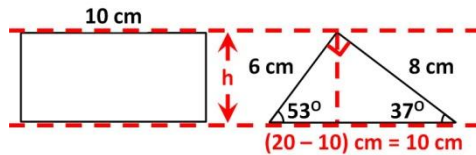
trapezium is 72 cm².

EXPLANATION



Draw lines as shown.

Separate the rectangle,
and join the 2 triangles.



There are now 2 figures
– the rectangle and a larger triangle.

Let's work on
the larger triangle.

In the larger triangle,
the angle at the top
is $180^\circ - 53^\circ - 37^\circ$,
that is 90° .

So, $B = 6$ and $H = 8$.

Larger triangle

$$\begin{aligned} &= \frac{1}{2} \times B \times H \\ &= \frac{1}{2} \times 6 \times 8 \\ &= 24 \end{aligned}$$

Let's work on
the rectangle.

$$\begin{aligned} \text{Larger triangle} &= \frac{1}{2} \times B \times H \\ &= \frac{1}{2} \times 10 \times h \\ &= 5 \times h \end{aligned}$$

We already know that
the larger triangle is 24 cm^2 .

$$\begin{aligned} 5 \times h &= 24 \\ h &= 24 \div 5 \\ &= 4.8 \end{aligned}$$

$$\begin{aligned} \text{Rectangle} &= B \times H \\ &= 10 \times 4.8 \\ &= 48 \end{aligned}$$

Let's combine.

$$\begin{aligned} \text{Trapezium} &= \text{Larger triangle} \\ &+ \text{Rectangle} \\ &= 24 + 48 \\ &= 72 \end{aligned}$$

Area of the
trapezium is 72 cm^2 .

ALTERNATIVE

$$\begin{aligned} \text{Length of the rectangle} &= \text{Base of the larger triangle} = 10 \end{aligned}$$

$$\begin{aligned} \text{Breadth of the rectangle} &= \text{Height of the larger triangle} = h \end{aligned}$$

$$\begin{aligned} \text{Larger triangle} &= \frac{1}{2} \times B \times H \\ &= \frac{1}{2} \times 10 \times h \end{aligned}$$

$$\begin{aligned} \text{Rectangle} &= L \times B \\ &= 10 \times h \end{aligned}$$

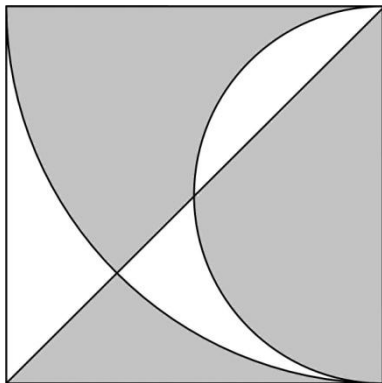
$$\begin{aligned} \text{So, rectangle} &= 2 \times \text{Larger triangle} \\ &= 2 \times 24 \\ &= 48 \end{aligned}$$

CHAPTER 2 REARRANGING PARTS: CIRCLES

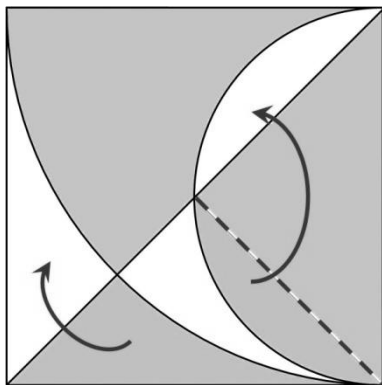
(For Primary 6 only)

EXAMPLES

1. The figure below shows a square, a quadrant and a semi-circle. The area of the square is 240 cm^2 . Find the area of the shaded parts. (Take $\pi = 3.14$)



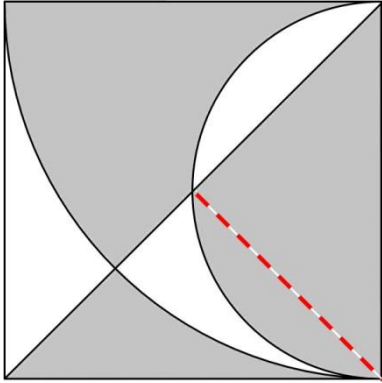
WORKING



$$\begin{aligned}\frac{3}{4} \text{ square} &= \frac{3}{4} \times 240 \\ &= 180\end{aligned}$$

Area of the shaded parts is 180 cm².

EXPLANATION

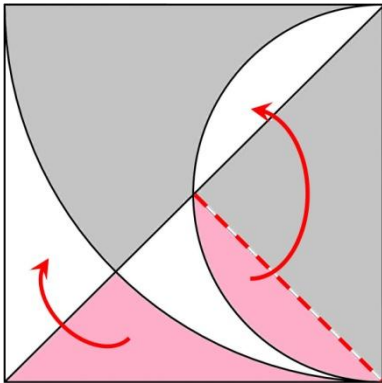


HINT

Clues are always in the question itself. Focus only on shapes mentioned in the question.

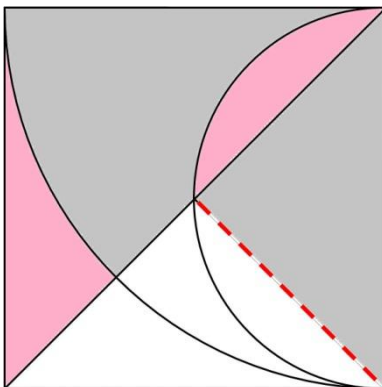
In this example, we can see triangles. However, since the question does not mention anything about triangles, we ignore the triangles. Instead, we focus on the square, quadrant and semi-circle which are mentioned in the question.

Draw half a diagonal line as shown.

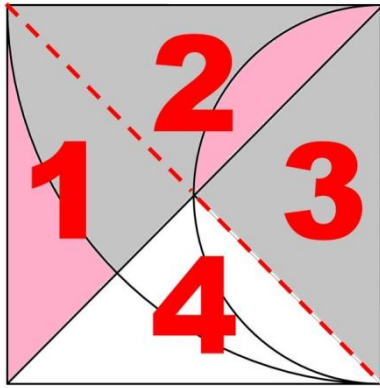


Pay attention to the 2 parts shaded pink.

Each pink-shaded part has a mirror unshaded part. Switch the corresponding parts.



This is what you will get.



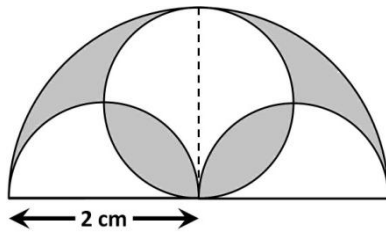
Extend the half diagonal into a full diagonal.
The square is now cut into 4 equal parts.

3 parts are shaded, while 1 part is unshaded.
 $\frac{3}{4}$ of the square is shaded.

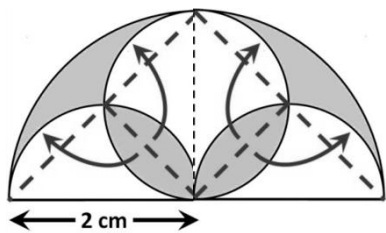
$$\begin{aligned}\text{Whole square} &= 240 \\ \frac{3}{4} \text{ square} &= \frac{3}{4} \times 240 \\ &= 180\end{aligned}$$

Area of the shaded parts is 180 cm².

2. The figure below shows a big semi-circle, with a circle and 2 identical smaller semi-circles within it. Find the area of the shaded parts. (Take $\pi=3.14$)



WORKING #1

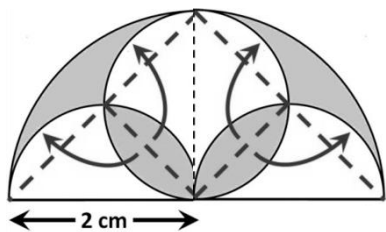


$$\begin{aligned} \frac{1}{2} \text{ shaded parts} &= \text{Quadrant} - \text{Triangle} \\ &= \left(\frac{1}{4} \times \pi r^2\right) - \left(\frac{1}{2} \times B \times H\right) \\ &= \left(\frac{1}{4} \times 3.14 \times 2 \times 2\right) - \left(\frac{1}{2} \times 2 \times 2\right) \\ &= 1.14 \end{aligned}$$

$$\begin{aligned} \text{Shaded parts} &= 1.14 \times 2 \\ &= 2.28 \end{aligned}$$

Area of the shaded parts is 2.28 cm².

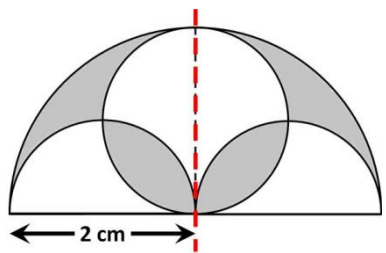
WORKING #2



$$\begin{aligned} \text{Shaded parts} &= \text{Semi-circle} - \text{Triangle} \\ &= \left(\frac{1}{2} \times \pi r^2\right) - (B \times H) \\ &= \left(\frac{1}{2} \times 3.14 \times 2 \times 2\right) - (4 \times 2) \\ &= 2.28 \end{aligned}$$

Area of the shaded parts is 2.28 cm².

EXPLANATION

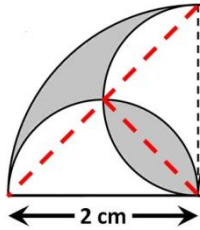


HINT

Clues are always in the question itself. Focus only on shapes mentioned in the question.

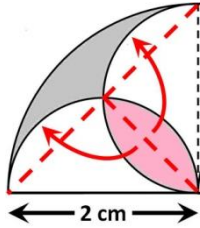
In this example, we focus on the big semi-circle, circle and 2 identical smaller semi-circles which are mentioned in the question.

Note how the semi-circle is made up of 2 quadrants which are mirror image of each other.



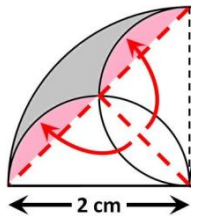
Let's look at one of the quadrants.

Draw lines as shown.



Pay attention to the 2 parts shaded pink.

Each pink-shaded part has a matching unshaded part. Switch the corresponding parts.

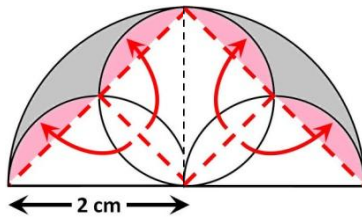


This is what you will get.

$$\begin{aligned}
 \text{Shaded parts} &= \text{Quadrant} - \text{Triangle} \\
 &= \left(\frac{1}{4} \times \pi r^2\right) - \left(\frac{1}{2} \times B \times H\right) \\
 &= \left(\frac{1}{4} \times 3.14 \times 2 \times 2\right) - \left(\frac{1}{2} \times 2 \times 2\right) \\
 &= 1.14
 \end{aligned}$$

Remember there are 2 identical quadrants.

$$\begin{aligned}
 \text{Total shaded parts} &= 1.14 \times 2 \\
 &= 2.28
 \end{aligned}$$

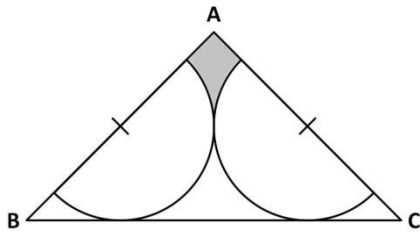


ALTERNATIVE

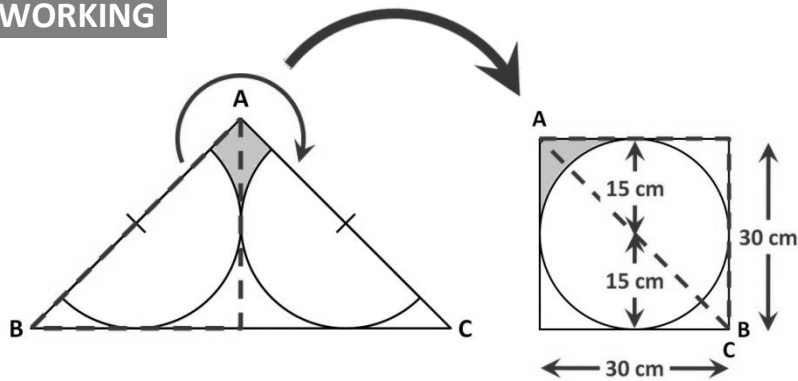
$$\begin{aligned}
 \text{Shaded parts} &= \text{Semi-circle} - \text{Triangle} \\
 &= \left(\frac{1}{2} \times \pi r^2\right) - (B \times H) \\
 &= \left(\frac{1}{2} \times 3.14 \times 2 \times 2\right) - (4 \times 2) \\
 &= 2.28
 \end{aligned}$$

Area of the shaded parts is 2.28 cm².

3. The diagram below shows an isosceles triangle ABC with 2 semi-circles within it. The semi-circles have a radius of 15 cm. Find the area of the shaded part. (Take π as 3.14)



WORKING



$$\begin{aligned} \text{Square} &= L \times B \\ &= 30 \times 30 \\ &= 900 \end{aligned}$$

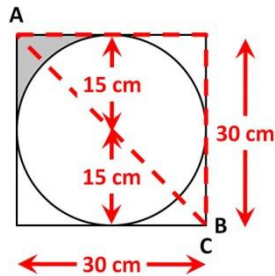
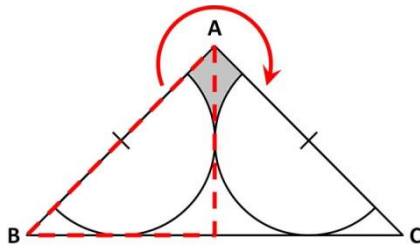
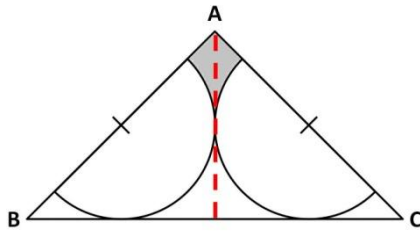
$$\begin{aligned} \text{Circle} &= \pi r^2 \\ &= (3.14 \times 15 \times 15) \\ &= 706.5 \end{aligned}$$

$$\begin{aligned} \text{4 corners} &= \text{Square} - \text{Circle} \\ &= 900 - 706.5 \\ &= 193.50 \end{aligned}$$

$$\begin{aligned} \text{1 corner} &= \frac{1}{4} \times 193.50 \\ &= 48.375 \end{aligned}$$

The shaded area is 48.475 cm².

EXPLANATION



HINT

Clues are always in the question itself. Focus only on shapes mentioned in the question.

In this example, we focus on the triangle and 2 semi-circles which are mentioned in the question.

Note how the triangle is made up of 2 smaller triangles which are mirror image of each other.

Rotate the left smaller triangle clockwise on Point A, until Point B touches Point C.

This is what you will get.

$$\begin{aligned} \text{Square} &= L \times B \\ &= 30 \times 30 \\ &= 900 \end{aligned}$$

$$\begin{aligned} \text{Circle} &= \pi r^2 \\ &= (3.14 \times 15 \times 15) \\ &= 706.5 \end{aligned}$$

$$\begin{aligned} 4 \text{ corners} &= \text{Square} - \text{Circle} \\ &= 900 - 706.50 \\ &= 193.50 \end{aligned}$$

$$\begin{aligned} 1 \text{ corner} &= \frac{1}{4} \times 193.50 \\ &= 48.375 \end{aligned}$$

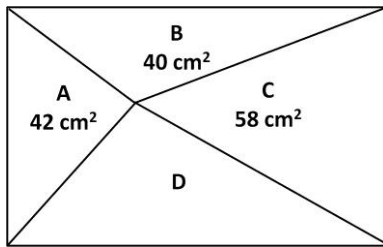
The shaded area is 48.475 cm².

CHAPTER 3 DRAWING LINES

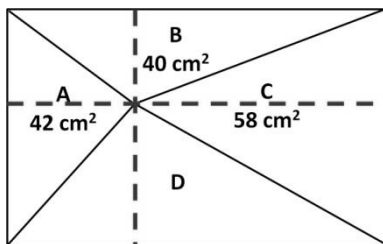
EXAMPLES

NON-CIRCLES SCENARIOS

1. The figure below shows a rectangle that is cut into 4 triangles A, B, C and D. Find the area of triangle D.



WORKING



$$A + C = \frac{1}{2} \text{ rectangle}$$

$$B + D = \frac{1}{2} \text{ rectangle}$$

$$B + D = A + C$$

$$40 + D = 42 + 58$$

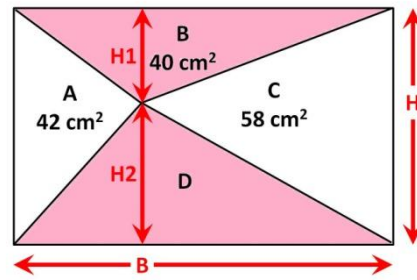
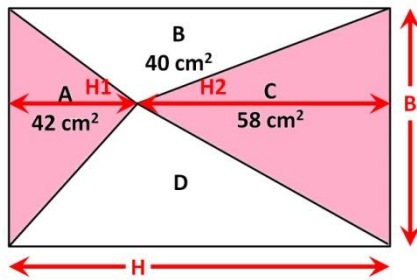
$$40 + D = 100$$

$$D = 100 - 40$$

$$D = 60$$

Area of triangle D is 60 cm².

EXPLANATION #1



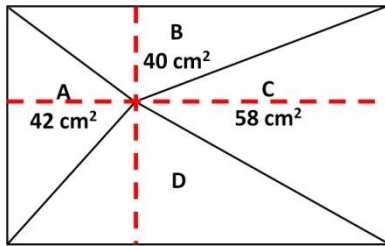
$$\begin{aligned}
 A + C &= \left(\frac{1}{2} \times B \times H1\right) + \left(\frac{1}{2} \times B \times H2\right) \\
 &= \frac{1}{2} \times B \times (H1 + H2) \\
 &= \frac{1}{2} \times B \times H \quad \leftarrow \text{Since } H = H1 + H2 \\
 &= \frac{1}{2} \text{ rectangle}
 \end{aligned}$$

$$\begin{aligned}
 B + D &= \left(\frac{1}{2} \times B \times H1\right) + \left(\frac{1}{2} \times B \times H2\right) \\
 &= \frac{1}{2} \times B \times (H1 + H2) \\
 &= \frac{1}{2} \times B \times H \quad \leftarrow \text{Since } H = H1 + H2 \\
 &= \frac{1}{2} \text{ rectangle}
 \end{aligned}$$

$$\begin{aligned}
 \text{That means } B + D &= A + C \\
 40 + D &= 42 + 58 \\
 40 + D &= 100 \\
 D &= 100 - 40 \\
 D &= 60
 \end{aligned}$$

Area of triangle D is 60 cm².

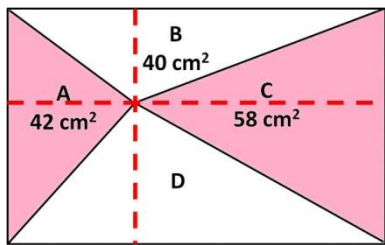
EXPLANATION #2



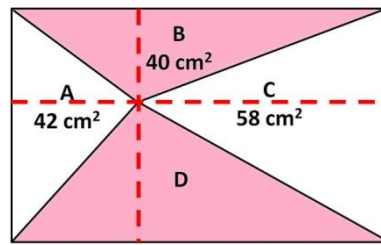
Draw lines as shown.
The rectangle is divided into
4 small rectangles of different sizes.

Within each small rectangle is a diagonal line.
Each diagonal line cuts its
respective small rectangle into 2 equal parts.
That means, any 1 part is $\frac{1}{2}$ its
respective small rectangle.

Logically, if we take $\frac{1}{2}$ of every small rectangle and
combine those areas, we get $\frac{1}{2}$ the rectangle.
You can take $\frac{1}{2}$ of every small rectangle as follows.



$$A + C = \frac{1}{2} \text{ rectangle}$$

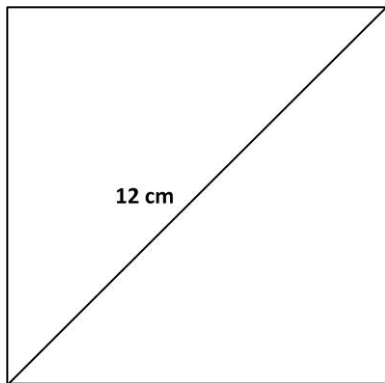


$$B + D = \frac{1}{2} \text{ rectangle}$$

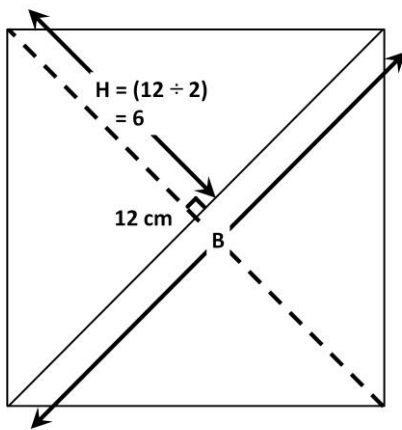
$$\begin{aligned} \text{That means } B + D &= A + C \\ 40 + D &= 42 + 58 \\ 40 + D &= 100 \\ D &= 100 - 40 \\ D &= 60 \end{aligned}$$

Area of triangle D is 60 cm².

2. Find the area of a square whose diagonal is 12 cm.



WORKING

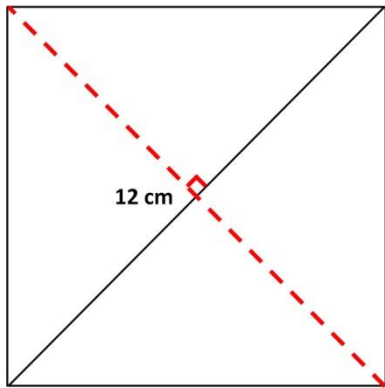


$$\begin{aligned} \text{Triangle} &= \frac{1}{2} \times 12 \times 6 \\ &= 36 \end{aligned}$$

$$\begin{aligned} \text{Square} &= 2 \times 36 \\ &= 72 \end{aligned}$$

Area of the square is 72 cm².

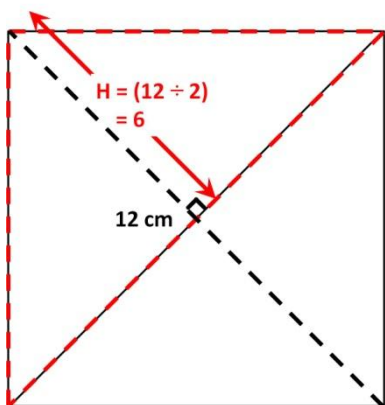
EXPLANATION



The diagonal divides the square into 2 identical triangles.

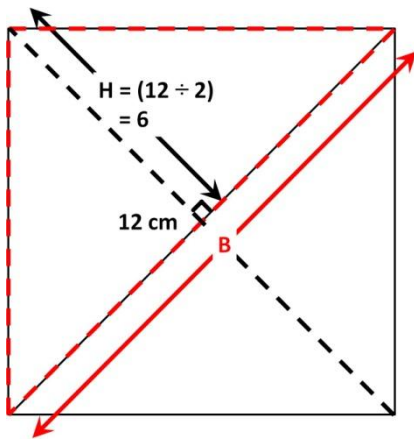
Draw line as shown.

This line is also a diagonal of the square, measuring 12 cm.



Let's look at one of the triangles.

$$\begin{aligned}\text{Height of triangle} &= \frac{1}{2} \text{ diagonal} \\ &= 12 \div 2 \\ &= 6\end{aligned}$$



$$\begin{aligned}\text{Triangle} &= \frac{1}{2} \times B \times H \\ &= \frac{1}{2} \times 12 \times 6 \\ &= 36\end{aligned}$$

$$\begin{aligned}\text{Square} &= 2 \times \text{Triangles} \\ &= 2 \times 36 \\ &= 72\end{aligned}$$

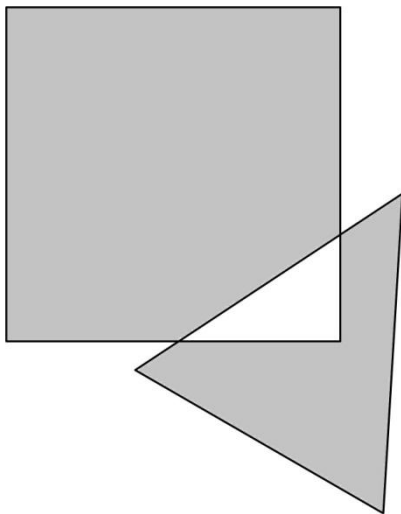
Area of the square is 72 cm².

CHAPTER 5 DIFFERENCE IN AREA

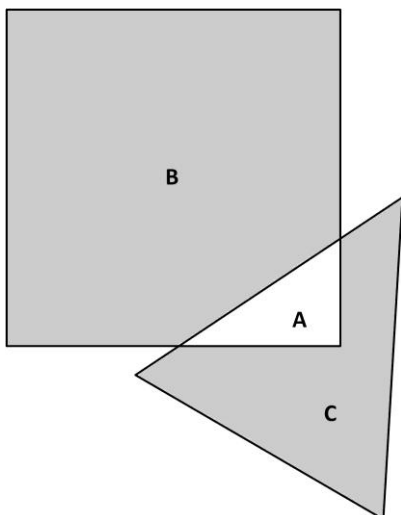
EXAMPLES

NON-CIRCLES SCENARIOS

1. The figure below shows a square and a triangle overlapping each other partially. The area of the square and triangle is 90cm^2 and 65cm^2 respectively. Find the difference between the two shaded areas.



WORKING

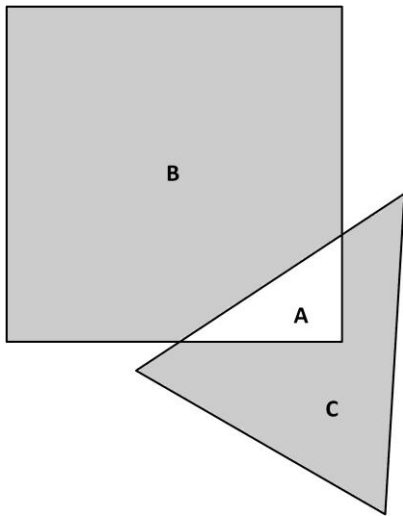


$$\begin{array}{|c|c|} \hline \text{A} & \text{B} \\ \hline \text{A} & \text{C} \\ \hline \end{array}$$

$\leftarrow 90 \rightarrow$
 $\leftarrow 65 \rightarrow$ $(B - C)$
 $= (90 - 65)$
 $= 25$

The difference between the two shaded areas is 25 cm^2 .

EXPLANATION

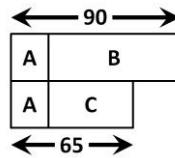


Label the parts as shown.

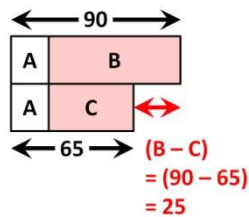
Using the model method,
draw all the information.

Square: $A + B = 90$

Triangle: $A + C = 65$



Visually, it is obvious that we can
find out what is $(B - C)$,
which is what we want
– the difference between B and C.



ALTERNATIVE

Square: $A + B = 90$

Triangle: $A + C = 65$

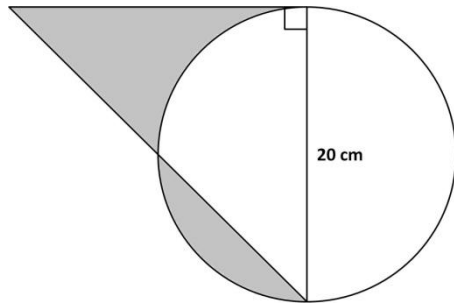
$$\begin{aligned} \text{Square} - \text{Triangle: } (A + B) - (A + C) &= (90 - 65) \\ B - C &= 25 \end{aligned}$$

The difference between
the two shaded areas is 25 cm².

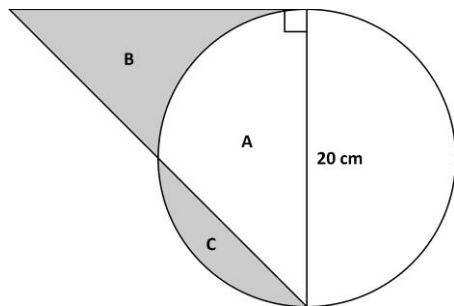
CIRCLES SCENARIOS

(For Primary 6 only)

2. The figure is formed by a circle of diameter 20 cm and a right-angled isosceles triangle. Find the difference in the area between the two shaded parts.
(Take $\pi = 3.14$)

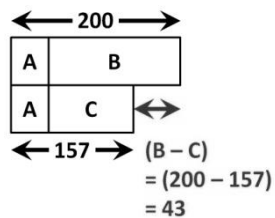


WORKING



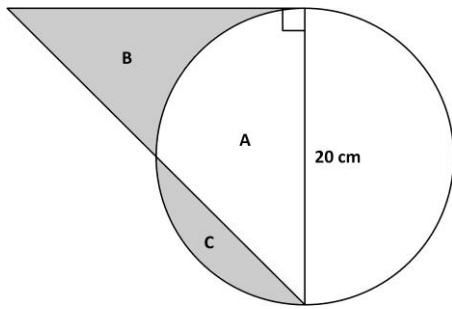
$$\begin{aligned} A + B &= \frac{1}{2} \times B \times H \\ &= \frac{1}{2} \times 20 \times 20 \\ &= 200 \end{aligned}$$

$$\begin{aligned} A + C &= \frac{1}{2} \times 3.14 \times 10 \times 10 \\ &= 157 \end{aligned}$$



The difference between the two shaded areas is 43 cm².

EXPLANATION



Label the parts as shown.

$$\begin{aligned} \text{Triangle: } A + B &= \frac{1}{2} \times B \times H \\ &= \frac{1}{2} \times 20 \times 20 \\ &= 200 \end{aligned}$$

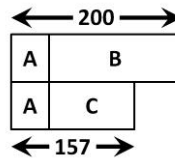
$$\begin{aligned} \text{Radius of circle} &= 20 \div 2 \\ &= 10 \end{aligned}$$

$$\begin{aligned} \text{Semi-circle: } A + C &= \frac{1}{2} \times \pi r^2 \\ &= \frac{1}{2} \times 3.14 \times 10 \times 10 \\ &= 157 \end{aligned}$$

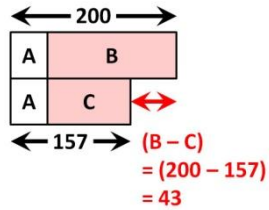
Using the model method,
draw all the information.

$$\text{Triangle: } A + B = 200$$

$$\text{Semi-circle: } A + C = 157$$



Visually, it is obvious that we can
find out what is $(B - C)$,
which is what we want
– the difference between B and C.



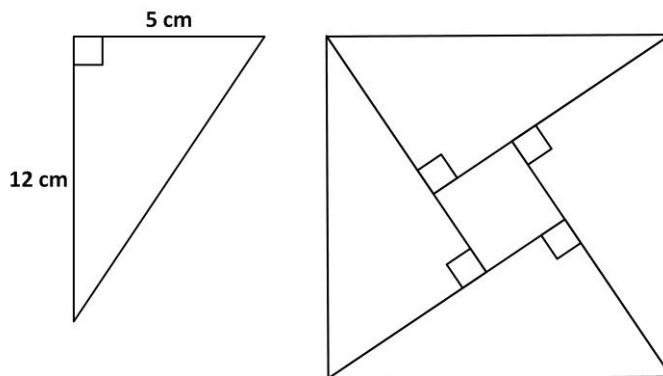
The difference between
the two shaded areas is 43 cm².

CHAPTER 7 VISUAL CLUES

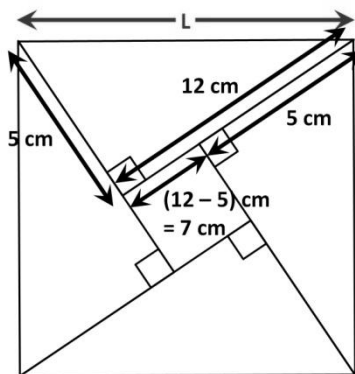
EXAMPLES

NON-CIRCLES SCENARIOS

1. The diagram below shows 4 right-angled triangles arranged to form a big square. Each triangle has a base of 5 cm and height of 12 cm. Find the perimeter of the big square.



WORKING



$$4 \text{ triangles:} \quad 4 \times \left(\frac{1}{2} \times 5 \times 12\right) = 120$$

$$\text{Small square:} \quad 7 \times 7 = 49$$

$$\text{Big square:} \quad 120 + 49 = 169$$

$$L \times L = 169$$

$$\text{If } L = 12 \quad 12 \times 12 = 144$$

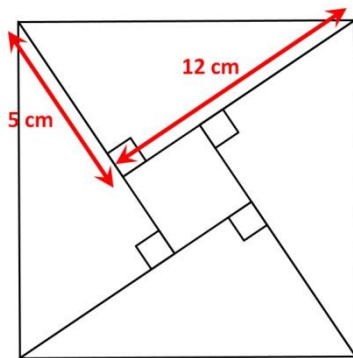
$$\text{If } L = 13 \quad 13 \times 13 = 169$$

Hence, $L = 13$.

$$\begin{aligned} \text{Perimeter of big square} &= 4 \times L \\ &= 4 \times 13 \\ &= 52 \end{aligned}$$

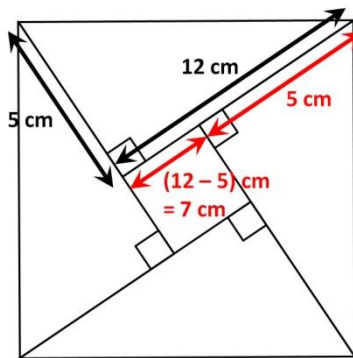
Perimeter of the big square is 52 cm.

EXPLANATION



1 triangle: $\frac{1}{2} \times B \times H = \frac{1}{2} \times 5 \times 12$
 $= 30$

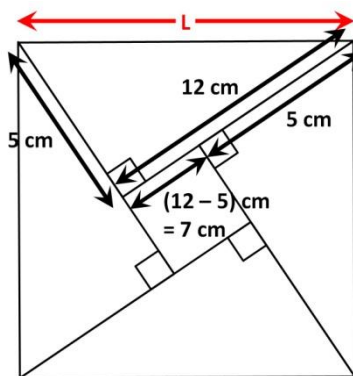
4 triangles: $4 \times 30 = 120$



Side of small square: $12 - 5 = 7$

Small square: $L \times L = 7 \times 7$
 $= 49$

Big square: $4 \text{ triangles} + \text{Small square}$
 $= 120 + 49$
 $= 169$



Let's look at the big square.

Big square = $L \times L$

So, $L \times L = 169$

Use Guess-and-Check method to find out what is L.

If $L = 12$ $12 \times 12 = 144 \rightarrow$ Need bigger number.

If $L = 13$ $13 \times 13 = 169$

Hence, $L = 13$

Perimeter of big square = $4 \times L$
 $= 4 \times 13$
 $= 52$

Perimeter of the big square is 52 cm.

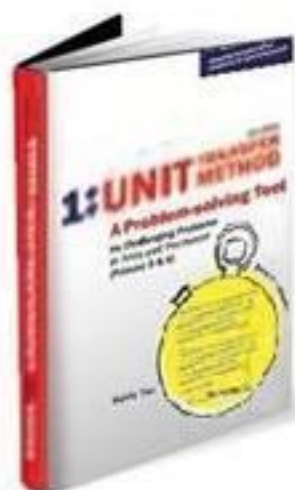
The Series

The Mastering Heuristics Series was conceptualised by Sunny Tan, Principal Trainer of mathsHeuristics™, to give parents and students a comprehensive guide to Heuristics. The Series neatly packages Heuristics techniques into a series of guidebooks with well-defined application scenarios. It offers many examples, showing the efficiency and step-by-step application of Heuristics techniques, plus opportunities to get in some practice.

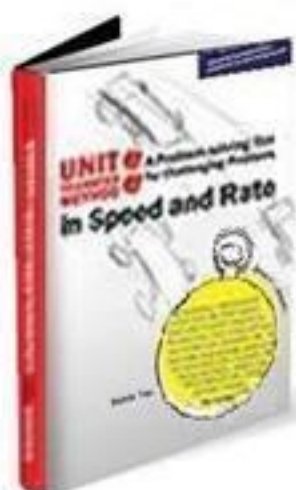
This particular guidebook, Spatial Visualisation teaches the use of visualisation techniques to effectively analyse, manipulate and solve challenging problems in Area & Perimeter.

Each guidebook in the series is a standalone publication. For students enrolled in mathsHeuristics™ programmes, each book serves as a study companion, while keeping parents well-informed of what their children are learning.

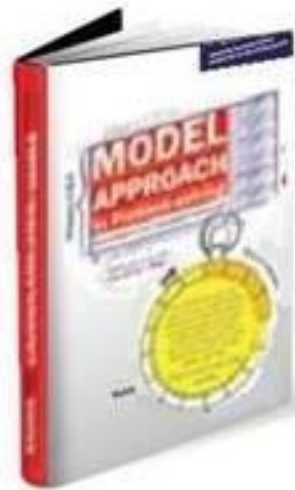
Other Titles in the Series



*Unit Transfer Method
@ P5 & P6*



*Unit Transfer Method
(Speed & Rate)
@ P6*



*Model Approach to
Problem-solving*



*Unit Transfer Method
@ P4*

Author

Sunny Tan trains students, especially those in the PSLE year, in the use of various Heuristics techniques. He also conducts Heuristics workshops for parents and educators.

In the 1990s, NIE-trained Sunny taught primary and secondary Maths in various streams. He observed how the transformed primary Maths syllabus stumped children, parents and, sometimes, even teachers. How do you teach young children to accurately choose and sequentially apply different situational logic in solving non-routine problems? Sunny resolved to simplify the learning and application of such skills. After a few years of research and development, Sunny eventually established the mathsHeuristics™ programme – and now the Mastering Heuristics Series – which has achieved consistent success and effectiveness.

Sunny's ingenious methodology has attracted much media interest – The Straits Times, The Business Times, The New Paper, TODAY, FM938 LIVE and major parenting magazines – as well as raving reviews by academia and parents.

