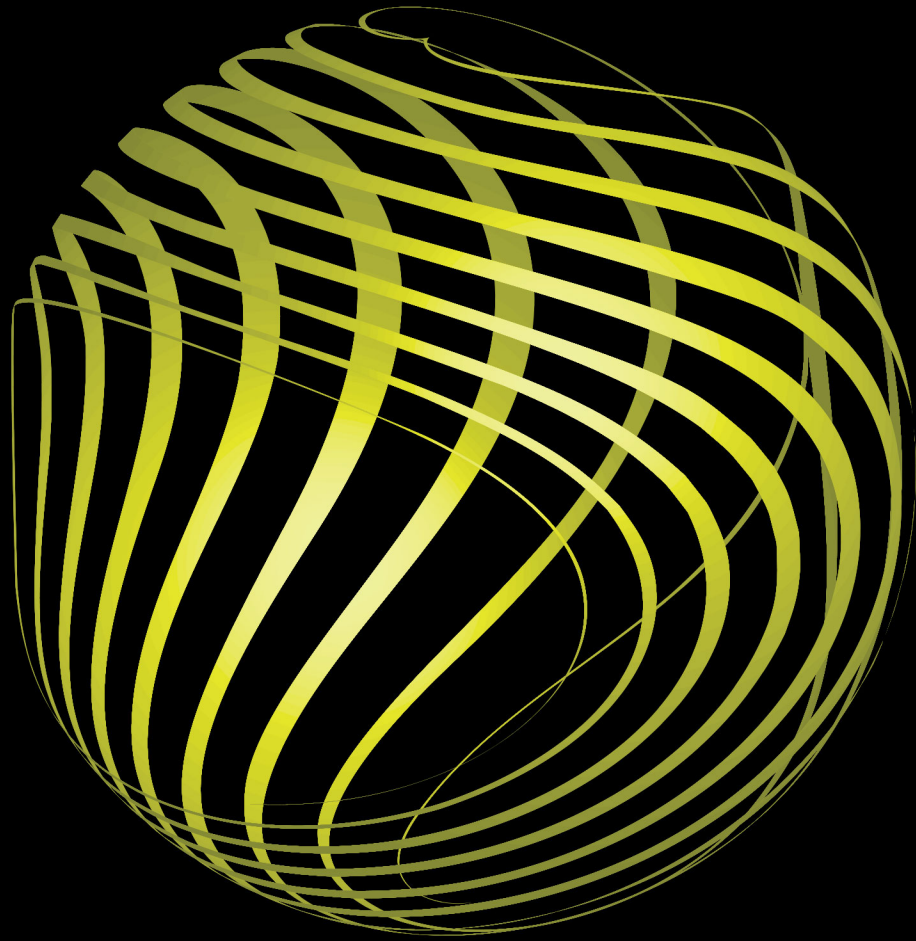# Mastering the STAAR

High School Algebra 1 Exam

Stephanie Cutler

I wish to personally thank the following people for their contributions in creating this book

William Cutler for drawing graphs and helping to create the math problems used in this book.

www.StaarGuides.com

Table of Contents

Introduction

This book is intended to aid the student preparing for the end of course STAAR Algebra 1 assessment. It is a concise review of key Algebra 1 material covered by the STAAR assessment and is intended to help the student recall material taught during the school year. It is not intended to substitute for comprehensive textbooks or teacher provided learning material.

Teachers: Your feedback on this study guide is appreciated. If you find an error in this book or a topic that needs clarification, please send your comments to: support@StaarGuides.com. Additionally, if you have material that you think would be useful to incorporate into this book, please email the material. Continuous improvement of this study guide is the goal.

Print Editions of this book can be purchased on StaarGuides.com

Reporting Categories

The content and skills tested on each STAAR assessment are grouped together. Each group is called a Reporting Category. The study material in this book is organized by Reporting Category. The table below shows the Reporting Categories for the Algebra 1 STAAR assessment. There are 54 multiple choice questions spread over 5 Reporting Categories.

Reporting Categories	Number of Questions
Reporting Category 1: Number and Algebraic Methods	11
Reporting Category 2: Describing and Graphing Linear Functions, Equations, and Inequalities	12
Reporting Category 3: Writing and Solving Linear Functions, Equations, and Inequalities	14
Reporting Category 4: Quadratic Functions and Equations	11
Reporting Category 5: Exponential Functions and Equations	6
Total Number of Questions on Test	54 Total

Category 1: Number and Algebraic Methods

Add and subtract polynomials of degree one and degree two.

To add polynomials, add the common terms. To subtract polynomials, subtract the common terms. When subtracting, change the sign of each term of the second operand and then add. The degree of a polynomial is the largest exponent of the variables. Example: $(4x - 2)$ is of degree one; $(2x^2 - 2x + 2)$ is of degree two; $(2x^7 - 2x + 2)$ is of degree seven.

Add polynomials of degree one:

$$(4x - 2) + (6x - 4) \qquad \text{expression}$$

$$4x - 2 \qquad \text{line up like terms}$$

$$\underline{+ \ 6x - 4}$$

$$10x - 6 \qquad \text{answer}$$

Subtract polynomials of degree one:

$$(4x - 2) - (6x - 4) \qquad \text{expression}$$

$$4x - 2 \qquad \text{line up like terms}$$

$$\underline{+ (-6x + 4)} \qquad \text{change signs and add}$$

$$-2x + 2 \qquad \text{answer}$$

Add polynomials of degree two:

$$(-3x^2 + 6) + (2x^2 - 2x + 2) \quad \text{expression}$$

$$-3x^2 \qquad + 6 \qquad \text{line up like terms}$$

$$\underline{+ \ 2x^2 - 2x + 2}$$

$$-x^2 - 2x + 8 \qquad \text{answer}$$

Subtract polynomials of degree two:

$$(-3x^2 + 6) - (2x^2 - 2x + 2) \quad \text{expression}$$

$$-3x^2 \qquad + 6 \qquad \text{line up like terms}$$

$$\underline{+ (-2x^2 + 2x - 2)} \qquad \text{change signs and add}$$

$$-5x^2 + 2x + 4 \qquad \text{answer}$$

Multiply polynomials of degree one and degree two.

The <u>FOIL</u> method is to multiply First terms, Outer terms, Inner terms, and Last terms.

Multiply polynomials of degree one:

$$(x + 4)(x + 2) = x^2 + 2x + 4x + 8 \qquad \text{use FOIL method}$$
$$= x^2 + 6x + 8 \qquad \text{combine like terms}$$

Multiply polynomials of degree two:

$$(x^2 + 4x - 2)(x^2 - x + 2) = \qquad \text{multiply two trinomials}$$

$$
\begin{array}{r}
x^2 + 4x - 2 \\
\times \quad x^2 - x + 2 \\
\hline
2x^2 + 8x - 4 \\
-x^3 - 4x^2 + 2x \\
x^4 + 4x^3 - 2x^2 \\
\hline
x^4 + 3x^3 - 4x^2 + 10x - 4
\end{array}
$$

line up terms to multiply

2 (x^2 +4x-2)

-x (x^2 +4x-2)

x^2 (x^2 +4x-2)

answer

Determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend.

Divide polynomial of degree one by polynomial of degree one:

$$(3x + 5) \div (x - 1) = \qquad \text{divide two binomials}$$

$$
\begin{array}{r}
3 + \dfrac{8}{x-1} \\
x - 1 \overline{\smash{)}\, 3x + 5} \\
-(3x - 3) \\
\hline
8
\end{array}
$$

answer

use long division

- x goes into 3x, 3 times
- 3 times (x – 1) ➔ 3x – 3
- subtract (3x – 3) from (3x + 5) by changing the signs and adding -(3x – 3) ➔ -3x + 3
- add the remainder of 8 / (x - 1) to 3

Divide polynomial of degree two by polynomial of degree one:

$$(6x^2 + 8x) \div (2x) =$$ divide binomial by monomial

$$\frac{6x^2 + 8x}{2x} = \frac{6x^2}{2x} + \frac{8x}{2x}$$ separate into two divisions

$$= 3x + 4$$ answer

Divide polynomial of degree two by polynomial of degree two:

$$(5x^2 + 3x + 12) \div (x^2 + 5) =$$ divide a trinomial by a binomial

$$\begin{array}{r} 5 + \dfrac{3x - 13}{x^2 + 5} \\[2mm] x^2 + 0x + 5 \overline{\smash{)}\; 5x^2 + 3x + 12} \\ \underline{-(5x^2 + 0x + 25)} \\ 3x - 13 \end{array}$$

answer

use long division, 0x is a placeholder

-(5x² + 0x +25) ➜ -5x² - 0x -25)

Rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property.

The Distributive Property: a(b + c) = ab + ac

Rewrite this linear expression of degree one into equivalent forms using the distributive property:

$$5x(x + 2)$$ expression

$$5x(x) + 5x(2)$$ equivalent form 1

$$5x^2 + 10x$$ equivalent form 2

Rewrite this linear expression of degree two into equivalent forms using the distributive property:

$$-5x^2 - 10x$$ expression

$$-5x(x) - 5x(2)$$ equivalent form 1

$$-5x(x + 2)$$ equivalent form 2

Factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two.

Factor this trinomial:

$$3x^2 + 12x + 9 = 3(x^2 + 4x + 3)$$ factor out GCF of 3

$$= 3(x + 3)(x + 1)$$ factor into two binomials

Factor this trinomial:

$$3x^2 + 8x + 4 = (3x + 2)(x + 2)$$ no GCF, so factor into two binomials
use FOIL to check the answer

Factor this trinomial:

$$3x^2 - 8x + 4 = (3x - 2)(x - 2)$$ no GCF, so factor into two binomials
use FOIL to check the answer

Factor this trinomial:

$$3x^2 - 4x - 4 = (3x + 2)(x - 2)$$ no GCF, so factor into two binomials
use FOIL to check the answer

Factor this trinomial:

$$3x^2 + 4x - 4 = (3x - 2)(x + 2)$$ no GCF, so factor into two binomials
use FOIL to check the answer

The Perfect Square Trinomial has this format: $a^2 + 2ab + b^2 = (a + b)^2$ OR $a^2 - 2ab + b^2 = (a - b)^2$

Factor this trinomial:

$$x^2 + 10x + 25 = x^2 + 2(x)(5) + 5^2$$ rewrite as $a^2 + 2ab + b^2$

$$= (x + 5)^2$$ rewrite as $(a + b)^2$

FOIL $(x + 5)(x + 5)$ to check the answer

Factor this trinomial:

$$x^2 - 10x + 25 = x^2 - 2(x)(5) + 5^2$$ rewrite as $a^2 - 2ab + b^2$

$$= (x - 5)^2$$

rewrite as $(a - b)^2$

FOIL $(x - 5)(x - 5)$ to check the answer

Decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial.

The Difference of Two Squares has this format: $a^2 - b^2 = (a + b)(a - b)$

Factor this binomial:

$$x^2 - 36 = x^2 - 6^2 \qquad \text{rewrite as } a^2 - b^2$$

$$= (x + 6)(x - 6) \qquad \text{rewrite as } (a + b)(a - b)$$

Factor this binomial:

$$16x^2 - 64 = (4x)^2 - 8^2 \qquad \text{rewrite as } a^2 - b^2$$

$$= (4x + 8)(4x - 8) \qquad \text{rewrite as } (a + b)(a - b)$$

Simplify numerical radical expressions involving square roots.

The Product Property of Square Roots states: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

Simplify:

$$\sqrt{75} = \sqrt{25 \cdot 3} \qquad \text{25 is a perfect square}$$

$$= \sqrt{25} \cdot \sqrt{3} \qquad \text{use Product Property of Square Roots}$$

$$= 5\sqrt{3} \qquad \text{answer}$$

The Quotient Property of Square Roots states: $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

Simplify:

$$\sqrt{\frac{3}{25}} = \frac{\sqrt{3}}{\sqrt{25}} \qquad \text{use Quotient Property of Square Roots}$$

$$= \frac{\sqrt{3}}{5} \qquad \text{answer}$$

Rationalize the Denominator (remove radical from denominator) by multiplying by a conjugate. Obtain a conjugate by reversing the sign of a binomial. Example: conjugate of $4 - \sqrt{6}$ is $4 + \sqrt{6}$

Simplify:

need to remove radical from denominator

$$\frac{10}{4 - \sqrt{6}} = \left(\frac{10}{4 - \sqrt{6}}\right)\left(\frac{4 + \sqrt{6}}{4 + \sqrt{6}}\right)$$

conjugate is $4 + \sqrt{6}$

$$= \frac{10(4 + \sqrt{6})}{4^2 - \left(\sqrt{6}\right)^2}$$

simplify

$$= \frac{10(4 + \sqrt{6})}{16 - 6}$$

square denominator terms

$$= \frac{10(4 + \sqrt{6})}{10}$$

simplify denominator

$$= 4 + \sqrt{6}$$

answer (10/10 cancels out)

Add Like Radicals: $a\sqrt{x} + b\sqrt{x} = (a + b)\sqrt{x}$

Simplify:

$$2\sqrt{3} + 3\sqrt{3} + 4\sqrt{5} = (2 + 3)\sqrt{3} + 4\sqrt{5}$$

combine like radicals

$$= 5\sqrt{3} + 4\sqrt{5}$$

answer

Subtract Like Radicals: $a\sqrt{x} - b\sqrt{x} = (a - b)\sqrt{x}$

Simplify:

$$2\sqrt{3} - 3\sqrt{3} + 4\sqrt{5} = (2 - 3)\sqrt{3} + 4\sqrt{5}$$

combine like radicals

$$= -\sqrt{3} + 4\sqrt{5}$$

answer

Multiply Radicals by using the Product Property of Square Roots: $\sqrt{a} \bullet \sqrt{b} = \sqrt{ab}$

Simplify:

$$\sqrt{5}\,(\sqrt{3} - \sqrt{5}) = (\sqrt{5})(\sqrt{3}) - (\sqrt{5})(\sqrt{5})$$

distributive property

$$= \sqrt{15} - \sqrt{25}$$

simplify

$$= \sqrt{15} - 5$$

answer

Simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

Any non-zero number raised to the 0th power is equal to 1: $a^0 = 1$, where a $\neq$ 0

Evaluate each expression:

$$2^0 = 1 \qquad\qquad a^0 = 1$$

$$-2^0 = 1 \qquad\qquad a^0 = 1$$

$$x^0 = 1 \qquad\qquad a^0 = 1$$

$$0^0 = undefined \qquad\qquad \text{can not be evaluated}$$

For numbers with negative exponents, take the reciprocal of the number: $a^{-n} = \dfrac{1}{a^n}$, where a $\neq$ 0

Evaluate each expression:

$$2^{-2} = \frac{1}{2^2} \qquad\qquad \text{take reciprocal}$$

$$= \frac{1}{4} \qquad\qquad \text{simplify}$$

$$\frac{3^2}{y^{-3}} = \frac{9}{\left(\frac{1}{y^3}\right)} \qquad\qquad \text{take reciprocal}$$

$$= 9y3 \qquad\qquad \text{simplify}$$

The Product of Powers Property states: $a^m \cdot a^n = a^{m+n}$

Evaluate each expression:

$$5^5 \cdot 5^3 = 5^8 \qquad\qquad \text{add exponents}$$

$$= 390{,}625 \qquad\qquad \text{simplify}$$

$$-2^4 \cdot -2^{-2} = -2^2 \qquad\qquad \text{add exponents}$$

$$= 4 \qquad\qquad \text{simplify}$$

$$x^4 \cdot x^2 = x^6 \qquad\qquad \text{add exponents}$$

The Quotient of Powers Property states: $\dfrac{a^m}{a^n} = a^{m-n}$**, where a $\neq$ 0**

Evaluate each expression:

$$\frac{5^5}{5^3} = 5^2 \qquad\qquad \text{subtract exponents}$$

$$= 25 \qquad\qquad \text{simplify}$$

$$\frac{-2^4}{-2^{-2}} = -2^6 \qquad\qquad \text{subtract exponents}$$

$$= 64 \qquad\qquad \text{simplify}$$

$$\frac{x^5 y}{x^2} = x^3 y \qquad\qquad \text{subtract exponents}$$

The Power of a Power Property states: $(a^m)^n = a^{mn}$

Evaluate each expression:

$$(5^2)^3 = 5^6 \qquad\qquad \text{multiply exponents}$$

$$= 15{,}625 \qquad\qquad \text{simplify}$$

$$(-2^4)^{-2} = -2^{-8} \qquad\qquad \text{multiply exponents}$$

$$= \frac{1}{-2^8} \qquad\qquad \text{take reciprocal}$$

$$= \frac{1}{256} \qquad\qquad \text{simplify}$$

$$(x^3)^7 = x^{21} \qquad\qquad \text{multiply exponents}$$

The Power of a Product Property states: $(ab)^m = a^m b^m$

Evaluate each expression:

$$(5 \bullet 2)^3 = 5^3 \bullet 2^3 \qquad\qquad \text{take power of each factor individually}$$

$$= 125 \bullet 8 \qquad\qquad \text{simplify}$$

$$= 1{,}000 \qquad\qquad \text{simplify}$$

$$(5 \cdot -2)^{-2} = 5^{-2} \cdot -2^{-2} \qquad \text{take power of each factor individually}$$

$$= \frac{1}{5^2} \cdot \frac{1}{-2^2} \qquad \text{simplify}$$

$$= \frac{1}{25} \cdot \frac{1}{4} \qquad \text{simplify}$$

$$= \frac{1}{100} \qquad \text{simplify}$$

$$(x^2 y^4)^3 = x^6 y^{12} \qquad \text{take power of each factor individually}$$

The Power of a Quotient Property states: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ **, where b ≠ 0**

Evaluate each expression:

$$\left(\frac{6}{2}\right)^3 = \frac{6^3}{2^3} \qquad \text{take power of each factor individually}$$

$$= \frac{216}{8} \qquad \text{simplify}$$

$$= 27 \qquad \text{simplify}$$

$$\left(\frac{6}{-2}\right)^{-2} = \frac{6^{-2}}{-2^{-2}} \qquad \text{take power of each factor individually}$$

$$= \frac{\frac{1}{6^2}}{\frac{1}{-2^2}} \qquad \text{simplify}$$

$$= \frac{\frac{1}{36}}{\frac{1}{4}} \qquad \text{simplify}$$

$$= \frac{1}{9} \qquad \text{simplify}$$

$$\left(\frac{2x^3 y^2 z^4}{xy}\right)^3 = \frac{2^3 x^9 y^6 z^{12}}{x^3 y^3} \qquad \text{take power of each factor individually}$$

$$= 8x^6 y^3 z^{12} \qquad \text{simplify}$$

A fractional exponent can be represented in two formats: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ **OR** $x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$

For example: $16^{\frac{1}{2}} = \sqrt[2]{16^1} = \sqrt[2]{16} = 4$ **OR** $16^{\frac{1}{2}} = \left(\sqrt[2]{16}\right)^1 = 4^1 = 4$

Evaluate each expression:

$$4^{\frac{3}{2}} = 4^{\left(\frac{1}{2}\right)(3)}$$ rewrite fractional exponent as ½ • 3

$$= (\sqrt{4})^3$$ $4^{\frac{1}{2}} = \sqrt{4}$

$$= 2^3$$ simplify

$$= 8$$ simplify

$$\frac{x^{\frac{3}{4}}}{x^{\frac{2}{4}}} = x^{\frac{1}{4}}$$ subtract exponents

$$= \sqrt[4]{x}$$ rewrite as a radical

$$256^{\frac{5}{4}} \cdot \sqrt[4]{x^4} = 256^{\left(\frac{1}{4}\right)(5)} \cdot x$$ rewrite fractional exponent as $\frac{1}{4}$ • 5

$$= \left(\sqrt[4]{256}\right)^5 \cdot x$$ $256^{\frac{1}{4}} = \sqrt[4]{256}$

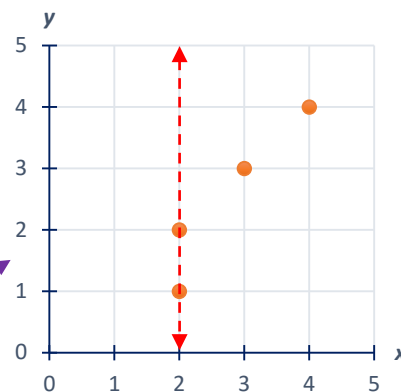$$= 4^5 \cdot x$$ simplify

$$= 1024x$$ simplify

Decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.

Vertical Line Test: a relation shown on a graph is a function if a vertical line drawn through the graph only intersects *one* point on the graph.
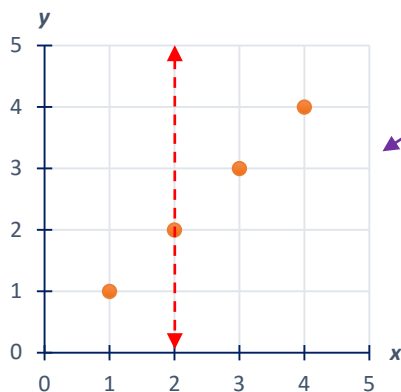
this graph is a function

(vertical line only intersects 1 point)

this graph is NOT a function

(vertical line intersects 2 points)

A relation is a set of input and output values. It can be represented in many ways.   A function has only one unique output value for each unique input value.
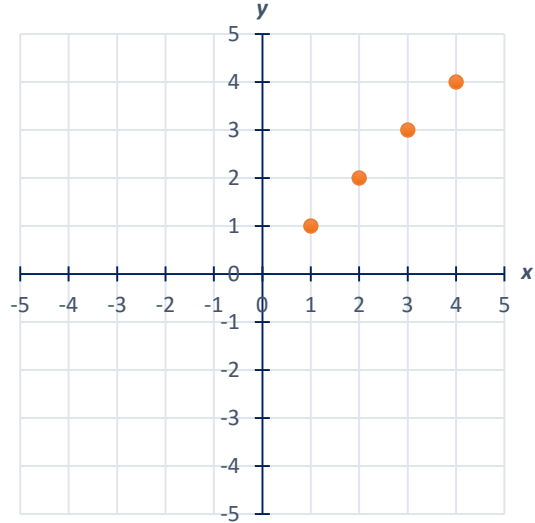
Here is the same relation represented in 5 different formats. Determine whether the relation is a function.

(1,1), (2,2), (3,3), (4,4)

$y = x$, with inputs 1, 2, 3, 4

input (x)	1	2	3	4
output (y)	1	2	3	4

x	y
1	1
2	2
3	3
4	4

This is a function. Every input value has only one unique output value.
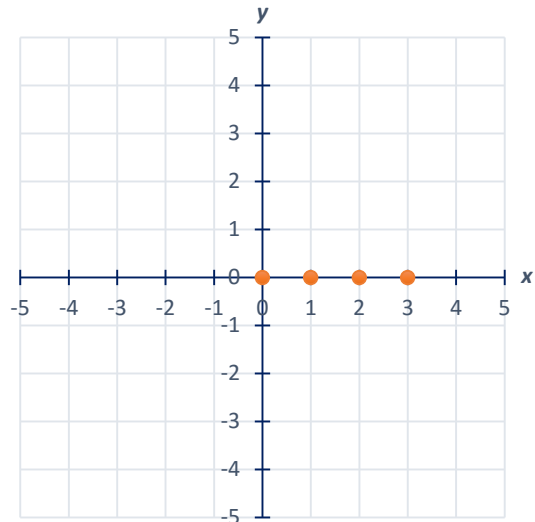
Passes Vertical Line Test

Here is the same relation represented in 5 different formats. Determine whether the relation is a function.

(0,0), (1,0), (2,0), (3,0)

$y = 0x$, with inputs 0, 1, 2, 3

input (x)	0	1	2	3
output (y)	0	0	0	0

x	y
0	0
1	0
2	0
3	0

Multiple 0's are OK if each is paired with a unique X input.

This is a function. Every input value has only one unique output value.
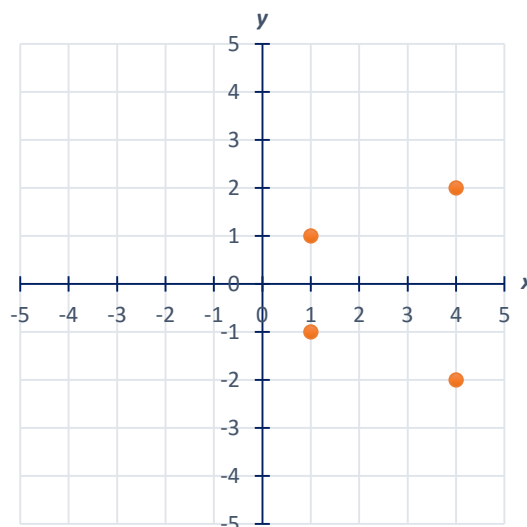
Passes Vertical Line Test

Here is the same relation represented in 5 different formats. Determine whether the relation is a function.

(1,-1), (1,1), (4,2), (4,-2)

x	y
1	-1
1	1
4	2
4	-2

$x = y^2$, with inputs 1, 4

input (x)	1	1	4	4
output (y)	-1	1	2	-2

Fails Vertical Line Test

This is NOT a function.
The input value 1 has two output values (1 and -1) and input value 4 has two output values (2 and -2).

NOTE: If the domain (X input) is unique, the relation is always a function. If the domain has duplicates, with different Y outputs, then the relation is NOT a function.

Evaluate functions, expressed in function notation, given one or more elements in their domains.

With function notation, "*f(x)*" is used for an equation. Evaluate the equation by substituting the specified domain element(s) for *x*. While "*f(x)*" is commonly used, other notations can also be used (*g(x), h(x)* etc).

Evaluate *f(x) = 4x + 2*, when x = 0, -2 and 2.

$$f(0) = 4(0) + 2$$ substitute 0 for x

$$f(0) = 2$$ answer

$$f(-2) = 4(-2) + 2$$ substitute -2 for x

$$f(-2) = -6$$ answer

$$f(2) = 4(2) + 2$$ substitute 2 for x

$$f(2) = 10$$ answer

Identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes.

Recursive Arithmetic Sequences will appear like this: a_1 = start, $a_n = a_{n-1} + d$, (d = common difference). The difference between each 2 terms will be the same, referred to as the common difference.

Compute the 4th term in this sequence: $a_1 = 3, a_n = a_{n-1} + 5$

$a_1 = 3$ 1st term is 3 as specified in problem

$a_2 = 3 + 5 = 8$ 2nd term is 1st term + 5

$a_3 = 8 + 5 = 13$ 3rd term is 2nd term + 5

$a_4 = 13 + 5 = 18$ 4th term is 3rd term + 5

NOTE: Another way to solve this problem is with this formula: $a_n = a_1 + d(n-1)$.
To compute the 4th term in the above problem: $a_4 = 3 + 5(4-1)$ $= 3 + 5(3)$ $= 18$

Recursive Geometric Sequences will appear like this: a_1 = start, $a_n = r(a_{n-1})$ (r = common ratio). The ratio between each 2 terms will be the same, referred to as the common ratio.

Compute the 4th term in this sequence: $a_1 = 1, a_n = 5a_{n-1}$

$a_1 = 1$ 1st term is 1 as specified in problem

$a_2 = 5(1) = 5$ 2nd term is 5 • 1st term

$a_3 = 5(5) = 25$ 3rd term is 5 • 2nd term

$a_4 = 5(25) = 125$ 4th term is 5 • 3rd term

NOTE: Another way to solve this problem is with this formula: $a_n = a_1 r^{(n-1)}$.
To compute the 4th term in the above problem: $a_4 = (1)5^{(4-1)} = 5^3$ $= 125$

Write a formula for the nth term of arithmetic and geometric sequences, given the value of several of their terms.

An Arithmetic Sequence is a list of numbers which have the same difference between two consecutive terms. Use this format to write an equation for an arithmetic sequence: $a_n = a_1 + d(n-1)$.

Write an equation for the nth term of this sequence: 20, 15, 10, 5, 0, -5

$$a_1 = 20$$ 1st term is 20 as specified in problem

$$d = -5$$ the difference between each term is -5

$$a_n = a_1 + d(n-1)$$ start with format

$$a_n = 20 - 5(n-1)$$ replace 20 for a_1 and -5 for d

$$a_n = 20 - 5n + 5$$ distributive property

$$a_n = -5n + 25$$ simplify

A Geometric Sequence is a list of numbers which have the same ratio between two consecutive terms. Use this format to write an equation for a geometric sequence: $a_n = a_1 r^{(n-1)}$.

Write an equation for the nth term of this sequence: 3, 15, 75, 375, 1875

$$a_1 = 3$$ 1st term is 3 as specified in problem

$$r = 5$$ the ratio between each term is 5

$$a_n = a_1 r^{n-1}$$ start with format

$$a_n = 3 \cdot 5^{n-1}$$ replace 3 for a_1 and 5 for r

NOTE: Do not multiply 3 and 5. That would be the wrong answer. The (n-1) exponent takes priority. For example, here is how to calculate the 5th term based on the equation above.

$$a_5 = 3 \cdot 5^{5-1}$$
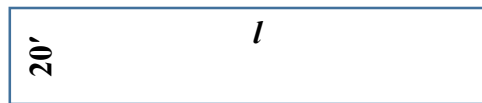$$a_5 = 3 \cdot 5^4$$
$$a_5 = 3 \cdot 625$$
$$a_5 = 1875$$

Solve mathematic and scientific formulas, and other literal equations, for a specified variable.

A rectangle has an area of 2,000 square feet and a width (*w*) of 20 feet. The formula for area of a rectangle is $A = lw$. Solve this mathematical equation for length *(l)*:

20'	*l*

$$A = lw \qquad \text{equation}$$

$$\frac{A}{w} = \frac{lw}{w} \qquad \text{divide each side by w so } \textit{l} \text{ is isolated}$$

$$\frac{A}{w} = l \qquad \text{simplify}$$

$$\frac{2000}{20} = l \qquad \text{substitute 2000 for A and 20 for w}$$

$$100 = l \qquad \text{answer is 100 feet in length}$$

The formula for the temperature conversion of Fahrenheit (F) to Celsius (C) is: $C = \frac{5}{9}(F - 32)$

Convert 35ºC to Fahrenheit. Solve this scientific formula for F:

$$C = \frac{5}{9}(F - 32) \qquad \text{equation}$$

$$\left(\frac{9}{5}\right)C = \left(\frac{9}{5}\right)\frac{5}{9}(F - 32) \qquad \text{multiply each side by the reciprocal 9/5}$$

$$\frac{9}{5}C = F - 32 \qquad \text{simplify}$$

$$+\,32 \qquad +\,32 \qquad \text{add 32 to both sides}$$

$$\frac{9}{5}C + 32 = F \qquad \text{simplify}$$

$$\frac{9}{5}(35) + 32 = F \qquad \text{substitute 35º for C}$$

$$95° = F \qquad \text{answer}$$

Solve this literal equation for y:

$$6x + 3y = 12 \qquad \text{equation}$$

$$-\,6x \qquad\qquad -\,6x \qquad \text{subtract 6x from both sides}$$

$$3y = 12 - 6x \qquad \text{simplify}$$

$$\frac{3y}{3} = \frac{12 - 6x}{3} \qquad \text{divide each side by 3}$$

$$y = 4 - 2x \qquad \text{answer}$$

Category 2: Describing and Graphing

Linear Functions, Equations, and Inequalities

Determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$.

The slope of a line is the rate of change between two points on the line. It measures the steepness and direction of the line. The slope can be determined using several different methods. The variable *m* is used to represent slope.

To determine the slope from a table of values use this method: $\boldsymbol{m = \dfrac{change\ in\ two\ y\ values,\ \ y_2 - y_1}{change\ in\ two\ x\ values,\ \ x_2 - x_1}}$

Choose any 2 points from the table and subtract the y and x values.
For this example, use (1,3) for the first point and (2,5) for the second point.

x	y
1	3
2	5
3	7
4	9

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ formula

$$= \frac{5 - 3}{2 - 1}$$ subtract the two y points

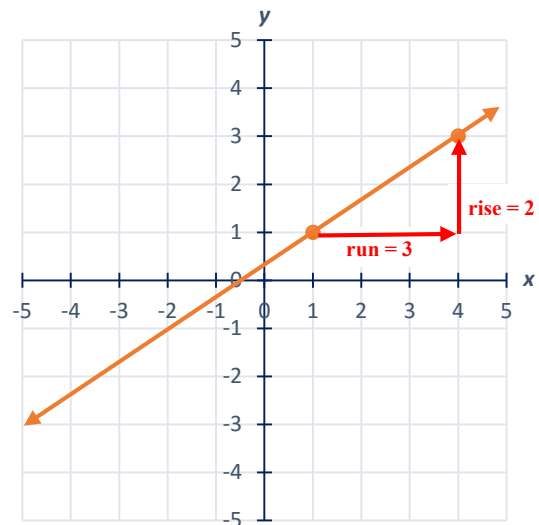   subtract the two x points

$$= \frac{2}{1}$$ simplify

$$= 2$$ answer

To determine the slope from a graph use this method:

$$\boldsymbol{m = \frac{rise}{run}}$$

Choose any 2 points from the graph. Subtract the y values to determine the **rise**. Subtract the x values to determine the r**un**. For this example, use (1,1) for the first point and (4,3) for the second point.

$$m = \frac{rise}{run}$$
$$m = \frac{2}{3}$$

A line that rises from left to right will have a **positive** slope.
A line that falls from left to right will have a **negative** slope.

To determine the slope from 2 points use this method: $m = \dfrac{change\ in\ two\ y\ values,\ \ y_2 - y_1}{change\ in\ two\ \ x\ values,\ \ x_2 - x_1}$

Determine the slope of the line passing through (-2, 2) and (0,0).

$$m = \dfrac{y_2 - y_1}{x_2 - x_1} \qquad\qquad \text{formula}$$

$$= \dfrac{0 - 2}{0 - (-2)} \qquad\qquad \text{subtract the two y points}$$

$$\qquad\qquad\qquad\qquad \text{subtract the two x points}$$

$$= \dfrac{-2}{2} \qquad\qquad\qquad \text{simplify}$$

$$= -1 \qquad\qquad\qquad \text{answer}$$

Given an equation in the format, $y = mx + b$, m is the slope, b is the y-intercept.

Determine the slope of this equation:

$$y = -5x - 3 \qquad\qquad \text{equation}$$

$$m = -5 \qquad\qquad\qquad \text{slope = -5}$$

Given an equation in the format, $Ax + By = C$, re-arrange the equation to $y = mx + b$, m is the slope.

Determine the slope of this equation:

$$4x + 3y = 15 \qquad\qquad\qquad \text{Standard Form equation}$$

$$-4x \qquad\qquad -4x \qquad\qquad \text{subtract 4x from both sides}$$

$$3y = -4x + 15 \qquad\qquad \text{simplify}$$

$$\dfrac{3y}{3} = -\dfrac{4x}{3} + \dfrac{15}{3} \qquad\qquad \text{divide both sides by 3}$$

$$y = -\dfrac{4}{3}x + 5 \qquad\qquad \text{simplify}$$

$$m = -\dfrac{4}{3} \qquad\qquad\qquad \text{slope = -4/3}$$

Given an equation in the format, $y - y_1 = m(x - x_1)$, m is the slope.

Determine the slope of this equation:

y + 3 is the same
as y – (-3)

$$y + 3 = -7(x - 2) \qquad\qquad \text{point-slope form}$$

$$m = -7 \qquad\qquad\qquad\qquad \text{slope = -7}$$

Calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems.

The rate of change of a linear function is the slope.

The minimum wage was set at $1.00 in 1956. The minimum wage was increased to $7.25 in 2009. What was the rate of change? Show the rate of change, based on a table, a graph and algebraically.

To determine the slope from a table of values use this method: $m = \dfrac{change\ in\ two\ y\ values,\ \ y_2 - y_1}{change\ in\ two\ x\ values,\ \ x_2 - x_1}$

Year (x)	Wage (y)
1956	1.00
2009	7.25

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{formula}$$

$$= \frac{7.25 - 1.00}{2009 - 1956} \qquad \begin{array}{l}\text{subtract the two y points} \\ \text{subtract the two x points}\end{array}$$

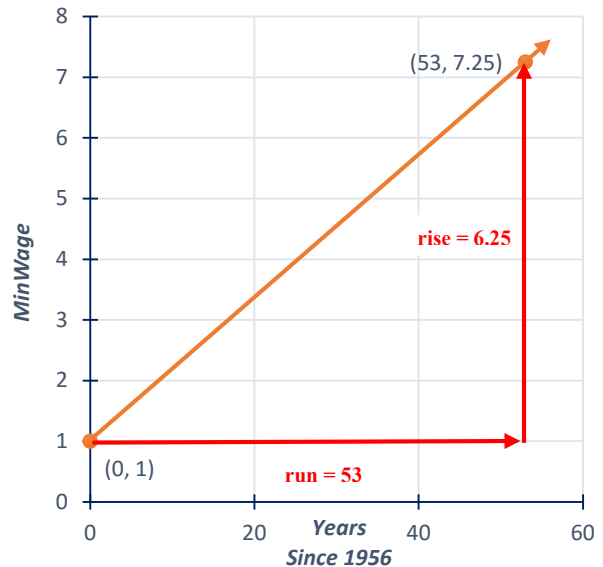$$= \frac{6.25}{53} \qquad \text{simplify}$$

$$= 0.1179 \qquad \text{answer (11.79 cents)}$$

To determine the slope from a graph use this method:

$$m = \frac{rise}{run}$$

Subtract the y values to determine the *rise*. Subtract the x values to determine the *run*. Use (0,1) for the first point and (53,7.25) for the second point.

$$m = \frac{rise}{run}$$

$$m = \frac{6.25}{53}$$

$$m = 0.1179$$

Using the graph, an equation can be determined using the slope-intercept form, $y = mx + b$. The slope is 0.1179 as calculated from rise ÷ run. The y-intercept is 1.00

$$y = mx + b$$
$$y = 0.1179x + 1.00$$

The rate of change is 0.1179 or 11.79 cents per year for all three methods

Graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.

The x-intercept of a graph is the x-coordinate where the graph intersects the x-axis. To dctermine the x-intercept, solve the equation when y = 0. For a linear function, the Zero is equal to the x-intercept

Determine the x-intercept of this equation: $y = -3x - 3$

$0 = -3x - 3$	substitute 0 for y
$+3 \qquad +3$	add 3 to both sides
$3 = -3x$	simplify
$\dfrac{3}{-3} = \dfrac{-3x}{-3}$	divide both sides by -3
$-1 = x$	**x-intercept** and **zero** is -1

The y-intercept of a graph is the y-coordinate where the graph intersects the y-axis. To dctermine the y-intercept, solve the equation when x = 0. The y-intercept is also the "b" value in $y = mx + b$.

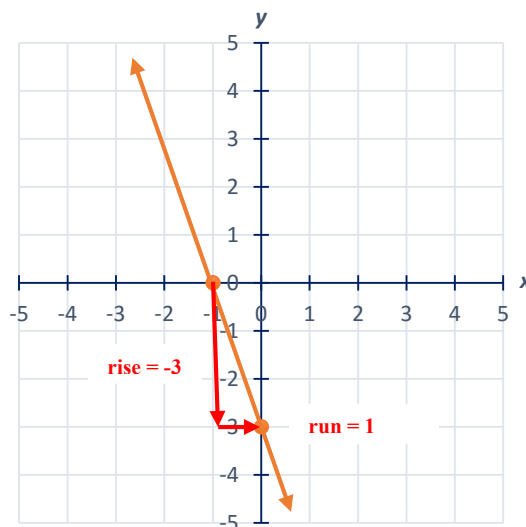Determine the y-intercept of this equation: $y = -3x - 3$

$y = -3(0) - 3$	substitute 0 for x
$y = -3$	**y-intercept** is -3

The slope is the "m" value in $y = mx + b$.

Determine the slope of this equation: $y = -3x - 3$
Slope is -3.

The graph can be drawn using the x-intercept, the y-intercept and the slope.

x-intercept (-1,0)

y-intercept (0,-3)

slope = -3

Graph the solution set of linear inequalities in two variables on the coordinate plane.

The graph of a linear inequality in two variables shows all the solutions which are possible. The shaded area will show the solutions. A solid line includes the points on the line (used for ≤ and ≥). A dashed line does not include the points on the line (used for < and >).

Graph the solution set for $3x + y > 3$

Determine the **x-intercept** of the equation $3x + y = 3$ (**use equal sign here**)

$$3x + 0 = 3 \qquad \text{substitute 0 for y}$$

$$3x = 3 \qquad \text{simplify}$$

$$\frac{3x}{3} = \frac{3}{3} \qquad \text{divide both sides by 3}$$

$$x = 1 \qquad \textbf{x-intercept} \text{ is 1: coordinate (1,0)}$$

Determine the **y-intercept** of the equation $3x + y = 3$ (**use equal sign here**)

$$3(0) + y = 3 \qquad \text{substitute 0 for x}$$

$$y = 3 \qquad \textbf{y-intercept} \text{ is 3: coordinate (0,3)}$$

The graph can be drawn using the x-intercept (1,0) and the y-intercept (0,3). Plot the two points and draw a dashed line to connect them. Use a dashed line since the ">" inequality is used.

Choose a point on either side of the dashed line and test to see whether that side of the graph is true or false.

For this example, use (0,0).

$3x + y > 3$ substitute 0 for x, 0 for y
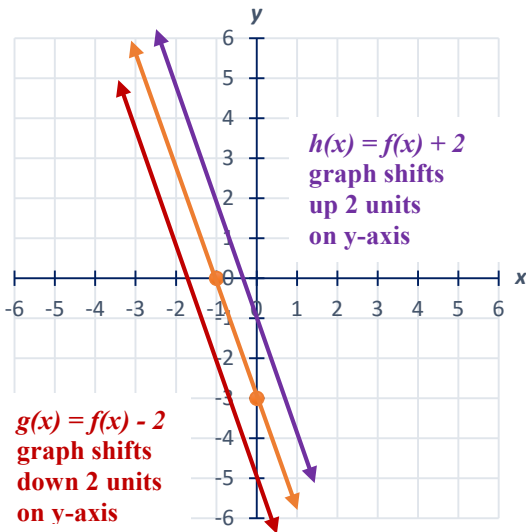$3(0) + (0) > 3$
$0 > 3$ false condition
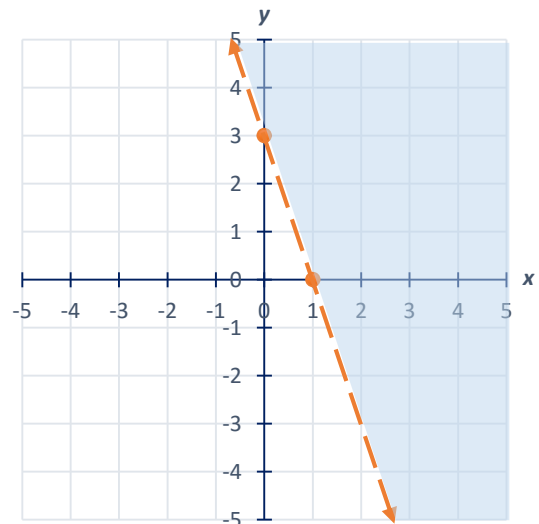
Since (0,0) produces a false condition, (0,0) is not in the solution set. Shade in the other side of the graph.

Determine the effects on the graph of the parent function f(x) = x when f(x) is replaced by af(x), f(x) + d, f(x − c), f(bx) for specific values of a, b, c, and d.
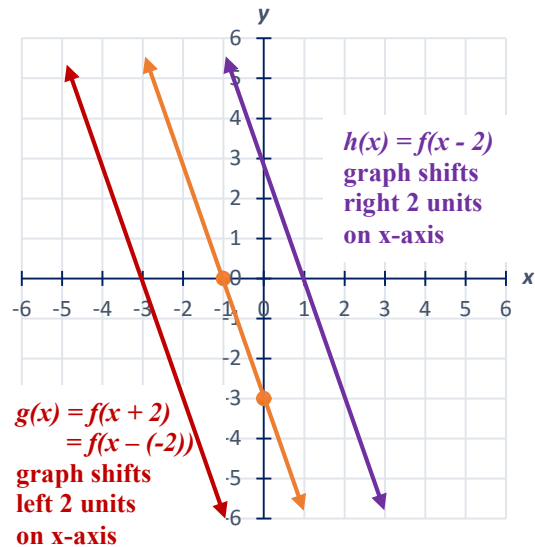
f(x)+d will translate a graph vertically.
Graph shifts down if d < 0, up if d > 0.
Vertical Translations: $f(x) = -3x - 3$
$$g(x) = f(x) - 2$$
$$h(x) = f(x) + 2$$

h(x) = f(x) + 2
graph shifts
up 2 units
on y-axis

g(x) = f(x) - 2
graph shifts
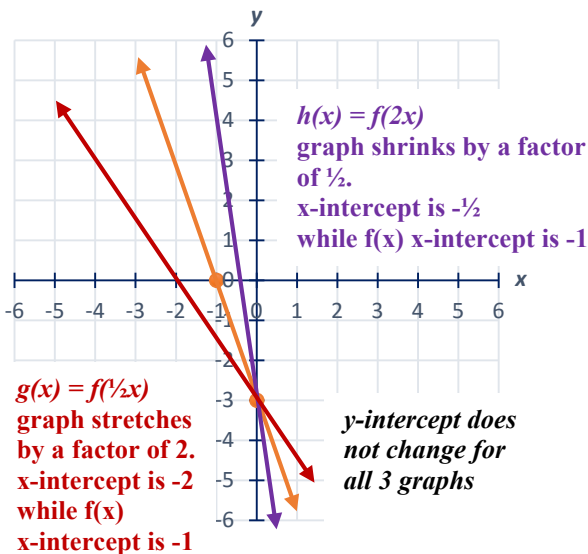*down 2 units*
on y-axis

f(x-c) will translate a graph horizontally.
Graph shifts left if c < 0, right if c > 0.
Horizontal Translations: $f(x) = -3x - 3$
$$g(x) = f(x + 2)$$
$$h(x) = f(x - 2)$$

h(x) = f(x - 2)
graph shifts
right 2 units
on x-axis

g(x) = f(x + 2)
= f(x − (-2))
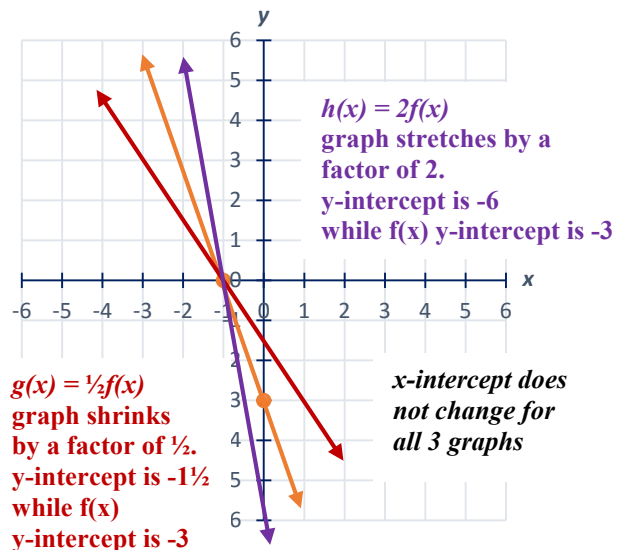*graph shifts*
left 2 units
on x-axis

f(bx) will stretch or shrink a graph horizontally.
Shrink if b > 1. Stretch if 0 < b < 1.
Horizontal Stretches/Shrinks: $f(x) = -3x - 3$
$$g(x) = f(\tfrac{1}{2}x)$$
$$h(x) = f(2x)$$

h(x) = f(2x)
graph shrinks by a factor
of ½.
x-intercept is -½
while f(x) x-intercept is -1

g(x) = f(½x)
graph stretches
by a factor of 2.
x-intercept is -2
while f(x)
x-intercept is -1

y-intercept does
not change for
all 3 graphs

af(x) will stretch or shrink a graph vertically.
Stretch if a > 1. Shrink if 0 < a < 1.
Vertical Stretches/Shrinks: $f(x) = -3x - 3$
$$g(x) = \tfrac{1}{2}f(x)$$
$$h(x) = 2f(x)$$

h(x) = 2f(x)
graph stretches by a
factor of 2.
y-intercept is -6
while f(x) y-intercept is -3

g(x) = ½f(x)
graph shrinks
by a factor of ½.
y-intercept is -1½
while f(x)
y-intercept is -3

x-intercept does
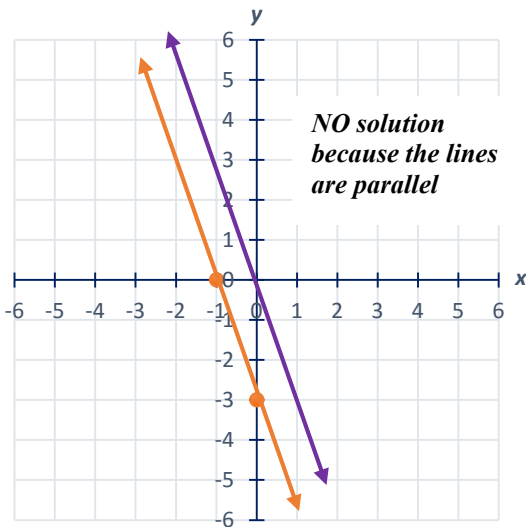*not change for*
all 3 graphs

Graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist.

If the graph of two linear equations in two variables are parallel, there is NO solution.

Solve this system.
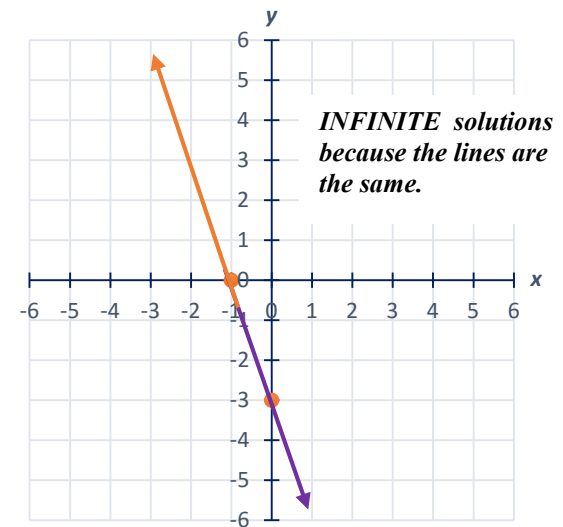**Equation 1:** $y = -3x - 3$
Equation 2: $y = -3x$

NO solution because the lines are parallel

If the graph of two linear equations in two variables represents the same line there are INFINITE solutions.

Solve this system.
Equation 1: $y = -3x - 3$
Equation 2: $2y = -6x - 6$

INFINITE solutions because the lines are the same.

If the graph of two linear equations in two variables intersect, there is ONE solution.

Solve this system.

Equation 1: $y = -3x - 3$
Equation 2: $y = x + 5$

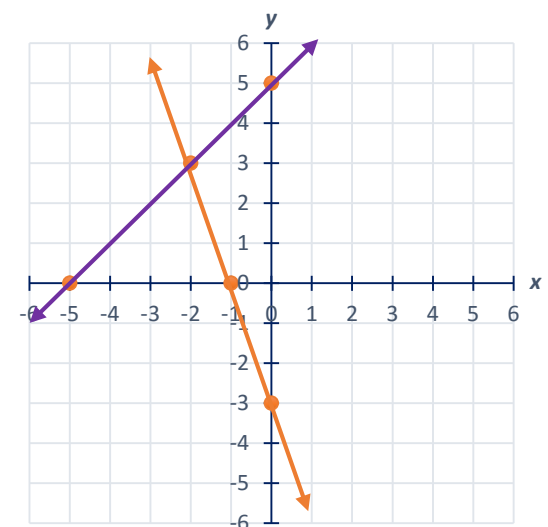**Equation 1:** $y = -3x - 3$
x-intercept is (-1,0)
y-intercept is (0,-3)

Equation 2: $y = x + 5$
x-intercept is (-5,0)
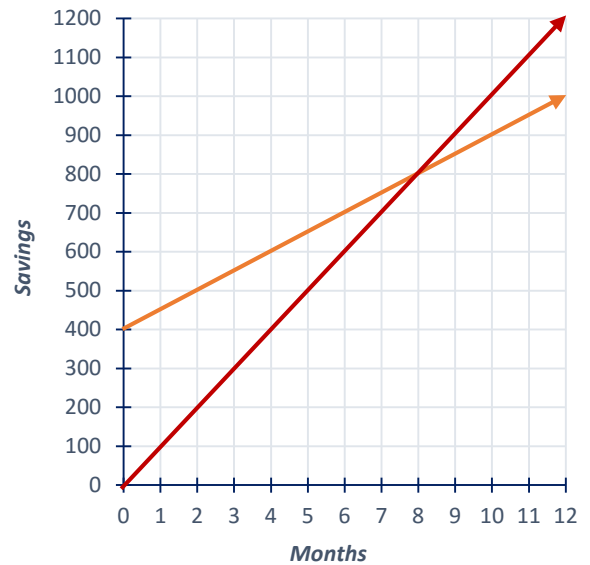y-intercept is (0,5)

The lines intersect at (-2,3). That is the solution.

Estimate graphically the solutions to systems of two linear equations with two variables in real-world problems.

Two high school students saved money for college over a 12 month period. One student started with $400 and added $50 each month. The second student started with no money, but added $100 each month.

Estimate at what month and what amount did the two students have an equal amount saved?

The two students had an equal amount saved at 8 months. The amount saved was $800.

Graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

Graph the solution set of this system of linear inequalities.

Equation 1: $y > -3x + 3$
Equation 2: $y > x + 1$

Equation 1: $y > -3x + 3$
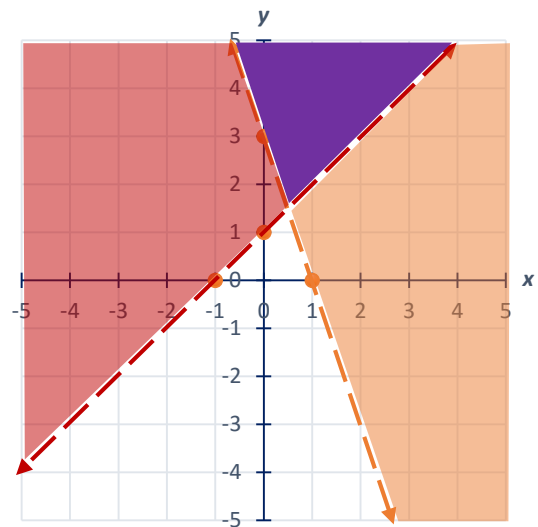x-intercept is (1,0)
y-intercept is (0,3)
Shown on graph as light orange in color.

Equation 2: $y > x + 1$
x-intercept is (-1,0)
y-intercept is (0,1)
Shown on graph as light pink in color.

The overlapping area shown in purple is the solution set.

NOTE: If there are no overlapping areas, then there is NO solution.

Calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association.

A study of how many hours students spend on homework each week at a high school was compared to the students GPA. Here is a table of the data. Determine the correlation coefficient.

Weekly Hours Spent On Homework (x)	Average Student GPA (y)
7	2.82
12	3.43
15	3.87
20	4.46
29	5.10
35	5.52
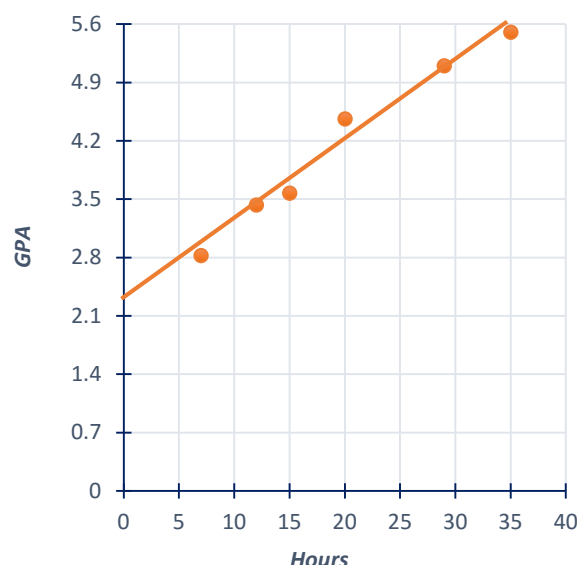
Enter the data in a graphing calculator. Use the calculator's Linear Regression option. The calculator should give the following results: y=ax + b
                a=.0952366864
                b=2.327011834
                r=.9887670571

The equation for the data is: y = 0.095x + 2.32.

The correlation coefficient is 0.988. Correlation coefficients vary between -1 and 1. Values near -1 show a strong negative correlation (negative slope). Values near 1 show a strong positive correlation (positive slope). Values near 0 show a weak correlation. A value of 0.988 shows a strong positive correlation.

Compare and contrast association and causation in real-world problems.

Causation is when a change in one variable *directly* causes a change in a second variable. Association is when a relationship exists between two variables *without one directly* affecting the second.

Are the following examples causation or association?

Smoking cigarettes and getting lung cancer?

This is causation. Smoking has been proven to cause lung cancer. More people smoking will cause more people to get lung cancer.

Number of schools and number of school age kids?

This is association. There is an association between the number of schools and school age kids. However, building more schools will not increase the number of school age kids.

Write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

This table shows the daily cost of electricity for a home owner's 1800 square foot house in Austin in July based on the outside temperature (F°). The thermostat is kept at a constant 78° (F).

Outside Temperature (x)	Daily Electricity Cost (y)
81°	$6.11
85°	$6.74
90°	$7.72
93°	$8.30
100°	$9.55
103°	$9.88
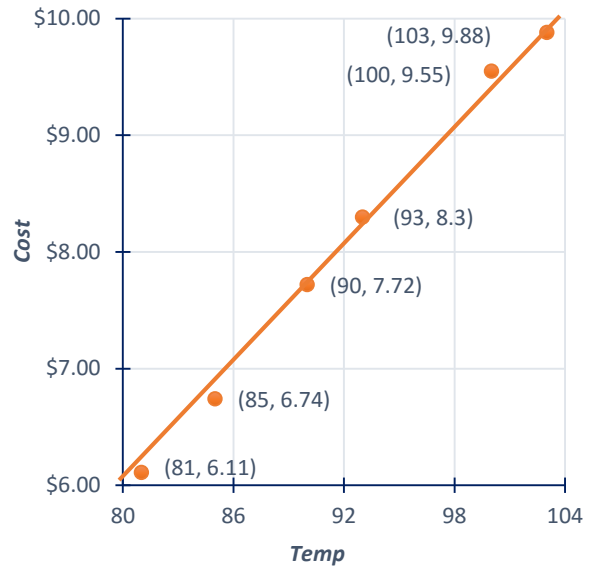
Write a linear function that provides a reasonable fit for the data. A line is drawn that reasonably fits the data. Two data points are above the line. Two data points are below the line and two data points are on the line. The two data points on the line are: (90, 7.72) and (103, 9.88).

The two points on the line can be used to calculate the slope.

$$m = \frac{y^2 - y^1}{x^2 - x^1}$$ formula

$$= \frac{9.88 - 7.72}{103 - 90}$$ subtract the two y points / subtract the two x points

$$= \frac{2.16}{13}$$ simplify

$$= 0.166154$$ answer for slope

Now write an equation using the slope and (103, 9.88).

$$y - y_1 = m(x - x_1)$$ formula

$$y - 9.88 = 0.166(x - 103)$$ subtract the two y points

$$y - 9.88 = 0.166x - 17.11$$ subtract the two x points

$$y = 0.166x - 7.23$$ answer for equation

Predict what the daily cost would be if the temperature is 98°.

$$y = 0.166(98) - 7.23$$ substitute 98 into the equation

$$y = 16.268 - 7.23$$ simplify

$$y = 9.038$$ answer for prediction = $9.03

Category 3: Writing and Solving
Linear Functions, Equations, and Inequalities

Determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real world situations, both continuous and discrete; and represent domain and range using inequalities.

A linear function has two variables, x and y. When graphed, a linear function will be a straight line. The domain of a function are all the input values (x). The range of a function are all the output values (y). A discrete domain are specific numbers in an interval. A continuous domain are all numbers in an interval.
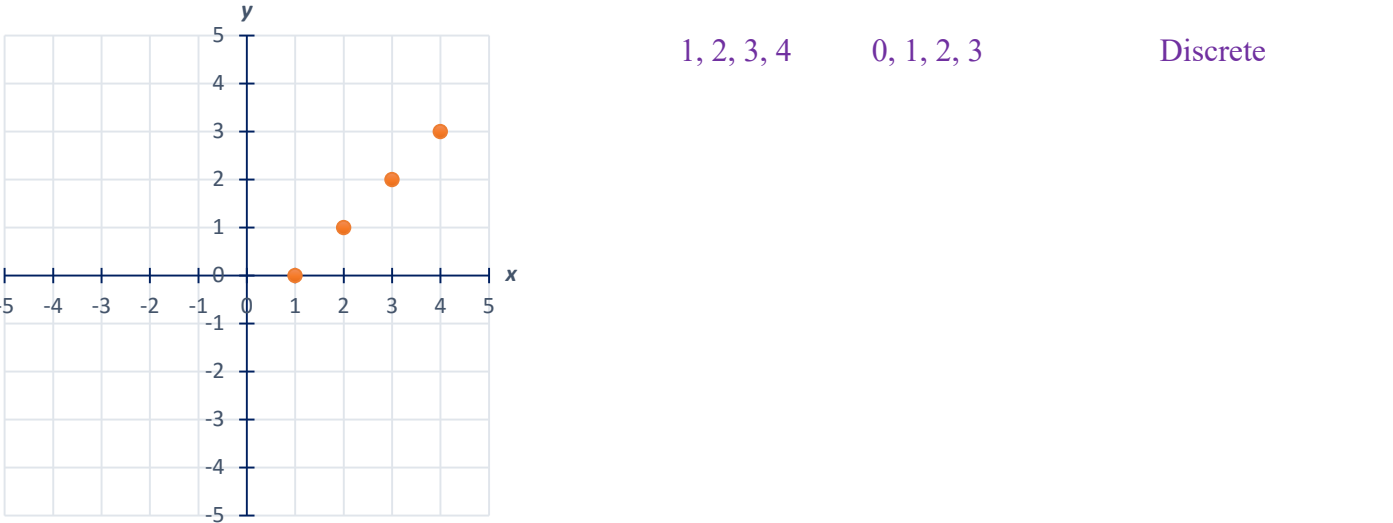
A linear function may appear in different formats. Determine the domain and range of the following linear functions and indicate whether the domain is continuous or discrete.

Linear Function	Domain	Range	Continuous or Discrete
(0,2), (2,4), (4,6), (6,8)	0, 2, 4, 6	2, 4, 6, 8	Discrete
y = x, with inputs 5, 6, 7, 8	5, 6, 7, 8	5, 6, 7, 8	Discrete

input (x)	1	2	3	4
output (y)	4	6	8	10

1, 2, 3, 4   4, 6, 8, 10   Discrete

x	y
2	6
3	9
4	12
5	15

2, 3, 4, 5   6, 9, 12, 15   Discrete

1, 2, 3, 4   0, 1, 2, 3   Discrete

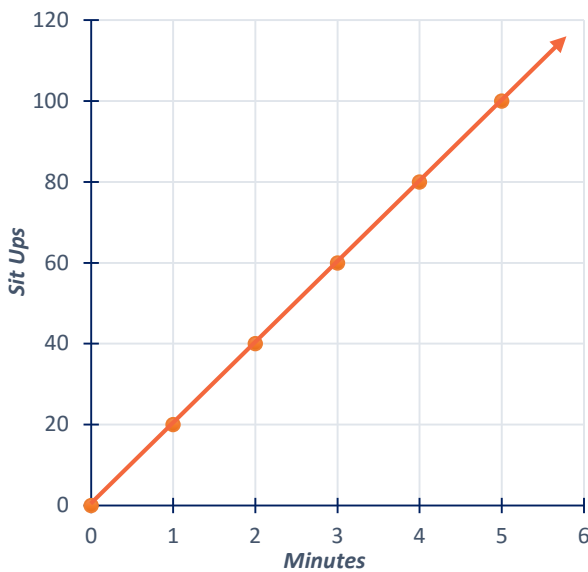| Linear Function	Domain	Range	Continuous or Discrete
	$1 \leq x \leq 4$	$0 \leq y \leq 3$	Continuous

The number of sit ups (y) an athlete can perform in (x) minutes is given by the linear function y=20x and shown by the graph below.
Determine the domain, range and indicate whether the domain is continuous or discrete.

	Domain	Range	Continuous or Discrete
	$x \geq 0$	$y \geq 0$	Continuous

Write linear equations in two variables in various forms, including y = mx + b, Ax + By = C, and y – y₁ = m(x – x₁), given one point and the slope and given two points.

The slope-intercept form is: $y = mx + b$
The standard form is: $Ax + By = C$
The point-slope form is: $y - y_1 = m(x - x_1)$
The slope formula (given two points): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Write an equation in slope-intercept form given a slope of -2 and a y-intercept of 3.

$$y = mx + b$$ slope-intercept form

$$y = -2x + 3$$ substitute **-2** for m and **3** for b

Write an equation in slope-intercept form given a slope of 2 and a point on the line of (1,3).

determine y-intercept first

$$y = mx + b$$ slope-intercept form

$$3 = 2(1) + b$$ substitute **2** for m. **3** for y, **1** for x

$$3 = 2 + b$$ simplify

$$-2 \quad -2$$ subtract 2 from both sides

$$1 = b$$ y-intercept is 1

now write equation

$$y = mx + b$$ slope-intercept form

$$y = 2x + 1$$ substitute **2** for m and **1** for b

Write an equation in standard form given two points on the line (1,3), and (2, 5).

determine slope first

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ slope formula given 2 points

$$m = \frac{5 - 3}{2 - 1}$$ replace with coordinates

$$m = 2$$ slope = 2

now write equation

$$y - y_1 = m(x - x_1)$$ point-slope form

$$y - 3 = 2(x - 1)$$ substitute **2** for m. **3** for y, **1** for x

$$y - 3 = 2x - 2$$ simplify

$$-2x + y = 1$$ rewrite in standard form

Write linear equations in two variables given a table of values, a graph, and a verbal description.

The slope-intercept form is: $y = mx + b$
The point-slope form is: $y - y_1 = m(x - x_1)$
The slope formula (given two points): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ OR $m = \dfrac{rise}{run}$

Write a linear function for this table.

x	y
2	5
3	8
4	11
5	14

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ determine slope first

      slope formula given 2 points

$m = \dfrac{8 - 5}{3 - 2}$ replace with (2,5) and (3,8)

$m = 3$ slope = 3

now write equation

$y - y_1 = m(x - x_1)$ point-slope form

$y - 5 = 3(x - 2)$ substitute **3** for m. **5** for y, **2** for x

$y - 5 = 3x - 6$ simplify

$y = 3x - 1$ simplify

Write an equation for this graph.

determine slope first $m = \dfrac{rise}{run}$

$m = \dfrac{-4}{2}$

$m = -2$

rise = -4

run = 2

use point-slope form with slope of -2 and point (-2,0)

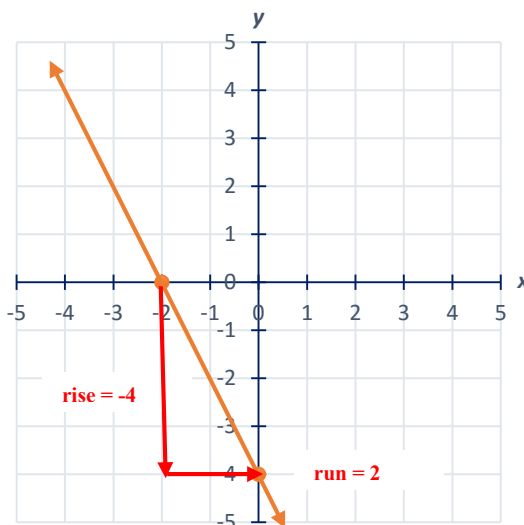$y - y_1 = m(x - x_1)$

$y - 0 = -2(x - (-2))$

$y - 0 = -2(x + 2)$

$y = -2x - 4$

OR

use slope-intercept form with slope of -2 and y-intercept of -4

$y = mx + b$

$y = -2x - 4$

The average price of a home in Texas was $87,300 in 1990. In 2010, the average price of a home in Texas increased to $192,300. Write a linear equation to represent the cost of a home between 1990 and 2010. Predict the cost of an average Texas home in 2030.

Let x represent the years since 1990. Let y represent the cost. The data can be represented as two coordinates (0,87300) and (20,192300).

determine slope first

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
slope formula given 2 points

$$m = \frac{192300 - 87300}{20 - 0}$$
replace with (0,87300) and (20,192300)

$$m = 5250$$
slope = 5250

now write equation

$$y - y_1 = m(x - x_1)$$
point-slope form

$$y - 87300 = 5250(x - 0)$$
substitute **5250** for m. **87300** for y, **0** for x

$$y - 87300 = 5250x - 0$$
simplify

$$y = 5250x + 87300$$
answer
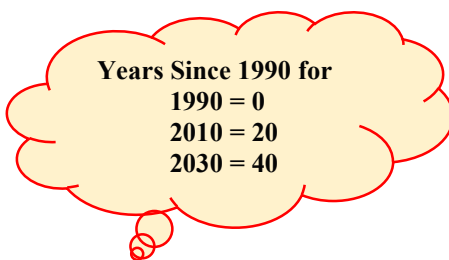
The prediction for 2030 is:

$$y = 5250(40) + 87300$$
substitute **40** for x

$$y = 297300$$
answer is $297,300

Years Since 1990 for
1990 = 0
2010 = 20
2030 = 40

Write and solve equations involving direct variation.

An equation of the form y = mx (where m ≠ 0) is a direct variation. m is the constant of variation and also the slope. It is a linear function which is a straight line on a graph. It always passes through the origin (0,0).

Write an equation for this scenario. Then graph the equation.
Betty Jones pays $1,155 in sales tax on a new car that cost $14,000.
Mary Wang pays $1,320 in sales tax on a new car that cost $16,000.
John Smith pays $1,650 in sales tax on a new car that cost $20,000.

Create a table of the data (make Price the x-axis, Tax the y-axis)

Price (x)	Tax (y)
14,000	1,155
16,000	1.320
20,000	1,650

Determine equation

$y = mx$ direct variation format

$1155 = m(14000)$ substitute in first row of data

$\dfrac{1155}{14000} = \dfrac{m(14000)}{14000}$ divide each side by 14000

$0.0825 = m$ answer (8.25% sales tax rate)

$y = mx$ repeat for second row of data

$1320 = m(16000)$

$\dfrac{1320}{16000} = \dfrac{m(16000)}{16000}$

$0.0825 = m$ answer (8.25% sales tax rate)

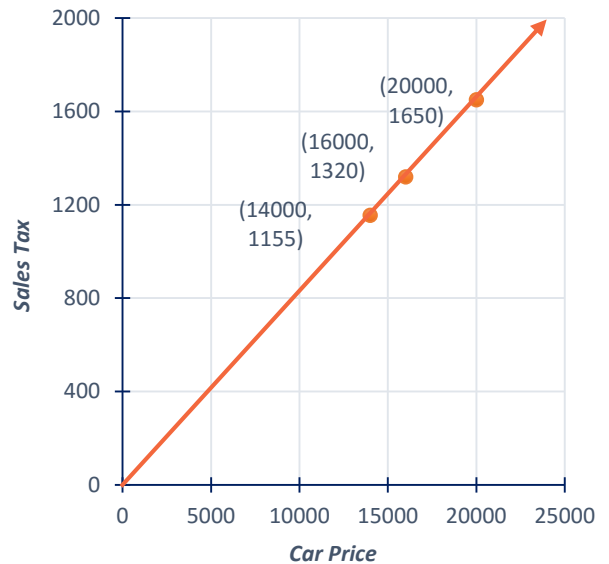$y = mx$ repeat for third row of data

$1650 = m(20000)$

$\dfrac{1650}{20000} = \dfrac{m(20000)}{20000}$

$0.0825 = m$ answer (8.25% sales tax rate)

All three sales tax rates are the same 0.0825 (8.25%). There is a direct variation. 0.0825 is the constant of variation (m). The equation is: **y = 0.0825x**

Graph equation

Write the equation of a line that contains a given point and is parallel to a given line.

Two lines are parallel if they have the same slope.

Write an equation for the line that passes through (-2,-3) and is parallel to the line *y = 3x + 1*.

$$y = mx + b$$
$$y = 3x + 1$$
$$m = 3$$

determine slope first

slope-intercept form

given equation

slope = 3

now write **parallel** equation

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = 3[x - (-2)]$$
$$y + 3 = 3x + 6$$
$$y = 3x + 3$$

point-slope form

substitute **3** for m. **-3** for y, **-2** for x

simplify

simplify

Write the equation of a line that contains a given point and is perpendicular to a given line.

Two lines are perpendicular if they have negative reciprocal slopes.

Write an equation for the line that passes through (-2,-3) and is perpendicular to the line *y = 3x + 1*.

$$y = mx + b$$
$$y = 3x + 1$$
$$m = 3$$

determine slope first

slope-intercept form

given equation

slope = 3, negative reciprocal is -1/3

now write **perpendicular** equation

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = -\frac{1}{3}[x - (-2)]$$
$$y + 3 = -\frac{1}{3}x - \frac{2}{3}$$
$$y = -\frac{1}{3}x - 3\frac{2}{3}$$

point-slope form

substitute **-1/3** for m. **-3** for y, **-2** for x

simplify

simplify

Write an equation of a line that is parallel or perpendicular to the x-axis or y-axis and determine whether the slope of the line is zero or undefined.

A line is parallel to the x-axis if it has this form, y = b, where b is a constant. All lines parallel to the x-axis have a slope of 0.

Write two equations parallel to the x-axis. What is the slope of each line?

$$y = -3 \qquad \text{slope is 0}$$

$$y = 3 \qquad \text{slope is 0}$$

A line is perpendicular to the x-axis if it has this form, x = a, where a is a constant. All lines perpendicular to the x-axis have an undefined slope.

Write two equations perpendicular to the x-axis. What is the slope of each line?

$$x = -3 \qquad \text{slope is undefined}$$

$$x = 3 \qquad \text{slope is undefined}$$

A line is parallel to the y-axis if it has this form, x = a, where a is a constant. All lines parallel to the y-axis have an undefined slope.

Write two equations parallel to the y-axis. What is the slope of each line?

$$x = -3 \qquad \text{slope is undefined}$$

$$x = 3 \qquad \text{slope is undefined}$$

A line is perpendicular to the y-axis if it has this form, y = b, where b is a constant. All lines perpendicular to the x-axis have a slope of 0.

Write two equations perpendicular to the y-axis. What is the slope of each line?

$$y = -3 \qquad \text{slope is 0}$$

$$y = 3 \qquad \text{slope is 0}$$

Write an equation parallel to the line, x = 3.

$$x = 10 \qquad \text{slope is undefined}$$

Write an equation parallel to the line, y = 3.

$$y = -20 \qquad \text{slope is 0}$$

Write an equation perpendicular to the line, x = 3.

$$y = 30 \qquad \text{slope is 0}$$

Write an equation perpendicular to the line, y = 3.

$$x = -40 \qquad \text{slope is undefined}$$

Write linear inequalities in two variables given a table of values, a graph, and a verbal description.

A custom made elevator can hold a maximum of 1250 lbs. This table shows the weight of each person in the elevator. Write a linear inequality to represent the maximum allowable weight of Jay and Ron in order to ride the elevator with the group.

Individual	Weight (lbs)
Tim	135
Jan	140
Sam	195
Jay	x
Lee	155
Ron	y
Sue	170

$$135 + 140 + 195 + x + 155 + y + 170 \leq 1250$$

$$795 + x + y \leq 1250$$

$$-795 \qquad \qquad -795$$

$$x + y \leq 455$$

Write a linear inequality to represent this graph.

determine slope first
$$m = \frac{rise}{run} = \frac{-2}{3} = -\frac{2}{3}$$
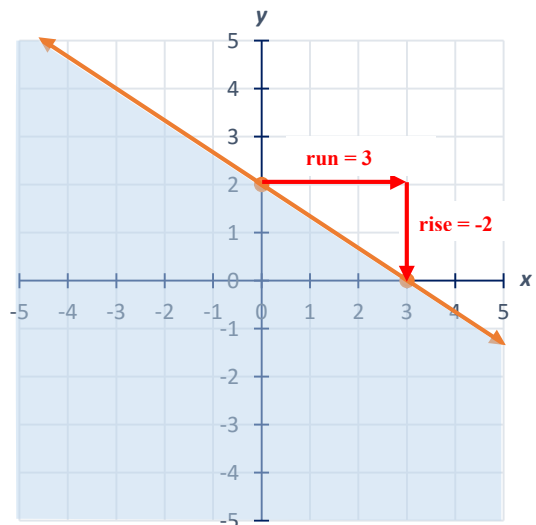
determine y-intercept
$$y - intercept = 2$$

write an equation of the line using the slope-intercept form with slope of -2/3 and y-intercept of 2
$$y = mx + b$$
$$y = -\frac{2}{3}x + 2$$

now, replace the "=" sign with an inequality sign using these rules:
$$y \leq -\frac{2}{3}x + 2$$

if graph has a dashed line, and is shaded below the line: replace "=" sign with "<".
if graph has a dashed line, and is shaded above the line: replace "=" sign with ">".
if graph has a solid line, and is shaded below the line: replace "=" sign with "≤".
if graph has a solid line, and is shaded above the line: replace "=" sign with "≥".

Maria and Carlos are getting married. You want to buy them items from their gift registry. The gift registry shows picture frames at $10 a piece and iced tea glassware at $12 a piece. You want to spend a maximum of $100 on gifts. Write a linear inequality to represent how many picture frames and iced tea glassware you can buy.

Let x represent picture frames.   Let y represent iced tea glassware.

$$10 \cdot x + 12 \cdot y \leq 100$$

$$10x + 12y \leq 100$$

Write systems of two linear equations given a table of values, a graph, and a verbal description.

A system of two linear equations uses the same variables (frequently x and y).

Sarah opened a savings account with $100 and added $50 each month. Frank opened a savings account with $50 and added $60 each month. Write a system of linear equations to model this scenario.

Let x represent months.   Let y represent total savings each month.

Month	Sarah's Savings	Frank's Savings
0	100	50
1	150	110
2	200	170
3	250	230
4	300	290
5	350	350

SYSTEM

equation 1 – Sarah's savings $y = 50x + 100$

equation 2 – Frank's savings $y = 60x + 50$

NOTE: The $50 and $60 added each month is the rate of change and thus the slope.

This graph represents yearly car sales by two local dealers since the year 2000. Dealership 1 is graphed in Orange. Dealership 2 is graphed in Blue.
Write a system of linear equations to model the graph lines.

The slope of Dealership 1 is: $m = \frac{rise}{run} = \frac{800-200}{8-0} = 75$

The slope of Dealership 2 is: $m = \frac{rise}{run} = \frac{600-300}{8-0} = 37.5$
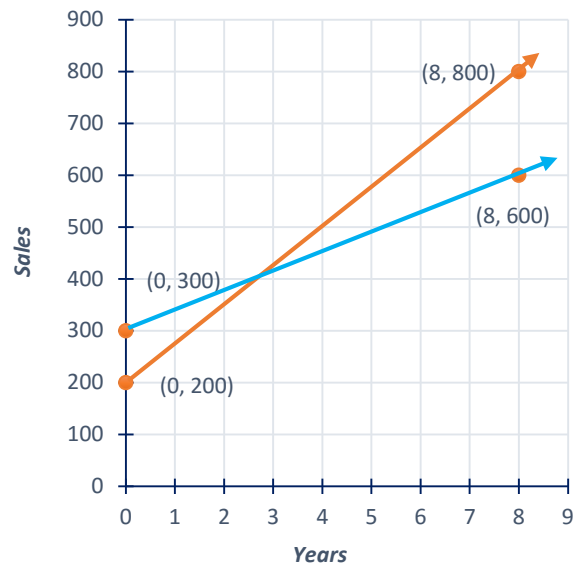
SYSTEM

equation 1 – Dealership 1 $y = 75x + 200$

equation 2 – Dealership 2 $y = 37.5x + 300$

NOTE: If the y-intercept isn't specified, use the point-slope form to write equation 1 and equation 2.

A teacher buys 20 rulers and 4 boxes of pencils for her students from a school supplies account that cost $60. The teacher buys another 4 rulers and 2 boxes of pencils a month later for $18.
Write a system of linear equations to find the cost of each ruler and each box of pencils.

Let x represent a ruler.   Let y represent a box of pencils.

SYSTEM

equation 1 – First Purchase $20x + 4y = 60$

equation 2 – Second Purchase $4x + 2y = 18$

Solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

The Distributive Property: $a(b + c) = ab + ac$

Solve this linear equation (one variable on one side) using the distributive property:

$5(2 + x) + 5 = 30$	equation
$5(2) + 5(x) + 5 = 30$	distributive property
$10 + 5x + 5 = 30$	multiply
$5x + 15 = 30$	combine like terms
$-15 \quad -15$	subtract 15 from both sides
$5x = 15$	simplify
$\dfrac{5x}{5} = \dfrac{15}{5}$	divide each side by 5
$x = 3$	answer

Solve this linear equation (one variable on both sides) using the distributive property:

$5(2 + x) + 5 = 4(5 - x) + 13$	equation
$5(2) + 5(x) + 5 = 4(5) - 4(x) + 13$	distributive property
$10 + 5x + 5 = 20 - 4x + 13$	multiply
$5x + 15 = -4x + 33$	combine like terms
$-15 \qquad -15$	subtract 15 from both sides
$5x = -4x + 18$	simplify
$+4x \quad +4x$	add 4x to both sides
$9x = 18$	simplify
$\dfrac{9x}{9} = \dfrac{18}{9}$	divide each side by 9
$x = 2$	answer

Solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

Change the inequality symbol direction if multiplying or dividing the inequality by a negative number.

Solve this linear inequality (one variable on one side) using the distributive property:

$-4(x-2) > 0$	equation
$-4(x) - 4(-2) > 0$	distributive property
$-4x + 8 > 0$	multiply
$-8 \quad -8$	subtract 8 from both sides
$-4x > -8$	simplify
$\dfrac{-4x}{-4} < \dfrac{-8}{-4}$	divide each side by -4 and change the inequality symbol direction
$x < 2$	answer

Solve this linear inequality (one variable on both sides) using the distributive property:

$4(x-2) > 2(x-2)$	equation
$4(x) + 4(-2) > 2(x) + 2(-2)$	distributive property
$4x - 8 > 2x - 4$	multiply
$+8 \quad\quad +8$	add 8 to both sides
$4x > 2x + 4$	simplify
$-2x \quad -2x$	subtract -2x to both sides
$2x > 4$	simplify
$\dfrac{2x}{2} > \dfrac{4}{2}$	divide each side by 2
$x > 2$	answer

If only constants remain after simplifying an inequality and the inequality is *false*, there is no solution.

$-4x + 8 > -4x + 10$	equation
$+4x \quad\quad +4x$	add 4x to both sides
$8 > 10$	false, so no solution

If only constants remain after simplifying an inequality and the inequality is *true*, then all real numbers are solutions.

$-4x + 8 < -4x + 10$	equation
$+4x \quad\quad +4x$	add 4x to both sides
$8 < 10$	true, so all real numbers are solutions

Solve systems of two linear equations with two variables for mathematical and real-world problems.

The solution to a system of two linear equations is an ordered pair that solves each equation. Solutions can be found by graphing, substitution and elimination.

To solve a system by graphing, graph the lines and find the intersecting point.

Solve this system of linear equations by graphing.

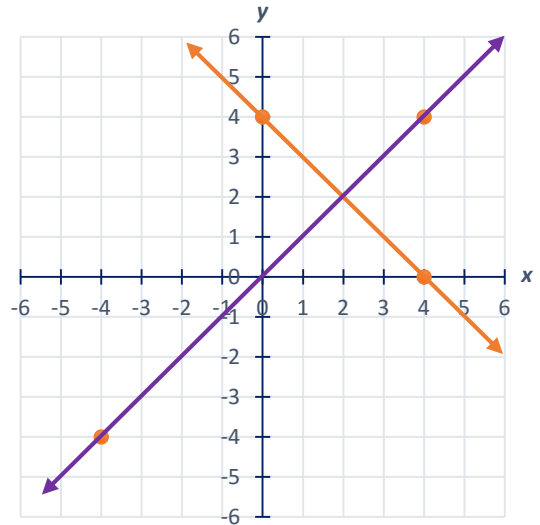**Equation 1:** $y = -x + 4$

x-intercept is (4,0)
y-intercept is (0,4)

Equation 2: $y = x$
x-intercept is (0,0), second point is (4,4)
y-intercept is (0,0), second point is (-4,-4)

The lines intersect at **(2,2)**. That is the solution.

To solve a system by substitution, solve one equation for "y". Substitute the expression into the other equation. This gives "x". Then substitute the value of "x" into the first equation and solve for "y".

Solve this system of linear equations by substitution.

$$20x + 4y = 60 \qquad \text{equation 1}$$
$$4x + 2y = 18 \qquad \text{equation 2}$$

$$y = \frac{-20x + 60}{4} \qquad \text{solve equation 1 for y}$$
$$y = -5x + 15 \qquad \text{simplify}$$

$$4x + 2(-5x + 15) = 18 \qquad \text{substitute -5x+15 into equation 2 for y}$$
$$4x - 10x + 30 = 18 \qquad \text{distributive property}$$
$$-6x = -12 \qquad \text{simplify}$$
$$x = 2 \qquad \text{solve for x}$$

$$20(2) + 4y = 60 \qquad \text{substitute 2 into equation 1 for x}$$
$$4y = 20 \qquad \text{simplify}$$
$$y = 5 \qquad \text{solve for y}$$

The solution is **(2,5)**.

To solve a system by elimination, multiply one equation (sometimes both equations) by a number so that the coefficients of one of the variables are the opposite. Add the equations together. Solve the equation for one of the variables. Substitute the value of the variable into one of the original equations and solve for the second variable.

Solve this system of linear equations by elimination.

$$14x + 2y = 36 \qquad \text{equation 1}$$

$$7x + 3y = 26 \qquad \text{equation 2}$$

$$-2(7x) + -2(3y) = -2(26) \qquad \text{multiply equation 2 by -2 so we}$$

$$-14x - 6y = -52 \qquad \text{have -14x}$$

$$14x + 2y = 36 \qquad \text{equation 1}$$

$$\underline{-14x - 6y = -52} \qquad \text{revised equation 2}$$

$$-4y = -16 \qquad \text{add the two equations}$$

$$y = 4 \qquad \text{solve for y}$$

$$7x + 3(4) = 26 \qquad \text{substitute 4 into equation 1 for y}$$

$$7x = 14 \qquad \text{simplify}$$

$$x = 2 \qquad \text{solve for x}$$

The solution is **(2,4)**.

Category 4: Quadratic Functions and Equations

Determine the domain and range of quadratic functions and represent the domain and range using inequalities.

A quadratic function is a function that can be written as $y = ax^2 + bx + c$. The graph is a parabola. The vertex is the lowest point of a parabola that opens up and the highest point of a parabola that opens down. The parabola opens up if *a* is positive (*a* > 0). The parabola opens down if *a* is negative (*a* < 0).

The *domain* for a quadratic equation is always *all real numbers.*
The *range* for a quadratic equation is determined by the vertex.
The formula for the x-coordinate of the vertex is: $-\dfrac{b}{2a}$ (this is also the axis of symmetry).
Replace the x-coordinate of the vertex back into the original equation and solve for y. y is the y-coordinate of the vertex.
If the parabola opens up, the range is *all real numbers ≥ the y-coordinate of the vertex.*
If the parabola opens down, the range is a*ll real numbers ≤ the y-coordinate of the vertex.*

Determine the domain and range of this equation: $y = 2x^2 + 4x - 3$

$$x = -\frac{b}{2a}$$ formula for vertex, x-coordinate

$$x = -\frac{4}{2(2)}$$ replace b with 4, replace a with 2

$$x = -1$$ vertex, x-coordinate is -1

$$y = 2x^2 + 4x - 3$$ original equation

$$y = 2(-1)^2 + 4(-1) - 3$$ replace x with -1

$$y = 2 - 4 - 3$$ simplify

$$y = -5$$ vertex, y-coordinate is -5

$$vertex = (-1, -5)$$

The domain is	***all real numbers***	domain is always all real numbers
The range is	**y ≥ -5**	since parabola opens up, use ≥ sign

Determine the domain and range of this equation: $y = -2x^2 + 4x - 3$

$$x = 1$$
$$y = -1$$
$$vertex = (1, -1)$$

x = -4 / [(2) (-2)]
y = -2 (1²) + 4(1) - 3

The domain is	***all real numbers***	domain is always all real numbers
The range is	**y ≤ -1**	since parabola opens down, use ≤ sign

Write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form (f(x) = a(x – h)² + k), and rewrite the equation from vertex form to standard form (f(x) = ax² + bx + c).

The standard form of a quadratic function is: $y = ax^2 + bx + c$
The vertex form of a quadratic function is: $y = a(x - h)^2 + k$

Write a quadratic equation in vertex form given the vertex of (2,3) and the point (4,1).

$y = a(x - h)^2 + k$	vertex form
$y = a(x - 2)^2 + 3$	replace h with 2, replace k with 3
$1 = a(4 - 2)^2 + 3$	replace x with 4, replace y with 1
$1 = 4a + 3$	simplify
$-\dfrac{1}{2} = a$	solve for a
$y = -\dfrac{1}{2}(x - 2)^2 + 3$	now write the equation replacing a with -1/2, h with 2 and k with 3

Rewrite equation $y = -\frac{1}{2}(x - 2)^2 + 3$ in standard form.

$y = -\dfrac{1}{2}(x - 2)^2 + 3$	vertex form
$y = -\dfrac{1}{2}(x^2 - 4x + 4) + 3$	square (x - 2)
$y = -\dfrac{1}{2}x^2 + 2x - 2 + 3$	distribute -1/2
$y = -\dfrac{1}{2}x^2 + 2x + 1$	simplify, now in standard form

Write quadratic functions when given real solutions and graphs of their related equations.

The standard form of a quadratic function is: $y = ax^2 + bx + c$
The intercept form of a quadratic function is: $y = a(x - p)(x - q)$

Write a quadratic equation in standard form given x-intercepts of -2 and 5.

$$y = a(x - p)(x - q)$$ intercept form

$$y = 1(x + 2)(x - 5)$$ replace p with -2, q with 5, a with 1

$$y = x^2 - 3x - 10$$ FOIL method

NOTE: "a" can be any non-zero value. Use "1" because it make the equation simple. If another point on the graph was provided, then "a" will have to be computed. (see next problem).

Write a quadratic equation for this graph.

The x-intercepts are 1 and 3.
The vertex is (2,2).

using the intercept form, substitute the x-intercepts in for p and q

$$y = a(x - p)(x - q)$$

$$y = a(x - 1)(x - 3)$$

next, determine the value of "a" by substituting a point on the graph (2,2) in for x and y

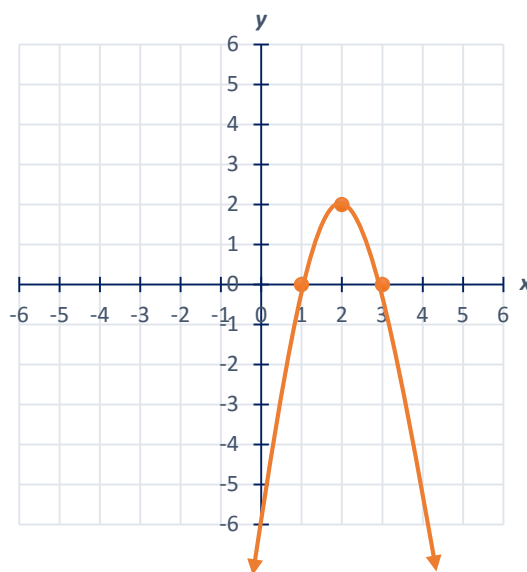$$2 = a(2 - 1)(2 - 3)$$

$$2 = a(1)(-1)$$

$$-2 = a$$

now write the equation replace a with -2, p with 1 and q with 3

$$y = a(x - 1)(x - 3)$$

$$y = -2(x - 1)(x - 3)$$

$$y = -2(x^2 - 4x + 3)$$

$$y = -2x^2 + 8x - 6$$

Graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

The standard form of a quadratic function is: $y = ax^2 + bx + c$
The intercept form of a quadratic function is: $y = a(x - p)(x - q)$
The x-intercept is the solution to the equation when y = 0. Also the x-intercept is the "p" and "q" when the function is written in intercept form.
The y-intercept is "c" from the standard form - coordinate (0,c).
The zeros of a function are the x-intercepts.

The vertex is the lowest point of a parabola that opens up and the highest point of a parabola that opens down. The parabola opens up if a is positive ($a > 0$). The parabola opens down if a is negative ($a < 0$).
The formula for the x-coordinate of the vertex is: $-\dfrac{b}{2a}$ (this is also the axis of symmetry).
Replace the x-coordinate of the vertex back into the original equation and solve for y. y is the y-coordinate of the vertex.
The y-coordinate of the vertex is the maximum value if the graph opens down. It is the minimum value if the graph opens up.

Graph the equation $y = 2x^2 - 8x + 6$.
Determine the x-intercepts, y-intercept, zeros, maximum or minimum value, vertex and axis of symmetry.

factor the equation into two bionomials.
using the intercept form, the x-intercepts are 1 and 3

$y = 2(x^2 - 4x + 3)$
$y = 2(x - 1)(x - 3)$

determine the x-coordinate of the vertex – this is also the axis of symmetry

$x = -\dfrac{b}{2a}$
$x = -\dfrac{(-8)}{2(2)}$
$x = 2$

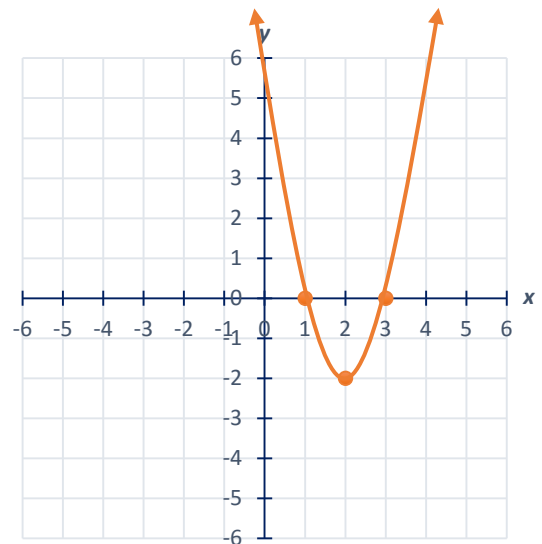determine the y-coordinate of the vertex by substituting 2 for x back into the original equation

$y = 2x^2 - 8x + 6$
$y = 2(2)^2 - 8(2) + 6$
$y = -2$

vertex is (2, -2)

the y-intercept is 6 (the "c" value from the original equation)

Summary

x-intercepts:	1 and 3
y-intercept:	6
zeros:	1 and 3
minimum value	-2
vertex:	(2, -2)
axis of symmetry:	x = 2

Describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions.

The standard form of a quadratic function is: $y = ax^2 + bx + c$
The intercept form of a quadratic function is: $y = a(x - p)(x - q)$

When a quadratic function can be rewritten into intercept form, the x-intercepts can be immediately determined. The x-intercepts are the "p" and "q" values of the intercept form. They are also the Zeros of the function.

Given the equation $y = x^2 - 8x + 15$, the equation can be factored into $y = (x - 5)(x - 3)$

When x is 5 or 3 the equation equals 0. Thus 5 and 3 are the x-intercepts and the Zeros of the function.

Determine the effects on the graph of the parent function f(x) = x² when f(x) is replaced by af(x), f(x) + d, f(x − c), f(bx) for specific values of a, b, c, and d.

af(x) will stretch or shrink a graph vertically.
Graph stretches if a > 1.
Graph shrinks if 0 < a < 1.
Graph stretches and reflects if a < -1.
Graph shrinks and reflects if -1 < a < 0.
Vertical Stretch/Shrink:

$$f(x) = x^2$$
$$g(x) = 2f(x)$$
$$h(x) = \frac{1}{2}f(x)$$
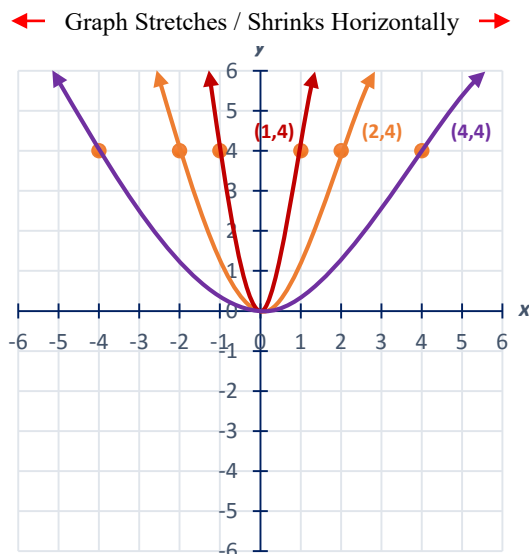$$i(x) = -x^2$$
$$j(x) = -2f(x)$$
$$k(x) = -\frac{1}{2}f(x)$$

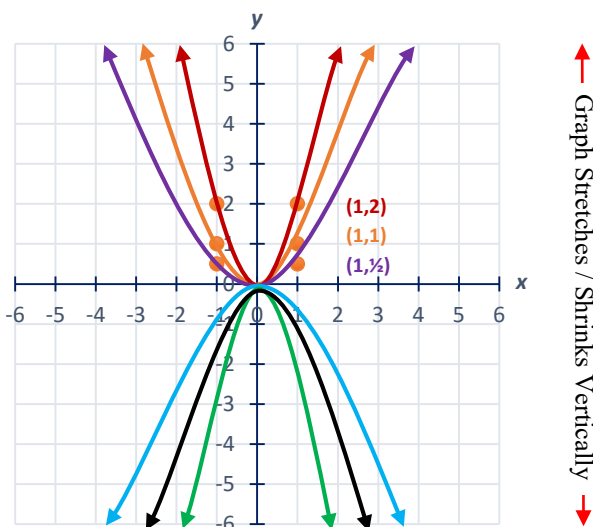**f(bx) will stretch or shrink a graph horizontally.**
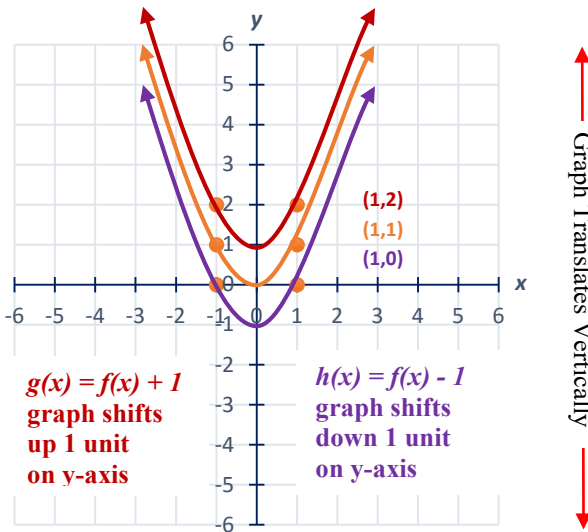**Graph shrinks if |a| > 1.**
Graph stretches if 0 < |a| < 1.

Horizontal Stretch/Shrink:

$$f(x) = x^2$$
$$g(x) = f(2x)$$
$$h(x) = f(\frac{1}{2}x)$$

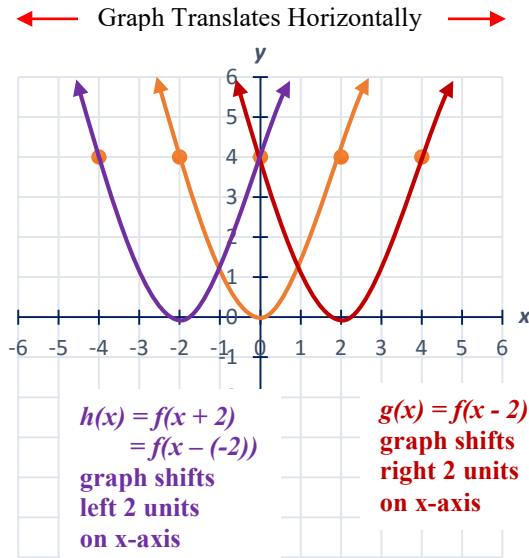← Graph Stretches / Shrinks Horizontally →

f(x)+d will translate a graph vertically.
Graph shifts down if d < 0, up if d > 0.
Vertical Translations: $f(x) = x^2$
$$g(x) = f(x) + 1$$
$$h(x) = f(x) - 1$$

f(x-c) will translate a graph horizontally.
Graph shifts left if c < 0, right if c > 0.
Horizontal Translations: $f(x) = x^2$
$$g(x) = f(x - 2)$$
$$h(x) = f(x + 2)$$

Graph Translates Horizontally

Graph Translates Vertically

(1,2)
(1,1)
(1,0)

g(x) = f(x) + 1
graph shifts
up 1 unit
on y-axis

h(x) = f(x) - 1
graph shifts
down 1 unit
on y-axis

h(x) = f(x + 2)
= f(x – (-2))
graph shifts
left 2 units
on x-axis

g(x) = f(x - 2)
graph shifts
right 2 units
on x-axis

Solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

To solve a quadratic equation by factoring, first set the equation equal to 0 (if not already equal to 0), then factor out the Greatest Common Factor (GCF) if any. Next, factor the equation into two binomials. Solve each binomial equation equal to 0 separately.

Solve this quadratic equation by factoring:

$4x^2 + 8x = 60$	equation
$-60 \quad -60$	subtract 60 from both sides
$4x^2 + 8x - 60 = 0$	simplify so equation is set equal to 0
$4(x^2 + 2x - 15) = 0$	factor out GCF of 4
$4(x + 5)(x - 3) = 0$	factor into 2 binomials
$(x + 5) = 0$	solve first binomial equal to 0
$x = -5$	subtract 5 from both sides
$(x - 3) = 0$	solve second binomial equal to 0
$x = 3$	add 3 to both sides
$x = 3 \ and -5$	answer

Taking the square root of a quadratic equation is possible for equations of this format: $ax^2 + c = 0$. (NOTE: This is not applicable if the equation has a "bx" term")

To solve a quadratic equation by taking square roots, first isolate the variable on one side of the equation. Then take the square root of both sides.

Solve this quadratic equation by taking square roots:

$2x^2 - 32 = 0$	equation
$+\,32 \quad +\,32$	add 32 to both sides
$2x^2 = 32$	simplify
$\dfrac{2x^2}{2} = \dfrac{32}{2}$	divide both sides by 2
$x^2 = 16$	simplify
$\sqrt{x^2} = \sqrt{16}$	take square root of both sides
$x = \pm\,4$	answer

Solve this quadratic equation by taking square roots:

$(x + 2)^2 = 36$	equation
$\sqrt{(x + 2)^2} = \sqrt{36}$	take square root of both sides
$x + 2 = \pm\,6$	simplify
$-\,2 \quad -\,2$	subtract 2 from both sides
$x = \pm\,6 - 2$	simplify
$x = 6 - 2$	solve with +6
$x = 4$	first answer
$x = -6 - 2$	solve with -6
$x = -8$	second answer
$x = 4 \text{ and } -8$	answer

Completing the square of a quadratic equation is possible for equations of this format: $x^2 + bx = c$.
To solve a quadratic equation by completing the square, add $\left(\dfrac{b}{2}\right)^2$ to both sides.

Solve this quadratic equation by completing the square:

$$x^2 - 12x = 28 \qquad \text{equation}$$

$$x^2 - 12x + \left(-\frac{12}{2}\right)^2 = 28 + \left(-\frac{12}{2}\right)^2 \qquad \text{add } (b/2)^2 \text{ to both sides}$$

$$x^2 - 12x + 36 = 64 \qquad \text{simplify}$$

$$(x - 6)^2 = 64 \qquad \text{factor into a perfect square}$$

$$\sqrt{(x - 6)^2} = \sqrt{64} \qquad \text{take square root of both sides}$$

$$x - 6 = \pm 8 \qquad \text{simplify}$$

$$+6 \quad +6 \qquad \text{add 6 to both sides}$$

$$x = \pm 8 + 6 \qquad \text{simplify}$$

$$x = 14 \text{ and } -2 \qquad \text{answer}$$

If the coefficient of the x^2 term isn't 1, make it 1 before completing the square.

Solve this quadratic equation by completing the square:

$$2x^2 + 16x = 40 \qquad \text{equation}$$

$$\frac{2x^2 + 16x}{2} = \frac{40}{2} \qquad \text{divide both sides by 2}$$

$$x^2 + 8x = 20 \qquad \text{simplify}$$

$$x^2 + 8x + \left(\frac{8}{2}\right)^2 = 20 + \left(\frac{8}{2}\right)^2 \qquad \text{add } (b/2)^2 \text{ to both sides}$$

$$x^2 + 8x + 16 = 36 \qquad \text{simplify}$$

$$(x + 4)^2 = 36 \qquad \text{factor into a perfect square}$$

$$\sqrt{(x + 4)^2} = \sqrt{36} \qquad \text{take square root of both sides}$$

$$x + 4 = \pm 6 \qquad \text{simplify}$$

$$-4 \quad -4 \qquad \text{subtract 4 from both sides}$$

$$x = \pm 6 - 4 \qquad \text{simplify}$$

$$x = 2 \text{ and } -10 \qquad \text{answer}$$

The quadratic formula is: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **where a $\neq$ 0 and b² – 4ac $\geq$ 0.**

Solve this quadratic equation by using the quadratic formula:

$$3x^2 + 2x - 5 = 0$$ equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ quadratic formula

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(-5)}}{2(3)}$$ substitute 3 for a, 2 for b, -5 for c

$$x = \frac{-2 \pm \sqrt{64}}{6}$$ simplify

$$x = \frac{-2 \pm 8}{6}$$ take square root of 64

$$x = 1 \ and \ -\frac{5}{3}$$ answer

Write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

A remote controlled helicopter flies in the air. An onboard computer tracks the helicopter's height and time in the air. Using a graphing calculator, find a quadratic model for the data. Is the model a good fit? Predict the height at 17 seconds.

Time in the air (seconds)	Height (inches)
4	90
6	120
8	150
10	158
12	146
14	120
16	90
18	50
20	0
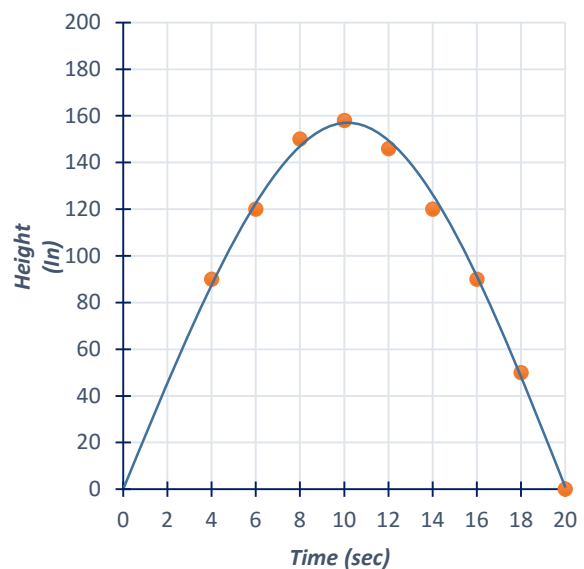
Enter the data in a graphing calculator. Use the calculator's Quadratic Regression option. The calculator should give the following results: y=ax² + bx + c

a= -1.584415584
b= 31.95930736
c= -10.43809524
R²= .9901094299   -- R² number close to 1 indicates a good fit
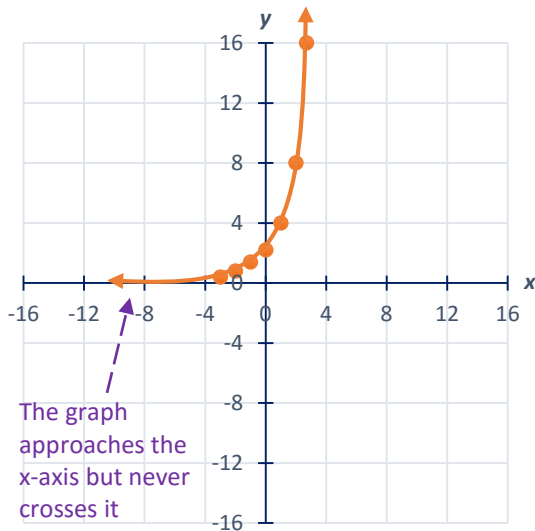
The equation for the data is: y = -1.58x² + 31.96x – 10.44   It is a good fit.
At 17 seconds, the height is: 76.26 inches   [y = -1.58(17)² + 31.96(17) – 10.44]

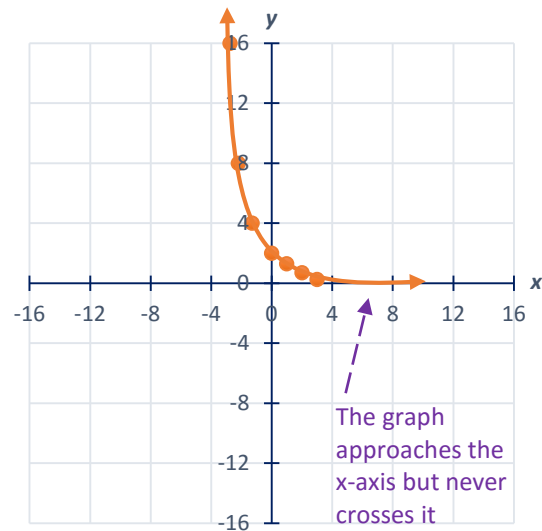# Category 5: Exponential Functions and Equations

Determine the domain and range of exponential functions of the form f(x) = abˣ and represent the domain and range using inequalities.

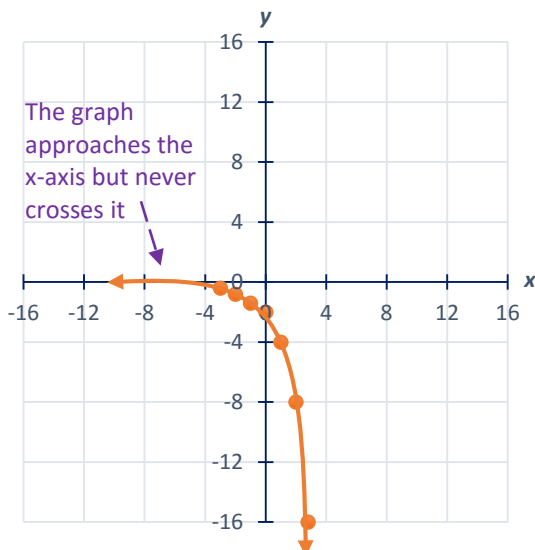**An exponential function has this form: $y = ab^x$,   where a ≠ 0, b ≠ 1 and b > 0.**

Determine the domain and range of $y = 2(2)^x$
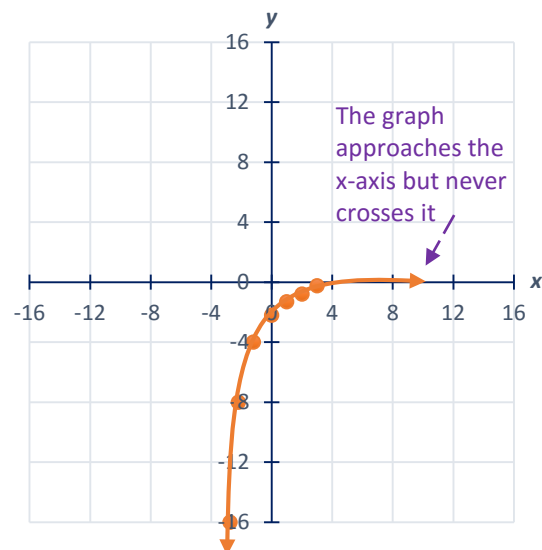Domain (x values)   =   All Real Numbers
Range   (y values)   =   y > 0

The graph approaches the x-axis but never crosses it

Determine the domain and range of $y = 2(½)^x$
Domain (x values)   =   All Real Numbers
Range   (y values)   =   y > 0

The graph approaches the x-axis but never crosses it

Determine the domain and range of $y = -2(2)^x$
Domain (x values)   =   All Real Numbers
Range   (y values)   =   y < 0

The graph approaches the x-axis but never crosses it

Determine the domain and range of $y = -2(½)^x$
Domain (x values)   =   All Real Numbers
Range   (y values)   =   y < 0

The graph approaches the x-axis but never crosses it

Interpret the meaning of the values of a and b in exponential functions of the form f(x) = abˣ in real-world problems.

The "a" value in $f(x) = ab^x$ represents the function output when x = 0. When x = 0, ab^0 is just a. It will represent the y-intercept of the graph.

The "b" value in $f(x) = ab^x$ represents the factor by which the output changes (increases or decreases) as x increases by 1. If b > 1, the output increases. If b < 1, the output decreases.

The "x" value in $f(x) = ab^x$ is ≥ 0 for the examples below.

Interpret the meaning of values *a* and *b* in this function: $y = 4(5)^x$

The value of *a* is 4. This is the starting point for the exponential function.
The value of *b* is 5. Since 5 > 1, each output increases by a factor of 5.

When x = 0, y = 4   4 is the starting point
When x = 1, y = 20   4 • 5 = 20
When x = 2, y = 100   20 • 5 = 100
When x = 3, y = 500   100 • 5 = 500
When x = 4, y = 2500   500 • 5 = 2500

Interpret the meaning of values *a* and *b* in this function: $y = 4\left(\frac{1}{5}\right)^x$
The value of *a* is 4. This is the starting point for the exponential function.
The value of *b* is 1/5. Since 1/5 < 1, each output decreases by 1/5ᵗʰ.

When x = 0, y = 4   4 is the starting point
When x = 1, y = 4/5   4 • 1/5 = 4/5
When x = 2, y = 4/25   4/5 • 1/5 = 4/25
When x = 3, y = 4/125   4/25 • 1/5 = 4/125
When x = 4, y = 4/625   4/125 • 1/5 = 4/625

Write exponential functions in the form f(x) = abˣ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay.

The equation for exponential growth is of the form f(x) = abˣ. b is replaced by (1+r), x is replaced by t giving *f(x) = a(1+r)ᵗ*.

a represents the starting point.
r represents the rate of growth (in decimal form)
t represents time

Write an exponential function to represent a savings account that starts with $1000 and earns 4% interest per year (compounded annually). How much will the account be worth after 10 years?

$$y = 1000(1 + 0.04)^t \qquad \text{equation}$$

$$y = 1000(1.04)^t \qquad \text{simplify}$$

$$y = 1000(1.04)^{10} \qquad \text{replace t with 10 to find amount after}$$

$$y = 1480.24 \qquad \text{10 years - answer is \$1,480.24}$$

The equation for exponential decay is of the form f(x) = abˣ. b is replaced by (1-r), x is replaced by t giving *f(x) = a(1-r)ᵗ*.

a represents the starting point.
r represents the rate of decay (in decimal form)
t represents time

Write an exponential function to represent a savings account that starts with $1000 and the owner spends 4% per year. How much will be left in the account after 10 years?

$$y = 1000(1 - 0.04)^t \qquad \text{equation}$$

$$y = 1000(0.96)^t \qquad \text{simplify}$$

$$y = 1000(0.96)^{10} \qquad \text{replace t with 10 to find amount after}$$

$$y = 664.83 \qquad \text{10 years - answer is \$664.83}$$

Graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems.

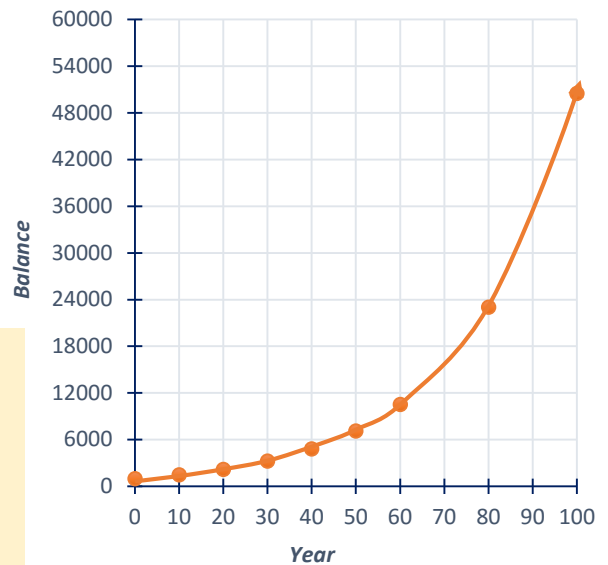**The asymptote of an exponential function is the line the graph approaches but never intersects.**

Graph an exponential growth function to represent a savings account that starts with $1000 and earns 4% interest per year (compounded annually). Identify the y-intercept, the asymptote (if any), the domain and the range.

From the previous page, the equation is: $y = 1000(1.04)^t$

Here is a table of data from the equation.

Year (t-axis)	Balance (y-axis)
0	1000
10	1480
20	2191
30	3243
40	4801
50	7106
60	10519
80	23049
100	50504

Key Features

y-intercept: (0,1000)
asymptote: none
domain: $t \geq 0$
range: $y \geq 1000$

Graph an exponential decay function to represent a savings account that starts with $1000 and the owner spends 4% per year. Identify the y-intercept, the asymptote (if any), the domain and the range.

From the previous page, the equation is: $y = 1000(0.96)^t$

Here is a table of data from the equation.

| Year (t-axis)	Balance (y-axis)
0	1000
10	664
20	442
30	293
40	195
50	129
60	86
80	38
100	16

Key Features

y-intercept: (0,1000)
asymptote: $y = 0$
domain: $t \geq 0$
range: $0 < y \leq 1000$
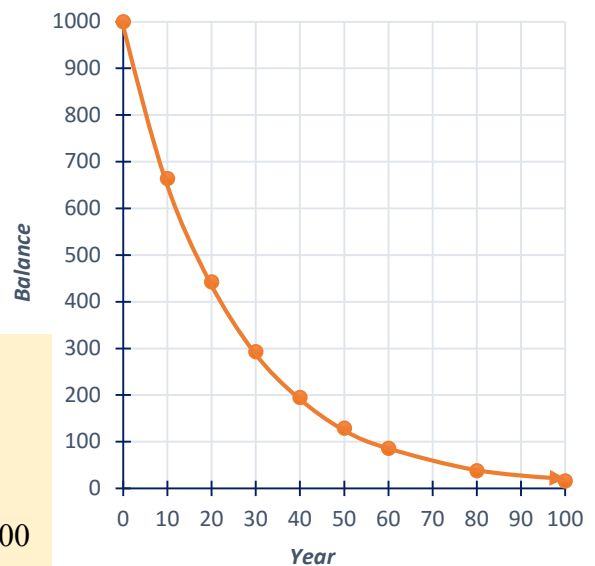
Write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.

The rabbit population on a farm grew rapidly. The farmer tracked the population over 5 years. Using a graphing calculator, find an exponential model for the data. Is the model a good fit? Predict the population at 7 years.

Rabbit Population	Years
4	1
21	2
154	3
861	4
5234	5
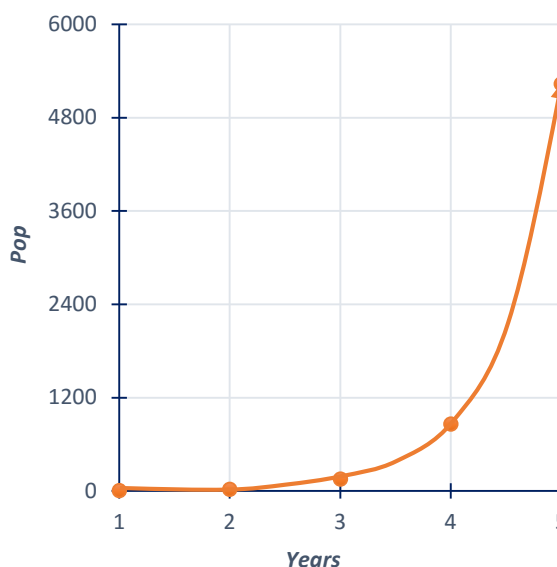
Enter the data in a graphing calculator. Use the calculator's Exponential Regression option. The calculator should give the following results: $y = ab^x$

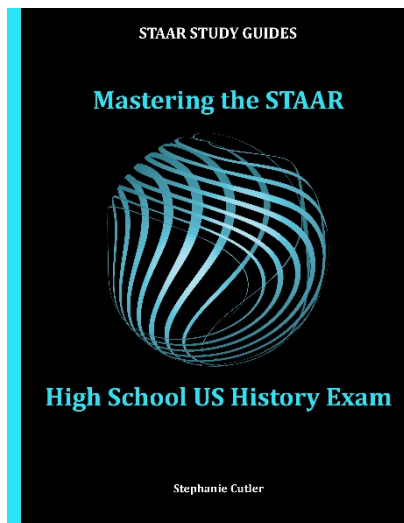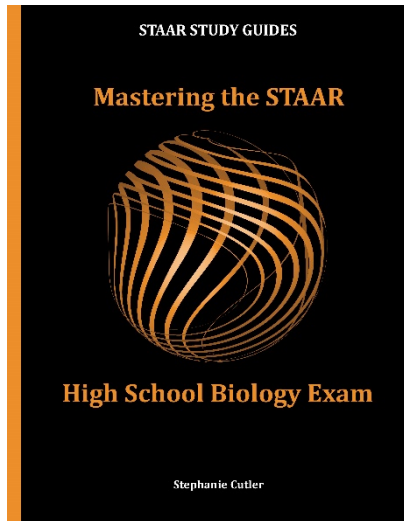$$a = .6298367278$$
$$b = 6.090222102$$
$$r^2 = .9994033038 \quad \text{-- } r^2 \text{ number close to 1 indicates a good fit}$$
$$r = .9997016074$$

The equation for the data is: $y = 0.629(6.09)^x$ It is a good fit.

At 7 years, the estimated population is: 195,421 rabbits $[y = 0.629(6.09)^7]$

Other STAAR Study Guides in this Series

STAAR STUDY GUIDES

Mastering the STAAR

High School Biology Exam

Stephanie Cutler

STAAR STUDY GUIDES

Mastering the STAAR

High School English 1 Exam

Stephanie Cutler

STAAR STUDY GUIDES

Mastering the STAAR

High School US History Exam

Stephanie Cutler

STAAR STUDY GUIDES

Mastering the STAAR

High School English 2 Exam

Stephanie Cutler

Mastering the STAAR
High School Algebra 1 Exam

Algebra 1 Topics

Number and Algebraic Methods

Describing and Graphing Linear Functions, Equations, and Inequalities

Writing and Solving Linear Functions, Equations, and Inequalities

Quadratic Functions and Equations

Exponential Functions and Equations

Print Editions of this book can be purchased on StaarGuides.com