

MATERIALS AND MECHANICS OF BENDING, AND CONCRETE SLAB SYSTEMS

1-1 Concrete

1-2 The ACI Building Code

1-3 Cement and Water

1-4 Aggregates

1-5 Concrete Mixes

1-6 Concrete in
Compression

1-7 Concrete in Tension

1-8 Reinforcing Steel

1-9 Concrete Cover

1-10 Beams: Mechanics of
Bending Review

1-11 Concrete Slab Systems

1-12 Gravity Load Distribution
in Concrete Slab Systems

1-1 CONCRETE

Concrete, the most commonly used construction material worldwide, is a composite material that consists primarily of a mixture of cement and fine and coarse aggregates (sand, gravel, crushed rock, and/or other materials) to which water has been added as a necessary ingredient for the chemical reaction during the curing process. The bulk of the mixture consists of the fine and coarse aggregates. The resulting concrete strength and durability are a function of the proportions of the mix as well as other factors, such as the concrete placing, finishing, and curing history.

The compressive strength of concrete is relatively high. Yet, it is a relatively brittle material, the tensile strength of which is small compared with its compressive strength. Hence steel reinforcing rods (which have high tensile and compressive strength) are used in combination with the concrete; the steel will resist the tension and the concrete the compression. *Reinforced concrete* is the result of this combination of steel and concrete. In many instances, steel and concrete are positioned in members so that they both resist compression.

1-2 THE ACI BUILDING CODE

The design and construction of reinforced concrete buildings is controlled by the *Building Code Requirements for Structural Concrete* (ACI 318-14) of the American Concrete Institute (ACI) [1]. The use of the term *code* in this text refers to the ACI Code unless otherwise stipulated. The code is revised, updated, and currently currently reissued on a 3-year cycle. It has been incorporated into the building codes of almost all states and municipalities throughout the United States, however. When

so incorporated, it has official sanction, becomes a legal document, and is part of the law controlling reinforced concrete design and construction in a particular area.

1-3 CEMENT AND WATER

Structural concrete uses, almost exclusively, hydraulic cement. With this cement, water is necessary for the chemical reaction of *hydration*. In the process of hydration, the cement sets and bonds the fresh concrete into one mass. *Portland cement*, which originated in England, is undoubtedly the most common form of cement. Portland cement consists chiefly of calcium and aluminum silicates. The raw materials are limestones, which provide calcium oxide (CaO), and clays or shales, which furnish silicon dioxide (SiO₂) and aluminum oxide (Al₂O₃). Following processing, cement is marketed in bulk or in 94-lb (1-ft³) bags.

In fresh concrete, the ratio of the amount of water to the amount of cement, by weight, is termed the *water-cementitious material ratio*. This ratio can also be expressed in terms of gallons of water per bag of cement. For complete hydration of the cement in a mix, a water-cementitious material ratio of 0.35 to 0.40 (4 to 4½ gal of water/bag of cement) is required. To increase the *workability* of the concrete (the ease with which it can be mixed, handled, and placed), higher water-cementitious material ratios are normally used or superplasticizers are added to the mix.

1-4 AGGREGATES

In ordinary structural concretes, the aggregates occupy approximately 70% to 75% of the volume of the hardened mass. Gradation of aggregate size to produce

close packing is desirable because, in general, the more densely the aggregate can be packed, the better are the strength and durability because of the inter-locking of the aggregates.

Aggregates are typically classified as fine or coarse. *Fine aggregate* is generally sand and may be categorized as consisting of particles that will pass a No. 4 sieve (four openings per linear inch). *Coarse aggregate* consists of particles that would be retained on a No. 4 sieve. The maximum size of coarse aggregate in reinforced concrete is governed by various ACI Code requirements. These requirements are established primarily to ensure that the concrete can be placed with ease into the forms without any danger of jam-up between adjacent bars or between bars and the sides of the forms. Section 26.4.2.1 of ACI 318 states that maximum size of coarse aggregate should not exceed the least of the following:

- (i) one-fifth the narrowest dimension between sides of forms
- (ii) one-third the depth of slabs
- (iii) three-fourths the minimum specified clear spacing between individual reinforcing bars or wires, bundles of bars, prestressed reinforcement, individual tendons, bundled tendons, or ducts

Note that smaller aggregate sizes have a relatively larger total surface area and thus require more cement paste to coat the surfaces of the aggregate which results in higher drying shrinkage. In typical building construction, the nominal maximum coarse aggregate size of $\frac{3}{4}$ " is commonly specified, though maximum aggregate size of up to $1\frac{1}{2}$ " is also sometimes specified to achieve reduced shrinkage.

1-5 CONCRETE MIXES

There are numerous factors to consider when determining all of the components of a concrete mix. The most basic mix consists of water, cement, coarse aggregate, and fine aggregate (sand). Beyond these basic components, one or more chemical admixtures can be used depending on the required performance of the concrete. Since the admixtures are a proprietary product, the dosage is based on the manufacturer's recommendations. Admixtures are often dosed in units of fluid ounces per 100 pounds of cementitious material (cwt). The strength of the concrete could be adversely impacted by certain admixtures, and there could also be compatibility issues with multiple admixtures. To control these issues and to also ensure the performance of the concrete, concrete suppliers will typically go through the process of creating batches of concrete and testing them. Once a certain mix is validated, it can then be used in the future using the same components and proportions. A sample mix design is shown in the below table.

The most critical parameter in the design and selection of a concrete mix is the design strength f'_c which is

defined as the compressive strength of the concrete at 28 days after placement, but there are other factors to consider in order to satisfy the concrete durability requirements of the ACI Code. For each group of structural members, the exposure classes must first be assigned based on ACI 318–14, Table 19.3.1.1 depending on the degree of exposure of the concrete to freeze/thaw (F), sulfate (S), water (W), and corrosion (C). Once the exposure classes are known, ACI 318–14, Table 19.3.2.1 is used to determine the minimum design strength and water-cementitious material ratio. This table is also used to determine whether or not air entrainment is needed. If air entrainment is needed, then ACI 318–14, Table 19.3.3.1 is used. The amount of air entrainment needed is a function of the exposure class and the aggregate size. Air entraining typically ranges between 4.5% and 7%. The design data for a sample mix is shown below:

Sample Mix Design Data

Mix Design Number: **4001-A** (this number is specific to the concrete supplier)

Minimum compressive strength at 28 days

$$f'_c = 4000 \text{ psi}$$

Used for: Footings, piers, and foundation walls

Slump: 4 in.; 8 in. with HRWR¹

Air content: 6.0% +/- 1.5%

water-to-cementitious material ratio: 0.40

Materials	Weight per Cubic Yard
Cement, Type I/II, ASTM C150	540 lbs
Fly Ash, ASTM C618	95 lbs
Natural Sand, ASTM C33	1230 lbs
#1 and #2 crushed stone, ASTM C33	1780 lbs
Potable Water, ASTM C1602	255 lbs
Air Entrainment, ASTM C260	1.5 oz./cwt
Water Reducer, ASTM C494 Type A	3.0 oz./cwt
High-Range Water Reducer, ASTM C494 Type F*	7 oz. to 10 oz./cwt
Non-Chloride Accelerator, ASTM C494*	0 oz. to 26 oz./cwt

*Added at the site upon request

The most common admixtures are outlined as follows:

Air Entrainment

Air entrainment provides small air bubbles in the concrete to act as a buffer against volumetric changes during freeze/thaw cycles and thus makes the concrete more durable. It also makes the concrete more workable. A rule of thumb is that the compressive strength of concrete will decrease by 5% for every 1% increase in entrained air.

Water Reducers

When it is desired to make the concrete more workable, a common cost-effective solution is to simply add water, but this will have a negative impact on

¹High range water reducer (HRWR)

the strength of the concrete. Water-reducing admixtures will increase the slump and thus the workability of concrete while not adversely impacting the water-cementitious material ratio.

When a much higher degree of workability is needed, a superplasticizer (high-range water reducer) can be used which will yield slumps around 8 in. and higher. There are also products available that are called mid-range water reducers that provide a more moderate amount of slump increase.

Accelerators and Retarders

It is sometimes desired to decrease or increase the setting time of the concrete, and so an accelerator or retarder product could be used. Accelerators can be used to increase the rate at which the concrete reaches its design strength. This may be desired due to weather conditions or to meet a certain construction schedule. Calcium chloride is a common accelerator component, but it cannot be used in reinforced concrete since the chloride will promote corrosion in the steel reinforcement.

Set retarders are used to delay the chemical reaction with the cement that allows the concrete to set. This might be used to offset the effect of high temperatures since high temperatures can produce a faster setting time. They might also be used, for example, during a paving operation to allow more time between concrete batches and reduce the possibility of a cold joint in the *pavement*.

Corrosion-Inhibitor

Corrosion-inhibiting admixtures are used to deter corrosion of reinforcing steel in concrete that is exposed to water or salt. This will increase the durability and life span of the concrete structure.

Other chemical admixtures include shrinkage compensating and permeability reducing admixtures. For more information on chemical admixtures, the reader should refer to ACI 212.3R-16-Report on Chemical Admixtures for Concrete.

Concrete that is placed between 50° F and 85° F do not require any special mix design or temperature considerations. When concrete is placed during extreme weather temperatures outside of these range, special considerations for cold and hot weather concrete must be accounted for [13, 14].

When concrete is poured in cold temperatures without any precautionary measures taken, the concrete will freeze and thus destroy the bond between the concrete and the rebar. Pouring concrete at excessively high temperatures causes the concrete to set faster than anticipated, leading to the formation of cold joints. High temperatures also lead to uncontrolled and significant cracking of the concrete which negatively impacts strength and durability. Detailed

information on the temperature requirements for concrete can be found in ACI 305 - Guide to Hot Weather Concreting and ACI 306 - Guide to Cold Weather Concreting. Special mix considerations also apply to mass concrete pours, such as thick mat foundations, where the high temperatures generated from the hydration process, if not adequately controlled, can lead to cracking of the concrete.

1-6 CONCRETE IN COMPRESSION

The theory and techniques relative to the design and proportioning of concrete mixes, as well as the placing, finishing, and curing of concrete, are outside the scope of this book and are adequately discussed in many other publications [2–5]. Field testing, quality control, and inspection are also adequately covered elsewhere. This is not to imply that these are of less importance in overall concrete construction technology but only to reiterate that the objective of this book is to deal with the design and analysis of reinforced concrete members.

We are concerned primarily with how a reinforced concrete member behaves when subjected to load. It is generally accepted that the behavior of a reinforced concrete member under load depends on the stress-strain relationship of the materials, as well as the type of stress to which it is subjected. With concrete used principally in compression, the compressive stress-strain curve is of primary interest.

The compressive strength of concrete is denoted as f'_c and is assigned the unit of *pounds per square inch* (psi). This is the unit for f'_c used in the ACI Code equations. For calculations, f'_c is frequently used with the unit *kips per square inch* (ksi).

A test that has been standardized by the American Society for Testing and Materials (ASTM C39) [6] is used to determine the compressive strength (f'_c) of concrete. The test involves compression loading to failure of a specimen cylinder of concrete. The compressive strength so determined is the highest compressive stress to which the specimen is subjected. Note in Figure 1-1 that f'_c is not the stress that exists in the specimen at failure but that which occurs at a strain of about 0.002 (though the concrete strain at f'_c may vary between approximately 0.0015 and 0.0025). Currently, 28-day concrete strengths (f'_c) range from 2500 to upwards of 10,000 psi, with 3000 to 4000 psi being common for reinforced concrete structures and 5000 to 6000 psi being common for prestressed concrete members, and higher strengths used for columns in high-rise buildings. High strength concrete with 12,000 psi compressive strength was used recently for the columns, shear walls, and drilled caissons for a project at the Hudson Yards in New York City. For normal weight concrete, the compressive strength at 28 days is specified in the ACI Code

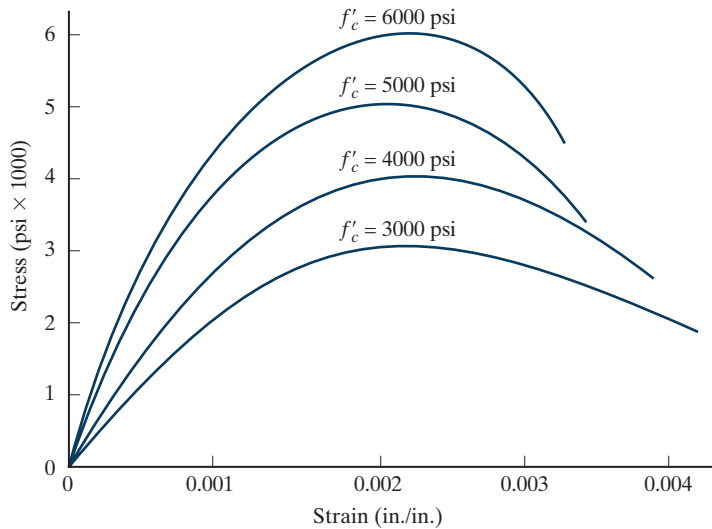


FIGURE 1-1 Typical stress–strain curves for concrete.

as the design strength. However, for higher strength concrete, the compressive strength at 56 or 90 days is commonly used [15]. The curves shown in Figure 1-1 represent the result of compression tests on 28-day standard cylinders for varying design mixes. The strain at rupture of concrete varies as indicated by the plots in Figure 1-1, but in developing the ACI Code equations for flexure, a limiting crushing strain of 0.003 is assumed (ACI 318-14, Section 22.2.2.1).

A review of the stress–strain curves for different-strength concretes reveals that the maximum compressive strength is generally achieved at a unit strain of approximately 0.002 in./in. Stress then decreases, accompanied by additional strain. Higher-strength concretes are more brittle and will fracture at a lower maximum strain than will the lower-strength concretes. The initial slope of the curve varies, unlike that of steel, and only approximates a straight line. For steel, where stresses are below the yield point and the material behaves elastically, the stress–strain plot will be a straight line. The slope of the straight line is the modulus of elasticity. For concrete, however, we observe that the straight-line portion of the plot is very short, if it exists at all. Therefore, there exists no constant value of modulus of elasticity for a given concrete because the stress–strain ratio is not constant. It may also be observed that the slope of the initial portion of the curve (if it approximates a straight line) varies with concretes of different strengths. Even if we assume a straight-line portion, the modulus of elasticity is different for concretes of different strengths. At low and moderate stresses (up to about $0.5f'_c$), concrete is commonly assumed to behave elastically.

The ACI Code, Section 19.2.2, provides the accepted empirical expression for *modulus of elasticity*:

$$E_c = w_c^{1.5} 33 \sqrt{f'_c}$$

where

E_c = modulus of elasticity of concrete in compression (psi)

w_c = unit weight of concrete (lb/ft³)

f'_c = compressive strength of concrete (psi)

This expression is valid for concretes having w_c between 90 and 160 lb/ft³. For normal-weight concrete, the unit weight w_c will vary with the mix proportions and with the character and size of the aggregates. If the unit weight is taken as 144 lb/ft³, the resulting expression for modulus of elasticity is

$$E_c = 57,000 \sqrt{f'_c} \quad (\text{see Table A-6 for values of } E_c)$$

It should also be noted that the stress–strain curve for the same-strength concrete may be of different shapes if the condition of loading varies appreciably. With different *rates of strain* (loading), we will have different-shape curves. Generally, the maximum strength of a given concrete is smaller at slower rates of strain.

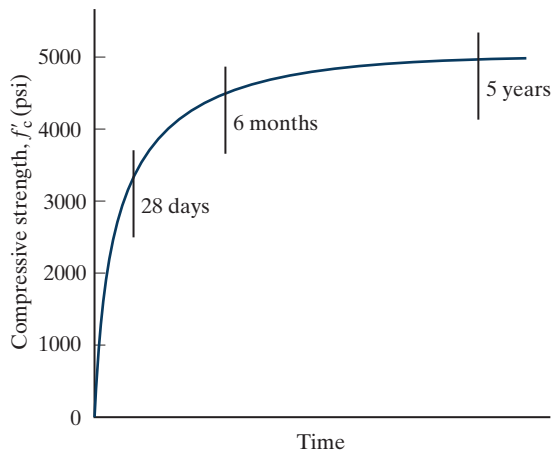


FIGURE 1-2 Strength–time relationship for concrete.

Concrete strength varies with time, and the specified concrete strength is usually that strength that occurs 28 days after the placing of concrete. A typical strength–time curve for normal stone concrete is shown in Figure 1-2. Generally, concrete attains approximately 70% of its 28-day strength in 7 days and approximately 85% to 90% in 14 days.

Concrete, under load, exhibits a phenomenon termed *creep*. This is the property by which concrete continues to deform (or strain) over long periods of time while under constant load. Creep occurs at a decreasing rate over a period of time and may cease after several years. Generally, high-strength concretes exhibit less creep than do lower-strength concretes. The magnitude of the creep deformations is proportional to the magnitude of the applied load as well as to the length of time of load application.

1-7 CONCRETE IN TENSION

The tensile and compressive strengths of concrete are not proportional, and an increase in compressive strength is accompanied by an appreciably smaller percentage increase in tensile strength. According to the ACI Code Commentary, the tensile strength of normal-weight concrete in flexure is about 10% to 15% of the compressive strength.

The true tensile strength of concrete is difficult to determine. The *split-cylinder test* (ASTM C496) [6] has been used to determine the tensile strength of lightweight aggregate concrete and is generally accepted as a good measure of the true tensile strength. The split-cylinder test uses a standard 6-in.-diameter, 12-in.-long cylinder placed on its side in a testing machine. A compressive line load is applied uniformly along the length of the cylinder, with support furnished along the full length of the bottom of the cylinder. The compressive load produces a transverse tensile stress, and the cylinder will split in half along a diameter when its tensile strength is reached.

The tensile stress at which splitting occurs is referred to as the *splitting tensile strength*, f_{ct} , and may be calculated by the following expression derived from the theory of elasticity:

$$f_{ct} = \frac{2P}{\pi LD}$$

where

f_{ct} = splitting tensile strength of lightweight aggregate concrete (psi)

P = applied load at splitting (lb)

L = length of cylinder (in.)

D = diameter of cylinder (in.)

Another common approach has been to use the *modulus of rupture*, f_r (which is the maximum tensile

bending stress in a plain concrete test beam at failure), as a measure of tensile strength (ASTM C78) [6]. The moment that produces a tensile stress just equal to the modulus of rupture is termed the *cracking moment*, M_{cr} , and may be calculated using methods discussed in Section 1-9. The ACI Code recommends that the modulus of rupture f_r be taken as $7.5\lambda\sqrt{f'_c}$, where f'_c is in psi (ACI 318 Equation 19.2.3.1). Greek lowercase lambda (λ) is a modification factor reflecting the lower tensile strength of lightweight concrete relative to normal-weight concrete. The values for λ are as follows:

Normal-weight concrete—1.0

Sand-lightweight concrete—0.85

All-lightweight concrete—0.75

Interpolation between these values is permitted. See ACI Code Table 19.2.4.2 for details. If the average splitting tensile strength f_{ct} is specified, then $\lambda = f_{ct}/(6.7\sqrt{f_{cm}}) \leq 1.0$, where f_{cm} is the average measured compressive strength, in psi.

1-8 REINFORCING STEEL

Concrete cannot withstand very much tensile stress without cracking; therefore, tensile reinforcement must be embedded in the concrete to overcome this deficiency. In the United States, this reinforcement is in the form of steel reinforcing bars or welded wire reinforcing composed of steel wire. In addition, reinforcing in the form of structural steel shapes, steel pipe, steel tubing, and high-strength steel tendons is permitted by the ACI Code. Many other approaches have been taken in the search for an economical reinforcement for concrete. Principal among these are the fiber-reinforced concretes, where the reinforcement is obtained through the use of short fibers of steel or other materials, such as fiberglass. For the purpose of this book, our discussion will primarily include steel reinforcing bars and welded wire reinforcing. High-strength steel tendons are used mainly in prestressed concrete construction (see Chapter 11).

The specifications for steel reinforcement published by the ASTM are generally accepted for the steel used in reinforced concrete construction in the United States and are identified in ACI 318 Section 20.2.

The ACI Code states that reinforcing bars should be secure and in place prior to the placement of concrete, thus the practice of *wet-setting* of rebar is not permitted. While it is not uncommon in practice to see some contractors wanting to place rebar in wet concrete, this should not be permitted because the rebar displaces the aggregates, and proper bond between the concrete and rebar cannot easily be assured.

The steel bars used for reinforcing are, almost exclusively, round deformed bars with some form of patterned ribbed projections rolled onto their surfaces. The patterns vary depending on the producer, but

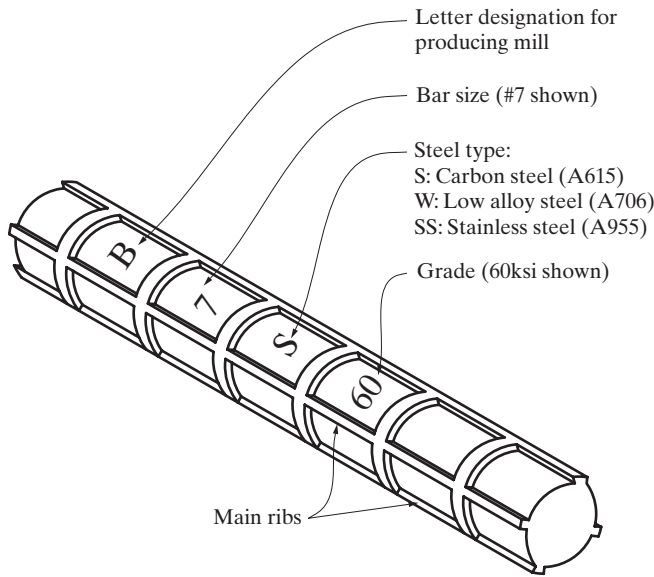


FIGURE 1-3 Rebar marks and designation.

all patterns should conform to ASTM specifications. Steel reinforcing bars are readily available in straight lengths of 60 ft. Smaller sizes are also available in coil stock for use in automatic bending machines. The bars vary in designation from No. 3 through No. 11, with two additional bars, No. 14 and No. 18. See Figure 1-3 for a sample rebar designation.

For bars No. 3 through No. 8, the designation represents the bar diameter in eighths of an inch. The No. 9, No. 10, and No. 11 bars have diameters that provide areas equal to 1-in.-square bars, 1 1/8-in.-square bars, and 1 1/4-in.-square bars, respectively. The No. 14 and No. 18 bars correspond to or have the same cross-sectional areas as 1 1/2-in.-square bars and 2-in.-square bars, respectively, and are commonly available only by special order. Round, plain reinforcing bars are permitted for spirals (lateral reinforcing) in concrete compression members.

ASTM specifications require that identification marks be rolled onto the bar to provide the following information: a letter or symbol indicating the producer's

mill, a number indicating the size of the bar, a symbol or letter indicating the type of steel from which the bar was rolled, and for grade 60 bars, either the number 60 or a single continuous longitudinal line (called a *grade line*) through at least five deformation spaces. The *grade* indicates the minimum specified yield stress in ksi. For instance, a grade 60 steel bar has a minimum specified yield stress of 60 ksi. No symbol indicating grade is rolled onto grade 40 or 50 steel bars (see Figure 1-3). Grade 75 bars can have either two grade lines through at least five deformation spaces or the grade mark 75. Reference [7] is an excellent resource covering the various aspects of bar identification.

Reinforcing bars are usually made from newly manufactured steel (billet steel). Steel types and ASTM specification numbers for bars are tabulated in Table A-1. Note that ASTM A615, which is billet steel, is available in grades 40, 60, 75, and 80. Grade 80 steel is allowed for non-seismic applications per ASTM 615 and ASTM 706 [8]. (The full range of bar sizes is not available in grades 40, 75, and 80, however.) Grade 75 steel is approximately 20% stronger than grade 60 steel, requiring a corresponding reduction in the required area of reinforcement, though the installed cost of grade 75 steel reinforcement is slightly higher than the cost for grade 60 steel. ASTM A706, low-alloy steel, which was developed to satisfy the requirement for reinforcing bars with controlled tensile properties and controlled chemical composition for weldability, is available in only one grade. Tables A-2 and A-3 contain useful information on cross-sectional areas of bars.

The most useful physical properties of reinforcing steel for reinforced concrete design calculations are yield stress (f_y) and modulus of elasticity. A typical stress–strain diagram for reinforcing steel is shown in Figure 1-4a. The idealized stress–strain diagram of Figure 1-4b is discussed in Chapter 2.

The yield stress (or yield point) of steel is determined through procedures governed by ASTM standards. For practical purposes, the yield stress may be thought of as that stress at which the steel

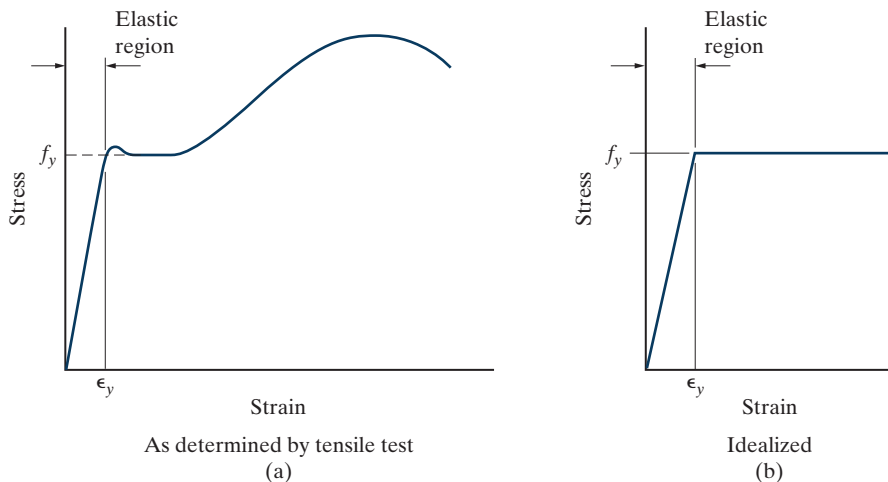


FIGURE 1-4 Stress–strain diagram for reinforcing steel.

exhibits increasing strain with no increase in stress. For reinforcement without a sharply defined yield point, ACI 318-14, similar to the ASTM standards, defines the yield strength as the 0.2% proof stress (that is, the offset stress at a 0.2% strain). The yield stress of the steel will usually be one of the known (or given) quantities in a reinforced concrete design or analysis problem. See Table A-1 for the range of f_y .

The modulus of elasticity of carbon reinforcing steel (the slope of the stress–strain curve in the elastic region) varies over a very small range and has been adopted as 29,000,000 psi (ACI Code Section 20.2.2.2).

Unhindered corrosion of reinforcing steel will lead to cracking and spalling of the concrete in which it is embedded. Quality concrete, under normal conditions, provides good protection against corrosion for steel embedded in the concrete with adequate cover (minimum requirements are discussed in Chapter 2). This protection is attributed to, among other factors, the high alkalinity of the concrete. Where reinforced concrete structures (or parts of structures) are subjected to corrosive conditions, however, some type of corrosion protection system should be used to prevent deterioration. Examples of such structures are bridge decks, parking garage decks, wastewater treatment plants, and industrial and chemical processing facilities.

One method used to minimize the corrosion of the reinforcing steel is to coat the bars with a suitable protective coating. The protective coating can be a nonmetallic material such as epoxy or a metallic material such as zinc (galvanizing). The ACI Code requires epoxy-coated reinforcing bars to comply with ASTM A775 or ASTM A934 and galvanized (zinc-coated) bars to comply with ASTM A767 (ACI 318 Section 20.6.2). The bars to be epoxy-coated or zinc-coated (galvanized) must meet the code requirements for uncoated bars as tabulated in Table A-1.

Welded wire reinforcing (WWR) (commonly called *mesh*) is another type of reinforcement. It consists of cold-drawn wire in orthogonal patterns, square or rectangular, resistance welded at all intersections. It may be supplied in either rolls or sheets, depending on

wire size. WWR with wire diameters larger than about $\frac{1}{4}$ in. is usually available only in sheets.

Both plain and deformed WWR products are available. According to ACI 318 Section 20.2.1.7, deformed wire, plain wire, welded deformed wire reinforcement, and welded plain wire reinforcement shall conform to ASTM A 1064 for carbon steel and ASTM A 1022 for stainless steel. Depending on the application, both materials have the maximum f_y permitted for design that varies from 60,000 psi to 100,000 psi (see ACI 318 Tables 20.2.2.4a and 20.2.2.4b). The deformed wire is usually more expensive, but it can be expected to have an improved bond with the concrete.

A rational method of designating wire sizes to replace the formerly used gauge system has been adopted by the wire industry. Plain wires are described by the letter W followed by a number equal to 100 times the cross-sectional area of the wire in square inches. Deformed wire sizes are similarly described, but the letter D is used. Thus a W9 wire has an area of 0.090 in.² and a D8 wire has an area of 0.080 in.² A W8 wire has the same cross-sectional area as the D8 but is plain rather than deformed. Sizes between full numbers are given by decimals, such as W9.5.

Generally, the material is indicated by the symbol WWR, followed by spacings first of longitudinal wires, then of transverse wires, and last by the sizes of longitudinal and transverse wires. Thus WWR6 × 12-W16 × W8 indicates a plain WWR with 6-in. longitudinal spacing, 12-in. transverse spacing, and a cross-sectional area equal to 0.16 in.² for the longitudinal wires and 0.08 in.² for the transverse wires.

Additional information about WWR, as well as tables relating size number with wire diameter, area, and weight, may be obtained through the Wire Reinforcement Institute [9] or the Concrete Reinforcing Steel Institute [9 and 10]. Table 1-1 contains common WWR sizes with the area of steel in in.²/ft width.

Most concrete is reinforced in some way to resist tensile forces (Figure 1-5). Some structural elements, particularly footings, are sometimes made of *plain concrete*, however. Plain concrete is defined as structural

TABLE 1-1 Welded Wire Reinforcement (Meets ASTM A185, $f_y = 60$ ksi)

Wire gauge	W-number	Area of steel in. ² /ft (each direction)	Wire diameter (in.)
6 × 6-10 × 10	6 × 6-W1.4 × W1.4	0.029	0.135 (10ga)
6 × 6-8 × 8	6 × 6-W2.0 × W2.0	0.041	0.162 (8ga)
6 × 6-6 × 6	6 × 6-W2.9 × W2.9	0.058	0.192 (6ga)
6 × 6-4 × 4	6 × 6-W4.0 × W4.0	0.080	0.225 (4ga)
4 × 4-10 × 10	4 × 4-W1.4 × W1.4	0.043	0.135 (10ga)
4 × 4-8 × 8	4 × 4-W2.0 × W2.0	0.062	0.162 (8ga)
4 × 4-6 × 6	4 × 4-W2.9 × W2.9	0.087	0.192 (6ga)
4 × 4-4 × 4	4 × 4-W4.0 × W4.0	0.120	0.225 (4ga)



FIGURE 1-5 Concrete construction in progress. Note formwork, reinforcing bars, and pumping of concrete.

(George Limbrunner)

concrete with no reinforcement or with less reinforcement than the minimum amount specified for reinforced concrete. Plain concrete is discussed further in Chapter 10.

1-9 CONCRETE COVER

The clear distance between the concrete surface and the face of the rebar is called the cover. The reinforcement in concrete needs to have this cover for several reasons:

- To protect the reinforcement against corrosion
- To provide adequate surface area for bond between the concrete and the reinforcement
- To protect the reinforcement against loss of strength in a fire

For protection against corrosion and to ensure adequate bond between the reinforcement and the surrounding concrete, ACI 318-14 Table 20.6.1.3.1 specifies the concrete cover requirements for cast-in-place reinforced concrete members. For slabs and walls not exposed to weather or in contact with the ground, the minimum clear cover to the outermost reinforcement surface is $\frac{3}{4}$ " for No. 11 and smaller bars, and 1.5" for No. 14 and larger bars. For beams and columns not exposed to weather or in contact with the ground, the minimum clear cover to the outermost reinforcement surface is 1.5". For all structural elements cast against and permanently in contact with soil or ground (e.g., footings, caissons, grade beams,

foundation and basement walls), the specified concrete cover is 3". These members are typically placed without the use of forms. For concrete elements not cast against soil or ground, but exposed to the weather or in contact with ground, the specified concrete cover is 2" for No. 6 through No. 18 bars, and 1.5" for No. 5 and smaller bars. The surfaces of these members would typically be formed. The presence of forms allows for greater accuracy in establishing the proper clear cover distance.

The fire protection requirements of the Code may sometimes necessitate a higher concrete cover than the above-specified concrete covers, depending on the required fire rating. See Chapter 14 for discussions on concrete cover requirements as a function of the fire ratings.

1-10 BEAMS: MECHANICS OF BENDING REVIEW

The concept of bending stresses in homogeneous elastic beams is generally discussed at great length in all strength of materials textbooks and courses. Beams composed of material such as steel or timber are categorized as homogeneous, with each exhibiting elastic behavior up to some limiting point. Within the limits of elastic behavior, the internal bending stress distribution developed at any cross section is linear (straight line), varying from zero at the neutral axis to a maximum at the outer fibers.

The accepted expression for the maximum bending stress in a beam is termed the *flexure formula*,

$$f_b = \frac{Mc}{I}$$

where

f_b = calculated bending stress at the outer fiber of the cross section

M = the applied moment

c = distance from the neutral axis to the outside tension or compression fiber of the beam

I = moment of inertia of the cross section about the neutral axis

The flexure formula represents the relationship between bending stress, bending moment, and the geometric properties of the beam cross section. By rearranging the flexure formula, the maximum moment that may be applied to the beam cross section, called the *resisting moment*, M_R , may be found:

$$M_R = \frac{F_b I}{c}$$

where F_b = the allowable bending stress.

This procedure is straightforward for a beam of known cross section for which the moment of inertia can easily be found. For a reinforced concrete beam, however, the use of the flexure formula presents some complications, because the beam is not homogeneous and concrete does not behave elastically over its full range of strength. As a result, a somewhat different approach that uses the beam's internal bending stress distribution is recommended. This approach is termed the *internal couple method*.

Recall from strength of materials that a couple is a pure moment composed of two equal, opposite, and parallel forces separated by a distance called the *moment arm*, which is commonly denoted Z . In the internal couple method, the couple represents an internal resisting moment and is composed of a compressive force C above the neutral axis (assuming a single-span, simply supported beam that develops compressive stress above the neutral axis) and a parallel internal tensile force T below the neutral axis.

As with all couples, and because the forces acting on any cross section of the beam must be in equilibrium, C must equal T . The internal couple must be equal and opposite to the bending moment at the same location, which is computed from the external loads. It represents a couple developed by the bending action of the beam.

The internal couple method of determining beam strength is more general and may be applied to

homogeneous or nonhomogeneous beams having linear (straight-line) or nonlinear stress distributions. For reinforced concrete beams, it has the advantage of using the basic resistance pattern found in the beam.

The following three analysis examples dealing with plain (unreinforced) concrete beams provide an introduction to the internal couple method. Note that the unreinforced beams are considered homogeneous and elastic. This is valid if the moment is small and tensile bending stresses in the concrete are low (less than the tensile bending strength of the concrete) with no cracking of the concrete developing. For this condition, the entire beam cross section carries bending stresses. Therefore, the analysis for bending stresses in the uncracked beam can be based on the properties of the gross cross-sectional area using the elastic-based flexure formula. The use of the flexure formula is valid as long as the maximum tensile stress in the concrete does not exceed the modulus of rupture f_r . If a moment is applied that causes the maximum tensile stress just to reach the modulus of rupture, the cross section will be on the verge of cracking. This moment is called the *cracking moment*, M_{cr} .

These examples use both the internal couple approach and the flexure formula approach so that the results may be compared.

Example 1-1

A normal-weight plain concrete beam is 6 in. \times 12 in. in cross section, as shown in Figure 1-6. The beam is simply supported on a span of 4 ft and is subjected to a midspan concentrated load of 4500 lb. Assume $f'_c = 3000$ psi.

- Calculate the maximum concrete tensile stress using the internal couple method.
- Repeat part (a) using the flexure formula approach.
- Compare the maximum concrete tensile stress with the value for modulus of rupture f_r using the ACI-recommended value based on f'_c .

Solution:

Calculate the weight of the beam (weight per unit length):

$$\begin{aligned} \text{weight of beam} &= \text{volume per unit length} \times \text{unit weight} \\ &= \frac{6 \text{ in.} (12 \text{ in.})}{144 \text{ in.}^2/\text{ft}^2} (150 \text{ lb/ft}^3) \\ &= 75 \text{ lb/ft} \end{aligned}$$

Calculate the maximum applied moment:

$$\begin{aligned} M_{max} &= \frac{PL}{4} + \frac{wL^2}{8} \\ &= \frac{4500 \text{ lb}(4 \text{ ft})}{4} + \frac{75 \text{ lb/ft}(4 \text{ ft})^2}{8} \\ &= 4650 \text{ ft}\cdot\text{lb} \end{aligned}$$

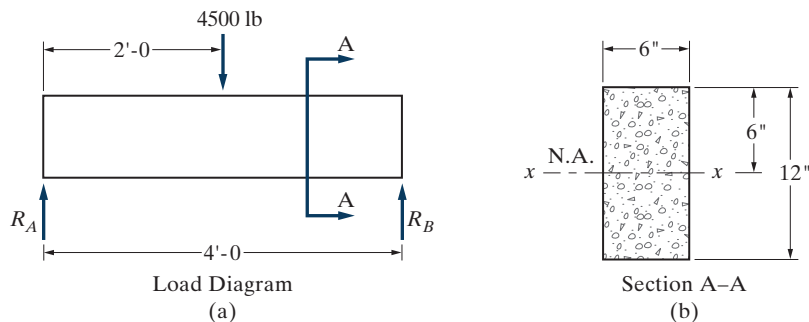


FIGURE 1-6 Loading diagram and section for Example 1-1.

a. Internal couple method

1. Because the beam is homogeneous, elastic, and symmetrical with respect to both the X-X and Y-Y axes, the neutral axis (N.A.) is at midheight. Stresses and strains vary linearly from zero at the neutral axis (which is also the centroidal axis) to a maximum at the outer fiber. As the member is subjected to positive moment, the area above the N.A. is stressed in compression and the area below the N.A. is stressed in tension. These stresses result from the bending behavior of the member and are shown in Figure 1-7.
2. C represents the resultant compressive force above the N.A. T represents the resultant tensile force below the N.A. C and T each act at the centroid of their respective triangles of stress distribution. Therefore, $Z = 8$ in. C and T must be equal (since $\Sigma H_F = 0$). The two forces act together to form the internal couple (or internal resisting moment) of magnitude CZ or TZ .
3. The internal resisting moment must equal the bending moment due to external loads at any section. Therefore,

$$M = CZ = TZ$$

$$4650 \text{ ft-lb (12 in./ft)} = C (8 \text{ in.})$$

from which

$$C = 6975 \text{ lb} = T$$

4. $C = \text{average stress} \times \text{area of beam on which stress acts}$

$$C = \frac{1}{2} f_{\text{top}}(y_{\text{top}})(b) = \frac{1}{2} f_{\text{top}}(6 \text{ in.})(6 \text{ in.}) = 6975 \text{ lb}$$

Solving for f_{top} yields

$$f_{\text{top}} = 388 \text{ psi} = f_{\text{bott}}$$

The modulus of elasticity of the concrete,

$$E_c = 57,000 \sqrt{f'_c} = 57,000 \sqrt{3000 \text{ psi}}$$

$$= 3,122,019 \text{ psi}$$

The concrete strain at the top of the beam is

$$\epsilon_{\text{top}} = \frac{f_{\text{top}}}{E_c} = \frac{388 \text{ psi}}{3,122,019 \text{ psi}} = 0.000124 \text{ in./in.}$$

b. Flexure formula approach

$$I = \frac{bh^3}{12} = \frac{6(12^3)}{12} = 864 \text{ in.}^4$$

$$f_{\text{top}} = f_{\text{bott}} = \frac{Mc}{I} = \frac{4650(12)(6)}{864} = 388 \text{ psi}$$

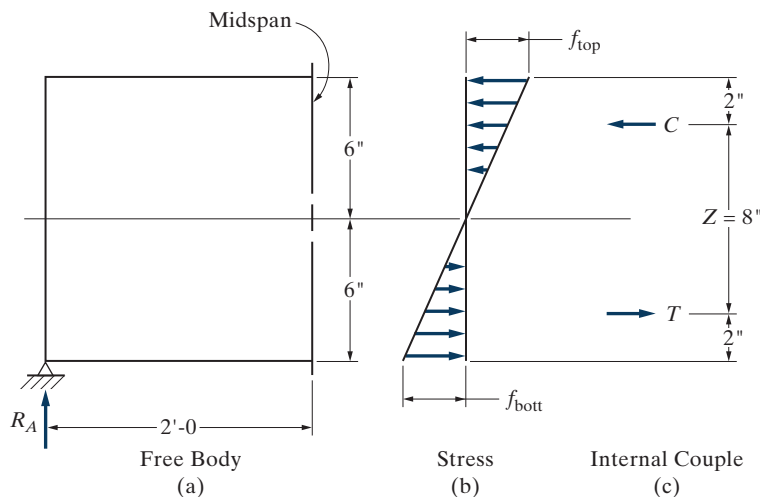


FIGURE 1-7 Stress and internal couple diagram for Example 1-1.

- c. The ACI-recommended value for the modulus of rupture (based on f'_c) is

$$f_r = 7.5\lambda \sqrt{f'_c} = 7.5(1.0)\sqrt{3000}$$

$$f_r = 411 \text{ psi}$$

The calculated tensile stress (f_{bot}) of 388 psi is about 6% below the modulus of rupture, the stress at which flexural cracking would be expected.

Example 1-1 is based on elastic theory and assumes the following: (1) a plane section before bending remains a plane section after bending (the variation in strain throughout the depth of the member is linear from zero at the neutral axis), and (2) the modulus of elasticity is constant; therefore, stress is proportional to strain and the stress distribution throughout the depth of the beam is also linear from zero at the neutral axis to a maximum at the outer fibers.

The internal couple approach may also be used to find the moment strength (resisting moment) of a beam.

Example 1-2

Calculate the cracking moment M_{cr} for the plain concrete beam shown in Figure 1-8. Assume normal-weight concrete and $f'_c = 4000$ psi.

- Use the internal couple method.
- Check using the flexure formula.

Solution:

The moment that produces a tensile stress just equal to the modulus of rupture f_r is called the cracking moment, M_{cr} . The modulus of rupture for normal-weight concrete is calculated from ACI Equation 19.2.3.1:

$$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{4000} = 474 \text{ psi}$$

For convenience, we will use force units of kips (1 kip = 1000 lb). Therefore, $f_r = 0.474$ ksi.

- Using the internal couple method

$$Z = 14 - 2(2.33) = 9.34 \text{ in.}$$

$$C = T = \frac{1}{2}(0.474)(8)(7) = 13.27 \text{ kips}$$

$$M_{cr} = CZ = TZ = \frac{13.27(9.34)}{12} = 10.33 \text{ ft.-kips}$$

- Check using the flexure formula

$$f = \frac{Mc}{I}$$

$$M_R = M_{cr} = \frac{f_r I}{c}$$

$$I = \frac{bh^3}{12} = \frac{8(14)^3}{12} = 1829 \text{ in.}^4$$

$$M_{cr} = \frac{f_r I}{c} = \frac{0.474(1829)}{7(12)} = 10.32 \text{ ft.-kips}$$

The internal couple method may also be used to analyze irregularly shaped cross sections, although for homogeneous beams it is more cumbersome than the use of the flexure formula.

Example 1-3

Calculate the cracking moment (resisting moment) for the T-shaped unreinforced concrete beam shown in Figure 1-9. Use $f'_c = 4000$ psi. Assume positive moment (compression in the top). Use the internal couple method and check using the flexure formula.

Solution:

The neutral axis must be located so that the strain and stress diagrams may be defined. The location of the neutral axis with respect to the noted reference axis is calculated from

$$\bar{y} = \frac{\sum(Ay)}{\sum A}$$

$$= \frac{4(20)(22) + 5(20)(10)}{4(20) + 5(20)}$$

$$= 15.33 \text{ in.}$$

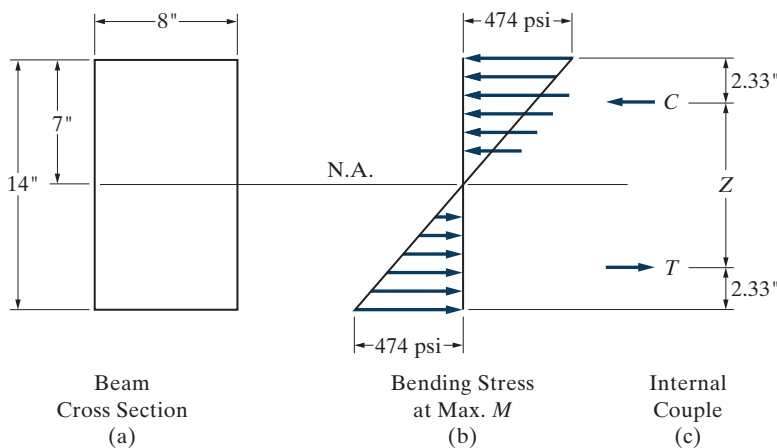


FIGURE 1-8 Sketch for Example 1-2.

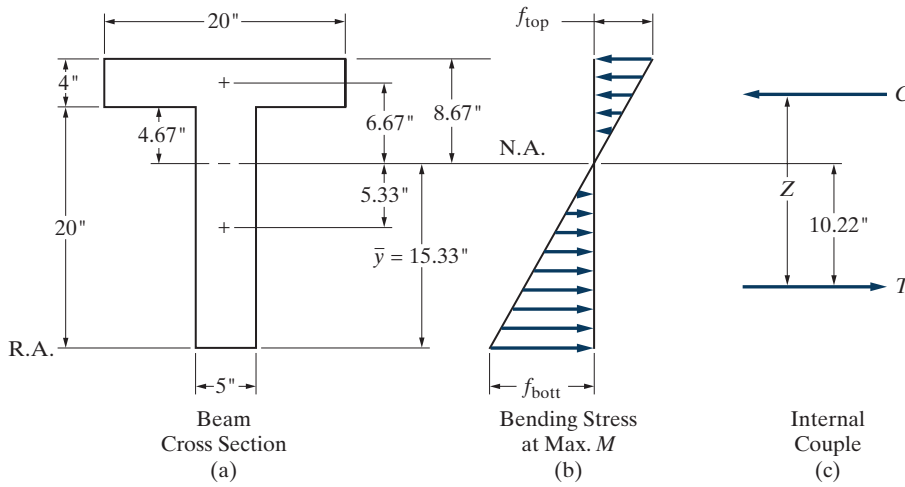


FIGURE 1-9 Sketch for Example 1-3.

The bottom of the cross section is stressed in tension. Note that the stress at the bottom will be numerically larger than at the top because of the relative distances from the N.A. The stress at the bottom of the cross section will be set equal to the modulus of rupture ($\lambda = 1.0$ for normal-weight concrete):

$$f_{bott} = f_r = 7.5\lambda\sqrt{f'_c} = 7.5(1.0)\sqrt{4000} = 474 \text{ psi} = 0.474 \text{ ksi}$$

Using similar triangles in Figure 1-9b, the stress at the top of the flange is

$$f_{top} = \frac{8.67}{15.33} (0.474) = 0.268 \text{ ksi}$$

The modulus of elasticity of the concrete,

$$E_c = 57,000\sqrt{f'_c} = 57,000\sqrt{4000} \text{ psi} = 3,605,000 \text{ psi}$$

The concrete strain at the top of the beam is

$$\epsilon_{top} = \frac{f_{top}}{E_c} = \frac{268 \text{ psi}}{3,605,000 \text{ psi}} = 0.000074 \text{ in./in.}$$

Similarly, the stress at the bottom of the flange is

$$f_{bott \text{ of flange}} = \frac{4.67}{15.33} (0.474) = 0.1444 \text{ ksi}$$

The total tensile force can be evaluated as follows:

$$T = \text{average stress} \times \text{area} \\ = \frac{1}{2}(0.474)(15.33)(5) = 18.17 \text{ kips}$$

and its location below the N.A. is calculated from

$$\frac{2}{3}(15.33) = 10.22 \text{ in. (below the N.A.)}$$

The compressive force will be broken up into components because of the irregular area, as shown in Figure 1-10. Referring to both Figures 1-9 and 1-10, the component internal compressive forces, component internal couples, and M_R may now be evaluated. The component forces are first calculated:

$$C_1 = 0.1444(20)(4) = 11.55 \text{ kips}$$

$$C_2 = \frac{1}{2} (0.1236)(20)(4) = 4.94 \text{ kips}$$

$$C_3 = \frac{1}{2} (0.1444)(5)(4.67) = 1.686 \text{ kips}$$

$$\text{total } C = C_1 + C_2 + C_3 = 18.18 \text{ kips}$$

$$C \approx T \text{ (O.K.)}$$

Next we calculate the moment arm distance from each component compressive force to the tensile force T :

$$Z_1 = 10.22 + 4.67 + \frac{1}{2} (4.00) = 16.89 \text{ in.}$$

$$Z_2 = 10.22 + 4.67 + \frac{2}{3} (4.00) = 17.56 \text{ in.}$$

$$Z_3 = 10.22 + \frac{2}{3} (4.67) = 13.33 \text{ in.}$$

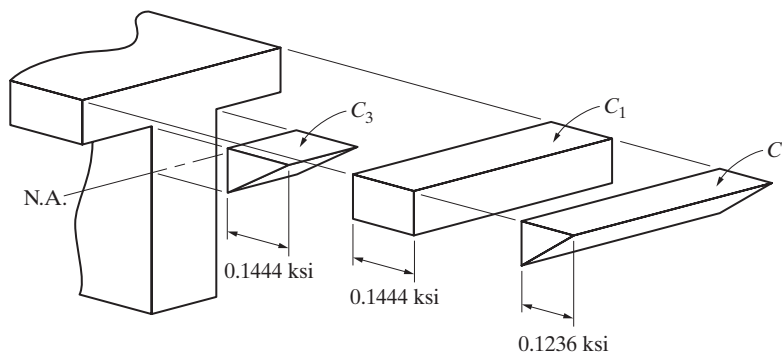


FIGURE 1-10 Component compression forces for Example 1-3.

The magnitudes of the component internal couples are then calculated from force \times moment arm as follows:

$$M_{R_1} = 11.55(16.89) = 195.1 \text{ in. -kips}$$

$$M_{R_2} = 4.94(17.56) = 86.7 \text{ in. -kips}$$

$$M_{R_3} = 1.686(13.33) = 22.5 \text{ in. -kips}$$

$$M_{cr} = M_R = M_{R_1} + M_{R_2} + M_{R_3} = 304 \text{ in. -kips}$$

Check using the flexure formula. The moment of inertia is calculated using the transfer formula for moment of inertia from statics:

$$I = \sum I_o + \sum Ad^2$$

$$I = \frac{1}{12}(20)(4^3) + \frac{1}{12}(5)(20^3) + 4(20)(6.67^2) + 5(20)(5.33^2)$$

$$= 9840 \text{ in.}^4$$

$$M_{cr} = M_R = \frac{f_r I}{c} = \frac{0.474(9840)}{15.33} = 304 \text{ in. -kips}$$

(Checks O.K.)

As mentioned previously, the three examples are for plain, unreinforced, and uncracked concrete beams that are considered homogeneous and elastic within the bending stress limit of the modulus of rupture. The internal couple method is also applicable to nonhomogeneous beams with nonlinear stress distributions of any shape, however. Because reinforced concrete beams are nonhomogeneous, the flexure formula is not directly applicable. Therefore, the basic approach used for reinforced concrete beams is the internal couple method (see Chapters 2 and 3).

1-11 CONCRETE SLAB SYSTEMS

The two types of floor systems used in reinforced and prestressed concrete structures are one way and two-way slab systems.

One-Way Slab Systems

One-way concrete floor systems are usually supported by stiff beams or walls and the slab spans or bends predominantly in one direction (usually in the shorter direction of the rectangular slab panel) and the load transfer to the members supporting the slab occurs predominantly in the shorter direction (see Figure 1-11). This is the case when the clear span of the longer side of the rectangular slab panel (ℓ_1) is greater than or equal to twice the clear span of the shorter side of the slab (ℓ_2); that is, $\ell_1/\ell_2 \geq 2$. In a reinforced or prestressed concrete one-way slab system, the curvature of the slab is predominantly in the shorter direction (i.e., parallel to the ℓ_2 dimension). The design of one-way reinforced concrete slab systems is covered in Chapter 2.

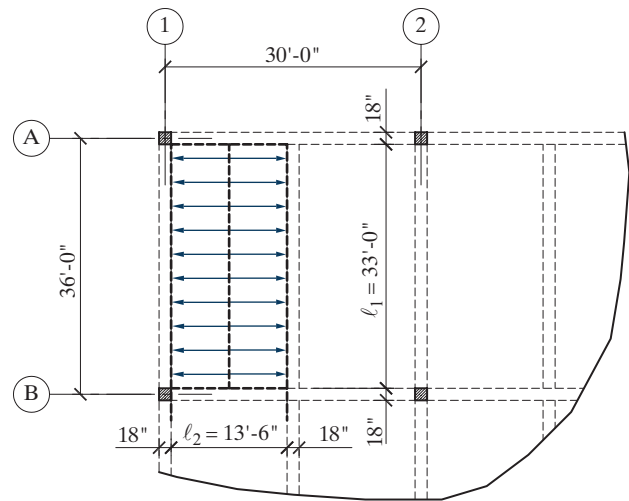


FIGURE 1-11 Load distribution in one-way slab systems.

Two-Way Slab Systems

Two-way concrete slab systems are supported on columns and span in two orthogonal directions, or the slab is supported on all four sides by beams or walls and the curvature of the slab, and the load transfer occurs in both orthogonal directions (see Figure 1-12). A two-way slab system occurs when the clear span of the longer side of the rectangular slab panel (ℓ_1) is less than twice the clear span of the shorter side of the slab panel (ℓ_2); that is, $\ell_1/\ell_2 < 2$. Examples of two-way slabs include flat plates, flat slabs, flat slabs with beams, and slabs supported on stiff beams or walls on all four sides of a rectangular slab panel. Note that ℓ_1 is the larger dimension of the rectangular slab panel bounded on all four sides by columns or beams or walls and ℓ_2 is the smaller dimension of the rectangular slab panel. Further treatment of two-way slab systems is covered in Chapter 6.

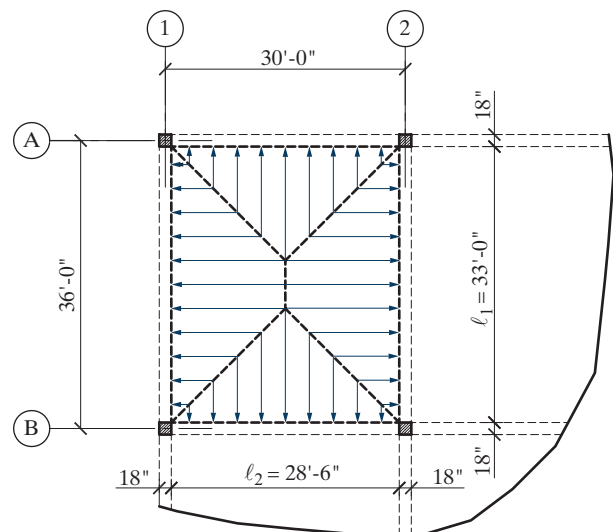


FIGURE 1-12 Load distribution in two-way slab systems.

1-12 GRAVITY LOAD DISTRIBUTION IN CONCRETE SLAB SYSTEMS

Reinforced concrete structures are subjected to a variety of loads which includes dead loads, floor live loads, roof live loads, snow loads, hydrostatic pressures, lateral soil pressures, wind loads, and seismic or earthquake loads. In this section, we introduce the reader to the distribution of gravity loads in concrete slab systems.

Gravity loads that act on structures include dead loads, floor live loads, snow loads, and roof live loads. In concrete floor and roof systems, the pertinent load combinations that usually govern for ultimate strength design are as follows:

Floor Slab:

$$1.4D$$

$$1.2D + 1.6L$$

Roof Slab:

$$1.4D$$

$$1.2D + 1.6(L_r \text{ or } S \text{ or } R) + 0.5W$$

$$1.2D + 1.0W + 0.5(L_r \text{ or } S \text{ or } R)$$

where,

- D = dead load
- L = floor live load
- L_r = roof live load
- S = snow load
- R = rain load
- W = wind load

When designing for serviceability limit state (e.g., deflections and vibrations), all the load factors in the above load combinations (i.e., the “1.2” and “1.6”) default to 1.0 since serviceability is checked at working or service loads. For a detailed treatment of the different structural loads and the load combinations prescribed in the Code, the reader should refer to References [11] and [12].

Dead Loads

The dead load is the weight of anything permanently attached to the structure, including the self-weight of the structure. Dead loads include the weight of the following:

- Floor finishes
- Partitions
- Mechanical and electrical equipment and conduits (M & E)

- Glazing
- Cladding

The weights of common building materials are given in Section C3 of ASCE 7-16. The density of unreinforced concrete is approximately 145 lb/ft³ and the density of reinforced concrete is approximately 150 lb/ft³.

A sample dead load calculation for a building is as follows:

Components of Roof Dead Load in a Reinforced Concrete Building:

$$5 \text{ ply} + \text{gravel} = 6.5 \text{ psf}$$

$$\text{Insulation and membrane} = 3.5 \text{ psf}$$

$$4'' \text{ concrete slab} \cong 50 \text{ psf}$$

$$(\text{slab weight} = 4''/12 \times 150 \text{ lb/ft}^3 = 50 \text{ psf})$$

$$\text{Mechanical/Electrical (M \& E)} \cong 10 \text{ psf (20 psf for industrial buildings)}$$

$$\text{Suspended ceiling} \cong 2 \text{ psf}$$

$$\text{Total roof dead load, } D \cong 72 \text{ psf (unfactored)}$$

Components of Floor Dead Load in a Reinforced Concrete Building:

$$6'' \text{ concrete slab}^1 = 75 \text{ psf}$$

$$\text{Floor finishes} \cong 8 \text{ psf (estimate weight of actual finishes specified by the architect)}$$

$$\text{M \& E} \cong 10 \text{ psf (20 psf for industrial building)}$$

$$\text{Suspended ceiling} \cong 2 \text{ psf}$$

$$\text{Partitions}^{2, 3} \cong 15 \text{ psf}$$

$$\text{Total floor dead load, } D = 115 \text{ psf (unfactored)}$$

Tributary Width and Areas

In this section, we introduce the concept of tributary area and tributary width. Beams in reinforced concrete floor or roof systems share the uniformly distributed loads in proportion to their distance from adjacent parallel beams. The tributary width (TW) of a beam is a measure of the total width of floor or roof supported by the beam. The tributary area (A_T) is the area of the floor supported by a structural element. For a beam, it is the product of

¹Weight of 6" slab = 6"/12 × 150 lb/ft³ = 75 psf

²The minimum value per ASCE 7-16, Section 4.3.2, is 15 psf. Actual weight of partitions may be higher.

³The Code allows partition loads to be neglected whenever the floor live load, L , is greater than or equal to 80 psf (ASCE 7-16, Section 4.3.2)

the tributary width and the span of the beam. For a column, it is the area bounded by lines mid-way to adjacent columns above, below, to the right, and to the left of the column under consideration. In a roof or floor system, the load path for gravity loads involves the beams supporting uniform loads from the floor and the beams transferring their reactions to the girders, which then in turn transfer their reactions to the columns.

Tributary width = Width of floor or roof supported
width (TW) by a **beam or girder**
= $(\frac{1}{2} \times \text{the distance to the adjacent beam on the right} + \frac{1}{2} \times \text{distance to the adjacent beam on the left})$

Tributary area = Area of roof or floor supported
(A_T) by a **beam, girder, or column**

The tributary area = TW of beam \times span of beam
of a beam, $A_{T(\text{beam})}$

The tributary area of a column,

$A_{T(\text{column})}$ = Floor or roof area supported by
the column
= the rectangular area bounded by
one-half the distance to adjacent
columns on all sides of the column
under consideration

Example 1-4

Load Calculations for One-Way Slabs and Supporting Beams

The elevation and section through a reinforced concrete building is shown in Figure 1-13. Determine the factored design loads for the floor slab, interior beam (A), and spandrel beam (B). Assume the following loads:

- Office building live load (i.e., floor live load = 50 psf)
- Block wall weighs 60 psf of vertical surface area
- Brick veneer weighs 40 psf of vertical surface area

Solution:

Floor Dead Load:

Finishes	8 psf
6" concrete slab	75 psf
(6"/12 \times 150 pcf)	
Suspended ceiling	= 2 psf
M & E	10 psf
Partitions	<u>20 psf</u>

Total dead load, \cong 115 psf

Live load, L = 50 psf (for office buildings)

Slab Loads:

$$\begin{aligned} \text{Factored load, } W_u &= 1.2 D + 1.6 L \\ &= 1.2 (115) + 1.6 (50) \\ &= \underline{218 \text{ psf}} \end{aligned}$$

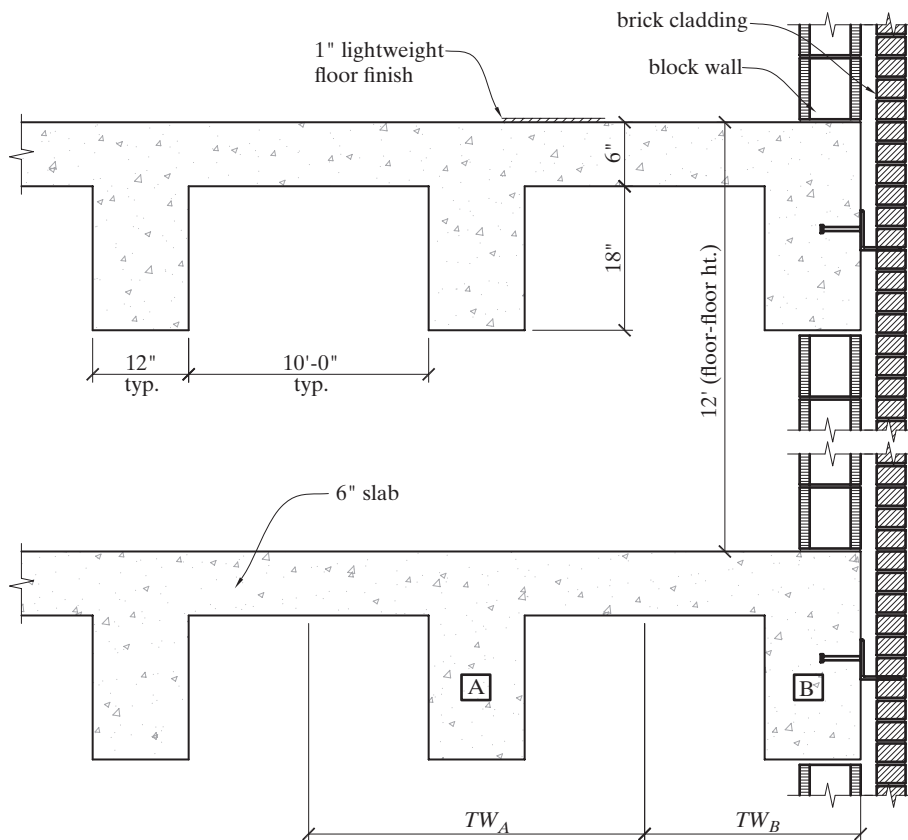


FIGURE 1-13 Building section for Example 1-4.

In the design of slabs, a 12-inch-wide strip is usually considered. Therefore, the load on the slab in pounds per linear foot (lb/ft) per foot width of slab is calculated as

$$w_u = (218 \text{ psf})(1 \text{ ft}) = 218 \text{ lb/ft per foot width of slab}$$

This load is used in the ultimate strength design of the slab.

$$\begin{aligned} \text{Service or unfactored load, } w_s &= 1.0 D + 1.0 L \\ &= 115 + 50 \\ &= 165 \text{ psf} \end{aligned}$$

$$w_s = (165 \text{ psf})(1 \text{ ft width}) = 165 \text{ lb/ft per foot width of slab}$$

This unfactored load will be used for the design of the slab for serviceability limit state such as deflections.

The loads to the interior and spandrel beams are based on the tributary widths of each beam as shown in Figure 1-14.

Typical Interior Beam Loads

$$\begin{aligned} \text{Tributary width of beam A, } TW_A &= \frac{10' - 0''}{2} + 1' - 0'' \\ &+ \frac{10' - 0''}{2} = 11' - 0'' \end{aligned}$$

$$\begin{aligned} \text{Unfactored weight of beam stem} &= \left(\frac{12''}{12}\right)\left(\frac{18''}{12}\right)(150 \text{ pcf}) \\ &= \mathbf{225 \text{ lb/ft}} \end{aligned}$$

$$\begin{aligned} \text{Factored load on beam A, } w_u &= w_u(\text{slab}) \times TW_A \\ &+ \text{factored dead weight of beam A stem} \\ &= (218 \text{ psf})(11') + (1.2)(225 \text{ lb/ft}) \\ &= 2668 \text{ lb/ft} \\ &= \mathbf{2.67 \text{ kip/ft}} \end{aligned}$$

$$\begin{aligned} \text{Service or unfactored load} &= w_s(\text{slab}) \times TW_A + \\ \text{on beam, } w_s &\quad \text{unfactored dead weight of} \\ &\quad \text{beam A stem} \\ &= (165 \text{ psf})(11') + 225 \text{ lb/ft} \\ &= 2040 \text{ lb/ft} \\ &= \mathbf{2.04 \text{ k/ft}} \end{aligned}$$

Spandrel Beam Loads

Tributary width of spandrel beam B,

$$TW_B = \frac{10' - 0''}{2} + 1' - 0'' = 6' - 0''$$

$$\begin{aligned} \text{Unfactored weight} &= \left(\frac{12''}{12}\right)\left(\frac{18''}{12}\right)(150 \text{ pcf}) = 225 \text{ lb/ft} \\ \text{of beam stem} & \end{aligned}$$

$$\text{Height of block back-up wall} = 12 \text{ ft} - 2 \text{ ft} = 10 \text{ ft}$$

$$\text{Height of exterior brick veneer/cladding} = 12 \text{ ft}$$

$$\begin{aligned} \text{Unfactored weight of exterior block and brick wall} \\ &= (60 \text{ psf})(10') + (40 \text{ psf})(12') \\ &= 1080 \text{ lb/ft} \end{aligned}$$

Factored load on beam,

$$\begin{aligned} w_u &= W_u, \text{ slab} \times TW_B + \text{factored weight of beam stem} \\ &\quad + \text{factored weight of wall} \\ &= (218 \text{ psf})(6') + (1.2)(225 \text{ lb/ft}) + (1.2)(1080 \text{ lb/ft}) \\ &= 2874 \text{ lb/ft} \\ &= \mathbf{2.87 \text{ kip/ft}} \end{aligned}$$

Service load on beam, w_s

$$\begin{aligned} w_{s, \text{slab}} \times TW_B &+ \text{unfactored weight of beam stem} \\ &+ \text{unfactored weight of wall} \\ &= (165 \text{ psf}) \times (6') + (225 \text{ lb/ft}) + (1080 \text{ lb/ft}) \\ &= 2295 \text{ lb/ft} \\ &= \mathbf{2.30 \text{ kip/ft}} \end{aligned}$$

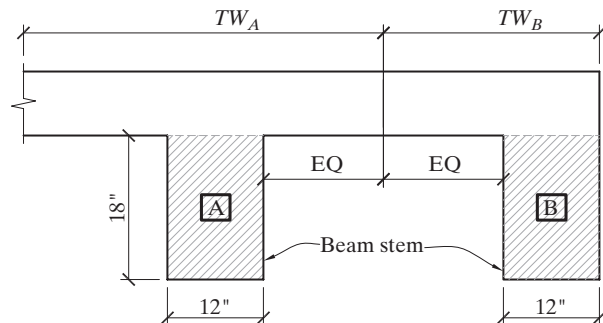


FIGURE 1-14 Tributary widths for Example 1-4.

References

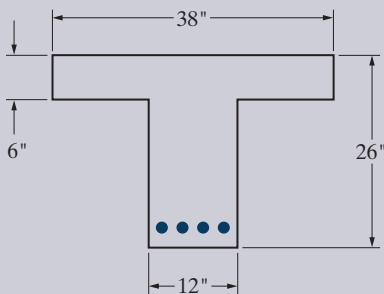
- [1] *Building Code Requirements for Structural Concrete* (ACI 318-14). American Concrete Institute, P.O. Box 9094, Farmington Hills, MI 48333-9094, 2014.
- [2] ACI Committee 211. *Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete* (ACI 211.1-91). American Concrete Institute, P.O. Box 9094, Farmington Hills, MI 48333-9094, 1991. (Reapproved 2002.)
- [3] George E. Troxell, Harmer E. Davis, and Joe W. Kelly. *Composition and Properties of Concrete*, 2nd ed. New York: McGraw-Hill Book Company, 1968.
- [4] Steven H. Kosmatka, Beatrix Kerkhoff, and William C. Panarese. *Design and Control of Concrete Mixtures*, 14th ed. Engineering Bulletin of the Portland Cement Association, 5420 Old Orchard Road, Skokie, IL 60077, 2002.
- [5] Joseph J. Waddell, ed. *Concrete Construction Handbook*, 3rd ed. New York: McGraw-Hill Book Company, 1993.
- [6] *ASTM Standards*. American Society for Testing and Materials, 100 Barr Harbor Drive, West Conshohocken, PA 19428-2959.
- [7] *Manual of Standard Practice*, 28th ed. Concrete Reinforcing Steel Institute, 933 North Plum Grove Road, Schaumburg, IL 60173, 2009.

- [8] Clifford W. Schwinger. "ASTM A615 Grade 75 Reinforcing Steel—When, Why & How to Use It," *STRUCTURE Magazine*, pp. 34–35, August 2011.
- [9] *Manual of Standard Practice (WWR-500-R-16)* Wire Reinforcement Institute, 942 Main Street, Suite 300, Hartford, CT 06103, 2016.
- [10] Concrete Reinforcing Steel Institute, 933 North Plum Grove Road, Schaumburg, IL 60173.
- [11] Abi Aghayere and Jason Vigil. *Structural Steel Design: A Practice-Oriented Approach*, 2nd ed. New York: Pearson, 2015.
- [12] David A. Fanella. *Structural Loads – 2012 IBC and ASCE/SEI 7-10*, International Code Council, 2012.
- [13] Cawsie Jijina and J. Benjamin Alper. "Cold and Hot Weather Concrete," *STRUCTURE Magazine*, pp. 14-16, September 2017.
- [14] S. H. Kosmatka and M. L. Wilson. "Design and Control of Concrete Mixtures," 16th Edition, Portland Cement Association (PCA), 2016.
- [15] Jeffrey Smilow and Ahmad Rahimian. "55 Hudson Yards," *STRUCTURE Magazine*, pp. 36–39, September 2017.

Problems

Note: In the following problems, assume plain concrete to have a weight of 145 pcf (conservative) unless otherwise noted.

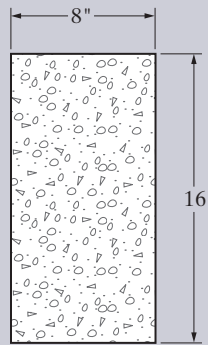
- 1-1. The unit weight of normal-weight reinforced concrete is commonly assumed to be 150 lb/ft³. Find the weight per lineal foot (lb/ft) for a normal-weight reinforced concrete beam that
- Has a rectangular cross section 16 in. wide and 28 in. deep.
 - Has a cross section as shown in the accompanying diagram.



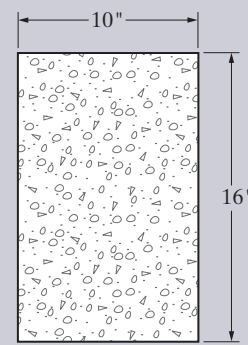
PROBLEM 1-1

- 1-2. Develop a spreadsheet application that will display in a table the values of modulus of elasticity E_c for concrete having unit weight ranging from 95 pcf to 155 pcf (in steps of 5 pcf) and compressive strength ranging from 3500 to 7000 psi (in steps of 500 psi). Display the modulus of elasticity rounded to the nearest 1000 psi.

- 1-3. A normal-weight concrete test beam 6 in. by 6 in. in cross section and supported on a simple span of 24 in. was loaded with a point load at midspan. The beam failed at a load of 2100 lb. Using this information, determine the modulus of rupture f_r of the concrete and compare with the ACI-recommended value based on an assumed concrete strength f'_c of 3000 psi.
- 1-4. A plain concrete beam has cross-sectional dimensions of 10 in. by 10 in. The concrete is known to have a modulus of rupture f_r of 350 psi. The beam spans between simple supports. Determine the span length at which this beam will fail due to its own weight. Assume a unit weight of 145 pcf.
- 1-5. The normal-weight plain concrete beam shown is on a simple span of 10 ft. It carries a dead load (which includes the weight of the beam) of 0.5 kip/ft. There is a concentrated load of 2 kips located at midspan. Use $f'_c = 4000$ psi. Compute the maximum bending stress. Use the internal couple method and check with the flexure formula.



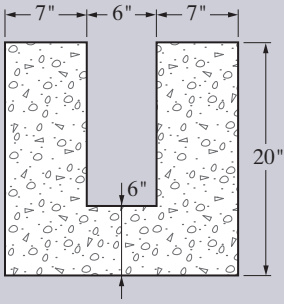
PROBLEM 1-5



PROBLEM 1-6

- 1-6. Calculate the cracking moment (resisting moment) for the unreinforced concrete beam shown. Assume normal-weight concrete with $f'_c = 3000$ psi. Use the internal couple method and check with the flexure formula.
- 1-7. Develop a spreadsheet application to solve Problem 1-6. Set up the spreadsheet so that a table will be generated in which the width of the beam varies from 8 in. to 16 in. (1-in. increments) and the depth varies from 12 in. to 24 in. (1-in. increments.) The spreadsheet should allow the user to input any value for f'_c between 3000 psi and 8000 psi.
- 1-8. Rework Example 1-3 but invert the beam so that the flange is on the bottom and the web extends vertically upward. Calculate the cracking moment using the internal couple method and check using the flexure formula. Assume positive moment.

1-9. Calculate the cracking moment (resisting moment) for the U-shaped unreinforced concrete beam shown. Assume normal-weight concrete with $f'_c = 3500$ psi. Use the internal couple method and check with the flexure formula. Assume positive moment.



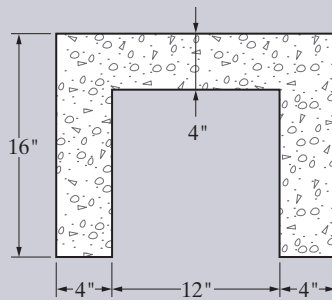
PROBLEM 1-9

1-10. The plain concrete beam shown is used on a 12-ft simple span. The concrete is normal weight with $f'_c = 3000$ psi. Assume positive moment.

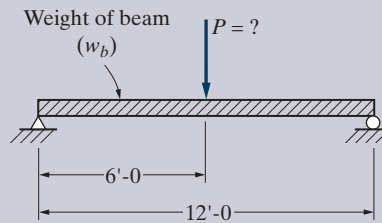
- a. Calculate the cracking moment.
- b. Calculate the value of the concentrated load P at midspan that would cause the concrete beam to crack. (Be sure to include the weight of the beam.)

1-11. Using the section below, determine the following:

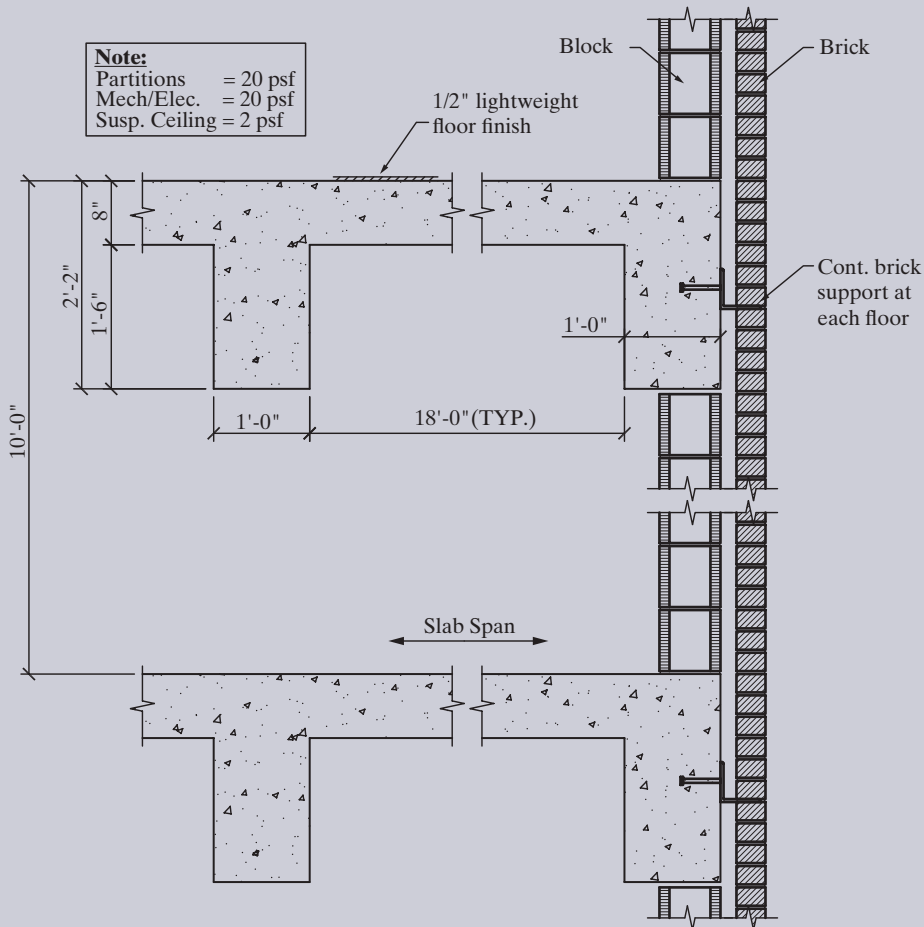
- a. The service dead load in psf supported by the slab.
- b. The factored total uniform load in psf supported by the slab. Assume a floor live load of 100 psf.



PROBLEM 1-10



Note:
 Partitions = 20 psf
 Mech/Elec. = 20 psf
 Susp. Ceiling = 2 psf

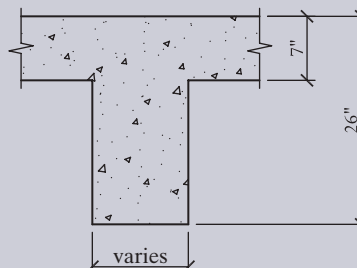
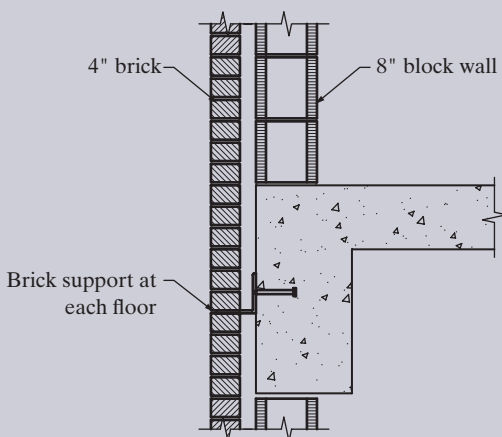
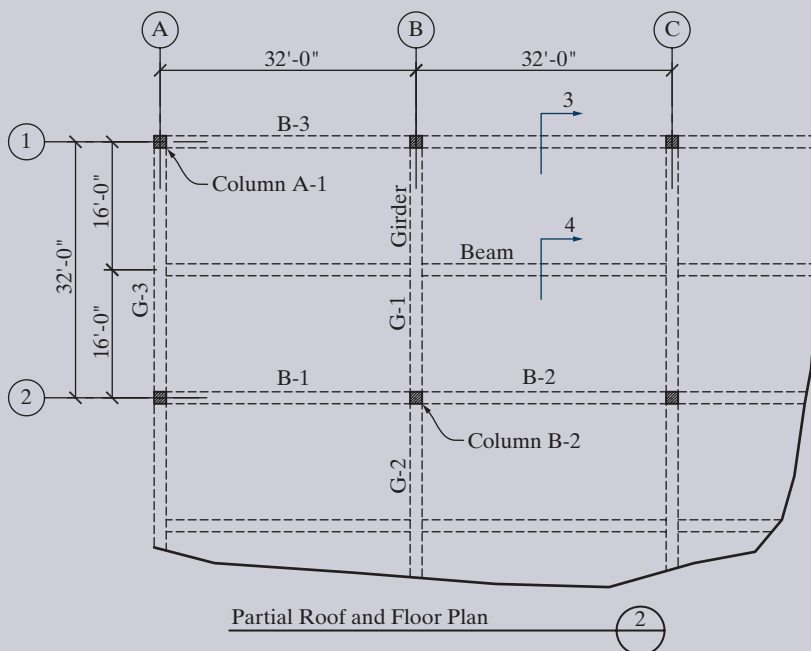


PROBLEM 1-11

- c. The service dead and service live load in kip/ft supported by the spandrel or edge beam. Assume the block backup wall weighs 55 psf and the brick cladding weighs 40 psf of the vertical surface are of the wall.
- d. The factored total uniform load in kip/ft supported by the spandrel beam.
- e. The factored total uniform load in kip/ft supported by a typical interior beam.

- 7" thick slab on the roof and floors
- 5-ply roofing plus gravel on the roof slab plus 3.5 psf for insulation
- 1/2 in. lightweight finish on the floor slabs
- 20 psf for the partitions
- 10 psf for mechanical and electrical equipment
- 2 psf for the suspended ceiling
- Exterior cladding:
 - 55 psf backup block wall—7 ft 10 in. high
 - 40 psf brick veneer—10 ft high
- Beams: 16" wide × 26" deep; Girders: 18" wide × 26" deep
- Ignore live load reduction

1-12. A partial floor plan for a three-story reinforced concrete office building with 10 ft floor-to-floor heights, located in Rochester, New York, is shown in the figure below. Assume the following:



Determine the following:

- a. The factored total uniform load in psf on the roof and floor slabs
- b. The factored total uniform load in kip/ft supported by the
 - i. Typical interior roof beam
 - ii. Typical interior floor beam
 - iii. Typical roof spandrel beam
 - iv. Typical floor spandrel beam
- c. The factored concentrated load from the beam reactions in kip on the following girders and the factored self-weight of the girder stem in kip/ft. For the purposes of calculating the beam reactions, assume the beams are simply supported:
 - i. Typical interior roof girder
 - ii. Typical interior floor girder
 - iii. Typical roof spandrel girder
 - iv. Typical floor spandrel girder
- d. The total factored load and the total service load in kip on Column B-2. Assume $16'' \times 16''$ columns. For the purposes of calculating the beam and girder reactions, assume the beams and girders are simply supported. Present your results in a tabular format.
- e. The total factored load and the total service load in kip on Column A-1. Assume $16'' \times 16''$ columns. For the purposes of calculating the beam and girder reactions, assume the beams and girders are simply supported. Present your results in a tabular format.