# Mastering the Basic Math 

# Facts in Multiplication and Division 

Strategies, Activities \& Interventions to Move Students Beyond Memorization

Susan O'Connell and John SanGiovanni

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## Dedication

To the little guys, Colin and Liam, with love S.O.

To Krissy, my favorite math teacher and wine connoisseur J.S.

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## How to Access the Downloadable Resources

Step 1: Go to www.heinemann.com.
Step 2: Click on "Login" to open or create your account. Enter your email address and password or click "Register" to set up an account.

Step 3: Enter keycode and click "Register."

## Foreword

Being able to add and subtract within 20 and multiply and divide within 100 is essential during the early years of schooling, and the basic facts of addition/ subtraction and multiplication/division are a critical baseline, not only then but also during later work with fractions, decimals, ratio, proportion, and more. Foundational? You better believe it. Essential? Absolutely.

That said, the basic facts are also problematic. The goal is for most students to know, fluently, and with automaticity, the addition/subtraction facts, typically by the end of second grade and the multiplication/division facts, typically by the end of third grade. But far too many teachers are unable to help their students reach these goals. "Not this year," they may mutter, or, "Not all my students," or worse, "Not ever." Why is fluency with the basic facts such a challenge for so many students? In our digit-conscious culture students can spout off multiple phone and pin numbers, but not the product of $6 \times 7$ ! I meet and work with middle school students who are still wondering about $8 \times 7$ or $48 \div 6$ and other basic facts. Why do far too many students fail to realize that the commutative property means that $9+7$ and $7+9$ get you to the same place, 16? This drives us all crazy! Have we neglected the basics? Is this about just having students memorize the facts? No, and no!

Over twenty years ago the Curriculum and Evaluation Standards for School Mathematics noted that "children should master the basic facts of arithmetic that are essential components of fluency with paper-pencil and mental computation and with estimation" (47). ${ }^{1}$ The National Research Council's Adding It Up dedicates almost ten pages to synthesizing the research dealing with basic fact acquisition. ${ }^{2}$ More recently, the Final Report of the National Mathematics Advisory Panel points out that computational proficiency with whole number operations depends on the practice (I prefer the term rehearsal) necessary to develop automatic recall of addition/subtraction and multiplication/ division facts. ${ }^{3}$ Nurturing computational facility in elementary school requires that students be fluent with the basic facts of arithmetic. How do we get this done?

Over the years teachers have tried and continue to use a myriad of practice activities-oral and written exercises, games, and classroom and homework

[^0]assignments, many of them now via the Internet. At last we have a more effective option-Susan O'Connell and John SanGiovanni's Mastering the Basic Facts in Addition and Subtraction: Strategies, Activities \& Interventions to Move Students Beyond Memorization and Mastering the Basic Facts in Multiplication and Division: Strategies, Activities \& Interventions to Move Students Beyond Memorization. What a find!

Based on Thornton's pioneering work ${ }^{4}$ emphasizing how thinking strategies facilitate fact acquisition, both books present activities that develop facility with the basic facts by building a conceptual understanding of the operations; following a teaching sequence designed to develop a sense of number using fact strategies and the commutative property; and using representational models and context-based problem solving. (The activities that link facts to their conceptual representations are also powerful diagnostic tools.) But there's more-related children's literature, partner activities, a professional-learning-community study guide. All these components add up to resources that engage students, from beginning activities that promote an understanding of arithmetic concepts, through fluency with the basic facts.

One final consideration: these books will be very helpful to teachers whose students' mathematical knowledge require some level of intervention. The powerful instructional opportunities these books provide not only make sense but also meet one of the key recommendations of the What Works Clearinghouse's Practice Guide Assisting Students Struggling with Mathematics. ${ }^{5}$

These books won't end up on a shelf at the back of your room. (And if you are a third/fourth-grade teacher you will probably need both of them.) You'll use them every day. You'll carry them home with you and talk about them in the faculty lounge. Just as the basic facts are "must haves" on the path to computational fluency, these books are "must haves" to help you navigate the route.

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[^1]
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## Introduction

As math teachers, we want our students to develop a quick recall of singledigit addition, subtraction, multiplication, and division facts. We label them basic math facts because they provide a foundation for math success. We expect that all students will master these basic skills, but that is not a simple goal to achieve. We watch some students effortlessly remember the facts and others struggle with the very same task. And we labor to find just the right strategies and activities to help all students succeed.

As teachers, we are constantly reminded that our students learn in a variety of ways. Although some students have very strong memory skills, others struggle to remember simple facts. Although some students make sense of math concepts on their own, others struggle to connect meaning to simple expressions like $3 \times 5$. Although some students intuitively use their knowledge of one math fact to solve a related fact, others simply get frustrated and discouraged when they cannot remember a specific sum or product. Our students are so different, and yet our goal for each of them is the same: to master basic math facts so they have a strong foundation for more complex math skills and procedures. The goal of this book is to explore numerous strategies and activities that support all students in understanding basic multiplication and division facts and committing those facts to memory. Whether you are introducing students to basic math facts, reviewing previously taught facts, or providing interventions for students who continue to struggle, this book supplies you with instructional considerations, practical strategies, and numerous classroom-tested activities.

## What are basic math facts?

For the purpose of this book, basic facts are considered to be facts with factors of $0-10$. In some programs, facts with single-digit factors ( $0-9$ ) are considered basic, but because of the significance of the $\times 10$ facts, they are included within this book. An understanding of $\times 10$ facts provides an important benchmark for understanding $\times 5$ facts or might suggest a strategy for determining $\times 9$ facts. The inclusion of $\times 10$ facts is based on providing a solid understanding of numbers as the foundation for our study of basic facts.

## What constitutes mastery of basic math facts?

In the past, much of mathematics was taught in a drill and practice style. Students were simply asked to memorize their math facts, often without much attention to conceptual understanding. Through worksheets filled with single-digit computations or lengthy flash card sessions, students were asked to memorize multiplication and division facts. Our goal in today's math classrooms has shifted from memorizing facts and procedures to increased understanding of math skills and concepts. We want our students to be able to do mathematics, but we also want them to understand the math they are doing. We recognize that as math tasks increase in complexity, an understanding of facts, formulas, and algorithms will help them experience continued success. We have not changed our view of the importance of basic math facts. We know that they are a foundational skill and that without that skill our students will view even simple math tasks as daunting. We have simply expanded our expectations to include understanding as an important component of our teaching of basic math facts. So, what do we expect of our students? Our goal is both automaticity and understanding. Automaticity is students' ability to effortlessly recall a fact. If students are automatic, they have successfully committed the facts to memory. In addition, we want our students to understand, not simply remember, these important math facts.

## Why memorize math facts?

Ask math teachers what they would like their students to know and be able to do, and the recall of basic math facts will undoubtedly rank high on most of their wish lists. Teachers recognize that once their students know $3 \times 5$, those students are better able to explore $3 \times 50$ or $3 \times 55$. Teachers recognize that students will have an easier time finding the solution to $3 \times \$ 5.00$ or $3 \times .5$. These teachers know that their students will be more successful when they are challenged with $\frac{2}{3} \times \frac{1}{5}$. As math tasks become more complex, we want our students to possess the foundational skills to be successful.

We have gained insights from brain research about demands on the working brain. As students begin to learn math facts, their brains are focused on those basic computations, but as students become automatic with basic facts, their brains are then able to focus on other aspects of the task like the challenges
of place value, decimals, or fractions. Being automatic with basic facts frees the brain to focus on other math processes.

Committing basic math facts to memory speeds up math tasks. As math tasks increase in complexity, they often require multiple steps to find the solution. Multiplication with three-digit factors and division with decimals are examples of more complex computational tasks. These tasks are timeconsuming, and often stressful, for students who must stop to figure out each basic fact along the way. And stopping to determine each fact disrupts the flow of the math procedure. The National Mathematics Advisory Panel (2008) urges that students develop automatic recall of multiplication and related division facts to be prepared for the study of algebra, in which solving multistep equations is a fundamental task.

Students who have committed basic math facts to memory are able to perform critical mental math tasks. They estimate answers prior to solving problems so they are able to compare their estimates to the actual answers and determine the reasonableness of their solutions. When browsing through a grocery store, students with mental math skills can determine the approximate cost of buying 4 bags of pretzels if each bag costs about $\$ 3.00$, or when counting the savings in their piggy banks, they can quickly determine that their 9 nickels amount to 45 cents in savings. As students determine how to fairly share 36 cookies among 9 friends, they can automatically determine the quantity each child will receive. Mastery of basic facts provides the foundation for everyday mental math tasks.

Automaticity means the quick and effortless recall of math facts. No need to count every object. No need to think about related facts. No need to extend patterns. The answer is automatically known. Although automaticity is a goal for our students, alone it is not enough. Students must first understand the facts that they are being asked to memorize.

## Why is it important to understand math facts?

During the memorization process, students are supported by an understanding of what they are being asked to memorize. Memorizing a chain of nonsensical words (e.g., sat chair red girl a in little the) is more difficult than memorizing a sentence in which the words have a meaning (e.g., A little girl sat in the red chair.). Asking students to memorize dozens of number facts can be discouraging and confusing when students view them simply as pairs of numbers. The
understanding that $7 \times 3$ represents 7 groups of 3 items aids the memorization process.

Students who rely solely on the memorization of math facts often confuse similar facts. Consider the multiplication facts $9 \times 5=45$ and $9 \times 6=54$. The products are commonly confused by students who have only memorized answers. Students often mix up the products and respond mistakenly that $9 \times$ $5=54$; however, for students who have explored an understanding of multiplication, $9 \times 5=54$ just doesn't make sense: $10 \times 5=50$, so how could $9 \times 5$ $=54$ ? The product has to be less than 50 ! A focus on understanding multiplication and division facts will provide students with a firm foundation rather than simply relying on memory.

Students who simply memorize math facts miss a prime opportunity to expand their understanding of equations. Problem solving is the central focus in today's math classrooms. To be a successful problem solver, students must be able to accurately compute answers, but more than that, they must be able to figure out how to build equations that correspond to problem situations.

## Colin was stacking books on the shelves of his brand-new bookcase. He put 7 books on each of the 4 shelves. How many books did he put on the shelves of his bookcase?

This problem certainly requires the student to know that $4 \times 7=28$, but even before the student can use his knowledge of math facts to find the answer, he must understand how to build an equation that works with this problem.

There are 4 shelves and each shelf has 7 books on it. Since each shelf has the same number of books, I can multiply. 4 groups of 7 or $4 \times 7$ is how I find the answer!

As we discuss the connection between the meaning of the equation and the basic math fact, we are supporting both students' computation skills as well as building a strong foundation for problem solving.

Both the Common Core State Standards (National Governors Association Center for Best Practices and Council of Chief State School Officers 2010) and the National Council of Teachers of Mathematics' Principles and Standards (2000) emphasize the importance of students understanding the concepts of multiplication and division. The Common Core State Standards recommend that second graders be given opportunities to explore problems with equal-sized groups of objects to build a foundation for multiplication. Third-grade students then continue to explore the concepts, work with properties of operations, and apply
their understanding of numbers and properties to develop effective strategies for multiplying and dividing. Understanding is developed first, with practice for fluency coming later.

## How can we help students master basic math facts?

We expect that our students will quickly recall facts without the need for manipulatives or number strategies. Although memorization is a part of the process, we recognize that students benefit from varied opportunities to explore basic math facts before being asked to memorize them. An instructional approach in which students investigate the meaning of facts through hands-on activities and thoughtful discussions, explore strategies to support their understanding of numbers, and then engage in strategic practice to memorize the facts provides students with a strong and balanced foundation for mastery.

Understanding operations is fundamental to understanding math facts. Students develop deeper understanding of operations through problem posing, hands-on explorations, real-world examples, classroom discussions, and exploring situations from children's literature. Division scenarios that show fair sharing and multiplication stories that demonstrate combining equal groups help students strengthen their understanding of operations, and students who understand operations will find that math facts make sense.

There are many ways that students might arrive at an answer to a math fact. When multiplying $4 \times 3$, Bailey might count every object in the 4 groups of 3 to find the total, and Liam might simply remember that $4 \times 3=12$. Math fact strategies lie somewhere between counting each object and simply memorizing the answer. They are predictable and efficient ways to find answers. Allison knows that $3 \times 4=12$, so $4 \times 3=12$, too. Brendan might recognize doubles-if $2 \times 3=6$, then $4 \times 3$ is twice that amount or 12 -and Katie knows that $3 \times 3=9$, which is 3 groups of 3 , so $4 \times 3$ is just 1 more group of 3 , which means 12. Strategies help students find an answer even if they forget what was memorized. Discussing math fact strategies focuses attention on number sense, operations, patterns, properties, and other critical number concepts. These big ideas related to numbers provide a strong foundation for the strategic reasoning that supports mastering basic math facts. For multiplication and division, strategic reasoning related to doubling and halving, the commutative property, zero and ones properties, recognizing patterns, and breaking
numbers apart to find related products provides students with a solid foundation for mastery of math facts.

Once an understanding of operations has been developed and students have explored strategic reasoning to find solutions to basic math facts, it is time to engage students in meaningful practice so they can commit the facts to memory. Rather than long practice sessions (Remember the lengthy flash card drills of days past?), consider activities that are short in duration but easy to implement, so students are frequently engaged in valuable practice. Scattered practice-five to ten minutes a day, spread throughout the school yearyields great results. And varying the practice activities so students remain motivated and engaged is essential to the process. Brief, frequent, interactive activities that provide students with repeated exposure to math facts support automaticity.

Because of the anxiety associated with memorization tasks for many students, the practice tasks in this book do not focus on speed or elimination. Although speed drills or elimination games may be enjoyed by some students, these types of activities often intensify the frustration and anxiety of others. Students who struggle with rote memory tasks, those students who are the reason we include math fact activities in our daily schedule, are just the ones who become discouraged by the speed drills or experience humiliation when they are the first to be eliminated. These are the students we want to motivate-the students we need to engage in repeated practice sessions. You will find that many practice activities are presented throughout this book. It is important to select the ones that work for your students. Although some students find competitive activities fun and motivating, others thrive on collegial tasks.

## How can this book help you?

This book is a practical guide for helping students master multiplication and division facts. It includes insights into the teaching of basic math facts including a multitude of instructional strategies, teacher tips, and classroom activities designed to help students master their facts. The emphasis is on strengthening students' understanding of numbers, patterns, and properties as an essential component of math fact teaching. Whether you are introducing your students to basic math facts, providing reviews to support their mastery, or looking for intervention strategies for students who have been exposed to math facts but have not reached mastery, this book will provide you with valuable resources, insights, and options.

You will find activities and resources for introducing students to basic math facts. You will find tips for generating student talk about math facts including examples of questions and prompts that direct students' thinking toward big ideas and lead them to insights that will simplify the task of mastering the facts. You will find activities to support varied levels of learners so that you can choose the right activity to extend learning for high-level students or modify skills to support struggling students. You will find strategies that are hands-on, engaging, and interactive to motivate reluctant students. You will find activities perfect for small-group interventions and others that work well for whole-class instruction or individual support. And you will find downloadable resources to ease your planning and preparation.

This book is a compilation of strategies and activities that are organized to provide a solid math facts program; however, the individual activities and strategies can be easily integrated into your existing math program to provide you with additional resources and varied instructional approaches. You may read the book from start to finish or you may focus on specific sections that address your needs. Consider your students and select the strategies and activities to match their needs, interests, learning styles, and abilities.

## How is this book organized?

Throughout the following chapters, multiple teaching strategies and activities are shared to build students' understanding and automaticity with math facts. Each chapter is organized to develop essential understanding and provide a menu of possible activities for instruction, practice, and assessment. Following are highlights of the key elements in Chapters 2 through 12.

## Making Connections and Focusing on the Big Ideas

Each chapter begins by connecting the new fact set to students' previous experiences and provides a brief overview of big ideas that play a key role in students' understanding of the facts and students' development of strategies related to the facts.

## Developing Understanding

Each chapter provides two introductory lessons that focus on developing conceptual understanding of the highlighted math facts. One lesson is a Literature Link, introducing the facts through a story context. The other lesson, Exploring the Facts, provides a language-based and/or hands-on exploration with the new set of facts. The activities in this book employ varied
instructional techniques, including the use of manipulatives, visuals, literature, and discussions, ensuring that students experience multiplication and division facts in diverse ways and that each student will be likely to experience these facts in a way that makes sense to him.

In Supporting All Learners, you will find more ideas for those students who may need additional or different types of experiences to develop understanding of the targeted facts. These activities might be done with the whole class but may also be perfect for small teacher-led groups of students. For some sets of facts, you may choose to use several of these activities; at other times, your students may not need the additional exposure. These activities simply provide you with more and varied possibilities for developing understanding.

## Building Automaticity

This section focuses on building students' fluency and is broken into two parts: Targeted Practice and Monitoring Progress. In Targeted Practice, a variety of activities are shared that provide practice for that specific set of math facts. Students will have fun rolling number cubes, spinning spinners, and pulling number cards from a deck as they engage in ongoing practice through interactive activities. It is through repeated and targeted practice that students gain fluency with math facts. Templates for these activities can be found among the accompanying downloadable resources.

Along with repeated practice to gain fluency with math facts, students need constant monitoring to ensure that they are progressing in their mastery of facts. Monitoring Progress provides ideas for monitoring students’ growth toward automaticity including ideas for conducting frequent Fact Checks and suggestions for varied ways to track students' progress including student conferences, progress graphs, and individual goal setting.

## Connecting to Division

Multiplication facts are the primary emphasis throughout this book because of our focus on building math fact fluency. When posed with a division math fact, the most efficient way to solve it is by knowing the related multiplication fact. When the recall of multiplication facts is automatic and students understand the connection between multiplication and division facts, their fluency with division facts naturally increases.

Lessons to develop students' understanding of related division facts are included in each chapter. In addition, you will find suggestions throughout the book of activities to build division fact fluency. To attain fluency with divi-
sion facts, students need ongoing opportunities to explore their connections to multiplication facts.

## What is the teaching sequence of math facts within this book?

Although the chapters are organized by specific math facts (e.g., multiplying by fives), you will notice that the focus of each chapter is the big ideas that guide students to understand that set of multiplication and division facts, to create effective math strategies related to the facts, and to ultimately commit those facts to memory. This book is not simply a collection of activities; it is intended to highlight big ideas that provide a perfect focus for math facts instruction, to broaden your repertoire of instructional strategies, to provide you with dozens of easy-to-implement activity ideas, and to stimulate your reflection related to the teaching of math facts. In reviewing the organization of this book, you may also notice that the multiplication facts do not appear in numerical order ( $0-10$ ), but rather appear in a sequence that focuses on the complexity of the number concepts and carefully links each new set of facts to previously explored facts, building upon students' prior knowledge.

The teaching sequence of facts suggested within this book begins with simpler facts and then connects each new set of facts to the previously mastered ones. A traditional sequence of learning math facts from 0 to 10 does not capitalize on students' prior knowledge, nor does it present simpler facts first. Beginning with $\times 2, \times 10$, and $\times 5$ facts allows students to explore multiplication with patterns that are familiar. $\times 1$ and $\times 0$ facts are addressed next. Although these facts are simple to memorize, they are a bit atypical of the grouping element of multiplication (i.e., What is a group of 1 ? What is 0 groups of 4?). Once $0,1,2,5$, and 10 are mastered, students have developed a strong foundation on which to build mastery of the remaining facts.

Figure 1 outlines a brief rationale for the sequence in which the facts are introduced within this book. We recognize, however, that students and instructional programs differ and that teachers might choose, or be required, to introduce facts in a different sequence. Although we believe that there is strong justification for this sequence, we have carefully developed strategies and activities that support instruction of math facts even if the order in which you present the facts differs from the sequence described in Figure 1.

| Foundation Facts |  |
| :---: | :---: |
| $\times 2$ | Students have extensive experience skip-counting by twos and grouping in twos (pairs) and have developed an understanding of doubling. This set of facts is a natural place to begin exploring multiplication facts. |
| $\times 10$ | The understanding of 10 is foundational in our number system. Students have experience skip-counting by 10 , grouping in tens, and working with models of 10 , such as ten-frames and base-ten blocks. |
| $\times 5$ | Students have extensive experience skip-counting by 5. They recognize connections with money concepts (nickels). Previous exploration with $\times 10$ facts leads to the insight that multiplying by 5 can be thought of as half of multiplying by 10 . |
| $\times 1$ | Although $\times 1$ facts are simple to memorize, we do not begin with $\times 1$ facts because of the confusion with the grouping aspect of multiplication (e.g., groups of 1 ?). Providing students with opportunities to explore groups of 2,5 , and 10 provides a stronger foundation for understanding multiplication facts. |
| $\times 0$ | $\times 0$ facts are easy for students to commit to memory because the product is always 0 , but this set of facts can be challenging for concrete thinkers. It is difficult to conceptualize a group of nothing. Once students have explored multiplication with 2, 10,5, and 1 , this set of facts becomes easier to understand. |
| Building on the Foundation |  |
| $\times 3$ | Multiplying by 3 can be thought of as multiplying by 2 and then adding 1 more group, or as tripling a number. |
| $\times 4$ | Multiplying by 4 can be thought of as doubling a double. The previous mastery of $\times 2$ facts allows students to double $\times 2$ products to find the $\times 4$ products. |
| $\times 6$ | Multiplying by 6 can be thought of as doubling a multiple of 3. Previous mastery of $\times 3$ facts allows students to see that $4 \times 6$ can be thought of as double $4 \times 3$, or ( $4 \times$ 3) $+(4 \times 3)$. Previous mastery of $\times 5$ facts also supports students with $\times 6$ facts, knowing that the product of a $\times 6$ fact is simply 1 set more than the product of the related $\times 5$ fact (e.g., the product of $6 \times 8$ is 8 more than the product of $5 \times 8$ ). |
| $\times 9$ | Building on knowledge of $\times 10$ facts, the product of a $\times 9$ fact is 1 group less than the product of the same $\times 10$ fact (e.g., $10 \times 5=50$, so $9 \times 5=45$, which is 5 less, or $10 \times$ $7=70$ and $9 \times 7=63$, which is 7 less). |
| $\times 8$ | Multiplying by 8 results in a product that is double that of multiplying by 4 . With the teaching sequence suggested in this book, only two of these facts have not been explored through a different strategy ( $7 \times 8$ and $8 \times 8$ ). |
| $\times 7$ | Multiplying by 7 may be the most difficult for students. Students can break apart the 7 (distributive property) to find that it is the sum of 5 times the factor and 2 times the factor (e.g., $7 \times 4$ is $(5 \times 4)+(2 \times 4)$. Although this works, it is more efficient to simply think commutative property and reverse the order of the factors. By doing this, students realize that they already know all of the $\times 7$ facts except $7 \times 7$. |

Figure 1. This suggested teaching sequence begins with simpler facts and then connects each new set of facts to students' previous experiences.

The lessons and activities in this book focus on strengthening students' number concepts to support their mastery of basic math facts. Teachers who have a deep understanding of big ideas related to numbers and the ways in which those big ideas relate to the teaching of math facts, and who have developed a repertoire of instructional techniques and classroom activities to highlight those big ideas, are able to simplify the task of mastering basic math facts for their students.

## What downloadable resources are available?

Along with the many easy-to-implement student activities within the book, customizable activities, templates, and recording sheets are available to download from www.heinemann.com (see instructions to the right). Because the activities are Microsoft Word documents, you can easily modify the activity page to make it simpler or more complex, personalize the tasks to motivate and engage your students, and adapt the activities to maintain your students' interest. A resource presented in one chapter to provide practice with a specific set of facts has often been modified for other fact sets (see Additional Resources) to provide you with a wide array of practice options. The downloadable resources also include teacher tools (e.g., hundred charts, multiplication tables, game templates, assessment options) to simplify your planning and reduce your preparation time.

## How to Access the Downloadable Resources

Step 1: Go to www.heinemann.com.

Step 2: Click on "Login" to open or create your account. Enter your email address and password or click
"Register" to set up an account.

Step 3: Enter keycode and click
"Register."

## Resource for Professional Learning Communities

Effective teachers constantly reflect on their own teaching. They gather new ideas, try them with students, reflect on their successes, and find ways to continually refine their teaching. At the conclusion of this book, questions are posed to stimulate reflection about the key points within the chapters. These guiding questions are designed for your personal reflection or for use in school-based study groups. Discussion about math facts instruction within our professional learning communities broadens our understanding and improves our teaching.

## Our Goal

The purpose of this book is to explore ways to support all students in mastering multiplication and division facts. By focusing on big ideas, strengthening students' understanding of math operations, developing strategic thinking, and providing varied and engaging practice tasks to promote fluency, our students will be better equipped to both understand math facts and commit the facts to memory. Whether you are introducing students to basic facts, reviewing facts, or providing remediation for struggling students, this book will provide you with insights and activities to simplify this complex, but critical, component of math teaching.

Math facts with 5 as a factor build on students' previously learned math facts skills. Just as with the $\times 2$ and $\times 10$ facts, patterns are a powerful way to learn $\times 5$ facts. Students' extensive experience skipcounting by fives provides them with essential prior knowledge to master these facts. In addition, realworld connections, like five fingers on a hand or five pennies in a nickel, are commonplace, so our students have experience thinking in fives. And now that students have explored the $\times 10$ facts, they can use this knowledge to better understand $\times 5$ facts, thinking of them as half of the related $\times 10$ fact.


## Focusing on the Big Ideas

When focusing on multiplication with 5 as a factor, some big ideas that support students in developing critical strategies include the following.

## Multiplication by 5 is like skip-counting by 5.

Students have already explored the connection between skip-counting and multiplication, and most students have had lots of experience skip-counting by fives. Reminding students of this connection, and helping them recognize that they already know the product of $5 \times 6$ if they can skip-count 5 , $10,15,20,25,30$, will help them build on their previous knowledge and ease their anxiety about learning this new set of math facts.

## Our number system is a system of patterns.

Patterns in multiplication by 5 are immediately evident to students. Students quickly notice that all of the products have either a 0 or 5 in the ones place and that the products alternate between even numbers and odd numbers. Observing, discussing, and generalizing about patterns helps students make sense of the $\times 5$ facts.

## 5 is half of 10. Multiplying a number by 5 will result in a product that is half of the product that results when the same number is multiplied by 10.

From their earlier experiences with counting, addition, and subtraction, students have developed an understanding of 10 and have internalized the concept that 5 is half of 10 . We have 10 fingers, 5 on each hand. A tenframe is made up of 2 five-frames. In a hundred chart, each row consists of 10 numbers, 2 sets of 5 . This understanding helps students figure out $\times 5$ facts from known $\times 10$ facts by simply cutting the product in half.

## The order of the factors does not change the product (the commutative property).

Whether students are visualizing 5 groups of a certain size (e.g., $5 \times 4$ or 5 groups of 4) or whether they are visualizing groups of 5 (e.g., $4 \times$ 5 or 4 groups of 5), they notice that the products are the same. Continued investigations with the commutative property build students' confidence that the order of the factors does not change the product.

These big ideas about numbers are central to students' understanding and should guide the types of questions we pose to stimulate discussions about $\times 5$ facts.

What patterns do you notice in the products when you multiply by 5 ?
How do the products for $\times 5$ facts connect to the products for $\times 10$ facts?
If you forget a $\times 5$ fact, could you find the answer by knowing the related $\times 10$ fact? How?

Does the order of the factors affect the product? Give examples to justify your thinking.

## Understanding $\times 5$ Facts

## Exploring the Facts: An Investigation with Pennies

To begin an exploration of $\times 5$ facts, students work with partners to determine the amount of money in sets of piggy banks. Provide partners with the piggy bank work mat (see DR ) and at least 50 pennies, then pose the following problem.

Mrs. Alexander bought each of her 7 grandchildren a brand new piggy bank. She went to the bank to get enough pennies to put 5 pennies in each of their piggy banks. How many pennies did she need?


Have students work together to solve the problem using their pennies, then have students share their solutions with the class. Probe with questions like the following.

> How many piggy banks did she have?
> How did you find the total number of pennies she needed?
> After you placed the pennies in each bank, did you count all of the pennies to find the total? Were there other ways you might have found the total?
> Did anyone skip-count to find the total? Which way would be faster, skip-counting or counting all of the pennies? Why?
> Did anyone add to find the total number of pennies? Why would addition work to solve this problem?
> Did anyone multiply? What multiplication math fact would help you solve this problem?

Record on the board $7 \times 5=35$ as you verbalize " 7 groups of 5 pennies is 35 pennies" or " 7 piggy banks with 5 pennies in each bank is 35 total pennies."

Challenge students to work with partners to find the total number of pennies needed for 1-10 piggy banks and to write multiplication equations to show their answers (e.g., $1 \times 5=5,2 \times 5=10$, etc.). Remind students that Mrs. Alexander wants each bank to hold exactly 5 pennies. Tell students that they may use the pennies and banks if it helps them find the totals, or they may find the totals in another way, but their goal is to find the products for the $\times 5$ facts.

Once students have completed their charts, have them share the $\times 5$ facts as you record them on the board. Then, ask students to work with partners to observe for patterns. Have them share their insights with the class. Their insights are likely to include:

- The ones digit is always 0 or 5 .
- The products alternate odd, then even.
- Some of the products are $\times 10$ products.
- Every other product is a $\times 10$ product.

Challenge students to talk about why these patterns appear.

Tip Connections to realworld experiences provide insights about $\times 5$ facts. Finding the value of sets of nickels or determining the minutes that have passed when the minute hand is on the 4 shows students everyday examples of multiplication by 5 .

Figure 4.1 This student shares insights for finding $\times 5$ products.

What can you do to help you remember the products of 5 ?


## Literature Link: Count on Pablo

In Count on Pablo, by Barbara deRubertis (1999), Pablo and his grandmother prepare and sell vegetables at an outdoor market. The story provides a review of previously taught math facts as well as an exploration of $\times 5$ facts as Pablo and his grandmother sell onions tied in pairs ( $\times 2$ facts), tomatoes in boxes of 10 ( $\times 10$ facts), and peppers in bags of 5 ( $\times 5$ facts). Pablo skip-counts to determine the number of vegetables being prepared for market, but through classroom explorations, the story allows for an easy transition from skip-counting to multiplication.

Before Reading Do a picture walk through the book, showing students the illustrations and asking them to predict what the story might be about. Discuss outdoor markets at which vendors sell their fresh fruits and vegetables.

Ask students to think about their experiences at grocery stores. Are fruits and vegetables sold individually or are they sometimes sold in packages? Why might they be sold in groups or packages?

During Reading As you read the story, be sure to emphasize the way in which the vegetables are packed for sale at the market (e.g., individually, pairs, groups of 5 or 10). Encourage students to join in as Pablo skip-counts throughout the story.

After Reading Ask students a few comprehension questions to be sure they understand some of the key aspects of the story.

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How did Pablo and Abuela sell the onions? Individually? In groups?
Groups of what size?
How did Pablo and Abuela package the tomatoes?
How did Pablo and Abuela package the peppers?
Provide students with a fifty chart (see DR) and some transparent counters. Have them choose a color counter (e.g., red) to show $\times 10$ facts by placing counters on the correct products as you review $1 \times 10,2 \times 10,3 \times$ $10,4 \times 10$, and $5 \times 10$. Ask students to think about the counters as Pablo's boxes of tomatoes. The first counter covers the number of tomatoes in 1 box, the second covers the number of tomatoes in 2 boxes, and so on. Have students refer to the fifty chart to tell you the number of tomatoes in $1,2,3,4$, and 5 boxes. This would be a great opportunity to introduce, or reinforce, the meaning of the word multiples. Explain that products are

| Fifty chart |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | also called multiples. Their red counters are covering multiples of 10 , or any

Tip
A classroom word wall is a great way to display new math vocabulary as it appears in your lessons.

If transparent counters are not available, students might place small paper clips or beans on each number. Selecting transparent or small manipulatives allows students to still view the numbers on the chart.
number that is a product when you multiply by 10 . For example $10,20,30,40$ are all multiples of 10 because they are products of a number multiplied by 10 .

Have students remove the counters from their fifty chart. Tell students they will be using a different-color transparent counter (e.g., green) to show the number of peppers in Pablo's bags. Guide students' thinking with the following questions.

## Will they be placing counters on every tenth number? Why or why not?

Where will they place counters?
How many peppers are in each bag?
---Have students show Pablo's counting of the peppers by placing green counters on the fifty chart (e.g., placing counters on $5,10,15,20$, etc.). Have students turn and share their observations of their charts with a partner. What do they notice? This time, counters are placed on every fifth number or each counter represents 5 numbers. Begin a chart of $\times 5$ facts on the board, asking students to help you complete it by finding the products on their fifty chart. As you record each $\times 5$ fact, ask students to tell you the product. Clarify each fact with words and symbols saying "So, one group of 5 is . . . ?" as you record $1 \times 5=5$. Continue to record and state each fact until the chart has facts from 1-10.

When the $\times 5$ fact list is completed, ask students to keep the green counters on the multiples of 5 on their fifty charts, but replace the red counters on the multiples of 10 . Inform them that there may be more than 1 counter on a number. Ask them to turn to their partners and share their observations about the charts. Have a class discussion of their observations, being sure to ask them to explain why some numbers are both multiples of 5 and 10 .

Extend the lesson by posing the following problem for students to discuss with their partners. Remind students that you will be asking them to share their solutions as well as how they arrived at their solutions.

## If Pablo sold 8 pairs of onions, how many onions did he sell?

Move through the room to listen to partners' discussions. Ask students to share their solutions and record some of their methods on the board (e.g., drawing pictures, skip-counting, adding, multiplying). Acknowledge that all of the methods work, but talk about how quickly students who knew the multiplication fact $(8 \times 2)$ were able to find the solution. Then, pose the following problem.

## If Pablo sold 5 boxes of tomatoes, how many tomatoes did he sell?

Before students begin the task, as them to silently show you, by holding up the correct number of fingers, how many tomatoes are in each box. Quickly scan the room to be sure students understand that each box holds 10 tomatoes. Then, move through the room to listen to partners' discussions as they solve the problem. Have them share their solutions with the class and again, record their methods and reinforce the efficiency of using multiplication $(5 \times 10)$ to find the solution. Finally, pose the following problem.

## If Pablo sold 4 bags of peppers, how many peppers did he sell?

Again check for understanding of peppers being in bags of 5, by asking students to show the number of peppers in a bag by holding up the appropriate number of fingers. Then, challenge students to decide on the multiplication fact that would solve the problem (see Figure 4.2).

Pose a final problem for students to turn and share with a partner.

Figure 4.2 Student $A$ uses pictures to make sense of the problem, and student $B$ uses an understanding of the connection to skip-counting.

## Student A



Student B

If Pablo sold 4 bags of peppers, how many peppers did he sell?
$5 \times 4=20$


## Pablo had 23 peppers and decided to sell them in groups of 5 . Could he sell all of his peppers that way? Why?

Move through the room to listen to students' mathematical talk. Are they using their fifty chart to see if 23 is covered? Are they skip-counting by fives to see if 23 is named? Are they referring to the list of $\times 5$ facts to see if 23 appears? Are they grouping the counters to test their thinking? Are they noticing that 23 does not end in a 0 or 5 ? Have students share their thoughts in a class discussion. Then, ask students to write in their journals with this prompt.

Danny says 23 is a multiple of 5 . Do you agree or disagree? Explain your thinking.

## Supporting All Learners

Team Problems Many students respond to team problem-solving tasks in which they can hear others' thinking and test their ideas. A problem card related to the $\times 5$ facts (see DR ) is given to each team of three or four students. Students work together to solve the problem. Team members must write a multiplication equation to solve their problem and must also show the solution in at least one other way (e.g., using manipulatives, drawing pictures, skip-counting, etc.). Team members raise their hands when a problem is solved, the teacher checks the solution and their thinking, and then hands the students another problem card. Groups move through the problems at their own pace. After a set amount of time, solutions and strategies can be shared with the class, but whole-class sharing is not necessary if the teacher has met with individual teams to hear students' thinking as they solve each problem. Varying the complexity of the problems will give you the flexibility to assign problems that best match the needs of each team. Problems may include:

Deryn has 5 bags of cookies. There are 4 cookies in each bag. How many cookies does she have?

Oscar reads for 10 minutes each night for 5 nights. How many minutes does he read altogether?

Kris has a photo album. It has 8 blue pages with 5 photos on each page, and 5 red pages with 6 photos on each page. How many photos are in her album?

Chanting Facts Rhythmically chanting math facts, or setting math facts to familiar songs, may be helpful for some students, particularly those with attention problems or other learning difficulties. Chanting math facts in a rhythmic way requires students to repeatedly verbalize a set of facts (i.e., 1 times 5 is 5 , 2 times 5 is 10,3 times 5 is 15 , etc.). Some students respond well to singing the facts to familiar tunes like "The Farmer in the Dell." Just be sure that students are focusing on the ideas and not simply chanting or singing words. Fluency is only one part of math fact mastery. Without a foundation of understanding, memory can be very fleeting.

Making Connection to Money Concepts Ideas for relating $\times 10$ facts to money were shared in Chapter Three and can be easily adapted for $\times 5$ facts. For $\times 5$ facts, students connect the number of nickels to the total amount of money, so 4 nickels would total $4 \times 5$ or 20 cents, because each nickel represents 5 cents. The Dimes and Cents activity for $\times 10$ facts has been modified to a Nickels and Cents activity (see DR) to strengthen $\times 5$ facts. Allowing students to use nickel manipulatives will support students who still need to explore the facts in a hands-on way.

The How Much Money? activity (see the DR) challenges students to compare the values of similar quantities of nickels and dimes. This activity continues to build students' number sense as it supports the development of the big idea that multiples of 5 are half of multiples of 10 .

Making Connections to Concepts About Time Facts that have 5 as a factor provide an excellent opportunity to connect the concepts of multiplication and telling time. The numbers on a clock symbolize 5 -minute increments (e.g., the 4 on a clock face represents 20 minutes because there are 4 groups of 5 minutes). Students can use movable clocks to represent different math facts. Students might begin by moving the minute hand of the clock to each number on the clock face while skip-counting by fives, then progress to writing multiplication equations that connect the number on the clock face to the number of minutes.


TipUsing beans or counters to cover game boards, rather than marking off products with pens or markers, allows students to clear the board and begin again each time they complete a game. Games that allow for repeated play are providing repeated practice with math facts!


Tip Once students have experience completing In/Out tables in which the In column is organized (numbers progressing from 1-10), mix the order of the numbers in the In column, so students cannot rely on patterns to complete the table and must rely on their math fact knowledge.

## Building Automaticity

## Targeted Practice

Introducing Math Games To maximize the impact of math fact games, model the games with the class before students have the opportunity to play games with partners. Games can be modeled on an overhead, on a SMART Board, or with a document camera to allow students to both see and hear the rules. During modeling, explain the rules and explore possible strategies, or thinking points, for the game. Field questions about the rules of the game or what to do when different scenarios arise. Have students play the game with partners as you move through the room observing students' understanding of the rules. A thorough introduction to each game will result in smoother play when students are working on their own.

Corners Corners (see DR) provides students with targeted practice for $\times 5$ facts. Players take turns spinning a spinner and multiplying the number by 5. Students then place a counter, or bean, on a space on the board with that product. The goal of this game is to cover 4 adjacent spaces to form the 4 corners of a square.

Ratio Tables Provide students with a vertical In/Out (ratio) table and ask them to record the product for each fact as in the Math Facts Column activity in the DR. These tables spur discussions and prompt writing about patterns seen in the rows or columns. Ratio tables can also be used to explore real-world connections (e.g., 5 points on a star or 5 toes on a foot (see DR activity)). And feel free to vary the format to create rows instead of columns. Whether tables are vertical or horizontal, they provide nice repetition of the facts.


Fact Card Arrays Provide students with a set of $\times 5$ fact cards and a set of 50 counters. Have students select a fact card and make an array to represent the fact. Students count the number of rows, the number of counters in each row, and the total number of counters, and then record their data. Remind students that skip-counting, or simply knowing the multiplication fact, are more efficient ways to find the total rather than counting every individual counter.

Independent Fact Card Reviews The following activities minimize the stress sometimes associated with fact card practice, as they emphasize knowledge of the answer rather than speed. Suggest one of the following for students who need additional practice with the $\times 5$ facts.

Pick a card, or a designated number of cards, and ...

- Draw an array of the fact.
- Write the repeated addition sentence that goes with it.
- Write the fact, with the product, three times.
- Write a story problem for the fact.


## Monitoring Progress: Working Toward Automaticity

Giving students frequent opportunities to engage in independent fact reviews provides them with repeated practice, supports fluency, and allows for ongoing monitoring of each student's progress toward automaticity. Fact Checks might focus specifically on the fact being taught (e.g., $\times 5$ facts) or might combine previously learned facts (e.g., $\times 2, \times 10$, and $\times 5$ facts). The same Fact Check can be done numerous times as students work toward fluency.

Our goal for students is automaticity with math facts, but automaticity takes time. Students benefit from frequent practice and lots of teacher support (see Figure 4.3). Students can be more successful if they learn their math facts in manageable pieces and progress at reasonable rates. In this book, fact groups are sequenced so that students have a manageable number of facts to focus on at any given time, and those facts have natural connections to students' previous learning. A large menu of possible activities allows you to continue work on a fact set if students are having difficulty mastering it. Don't rush! Long-term retention is the goal.

Tip
Practice activities from previous chapters can be easily adapted for $\times 5$ facts. You might want to try:

Fact Card Jumps (Chapter Two) Fact Grids (Chapter Three)

TipFollowing Fact Checks, have students use yellow highlighters to indicate known facts on a completed multiplication chart (see DR). By highlighting known facts, students will also be able to quickly focus on facts that still need to be mastered. Encourage students to select a few facts as their goal for the next Fact Check.

Figure 4.3 This teacher frequently monitors student progress and provides ongoing support.


## Connecting to Division

## Linking to Count on Pablo

The activities for Count on Pablo (deRubertis 1999) can be easily modified to focus on division facts. Pablo and his grandmother put 5 peppers in each bag to take to the market. Possible division problems include:

Pablo has 30 peppers. How many bags does he need?

Pablo has 45 peppers in bags. How many bags of peppers does Pablo have?

These problems are wonderful opportunities for students to work with partners, use manipulatives, draw pictures, and write equations to solve division problems. They also provide a wonderful opportunity for students to connect their ideas about division with their understanding of multiplication.


## Divide and Go

Students can practice their division facts by playing Divide and Go (see DR). Players spin a spinner and divide by 5. The quotient is the amount of points they earn for that turn. Players accumulate and keep track of their points using a counter on a hundred chart. For example, a player has her counter on 24. She spins 45 and divides by 5 to get 9 points. She then moves her counter to 33 . The first player to get to exactly 100 , without going over, is the winner.

## Modifying Multiplication Activities

An In/Out table can be used to help students build automaticity with multiplication facts. By changing the function on the table, it becomes an opportunity for practicing division facts (see the Math Facts Column activity in the DR).



[^0]:    ${ }^{1}$ National Council of Teachers of Mathematics. 1989. Curriculum and Evaluation Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics.
    ${ }^{2}$ National Research Council. 2001. Adding It Up: Helping Children Learn Mathematics. Washington, DC: National Academy Press.
    ${ }^{3}$ National Mathematics Advisory Panel. 2008. Foundations for Success: The Final Report of the National Mathematics Advisory Panel. Washington, DC: U.S. Department of Education.

[^1]:    4 Thornton, C.A. 1978. "Emphasizing Thinking Strategies in Basic Fact Instruction." Journal for Research in Mathematics Education. 16: 337-355.

    5 Gersten et al. 2009. Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools. Washington, DC: Institute of Education Sciences.

