



# MAT-TRIAD 2007

three days full of matrices

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Program and abstracts

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Bełdewo, Poland, March 22–24, 2007

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Part I

**Information**



The Mat-Triad 2007 (<http://mtriad07.amu.edu.pl/>) is the second workshop in a series started in 2005 (<http://mtriad05.amu.edu.pl/>). The place where the workshops are held is the Mathematical Research and Conference Center of the Institute of Mathematics of the Polish Academy of Sciences at Będlewo (about 30 km from Poznań). The setting is similar to Oberwolfach, with accommodation on site (for further info about this place please visit <http://www.impan.gov.pl/Bedlewo>). The workshop will focus on a number of topics, including: matrix analysis, methods of linear algebra, algorithms of numerical linear algebra, matrix games, as well as the analysis of and solutions to problems in mathematical statistics with the use of matrix algebra. It is expected that the workshop will provide a forum for the participants' presentations of current trends, the latest developments and recent techniques, as well as for an exchange of new ideas.

The workshop will include the presentation of both Invited and Contributed papers. Among the participants there are many young researchers (partially supported). The activity of young scientists has a special position in the Mat-Triad Workshop in order to encourage and promote them. Selected talks and posters will be awarded. Prizes will be given to graduate students or scientists with a recently completed Ph.D. thesis, in the area of Applications of the Methods of Matrix Algebra with special emphasis on Statistics. Prize-winning works will be widely publicized and promoted by the Workshop.

### **Invited speakers:**

- Rafael Bru (Spain),
- Ludwig Elsner (Germany),
- Daniel A. Griffith (USA),
- Charles R. Johnson (USA),
- Thomas Klein (Germany),
- João T. Mexia (Portugal),
- Juan M. Peña (Spain),
- Friedrich Pukelsheim (Germany),
- Siegfried Rump (Germany),
- George P.H. Styan (Canada),
- Goetz Trenkler (Germany),
- Roman Zmysłony (Poland).



Part II

**Program**





# Program

**Thursday, March 22, 2007**

**8:00–8:45 Breakfast**

**8:45 – Opening**

## **Session I (LA)**

9:00–9:40 *L. Elsner*: On Hessenberg matrices and their generalizations

9:40–10:05 *I. Gimenez*: On singular and nonsingular H-matrices

10:05–10:25 *V. Kostic*: New results on subdirect sums of  $\mathcal{S}$ -strictly diagonally dominant matrices

**10:25–10:40 Coffee break**

## **Session II (LA)**

10:40–11:20 *J.M. Pena*: On bounds and localization results for the eigenvalues of structured matrices

11:25–11:45 *V. Cortes*: Pivoting strategies and growth factor

**11:45–12:00 Coffee break**

## **Session III (LA)**

12:00–12:20 *M. Kovacevic*: Some invariants of Schur complement derived from scaling approach

12:20–12:40 *B. Ricarte*: The positive realization problem in normal transfer matrices with simple real poles

12:40–13:00 *F. Pedroche*: Characterization of  $\alpha$ -matrices

**13:00 Lunch**

## **Session IV (LA)**

15:00–15:40 *S. Rump*: Toeplitz structured perturbations

15:40–16:05 *D. von Rosen*: Singular structured matrix with integer spectrum

**16:05–16:20 Coffee break**

**Session V (Stat.)**

16:20–16:50 *C. Coelho*: Testing for structure in covariance matrices

16:50–17:10 *P. Druilhet*: A statistical interpretation of Krylov subspaces decomposition

17:10–17:30 *A. Michalski*: On using generalized Cramer-Rao inequality to REML estimation in linear models

**17:30–17:45 Coffee break**

**Session VI (Stat.)**

17:45–18:25 *S. Puntanen*: A conversation with Sujit Kumar Mitra in 1993 and some comments on his research publications

18:25–19:15 *G.P.H. Styan*: A philatelic introduction to matrices and magic squares

**19:30 Dinner**

**Friday, March 23, 2007**

**8:00–9:00 Breakfast**

**Session VII (Stat.)**

9:00–9:40 *F. Pukelsheim*: The European Parliament election in the Treaty on a Constitution for Europe: An electorate of degressive valencies

9:45–10:05 *P. Jiraneek*: On the limiting accuracy of segregated saddle point solvers

10:05–10:25 *T. Ostrowski*: On properties of special saddle point matrices

**10:25–10:40 Coffee break**

**Session VIII (Stat.)**

10:40–11:20 *D.A. Griffith*: A numerical study of planar and near-planar adjacency matrices used in geographical analysis

11:25–11:45 *F.J. Marques*: Near-exact distributions for testing the equality and sphericity of several covariance matrices: the multi-sample sphericity test

**11:45–12:00 Coffee break**

**Session IX (Stat.)**

12:00–12:40 *T. Klein*: Loewner comparability of moment matrices in cubic mixture models

12:45–13:05 *E. Liski*: Model averaging for linear regression

**13:10 Lunch****Session X (LA & Stat.)**

15:00–15:20 *F. Blömeling*: MLS model reduction for second-order time-invariant dynamical systems

15:20–15:40 *W. Tadej*: Continuous affine families of unitary Hadamard matrices

15:40–16:00 *M. Ohlson*: Distributions of quadratic forms

16:00–16:20 *M. Grządziel*: On determinant maximization with linear matrix inequality constraints and REML estimators of variance components

**16:20–16:50 Coffee break – Poster session**

*R. Rózański, A. Szczepańska, M. Beim, T. Kossowski*

**Session XI Statistics and Linear Algebra in Practice**

16:50–17:10 *S. Galas (Glaxo)*: Responsible for evaluating process capability and implementing statistical methods in the Site

17:10–17:30 *H. Michalska (Glaxo)*: Responsible for measurement systems improvement and implementing Lean Manufacturing

17:30–18:00 *J. Jackowski (Glaxo)*: Six Sigma. What Employers Need - Discussion

**18:00–18:15 Coffee break**

18:15–18:40 *A. Liski*: A propensity score approach to comparing costs between hospital districts

18:40–19:05 *A. Dąbrowski*: Process monitoring based on multivariate statistical analysis

19:05–19:15 Discussion

**19:30 Dinner** - Banquet speaker: Simo Puntanen

**Saturday, March 24, 2007****8:00–9:00 Breakfast****Session XII (LA)**

9:00–9:40 *C.R. Johnson*: Possible Jordan Structures for  $A$ ,  $B$ , and  $C$ ,  
when  $C = AB$

9:45–10:05 *Z. Woźnicki*: The solution of continuous-time algebraic Riccati equations by means of the SOR-LIKE method

10:05–10:25 *D. Kubalińska*: Interpolation based model reduction

**10:25–10:40 Coffee break****Session XIII (LA)**

10:40–11:20 *G. Trenkler*: On the distance and the angle of subspaces

11:25–11:45 *O.M. Baksalary*: Further properties of a pair of orthogonal projectors

**11:45–12:00 Coffee break****Session XIV (LA)**

12:00–12:20 *A. Smoktunowicz*: How and how not to compute the Moore-Penrose inverse

12:20–12:40 *I. Wróbel*: Numerical ranges in a strip

12:40–13:00 *K. Vehkalahti*: Enhancing the documentation by leaving useful traces

**13:00 Lunch****Session XV (LA)**

15:00–15:40 *R. Bru*: On incomplete decompositions of inverses

15:45–16:05 *P. Batra*: Infinite matrices - computable stability conditions for timedelay systems

16:05–16:25 *A. Suchocka*: Incomplete alternating projection method for large inconsistent linear systems

**16:25–16:40 Coffee break****Session XVI (Stat.)**

16:40–17:20 *J.T. Mexia / M. Fonseca*: Canonical forms for factorial and related models

17:25–17:45 *F. Carvalho*: Canonic inference and commutative orthogonal block structures

**17:45–18:00 Coffee break**

**Session XVII (Stat.)**

18:00–18:40 *R. Zmysłony*: Jordan algebra and statistical inference in linear mixed models

18:45–19:05 *R. Covas*: Lattices of commutative Jordan algebras, remarks and some applications

19:05–19:15 **Closing**

**19:30 Dinner – Prize ceremony**



Part III

**Abstracts**





# Further properties of a pair of orthogonal projectors

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<sup>2</sup> University of Dortmund, Germany

## Abstract

Representing two orthogonal projectors on a finite dimensional vector spaces (i.e., Hermitian idempotent matrices) as partitioned matrices turns out to be very powerful tool in considering properties of such a pair. The usefulness of this representation is discussed and several new characterizations of a pair of orthogonal projectors are provided, with particular attention paid to the spectral properties.

## Keywords

Hermitian idempotent matrix, Partitioned matrix.

# Infinite matrices - computable stability conditions for time-delay systems

Prashant Batra

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## Abstract

Hurwitz' stability criterion for polynomials does not carry over to general entire functions (i.e. functions holomorphic everywhere in the complex plane). Entire functions connected to delay-difference equations of retarded type still possess many properties of polynomials. It is known, e.g., that the Hermite-Biehler criterion for stability carries over to this class while the Hurwitz' criterion does not. Alas, non-trivial, non-polynomial solutions of systems of retarded type have infinitely many zeros which renders the Hermite-Biehler criterion impractical. We show how to work with the infinite analogues of the Hurwitz and Bezout matrices to obtain finite, effectively computable, sufficient stability tests.

## Keywords

Totally positive matrices, Linear time-delay systems, Constructive computable criterion.

## References

- Bellman, R. and K.L. Cooke (1963). *Differential-Difference Equations*. New York: Academic Press.
- Gu, K., V.L. Kharitonov and J. Chen (2003). *Stability of Time-Delay Systems*. Boston: Birkhäuser.
- Krein, M. Concerning a special class of entire and meromorphic functions. In Ahiezer, N.I. and M. Krein (1962)., eds., *Some questions in the theory of moments*, Ch. 3. Rhode Island: AMS.
- Lipatov, A.V. and N.I. Sokolov (1978). Some sufficient conditions for stability and instability of continuous linear stationary systems. *Autom. Remote Control*, 1285–1291.

# Application of numerical methods to the modeling of suburbs' growth

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## Abstract

Many factors (e.g. level of street noise, air pollution, prices of flats or a wish to have a private garden) influence inhabitants' choice of a place to live. So, it would appear that the complexity of urban growth cannot be efficiently described by formal models. However, this paper presents a model based on an artificial neural network and a cellular automaton, which proved efficient in this situation. The area of study, the Poznań agglomeration, was divided into squares 500 x 500 m of size. Each square was equipped with data presenting the intensity of factors influencing peoples willingness to live in this area. The factor modeled was not, as used to be in previous models, the built-up area but population density. After the socio-economic changes following 1989 in Poznań as well as in other Polish cities, the number of buildings in their central areas has been rising, but the population density has been decreasing. For this reason the model based only on the built-up area would give incorrect results.

The simulation of the growth of suburbs in the Poznań agglomeration takes into account legal barriers to the development of new residential areas (e.g. protection of forests, fields), prices of plots of land, accessibility to roads and to public transport, distance from sources of noise, and the physical barriers like rivers and lakes. In the first approach all these factors were interrelated by artificial neural networks with an outcome in the form of a matrix of the probability of change in population density. The simulation was made by cellular automata which took into account not only the nearest Moore neighborhoods, but also extended neighborhoods. In the second approach, the artificial neural networks of the model were conjoined into rules for updating cells in the cellular automaton. These models brought good results applicable in socio-economic geography and physical planning.

## **Keywords**

Artificial neural network, Cellular automaton, Suburbanization.

## **References**

Betty, M. (2005). *Cities and Complexity. Understanding Cities with Cellular Automata, Agent-Based Models and Fractals*. Cambridge - Massachusetts - London: Academic Press.

# MLS model reduction for second-order time-invariant dynamical systems

Frank Blömeling

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## Abstract

Time-invariant dynamical systems play an important role in the modelling of physical processes. Particularly, a variety of problems lead to second-order systems. Very often necessary discretizations yield large systems. In general it is required to reduce the complexity of such systems by dimension reduction techniques for numerical treatment. Additionally, it is desirable to preserve the second-order structure of the system. Hence, special reduction methods for second-order systems have been developed. Nevertheless the applicability of the methods to very large systems is often limited.

Multi-level substructuring (MLS) is a hierarchical substructuring concept for the decomposition of huge systems into several smaller systems. In the symmetric case it can be seen as an orthogonal decomposition of the state space. Using this framework it is possible to apply the second-order methods to the smaller subsystems to reduce the dimension of the overall system.

We present a way to apply multi-level substructuring to second-order systems which turns out to be strongly connected to the application of MLS to first-order systems. Furthermore we discuss the error due to the MLS framework and show numerical examples.

## Keywords

Model reduction, Multi-level substructuring, Second-order systems.

## References

- Bai, Z. and Y. Su (2005). Dimension Reduction of Large-Scale Second-Order Dynamical Systems Via a Second-Order Arnoldi Method. *SIAM J. Sci. Comput.* 26, 1692–1709.
- Benner, P., V. Mehrmann and D.C. Sorensen (Eds) (2005). *Dimension Reduction of Large-Scale Systems*. Berlin: Springer.

- Blömeling, F. (2006). Hierarchical substructuring combined with SVD-based model reduction methods. *Technical report 102*  
<http://www.tu-harburg.de/mat/schriften/rep/rep102.pdf>.
- Chahlaoui, Y., D. Lemonnier, A. Vandendorpe and P. Van Dooren (2005). Second-Order Balanced Truncation Linear Algebra and its Applications. To appear.
- Li, R.C. and Z. Bai (2005). Structure-Preserving Model Reduction using a Krylov Subspace Projection Formulation. *Comm. Math. Sci.* 3, 179–199.

## On incomplete decompositions of inverses

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and Miroslav Tůma<sup>2</sup>

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<sup>2</sup> Academy of Sciences, Prague, Czech Republic

### Abstract

In this work we consider the approximate inverse decomposition AISM based on the Sherman-Morrison formula [1]. That decomposition is written as a product of three factors and a parameter  $s$  should be chosen. The existence of the AISM decomposition of a square matrix is studied in function of the parameter  $s$  of the decomposition. The factors obtained for two different values of the parameter are related. The relationship with the ILU decomposition is deduced for each factor.

### Keywords

Preconditioning, Approximate inverses, Incomplete decompositions, Sherman-Morrison formula.

### References

Bru, R., J. Cerdán, J. Marín and J. Mas (2003). Preconditioning nonsymmetric linear systems with the Sherman-Morrison formula, *SIAM J. Sci. Comput.* 25(2), 701–715.

# Canonic inference and commutative orthogonal block structures

Francisco Carvalho<sup>1</sup> and João T. Mexia<sup>2</sup>

<sup>1</sup> Polytechnic Institute of Tomar, Portugal

<sup>2</sup> New University of Lisbon, Portugal

## Abstract

It is shown how to define the canonic formulation for orthogonal models associated to comutative Jordan algebras. This canonic formulation is then used to carry out inference. The case of models with commutative orthogonal block structures is stressed out.

## Keywords

COBS, Commutative orthogonal block structures, Linear models, Variance components.

## References

- Carvalho, F. and J.T. Mexia (2005).  $\mathcal{F}$  tests in models with Commutative Orthogonal Block Structures. SCRA'06
- Fonseca, M., J.T. Mexia and R. Zmyslony (2006). Binary operations on Jordan algebras and orthogonal normal models. *Linear Algebra Appl.* 417, 75–86



## Testing for structure in covariance matrices

The advantage of the use of conditionally independent tests in building near-exact distributions for the test statistics

Carlos A. Coelho and Filipe J. Marques

New University of Lisbon, Portugal

### Abstract

We show how several rather elaborate structures for covariance matrices may be tested by decomposing the null hypotheses under study in a number of 'partial' hypotheses which are conditionally independent. Using this technique we are able not only to construct quite easily the overall likelihood ratio test statistics but also to obtain, from the decomposition of the characteristic function of the statistics used to test the 'partial' hypotheses, several near-exact distributions for such overall test statistics. These distributions are of great importance given the intractability of the exact distributions and the non-existence of asymptotic distributions for most cases.

Among the tests studied are the tests of multisample sphericity, block sphericity and multisample block sphericity. As partial tests we use the test for independence of several sets of variates, the test of equality of several covariance matrices and the sphericity test.

Two measures based on the Berry-Esseen inequality and on the inversion formulas are considered to assess the quality of the near-exact distributions developed.

### Keywords

Near-exact distributions, Likelihood ratio test, Asymptotic distributions, Sphericity, Block sphericity, Structure, Covariance matrices.

### References

Anderson, T. W. (2003). *An Introduction to Multivariate Statistical Analysis*. 3rd ed., J. Wiley & Sons, New York.

- Berry, A. (1941). The accuracy of the Gaussian approximation to the sum of independent variates. *Trans. Amer. Math. Soc.* 49, 122-136.
- Box, G. E. P. (1949). A general distribution theory for a class of likelihood criteria. *Biometrika* 36, 317-346.
- Cardeno, L. and D.K. Nagar (2001). Testing block sphericity of a covariance matrix. *Divulg. Mat.* 9, 25-34.
- Coelho, C. A. (2004). The Generalized Near-Integer Gamma distribution: a basis for 'near-exact' approximations to the distributions of statistics which are the product of an odd number of independent Beta random variables. *J. Multivariate Anal.* 89, 191-218.
- Esseen, C.-G. (1945). Fourier analysis of distribution functions. A Mathematical Study of the Laplace-Gaussian Law. *Acta Math.* 77, 1-125.
- Grilo, L. and C.A. Coelho (2006). Development and study of two near-exact approximations to the distribution of the product of an odd number of independent Beta random variables. *J. Statist. Plann. Inference* (in print).
- Mathai, A. M. (1986). Hypothesis of multisample sphericity. *J. Sov. Math.* 33, 792-796.
- Moschopoulos, P. G. (1988). Asymptotic expansions of the non-null distribution of the likelihood ratio criterion for multisample sphericity. *Amer. J. Math. Management Sci.* 8, 135-163.
- Moschopoulos, P. G. (1992). The hypothesis of multisample block sphericity. *Sankhyā* 54, 260-270.

## Pivoting strategies and growth factor

Vanesa Cortés and Juan M. Peña

University of Zaragoza, Spain

### Abstract

Several pivoting strategies for Gauss elimination and Neville elimination are considered. Special emphasis is given to scaled partial pivoting strategies. Several definitions of growth factors are compared, including an approximation of the average normalized growth factor for random matrices.

### Keywords

Gauss elimination, Neville elimination, Pivoting strategy, Growth factor, Scaled partial pivoting.

### References

- Cortés, V. and J.M. Peña. Growth factor and expected growth factor of some pivoting strategies. To appear in *J. Comput. Appl. Math.*
- Cortés, V. and J.M. Peña (2007). Sign regular matrices and Neville elimination. *Linear Algebra Appl.* 421, 53–62.
- Peña, J.M. (2003). Scaled pivots and scaled partial pivoting strategies. *SIAM J. Numer. Anal.* 41, 1022–1031.
- Trefethen, L.N. and R.S. Schreiber (1990). Average case stability of Gaussian elimination. *SIAM J. Matrix Anal. Appl.* 11, 335–360.

## Lattices of commutative Jordan algebras, remarks and some applications

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### Abstract

Commutative Jordan Algebras provide a convenient algebraic framework for linear models.

We intend to extend this framework by considering lattices of Commutative Jordan Algebras which will be helpful for studying factor aggregation and desegregation as well as invariance properties.

### Keywords

Jordan algebra, Lattice, Isomorphism.

### References

- Jordan, P., J. Von Neumann and E. Wigner (1934). On an algebraic generalization of the quantum mechanical formulation. *Ann. Math.* 36, 26–64.
- Malley, J.D. (1994). *Statistical applications of Jordan algebras*. Springer-Verlag.

# Process monitoring based on multivariate statistical analysis

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## Abstract

In recent years interests in multivariate extension of one-dimensional statistical process control have increased. In this paper some methods of detecting shifts in mean vector and covariance matrix of a multivariate process: the  $T^2$  charts with Bonferroni and Roy-Bose control limits and multivariate CUSUM charts are reviewed. The difficulty with  $T^2$  chart is that it gives no indication of which variables are causing the problem. Various techniques using singular value decomposition of data matrix such as biplot and Procrustes analysis may be helpful. These methods provide an elegant way to extract some interesting patterns showing the dynamic of batches of multidimensional processes.

## Keywords

Biplot, CUSUM, Multivariate SPC, Procrustes analysis, SPC,  $T^2$  charts.

## References

- Cheng, S. W. and K. Thaga (2005). Multivariate Max-CUSUM Chart. *Qual. Technol. Quant. Manag. Vol. 2, No. 2*, 221–235.
- Lavit, C., Y. Escoufier, R. Sabatier and P. Traissac (1994). The ACT (STATIS) method. *Comput. Statist. Data Anal. 18*, 97–119.
- Sparks, R.S., A. Adolphson, and A. Phatak (1997). Multivariate Process Monitoring Using the Dynamic Biplot. *International Statistical Review 65*, 325–349.

## A statistical interpretation of Krylov subspaces decomposition

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### Abstract

Consider the standard linear model

$$Y = X\beta + \varepsilon.$$

where  $\beta$  the  $p$ -vectors of unknown parameters. To stabilize the OLS estimator when explanatory variables are highly correlated, PLS uses a Gram-Schmidt decomposition of the Krylov subspaces

$$K_q = \text{span}(s, S s, S^2 s, \dots, S^{q-1} s)$$

generated by  $S = X'X$  and  $s = X'Y$ . We show how this decomposition can be obtained from an algorithm that iteratively maximizes the directional signal-to-noise ratio (SNR) applied to the least squares estimator under orthogonality constraints. The SNR on the direction given by  $x \in R^p$  is defined by

$$\text{SNR} = \frac{|x' \widehat{\beta}^{ols}|}{\sigma \sqrt{x' S^{-1} x}},$$

and is related with optimal shrinkage factors that realize optimal trade-off between bias and variance.

### Keywords

PLS regression, Krylov subspaces, Signal-to-noise ratio.

### References

- Druilhet, P. and A. Mom (2006). PLS regression: a signal-to-noise ratio approach. *J. Multivariate Anal.* 97, 1313–329.
- Holland, I.S. (1988). On the structure of partial least squares regression. *Comm. Statist. Simulation Comput.* 17, 581–607.

# On Hessenberg matrices and their generalizations

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## Abstract

Recently, two different generalizations of the concept of a Hessenberg matrix have appeared in the literature. After discussing their connection and canonical forms we study the class of generalized Hessenberg matrices where the matrix and the transposed of its inverse have the same form.

## Keywords

Submission of abstract, Reference style, Submission date, How to submit.

## References

- Chouakria, A., E. Diday, and P. Cazes (1998). Vertices principal components with an improved factorial representation. In: A. Rizzi, M. Vichi, and H.-H. Bock (Eds.), *Advances in Data Science and Classification* (pp. 397–402). Heidelberg: Springer.
- Marshall, A.W. and I. Olkin (1979). *Inequalities: Theory of Majorization and its Applications*. New York: Academic Press.
- Olkin, I. (1997). A determinantal proof of the Craig-Sakamoto theorem. *Linear Algebra Appl.* 264, 217–223.
- Johnson, C., T.J. Laffey, and R. Loewy (1996). The real and the symmetric nonnegative inverse eigenvalue problems are different. *Proc. Amer. Math. Soc.* 124, 3647–3651.

## **Responsible for evaluating process capability and implementing statistical methods in the Site**

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### **Abstract**

The speech is about the usage of statistical methods in tablet manufacturing at the GSK Site in Poznań. The author is involved in evaluating process capability for products in Solid Forms Value Stream. In the presentation an explanation of goals set for a statistical analysis is given plus a quick comparison between statistics in theory and practice.

The primary aim of the presentation is to show how samples are taken from every batch and how they are being analyzed using popular statistical software. In addition there is also an example of a current project that is right now taking place at the GSK Site in Poznań.



## On singular and nonsingular $H$ -matrices

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and José Mas

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### Abstract

Let denote by  $\mathcal{M}(A)$  the comparison matrix of a square  $H$ -matrix  $A$ , that is,  $\mathcal{M}(A)$  is an  $M$ -matrix.  $H$ -matrices such that  $\mathcal{M}(A)$  is nonsingular are well studied in the literature. In this work, we study some characterizations of singular and nonsingular  $H$ -matrices when  $\mathcal{M}(A)$  is singular. The spectral radius of the Jacobi matrix and the generalized diagonal dominance property are used in the characterizations. In particular, we study the case when  $A$  is irreducible and then give some insights to the reducible case.

### Keywords

$H$ -matrix, Comparison matrix, Equimodular matrices, Generalized diagonally dominant matrices.

### References

- Berman, A. and R.J. Plemmons R.J. (1994). *Nonnegative Matrices in the Mathematical Sciences*, Philadelphia: SIAM.
- Varga, R.S. (1976). On recurring theorems on diagonal dominance. *Linear Algebra Appl.* 13, 1–9.
- Varga, R.S. (2004). *Geršgorin and its Circles*, Berlin: Springer.

## A numerical study of planar and near-planar adjacency matrices used in geographical analysis\*

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### Abstract

Geographical analysis most frequently involves irreducible, non-negative (i.e., 0-1 binary) square adjacency matrices associated with planar or near-planar [i.e., a total number of 1s only slightly exceeding  $6(n-2)$ , where  $n$  is the number of nodes] graphs that can reach sizes for which  $n$  is at least 1,000,000. Eigenvalues of these matrices are needed for a variety of spatial statistical analysis purposes. Current computer technology and linear algebra theory support calculation of these eigenvalues for such a matrix as long as its size does not exceed approximately  $n = 10,000$ . Quantum physics encounters a similar problem, but for regular, random matrices, and as such, furnishes some useful guidelines for numerically studying the eigenfunctions of (near-)planar graph adjacency matrices that are expressed in terms of C-, S- and W-coding schemes. This paper begins by addressing the numerical preliminaries for sparse matrices of: confirming irreducibility, simple evaluations of the presence of 0 eigenvalues, and estimating the extreme eigenvalues. Next, based upon a database comprising more than seven dozen regular and irregular surface partitionings, most of which have been used in published spatial statistical analyses, both power law descriptions of eigenvalue distributions and time-series ARIMA descriptions of eigenvalue spacings are summarized.

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# On determinant maximization with linear matrix inequality constraints and REML estimators of variance components

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## Abstract

It is shown how modern interior point methods for solving a class on convex optimization problems over the cone of nonnegative definite matrices ("determinant maximization with linear matrix inequality constraints", see Vandenberghe et al. (1998)) can be used for finding residual maximum likelihood (REML) estimators of variance components in linear mixed models. The class of mixed models considered can be defined in terms of quadratic subspaces (or Jordan algebras) of symmetric matrices (see Rao et al., 1998, Chap. 13; Grządziel, 2007) and includes many balanced models with random effects. The efficiency of our approach is evaluated by a simulation experiment.

## Keywords

Cone of nonnegative definite matrices, Linear mixed model, Quadratic subspace of symmetric matrices.

## References

- Grządziel, M. (2007). Quadratic subspaces and construction of Bayes invariant quadratic estimators in mixed linear models. Accepted for publication in *Statist. Papers*.
- Rao, C.R. and M.B. Rao (1998). *Matrix algebra and its applications to statistics and econometrics*. Singapore:World Scientific Publishing.
- Vandenberghe, L., S. Boyd and S.-P. Wu (1998). Determinant maximization with linear matrix inequality constraints. *SIAM J. Matrix Anal. Appl.* 19, 499–533.

## Comparison of values of Pearson's and Spearman's correlation coefficient on the same sets of data

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### Abstract

Spearman's rank correlation coefficient (denoted here by  $r_s$ ) is a non-parametric (distribution-free) rank statistic proposed by Charles Spearman as a measure of the strength of the associations between two variables. It is a measure of a monotone association that is used when the distribution of the data makes Pearson's correlation coefficient undesirable or misleading. Spearman's coefficient is not a measure of the linear relationship between two variables, as some "statisticians" declare. It assesses how well an arbitrary monotonic function can describe the relationship between two variables, without making any assumptions about the frequency distribution of the variables. Unlike Pearson's product-moment correlation coefficient, it does not require the assumption that the relationship between the variables is linear, nor does it require the variables to be measured on interval scales; it can be used for variables measured at the ordinal level.

In principle,  $r_s$  is simply a special case of Pearson's product-moment coefficient in which the data are converted to ranks before calculating the coefficient.

Charles Spearman developed his rank correlation in 1904. However, his statistical work was not appreciated by his University College colleague Karl Pearson and there was a long feud between them. Nowadays, the coefficient  $r_s$  is widely used in statistical analysis. In the presentation we would like to compare it values and significance for different sets of data (original - for Pearson's coefficient and ranked data for Spearman's coefficient).

What inspired this presentation was a paper by Plata (2006).

### Keywords

Pearson's and Spearman's correlation coefficient.

## References

- Plata, S. (2006). A note on Fisher's correlation coefficient. *Applied Mathematical Letters* 19, 499–502.
- Spearman, Ch. (1904). Proof and measurement of association between two things. *American Journal of Psychology* 15, 72–101.

## On the limiting accuracy of segregated saddle point solvers

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<sup>2</sup> Czech Academy of Sciences, Czech Republic

### Abstract

Saddle point problems arise in a wide variety of applications in computational science and engineering. The aim of this paper is to discuss numerical behavior of several iterative methods applied for solving the saddle point systems via the Schur complement reduction or the null-space projection approach. Krylov subspace methods often produce the iterates which fluctuate rather strongly. Here we address the question whether large intermediate approximate solutions reduce the final accuracy of these two-level (inner-outer) iteration algorithms. We distinguish between three mathematically equivalent back-substitution schemes which lead to a different numerical behavior when applied in finite precision arithmetic. Theoretical results are then illustrated on a model example.

### Keywords

Saddle point problems, Schur complement reduction method, Null-space projection method, Rounding error analysis.

### References

- Jiránek, P. and M. Rozložník (2006). Maximum attainable accuracy of inexact saddle point solvers. *Technical report, Institute of Computer Science, Prague* <http://www.cs.cas.cz/mweb/download/publi/JiRo2006.pdf>.
- Jiránek, P. and M. Rozložník (2006). Limiting accuracy of segregated solution methods for nonsymmetric saddle point problems. *Technical report, Institute of Computer Science, Prague* <http://www.cs.cas.cz/mweb/download/publi/JirRoz2006.pdf>.

**Possible Jordan Structures for  $A$ ,  $B$ , and  $C$ ,  
when  $C = AB$**

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**Abstract**

We survey very recent progress on this problem, focussing upon the diagonalizable case, which is most difficult.

# Loewner comparability of moment matrices in cubic mixture models

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## Abstract

Mixture experiments are experiments in which the experimental conditions are the relative proportions of ingredients of a whole. Given a second-degree polynomial regression model for such experiments, Draper and Pukelsheim (1999) and Draper, Heiligers, and Pukelsheim (2000) have identified a class of designs (the class of so-called weighted centroid designs) that is essentially complete with respect to many popular design criteria, that is, for any given design there is a weighted centroid design that is at least as good as the given design relative to the given design criterion. The crucial steps towards this result are: (a) understanding the inherent symmetry of moment matrices in this model (invariance with respect to a subgroup of permutations), and (b) a characterization of Loewner comparability of moment matrices. One natural question is whether a similar completeness result can be reproduced for cubic mixture models. Mikaeili's (1993) result on D-optimal designs for a cubic mixture model shows that the class of weighted centroid designs cannot be essentially complete in this case. Instead, his result gives rise to the conjecture that the class of weighted centroid designs needs to be augmented in order to obtain an essentially complete class.

As in the quadratic case, understanding symmetry and Loewner comparability of moment matrices is essential. I will show how Andersson's (1975) result on variance-covariance matrices of invariant distributions can be put to use here, and I will discuss the completeness issue for mixture experiments with a small number of ingredients.

## References

Andersson, S.A. (1975). Invariant normal models. *Ann. Statist.* 3, 132–154.



- Draper, N.R., B. Heiligers and F. Pukelsheim (2000). Kiefer-ordering of simplex designs for second-degree mixture models with four or more ingredients. *Ann. Statist.* 28, 578–590.
- Draper, N.R. and F. Pukelsheim (1999). Kiefer ordering of simplex designs for first- and second-degree mixture models. *J. Statist. Plann. Inference* 79, 325–348
- Mikaeili, F. (1993). D-optimum design for full cubic on  $q$ -simplex. *J. Statist. Plann. Inference* 35, 121–130.

## New results on subdirect sums of $\mathcal{S}$ -strictly diagonally dominant matrices

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<sup>2</sup> University of Novi Sad, Serbia

### Abstract

The question when subdirect sum of two  $H$ -matrices is an  $H$ -matrix for the case of  $\mathcal{S}$ -SDD matrices was treated in the paper by Bru et al. (2006), where some sufficient conditions were given. Motivated by the same question, using a different technique, we are going to give a simplified proof of the main theorem from the paper above followed by its generalization. As the main result, for the class of  $\mathcal{S}$ -SDD, defined as a subclass of  $H$ -matrices which is a kind of broader concept of  $\mathcal{S}$ -SDD matrices, we are going to give the exact answer to the question when the subdirect sum of two  $\mathcal{S}$ -SDD matrices is an  $\mathcal{S}$ -SDD matrix.

### Keywords

Subdirect sum,  $H$ -matrices, Overlapping blocks.

### References

- Bru, R., F. Pedroche and D. B. Szyld. (2006) Subdirect sums of  $\mathcal{S}$ -Strictly Diagonally Dominant matrices. *Electron. J. Linear Algebra* 15, 201–209.
- Bru, R., F. Pedroche and D. B. Szyld. (2005) Subdirect sums of nonsingular  $M$ -matrices and of their inverses. *Electron. J. Linear Algebra* 13, 162–174.
- Cvetković, L. (2006).  $H$ -matrix theory vs. eigenvalue localization. *Numerical Alg.* 42, 229–245.
- Cvetković, L. and V. Kostić (2006). Between Geršgorin and minimal Geršgorin set. *J. Comput. Appl. Math.* 196, 452–458.
- Cvetković, L., V. Kostić and R. S. Varga (2004). A new Geršgorin-type eigenvalue inclusion set. *Electron. Trans. Numer. Anal.* 18, 73–80.
- Fallat, S.M. and C.R. Johnson (1999). Sub-direct sums and positivity classes of matrices. *Linear Algebra Appl.* 288, 149–173.

## Some invariants of Schur complement derived from scaling approach

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### Abstract

It is well-known that the Schur complement of a strictly diagonally dominant matrix is strictly diagonally dominant, too. Also, if a matrix is an H-matrix, then its Schur complement is an H-matrix, too. Recent research showed that the same type of statement holds for some special subclasses of H-matrices, see, for example, Liu et al. (2004). The aim of this paper is twofold: first, to show that the proof of these results can be significantly simplified by using "scaling" approach, and second, to prove more similar invariant results.

### Keywords

H-matrices, Schur complement, Diagonal scaling blocks.

### References

- Dashnic, L.S. and M.S. Zusmanovich (1970). O nekotoryh kriteriyah reguljarnosti matric i lokalizacii ih spectra. *Zh. vychisl. matem. i matem. fiz.* 5, 1092–1097.
- Dashnic, L.S. and M.S. Zusmanovich (1970). K voprosu o lokalizacii harakteristicheskikh chisel matricy. *Zh. vychisl. matem. i matem. fiz.* 6, 1321–1327.
- Liu, J., Y. Huang and F. Zhang (2004). The Schur complements of generalized doubly diagonally dominant matrices. *Linear Algebra Appl.* it 378, 231–244.
- Cvetković, L. (2006). H-matrix theory vs. eigenvalue localization. *Numerical Alg.* 42, 229–245.
- Cvetković, L. and V. Kostić (2005). New criteria for identifying H-matrices. *J. Comput. Appl. Math.* 180, 265–278.
- Cvetković, L. and V. Kostić (2006). Between Geršgorin and minimal Ger'vsgorin set. *J. Comput. Appl. Math.* 196, 452–458.
- Cvetković, L., V. Kostić and R.S. Varga (2004). A new Geršgorin-type eigenvalue inclusion set. *Electron. Trans. Numer. Anal.* 18, 73–80.

## Interpolation based model reduction

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### Abstract

State-space models of dynamical systems are often of very high dimension. It is then desirable to find systems of the same form but of lower complexity, whose input-output behaviour approximates the behaviour of the original system. Such reduced order models are typically constructed by projecting the state space of the system. There are several approaches to find such projective subspaces. In this talk we give a brief survey on a rational Hermite interpolation approach and its generalization to tangential interpolation. We show also how the projection matrices can be computed via solving Sylvester equations. First order necessary conditions for  $H_2$ -optimal model reduction are presented for SISO (single-input-single-output) as well as for MIMO (multiple-input-multiple-output) discrete and continuous systems. From the  $H_2$ -optimal conditions we derive a good choice of interpolation data. Numerical experiments report the efficiency of the proposed method.

### Keywords

Model reduction, Rational interpolation, Tangential interpolation, Sylvester equation,  $H_2$  approximation.

### References

- Antoulas, A.C. (2005). Approximation of large-scale dynamical systems. *Adv. Des. Control DC-06*, SIAM, Philadelphia.
- Gallivan K., A. Vandendorpe and P. Van Dooren (2004). Model Reduction of MIMO Systems via Tangential Interpolation. *SIAM J. Matrix Anal. Appl.* 26, No. 2, 328–349.
- Gugercin, S., C. Beattie and A.C. Antoulas (2006). Rational Krylov Methods for Optimal  $H_2$  Model Reduction. *ICAM Technical Report, Virginia Tech.*
- Wilson, D.A. (1970). Optimum Solution of Model Reduction Problem. *Proc. Inst. Elec. Eng.* 117, No. 6, 1161–1165.

## A propensity score approach to comparing costs between hospital districts

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### Abstract

In order to enhance the performance of the health care system there is a need for comparative information suitable for benchmarking purposes. One widely used approach in the performance assessment is the disease-based comparison of episodes of care between providers (hospitals or care districts). The basic idea is to produce performance indicators which are adjusted for confounding factors (differences in population characteristics). Costs are obviously an important performance measure, but the adjustment has turned out to be methodologically challenging.

The data consists of the population of 16881 hip fracture patients aged 65 or older in 1998-2001 and includes information about patient characteristics (age, sex, a set of diseases diagnosed before the fracture) to be adjusted as well as the cumulative costs of treatment at one year after the fracture. Patient characteristics vary between hospital districts, which must be taken into account. It has also effect on the costs if a patient has died during the follow up period and this effect should be adjusted. The size of the data is an advantage but on the other hand presents some computational problems.

To make the results easily interpretable we decided to use the propensity score method in comparing the districts with each other. In this method we first take a subdata which contains the observations from the two districts which we wish to compare. Then the propensity score is estimated using a logistic regression model with the hospital district as the dependent variable and the patients characteristics as explanatory variables.

The data is sorted by the propensity score (which are the predicted values from the logistic model estimated) and the costs are smoothed over the propensity score by using local linear regression with kernel

weights. This results in smoothed cost-curves over the scale of the propensity score. These cost-curves can be compared with each other or we can integrate the curves over the propensity score to obtain cost estimates for each district.

It turns out that there are clear differences between the cost-levels of various care districts. Thus the cost variation of hip fracture treatments between care districts can not be explained by patients characteristics only, but is also attributable to different treatment practices. This is an important finding because the identification of good practices can lead to learning from them, and consequently also to savings in treatment costs.

### **Keywords**

Hip fracture, Health care costs, Kernel smoothing, Propensity score.

### **References**

Rosenbaum, P.R. and D.B. Rubin (1983). The central role of the propensity score in observational studies for causal effects. *Biometrika* 70, 41–55.

# Model averaging for linear regression

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## Abstract

Estimators formed after model selection really are like mixtures of many potential estimators. Sometimes it is advantageous to smooth estimators across several models, rather than rely only on the model that is suggested by a single selection criterion. The main theme of this paper is the problem of selecting of weights for averaging across least squares estimates obtained from a set of models. Existing model average (MA) methods are based on exponential AIC or BIC weights and on Mallows' criterion.

We propose selecting the weights by using the MDL (Minimum Description Length) criterion and compare this new model average estimator with those based on AIC and BIC weights. The model is homoscedastic linear regression and the set of potential explanatory variables is countable infinite. For MA estimation we construct first a sequence of approximating models and then derive an MA estimator of the regression coefficients as a convex combination of estimators for single models. The implied "hat" matrix  $P_w$  is symmetric but generally not idempotent. An MA estimator shows a certain resemblance to a shrinkage estimator and  $P_w$  has a central role in the structure of an MA estimator. Some of the properties of  $P_w$  are discussed.

## Keywords

Approximating models, MDL criterion, Shrinkage estimator, Hat matrix.

## References

- Burnham, K.P. and D.R. Anderson (2002). *Model Selection and Multi-model Inference*. New York: Springer-Verlag.
- Hjort, L.H. and G. Claeskens (2003). Frequentist Model Average Estimators. *J. Amer. Statist. Assoc.* 98, 879–899.

- Liski, E.P. (2006). Normalized ML and the MDL Principle for Variable Selection in Linear Regression. In: Liski, E.P., Isotalo, J., Niemel, J., Puntanen, S., and Styan, G.P.H. (eds), *Festschrift for Tarmo Pukkila on His 60th Birthday* (pp. 159–172). Tampere: Department of Mathematics, Statistics and Philosophy.
- Rissanen, J. (2000). MDL Denoising. *IEEE Trans. Inform. Theory*, *IT-46*, No. 1, 2537–2543.



# Near-exact distributions for testing the equality and sphericity of several covariance matrices: the multisample sphericity test

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## Abstract

The multisample sphericity hypothesis can be decomposed in two nested hypotheses, the first one corresponding to the equality of several covariance matrices and the second to the sphericity test. This procedure allows us to work with the decompositions of the characteristic functions for the logarithm of those two test statistics in order to build a near-exact characteristic function that corresponds to a manageable and well known distribution. Using this technique we are able to obtain near-exact distributions for the likelihood ratio test statistic of the multisample sphericity test.

Two measures that are upper bounds on the absolute value of the difference between the exact and the approximate distribution and density functions are used to evaluate the quality of these near-exact approximations and also to compare them with asymptotic approximations for this statistic.

## Keywords

Multisample sphericity, Near-exact distributions, Asymptotic approximations, Covariance matrices, Structure.

## References

- Coelho, C.A. (2004). The Generalized Near-Integer Gamma distribution: a basis for 'near-exact' approximations to the distributions of statistics which are the product of an odd number of independent Beta random variables. *J. Multivariate Anal.* 89, 191–218.
- Huynh, H., and L.S. Feldt (1970). Conditions under which mean square ratios in repeated measurements designs have exact F-distributions. *J. Amer. Statist. Assoc.* 65, 1582–1585.

- Marques, F.J., and C.A. Coelho (2006). Near-exact distributions for the sphericity likelihood ratio test statistic. *Math. Dep., Fac. of Science and Technology, New University of Lisbon, Technical report 04/06*.
- Mendoza, J.L. (1980). A significance test for the multisample sphericity. *Psychometrika* 45, 495–498.
- Moschopoulos, P.G. (1988). Asymptotic expansions of the non-null distribution of the likelihood ratio criterion for multisample sphericity. *Amer. J. Math. Management Sci.* 8, 135–163.

# Canonical forms for factorial and related models

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## Abstract

Commutative Jordan algebras are used to present factorial and related models in canonical form. Binary operations defined on the algebras enable the derivation of complex models from simple ones: prime basis factorials and their fractional replications. Sub-algebras are then used to study the extension to models with an arbitrary number of levels for the factors.

Canonic formulation enables the study of mixed models, leads to BLUE for all estimable vectors and to UMVUE both for estimable vectors and variance components once normality is assumed.

## **Responsible for measurement systems improvement and implementing Lean Manufacturing**

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### **Abstract**

The author is responsible for measurement systems improvement and implementing Lean Manufacturing primarily in Quality Control Department. The presentation will address the issues concerning the need of having a good measurement system to keep high quality and dealing with growing complexity of products manufactured at the GSK Site in Poznań. Therefore basic ideas of MSA are explained with reference to producer's and consumer's risk. An example of an MSA study for one of the products is included in the presentation, to illustrate a successful usage of statistical methods in manufacturing.

## On using generalized Cramér-Rao inequality to REML estimation in linear models

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### Abstract

The main aim of considerations in the problem of estimation of variance components  $\sigma_1^2$  and  $\sigma_2^2$  by using the ML-method and REML-method in normal mixed linear model  $N\{Y, E(Y) = X\beta, Cov(Y) = \sigma_1^2 V + \sigma_2^2 I_n\}$  was concerned in the examination of their efficiency. It is particularly important when an explicit form of these estimators is unknown and we search for the solutions of the likelihood equations system by using different iterative procedures e.g.: Newton-Raphson method, Fisher's method of scoring, EM algorithm, Monte Carlo methods, gradient procedure (see papers: [4], [5], [6], [8]). For the sake of statistical properties of obtained solutions, unusually useful, are the numerical procedures which allow to control the biases and variances of these estimators. Some approximations of these parameters obtained without simulations for ML- and REML-estimators of variance components are presented in the paper [2]. Residual maximum likelihood estimation is often preferred to maximum likelihood estimation as a method of estimating covariance parameters in linear models because it takes account of the loss of degrees of freedom in estimating the mean and produces unbiased estimating equations for the variance parameters (cf. papers [2], [7]). Since the REML-estimators are "almost unbiased" for variance components  $\sigma_1^2$  and  $\sigma_2^2$  we may apply the generalized Cramér-Rao inequality to determine upper bounds of variances of the estimators (see [3]). This fact can be used to making alternative iterative procedures giving good solutions of the likelihood equations system (cf. [1], [2]). In the paper the numerical calculations for unbalanced random models corresponding to one-way layouts are also given.

### Keywords

Mixed linear models, ML-and REML estimation, Variance components, Fisher's information, Cramér-Rao inequality, Iterative MIVQUE.

## References

- [1] Cheang, W.K. and G.C. Reinsel (2003). Approximate ML and REML estimation for regression models with spatial or time series AR(1) noise. *Statist. Probab. Lett.* 62, 123–135.
- [2] Gnot, S., A. Michalski and A. Urbańska-Motyka (2004). On some properties of ML and REML estimators in mixed normal models with two variance components. *Discuss. Math. Probab. Stat.* 24, 109–126.
- [3] Lehman, E.L. (1983). *Theory of Point Estimation*. New York: Wiley.
- [4] Neumaier, A. and E. Groeneveld (1998). Restricted maximum likelihood estimation of covariances in sparse linear models. *Genet. Sel. Evol.*, 1–32.
- [5] Rao, C.R. (1979). MINQUE theory and its relation to ML and MML estimation of variance components. *Sankhyā Ser. B* 41, 138–153.
- [6] Searle, S.R., G. Casella and Ch. E. McCulloch (1992). *Variance Components* New York: Wiley.
- [7] Smyth, G.K. and A.P. Verbyla (1996). A conditional approach to residual maximum likelihood estimation in generalized linear models. *J. Roy. Statist. Soc. B* 58, 565–572.
- [8] Swallow, W.H. and J.F. Monahan (1984). Monte Carlo comparison of ANOVA, MIVQUE, REML, and ML estimators of variance components. *Technometrics* 26, 47–57.

# Distributions of quadratic forms

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## Abstract

A well known fact is that when testing hypotheses for covariance matrices, distributions of quadratic forms arise. A generalization of the distribution of the multivariate quadratic form  $XAX'$ , where  $X$  is a  $(p \times n)$  normally distributed matrix and  $A$  is a  $(n \times n)$  symmetric real matrix, is presented. It is shown that the distribution of the quadratic form is the same as the distribution of a weighted sum of noncentral Wishart distributed matrices.

Using this characterization of the distribution several properties of the quadratic form  $XAX'$  will be shown.

## Keywords

Quadratic forms, Latent roots, Latent vectors, Noncentral chi-square distribution, Noncentral Wishart distribution.

## References

- Graybill, F.A. (1976). *Theory and Application of the Linear Model*. Massachusetts, North Scituate, Duxbury Press.
- Gupta, A.K. and D.K. Nagar (2000). *Matrix Variate Distributions*. Chapman and Hall.
- Muirhead, R.J. (1982). *Aspects of Multivariate Statistical Theory*. New York: John Wiley & Sons.
- Ohlson, M. (2007). *Testing Spatial Independence using a Separable Covariance Matrix*. Licentiate Thesis No 1299, Department of Mathematics, Mathematical Statistics, Linköping University, Sweden.
- Srivastava, M.S. and C.G. Khatri (1979). *An Introduction to Multivariate Statistics*. New York: North Holland.

## On properties of special saddle point matrices

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### Abstract

Some saddle point matrices with two vector blocks are considered. These kind of matrices arise in different fields; for instance, in standard quadratic program on standard simplex if the left-up matrix is symmetric, in game theory if the left-up matrix (not necessarily symmetric) is interpreted as the payoff matrix of the game.

The considered class of saddle point matrices allows to formulate a simple form of necessary and sufficient conditions for existence of an interior solution of standard quadratic program or a completely mixed Nash equilibrium of a bi-matrix game. These special block matrices give also possibility to formulate simple formulas for computing the interior solution or for computing the completely mixed equilibrium.

### Keywords

Saddle point matrix, Standard quadratic program, Bimatrix game.

### References

- Benzi, M., G. Golub and J. Liesen (2005). Numerical solution of saddle point problems. *Acta Numer.* 14, 1–137.
- Ostrowski, T. (2006). Population equilibrium with support in evolutionary matrix games. *Linear Algebra Appl.* 417, 211–219.



## Characterization of $\alpha$ -matrices\*

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### Abstract

In this talk we show practical characterizations for the classes of  $\alpha 1$ -matrices and  $\alpha 2$ -matrices, which are subclasses of (nonsingular)  $H$ -matrices. The characterizations introduced allow a better comprehension of these kind of matrices in the sense that they bound the parameter  $\alpha$  that actually defines each class. As an application of these results, and following the technique introduced in previous works, we also show some properties of the subdirect sum (a generalization of the usual sum) of  $\alpha 1$ -matrices and the subdirect sum of  $\alpha 2$ -matrices.

### Keywords

$H$ -matrices,  $\alpha$ -matrices, Overlapping blocks, Subdirect sum.

### References

- Bru, R., F. Pedroche, and D.B. Szyld (2006). Subdirect sums of S-Strictly Diagonally Dominant matrices. *Electron. J. Linear Algebra* 15, 201–209.
- Bru, R., F. Pedroche, and D.B. Szyld (2005). Subdirect sums of nonsingular  $M$ -matrices and of their inverses. *Electron. J. Linear Algebra* 13, 162–174.
- Cvetković, L. (2006).  $H$ -matrix theory vs. eigenvalue localization. *Numerical Algorithms* 42, 229–245.
- Cvetković, L. and V. Kostić (2005). New criteria for identifying  $H$ -matrices. *J. Comput. Appl. Math.* 180, 265–278.
- Cvetković, L., V. Kostić and R.S. Varga (2004). A new Geršgorin-type eigenvalue inclusion set. *Electron. Trans. Numer. Anal.* 18, 73–80.
- Fallat, S.M. and C.R. Johnson (1999). Sub-direct sums and positivity classes of matrices. *Linear Algebra Appl.* 288, 149–173.
- Varga, R.S. (2004). *Geršgorin and his circles*. Springer Series in Computational Mathematics, vol. 36. Springer, Berlin, Heidelberg.

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## On bounds and localization results for the eigenvalues of structured matrices

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### Abstract

Nonsingular classes of matrices lead to localization results of the eigenvalues of a matrix. These localization result will be in turn more useful when applied to special classes of matrices (see [1-4]). We illustrate with new results the previous fact. We also present bounds for the eigenvalues of structured classes of matrices.

### Keywords

Gerschgorin disks, Nonsingularity criteria, Positive matrices, M-matrices, Totally positive matrices, Eigenvalues location, Exclusion sets.

### References

- [1] Peña, J. M. (2001). A class of P-matrices with applications to the localization of the eigenvalues of a real matrix. *SIAM J. Matrix. Anal. Appl.* 22, 1027–1037.
- [2] Peña, J. M. (2003). On an alternative to Gerschgorin circles and ovals of Cassini. *Numer. Math.* 95, 337–345.
- [3] Peña, J. M. (2005). Inclusion and exclusion intervals for the real eigenvalues of positive matrices. *SIAM J. Matrix. Anal. Appl.* 26, 908–917.
- [4] Varga, R. S. (2004). Geršgorin and his circles. Berlin: Springer.

# The European Parliament election in the treaty on a constitution for Europe: an electorate of degressive valencies

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## Abstract

Electoral equality is considered of high constitutional value in all of the 25 member states of the European Union. When applied to Proportional Representation systems, electoral equality is generally understood in a way that the proportion of seats should fit the proportion of votes of a party, up to the inevitable rounding errors. We present a discussion which apportionment methods are more suitable to achieve the goal than others. A special emphasize is laid on biproportional apportionment methods archiving a two-way fit, namely relative to the vote distribution for the political parties, but also relative to the population distribution across various electoral districts. A biproportional apportionment method might also be considered suitable for the election of the European Parliament. However, the Treaty on a Constitution for Europe, signed by the government leaders on 29 October 2004, remains silent on the principle of electoral equality, and instead introduces the concept of "degressive proportionality" whose content remains nebulous to date.

## A conversation with Sujit Kumar Mitra in 1993 and some comments on his research publications

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### Abstract

Sujit Kumar Mitra was born on 23 January 1932 in Calcutta. He earned his B.Sc. degree in Statistics from Presidency College, Calcutta, in 1949, and his M.Sc. degree in Statistics, from Calcutta University, in 1951. He spent two years, August 1954–August 1956, in the Department of Statistics at the University of North Carolina at Chapel Hill, completing his Ph.D. thesis entitled “Contributions to the statistical analysis of categorical data” under the guidance of Samarendra Nath Roy, in 1956. Sujit Kumar Mitra passed away at his home in New Delhi on 18 March 2004.

In this talk we present a conversation that took place at the Indian Statistical Institute, New Delhi, in early February 1993, while Simo Puntanen and George Styan were visiting the Institute.

Moreover, we present several tables based on the publications by Sujit Kumar Mitra. We begin with a list of 7 books by Sujit Kumar Mitra and then continue with an annotated list of 102 publications in research journals and collections. The 102 publications in research journals and collections are listed chronologically, and by journal/collection within year, and may be classified as follows: 88 papers in 24 peer-refereed research journals, 14 papers in 14 research collections.

Associated with a bibliography by a particular author (with several coauthors), we follow Baksalary and Styan (2005) and Puntanen and Styan (2006) by defining an “authorship matrix”  $A = \{a_{ij}\}$ , where  $a_{ij} = 1$  if bibliographic entry number  $i$  is written with coauthor number  $j$  and  $a_{ij} = 0$  otherwise. The authorship matrix  $A$  for Sujit Kumar Mitra is  $109 \times 38$ . The diagonal entries of the  $38 \times 38$  matrix  $A'A$  represent the numbers of bibliographic entries written with each of the 38 coauthors.

## Keywords

Authorship matrix, Bibliography, Bibliometrics, Eigenvalues, Eigenvectors, Indian Statistical Institute, University of North Carolina at Chapel Hill, Debabrata Basu (1924–2001), Raj Chandra Bose (1901–1987), Radha Govind Laha (1930–1999), Prasantha Chandra Mahalanobis (1893–1972), Calyampudi Radhakrishna Rao (b. 1920), Samarendra Nath Roy (1906–1964).

## References

- Baksalary, Oskar Maria and Styan, George P. H., eds. (2005). Some comments on the life and publications of Jerzy K. Baksalary (1944–2005). *Linear Algebra Appl.*, 410, 3–53.
- Puntanen, S. and G.P.H. Styan (2006). Some comments on the research publications of Tarmo Mikko Pukkila. In: Erkki P. Liski, Jarkko Isotalo, Jarmo Niemelä, Simo Puntanen, and George P. H. Styan (Eds.), *Festschrift for Tarmo Pukkila on his 60th Birthday* pp. 45–62. Dept. of Mathematics, Statistics and Philosophy, University of Tampere.

## The positive realization problem in normal transfer matrices with simple real poles\*

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### Abstract

The positive realization problem is formulated as follows: Let  $H(z) \in \mathbb{R}^{r \times s}(z)$  be a transfer matrix, it is said to admit a positive realization  $(A, B, C)$  if we find nonnegative matrices  $A \in \mathbb{R}^{N \times N}$ ,  $B \in \mathbb{R}^{N \times s}$ ,  $C \in \mathbb{R}^{r \times N}$  and  $D \in \mathbb{R}^{r \times s}$  such that  $H(z) = C[zI - A]^{-1}B + D$ . Moreover, this realization will be minimal if it has the minimal dimension.

It has been studied for several authors although, they are mainly concentrated on single-input single-output (SISO) systems. The positive realization problem for multi-input multi-output (MIMO) linear systems is less studied. For instance, K. H. Forster and B. Nagy studied the nonnegative realizability of transfer matrices of discrete-time systems with a primitive state matrix. And in continuous-time systems, R. Cantó, B. Ricarte and A. M. Urbano computed a procedure to obtain a minimal positive realization  $(A, B, C, D)$  with the matrix  $A$  in the Jordan form.

In this work we deal with MIMO systems and solve the positive realization problem of certain *normal transfer matrices* with simple real poles. Moreover, we give necessary and sufficient conditions to obtain a minimal positive realization.

The concept of *normal rational matrix* was introduced by Lampe and Rosenwasser in multivariable linear control theory with the purpose to advance in the realization problem. Given a transfer matrix expressed by its standard rational form  $H(z) = \frac{M(z)}{d(z)}$ , where  $M(z) \in \mathbb{R}^{r \times s}[z]$  is a polynomial matrix and  $d(z)$  is a scalar polynomial (the minimal common denominator), they define it as a *normal transfer matrix* if any second order minor of  $M(z)$  is zero or is divisible by  $d(z)$ .

Later, it has been proved that in transfer matrices, *normal transfer matrix* is equivalent to *irreducible transfer matrix*, which means that for any root  $z_i$  of  $d(z) = 0$ , the matrix  $M(z_i)$  is not equal to the  $r \times s$  zero matrix ( $M(z_i) \neq 0$ ). Therefore, the dimension  $N$  of any minimal

realization  $(A, B, C, D)$  will be equal to the order  $n$  of the transfer matrix  $H(z)$ .

## Keywords

Minimal realizations, Multivariable systems, Transfer matrices, Positive realizations.

## References

- Benvenuti, L. and L. Farina (2004). A Tutorial on the Positive Realization Problem. *IEEE Trans. Automatic Control* 49(5), 651–664.
- Cantó, R., B. Ricarte and A. M. Urbano (2006). On Positive Realizations of Irreducible Transfer Matrices. *Lecture Notes in Control and Information Sciences* 341, 41–48.
- Farina, L. and S. Rinaldi (2000). *Positive Linear Systems: Theory and Applications*. New York: John Wiley & Sons, Inc.
- Forster, K. H. and B. Nagy (2000). Nonnegative realizations of matrix transfer functions. *Linear Algebra Appl.* 311, 107–129.
- Hadjicostis, C. (1999). Bounds on the size of minimal nonnegative realizations for discrete-time LTI systems. *Systems and Control Letters* 37, 39–43.
- Halmschlager, A and M. Matolcsi (2005). Minimal Positive Realizations for a Class of Transfer Functions. *IEEE Trans. Circuits Syst. II, Expr. Briefs* 52(4), 177–180.
- Kaczorek, T. (2002). *Positive 1D and 2D Systems*. London: Springer.
- Lampe, B. P. and E. N. Rosenwasser (2000). Algebraic Properties of Irreducible Transfer Matrices. *Autom. Remote Control* 61(7), 1091–1102.

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## Singular structured matrix with integer spectrum

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### Abstract

The objective of the paper is to consider a class of singular structured non-symmetric matrices with integer spectrum. An interesting factorization of the matrices from that class, with the help of triangular matrices, will be provided. A special case of the class originates from statistical survey sampling theory.



# Some properties of information matrices of complete designs under an interference model

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## Abstract

We consider an experiment with fixed number of blocks, in which a response to a treatment can be affected by treatments from neighboring units. For such experiment the interference model with neighbor effects can be used. Under this model we study some properties of information matrices of complete block designs, according to connectedness and optimality of designs. Assuming the circular interference model with left-neighbor effects we give the necessary and sufficient conditions of connectedness of complete block designs with arbitrary, fixed number of blocks.

It is known that circular neighbor balanced designs (CNBD) are universally optimal in a circular interference model. However, CNBDs cannot exist for each combination of design parameters. In such a situation, only optimality with respect to the specified optimality criteria can be studied. Our aim is to characterize E-optimal allocation of treatments in blocks with respect to the number of blocks under the interference model with left-neighbor effects. For the CNBD extended and abridged by one certain block, the resulting structure is proved E-optimal.

## Keywords

Information matrix, Interference model, Circular design, Optimality, Connectedness.

## Toeplitz structured perturbations

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### Abstract

We will investigate the condition number, eigenvalue perturbations and pseudospectrum of Toeplitz matrices under structured perturbations. Sometimes we will see few changes, sometimes, although provably rare, dramatic differences in the structured and unstructured view. Connections to minimization problems concerning polynomials are shown. One result is that the structured distance to the nearest singular matrix equals the reciprocal of the structured condition number, a nice generalization of the well known result by Eckart and Young.

### Keywords

Submission of abstract, Reference style, Submission date, How to submit.

### References

- Chouakria, A., E. Diday, and P. Cazes (1998). Vertices principal components with an improved factorial representation. In: A. Rizzi, M. Vichi, and H.-H. Bock (Eds.), *Advances in Data Science and Classification* (pp. 397–402). Heidelberg: Springer.
- Marshall, A.W. and I. Olkin (1979). *Inequalities: Theory of Majorization and its Applications*. New York: Academic Press.
- Olkin, I. (1997). A determinantal proof of the Craig-Sakamoto theorem. *Linear Algebra Appl.* 264, 217–223.
- Johnson, C., T.J. Laffey, and R. Loewy (1996). The real and the symmetric nonnegative inverse eigenvalue problems are different. *Proc. Amer. Math. Soc.* 124, 3647–3651.

# How and How Not to Compute the Moore-Penrose Inverse

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## Abstract

A number of fast algorithms for computing the Moore-Penrose inverse of structured and block matrices have been designed. However, very often they are not accurate up to the limitations of data and conditioning of the problem. Thus procedures of improving the accuracy and stability of algorithms are necessary. We consider some ways in which iterative refinement may be used to improve the computed results. Extensive numerical testing was done in Matlab to compare the performance of some direct and iterative methods for computing the Moore-Penrose inverse of special matrices.

## Keywords

Block matrix, Moore-Penrose inverse, Singular values, Least squares solutions, Condition numbers, Stability of algorithms, Iterative methods.

## References

- Ben-Israel, A. and T.N.E. Greville (2003). *Generalized Inverses: Theory and Applications, 2nd edn.* New York: Springer.
- Golub, G.H. and C.F. Van Loan (1996). *Matrix Computations, 3rd edn.* Johns Hopkins University Press, Baltimore, MD.
- Smoktunowicz, A. (1999). Blockwise analysis for solving linear systems of equations. *J. KSIAM. 3, No. 1*, 31–41.
- Xu, W., Y. Wei and S. Qiao (2006). Condition numbers for structured least squares problems. *BIT Numerical Mathematics. 46*, 203–225.
- Zielke, G. (1986). Report on test matrices for generalized inverses. *Computing 36*, 105–162.

## An illustrated philatelic introduction to magic square matrices

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### Abstract

In this talk we focus on magic square matrices and associated postage stamps. As noted by Barnard (1888), “The construction of magic squares has been practiced earlier than the period of authentic history and it has preoccupied the attention of the curious in every age, among them men of high scientific eminence.” The magic square associated with the matrix

$$L = \begin{pmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{pmatrix}$$

is, according to legend, over 4000 years old and is known as the *luoshu* or “Luo River Writing” (Needham 1970, page 56; Swetz 2003, page 2). It was apparently first considered by the Chinese engineer-emperor Yü the Great (*fl.* 21st century BC) in connection with the arrangement of dots on the shell of a turtle from the River Luo, a tributary of the Yellow River.

While magic squares have a very long history, postage stamps are less than 200 years old; the first postage stamp, the “Penny Black”, was issued by Great Britain in May 1840 (Arthur Cayley introduced matrices in 1857). Since 1840 many thousands of stamps have been issued, catalogued and collected. As Schaaf (1978, page xiiii) observed, stamp collectors and philatelists deal with a rich and fascinating world which is “a mirror of civilization”. Schaaf’s *Adventure in Postage Stamps* considered “the impact of mathematics and science on society”. As for science, the nuclear physicist Ernest Rutherford (1871–1937) said “All science is either physics or stamp collecting” (Blackett 1962, page 108; Wilson 2001, book title page)—Rutherford won the Nobel Prize in chemistry in 1908.

While we have not found any matrices actually depicted on postage stamps, we have found three stamps which depict magic squares; the associated matrices are:

$$A = \begin{pmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 52 & 61 & 4 & 13 & 20 & 29 & 36 & 45 \\ 14 & 3 & 62 & 51 & 46 & 35 & 30 & 19 \\ 53 & 60 & 5 & 12 & 21 & 28 & 37 & 44 \\ 11 & 6 & 59 & 54 & 43 & 38 & 27 & 22 \\ 55 & 58 & 7 & 10 & 23 & 26 & 39 & 42 \\ 9 & 8 & 57 & 56 & 41 & 40 & 25 & 24 \\ 50 & 63 & 2 & 15 & 18 & 31 & 34 & 47 \\ 16 & 1 & 64 & 49 & 48 & 33 & 32 & 17 \end{pmatrix}.$$

The magic square associated with the matrix  $A$  appears in Albrecht Dürer's engraving *Melencolia I*, which is depicted on a 1986 stamp issued by Aitutaki–Cook Islands and on a 1978 stamp from Mongolia (Wilson 2001, frontispiece). The magic square associated with the matrix  $B$  was published in 1769 by Benjamin Franklin (1706–1790) and is shown on a stamp issued by the USA in 2006.

In this talk we also look philatelically at several other magic squares, including two to be found in Europe: in the Sagrada Familia basilica in Barcelona and in the Villa Albani in Rome, and three in India: in the Gwalior Fort in Madhya Pradesh, in a Hindu temple in Dudhai (Jhansi District), and in the Jain Parshvanath Temple in Khajuraho.

## Keywords

Arthur Cayley (1821–1895), Albrecht Dürer (1471–1528), Benjamin Franklin (1706–1790), Frederick Augustus Porter Barnard (1809–1889), Ernest Rutherford (1871–1937); Franklin squares, History of mathematics, *Luoshu*, Magic squares, Philately, Postage stamps.

## References

- Barnard, F.A.P. (1888). Theory of magic squares and of magic cubes. *Memoirs of the National Academy Sciences USA*, vol. IV, Sixth Memoir, pp. 207–270.
- Blackett, P.M.S. (1962). Memories of Rutherford. In *Rutherford at Manchester* edited by J. B. Birks, London: Heywood, pp. 102–113.
- Needham, Joseph, with the collaboration of Wang Ling (1970). *Science and Civilisation in China, Volume 3: Mathematics and the Science of the Heavens and Earth*, Reprint edition. Cambridge University Press. [Originally published: 1959.]

- Schaaf, W.L. (1978). *Mathematics and Science: An Adventure in Postage Stamps*. Reston, Virginia: National Council for Teachers of Mathematics.
- Swetz, F.J. (2002). *Legacy of the Luoshu: The 4,000 Year Search for the Meaning of the Magic Square of Order Three*. Chicago: Open Court.
- Wilson, R.J. (2001). *Stamping Through Mathematics*. New York: Springer-Verlag.

# Incomplete alternating projection method for large inconsistent linear systems

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## Abstract

Let  $A$  be an  $n \times m$  real matrix and let  $b \in \mathbb{R}^m$ . We consider the problem of finding an approximative solution  $x \in \mathbb{R}^n$  of a large scale system of linear equations

$$A^\top x = b,$$

if such a solution exists. In practice these systems are often inconsistent. The problem is a special case of the following problem: Let  $\mathcal{P}, \mathcal{Q}$  be nonempty and affine subspaces. In practice we want to find an element of the intersection  $\mathcal{P} \cap \mathcal{Q}$  or find points  $p \in \mathcal{P}$  and  $q \in \mathcal{Q}$  which realize the distance between these two subspaces. In order to solve the problem we deal with a modification of the so called *alternating projection method* (APM). APM generate a sequence  $(x_k)$  by the following iterative scheme:

$$x_{k+1} = P_{\mathcal{P}}P_{\mathcal{Q}}x_k.$$

We take in the modification an approximative projection  $\tilde{P}_{\mathcal{P}}$  instead of an exact projection  $P_{\mathcal{P}}$  with appropriate stopping criteria. We modify the APM in such a way that the Fejér monotonicity with respect to  $\text{Fix}P_{\mathcal{P}}P_{\mathcal{Q}}$  and the convergence of  $(x_k)$  to an element of  $\text{Fix}P_{\mathcal{P}}P_{\mathcal{Q}}$  is preserved.

We present preliminary numerical results for our method.

## Keywords

Alternating projection method, Fejér monotonicity, Approximative projection, Residual projection.

## References

- Bauschke, H.H. and J.M. Borwein (1994). Dykstra's alternating projection algorithm for two sets. *J. Approx. Theory* 79, 418–443.

- Cegielski, A. Convergence of a relaxed alternating projection method. Submitted.
- Cegielski, A. and A. Suchocka. Incomplete alternating projection method for large inconsistent linear systems. Submitted.
- Scolnik, H. D., N. Echebest, M.T. Guardarucci and M.C. Vacchino (2002). A class of optimized row projection methods for solving large nonsymmetric linear systems. *Appl. Numer. Math.* 41, 499–513.
- Yang, K. and K.G. Murty (1992). New iterative methods for linear inequalities. *JOTA* 72, 163–185.



# Optimal designs in multivariate linear models

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## Abstract

We study optimality of designs under the multivariate model of the form

$$\mathbf{Y} = \mathbf{A}_{1,d} \mathbf{B}_1 \mathbf{C}_1 + \mathbf{A}_{2,d} \mathbf{B}_2 \mathbf{C}_2 + \mathbf{A}_3 \mathbf{B}_3 \mathbf{C}_3 + \mathbf{E}, \quad \mathbf{E} \sim N_{n,q}(\mathbf{0}, \boldsymbol{\Sigma} \otimes \mathbf{V}),$$

$$\mathcal{R}(\mathbf{C}'_1) \subseteq \mathcal{R}(\mathbf{C}'_3), \quad \mathcal{R}(\mathbf{C}'_2) \subseteq \mathcal{R}(\mathbf{C}'_3), \quad \mathcal{R}(\mathbf{C}'_2) \subseteq \mathcal{R}(\mathbf{C}'_1).$$

Additionally, we assume that the dispersion matrix of matrix of random errors is known or partially unknown. When dispersion matrix is known we determine optimal designs using Kiefer optimality. In the case of unknown dispersion matrix optimality is considered with respect to the precision in maximum likelihood estimation.

# Continuous affine families of unitary Hadamard matrices

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## Abstract

A unitary Hadamard matrix is understood here as an  $N \times N$  unitary matrix with all its entries having moduli equal to  $1/\sqrt{N}$ .

By an affine family of unitary Hadamard matrices stemming from  $H$  a set of the form:  $\{H \circ \mathbf{EXP}(\mathbf{i}R) : R \in \mathcal{R}\}$ , is meant, where  $H$  is an  $N \times N$  unitary Hadamard matrix,  $\circ$  denotes the Hadamard product,  $\mathbf{EXP}$  denotes the entrywise exp operation on a matrix, and  $\mathcal{R}$  is a subspace of real  $N \times N$  matrices.

The most general way of constructing such affine families will be presented, being a serious combinatorial problem. A conjecture on the relevant calculational method, involving solution of an algebraic system, will be discussed.

## Keywords

Hadamard matrices, complex Hadamard matrices.

# On the distance and the angle of subspaces

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## Abstract

For two given subspaces of  $\mathbb{C}^n$  we investigate their distance and angle on the basis of a joint decomposition of the corresponding orthogonal projectors. Further attention is paid to the notions inclinedness, orthogonal incidence and minimal angle. Some formulas for the spectral norm of orthogonal projectors or functions thereof are derived. Also the proofs of results known from Hilbert space theory, related to orthogonal projectors, are simplified considerably.

## Enhancing the documentation by leaving useful traces

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### Abstract

An appropriate documentation of the scientific work process 1) advocates the reproducibility of the research, 2) supports backtracking and following side tracks, 3) improves the data quality, and 4) helps to avoid reinventing the wheel.

Instead of formal documentation, we focus on *traces* that are left behind when working. By traces we refer to free-form notes and comments written down explicitly, but also to expressions, commands and work schemes created primarily to utilize various operations of the software. These pieces together reflect the *thinking process*, which needs to be retraced, say, when revising a paper after a possibly long review process. Leaving useful traces enhances the documentation and helps to get back on the track.

We demonstrate these ideas especially with the matrix operations of SURVO MM software. Its unique *editorial interface* promotes building and maintaining the work process so that each step is appropriately documented.

Our examples are taken from certain new developments in the area of multivariate statistical modeling with measurement errors.

### Keywords

Work process, Documentation, Reproducibility, Data quality, Matrix computations, Multivariate methods, Measurement error.

### References

- Dasu, T. and T. Johnson (2003). *Exploratory Data Mining and Data Cleaning*. Hoboken, New Jersey: John Wiley & Sons.
- Gentleman, R. and D. Temple Lang (2004). Statistical analyses and reproducible research. Bioconductor Project Working Papers, Working Paper 2 <http://www.bepress.com/bioconductor/paper2>.

- Mustonen, S. (2001). *SURVO MM: Computing Environment for Creative Processing of Text and Numerical Data*  
<http://www.survo.fi/mm/english.html>.  
Freeware version *Survo Editor*: <http://www.survo.fi/english/download>.
- Tarkkonen, L. and K. Vehkalahti (2005). Measurement errors in multivariate measurement scales. *J. Multivariate Anal.*, 96, 172–189.
- Vehkalahti, K. (2005). Leaving useful traces when working with matrices. *Research Letters in the Information and Mathematical Sciences*, 8, 143–154. Proceedings of the 14th International Workshop on Matrices and Statistics. (Paul S.P. Cowpertwait, ed.) Massey University, Auckland, New Zealand, March 29 – April 1, 2005  
<http://iims.massey.ac.nz/research/letters/volume8/>.
- Vehkalahti, K., S. Puntanen and L. Tarkkonen (2006). Effects of measurement errors in predictor selection of linear regression model. *Reports on Mathematics, Preprint 439, Department of Mathematics and Statistics, University of Helsinki, Finland*  
<http://mathstat.helsinki.fi/reports/Preprint439.pdf>.
- Vehkalahti, K., S. Puntanen and L. Tarkkonen (2006). Estimation of reliability: a better alternative for Cronbach's alpha. *Reports on Mathematics, Preprint 430, Department of Mathematics and Statistics, University of Helsinki, Finland*  
<http://mathstat.helsinki.fi/reports/Preprint430.pdf>.

## The solution of continuous-time algebraic Riccati equations by means of the SOR-LIKE method

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### Abstract

The main subject of the paper is to demonstrate the efficiency of the SOR-LIKE method, introduced recently by the author for solving Sylvester equations [1,2], with solving the continuous-time algebraic Riccati equation (CARE)

$$\mathbf{A}^T \mathbf{X} + \mathbf{X} \mathbf{A} - \mathbf{X} \mathbf{S} \mathbf{X} + \mathbf{Q} = \mathbf{0}, \quad (1)$$

where  $\mathbf{A}, \mathbf{Q}, \mathbf{S}, \mathbf{X} \in \mathbb{R}^{n \times n}$  and the matrix  $\mathbf{S}$  is usually factored in the following form  $\mathbf{S} = \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T$  with  $\mathbf{B} \in \mathbb{R}^{n \times m}$  and  $\mathbf{R} \in \mathbb{R}^{m \times m}$ . Evidently, when  $\mathbf{S}$  is the null matrix, the above equation reduces to the well known Lyapunov equation considered also in [1,2].

Substituting  $\mathbf{F} = \mathbf{A}^T$  for the simplicity of notation and assuming that the matrix  $\mathbf{F}$  is defined by the following decomposition

$$\mathbf{F} = \mathbf{K} - \mathbf{L} - \mathbf{U}, \quad (2)$$

where  $\mathbf{K}, \mathbf{L}$  and  $\mathbf{U}$  are nonsingular diagonal, strictly lower triangular and strictly upper triangular parts of  $\mathbf{A}$  respectively; Eq.(1) can be rewritten as follows

$$\mathbf{K} \mathbf{X} = \mathbf{L} \mathbf{X} + \mathbf{U} \mathbf{X} - \mathbf{X} \mathbf{A} + \mathbf{X} \mathbf{S} \mathbf{X} - \mathbf{Q} \quad (3)$$

or equivalently

$$\mathbf{X} = \mathbf{K}^{-1} \{ \mathbf{L} \mathbf{X} + \mathbf{U} \mathbf{X} - \mathbf{X} \mathbf{A} + \mathbf{X} \mathbf{S} \mathbf{X} - \mathbf{Q} \}. \quad (4)$$

We assume that the iterative scheme has the following form

$$\mathbf{X}^{(t)} = \mathbf{K}^{-1} \{ \mathbf{L} \mathbf{X}^{(t)} + \mathbf{U} \mathbf{X}^{(t-1)} - \mathbf{X}^{(t-1,t)} \mathbf{A} + \mathbf{X}^{(t-1,t)} \mathbf{S} \mathbf{X}^{(t-1,t)} - \mathbf{Q} \} \\ \text{for } t = 1, 2, \dots (5)$$

where the terms  $\mathbf{X}^{(t-1,t)}\mathbf{A}$  and  $\mathbf{X}^{(t-1,t)}\mathbf{S}\mathbf{X}^{(t-1,t)}$  mean that for computing the products  $\mathbf{X}\mathbf{A}$  and  $\mathbf{X}\mathbf{S}\mathbf{X}$ , the entries of  $\mathbf{X}^{(t-1)}$  and  $\mathbf{X}^{(t)}$  are used in the current iteration  $t$ .

For the acceleration of convergence in the above iterative scheme, the overrelaxation procedure can be used as follows

$$\begin{aligned} \mathbf{X}^{(t)} = & \omega \mathbf{K}^{-1} \{ \mathbf{L}\mathbf{X}^{(t)} + \mathbf{U}\mathbf{X}^{(t-1)} - \mathbf{X}^{(t-1,t)}\mathbf{A} + \mathbf{X}^{(t-1,t)}\mathbf{S}\mathbf{X}^{(t-1,t)} - \mathbf{Q} \} \\ & - (\omega - 1)\mathbf{X}^{(t-1)} \quad \text{for } t = 1, 2, \dots \end{aligned} \quad (6)$$

or written equivalently

$$\begin{aligned} \mathbf{X}^{(t)} = & [\mathbf{I} - \omega \mathbf{K}^{-1}\mathbf{L}]^{-1} \{ [(1 - \omega)\mathbf{I} + \omega \mathbf{K}^{-1}\mathbf{U}]\mathbf{X}^{(t-1)} \\ & + \omega \mathbf{K}^{-1}[-\mathbf{X}^{(t-1,t)}\mathbf{A} + \mathbf{X}^{(t-1,t)}\mathbf{S}\mathbf{X}^{(t-1,t)} - \mathbf{Q}] \} \quad \text{for } t = 1, 2, \dots \end{aligned} \quad (7)$$

Since the exact solution  $\mathbf{X}$  satisfies the above equation, then with the error solution matrix  $\mathbf{E}^{(t)} = \mathbf{X} - \mathbf{X}^{(t)}$ , we have

$$\begin{aligned} \mathbf{E}^{(t)} = & [\mathbf{I} - \omega \mathbf{K}^{-1}\mathbf{L}]^{-1} \{ [(1 - \omega)\mathbf{I} + \omega \mathbf{K}^{-1}\mathbf{U}]\mathbf{E}^{(t-1)} \\ & + \omega \mathbf{K}^{-1}[-\mathbf{E}^{(t-1,t)}\mathbf{A} + \mathbf{E}^{(t-1,t)}\mathbf{S}\mathbf{E}^{(t-1,t)} - \mathbf{Q}] \} \quad \text{for } t = 1, 2, \dots \end{aligned} \quad (8)$$

Similarly as in the case of Sylvester equation solutions analyzed in [1,2], the error solution matrix  $\mathbf{E}^{(t)}$  can not be expressed explicitly in dependence on  $\mathbf{E}^{(0)}$  however, we can assume that there exists an implicit iteration matrix  $\mathcal{T}_{impl}$  which form can not be expressed explicitly but we are able to compute its spectral radius according to the following equation derived from (6)

$$\begin{aligned} \omega \mathbf{K}^{-1} \{ \mathbf{L}\mathbf{Y}^{(t)} + \mathbf{U}\mathbf{Y}^{(t-1)} - \mathbf{Y}^{(t-1,t)}\mathbf{A} + \mathbf{Y}^{(t-1,t)}\mathbf{S}\mathbf{Y}^{(t-1,t)} \} \\ - (\omega - 1)\mathbf{Y}^{(t-1)} = \Lambda \mathbf{Y}^{(t)} \quad \text{for } t = 1, 2, \dots, \end{aligned} \quad (9)$$

where  $\Lambda$  is an eigenvalue of the implicit iteration matrix  $\mathcal{T}_{impl}$  and the matrix  $\mathbf{Y}$  plays a role of an "eigenvector". When  $\Lambda$  is a real eigenvalue, the above equation represents the algorithm of the power method providing us the spectral radius  $\varrho(\mathcal{T}_{impl}) = |\Lambda|$ , where the value of  $\varrho(\mathcal{T}_{impl})$  is frequently minimized for  $0 < \omega < 1$  and method is not sensitive to the choice of the accurate value of  $\omega_{best}$ .

As is observed in numerical experiments presented in the paper, the proposed method, based on a simple algorithm, provides very accurate solutions with a low computational work because the arithmetical effort in one iteration is roughly equivalent to that required for computing the residual matrix

$$\mathbf{R}^{(t)} = \mathbf{F}(\equiv \mathbf{A}^T)\mathbf{X}^{(t)} + \mathbf{X}^{(t)}\mathbf{A} - \mathbf{X}^{(t)}\mathbf{G}\mathbf{X}^{(t)} + \mathbf{Q}. \quad (10)$$

## Keywords

Algebraic Riccati equation, SOR-LIKE iterative method, Spectral radius, Implicit iteration matrix.

## References

- [1] Woźnicki, Z.I. (2003). A SOR-like Method for Solving the Sylvester Equation. *Annals the European Academy of Sciences*. 335–344, ISSN 1379-1982.
- [2] Woźnicki, Z.I. (2007). On the Iterative Solution of Sylvester Equation  $AX - XB = C$ . An invited talk, *Proc. International Workshop on Numerical Linear Algebra in Signals, Systems and Control*, January 9-11, 2007, IIT Kharagpur, India.



## Numerical ranges in a strip

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### Abstract

It is shown that a matrix is power bounded if and only if the numerical range of all its powers lies in a fixed strip of the complex plane. In particular, a matrix  $A$  is power bounded if and only if the sums  $A^k + A^{*k}$  are uniformly bounded with respect to  $k = 1, 2, \dots$ . A number of analogous splittings are also obtained, especially for the Cesàro means of powers. The method is based on the concept of numerical range, the Schur triangularization of matrices, and a recent inequality of Kittaneh. The extension of this result to linear operators on a Hilbert space is given. Analogous problems in the Gerschgorin set setting are also investigated.

## Jordan algebra and statistical inference in linear mixed models

Miguel Fonseca<sup>1</sup>, João T. Mexia<sup>1</sup>  
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### Abstract

In the presentation we consider the well known mixed linear model and the problems of estimation and testing hypotheses both for fixed parameters and variance components. Necessary and sufficient conditions for existence of best linear unbiased estimators BLUE's for linear functions of fixed effect and best quadratic unbiased estimators BQUE's for variance components of random effects will be given. It will be shown, that Jordan Algebras play very important role in such characterization. Under normality assumption those conditions are equivalent for existence of BUE's. Moreover, in the case of normality assumptions, we can get some optimal tests based on those estimators and it can be shown that the some tests are based on statistics as a functions of BLUE's and/or BQUE's which are F distributed. Also, in this case the assumptions on the covariance structure is very important and Jordan Algebras play main role. We would like to refer readers to the following papers and to the references of those papers.

### References

- Fonseca, M., J. T. Mexia and R. Zmyślony (2006). Binary operations on Jordan algebras and orthogonal normal models. *Linear Algebra Appl.* 417, 75-97.
- Fonseca, M., J. T. Mexia and R. Zmyślony (2002). Exact distributions for the generalized F tests. *Discuss. Math. Probab. Stat.* 26, 37-51.
- Gnot, S., W. Klonecki and R. Zmyślony (1978). Linear spaces and the theory of best linear unbiased estimation. *Bull. Pol. Acad. Sci. Math.* 26, 69-72.
- Gnot, S., W. Klonecki and R. Zmyślony (1978). Best linear unbiased estimation, a coordinate free approach. *Probab. Math. Statist.* 1, 1-13.
- Michalski, A. and R. Zmyślony (1996). Testing hypothesis for variance components in mixed linear models. *Statistics* 27, 297-310.
- Michalski, A. and R. Zmyślony (1996). Testing hypothesis for linear functions of parameters in mixed linear models. *Tatra Mt. Math. Publ.* 17, 103-110.

- Jordan, P., J. von Neumann and E.P. Wigner (1934). On an algebraic generalization of the quantum mechanical formalism, *Ann. Math. II Ser.* 35, 29-64.
- Seely, J. (1971). Quadratic subspaces and completeness. *Ann. Math. Stat.* 42, 1735-1748.
- Zmyślony, R. (1980). A characterization of best linear unbiased estimators in the general linear model. *Mathematical Statistics and Probability Theory*, Proc. 6th Int. Conf. Wiśna, Poland, 1978, Lecture Notes in Statist., 365-373.
- Zmyślony, R. (1980). Completeness for a family of normal distributions. *Mathematical Statistics*, Banach Center Publ. 6, 355-357



Part IV

**Two birthday boys**



## Many happy returns, George Peter Hansbenno Styan\*

Simo Puntanen

University of Tampere, Finland

### Abstract

George Peter Hansbenno Styan, uniquely known as GPHS, was born almost 70 years ago, 10 September 1937; in Hendon, Middlesex (now part of Greater London), England, UK.

Our society knows George pretty well. In early April in 1985, in Budapest, I met Professor Shayle R. Searle and I asked whether he knows GPHS. Shayle's wife Helen replied immediately "Oh yes! Everybody knows him, except maybe somebody in the moon". Almost exactly 20 years later, in Auckland, New Zealand, in the 14th International Workshop on Matrices and Statistics (IWMS-14), Shayle still knew George very well.

I met George first time at the Tampere Airport on 22 January 1976, at 12:15. (Since then, we've met in numerous airports—two highlights being the 7 a.m. meeting in the New Delhi Airport on 9 January 1993, and on 10 April 2005 at 6:30 a.m. in the Airport of Auckland, from where our trio, Jarkko Isotalo as a personal driver, headed towards Papatoetoe, the Waikato University, and Lagavulin in Raglan to be provided by Murray Jorgensen.) On 22 January 1976, George was flying from Oulu to Tampere to give a 10 hours lecture series at our place. He was spending a sabbatical year 1975–76 in Helsinki, but there his lectures started at 8:15 on Friday mornings and that was too tricky for me and Erkki Liski. Hence we invited him to Tampere. In a nutshell, our cooperation yielded the Honorary Doctorate to George, "for his great scientific contributions and merits in mathematics and statistics, and in the promotion of research in the University of Tampere", awarded in Tampere, on 19 May 2000.

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\* George P. H. Styan is Professor Emeritus at McGill University, Montréal, Canada. Ph.D. thesis in 1969 entitled "Multivariate normal inference with correlation structure" (T. W. Anderson, advisor) from Columbia University, New York City. Since 1969, worked at the Department of Mathematics and Statistics, McGill University; promoted to rank of Professor Emeritus in 2005.

Why do we know George so well? (To go the other way round, I recall George saying that with Alastair J. Scott they figured out that together they know anyone who is anyone.)

I think the answer is that George is a “man of the society”, so to say. George likes to run the show, and there are many shows in academic life.

From 24 February to 18 March 1976, Professor C. Radhakrishna Rao, a birthday-buddy to George (being 17 years senior to George), was hosting George and Evelyn in New Delhi. After returning back to Finland (via Calcutta, Teheran, Istanbul, . . .) in April, George and Evelyn invited Erkki and me for dinner to their place in Helsinki (Kontula, Naapurintie) and Evelyn cooked us outstanding chicken (first of many superb dinners to come). After the dinner, George drove us to the train station but I had taken a careless look at the time table and as a result we had to kill some time in a restaurant at the station—beginning of many memorable railway-related activities with George.

Regarding important airport visits, I have to add the event of the 2nd of December 1982, when George and I went to pick up Professor Peter Bloomfield at the Montréal–Dorval Airport. George had agreed that Peter would carry the latest issue of the *JRSS-Series A* in his left hand upon arrival so that we would recognize him. Indeed we had no problems in getting Peter back to town but this was not so on the next day when we all three went shopping oysters (for 60 people) and George’s famous red Saab was hit by some idiot’s lousy old car. (Note that in the trunk George had a remarkable amount of Beaujolais Nouveau, some of which gave unusual colour to a recently bought copy of Cook & Weisberg’s *Residuals and Influence in Regression*.)

One further airport trip should be mentioned: on 3 May 1988, George and I went to the Montréal–Mirabel Airport to welcome Jerzy K. Baksalary for his first North-American visit. Jerzy was rather exhausted after the long flight, telling that all his flight through he had been reading and correcting the thesis of Augustyn Markiewicz. “Oh, those youngsters and their writings . . .”, was Jerzy’s comment.

I’m happy to tell that George has taught me a lot about the art of traveling. Yet, another necessity of life, eating, that is the area where I am not his best student. (Knut Conradsen is George’s best eating buddy if somebody does not know that.) Having an absolute restaurant-ear (tongue?) George is of great company around lunch and dinner time, where ever you are. Professor C. R. Rao told that when in Delhi some-



body was asking him for advice about the decent dinner places, he asked him to go and ask George.

As for eating in Delhi in January 1993, I have warm memories about our regular evening diet: George, Ludwig Elsner and I, we had some nuts in a balcony in the guesthouse of the Indian Statistical Institute, a drop of pre-dinner refreshments, and then we took a 3-wheeler (after heavy negotiations with a driver) to a place that George had chosen for that evening. Enjoyable!

Earlier was mentioned that George is a society man. He enjoys doing things together, as many of us have experienced. To describe his contributions in a nutshell in this frontier, it is best to refer to his 2003 Harry C. Carver Medal for exceptional service to the Institute of Mathematical Statistics (IMS): “For dedicated and enthusiastic service to the IMS and the statistics profession throughout his career . . . and for elevating the taste of the statistical community with *élan* and *savoir faire* as a gourmet par excellence”. Well put.

Many happy returns, George!





Fig. 1. George P.H. Styan, Montréal, October 1982.





**Fig. 2.** Keith J. Worsley, Alastair J. Scott and George P. H. Styan; Auckland, New Zealand, May 1985.





**Fig. 3.** James V. Zidek, George P. H. Styan and David F. Andrews; Banff, Canada, May 1986.



**Fig. 4.** Jerzy K. Baksalary and George P. H. Styan; Wilderton Avenue, Montréal, May 1988.





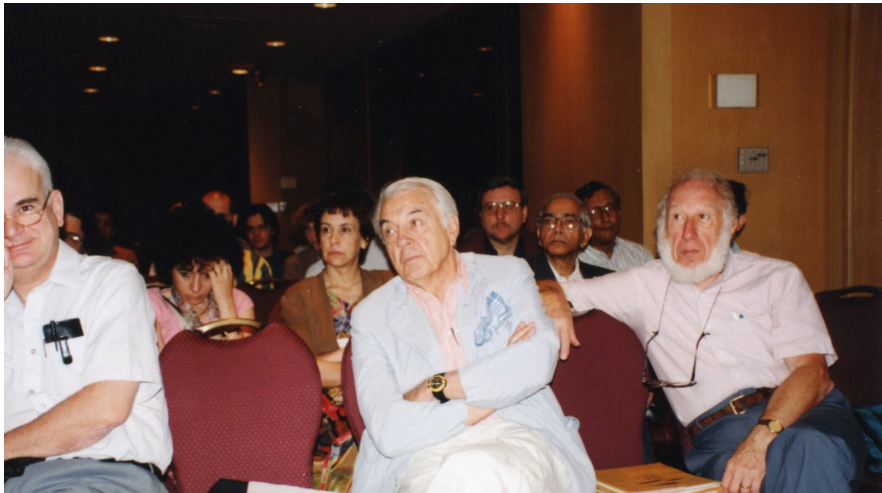


**Fig. 5.** Lobsters, Jerzy K. Baksalary and George P.H. Styan; Wilderton Avenue, Montréal, May 1988.





**Fig. 6.** Oysters, Jerzy K. Baksalary and George P.H. Styan; Wilderton Avenue, Montréal, May 1988.



**Fig. 7.** George P.H. Styan, Geoffrey S. Watson, Ingram Olkin (2nd row: C.R. Rao, 3rd row: Sujit Kumar Mitra); IWMS-4, Montréal, July 1995.





Fig. 8. George P. H. Styan, Tampere, 19 May 2000.





**Fig. 9.** Soile Puntanen, Evelyn Matheson Styan and George P. H. Styan; Prince Edward Island, Canada, June 2003.



**Fig. 10.** Jarkko Isotalo, Simo Puntanen, George P. H. Styan and T. W. Anderson; IWMS-15, Uppsala, Sweden, 15 June 2006.





## Many happy returns, Erkki Pellervo Liski\*!

Simo Puntanen

University of Tampere, Finland

### Abstract

Erkki Pellervo Liski, commonly known as Sir ... (skip this please) was born almost precisely 60 years ago, 12 March 1947; in Hämeenlinna, Finland. He is a member of the largest Finnish age group, comprising 108 000 children.

Thirty-seven years later, that is, 23 years ago, on 3 June 1984, Erkki and I stepped out of the train in the Poznań Railway Station and were friendly welcomed by Barbara Bogacka. She kindly guided us to the conference site: International Statistical Conference in Linear Inference, held in Poznań, 4–9 June 1984.

That event was the first time we had an opportunity to meet alive some famous Polish stars of linear statistical models, like Jerzy K. Baksalary. The Dynamic Duo, Baksalary–Kala, was known to everyone interested in linear models.

Five years later, 1989–90, Jerzy spent one year in Tampere, as a research professor of the Academy of Finland. Oh boys, wasn't he working hard! Oskar Maria Baksalary, very appropriately, started his academic career with a sabbatical leave in Tampere while accompanying his father. Antti Mikael Liski did almost the same thing while staying 19 months in Dortmund with his father in 1986–87.

The train trip from Berlin to Poznań in June 1984 was a pleasant experience for such youngsters as Erkki and I used to be those days; no worries made. Now that I'm writing this I wonder how Erkki and I will survive from the forthcoming train trip Warsaw–Poznań on Wednesday, 21 March 2007, arriving at 16:26 (EX 1811). Life is now very different and I doubt whether Barbara Bogacka will care to meet us anymore. Anyways, since we will be accompanied by Antti Liski (who was four

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\* Erkki P. Liski is Professor of Statistics, University of Tampere, Tampere, Finland. Ph.D. thesis in 1979 entitled "On reduced risk estimation in linear models" from the University of Tampere. Since 1969, worked at the Department of Mathematics, Statistics and Philosophy at the University of Tampere; Acting Professor 1988–95, Professor since 1996.

when his father first time took a train towards Poznań) and Kimmo Vehkalahti (who is from a big city of Helsinki), I trust we should make it to Poznań at 16:26 smoothly.

The Poznań Conference in June 1984 had a great social program, up to highest Polish standard. The highlight for Erkki and me was the Nordic-Polish Session at the apartment of Jan Hauke. That Session would be worth a separate Abstract and even *Proceedings*: memorable contributions!

It was mentioned above that life is now different from what it used to be in old (good—this adjective is frequently used by serious authors) times. You want hear examples: In 1975, on the Midsummer Eve, Erkki beat me by ca. 3 minutes. This was *Poronkusema Run* in Kemijärvi, Lapland. My time was 1.16.17 while Erkki made it in 1.13.57. The running distance in *Poronkusema* is the distance that a reindeer is capable to do without replying his/her number 1 call of nature. —Now you see that I'm not joking when saying that times are different. However, some things do not change; to give an example, when the Statistics Professorship was open at our department in 1995, 20 years after the *Poronkusema*, Erkki beat me by ca. three papers. (Now you see that I can make jokes about serious matters too.)

My cooperation with Erkki has not always been pure celebration. Here comes an example. On 25th of October 1985, Erkki was invited to meet Maurits Dekker, Chairman of Marcel Dekker Inc., in New York. However, we ended up going to the office of Maurits Dekker together. Mr. Dekker, 85, warmly welcomed us with his young female secretary, Vicky. But immediately, when recognizing me, Mr. Dekker announced his deep disappointment in seeing me with Erkki since he was fully prepared to meet a youngish female person. (My youngish outlooks were clearly not enough, once again.)

Another drawback we experienced in Manchester, UK, on 12 October 1988. Erkki and I had been visiting Bill and Sheila Farebrother and were about to take a train from Manchester to Sheffield. At the Manchester Train Station's ticket office Erkki politely asked for two tickets to Manchester. The lady of the office in her excellent English replied that she cannot, unfortunately, sell the tickets asked for. Erkki and I looked at each other and felt so disappointed with the rather limited service of the British Railways.

As is mentioned earlier, Erkki was an exceptionally talented runner (I was not so talented and hence I had to train like crazy.) I well

remember how in August 1984, we went to the 7th World Congress in Applied Linguistics in Brussels. We stayed in Sheraton Hotel—“Erkki Liski stays always in Sheraton”, had the hotel replied to me a couple of weeks earlier when I told them that my colleague E.P. Liski has booked a room from them and I’d like to do the same—and we were watching TV when a 5,000 and 10,000 meter Finnish runner was caught using steroids in the Los Angeles Olympic games. Only a Finnish man (most of us has a kind of long-distance-running history behind) can understand what such an experience can internally mean.

Erkki Liski belongs to a particular group who together built up the Statistics Group in the University of Tampere. It all has its start in Professor Eino Haikala (1913–93) who was the first professor of statistics in Tampere. We had an enthusiastic group of youngsters, including Tarmo Pukkila and Pentti Huuhtanen. Later we have learnt to appreciate those days even more. We somehow grew together, learnt, pretty much on our own, the rules of academic life style, including the appreciation of international cooperation.

Poland is a very appropriate place to propose a toast to Erkki Liski. Many happy returns, Erkki!





Fig. 1. Erkki Liski, Antti Liski and C. Radhakrishna Rao; Tampere, August 1983.





**Fig. 2.** Leena, Anni, Antti and Erkki Liski, C. Radhakrishna Rao; Tampere, August 1983.







**Fig. 3.** Erkki Liski and Simo Puntanen in Kemijärvi, Midsummer Eve 1975.





**Fig. 4.** Erkki Liski, Radosław Kala, Hans Nyquist, Jerzy K. Baksalary; Poznań, June 1984.



**Fig. 5.** Jerzy K. Baksalary and Erkki Liski in Poznań, June 1984.





Fig. 6. Erkki Liski, Pentti Huuhtanen and Simo Puntanen; JFK Airport, New York, October 1985.



Fig. 7. Tarmo Pukkila, Sergio Koreisha, Pentti Huuhtanen and Erkki Liski; Nokia, July 1992.





**Fig. 8.** Paweł Pordzik, Erkki Liski and Augustyn Markiewicz; Tampere, December 1989.



**Fig. 9.** Tarmo Pukkila, Simo Puntanen, George P. H. Styan, Erkki Liski, Pentti Huuhtanen, Tapio Nummi, Lasse Koskinen; Tampere, May 2000.







**Fig. 10.** Augustyn Markiewicz, Erkki Liski and Friedrich Pukelsheim; IWMS-15, Uppsala, Sweden, 15 June 2006.



Part V

**List of Participants**



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