MASTERING MATH FACTS

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Research and Results

Thanks for your interest in the research base behind ***Mastering Math Facts*** and ***Mastering Math Fact Families***—published by Otter Creek Institute.

We have two parts to respond to your question. The first part is an article entitled, The third stage of learning math facts: Developing automaticity (submitted for publication) that reviews the literature on learning math facts. Here's a summary of the article.

> Learning math facts proceeds through three stages: 1) procedural knowledge of figuring out facts; 2) strategies for remembering facts based on relationships; 3) automaticity in math facts—declarative knowledge. Students achieve automaticity with math facts when they can directly retrieve the correct answer, without any intervening thought process. The development of automaticity is critical so students can concentrate on higher order thinking in math. Students who are automatic with math facts answer in less than one second, or write between 40 to 60 answers per minute, if they can write that quickly. Research shows that math facts practice that effectively moves students towards automaticity proceeds with small sets of no more than 2 –4 facts at a time. During practice, the answers must be remembered rather than derived. Practice must limit response times and give correct answers immediately if response time is slow. Automaticity must be achieved with each small set of facts, and maintained with the facts previously mastered, before more facts are introduced. Suggestions for doing this with flashcards or with worksheets are offered.

What you will note is that these recommendations form the basis of how ***Mastering Math Facts*** (***MMF***) and ***Mastering Math Fact Families*** were designed, and how we teach that the programs should be implemented.

The second part of our answer, in the Appendix, is to offer data from two informal implementations of MMF. While these were not peer-reviewed studies, they do indicate good results from use of the program. We would encourage local schools and districts to collect their own evaluative data and to publish their findings.

Sincerely,

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The third stage of learning math facts: Developing automaticity

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"Everyone knows what automaticity is. It is the immediate, obligatory way we apprehend the world around us. It is the fluent, effortless manner in which we perform skilled behaviors. It is the 'popping into mind' of familiar knowledge at the moment we need it (Logan, 1991a, p. 347)."

Fluency in computation and knowledge of math facts are part of the NCTM Math Standards. As the explanation of the Number and Operations Standard states, "Knowing basic number combinations—the single-digit addition and multiplication pairs and their counterparts for subtraction and division—is essential. Equally essential is computational fluency—having and using efficient and accurate methods for computing (NCTM, 2003, p. 32)..

Later within the explanation of this standard, it is stated that, "By the end of grade 2, students should know the basic addition and subtraction combinations, should be fluent in adding two-digit numbers, and should have methods for subtracting two-digit numbers. At the grades 3–5 level, as students develop the basic number combinations for multiplication and division, they should also develop reliable algorithms to solve arithmetic problems efficiently and accurately. These methods should be applied to larger numbers and practiced for fluency (NCTM, 2003, p. 35).

For children to be fluent in computation and to be able to do math mentally requires that they become fluent in the basic arithmetic facts. Researchers have long maintained there are three general stages of "learning" math facts and different types of instructional activities to effect learning in those stages (Ando & Ikeda, 1971; Ashlock, 1971; Bezuk & Cegelka, 1995; Carnine & Stein, 1981; Garnett, 1992; Garnett & Fleischner, 1983). Researchers such as Garnett (1992) have carefully documented the developmental sequence of procedures that children use to obtain the answers to math facts. Children begin with a first stage of counting, or procedural knowledge of math facts. A second stage consists of developing ways to "remember" math facts by relating them to known facts. Then the third and final stage is the declarative knowledge of the "knowing" the facts or "direct retrieval." Another line of research has established that most children move from using procedures to "figure out" math facts in the first grade to the adult model of retrieval by fifth grade (Koshmider & Ashcraft, 1991). The switch from predominantly procedural to predominantly declarative knowledge or direct retrieval of math facts normally begins at around the 2nd to 3rd grade level (Ashcraft, 1984).

If these three levels of knowing math facts are attainable, what does that imply about how to teach math facts? Different teaching and practice procedures apply to each of these stages. Meyers & Thorton (1977) suggested activities specifically geared toward different levels, including activities for "overlearning" intended to promote rapid recall of a set or cluster of facts. As Ashcraft (1982) points out there is much more evidence of direct retrieval of facts when you study adults or children in 4th grade or above. There is much research on the efficacy of deriving facts from relationships developed in studies on addition and subtraction with primary age children (Carpenter & Moser, 1984; Garnett, 1992; Steinberg, 1985; Thorton, 1978). However, it is not always clear that this research focuses on "soon-to-be-discarded" methods of answering math facts

Some of the advice on "how to teach" math facts is unclear about which stage is being addressed. The term "learning" has been variously applied to all three stages, and the term "memorization" has been used to apply to both the second and third stages (Stein, Silbert & Carnine, 1997). What is likely is that teaching strategies that address earlier stages of learning math facts may be counterproductive when attempting to develop automaticity. This paper will examine research on math fact learning as it applies to each of the three stages, with an emphasis on the third stage, the development of automaticity.

First stage: Figuring out math facts

The first stage has been characterized as "understanding," or "conceptual" or "procedural." This is the stage where the child must count or do successive addition, or some other strategy to "figure out" the answer to a fact (Garnett, 1992). "Specifically in view is the child's ability to associate the written number sentence with a physical referent (Ashlock, 1971, p. 359).

There is evidence that problems with learning mathematics may have their origins in failure to completely master counting skills in kindergarten and first grade (Geary, Bow-Thomas, & Yao, 1992). At the beginning stage arithmetic facts are problems to be solved (Gersten & Chard, 1999). If students cannot solve basic fact problems given plenty of time—then they simply do not understand the process, and certainly are not ready to begin memorization (Kozinski & Gast, 1993). Students must understand the process well enough to solve any fact problem given them before beginning memorization procedures. "To ensure continued success and progress in mathematics, students should be taught conceptual understanding prior to memorization of the facts (Miller, Mercer, & Dillon, 1992, p. 108)." Importantly, the conceptual understanding, or procedural knowledge of counting should be developed prior to memorization, rather than in place of it. "Prior to teaching for automaticity… it is best to develop the conceptual understanding of these math facts as procedural knowledge (Bezuk & Cegelka, 1995, p. 365)."

Second stage: Strategies for remembering math facts

The second stage has been characterized as "relating" or as "strategies for remembering." This can include pairs of facts related by the commutative property, e.g., 5 + 3 = 3 + 5 = 8. This can also include families of facts such as 7 + 4 = 11, 4 + 7 = 11, 11 - 4 = 7, and 11 – 7 = 4. Garnett characterizes such strategies as more "mature" than counting procedures indicating, "Another mature strategy is 'linking' one problem to a related problem (e.g., for 5 + 6, *thinking '5 + 5 =10, so 5 + 6 = 11'*) (Garnett, 1992, p. 212)." The goal of the second stage is simple accuracy rather than developing fluency or automaticity. Studies looking at use of "strategies for remembering" seldom use timed tests, and never have rigorous expectations of highly fluent performance. As long as students are accurate, a remembering strategy is considered successful.

Carpenter and Moser (1984) reported on a longitudinal study of children's evolving methods of solving simple addition and subtraction problems in first through 3rd grade. They identified sub-stages within counting as well as variations on recall strategies. The focus of their study was on naturalistically developed strategies that children had made up to solve problems they were presented, rather than evaluating a teaching sequence for its efficiency. For example, they noted that children were not consistent in using the most efficient strategy, even when they sometimes used that method. Carpenter and Moser did caution that, "It has not been clearly established that instruction should reflect the natural development of concepts and skills (1984, p. 200)." Garnett (1992) identified similar stages and sub-stages in examining children with learning disabilities.

Successful students often hit early on a strategy for remembering simple facts, where less successful students lack such a strategy and may simply guess (Thorton, 1978; Carnine & Stein, 1981). Research has demonstrated that teaching the facts to students, especially those who don't develop their own strategies, in some logical order that emphasizes the relationships, makes them easier to remember. Many studies have been reported in which various rules and relationships are taught to help children "derive" the answers to math facts (Carnine & Stein, 1981; Rightsel & Thorton, 1985; Steinberg, 1985; Thorton, 1978; Thorton & Smith, 1988; Van Houten, 1993).

An example of such a relationship rule is the, "doubles plus 1" rule. This rule applies to facts such as 8 + 9 which is the same as 8 + 8 plus 1, therefore because 8 + 8 = 16 and 16 + 1 = 17, therefore 8 + 9 = 17 (Thorton, 1978). Most authors suggest that learning the facts in terms of relationships is an intermediate stage between the stage of conceptual development that involves using a procedure and the stage of practicing to develop automatic recall on the other hand (Steinberg, 1985; Stein et al., 1997; Suydam, 1984; Thorton, 1978). "Research has indicated that helping them [students] to develop thinking strategies is an important step *between* the development of concepts with manipulative materials and pictures and the mastery of the facts with drill-and-practice activities [emphasis added] (Suydam, 1984, p. 15)." Additional practice activities are needed (after teaching thinking strategies) in order to develop automatic, direct retrieval of facts.

Thorton suggested that "curriculum and classroom efforts should focus more carefully on the development of strategy prior to drill on basic facts (1978, p. 226)." Thorton's (1978) research showed that using relationships such as doubles and doubles plus 1, and other "tricks" to remember facts as part of the teaching process resulted in more facts being learned after eight weeks than in classes where such aids to memory were not taught. Similarly, Carnine and Stein (1981) found that students instructed with a strategy for remembering facts learned a set of 24 facts with higher accuracy than students who were presented the facts to memorize with no aids to remembering (84% vs. 59%).

Steinberg (1985) studied the effect of teaching "noncounting, derived facts strategies in which the child uses a small set of known number facts to find or derive the solution to unknown number facts (1985, p. 337)." The subjects were second graders and the focus was on addition and subtraction facts. The children changed strategies from counting to using the derived facts strategies and did write the answers to more of the fact problems within the 2 seconds allotted per problem than before such instruction. Steinberg did not find that teaching derived facts strategies necessarily led to automaticity of number facts.

Thorton and Smith (1988) found that after teaching strategies and relationship activities to first graders, the students correctly answered more facts than a control group. In the timed tests rates were about 20 problems per minute in the experimental group and about 10 per minute in the traditional group. Although rates of responding did not indicate automaticity, approximately 55% of the experimental group, compared to 12% in the control group, reported that rather than using counting strategies, they had mostly "memorized" a set of 9 target subtraction facts.

Van Houten (1993) specifically taught rules to children (although the rules were unlike those normally used by children, or discussed in the literature) and found that the children "learned" a set of seven facts more rapidly when there was a "rule" to remember them with compared to a random mix of seven facts with no relationships. However, as in the other studies, Van Houten did not measure development of automaticity through some kind of timing criteria. Instead accuracy was the only measure of learning.

Isaacs & Carroll (1999) listed a potential instructional sequence of relationships to be explored for addition and subtraction facts:

"1. Basic concepts of addition; direct modeling and 'counting all' for addition
2. The 0 and 1 addition facts; 'counting on'; adding 2
3. Doubles (6 + 6, 8 + 8, etc.)
4. Complements of 10 (9 + 1, 8 + 2, etc.)
5. Basic concepts of subtraction; direct modeling for subtraction

6. Easy subtraction facts (-0, -1, and -2 facts); 'counting back' to subtract
7. Harder addition facts; derived-fact strategies for addition (near doubles, over-10 facts)
8. 'Counting up' to subtract
9. Harder subtraction facts; derived-fact strategies for subtraction (using addition facts, over-10 facts) (Isaacs & Carroll, 1999, p. 511-512)."

Baroody noted that, " 'mastery of the facts' would *include* discovering, labeling, and internalizing *relationships*. Meaningful instruction (the teaching of thinking strategies) would probably contribute more directly to this process than drill approach alone (1985, p. 95)." Certainly the research is clear that learning rules and relationships in the second stage helps students learn to give the correct answer to math facts. And with the variety of rules that have been researched and found to be effective, the exact nature of the rule children use is apparently immaterial. It appears that any rule or strategy that allows the child to remember the correct answer, insures that practice will be "perfect practice" and learning will be optimized. Performance may deteriorate if practice is not highly accurate; because if the student is allowed to give incorrect answers, those will then "compete" with the correct answer in memory (Goldman & Pellegrino, 1986). Learning facts in this second stage develops accurate but not necessarily automatic responding—which would be measured by rapid rates of responding.

Third stage: Developing automaticity with math facts

The third stage of learning math facts has been called mastery, or overlearning, or the development of automaticity. In this stage children develop the capacity to simply recall the answers to facts without resorting to anything other direct retrieval of the answer. Ashlock (1971) indicated that children must have "immediate recall" of the basic facts so they can use them "with facility" in computation. "If not, he may find it difficult to develop skill in computation for he will frequently be diverted into tangential procedures (1971, p. 363)."

When automaticity is developed, one of its most notable traits is speed of processing. "Proficient levels of performance go beyond the accuracy (quality) of an acquired skill to encompass sufficient speed (quantity) of performance. It is this sort of proficiency with basic facts, rather than accuracy per se, which is so notably lacking in many learning disabled children's computation performance (Garnett & Fleischner, 1983, p. 224)."

The development of automaticity has been studied extensively in the psychology literature. Research by psychologists on math facts and other variations on automatic fact retrieval has been extensive, probably because "Children's acquisition of skill at mental addition is a paradigm case of automaticity (Logan & Klapp, 1991a, p.180).

While research continues to refine the details of the models of the underlying processes, there has for some time been a clear consensus that adults "retrieve" the answers to math facts directly from memory. Psychologists "…argue that the process underlying automaticity is memory retrieval: According to these theories, performance is automatic when it is based on direct-access, single-step retrieval of solutions from memory rather than some algorithmic computation (Logan & Klapp, 1991a, p. 179)." When facts have been well practiced, they are "remembered" quickly and automatically—which frees up other mental processes to use the facts in more complex problems (Ashcraft, 1992; Campbell, 1987b; Logan, 1991a). A variety of careful psychological experiments have demonstrated that adults no longer use rules, relationships, or procedures to derive the answers to math facts—they simply remember or retrieve them (Ashcraft, 1985; Campbell, 1987a; Campbell, 1987b; Campbell & Graham, 1985; Graham, 1987; Logan, 1988; Logan & Klapp, 1991a; Logan & Klapp, 1991b; McCloskey, Harley, & Sokol, 1991).

For example, Logan and Klapp (1991a) did experiments with "alphabet arithmetic" where each letter corresponded to a number (A=1, B=2, C=3, etc.). In these studies college students initially experienced the same need to count to figure out sums that elementary children did. In the initial-counting stage the time to answer took longer the higher the addends (requiring more time to count up). However, once these "alphabet facts" became automatic—they took the same amount of time regardless of the addends, and they took less time than they could possibly have counted. In addition, they found that when students were presented with new items they had not previously had—they reverted to counting and the time to answer went up dramatically and was again related to the size of the addends. When the students were presented with a mix of items, some which required counting and others, which they had already learned, they were still able to answer the ones they had previously learned roughly as fast as they had before. Response times were able to clearly distinguish between the processes used on facts the students were "figuring out" vs. the immediate retrieval process for facts that had become automatic.

Does the same change in processes apply to children as they learn facts? Another large set of experiments has demonstrated that as children mature they transition from procedural (counting) methods of solving problems to the pattern shown by adults of direct retrieval. (Ashcraft, 1982; Ashcraft, 1984; Ashcraft, 1992; Campbell, 1987; Campbell & Graham, 1985; Geary & Brown, 1991; Graham, 1987; Koshmider & Ashcraft, 1991; Logan, 1991b; Pelegrino & Goldman, 1987). There is evidence that adults and children use strategies and rules for certain facts (Isaacs & Carroll, 1999). However, this phenomena appears to be limited to facts adding "+ 0" or "+ 1" or multiplying "x 0" or "x 1" and therefore does not indicate adult use of such strategies and rules for any of the rest of the facts (Ashcraft, 1985).

Interestingly, most of the research celebrating "strategies and rules" relates to addition and subtraction facts. Once children are expected to become fluent in recalling multiplication facts such strategies are inadequate to the task, and memorization seems to be the only way to accomplish the goal of mastery. Campbell and Graham's (1985) study of error patterns in multiplication facts by children demonstrated that as children matured errors became limited to products related correctly to one or the other of the factors, such as 4 X 7 = 24 or 6 X 3 = 15. See also (Campbell, 1987b; Graham, 1987). They concluded that "the acquisition of simple multiplication skill is well described as a process of associative bonding between problems and candidate answers....determined by the relative strengths of correct and competing associations ... not a consequence ... of the execution of reconstructive arithmetic procedures (Campbell & Graham, 1985, p. 359)." In other words, we remember answers based on practice with the facts rather than using some procedure to derive the answers. And what causes difficulty in learning facts is that we when we mistakenly bring up the answers to other similar math fact problems we sometimes fail to correctly discard these errors (Campbell, 1987b).

Graham (1987) reported on a study where students who learned multiplication facts in a mixed order had little or no difference in response times between smaller and larger fact problems. Graham's recommendation was that facts be arranged in sets where commonly confused problems are not taught together. In addition he suggested, "it may be beneficial to give the more difficult problems (e.g. 3 x 8, 4 x 7, 6 x 9, and 6 x 7) a head start. This could be done by placing them in the first set and requiring a strict performance criterion before moving on to new sets (Graham, 1987, p. 139)."

Ashcraft has studied the development of mental arithmetic extensively (Ashcraft, 1982; Ashcraft, 1984; Ashcraft, 1992; Koshmider & Ashcraft, 1991). These studies have examined response times and error rates over a range of ages. The picture is clear. In young children who are still counting, response rates are consistent with counting rates. Ashcraft found that young children took longer to answer facts—long enough to count the answers. He also found that they took longer in proportion to the sum of the addends—an indication that they were counting. After children learn the facts, the picture changes. Once direct retrieval is the dominant mode of answering, response rates decrease from over 3 seconds down to less than one second on average. Ashcraft notes that other researchers have demonstrated that while young children use a variety of approaches to come up with the answer (as they develop knowledge of the math facts), eventually these are all replaced by direct retrieval in 5^{th} and 6^{th} grade normally achieving children.

Ashcraft and Christy (1995) summarized the model of math fact performance from the perspective of cognitive psychology. "...the whole-number arithmetic facts are eventually stored in long-term memory in a network-like structure .... at a particular level of strength, with strength values varying as a function of an individual's practice and experience....because these variables determine strength in memory (Ashcraft & Christy, 1995, p. 398)." Interestingly enough, Ashcraft and Christy's (1995) examination of math texts led to the discovery in both addition and multiplication that smaller facts (2s-5s) occur about twice as frequently as larger facts (6s-9s). This may in part explain why larger facts are less well learned and have slightly slower response times.

Logan's (1991b) studies of the transition to direct retrieval show that learners start the process of figuring out the answer and use retrieval if the answer is remembered before the process is completed. For a time there is a mixture of some answers being retrieved from memory while others are being figured out. Once items are learned to the automatic level, the answer "occurs" to the learner before they have time to count. Geary and Brown (1991) found the same sort of evolving mix of strategies among gifted, normal, and math-disabled children in the third and fourth grades.

Why is automaticity with math facts important?

Psychologists have long argued that higher-level aspects of skills require that lower level skills be developed to automaticity. Turn-of-the-century psychologists eloquently captured this relationship in the phrase, "Automaticity is not genius, but it is the hands and feet of genius." (Bryan & Harter, 1899; as cited in Bloom, 1986). Automaticity occurs when tasks are learned so well that performance is fast, effortless, and not easily susceptible to distraction (Logan, 1985).

An essential component of automaticity with math facts is that the answer must come by means of direct retrieval, rather than following a procedure. This is tantamount to the common observation that students who "count on their fingers" have not mastered the facts. Why is "counting on your fingers" inadequate? Why isn't procedural knowledge or counting a satisfactory end point? "Although correct answers can be obtained using procedural knowledge, these procedures are effortful and slow, and they appear to interfere with learning and understanding higher-order concepts (Hasselbring, Goin and Bransford, 1988, p. 2)." The notion is that the mental effort involved in figuring out facts tends to disrupt thinking about the problems in which the facts are being used.

Some of the argument of this information-processing dilemma was developed by analogy to reading, where difficulty with the process of simply decoding the words has the effect of disrupting comprehension of the message. Gersten and Chard illuminated the analogy between reading and math rather explicitly. "Researchers explored the devastating effects of the lack of automaticity in several ways. Essentially they argued that the human mind has a limited capacity to process information, and if too much energy goes into figuring out what 9 plus 8 equals, little is left over to understand the concepts underlying multi-digit subtraction, long division, or complex multiplication (1999, p. 21)."

Practice is required to develop automaticity with math facts. "The importance of drill on components [such as math facts] is that the drilled material may become sufficiently over-learned to free up cognitive resources and attention. These cognitive resources may then be allocated to other aspects of performance, such as more complex operations like carrying and borrowing, and to self-monitoring and control (Goldman & Pellegrino, 1986, p. 134)." This suggests that even mental strategies or mnemonic tricks for remembering facts must be replaced with simple, immediate, direct retrieval, else the strategy for remembering itself will interfere with the attention needed on the more complex operations. For automaticity with math facts to have any value, the answers must be recalled with no effort or much conscious attention, because we want the conscious attention of the student directed elsewhere.

"For students to be able to recall facts quickly in more complex computational problems, research tells us the students must know their math facts at an acceptable level of 'automaticity.' Therefore, teachers...must be prepared to supplement by providing more practice, as well as by establishing rate criteria that students must achieve. (Stein et al., 1997, p. 93)." The next question is what rate criteria are appropriate?

How fast is fast enough to be automatic?

Some educational researchers consider facts to be automatic when a response comes in two or three seconds (Isaacs & Carroll, 1999; Rightsel & Thorton, 1985; Thorton & Smith, 1988). However, performance is, by definition, not automatic at rates that purposely "allow enough time for students to use efficient strategies or rules for some facts (Isaacs & Carroll, 1999, p. 513)."

Most of the psychological studies have looked at automatic response time as measured in milliseconds and found that automatic (direct retrieval) response times are usually in the ranges of 400 to 900 milliseconds (less than one second) from presentation of a visual stimulus to a keyboard or oral response (Ashcraft, 1982; Ashcraft, Fierman & Bartolotta, 1984; Campbell, 1987a; Campbell, 1987b; Geary & Brown, 1991; Logan, 1988). Similarly, Hasselbring and colleagues felt students had automatized math facts when response times were "down to around 1 second" from presentation of a stimulus until a response was made (Hasselbring et al. 1987)." If however, students are shown the fact and asked to read it aloud then a second has already passed in which case no delay should be expected after reading the fact. "We consider mastery of a basic fact as the ability of students to respond immediately to the fact question. (Stein et al., 1997, p. 87)."

In most school situations students are tested on one-minute timings. Expectations of automaticity vary somewhat. Translating a one-second-response time directly into writing answers for one minute would produce 60 answers per minute. Sixty problems per minute is exactly in the middle of the range of 40 to 80 problems per minute shown by adult teachers in this author's workshops. However, some children, especially in the primary grades, cannot write that quickly. "In establishing mastery rate levels for individuals, it is important to consider the learner's characteristics (e.g., age, academic skill, motor ability). For most students a rate of 40 to 60 correct digits per minute [25 to 35 problems per minute] with two or few errors is appropriate (Mercer & Miller, 1992, p.23)."

Howell and Nolet (2000) recommend an expectation of 40 correct facts per minute, with a modification for students who write at less than 100 digits per minute. The number of digits per minute is a percentage of 100 and that percentage is multiplied by 40 problems to give the expected number of problems per minute; for example, a child who can only write 75 digits per minute would have an expectation of 75% of 40 or 30 facts per minute.

One goal is to develop enough math fact skill in isolation, so that math fact speed will continue to improve as a result of using facts in more complex problems. Miller and Heward discussed the research finding that "students who are able to compute basic math facts at a rate of 30 to 40 problems correct per minute (or about 70 to 80 digits correct per minute) continue to accelerate their rates as tasks in the math curriculum become more complex....[however]...students whose correct rates were lower than 30 per minute showed progressively decelerating trends when more complex skills were introduced. The *minimum* correct rate for basic facts should be set at 30 to 40 problems per minute, since this rate has been shown to be an indicator of success with more complex tasks (1992, p. 100)." If a range from 30 to 40 problems per minute is recommended the higher end of 40 would be more likely to continue to accelerate than the lower end at 30, making 40 a better goal.

Another recommendation was that "the criterion be set at a rate [in digits per minute] that is about 2/3 of the rate at which the student is able to write digits (Stein et al., 1997, p. 87)." For example a student who could write 100 digits per minute would be expected to write 67 digits per minute, which translates to between 30 and 40 problems per minute.

In summary, if measured individually, a response delay of up to 1 second would be automatic. In writing the range of 30 to 40 seems to be the minimum, although a good case can be made that setting an expectation of 40 would be more prudent. However, students can be expected to be able to develop their fluency up to about 60 per minute, for students who can write that quickly. Sadly, many school districts have expectations as low as 16 to 20 problems per minute (from standards of 50 problems in 3 minutes or 100 problems in five minutes). Children can count answers on their fingers at the rate of 20 per minute. Such low expectations "pass" children who have only developed procedural knowledge of how to figure out the facts, rather than the direct recall of automaticity.

What type of practice effectively leads to automaticity?

Some children, despite a great deal of drill and practice on math facts, fail to develop fluency or automaticity with the facts. Hasselbring and Goin (1988) reported that researchers, who examined the effects of computer delivered drill-and-practice programs, have generally reported that computer-based drill-and-practice seldom leads to automaticity, especially amongst children with learning disabilities.

Ashlock explained the reason was that, "the major result of practice is increased rate and accuracy in doing the task that is *actually* practiced. For example, the child who practices counting on his fingers to get missing sums usually learns to count on his fingers more quickly and accurately. The child who uses skip counting or repeated addition to find a product learns to use repeated addition more skillfully—but he continues to add. Practices does not necessarily lead to more mathematically mature ways of finding the missing number or to immediate recall as such....If the process to be reinforced is recalling, then it is important that the child feel secure to state what he recalls, even if he will later check his answer...(1971, p. 363)"

Hasselbring et al. stated that, "From our research we have concluded that if a student is using procedural knowledge (i.e., counting strategies) to solve basic math facts, typical computer-based drill and practice activities do not produce a developmental shift whereby the student retrieves the answers from memory (1988, p. 4)." This author experienced the same failure of typical practice procedures to develop fluency in his own students with learning disabilities. Months and sometimes multiple years of practice resulted in students who barely more fluent than they were at the start.

Hasselbring and Goin noted that the minimal increases in fluency seen in such students

> "can be attributed to students becoming more efficient at counting and not due to the development of automatic recall of facts from memory....We found that if a learner with a mild disability has not established the ability to retrieve an answer from memory before engaging in a drill-and-practice activity then time spend in drill-and-practice is essentially wasted. On the other hand, if a student can retrieve a fact from memory, even slowly, then use of drill-and-practice will quickly lead to the fluent recall of that fact....the key to making computer-based drill an activity that will lead to fluency is through additional instruction that will establish information in long-term memory....Once acquisition occurs (i.e., information is stored in long term memory), then drill-and-practice can be used effectively to make the retrieval of this information fluent and automatic (1988, p. 203)."

For practice to lead to automaticity, students must be "recalling" the facts, rather than "deriving" them.

Other researchers have found the same phenomena, although they have used different language. If students can recall answers to fact problems, rather than derive them from a procedure, those answers have been "learned." Therefore continued practice could be called "overlearning." For example, "Drill and practice software is most effective in the overlearning phase of learning—i.e., effective drill and practice helps the student develop fast and efficient retrieval processes (Goldman & Pelegrino, 1986).

If students are not recalling the answers and recalling them correctly, accurately, while they are practicing—the practice is not valuable. So teaching students some variety of memory aid would seem to be necessary. If the students can't recall a fact directly or instantly, if they recall the "trick" they can get the right answer—and then continue to practice remembering the right answer.

However, there may be another way to achieve the same result. Garnett, while encouraging student use of a variety of relationship strategies in the second stage, hinted at the answer when she suggested that teachers should "press for speed (direct retrieval) with a few facts at a time (1992, p. 213)."

How efficient is practice when a small set of facts is practiced (small enough to remember easily)? Logan and Klapp (1991a) found that college students required less than 15 minutes of practice to develop automaticity on a small set of six alphabet arithmetic facts. "The experimental results were predicted by theories that assume that memory is the process underlying automaticity. According to those theories, performance is automatic when it is based on single-step, direct-access retrieval of a solution from memory; in principle, that can occur after a single exposure. (Logan & Klapp, 1991a, p. 180)."

This suggests that if facts were learned in small sets, which could easily be remembered after a couple of presentations, the process of developing automaticity with math facts could proceed with relatively little pain and considerably less drill than is usually associated with learning "all" the facts. "The conclusion that automatization depends on the number of presentations of individual items rather than the total amount of practice has interesting implications. It suggests that automaticity can be attained very quickly if there is not much to be learned. Even if there is much to be learned, parts of it can be automatized quickly if they are trained in isolation (Logan and Klapp, 1991a, p. 193)."

Logan & Klapp's studies show that, "The crucial variable [in developing automaticity] is the number of trials per item, which reflects the opportunity to have memorized the items (1991a, p. 187)." So if children are only asked to memorize a couple of facts at a time, they could develop automaticity fairly quickly with those facts. "Apparently extended practice is not necessary to produce automaticity. Twenty minutes of rote memorization produced the same result as 12 sessions of practice on the task! (Logan, 1991a, p. 355)."

Has this in fact been demonstrated with children? As has been noted earlier few researchers focus on the third stage development of automaticity. And there are even fewer studies that instruct on anything less than 7 to 10 facts at a time. However, in cases where small sets of facts have been used, the results have been uniformly successful, even with students who previously had been "unable" to learn math facts. Cooke and colleagues also found evidence in practicing math facts to automaticity, "suggesting that greater fluency can be achieved when the instructional load is limited to only a few new facts interspersed with a review of other fluent facts (1993, p. 222)." Stein et al. indicate that a "set" of "new facts" should consist of no more than four facts (1997, p. 87). Hasselbring et al. found that,

> "Our research suggest that it is best to work on developing declarative knowledge by focusing on a very small set of new target facts at any one time—no more than two facts and their reversals. Instruction on this target set continues until the student can retrieve the answers to the facts consistently and without using counting strategies. …
>
> We begin to move children away from the use of counting strategies by using 'controlled response times.' A controlled response time is the amount of time allowed to retrieve and provide the answer to a fact. We normally begin with a controlled response time of 3 seconds or less and work down to a controlled response time around 1.25 seconds.
>
> We believe that the use of controlled response times may be the most critical step to developing automatization. It forces the student to abandon the use of counting strategies and to retrieve answers rapidly from the declarative knowledge network.
>
> If the controlled response time elapses before the child can respond, *the student is given the answer* and presented with the fact again. This continues until the child gives the correct answer within the controlled response time. (1988, p.4)." [emphasis added].

Giving the answer to the student, rather than allowing them to derive the answer changes the nature of the task. Instead of simply *finding the answer* the student is involved in checking to see if he or she *remembers* the answer. If not, the student is reminded of the answer and then gets more opportunities to practice "remembering" the fact's answer. Interestingly this finding—that it is important to allow the student only a short amount of time before providing the answer—has been studied extensively as a procedure psychologists call "constant time delay."

Several studies have found that using a constant time delay procedure alone for teaching math facts is quite effective (Bezuk & Cegelka, 1995). This "near-errorless technique" means that if the student does not respond within the time allowed, a "controlling prompt" (typically a teacher modeling the correct response) is provided. The student then repeats the task and the correct answer (Koscinski & Gast, 1993b)." The results have shown that this type of near-errorless practice has been effective whether delivered by computer (Koscinski & Gast, 1993a) or by teachers using flashcards (Koscinski & Gast, 1993b).

Two of the aspects of the constant time delay procedures are instructionally critical. One is the time allowed is on the order of 3 or 4 seconds, therefore ensuring that students are using memory retrieval rather than reconstructive procedures, in other words, they are remembering rather than figuring out the answers. Second, if the student fails to "remember" he or she is immediately told the answer and asked to repeat it. So it becomes clear that the point of the task is to "remember" the answers rather than continue to derive them over and over.

These aspects of constant time delay teaching procedures are the same ones that were found to be effective by Hasselbring and colleagues. Practicing on a small set of facts that are recalled from memory until those few facts are answered very quickly contrasts sharply with the typical practice of timed tests on all 100 facts at a time. In addition, the requirement that these small sets are practiced until answers come easily in a matter of one or two seconds, is also unusual. Yet, research indicates that developing very strong associations between each small set of facts and their answers (as shown by quick, correct answers), before moving on to learn more facts will improve success in learning.

Campbell's research found that most difficulty in learning math facts resulted from interference from correct answers to "allied" problems—where one of the factors was the same (Campbell, 1987a; Campbell, 1987b; Campbell & Graham, 1985). He asked the question, "How might interference in arithmetic fact learning be minimized?…If strong correct associations are established for problems and answers encountered early in the learning sequence (i.e. a high criterion for speed and accuracy), those problems should be less susceptible to retroactive interference when other problems and answers are introduced later. Furthermore, the effects of proactive interference on later problems should also be reduced (Campbell, 1987b, p. 119-120)."

Because confusion among possible answers is the key problem in learning math facts, the solution is to establish a mastery-learning paradigm, where small sets of facts are learned to high levels of mastery, before adding any more facts to be learned. Researchers have found support for the mastery-learning paradigm, when they have looked at the development of automaticity. The mechanics of achieving this gradual mastery-learning paradigm have been outlined by two sets of authors. Hasselbring et al. found that,

"As stated, the key to making drill and practice an activity that will lead to automaticity in learning handicapped children is additional instruction for establishing a declarative knowledge network. Several instructional principles may be applied in establishing this network:

1. Determine learner's level of automaticity.
2. Build on existing declarative knowledge.
3. Instruct on a small set of target facts.
4. Use controlled response times.
5. Intersperse automatized with targeted nonautomatized facts during instruction.

(1988, p. 4)."

The teacher cannot focus on a small set of facts and intersperse them with facts that are already automatic until after an assessment determines which facts are already automatic. Facts that are answered without hesitation after the student reads them aloud would be considered automatic. Facts that are read to the student should be answered within a second.

Silbert, Carnine, and Stein recommended similar make-up for a set of facts for flashcard instruction. "Before beginning instruction, the teacher makes a pile of flash cards. This pile includes 15 cards: 12 should have the facts the student knew instantly on the tests, and 3 would be facts the student did not respond to correctly on the test. (1990, p. 127)."

These authors also recommended procedures for flashcard practice similar to the constant time delay teaching methods. "If the student responds correctly but takes longer than 2 seconds or so, the teacher places the card back two or three cards from the front of the pile. Likewise, if the student responds incorrectly, the teacher tells the student the correct answer and then places the card two or three cards back in the pile. Cards placed two or three back from the front will receive intensive review. The teacher would continue placing the card two or three places back in the pile until the student responds acceptably (within 2 seconds) four times in a row (Silbert et al., 1990, p. 130)."

Hasselbring et al. described how their computer program "Fast Facts" provided a similar, though slightly more sophisticated presentation order during practice. "Finally, our research suggests that it is best to work on developing a declarative knowledge network by interspersing the target facts with other already automatized facts in a prespecified, expanding order. Each time the target fact is presented, another automatized fact is added as a 'spacer' so that the amount of time between presentations of the target fact is expanded. This expanding presentation model requires the student to retrieve the correct answers over longer and longer periods (1988, p. 5)."

Unfortunately the research-based math facts drill-and-practice program "Fast Facts" developed by Hasselbring and colleagues was not developed into a commercial product. Nor were their recommendations incorporated into the design of existing commercial math facts practice programs, which seldom provide either controlled response times or practice on small sets. What practical alternatives to cumbersome flashcards or ineffective computer drill-and-practice programs are available to classroom teachers who want to use the research recommendations to develop automaticity with math facts?

How can teachers implement class-wide effective facts practice?

Stein et al. tackled the issue of how teachers could implement an effective facts practice program consistent with the research. As an alternative to total individualization, they suggest developing a sequence of learning facts through which all students would progress. Stein et al. describe the worksheets that students would master one at a time.

> "Each worksheet would be divided into two parts. The top half of the worksheets should provide practice on new facts including facts from the currently introduced set and from the two preceding sets. More specifically, each of the facts from the new set would appear four times. Each of the facts from the set introduced just earlier would appear three times, and each of the facts from the set that preceded that one would appear twice….The bottom half of the worksheet should include 30 problems. Each of the facts from the currently introduced set would appear twice. The remaining facts would be taken from previously introduced sets. (1997, p. 88)."

The daily routine consists of student pairs, one of which does the practicing while the other follows along with an answer key. Stein et al. specify:

"The teacher has each student practice the top half of the worksheet twice. Each practice session is timed…the student practices by saying complete statements (e.g., 4 + 2 = 6), rather than just answers….If the student makes an error, the tutor corrects by saying the correct statement and having the student repeat the statement. The teacher allows students a minute and a half when practicing the top part and a minute when practicing the bottom half. (1997, p. 90)."

After each student practices, the teacher conducts a timed one-minute test of the 30 problems on the bottom half. Students who correctly answered at least 28 of the 30 items, within the minute allowed on the bottom half test, have passed the worksheet and are then given the next worksheet to work on (Stein, et al., 1997). A specific performance criterion for fact mastery is critical to ensure mastery before moving on to additional material. These criteria, however, appear to be lower than the slowest definition of automaticity in the research literature. Teachers and children could benefit from higher rate of mastery before progressing, in the range of 40 to 60 problems per minute—if the students can write that quickly.

Stein et al. also delineate the organizational requirements for such a program to be effective and efficient: "A program to facilitate basic fact memorization should have the following components:

1. a specific performance criterion for introducing new facts.
2. intensive practice on newly introduced facts
3. systematic practice on previously introduced facts
4. adequate allotted time
5. a record-keeping system
6. a motivation system (1997, p. 87)."

The worksheets and the practice procedures ensure the first three points. Adequate allotted time would be on the order of 10 to 15 minutes per day for each student of the practicing pair to get their three minutes of practice, the one-minute test for everyone and transition times. The authors caution that "memorizing basic facts may require months and months of practice (Stein et al., 1997, p. 92)." A record-keeping system could simply record the number of tries at each worksheet and which worksheets had been passed. A motivation system that gives certificates and various forms of recognition along the way will help students maintain the effort needed to master all the facts in an operation.

Such a program of gradual mastery of small sets of facts at a time is fundamentally different than the typical kind of facts practice. Because children are learning only a small set of new facts it does not take many repetitions to commit them to memory. The learning is occurring during the "practice" time. Similarly because the timed tests are only over the facts already brought to mastery, children are quite successful. Because they see success in small increments after only a couple of days practice, students remain motivated and encouraged.

Contrast this with the common situation where children are given timed-tests over all 100 facts in an operation, many of which are not in long term memory (still counting). Students are attempting to become automatic on the whole set at the same time which requires "brute force" to master. Because children's efforts are not focused on a small set to memorize, students often just become increasingly anxious and frustrated by their lack of progress. Such tests do not teach students anything other than to remind them that they are unsuccessful at math facts. Even systematically taking timed tests daily on the set of 100 facts is ineffective as a teaching tool and is quite punishing to the learner. “Earlier special education researchers attempted to increase automaticity with math facts by systematic drill and practice...But this “brute force” approach made mathematics unpleasant, perhaps even punitive, for many (Gersten & Chard, 1999, p. 21).”

These inappropriate type timed tests led to an editorial by Marilyn Burns in which she offered her opinion that, “Teachers who use timed tests believe that the tests help children learn basic facts....Timed tests do not help children learn (Burns, 1995, p. 408-409).” Clearly timed tests only establish whether or not children have learned—they do not teach. However, if children are learning facts in small sets, and are being taught their facts gradually, then timed tests will demonstrate this progress. Under such circumstances, if children are privy to graphs showing their progress, they find it quite motivating (Miller, 1983).

Given the most common form of timed tests, it is not surprising that Isaacs and Carroll argue that “...an over reliance on timed tests is more harmful than beneficial (Burns, 1995), this fact has sometimes been misinterpreted as meaning that they should never be used. On the contrary, if we wish to assess fact proficiency, time is important. Timed tests also serve the important purpose of communicating to students and parents that basic-fact proficiency is an explicit goal of the mathematics program. However, daily, or even weekly or monthly, timed tests are unnecessary (1999, p. 512).”

In contrast, when children are successfully learning the facts, through the use of a properly designed program, they are happy to take tests daily, to see if they’ve improved. “Results from classroom studies in which time trials have been evaluated show…Accuracy does not suffer, but usually improves, and students enjoy being timed...When asked which method they preferred, 26 of the 34 students indicated they liked time trials better than the untimed work period (Miller & Heward, 1992, p. 101-102).”

In summary, the proper kind of practice can enable students to develop their skill with math facts beyond strategies for remembering facts into automaticity, or direct retrieval of math fact answers. Automaticity is achieved when students can answer math facts with no hesitation or no more than one second delay—which translates into 40 to 60 problems per minute. What is required for students to develop automaticity is a particular kind of practice focused on small sets of facts, practiced under limited response times, where the focus is on remembering the answer quickly rather than figuring it out. The introduction of additional new facts should be withheld until students can demonstrate automaticity with all previously introduced facts. Under these circumstances students are successful and enjoy graphing their progress on regular timed tests. Using an efficient method for bringing math facts to automaticity has the added value of freeing up more class time to spend in higher level mathematical thinking.

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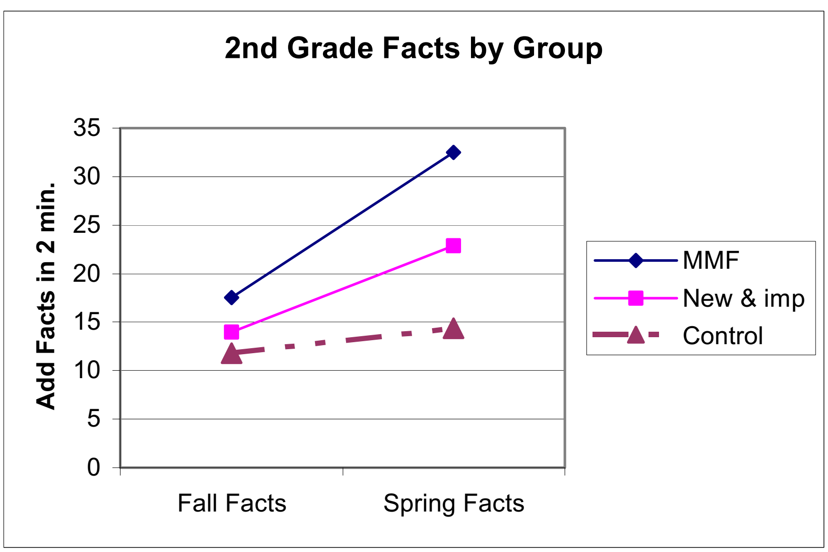
All groups: Testing

1. *Took the Stanford Diagnostic Math Test –Computation Subtest: 25 minutes, multiple choice in September and again in May. Use the answer form provided.*
2. *Chose one operation to study and took 3 math fact timings (two minutes in length) of all students in September and 3 timings again in May. Score was the average number of problems correctly answered in two minutes.*

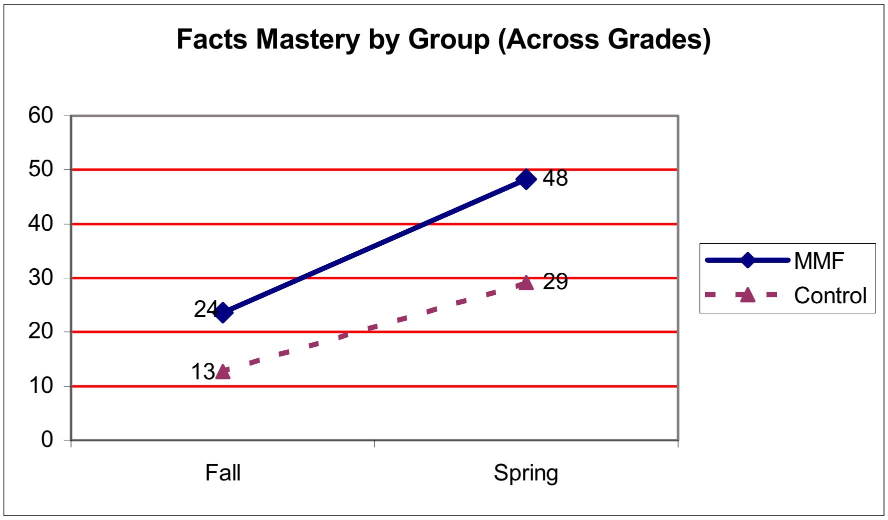
Three groups were set up.
The control group stayed as they were. They used whichever methods of teaching/learning math facts they were using before the study. The second group, was the standard" *Mastering Math Facts* group. This group followed ***MMF*** standard directions and procedures as written. They took the time to do MMF daily (either practice or two-minute progress monitoring sheet). They began all students in the chosen operation. They gave the two-minute timing in the chosen operation once per week. They collected that data on the student graph (writing down the number correct in two minutes as well as graphing it). A third group was designated the New and Improved" *Mastering Math Facts* group. They followed standard ***Mastering Math Facts*** procedures with some modifications. They cut time in half for timings, to allow for higher number per minute. They kept raising goals with no upper limit. They didn't move students on until they had passed three times instead of once.

Findings of improvement on Math Facts

A comparison among the three groups was possible only amongst 2nd grade students where there were 3 classes, one in each condition, all of whom studied addition facts. No other grade offered such a clear comparison. The results show suggest that MMF in its standard set of directions performs best. It is notable that the control group made almost no progress in math facts relative to either of the groups using the MMF materials.



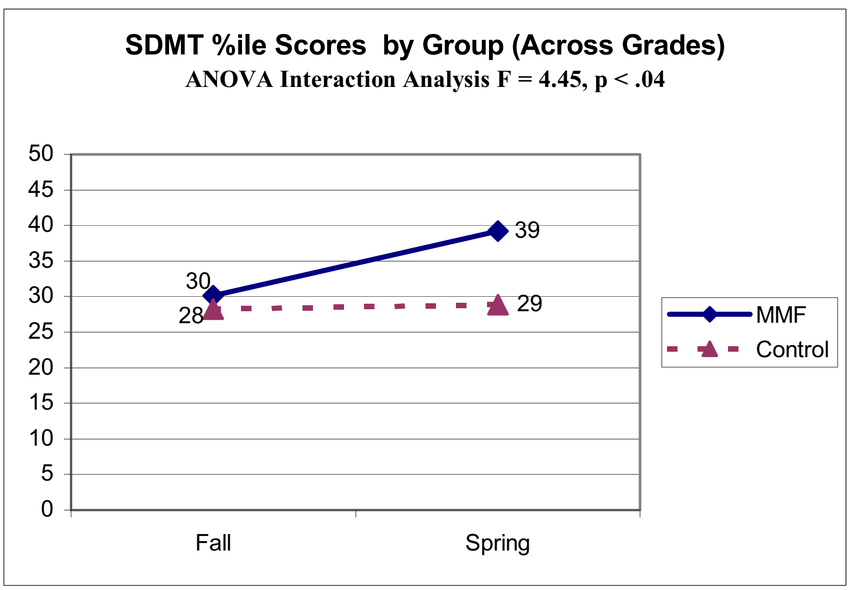
Because there was no significant difference between the Standard ***MMF*** group and the New and Improved ***MMF*** group in the school as a whole, those results were collapsed into one group.



As can be seen from the graph, students did improve in math fact fluency, and as a result of using Mastering Math Facts students improved significantly more (approximately twice) the improvement of students in the control condition. Students who used ***MMF*** also ended the year significantly better at facts than the control group children.

Effects on Math Achievement

Perhaps more interesting are the effects upon achievement as measured by scores on the Stanford Diagnostic Math Test – computation subtest.



Scores reported are the national percentile scores with the different norms appropriate to the time of testing. Students in the control group did not significantly change their ranking from fall to spring against the expected improvement as reflected in the spring norms. Students in the MMF group however did show a significant rise in their national ranking in math achievement during the year.

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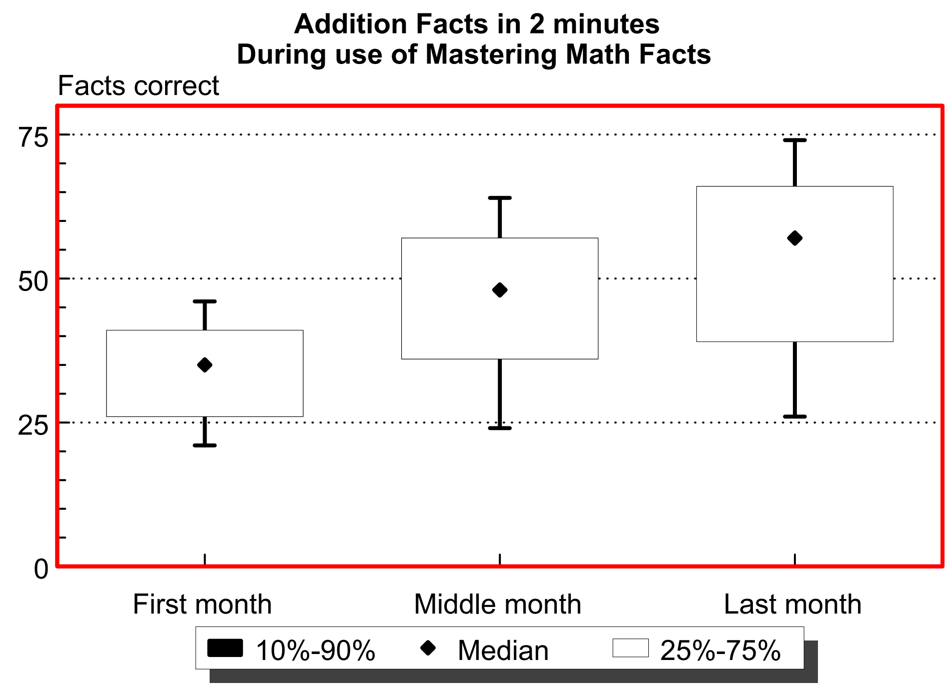
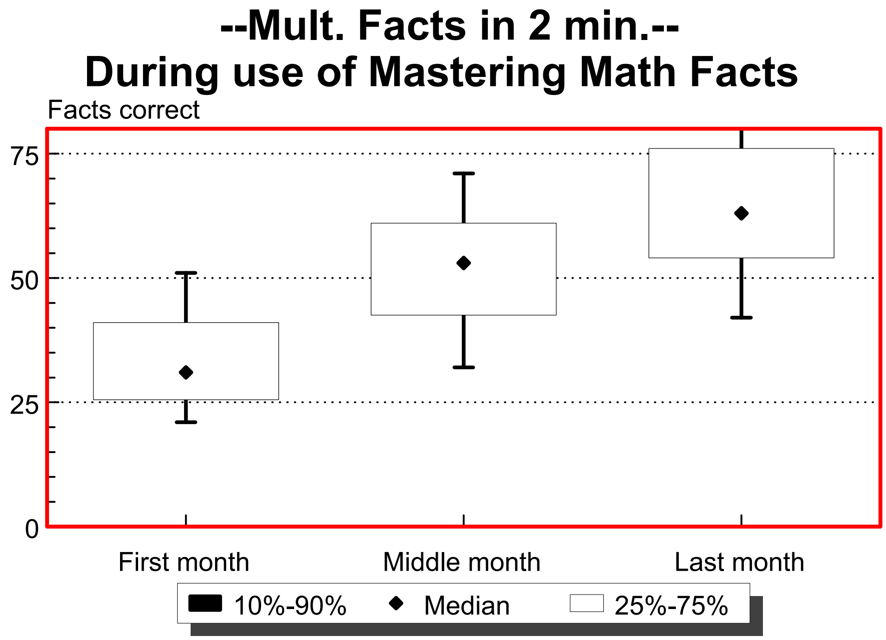
This school did not wish to employ a control group. Several teachers across all grades implemented Mastering Math Facts with varying levels of fidelity to program guidelines. Data were collected from math timings, scores representing the number of facts correctly answered in writing during a 2-minute test. The data from all students who were working on a given operation were included without regard to grade level. Data were collected from the beginning of practice on that operation, the middle month of practice and the end of practice for each student.

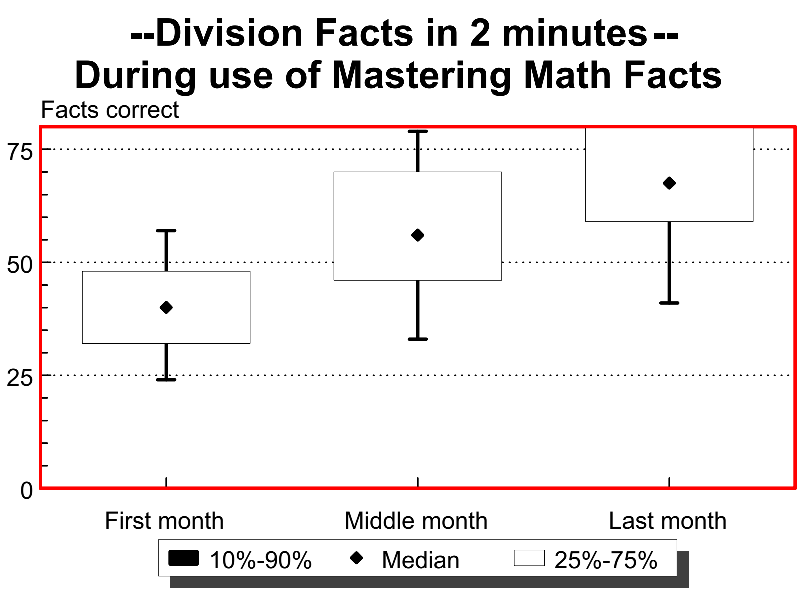
The data clearly show improvement in fluency of math facts during the course of working through the program of ***Mastering Math Facts***.



The sample size for the addition operation was 115 students. The mean number of facts correct at the outset was 34 facts in two minutes (with a standard deviation of 9.6). The middle month mean was up to 46 facts in two minutes (with a standard deviation of 14.3). The last month mean was 52 (with a standard deviation of 17.1).



The sample size for multiplication was 61 students. The mean number of facts correct at the outset was 34 facts in two minutes (with a standard deviation of 12.7). The middle month mean was up to 52 facts in two minutes (with a standard deviation of 13.4). The last month mean was 62 (with a standard deviation of 13.7).



The sample size for division was 50 students. The mean number of facts correct at the outset was 41 facts in two minutes (with a standard deviation of 12.2). The middle month mean was up to 56 facts in two minutes (with a standard deviation of 15.4). The last month mean was 66 (with a standard deviation of 14.9).