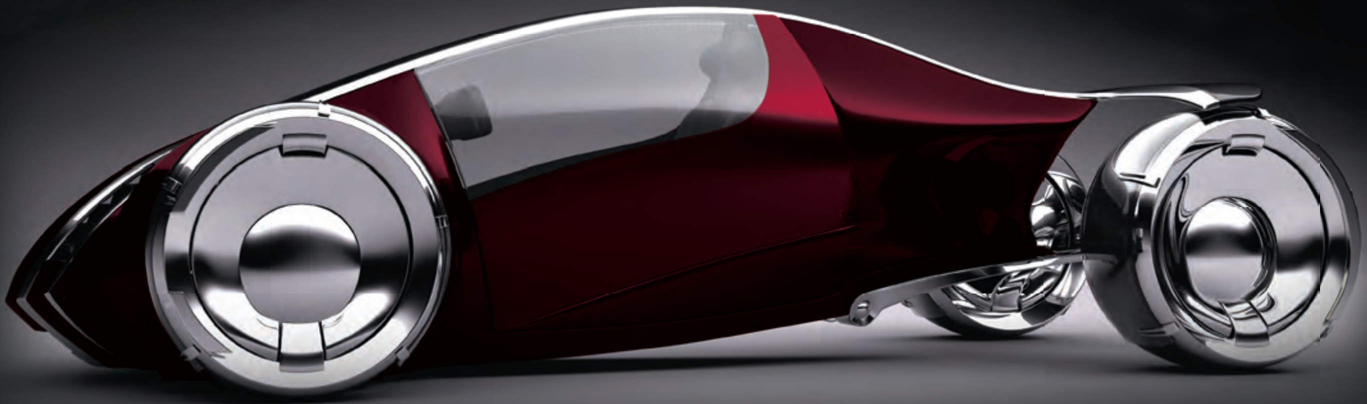
THIRD EDITION



Materials

NORTH
AMERICAN
EDITION

ENGINEERING, SCIENCE, PROCESSING AND DESIGN

Michael Ashby, Hugh Shercliff and David Cebon

Materials: engineering, science, properties, and design
3e Solution manual

Chapter 1: Exercises with worked solutions

Exercise E1.1 Use a search engine such as Google to research the history and uses of one of the following materials:

- Tin
- Glass
- Cement
- Titanium
- Carbon fiber

Present the result as a short report of about 100 - 200 words (roughly half a page).

Specimen answer: Tin. Tin (symbol Sn), a silver-white metal, has a long history. It was traded in the civilizations of the Mediterranean as early as 1500 BC (the Old Testament of the Christian bible contains many references to it). Its importance at that time lay in its ability to harden copper to give *bronze* (copper containing about 10% tin), the key material for weapons, tools and statuary of the Bronze age (1500 BC – 500 BC). Today tin is still used to make bronze, for solders and as a corrosion resistant coating on steel sheet ("tin plate") for food and drink containers – a "tinnie", to an Australian, is a can of beer. Plate glass is made by floating molten glass on a bed of liquid tin (the Pilkington process). Thin deposits of tin compounds on glass give transparent, electrically conducting coatings used for frost-free windshields and for panel lighting.

Exercise E1.2 What is meant by the *design-limiting properties* of a material in a given application?

Answer. A design-limiting property in a given application is one that limits the performance of the application. To meet a required level of performance, the design-limiting properties must meet or exceed target values.

Exercise E1.3 There have been many attempts to manufacture and market plastic bicycles. All have been too flexible. Which design-limiting property is insufficiently large?

Answer. Flexibility is lack of stiffness. The stiffness of a structure depends on its shape and size, and on the value of Young's modulus E of the material of which it is made. In design for stiffness E is a design-limiting property. In bicycle design the values of E offered by plastics are insufficiently large. However if the plastic is reinforced with carbon or glass fiber the stiffness can be increased to a useful level – high performance bicycles are made of carbon-reinforced plastic.

Exercise E1.4 What, in your judgment, are the design-limiting properties for the material for the blade of a knife that will be used to gut fish?

Answer. Hardness (to give a wear resistant, sharp edge); ability to be shaped to a blade; resistance to corrosion in fresh and salt water; stiffness (meaning modulus) to ensure that the thin blade does not bend or buckle during use.

Exercise E1.5 What, in your judgment, are the design-limiting properties for the material of an oven glove?

Answer. Flexibility (to allow weaving or shaping, and motion in use); low thermal conductivity (to insulate); maximum operating temperature > 200C (high oven setting); and ability to be washed, meaning tolerance of water.

Exercise E1.6 What, in your judgment, are the design-limiting properties for the material of an electric lamp filament?

Answer. The filament must be a good electrical conductor; maximum operating temperature > 2000^{o}C (or a temperature of that order); ductility to enable it to be drawn to fine wire.

Exercise E1.7 A material is needed for a tube to carry fuel from the fuel tank to the carburetor of a motor-powered mower. The design requires that the tube be flexible, and that the fuel be visible. List what you think would be the design-limiting properties.

Answer. Flexible (meaning low modulus E); transparency (to allow fuel to be visible); very good resistance to organic solvents (gasoline and oil); ability to be formed into tube.

Exercise E1.8 A material is required as the magnet for a magnetic soap holder. Soap is mildly alkaline. List what you would judge to be the design-limiting properties.

Answer. Ferromagnetic; very good resistance to fresh water and mild alkali.

Exercise E1.9 The cases in which most CDs are sold have an irritating way of cracking and breaking. Which design-limiting property has been neglected in selecting the material of which they are made?

Answer. Fracture toughness

Exercise E1.10 List three applications that, in your judgment, need high stiffness and low weight. Think of things that must be light (as they are moved, perhaps rapidly) but must not be too 'bendy'.

Possible answers. Racing bicycle frames; aircraft wing spars; car wheels; sports equipment; precision machine tools; radio-telescope dishes; high-speed printing presses.

Exercise E1.11 List three applications that, in your judgment, need optical quality glass. Think of products that rely on distortion-free imaging.

Possible answers. Binoculars; cameras; contact lenses; microscopes; telescopes; fiber-optic cables.

Exercise E1.12 List three applications that you think would require high thermal conductivity. Think of things that you have to get heat into or out of.

Possible answers. Cooking utensils; heat exchangers (car radiators, air-conditioning units); heat sinks (like those that conduct the heat from the processor in your PC); thermal sensors and thermally activated safety equipment.

Exercise E1.13 List three applications that you think would require low thermal expansion. Think of things that you that will lose accuracy or won’t work if they distort.

Possible answers. Optical benches; precision watches; gyroscopes such as those in inertial guidance systems; precision equipment generally.

Materials: engineering, science, properties, and design
3e Solution manual

Chapter 2. Exercises with worked solutions

Exercise E2.1 ***Material properties from experience.*** List the six main classes of engineering materials. Use your own experience to rank them approximately:

(a) by stiffness (modulus, E). A sheet of a material that has a high modulus is hard to bend when in the form of sheet. A sheet of material with a low modulus is floppy.

(b) by thermal conductivity (λ). Materials with high conductivity feel cold when you pick them up on a cold day. Materials with low conductivity may not feel warm, but they don't freeze your hands.

Answer. The main classes of engineering materials are ceramics, glasses, metals, polymers, elastomers and hybrids that include composites, foams and natural materials.

(a) Metals, ceramics and glasses and composites are stiff; polymers, elastomers and foams are less stiff.

(b) Metals are cold to touch - they conduct heat well; polymers, many hybrids (like wood and foams) have low thermal conductivity.

Exercise E2.2 ***Classification (1).*** A good classification looks simple – think, for instance, of the periodic table of the elements. Creating it in the first place, however, is another matter. This chapter introduced two classification schemes that work, meaning that every member of the scheme has a unique place in it, and any new member can be inserted into its proper position without disrupting the whole. Try one for yourself. Here are some scenarios. Make sure that each level of the hierarchy properly contains all those below it. There may be more than one way to do this, but one is usually better than the others. Test it by thinking how you would use it to find the information you want.

How many different ways can two sheets of paper be attached to each other, temporarily or permanently? Classify these, using 'Mechanism of joining' as the top level of the classification. Then try to develop the next level, based on your observations of the ways in which sheets of paper are joined.

Possible answer

- **Adhesives**
  - Water based
  - Solvent based
- **Staples**
  - Steel
  - Brass
- **Paper clips**
  - Metal
  - Plastic
- **Sewing**
  - Natural fibers
  - Man-made fibers

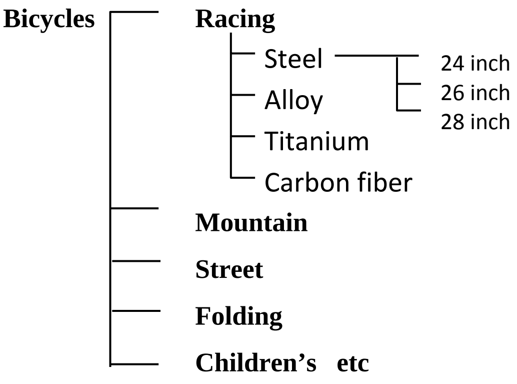
Exercise E2.3 ***Classification (2).*** In how many ways can wood be treated to change its surface appearance? Classify these, using the generic finishing technique as the top level of the classification. Then try to develop the next level, based on your observations of the ways in which wood products are finished.

Possible answer

- **Varnish**
  - Clear
  - Tinted
- **Paint**
  - Water-based
  - Solvent based
- **Polish**
  - Wax
  - Spray
- **Dye, stain**
  - Natural,
  - Synthetic
- **Texture**
  - Sanded, sand-blasted
  - Weathered.

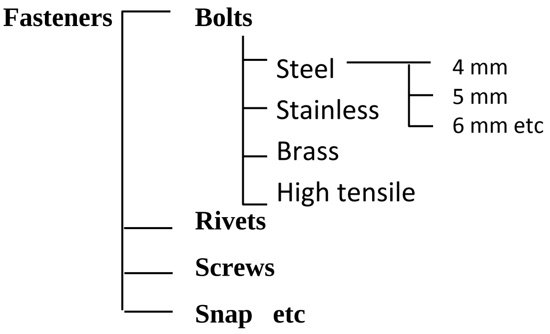
Exercise E2.4 ***Classification (3).*** You run a bike shop that stocks bikes of many types (children's bikes, street bikes, mountain bikes, racing bikes, folding bikes etc), prices and sizes. You need a classification system to allow customers to look up your bikes on the internet. How would you do it?

Possible answer.



Exercise E2.5 ***Classification (4).*** You are asked to organize the inventory of fasteners in your company. There are several types (snap, screw, bolt, rivet) and within each, a range of materials and sizes. Devise a classification scheme to store information about them.

Possible answer



Exercise E2.6 ***Shaping.*** What is meant by a shaping process? Look around you and ask yourself how the things you see were shaped.

Answer. A shaping process takes a raw material and gives it shape (eg casting or molding), or modifies the shape of an already shaped component (eg machining or grinding).

Exercise E2.7 ***Joining.*** Almost all products involve several parts that are joined. Examine products immediately around you and list the joining methods used to assemble them.

Answer. Find examples of joining by: adhesives, fasteners, snap-fits and welding.

Exercise E2.8 ***Surface treatment.*** How many different surface treatment processes can you think of, based on your own experience? List them and annotate the list with the materials to which they are typically applied.

Possible answers. Painting (applied to most materials); electroplating (e.g. chromium plating for appearance and protection); polishing (of wood, brass, silver); anodizing (of aluminium); texturing (metals, polymers); enameling (of copper, cast iron cooking ware, steel).

Exercise E2.9 ***Use of bar-charts.*** Examine the material property chart of Figure 2.8. By what factor, on average, are polymers less stiff than metals? By what factor is Neoprene less stiff than Polypropylene (PP)?

Answer. On average polymers are about 50 times less stiff than metals. Neoprene has a modulus that is about 1000 times smaller than that of polypropylene.

Exercise E2.10 ***Use of bar and bubble-charts (1).*** Wood is a natural polymer, largely made up of cellulose and lignin, and they, like engineering polymers, are almost entirely hydrogen, carbon, oxygen and a little nitrogen. Has nature devised a cocktail of these elements that has a higher modulus than any bulk man-made polymer? Engineering polymers and woods appear on the charts of Figures 2.8 and 2.9. Use either one to answer the question.

Answer. The charts show that woods, have a higher modulus parallel to the grain than any bulk commercial polymer.

Exercise E2.11 ***Use of bubble-charts (2).*** Windows can be made of glass (houses), polycarbonate, PC (conservatories) or poly-methyl-methacrylate, PMMA (aircraft). Would a glass window be more flexible than a replacement of the same thickness made of polycarbonate? Would it be heavier? Use the bubble chart of Figure 2.9 to find out.

Answer. Polycarbonate, PC, lies in upper end of the blue envelope. Glass lies at the lower left end of the yellow one. Glass has a modulus that is about 30 times greater that PC, so a glass pane is 30 times stiffer than a PC one of the same thickness. The density of glass is about twice that of PC – the glass pane will be twice as heavy as the one made of PC.

Exercise E2.12 ***Use of bubble-charts (3).*** Do zinc alloys have a higher specific stiffness E/ρ than Polypropylene (PP)? Use the bubble chart of Figure 2.9 to find out.

Answer. Yes, zinc alloys lie on a higher line of E/ρ than Polypropylene. Reading values of E and ρ from the chart we find the specific stiffness of zinc alloys is approximately 0.02 GPa/(kg/m^3), that of Polypropylene is approximately 0.003 in the same units.

Exploring design with CES.

Designers need to be able to find data quickly and reliably. That is where the classifications come in. The CES system uses the classification scheme described in this chapter. Before trying these exercises, open the Materials Universe in CES and explore it. The opening menu offers three or more options – take the first, 'Level 1'.

Exercise E2.13 Use the 'Browse' facility in Level 1 of the CES Software to find the record for copper. What is its thermal conductivity? What is its price?

Answer. The thermal conductivity of copper is 160 – 390 W/m.K, depending on purity. Its price is 4.9 – 6.3 $/kg (the price of copper fluctuates considerable – hence the wide range).

Exercise E2.14 Use the 'Browse' facility in Level 1 of the CES Software to find the record for the thermosetting polymer *Phenolic*. Are they cheaper or more expensive than *Epoxies*?

Answer. Phenolics are cheaper.

Exercise E2.15 Use the 'Browse' facility to find records for the polymer-shaping processes *Rotational molding*. What, typically, is it used to make?

Answer. Tanks, food and shipping containers, housings, portable lavatories, traffic cones, toys (e.g. balls), dustbins, buckets, boat hulls and pallets.

Exercise E2.16 Use the 'Search' facility to find out what *Plexiglas* is. Do the same for *Pyroceram*.

Answer. Plexiglass is PMMA, polymethyl methacrylate, or acrylic. Pyroceram is a glass ceramic.

Exercise E2.17 Use the 'Search' facility to find out about the process *Pultrusion*. Do the same for *TIG welding*. Remember that you need to search the Process Universe, not the Material Universe. To do this change the Table from 'MaterialUniverse' to 'ProcessUniverse' using the tab at the top of the Browse window.

Answer. Pultrusion is a way of making continuous fiber-reinforced sections such as tube and sheet. TIG welding is Tungsten Inert Gas welding – electric arc welding with a tungsten electrode in an atmosphere of argon.

Exercise E2.18 Compare Young's modulus E (the stiffness property) and thermal conductivity λ (the heat-transmission property) of *aluminum alloys* (a non-ferrous metal), *alumina* (a technical ceramic), *polyethylene* (a thermoplastic polymer) and *neoprene* (an elastomer) by retrieving values from CES Level 1. Which has the highest modulus? Which has the lowest thermal conductivity?

Answer.

Material	**Young's modulus , E GPa**	**T-conductivity λ , W/m.K**
Aluminum alloys	112 - 148	160 - 390
Alumina	343 - 390	26 – 38.5
Polyethylene	0.62 – 0.9	0.4 – 0.44
Neoprene	0.0007 – 0.002	0.1 – 0.12

All material properties have ranges. At Level 1 of the software, in which one record describes a class of materials, the ranges are wide because they encompass the whole class. At Level 2 they are a little narrower because it subdivides classes into sub-classes. At Level 3 they are much narrower because a Level 3 record describes a single grade of a material.

The ceramic alumina has by far the highest value of Young's modulus of the four materials. The elastomer neoprene has the lowest value of thermal conductivity.

Exploring the science with CES Elements

The CES system contains a database for the periodic table called 'Elements'. The records contain fundamental data for each of the elements. We will use this in the book to delve a little deeper into the science that lies behind material properties.

Exercise E2.19 Refresh your memory of the periodic table, perhaps the most significant classification system of all time. Open the *Elements* database by clicking on Change in the Browse window and selecting Elements from the menu that is offered. (or go to *File – Change database – Elements*) and double click on Periodic Table to see the table. This database, like the others described in this chapter, has a tree-like structure. Use this to find the record for Aluminum (Row 3, Atomic number 13) and explore its contents. Many of the properties won't make sense yet. We introduce them gradually throughout the book.

Materials: engineering, science, properties, and design
3e Solution manual

Chapter 3: Exercises with worked solutions

Exercise E3.1 What are the steps in developing an original design?

Answer.

- Identify market need, express as *design requirements*.
- Develop *concepts*: ideas for the ways in which the requirements might be met
- *Embodiment*: a preliminary development of a concept to verify feasibility and show layout.
- *Detail design*: the layout is translated into detailed drawings (usually as computer files), stresses are analyzed and the design is optimized.
- *Prototyping*: a prototype is manufactured and tested to confirm viability.

Exercise E3.2 What is meant by an *objective* and what by a *constraint* in the requirements for a design? How do they differ?

Answer. A *constraint* is an essential condition that must be met, usually expressed as a limit on a material or process attribute. An *objective* is a quantity for which an extreme value (a maximum or minimum) is sought, frequently cost, mass or volume, but there are others. Constraints are used for *screening* to isolate candidates that are capable of doing the job; objectives are used for *ranking* to identify those among them that can do the job best.

Exercise E3.3 Describe and illustrate the ‘translation’ step of the material selection strategy.

Answer. *Translation* is the conversion of design requirements for a component into a statement of function, constraints, objectives and free variables.

Function	• What does the component do?
Constraints	• What non-negotiable conditions must the material meet?
Objectives	• What aspects of performance are to be maximized or minimized?
Free variables	• What parameters of the problem is the designer free to change?

Exercise E3.4 ***Translation (1).*** The teeth of a scoop for a digger truck, pictured in Example 1.1 of Chapter 1, must cut earth, scoop stones, crunch rock, often in the presence of fresh or salt water and worse. Translate these requirements into a prescription of Function, Constraints, Objectives and Free variables.

Answer.

Function	• Penetrate earth, gravel and stone
Constraints	• High wear resistance, thus high hardness H • High toughness, thus high fracture toughness K_{1c} • Corrosion resistance in fresh and salt water.
Objectives	• Maximize life-time to avoid down-time
Free variables	• Choice of material

Exercise E3.5 ***Translation (2).*** A material for an energy-efficient saucepan, pictured in Example 1.2 of Chapter 1, must transmit and spread the heat well, resist corrosion by foods, and withstand the mechanical and thermal loads expected in normal use. The product itself must be competitive in a crowded market. Translate these requirements into a prescription of Function, Constraints, Objectives and Free variables.

Answer.

Function	Contain foodstuffs during cooking
Constraints	Maximum working temperature above 300°C (boiling and frying) High thermal conductivity, λ , to transmit and spread the heat well Excellent corrosion resistance in salt water Excellent corrosion resistance in mild acids (e.g. Vinegar) Excellent corrosion resistance in alkalis (e.g. Baking soda) Resistant to thermal shock (sudden change of temperature)
Objectives	Minimize material cost
Free variables	Thickness of the pan Choice of material

Exercise E3.6 ***Translation (3).*** A material for eyeglass lenses, pictured in Example 1.3 of Chapter 1, must have optical-quality transparency. The lens may be moulded or ground with precision to the required prescription. It must resist sweat and be sufficiently scratch-resistant to cope with normal handling. The mass-market end of the eyeglass business is very competitive so price is an issue. Translate these requirements into a prescription of Function, Constraints, Objectives and Free variables.

Answer.

Function	Refract light without loss of clarity
Constraints	Optical-quality transparency Ability to be molded or ground with precision Excellent resistance to fresh and salt water (rain, sweat) High hardness, or ability to carry a hard transparent coating
Objectives	Minimize cost of material and of molding or grinding process
Free variables	Choice of material Choice of coating to improve scratch resistance and reflectivity

Exercise E3.7 Bikes come in many forms, each aimed at a particular sector of the market:

- Sprint bikes
- Touring bikes
- Mountain bikes
- Shopping bikes
- Children's bikes
- Folding bikes

Use your judgment to identify the primary objective and the constraints that must be met for each of these.

Answer.

Take the example of a bicycle frame. It must have a certain stiffness and strength. If it is not stiff and strong enough it will not work, but it is never required to have infinite stiffness or strength. Stiffness and strength are therefore constraints that become limits on modulus, elastic limit and shape. If the bicycle is for sprint racing, it should be as light as possible – if you could make it infinitely light, that would be best of all. Minimizing mass, here, is the objective, perhaps with an upper limit (a constraint) on cost. If instead it is a shopping bike to be sold through supermarkets it should be as cheap as possible – the cheaper it is, the more will be sold. This time minimizing cost is the objective, possibly with an upper limit (a constraint) on mass. For most bikes, of course, minimizing mass and cost are both objectives, and then trade-off methods are needed. They come later. For now use judgment to choose the single most important objective and make all others into constraints.

Exercise E3.8 A material is required for the windings of an electric air-furnace capable of operating at temperatures up to 1000ºC. Think out what attributes a material must have if it is to be made into windings and function properly in a furnace. List the function and the constraints; set the objective to 'minimize material price' and the free variables to 'choice of material'.

Answer.

Function	• *Windings for 1000 ºC electric air furnace*
Constraints	• *Good electrical conductor* • *Capable of being shaped to wire* • *Sufficient ductility to allow a coil to be wound* • *Maximum service temperature > 1200ºC (allowing 200ºC over operating temperature of the furnace itself)* • *Resistance to oxidation: very good*
Objective	• *Minimize material price*
Free variables	• *Choice of material*

Exercise E3.9 A material is required to manufacture office scissors. Paper is an abrasive material, and scissors sometimes encounter hard obstacles like staples. List function and constraints; set the objective to 'minimize material price' and the free variables to 'choice of material'.

Answer.

Function	• *Blades for office scissors*
Constraints	• *High hardness(for wear resistance)* • *Some toughness (to avoid chipping the blade edge)* • *High stiffness to ensure that the blades do not bend in service* • *Able to be ground to a sharp edge*
Objective	• *Minimize material price*
Free variables	• *Choice of material*

Exercise E3.10 A material is required for a heat exchanger to extract heat from geothermally heated, saline, water at 120ºC (and thus under pressure). List function and constraints; set the objective to 'minimize material price' and the free variables to 'choice of material'.

Answer.

Function	• *Geothermal heat exchanger*
Constraints	• *Good thermal conductivity* • *Durablity in hot salt water: very good* • *Maximum service temperature > 120ºC* • *Sufficient strength to carry the pressure of the super-heated water* • *Sufficient ductility to allow shaping to form tubes*
Objective	• *Minimize material price*
Free variables	• *Choice of material*

Exercise E3.11 A material is required for a disposable fork for a fast-food chain. List the objective and the constraints that you would see as important in this application.

Answer.

Function	*Disposable cutlery*
Constraints	- *Easy(and thus cheap) to mold* - *Non-toxic* - *Sufficient strength and stiffness (modulus) to withstand loads in use* - *Recyclable or renewable (e.g. wooden cutlery)* - *Ability to be colored (?)*
Objective	*Minimize material price*
Free variables	*Choice of material*

Exercise E3.12 Formulate the constraints and objective you would associate with the choice of material to make the forks of a racing bicycle.

Answer. Both stiffness and strength are important in bicycle design. But the first consideration in the design of the forks is strength – it is essential that they do not fail in normal use – which includes riding over curbs and other irregularities. The loading on the forks is predominantly *bending*. If the bicycle is for racing, then the mass is a primary consideration: the forks should be as light as possible.

Function	*Forks for racing bike*
Constraints	- *Must not fail under design loads – a constraint bending strength* - *Dimensions specified*
Objective	*Minimize weight*
Free variables	*Choice of material*

Exercise E3.13 Cheap coat hangers used to be made of wood, but now only expensive ones use this material. Most coat hangers are now metal or plastic, and both differ in shape from the wooden ones, and from each other. Examine wood, metal and plastic coat hangers, comparing the designs, and comment on the ways in which the choice of material has influenced them.

Answer. Coat hangers often have three components: the hook, the 'shoulders' and the 'trouser bar'. To make this structure entirely from wood is not practical: it is difficult to make the thin curved shape of the hook from wood. So wooden hangers normally have steel wire hooks. This has the added advantage that the hooks can be made to rotate about a vertical axis in the shoulders, making it easier to hook the hanger over the rail. Joining the trouser bar to the shoulders requires machining and gluing, or additional wire joining pieces and surface has to be shaped, sanded and varnished for aesthetic reasons. The whole process is expensive: wooden hangars typically cost $1 or more. Many different types of plastic hangers are available, in a wide variety of different polymers, shapes, sizes and colors. The cheapest are molded in one piece. Consequently their hooks do not rotate. More expensive ones have metal wire hooks that can rotate. Some are adjustable, some have additional garment hooks or textured shoulders to make them more versatile. Simple plastic hangers cost about 40c each. If you are a drycleaner even this is too much. The cheapest hangers are made of bent steel wire. Steel wire is cheap and the manufacturing operations – cutting the wire, bending it into a triangular shape and twisting the two ends together – are fast and inexpensive. The hooks don't rotate, but at 10c each, you can't be too choosy!

Exercise E3.14 Cyclists carry water in bottles that slot into bottle holders on their bicycles. Examine metal and plastic bottle holders, comparing the designs, and comment on the ways in which the choice of material has influenced them.

Answer. The main criteria for 'bottle cages' are to hold the bottle securely, while being light weight and aesthetically pleasing. The importance of each of these factors varies, depending on the application. Mountain bikes have an emphasis on 'secure'.. they traverse rough terrain, and the riders occasionally fall off at high speed – the resulting impact can test even the most robust bottle holder. Aluminum alloy and stainless steel wire are the favoured materials: aluminum for its lower weight, stainless steel for its aesthetic appeal. The designs often make use of the 'springiness' of the metal wire to clamp the bottle in place. Racing cyclists need the lightest possible accessories – their bottle cages are often made of carbon-fiber reinforced plastics (CFRP). These come in a variety of shapes that tend to surround the bottle, rather than clamping it. They are significantly more expensive than their aluminum counterparts – about a factor of 5 to 10 times more.

Exploring design with CES

Exercise E3.15 A company wishes to enhance its image by replacing oil-based plastics in its products by polymers based on natural materials. Use the 'Search' facility in CES to find *biopolymers*. List the materials you find.

Answer.

- Natural Rubber (NR)
- Cellulose polymers (CA)
- Polylactide (PLA)
- Polyhydroxyalkanoates (PHA, PHB)
- Starch-based thermoplastics (TPS)

Exercise E3.16 A maker of garden furniture is concerned that the competition is stealing part of his market with furniture made by RTM, a term with which he is unfamiliar. Use the 'Search' facility in CES to find out what *RTM* is, and whether it is used to make things like garden furniture.

Answer (copied from the record for RTM)

The Process. RESIN TRANSFER MOLDING (RTM) allows manufacture of complex shapes in fiber-reinforced composites without high tooling costs. It uses a closed mold, in two or more parts, usually made of glass-reinforced polymers or light metal alloys, with injection points and vents to allow air to escape. Reinforcement is cut to shape and placed in the mold, together with any inserts or fittings. The mold is closed and a low viscosity thermosetting resin (usually polyester) is injected under low pressure (roughly 2MPa) through a mixing head in which hardener is blended with the resin. The mold is allowed to cure at room temperature. The fluidity of the resin and the low molding pressure allows long tool life at low cost.

Typical uses. RTM is used to make large composite structures such as manhole covers, compress soil casings, car doors and side panels, propeller blades, boat building, hulls, canoe paddles, water tanks, bath tubs, roof sections, airplane escape doors.

Exercise E3.17 Use the 'Search' facility in CES Level 2 to find materials for *furnace windings.*

Answer.

- Nickel-chromium alloys
- Tungsten alloys

Exercise E3.18 Use the 'Search' facility in CES Level 2 to find materials for *scissors* and *knife blades.*

Answer.

- High carbon steel
- Low alloy steel
- Medium carbon steel
- Stainless steel

Exercise E3.19 Use the 'Search' facility in CES Level 2 to find materials for *heat exchangers.*

Answer.

- Copper alloys
- Titanium alloys
- Bronze
- Brass
- Commercially pure titanium
- Metal foam

Exercise E3.20 Use the 'Search' facility in CES Level 2 to find materials for *flooring*.

Answer.

- Lead alloys
- Bamboo
- Cork
- Hardwood: oak, along grain
- Hardwood: oak, across grain M
- Polyvinylchloride (tpPVC)
- Marble
- Commercially pure lead
- Softwood

(This illustrates a weakness of selecting materials by Searching. Many other materials in the CES database are in fact used for flooring: concrete, brick, stone, glass, granite, sandstone….. The Search procedure does not find them because their records do not contain the specific word 'flooring')

Exercise E3.21 Eye-glass lenses require a material with optical-quality transparency that can be molded with precision and will resist sweat. A high refractive index (greater than 1.5, say) allows thinner, lighter lenses. Open CES Edu Materials Level 2, choose the data subset Materials with Durability properties, and open a Limit stage. Open *Optical properties* and select Optical quality and set a lower limit of 1.5 for the refractive index. Open *Processability* and put a lower limit of 5 (the best possible) on Moldability. Open *Durability: water and aqueous environments* and click on Excellent in Fresh water and in Salt water (sweat). Finally, click *Apply*. What is the selected short-list? (It appears in the lower left of the CES user-interface).

Answer.

The software delivers the following list, all of which are practical choices for eyeglasses.

- Polycarbonate (PC)
- Polymethyl methacrylate (Acrylic, PMMA)
- Polystyrene (PS)
- Soda-lime glass

Materials: engineering, science, properties, and design
3e Solution manual

Chapter 4: Exercises with worked solutions

Exercise E4.1 Identify which of the five modes of loading (Figure 4.2) is dominant in the following components:

- Fizzy drinks container
- Overhead electric cable
- Shoe soles
- Wind turbine blade
- Climbing rope
- Bicycle forks
- Aircraft fuselage

Can you think of another example for each mode of loading?

Answer.

- Fizzy drinks container: bi-axial tension in the wall of the container caused by the hydrostatic pressure of the pressurized liquid.
- Overhead electric cable: tension
- Shoe soles: compression when standing still; some superimposed shear when running.
- Wind turbine blade: bending, caused by wind loading and the self-weight of the blades
- Climbing rope: tension.
- Bicycle forks: bending and compression
- Aircraft fuselage: bi-axial tension in the fuselage wall caused by the hydrostatic pressure of pressurized cabin

Other examples for each mode of loading:

- Bi-axial tension: balloons, bicycle tyre tubing, bouncy castle.
- Tension: cabling/chains on suspension bridges, shop moorings etc; tie members in truss structures.
- Compression: mounting supports under bridges
- Bending: sports goods (golf clubs, vaulting poles, fishing rods, rowing oars)

Exercise E4.2 The cable of a hoist has a cross section of 80 mm^2. The hoist is used to lift a crate weighing 500 kg. What is the stress in the cable? The free length of the cable is 3 m. How much will it extend if it is made of steel (modulus 200 GPa)? How much if it is made of polypropylene, PP (modulus 1.2 GPa)?

Answer. The stress is 61.3 MPa. The steel cable will stretch by 0.92 mm. The polypropylene cable will stretch by 153 mm.

Exercise E4.3 Water has a density of 1000 kg/m^3. What is the hydrostatic pressure at a depth of 100m? How many times atmospheric pressure is this (1 atm = 0.1 MPa)?

Answer. 9.8 MPa, 10 × atmospheric pressure.

Exercise E4.4   Figure 4.5 shows a cubic element of material with Young's modulus E and Poisson's ratio ν which is subjected to normal stresses σ_1, σ_2 and σ_3, resulting in strains ε_1, ε_2 and ε_3 (equation (4.13)).

(a) Find an expression for the dilatation Δ when a unit cube experiences strains of ε_1, ε_2 and ε_3, assuming that the strains are elastic and therefore small (<< 1).

(b) Find the dilatation Δ for an element subjected to a uniaxial tensile stress $\sigma_1 = \sigma$ (i.e. $\sigma_2 = \sigma_3 = 0$). For what value of Poisson's ratio is volume conserved?

(c) Find the dilatation Δ for an element subjected to equal pressure p in all directions, i.e. $\sigma_1 = \sigma_2 = \sigma_3 = -p$. Hence derive the formula relating the bulk modulus, K, to Young's modulus, E and Poisson's ratio, ν (equation (4.10)).

Answer. (a) The new dimensions of a unit cube subjected to strains of ε_1, ε_2 and ε_3 will be $(1 + \varepsilon_1, 1 + \varepsilon_2, 1 + \varepsilon_3)$. So the new volume $= (1 + \varepsilon_1) \times (1 + \varepsilon_2) \times (1 + \varepsilon_3) = 1 + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$ + (higher order terms in ε, which are negligible). The change in volume is therefore $\Delta V = [1 + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)] - 1 = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$. Since the initial volume is equal to unity, this is also the dilatation, $\Delta = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$.

(b) For uniaxial stress in the 1-direction, σ_1, the strains are: $\varepsilon_1 = \dfrac{\sigma_1}{E}$, $\varepsilon_2 = \varepsilon_3 = \dfrac{-\nu\sigma_1}{E}$

Hence the dilatation $\Delta = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) = \dfrac{\sigma_1}{E}(1-2\nu)$.

Volume is conserved when $\Delta = 0$, hence when $\nu = 0.5$.

(c) From Hooke's Law in 3D:

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \frac{1}{E}(-p + \nu p + \nu p) = -\frac{p}{E}(1 - 2\nu)$$

Hence volumetric strain $\Delta = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) = -\dfrac{3p}{E}(1 - 2\nu)$

So the bulk modulus is given by: $K = \dfrac{p}{\Delta} = \dfrac{E}{3(1 - 2\nu)}$

Exercise E4.5 A mounting block made of butyl rubber of modulus = 0.0015 GPa is designed to cushion a sensitive device against shock loading. It consists of a cube of the rubber of side-length L = 40mm located in a slot in a rigid plate, exactly as shown in Figure 4.6. The slot and cube have the same cross-sectional dimensions, and the surrounding material is much stiffer, so the strain in one direction is constrained to be zero. A vertical compressive load F is applied, and the deflection of the top block is δ. The maximum expected value of F is 50 N.

Use equation (4.14) to find the stiffness of the cube (F/δ) in this constrained condition, assuming $\nu \approx 0.5$. Hence find the maximum deflection δ of the top face of the block. What would this deflection be without any constraint?

Answer. From equation (4.14), the effective modulus for constraint in one transverse direction is:

$$\frac{\sigma_1}{\varepsilon_1} = \frac{E}{(1 - \nu^2)}$$

Hence for a constrained cube of side-length L: $\dfrac{F/L^2}{\delta/L} = \dfrac{E}{(1 - \nu^2)}$ so $\dfrac{F}{\delta} = \dfrac{E\,L}{(1 - \nu^2)}$

For E = 0.0015 GPa, L = 40mm and $\nu = 0.5$:

$$\frac{F}{\delta} = \frac{0.0015\times10^9 \times 40\times10^{-3}}{(1 - 0.5^2)} = 80\,kN/m = 80\,N/mm$$

The deflection in the vertical direction: $50 \;/\; 80 \;=\; 0.625$ mm

Constraint increases the stiffness by a factor of 1.33 (for ν = 0.5), so the deflection without constraint = 1.33 × 0.625 = 0.833 mm.

Exercise E4.6 A cube of linear elastic material is again subjected to a vertical compressive stress σ_1 in the 1-direction, but is now constrained ($\varepsilon = 0$) in both the 2 *and* the 3 directions.

(a) Find expressions for the induced transverse stresses, σ_2 and σ_3 in terms of σ_1 . Hence derive an expression for the "effective modulus" (σ_1/ε_1) in this case.

(b) Sketch the variation of effective modulus with ν and comment on the limiting values when $\nu = 0$ (foam) and $\nu \approx 0.5$ (rubber).

(c) Explain why the rubber soles of running shoes are designed with some combination of air or gel pockets, partially foamed rubber, and a tread.

Answer.

(a) For full transverse constraint: $\varepsilon_2 = \varepsilon_3 = 0$

Hence from Hooke's Law in 3D: $\varepsilon_2 = 0 = \dfrac{1}{E}\left(-\nu\sigma_1 + \sigma_2 - \nu\sigma_3\right)$

$$\varepsilon_3 = 0 = \frac{1}{E}\left(-\nu\sigma_1 - \nu\sigma_2 + \sigma_3\right)$$

So $\sigma_2 - \nu\sigma_3 = -\nu\sigma_2 + \sigma_3$, i.e. $\sigma_2 = \sigma_3$ (as it must, from symmetry)

Thus for the 2 direction: $\varepsilon_2 = 0 = \dfrac{1}{E}\left(-\nu\sigma_1 + \sigma_2 - \nu\sigma_2\right) \Rightarrow \sigma_2 = \dfrac{\nu\sigma_1}{(1-\nu)}$

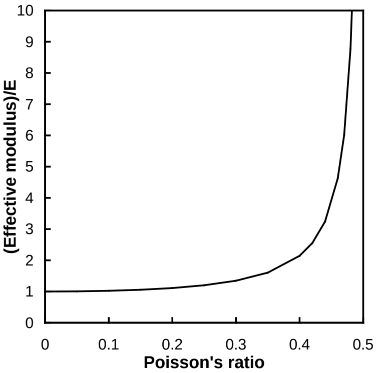
and from symmetry, for the 3 direction: $\sigma_3 = \dfrac{\nu\sigma_1}{(1-\nu)}$.

Hence in the 1 direction: $\varepsilon_1 = \dfrac{1}{E}\left(\sigma_1 - \nu\sigma_2 - \nu\sigma_3\right) = \dfrac{\sigma_1}{E}\left(1 - \dfrac{2\nu^2}{1-\nu}\right) = \dfrac{\sigma_1}{E}\left(\dfrac{1-\nu-2\nu^2}{1-\nu}\right)$

Hence the "effective modulus" in the 1 direction is given by: $\dfrac{\sigma_1}{\varepsilon_1} = \dfrac{E(1-\nu)}{(1-\nu-2\nu^2)}$

The expression is a little more complicated than for constraint in one direction, but has the same effect of multiplying the modulus by a factor greater than unity.

(b)



For $\nu = 0$ (foam), $(\sigma_1/\varepsilon_1) = E$ (i.e. constraint has no effect, as no lateral expansion).

For $\nu \to 0.5$ (rubber), $(\sigma_1/\varepsilon_1) \to \infty$ (i.e. incompressible, but with lateral expansion prevented). So rubber, which has the lowest Young's modulus of all solid materials, has a very stiff response when constrained.

(c) Running shoe treads not only provide grip (by allowing the shoe sole to deform around surface irregularities, giving higher friction) but must also provide an elastic cushion to reduce the impact forces (and avoid injury). Solid rubber would be very uncomfortable – the sole is thin relative to its length and width and would be unable to strain laterally under the foot (giving the high constraint effect above). Foaming the rubber reduces Poisson's ratio, reducing the effective modulus; treads allow local transverse Poisson strain of the rubber; and air and gel pockets are compressible – all giving a much more elastic response (and a great deal of freedom to customise the design – it's not all marketing hype).

Exercise E4.7 A catapult has two rubber arms, each with a square cross-section with a width 4 mm and length 300 mm. In use its arms are stretched to three times their original length before release. Assume the modulus of rubber is 10^{-3} GPa and that it does not change when the rubber is stretched. How much energy is stored in the catapult just before release?

Answer. 17.28 kJ

Exercise E4.8 Use the modulus-density chart of Figure 4.8 to find, from among the materials that appear on it:

(a) the material with the highest density
(b) the metal with the lowest modulus
(c) the polymer with the highest density
(d) the approximate ratio of the modulus of woods measured parallel to the grain and perpendicular to the grain
(e) the approximate range of modulus of elastomers

Answer. (a) tungsten; (b) lead; (c) PTFE, Teflon; (d) about 50; (e) 10^{-4} to 10^{-1} GPa.

Exercise E4.9 Use the modulus-relative cost chart of Figure 4.9 to find, from among the materials that appear on it

(a) the cheapest material with a modulus greater than 1 GPa
(b) the cheapest metal
(c) the cheapest polymer
(d) whether magnesium alloys are more or less expensive than aluminum alloys
(e) whether PEEK (a high performance engineering polymer) is more or less expensive than PTFE

Answer. (a) concrete; (b) cast iron; (c) polypropylene; (d) magnesium alloys are more expensive than aluminum alloys; (e) PEEK is much more expensive than PTFE, which itself is an expensive polymer.

Exercise E4.10 What is meant by:

- a crystalline solid
- an amorphous solid
- a thermoplastic
- a thermoset
- an elastomer?

Answer.

- A crystalline solid is one in which the atoms or molecules are arranged in an ordered way that can be described by a geometric lattice.
- An amorphous solid (or glassy solid) is one in which crystallographic order is absent.
- A thermoplastic is one in which the bonding along the polymer chains is covalent and strong but the bonding between them is Van der Waals or hydrogen bonding and is weak. This causes the polymer to melt when heated, allowing thermal molding.
- A thermosetting plastic is one in which the polymer chains are cross-linked by strong covalent bonds that do not melt when the polymer is heated.
- An elastomer is a slightly cross-linked polymer which, at room temperature, is above its glass-transition temperature.

Exercise E4.11 The stiffness S of an atomic bond in a particular material is 50 N/m and its center-to-center atom spacing is 0.3 nm. What, approximately, is its elastic modulus?

Answer. 170 GPa.

Exercise E4.12 Derive the upper and lower bounds for the modulus of a composite quoted as equations (4.24) and (4.25) of the text. To derive the first, assume that the matrix and reinforcement behave like two springs in parallel (so that each must strain by the same amount), each with a stiffness equal to its modulus E multiplied by its volume fraction, f. To derive the second, assume that the matrix and reinforcement behave like two springs in series (so that both are stressed by the same amount), again giving each a stiffness equal to its modulus E multiplied by its volume fraction, f.

Answer. The composite has a volume fraction f of fibers of modulus E_r contained in a matrix occupying a volume fraction $(1-f)$, and of modulus E_m. If the strain ε in the fibers and matrix are the same (as in springs in parallel), then the stress is the sum of that carried by fibers and matrix:

$$\sigma = \sigma_r + \sigma_m = f\,E_r\,\varepsilon + (1-f)E_m\,\varepsilon$$

Dividing through by ε gives

$$\tilde{E}_u = f\,E_r + (1-f)E_m .$$

If instead the stress σ in fibers and matrix is the same (as in springs in series), the strain is the sum of that in the matrix and that in the fibers:

$$\varepsilon = \varepsilon_r + \varepsilon_m = \frac{\sigma}{f\,E_r} + \frac{\sigma}{(1-f)E_m}$$

Again dividing through by ε gives

$$\tilde{E}_L = \frac{E_m E_r}{f\,E_m + (1-f)E_r}$$

Exercise E4.13 A volume fraction $f = 0.2$ of silicon carbide (SiC) particles is combined with an aluminum matrix to make a metal matrix composite. The modulus and density of the two materials are listed in the table. The modulus of particle-reinforced composites lies very close to the lower bound, equation (4.25), discussed in the text. Calculate the density and approximate modulus of the composite. Is the specific modulus, E/ρ, of the composite greater than that of unreinforced aluminum? How much larger is the specific modulus if the same volume fraction of SiC in the form of continuous fibers is used instead? For continuous fibers the modulus lies very close to the upper bound, equation (4.24).

	Density, Mg/m^3	Modulus, GPa
Aluminum	2.70	70
Silicon carbide	3.15	420

Answer.

	Density, Mg/m^3	Modulus, GPa	Modulus/Density, GPa/(Mg/m^3)
Aluminum	2.70	70	26
Silicon carbide	3.15	420	133
Al-20% SiC, particles	2.79	84	30.1
Al-20% SiC, fibers	2.79	140	50.2

Particles of SiC give a modest improvement of about 15% in E/ρ compared to unreinforced aluminium; fibres on the other hand almost double it (in the direction of the fibres).

Exercise E4.14 Medical prosthetic implants such as hip replacements have traditionally been made of metals such as stainless steel or titanium. These are not ideal as they are much stiffer than the bone, giving relatively poor transfer of load into the bone. New composite materials are being developed to provide a much closer stiffness match between the implant material and the bone. One possibility uses high density polyethylene (HDPE) containing particulate hydroxyapatite (HA, the natural mineral in bone, which can be produced artificially). Data for some experimental composites are provided in the table below, and those for the bulk materials are: HDPE 0.65 GPa, Hydroxyapatite 80 GPa.

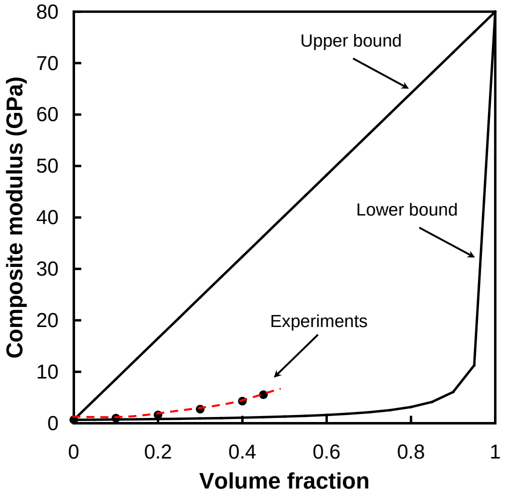
(a) Plot the upper and lower bounds for the Young's modulus of HDPE-HA composites against the volume fraction of reinforcement, *f* (from 0 to 1), together with the experimental data.

(b) Which of the bounds is closer to the data for the particulate composite? Use this bound as a guide to extrapolate the experimental data to identify the volume fraction of hydroxyapatite particulate that is required to match the modulus of bone, *E* = 7 GPa. Is this practical?

Volume fraction of HA	Young's modulus (GPa)
0	0.65
0.1	0.98
0.2	1.6
0.3	2.73
0.4	4.29
0.45	5.54

Answer.

PE-hydroxyapatite composite: bounds for Young's modulus plotted below.



Experimental data is closer to the lower bound, but this underestimates the modulus by up to a factor of 5 (by *f* = 0.45). Extrapolating the data to a modulus of 7 GPa, following the shape of the lower bound, the required volume fraction is around 50%. This is just practical, as mixing volume fractions of ceramic greater than 50% into a polymer is difficult – probably why the experiments stop at 45%. The particles end up in contact with one another, without complete infill with the polymer matrix. This reduces the stiffness, but more seriously the strength. 40-45% HA gives 4-5 GPa, close enough to the value for bone (and a great improvement on steel, at 210 GPa).

Exercise E4.15 (a) Use the material property data in the table below to find the composite density $\tilde{\rho}$ and upper bound modulus $\tilde{E}_U$ for the following composites: (i) carbon-fibre/epoxy resin (*f* = 0.5), (ii) glass-fibre/polyester resin (*f* = 0.5).

Material	Density (Mg m^{-3})	Young's modulus (GPa)
Carbon fibre	1.90	390
Glass fibre	2.55	72
Epoxy resin	1.15	3
Polyester resin	1.15	3

(b) A magnesium company has developed an experimental metal-matrix composite (MMC), by casting magnesium containing 20% (by volume) of particulate SiC. Use the lower bound estimate for Young's modulus to estimate $\tilde{E}_L$ for this MMC, with the following data for matrix and particles: Mg alloys, E = 44.5 GPa, ρ = 1.85 Mgm^{-3}; SiC, E = 380 GPa, ρ = 3.1 Mgm^{-3}.

(c) Find the *specific stiffness* $\tilde{E}/\tilde{\rho}$ for the three composites in (a,b). Compare the composites with steels, for which $E/\rho \approx$ 28 GPa Mg^{-1} m^3.

Answer.

(a) Density and Young's moduli of the fibre composites are both derived using the rule of mixtures:

Carbon-fibre epoxy: $\tilde{\rho} = (0.5 \times 1.90) + (0.5 \times 1.15) = 1.53$ Mgm^{-3}

$\tilde{E}_U = (0.5 \times 390) + (0.5 \times 3) = 197$ GPa

Glass-fibre polyester: $\tilde{\rho} = (0.5 \times 2.55) + (0.5 \times 1.15) = 1.85$ Mgm^{-3}

$\tilde{E}_U = (0.5 \times 72) + (0.5 \times 3) = 37.5$ GPa

(b) Magnesium – SiC: $\tilde{\rho} = (0.2 \times 3.1) + (0.8 \times 1.85) = 2.1$ Mg m^{-3}

$\tilde{E}_L = 1/[(0.2/380) + (0.8/44.5)] = 54.0$ GPa

(c)

Composite	$\tilde{E}/\tilde{\rho}$ (GPa Mg^{-1}m^3)
Carbon-fibre/epoxy resin	129
Glass-fibre/polyester resin	20
Mg/SiC particulate	26
Steels	28

Carbon-fibre/epoxy resin has much higher specific stiffness than steels, while the MMC is comparable to steels, and the glass-fibre polyester has a lower value.

[*Footnote*: specific stiffness is a good indicator for tensile loading, but remember that other combinations of E and ρ are usually appropriate in *bending* – e.g. $E^{1/3}/\rho$. In this case it is readily shown that all of the composites perform better than steels. But there will be other important factors to consider in using one of the composites instead of steel: geometry and shape limits, cost, strength, durability, ability to manufacture and join etc.]

Exercise E4.16 Read the modulus E for polypropylene (PP) from the $E-\rho$ chart of Figure 4.8. Estimate the modulus of a PP foam with a relative density $\tilde{\rho}/\rho_s$ of 0.2.

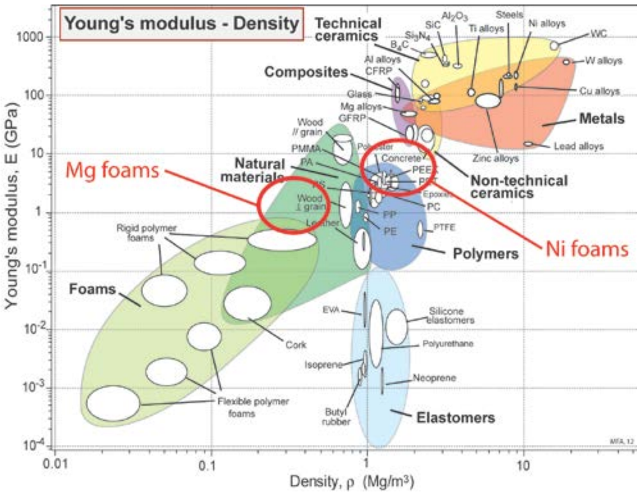
Answer. Figure 4.8 shows polypropylene at a modulus of about 1.5 GPa. Using equation (4.27) the modulus of a polypropylene foam of relative density $\tilde{\rho}/\rho_s$ of 0.2 is 60 MPa.

Exercise E4.17 Use the relationship $\tilde{E} = E_s\left(\frac{\tilde{\rho}}{\rho_s}\right)^2$ (equation (4.27)) to estimate the ranges of modulus and density that might be achievable by foaming the following metals to relative densities in the range 0.1 – 0.3: (a) Ni alloys (E = 205 GPa, ρ = 8.89 Mg/m^3); (b) Mg alloys (E = 44.5 GPa, ρ = 1.85 Mg/m^3). Locate approximate 'property bubbles' for these new metallic foams on a copy of the modulus-density chart. With which existing materials would these foams appear to compete, in terms of modulus and density? Can you think of a property for which the metallic foams will differ significantly from the competition?

Answer.

	E (GPa)	ρ (Mg/m^3)	$\tilde{\rho}$ (foam) (Mg/m^3)	$\tilde{E}$ (foam) (GPa)
Ni alloy	205	8.89	0.89 – 2.7	2.05 – 18.5
Mg alloy	44.5	1.85	0.19 – 0.56	0.45 – 4.0

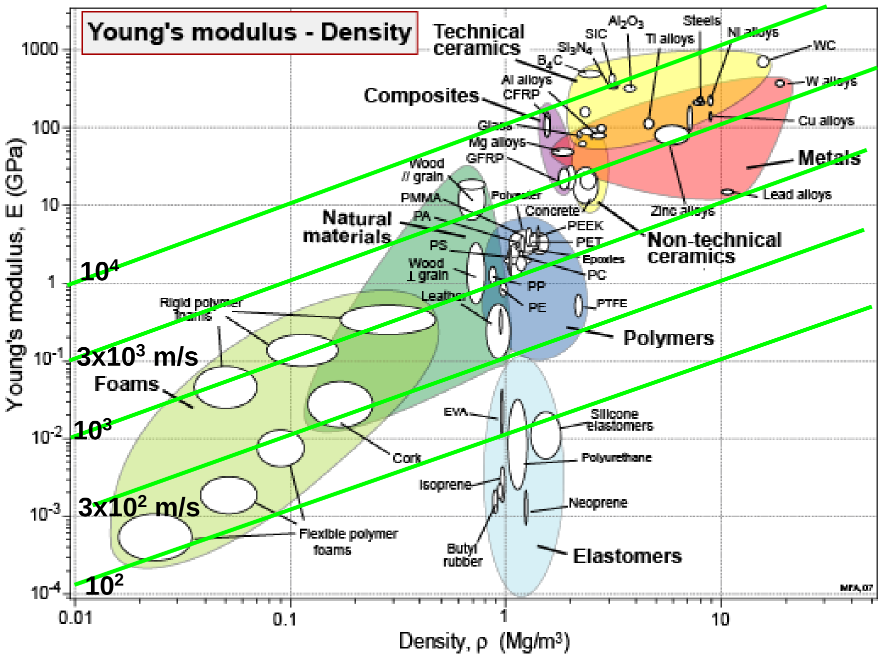
The prototype metallic foams are shown on the Figure below. Ni foams of this relative density are similar to thermosets and a little heavier than the denser woods; Mg foams of this relative density are a little stiffer than the lower density woods and rigid polymer foams.



One property which would be significantly different for the metallic foams would be their *thermal* properties – higher thermal conductivity than woods, thermosets or polymer foams. Ni foams in particular would also be resistant to much higher temperatures. Mg foams might actually present something of a fire hazard (reactive metal, and large surface area in a foam).

Exercise E4.18 The speed of longitudinal waves in a material is proportional to $\sqrt{E/\rho}$. Plot contours of this quantity onto a copy of a modulus-density chart allowing you to read off approximate values for any material on the chart. Which metals have the about the same sound velocity as steel? Does sound move faster in titanium or glass?

Answer. The figure shows the chart of Figure 4.6 onto which contours of $\sqrt{E/\rho}$ are plotted (remember to multiply E by 10^9 and ρ in the units of this chart by 10^3 in order to get the velocity in m/s). Tungsten, titanium, nickel, aluminum, magnesium and steel all have about the same value of $\sqrt{E/\rho}$, and thus similar sound velocities. The sound velocity in glass is a little higher than that in titanium.



Exercise E4.19 The density ρ of carbon steel is 7,500 kg/m^3 and its Young's E modulus is 210 GPa. What is the velocity of longitudinal sound waves in steel? Watch the units!

Answer. The velocity of sound waves in steel is $v_1 = \sqrt{\dfrac{E}{\rho}} = \sqrt{\dfrac{210\times10^9}{7{,}500}} = 5{,}290\ m/s$

Exercise E4.20 Given the data in the previous question, what is the acoustic impedance of steel?

Answer. The acoustic impedance $Z = \sqrt{\rho E} = \sqrt{210\times10^9\times7{,}500} = 4.0\times10^7\ \ N.s/m^3$

Exercise E4.21 The acoustic impedance of water is 1.5×10^6 N.s/m^3. If a sound wave of unit power in water impinges on a steel ship's hull with an impedance of 4×10^7 N.s/m^3, what fraction of the power is transmitted into the steel?

Answer. The fraction of the amplitude of the sound wave that is transmitted is $T = \dfrac{4\,Z_1 Z_2}{(Z_2+Z_1)^2}$ where Z_1 is the impendence in the medium in which the sound originates and Z_2 is that of the material into which it is transmitted. Thus the transmitted fraction is

$$T = \frac{4\times1.5\times10^6\times4\times10^7}{\left(4\times10^7+1.5\times10^6\right)^2} = 0.14$$

Thus 14% of the acoustic power is transmitted into the steel, 86% is reflected.

Exercise E4.22 Ultrasonic testing relies on reflection of sound wave from cracks or imperfections in castings and forgings. If the resolution (the smallest imperfection it can detect) of the method is the wavelength, will a tester operating at 1 megahertz be able to detect a 1 mm sized imperfection an aluminum casting?

Answer. The wavelength of 1 megahertz sound in steel is

$$\lambda = \frac{v}{f} = \frac{\sqrt{E/\rho}}{f} = \frac{5{,}290}{1\times10^6} = 5.3\times10^{-3}m = 5.3\,mm$$

Thus the 1 megahertz tester will not detect a 1 mm crack. A 5 megahertz tester might just do so.

Exercise E4.23 Sonar depends of the reflection of sound waves in water. Would sonar more easily detect a large object made of polypropylene (PP) or one of similar size made of aluminum? Use Figure 4.29 to find out.

Answer. Figure 4.29 shows that the acoustic impedance of polypropylene is almost identical with that of water so the PP object will reflect very little of the acoustic energy. Aluminum, by contrast, has an impedance that is nearly 10 times larger than water, and will reflect well.

Exercise E4.24 Practice visualizing radiation factor and acoustic impedance. Make a copy of the modulus–density chart of Figure 4.8 and plot on it a set of contours showing the radiation factor

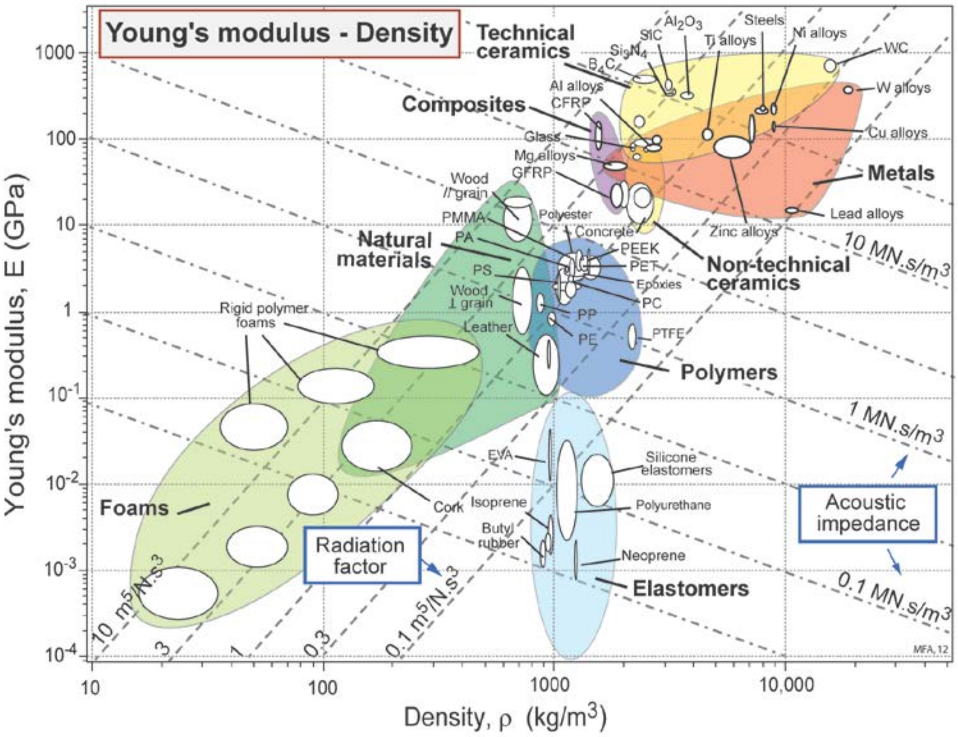
$$I = \sqrt{\frac{E}{\rho^3}}$$

and a set for acoustic impedance

$$Z = \sqrt{\rho E}$$

An innovative guitar-maker plans to make acoustic guitars with CFRP or GFRP composite front faces instead of the more usual choice of spruce, chosen for its high radiation factor. What can you learn from the chart that might help choose between CFRP and GFRP?

Answer. The figure shows the chart with contours of the two acoustic properties. The relevant modulus for wood is $\overline{E} = \sqrt{E_{||} E_{\perp}}$ where $E_{||}$ is the modulus parallel to the grain and $E_{\perp}$ is that across the grain. Both are shown on the chart; the geometric mean $\overline{E}$ lies half way between them, approximately on the contour of $I = 3\, m^5 / N.s^3$. CFRP has a somewhat higher radiation factor; that of GFRP is lower. We would expect the CFRP instrument to have the greater volume. The GFRP has an impedance that more closely matches that of wood.



Exercise E4.25 High buildings on either side of busy streets reflect the noise of traffic and add to the stress of city life. Buildings can be faced with wood, concrete or glass. Use approximate data read from Figure 4.29 to estimate the relative reflectance of the three types of facing.

Answer. The reflectance depends on the difference in acoustic impedance between the material in which the sound arrives and that into which it is transmitted. The acoustic impedance of air is very small, so the material with the largest impedance reflects the most sound. From Figure 4.29 approximate acoustic impedance of wood, concrete and glass are:

Material	Acoustic impedance (MN.s/m^3)
Wood	3
Concrete	7
Glass	15

Thus we anticipate that wood reflects the least sound, glass reflects the most.

Exploring design with CES. *Use Level 2, Materials, throughout.*

Exercise E4.26 Make an $E - \rho$ chart using the CES software. Use a box selection to find three materials with densities between 1000 and 3000 kg/m^3 and the highest possible modulus.

Answer.

- Boron carbide
- CFRP, epoxy matrix (isotropic)
- Silicon

Exercise E4.27 Explore *data estimation*. The modulus E is approximately proportional to the melting point T_m in Kelvin (because strong inter-atomic bonds give both stiffness and resistance to thermal disruption). Use CES to make an $E - T_m$ chart for metals and estimate a line of slope 1 through the data. Use this line to estimate the modulus of cobalt, given that it has a melting point of 1760 K.

Answer. A line of slope 1, sketched through the data for Young's modulus E (GPa) plotted against absolute melting point T_m (K) for metals, gives the relationship

$$E / T_m \approx 0.1$$

Thus cobalt, with a melting point of 1760 K, should have a modulus of about 180 GPa. (It's modulus is, in fact, 205 GPa).

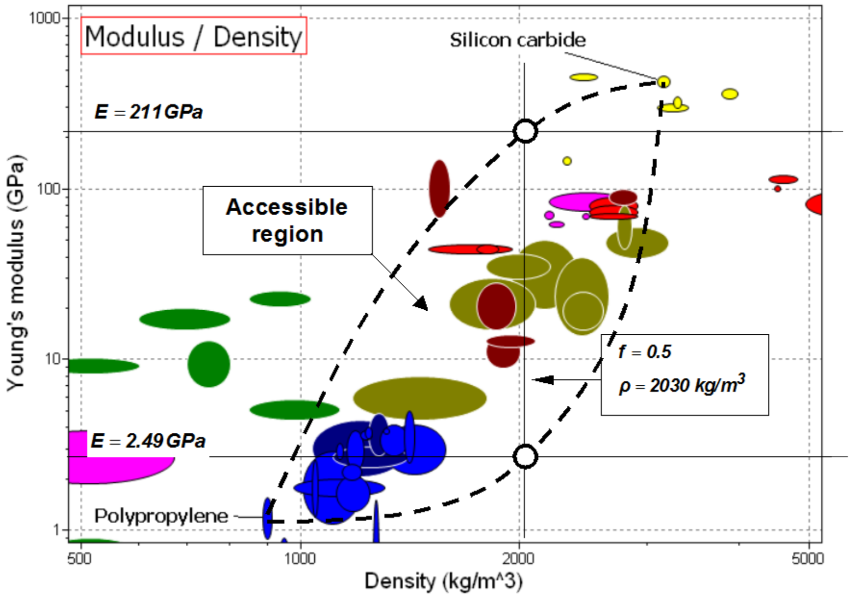
Exercise E4.28 Using *sanity checks for data*. A text reports that nickel, with a melting point of 1720 K has a modulus of 5500 GPa. Use the $E - T_m$ correlation of the previous question to check the sanity of this claim. What would you expect it to be?

Answer. Based on the correlation of Exercise E4.27, we would expect the modulus of nickel to be in the vicinity of 170 GPa – the value of 5500 is clearly incorrect.

Exercise E4.29 Explore the potential of PP-SiC (polypropylene – silicon carbide) fiber composites in the following way. Make a modulus – density ($E - \rho$) chart and change the axis ranges so that they span the range $1 < E < 1000$ GPa and $500 < \rho < 5000$ kg/m^3. Find and label PP and SiC, then print it. Retrieve values for the modulus and density of PP and of SiC from the records for these materials (use the means of the ranges). Calculate the density $\tilde{\rho}$ and upper and lower bounds for the modulus $\tilde{E}$ at a volume fraction f of SiC of 0.5 and plot this information on the chart. Sketch by eye two arcs starting from (E, ρ) for PP, passing through each of the $(\tilde{E}, \tilde{\rho})$ points you have plotted and ending at the (E, ρ) point for SiC. PP-SiC composites can populate the area between the arcs roughly up to $f = 0.5$ because it is not possible to insert more than this.

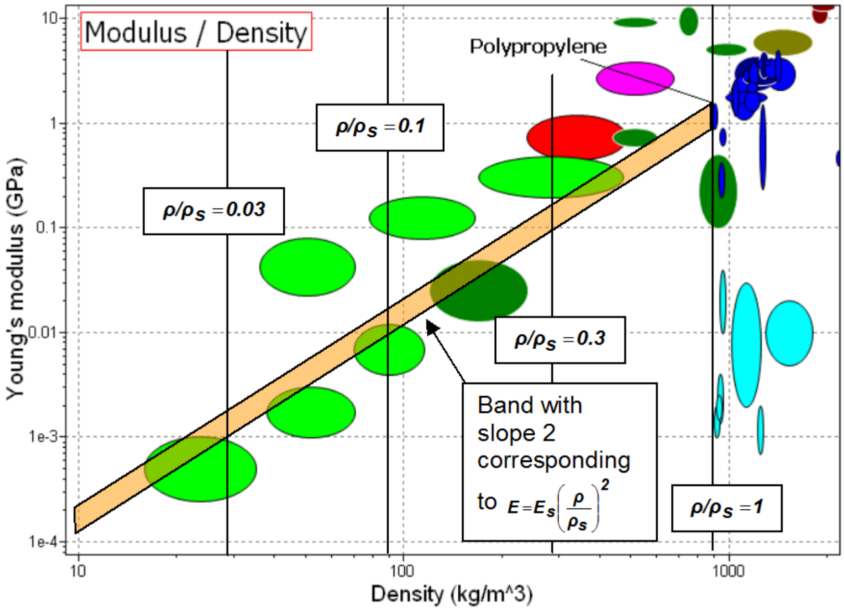
Answer. Data retrieved from the records for SiC and Polypropylene, and moduli and densities calculated form equations (4.23)–(4.25) are tabulated. The plot, with the bounds sketched in is shown.

	Density, kg/m^3	Modulus, GPa
Polypropylene	900	1.2
Silicon carbide	3150	420
Upper bound, $f = 0.5$	2030	211
Lower bound, $f = 0.5$	2030	2.49



Exercise E4.30 Explore the region that can be populated by making PP foams. Expand an $E-\rho$ plot so that it spans the range $10^{-4} < E < 10$ GPa and $10 < \rho < 2000$ kg/m^3. Find and label PP, then print the chart. Construct a band starting with the PP bubble by drawing lines corresponding to the scaling law for foam modulus $\tilde{E} \propto \tilde{\rho}^2$ (equation 4.27) touching the top and the bottom of the PP bubble. The zone between these lines can be populated by PP foams.

Answer. The figure shows the construction and the range of moduli available to polypropylene foams.



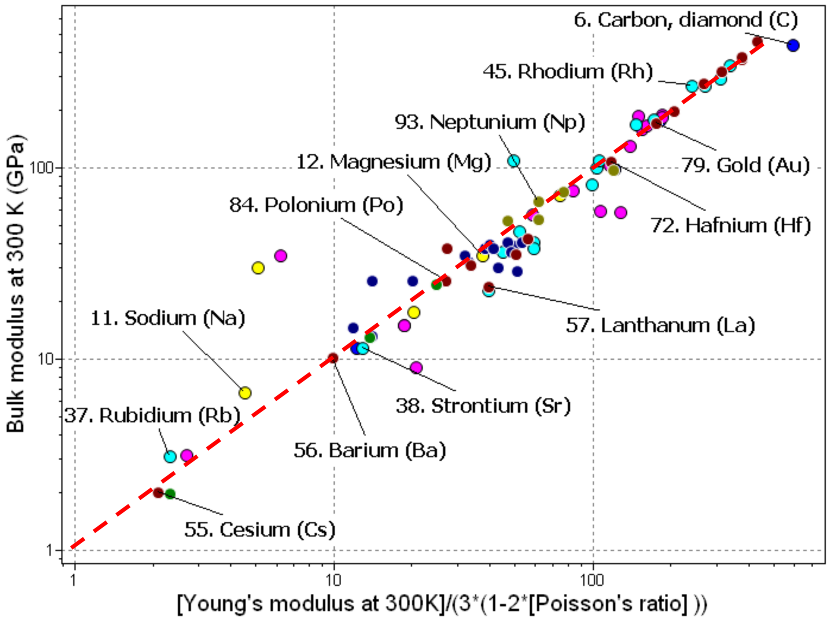
Exploring the science with CES Elements

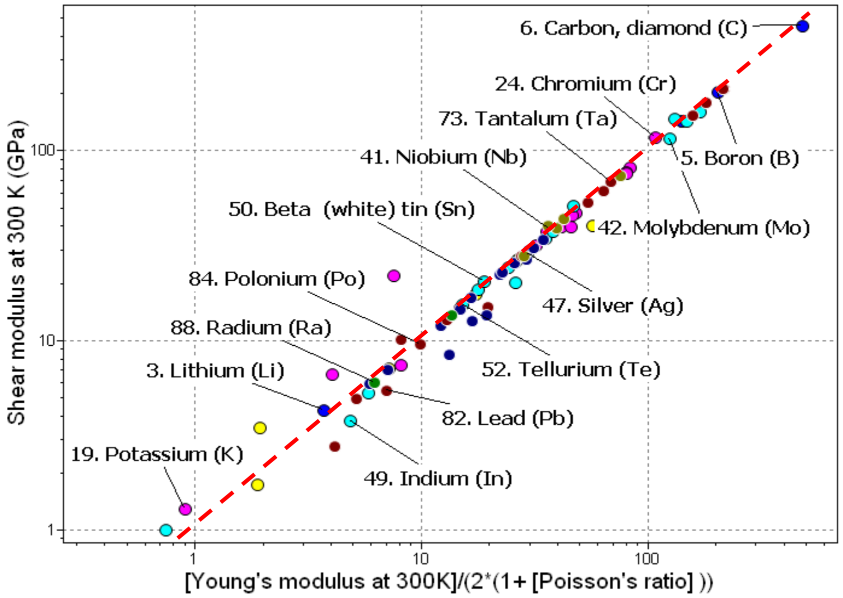
Exercise E4.31 The text cited the following approximate relationships between the elastic constants Young's modulus, E, the shear modulus, G, the bulk modulus K and Poisson's ratio, ν.

$$G = \frac{E}{2(1+\nu)}; \qquad K = \frac{E}{3(1-2\nu)}$$

Use CES to make plots with the bit on the left hand side of each equation on one axis and the bit on the right on the other. To do this you will need to use the 'Advanced' facility in the dialog box for choosing the axes to create functions on the right of the two equations. How good an approximation are they?

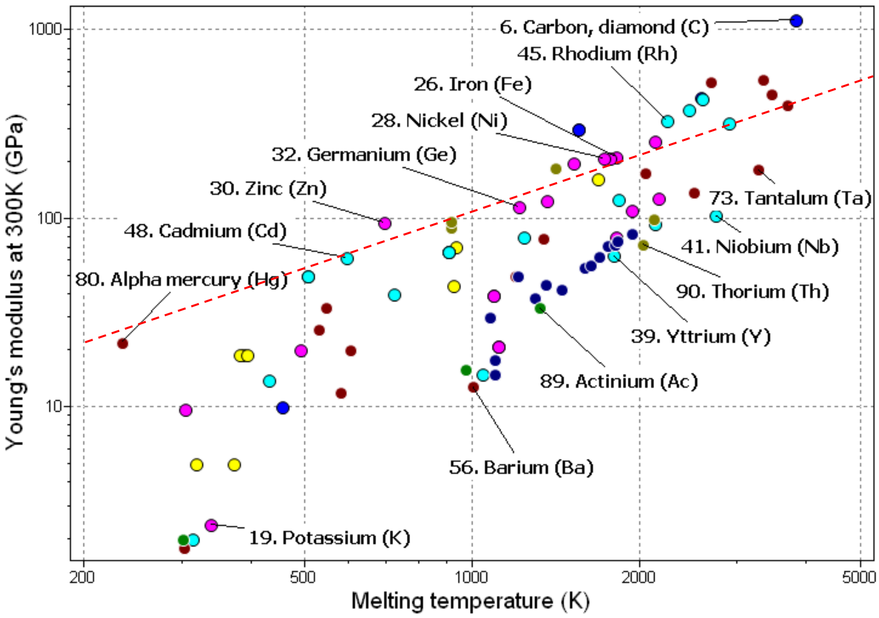
Answer. See the two charts. The broken red lines plot the equations. The approximations are, for most elements, very good.





Exercise E4.32 The cohesive energy H_c is the energy that binds atoms together in a solid. Young's modulus E measures the force needed to stretch the atomic bonds and the melting point, T_m, is a measure of the thermal energy needed to disrupt them. Both derive from the cohesion, so you might expect E and T_m to be related. Use CES to plot one against the other to see (use absolute melting point, not centigrade or Fahrenheit).

Answer. There is clearly a relationship between the two. It is not, however, a completely linear one; different bond-types have different ratios of modulus to melting point. If the plot is restricted to one type only, the relationship is more nearly linear.



Exercise E4.33 The force required to stretch an atomic bond is

$$F = \frac{dH}{da}$$

where dH is the change in energy of the bond when it is stretched by da. This force corresponds to a stress

$$\sigma = \frac{F}{a_o^2} = \frac{1}{a_o^2}\frac{dH}{da} = \frac{1}{a_o^3}\frac{dH}{d(a/a_o)}$$

The modulus E is

$$E = \frac{d\sigma}{d\varepsilon} = \frac{d\sigma}{d(a/a_o)} = \frac{1}{a_o^3}\frac{d^2H}{d(a/a_o)^2}$$

The binding energy per atom in a crystal, H_a is

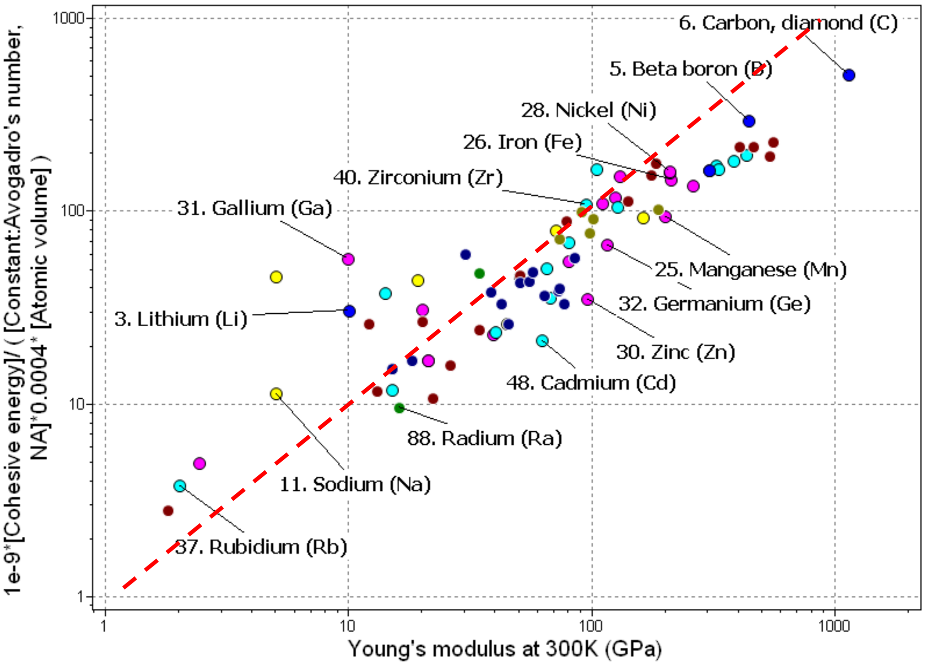
$$H_a = \frac{H_c}{N_A}$$

where H_c is the cohesive energy and N_A is Avogadro's number (6.022×10^{23}). If we assume that a stretch of 2% is enough to break the bond, we can make the approximation

$$\frac{d^2H}{d(a/a_o)^2} \approx \frac{H_c}{(0.02)^2} \quad \text{giving} \quad E \approx \frac{1}{a_o^3}\left(\frac{N_c}{0.0004.N_A}\right)$$

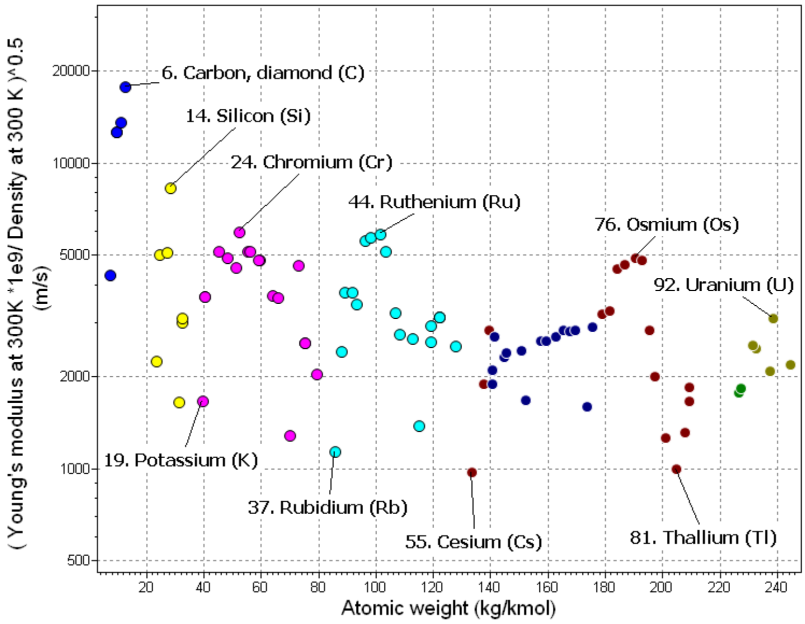
Make a plot of Young's modulus E against the quantity on the right of the equation (using the 'Advanced' facility in the dialog box for choosing the axes) to see how good this is. (You will need to multiply the right by 10^{-9} to convert it from Pascals to GPa.)

Answer. The chart shows that there is an approximately linear relationship between the left and right hand sides of the equation. The red line is a plot of the equation above. The correlation is pretty good.



Exercise E4.34 Make a chart of the sound velocity $(E/\rho)^{1/2}$ for the elements. To do so, construct the quantity $(E/\rho)^{1/2}$ on the y-axis using the 'Advanced' facility in the axis-choice dialog box, and plot it against atomic number A_n. Use a linear scale for A_n. To do so, change the default log scale to linear by double clicking on the axis name to reveal the axis-choice dialog box and choose 'Linear'. And multiply E by 10^9 to give the sound velocity in m/s.)

Answer. The values of sound velocity, $(E/\rho)^{1/2}$, range from 1000 m/s (Cesium) up to nearly 20,000 m/s (Diamond). Most cluster around 3000m/s. The velocity peaks in the middle of each row of the Periodic table.

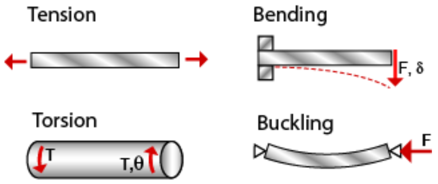


Materials: engineering, science, properties, and design
3e Solution manual

Chapter 5: Exercises with worked solutions

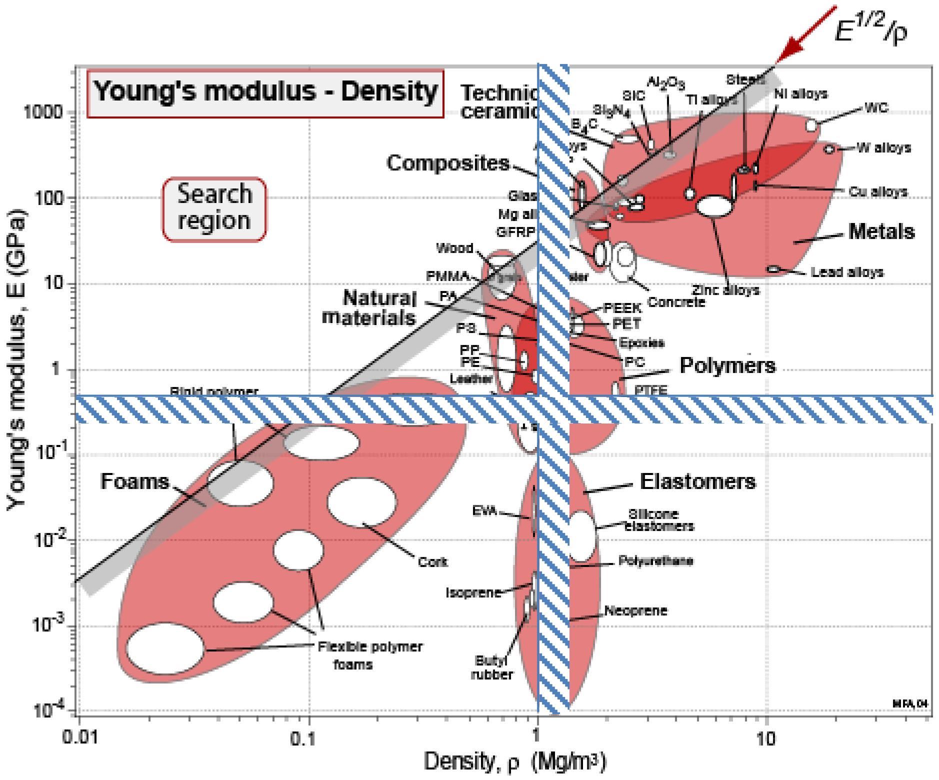
Exercise E5.1 Distinguish between tension, torsion, bending and buckling.

Answer. This is best answered by a sketch.



Exercise E5.2 Polymer ropes and lines for use on water are often designed to float, to aid in their retrieval and to avoid applying a downwards load to an object or person attached to them in the water. Excessive stretch is undesirable, so a lower limit of 0.5 GPa is also imposed on Young's modulus. Identify suitable polymers, using Figure 5.14.

Answer. On Figure 5.14, draw a vertical line corresponding to a density of 1 Mg/m^3 (the density of water). Draw a horizontal line with a modulus of 0.5 GPa. The polymers in the top left quadrant of the chart satisfy the modulus and density criteria – they are candidates for ropes. Polyethylene PE and Polypropylene PP would be suitable choices.

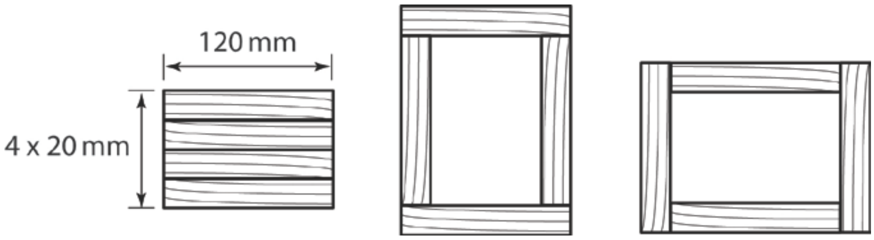


Exercise E5.3 A beam is to be made by gluing and screwing-together four wooden planks of length 2m, thickness 20mm and width 120mm.

(a) Determine the second moment of area I for the three configurations shown in cross-section in the figure below.

(b) Calculate the mid-span deflection δ of each of the beams when they are simply supported at their ends and loaded at mid-span with a force F of 100N.

(c) Determine the stiffness F/δ for each of the beam configurations.



Answer.

(a) (i) Use the second moment of area for a rectangular shape of width *b* and depth *d*:

$$I = \frac{b\,d^3}{12} \text{ with b = 120mm, d = 80mm, giving: } I = \frac{120(80)^3}{12} = 5.12\times10^6\ mm^4$$

(ii) The second moment of area of the outer rectangle is $I_{outer} = \frac{bd^3}{12} = \frac{120(120+2\times20)^3}{12}$

The second moment of area of the 'hole' is $I_{hole} = \frac{bd^3}{12} = \frac{(120-2\times20)(120)^3}{12}$

The total moment of area is the difference: $I_{total} = I_{outer} - I_{hole} = 29.44\times10^6\ mm^4$

(iii) $I_{total} = I_{outer} - I_{hole} = \frac{160(120)^3}{12} - \frac{120(80)^3}{12} = 17.92\times10^6\ mm^4$

The conclusion is that for a fixed cross section area – i.e. the same mass of timber – the tall fabricated box section (the middle figure) has nearly six times the second moment of area of the simple stack of planks on the left and 1.6 times the second moment of area of the fabricated box on the right. This is because the middle beam has more material located further from the axis of bending.

(b) Convert to 'base' SI units: m, N and N/m^2: For a simply-supported beam, loaded in mid-span is given by equation (5.5) with C_1 =48, from Figure 5.3: $\delta = \frac{F\,L^3}{48\,E\,I}$. Taking the Young's Modulus of wood to be 10 GPa = 10 × 10^9 N/m^2:

(i) I = 5.12 × 10^{-6} m^4, giving $\delta = \frac{1000\,(2)^3}{48\left(10\times10^9\right)\left(5.12\times10^{-6}\right)} = 0.0033\,m = 3.3\,mm.$

(ii) I = 29.44 × 10^{-6} m^4, giving $\delta = \frac{1000\,(2)^3}{48\left(10\times10^9\right)\left(29.44\times10^{-6}\right)} = 0.57\,mm.$

(iii) I = 17.92 × 10^{-6} m^4, giving $\delta = \frac{1000\,(2)^3}{48\left(10\times10^9\right)\left(17.92\times10^{-6}\right)} = 0.93\,mm.$

(c) Stiffness F/δ: (i) 1000/0.0033 = 3 × 10^5 *N/m*; (ii) 1.75 × 10^6 *N/m;* (iii) 1.08 × 10^6 *N/m*

Exercise E5.4 What is meant by a *material index*?

Answer. A material index measures the excellence of a material in a given application. Material indices are used to rank materials that meet the constraints imposed by a design. Sometimes the material index for a design is a single property, like strength σ_y or thermal conductivity λ. More often it is a combination of properties, like the product $\sigma_y\,\lambda$.

Exercise E5.5 Your task is to design a light-weight tie of length L with a circular cross-section of radius R. It has to carry an axial force F, without stretching by more than δ. You will need to choose the material (with Young's modulus E and density ρ) and the corresponding cross section radius to suit your choice of material, i.e. R is a 'free variable'.

(a) Show that the extension of the tie is given by $\delta = FL/AE$, where $A = \pi R^2$

(b) Rearrange this equation to find an expression for the radius R of the tie that will carry the load without excessive deflection.

(c) If the length of the tie is L=0.3m and the extension is not to exceed 0.1mm for a load of F=100N, what value of R is needed if the tie is made of: (i) PEEK; (ii) Butyl Rubber; (iii) Titanium; (iv) Copper? Use the material properties in the table below.

(d) Write an expression for the mass m of the tie and determine the mass of each of the ties in (c). Which one would you choose?

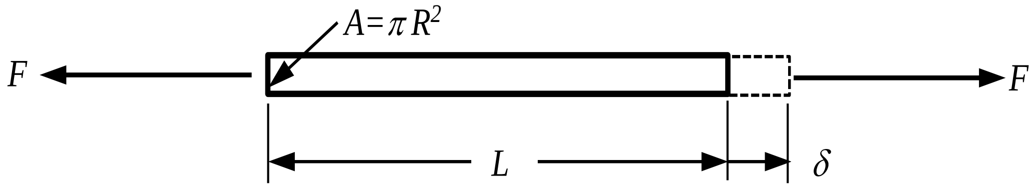
(e) Substitute the expression for radius R from (b) into the expression for mass m from (d) to show that the material index to be *minimised* is $\rho/$E. Determine the value of the material index for the four ties.

(f) Comment on the relationship between the mass of each beam and its material index $\rho/$E.

(g) Examine Figure 5.14 to see how these three materials compare in terms of the reciprocal material index E/ρ.

	Young's Modulus E (GPa)	Density ρ (kg/m^3)	Radius R (mm)	Mass m (kg)	Material index $\rho/$E
PEEK	3.8	1300			
Butyl Rubber	0.0015	2400			
Titanium alloy	110	4600			
Copper alloy	120	8900			

Answer.



(a) Stress: $\dfrac{\sigma}{\varepsilon} = E$; $\sigma = \dfrac{F}{A}$; $\varepsilon = \dfrac{\delta}{L}$. So: $\dfrac{F/A}{\delta/L} = E$; giving $\delta = \dfrac{FL}{AE}$ (1)

(b) $A = \pi R^2 = \dfrac{F\,L}{\delta\,E}$; giving $R = \sqrt{\dfrac{F\,L}{\pi\,\delta\,E}}$ (2)

(c) Use values in the table below with F = 1000 N, δ = 0.1mm and L = 0.3 m. For PEEK, this gives:

$$R = \sqrt{\frac{1000 \times 0.3}{\pi\left(0.1\times10^{-3}\right)\left(3.8\times10^{9}\right)}} = 15.9\,mm$$

(d) $m = \rho\,A\,L = \rho\,\pi\,R^2\,L$. For PEEK, $m = 1300\,\pi\left(15.9\times10^{-3}\right)^2(0.3) = 0.31\,kg$.

(e) $m = \rho\,\pi\,R^2\,L$ Using (2) this gives $m = \rho\,\pi\left[\dfrac{F\,L}{\pi\,\delta\,E}\right]L = \dfrac{F\,L^2}{\delta}\left(\dfrac{\rho}{E}\right)$. So the material index to be minimized is ρ/E.

(f) The material indices are proportional to the mass of the tie, and consequently they rank the materials in the same order. For example, compare copper and titanium in the table: $m_{Cu}/m_{Ti} = 1.6$;

$$\left(\frac{\rho}{E}\right)_{Cu} \Big/ \left(\frac{\rho}{E}\right)_{Ti} = 1.6, \quad \text{etc.}$$

(g) Selecting materials to <u>minimize</u> ρ/E is the same as selecting materials to <u>maximize</u> E/ρ. Materials with high values of E/ρ are furthest to the top left of Figure 5.14 above a line of slope 1. Titanium is the best of the bunch.

Material	Youngs Modulus E (GPa)	Density ρ (kg/m^3)	R (mm)	m (kg)	$\rho/$E
PEEK	3.8	1300	15.9	0.31	342
Butyl Rubber	0.0015	2400	798	1440	1.6×10^6
Titanium alloy	110	4600	2.9	0.038	41.8
Copper alloy	120	8900	2.8	0.067	74.2

Exercise E5.6 You are asked to design a light-weight cantilever beam of length L with a square cross-section of width b and depth b. The beam is to be built-in to a wall at one end and loaded at the free end with a point load F. The tip deflection is δ. When designing the beam, you are free to specify the value of b to suit your choice of material, i.e. b is a ‘free variable’.

(a) Find an expression for the size of the beam b that can carry F with deflection δ.

(b) If the length of the cantilever is L=1m and the deflection δ is not to exceed 5mm for a load of F=1000N, what value of b is needed if the beam is made of: (i) Steel; (ii) Aluminium; (iii) CFRP? Use the material properties in the table below.

(c) Write an expression for the mass m of the beam and determine the mass of each of the beams in (b). Which one would you choose?

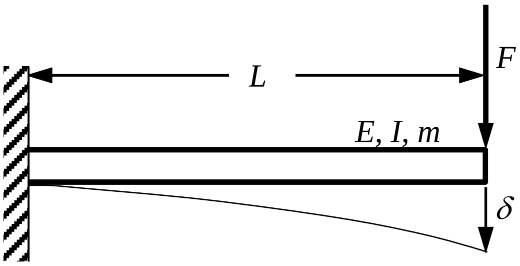
(d) Substitute the expression for beam dimension b from (a) into the expression for mass m from (c) to show that the material index to be *minimised* is $\rho/\mathrm{E}^{1/2}$. Determine the value of the material index for the three beams.

(e) Comment on the relationship between the mass of each beam and its material index $\rho/\mathrm{E}^{1/2}$.

(f) Examine Figure 5.14 to see how these three materials compare in terms of the reciprocal material index $\mathrm{E}^{1/2}/\rho$.

	Young’s Modulus E (GPa)	Density ρ (kg/m^3)	Section size b (mm)	Mass m (kg)	Material index $\rho/\mathrm{E}^{1/2}$
Steel	210	7800			
Aluminium	80	2600			
CFRP	100	1500			

Answer.



(a) The deflection of the cantilever beam is given by equation 5.5 with C_1 =3, from Figure 5.3, i.e.

$$\frac{F}{\delta}=\frac{3EI}{L^3} \qquad (1)$$

For a beam with cross section b × b,

$$I=b^4/12 \qquad (2)$$

Combining (1) and (2) gives

$$I=\frac{b^4}{12}=\frac{FL^3}{3E\delta} \quad \text{and so} \quad b=\left[\frac{12FL^3}{3E\delta}\right]^{1/4} \qquad (3)$$

(b) The numbers for steel are: $b=\left[\frac{12(1000)1^3}{3\left(210\times10^9\right)\left(5\times10^{-3}\right)}\right]^{1/4}=0.044m=44mm.$

(c) The mass of the beam is $m=b^2L\rho$ (4)

For steel, $m=(0.044)^2(1)(7800)=15.2$ kg. The CFRP beam is the lightest by far: 4.2 kg.

(d) Substitute for b from (3) into (4): $m=\left[\frac{12FL^3}{3E\delta}\right]^{1/2}L\rho=\left[\frac{2F^{1/2}L^{5/2}}{\delta^{1/2}}\right]\left[\frac{\rho}{E^{1/2}}\right]$

For steel the material index is $\frac{\rho}{E^{1/2}}=\frac{7800}{\left(210\times10^9\right)^{1/2}}=0.017.$

(e) The masses of the beams are proportional to $\rho/E^{1/.2}$, so you don't need to calculate b and m in order to know that the CFRP beam will be the lightest… just compare values of $\rho/E^{1/.2}$ for the three materials. The same result holds if you change the boundary conditions – for example, use simple supports instead of a cantilever… All that changes is the value of C_1 in Figure 5.3 - and that doesn't affect the material performance index. Of course, the boundary conditions do affect the deflection – so if you have different boundary conditions, you will need to calculate a new value for the cross section of the beam, *b*.

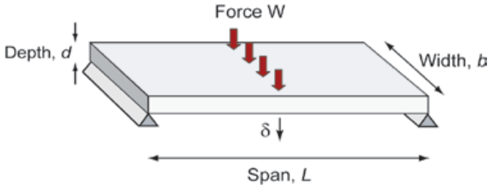
(f) Selecting materials to minimize $\rho/E^{1/.2}$ is the same as selecting materials to maximize $E^{1/2}/\rho$. The best materials are ones nearest to the top left of Figure 5.14 above a line of slope 2. CFRP is best followed by Aluminium and steel. It isn't surprising that aircraft wings are made of Aluminium or, more recently, CFRP.

Material	Youngs Modulus E (GPa)	Density ρ (kg/m^3)	b (mm)	m (kg)	$\rho/E^{1/2}$
Low Alloy Steel	210	7800	44	15.2	0.017
Aluminium	80	2600	56.2	8.22	0.0092
CFRP	100	1500	53.2	4.24	0.0047

Exercise E5.7 The objective in selecting a material for a panel of given in-plane dimensions for the casing of a portable computer is that of minimizing the panel thickness d while meeting a constraint on bending stiffness, S^*. What is the appropriate material index?

Answer. The material index is simply the modulus: $M = E$.

Exercise E5.8 A sailing enthusiast is seeking materials for lightweight panels to use in a sea-going yacht. The panels are of rectangular cross-section and will be loaded in bending, as shown in the figure. The span L and width b of the panels are fixed, but the depth d may vary (up to a maximum specified value). The required stiffness is specified as a maximum allowable deflection δ under a given central load W.



The central deflection for the simply supported span shown is given by:

$$\delta = \frac{W L^3}{48 E I} \quad \text{where} \quad I = \frac{b d^3}{12}$$

The designer is interested in two scenarios: (i) minimum mass; (ii) minimum material cost.

(a) Show that the stiffness and geometric constraints lead to the relationship $E d^3 = \text{constant}$, where E is Young's modulus. Explain why the design specification leads to a minimum allowable value for E, and find an expression for this minimum value.

(b) Write down an expression for the first objective to be minimised (mass) and use the stiffness constraint to eliminate the free variable (thickness). Hence define the material performance index to *maximise* for a minimum mass design.

(c) On a Young's modulus – density property chart (Figure 5.14), how are materials with the best values of the performance index identified? Use this method to identify a short-list of candidate materials (excluding ceramics and glasses - why is this?)

(d) How does the short-list of materials in (c) change if the limit on depth leads to a requirement for a minimum required Young's modulus of 5 GPa?

(e) Modify your material performance index to describe the alternative objective of minimum material cost. Use the material property chart in Figure 5.16 to identify a short-list of materials, noting any which may be excluded because they have excessive thickness.

Answer.

(a) For the stiffness constraint: the panel deflection $\delta = \dfrac{W L^3}{48 E I}$.

Substituting for $I = \dfrac{b d^3}{12}$: $\delta = \dfrac{12\, W L^3}{48 E b d^3} \Rightarrow E d^3 = \dfrac{12\, (W/\delta)\, L^3}{48 b} = \text{constant}$, since all the parameters on the RHS are specified.

Since an upper limit on d is specified, this translates into a minimum E to satisfy this relationship.

(b) Objective: minimum mass. Mass = $m = \rho b d L$

From constraint equation, free variable $d = \left(\dfrac{12\, (W/\delta)\, L^3}{48 b E} \right)^{1/3}$

Substituting for free variable d in objective equation: $m = \rho\, b\, L \left(\dfrac{12\,(W/\delta)\,L^3}{48\, b\, E} \right)^{1/3}$

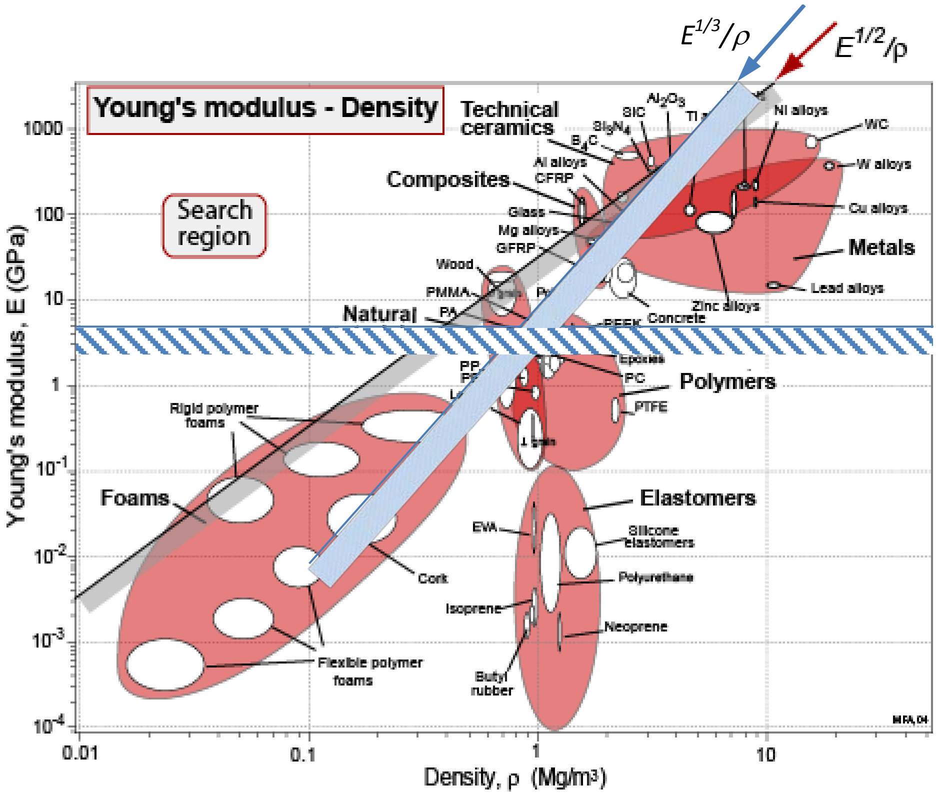
Hence $m = \left(\dfrac{\rho}{E^{1/3}} \right) \left[b\, L \left(\dfrac{(W/\delta)\,L^3}{4b} \right)^{1/3} \right] = \left(\dfrac{\rho}{E^{1/3}} \right) \times \text{constant}$

So material performance index to maximise for minimum mass is $M_1 = \left(\dfrac{E^{1/3}}{\rho} \right)$.

(c) On an E-ρ chart, constant values of the performance index have slope = 3, since:

$$\log(M_1) = \frac{1}{3}\log E - \log\rho \quad \Rightarrow \quad \log E = 3\log\rho + \log(M_1)$$

On Figure 5.14, plot a line with slope 3 positioned to leave a reasonable shortlist. Ceramics and glasses are excluded because of their brittleness. Plot another (horizontal) line with E > 5GPa. If you are adventurous, you can check the environmental resistance from the table in Chapter 17 (or use CES).



The selection?...

Material	**Comments**
CFRP	The best.
Wood	Inadequate sea water resistance… though a coat or two of durable paint might help here.
Foams	Modulus too low – would have to be too thick
Mg alloys	Good, but inadequate sea water resistance
Al alloys, GFRP	Close competition, and cheaper options than CFRP

(d) Material cost = mass $\times\, C_m$ where C_m is the material cost/kg. Since, everything else is constant, this can be incorporated directly into the performance index:

$$\text{Material cost } = m \times C_m = \left(\frac{\rho\, C_m}{E^{1/3}}\right) \times \text{constant}$$

Hence for minimum material cost, maximise the index $M_2 = \left(\frac{E^{1/3}}{\rho\, C_m}\right)$.

Use the chart in Figure 5.16 and plot a line of slope 3. The conclusions are:

Material	**Comments**
Al alloys	The best.
GFRP	More expensive, but a popular alternative for yachts
Al/SiC composite, Cu alloys, SS, POM	Possible, but 5-10 times more expensive. Panel weight may be high (e.g. Cu)
Wood	Very cheap, but inadequate sea water resistance… again, paint can help, which is the traditional solution.
Foams	Very cheap, but modulus too low – would have to be too thick

Exercise E5.9 Derive the material index for a torsion bar with a solid circular section. The length L and the stiffness S^* are specified, and the torsion bar is to be as light as possible. Follow the steps used in the text for the beam, but replace the bending stiffness $S^* = F/\delta$ with the torsional stiffness $S^* = T/(\theta/L)$ (equation 5.8), using the expression for K given in Figure 5.2.

Answer. The material index is $M = \dfrac{G^{1/2}}{\rho}$ where G is the shear modulus and ρ is the density.

Exercise E5.10 The body of a precision instrument can be modelled as a beam with length L, second moment of area I, and mass per unit length m_o. It is made of a material with density ρ and Young's modulus E. For the instrument to have highest possible accuracy, it is desired to maximize the first natural frequency of vibration, which is given by equation 5.11, with the value of C_2 from Figure 5.6:

$$f = \frac{C_2}{2\pi}\sqrt{\frac{EI}{m_o L^4}}$$

If the beam has a square cross section $b \times b$, where the value of b is free to be selected by the designer, and the stiffness S is fixed by the design, show that the best material is one with a high value of the material index $E^{1/2}/\rho$.

Answer.

From equation 5.11, the natural frequency of bending vibration is given by $f = \dfrac{C_2}{2\pi}\sqrt{\dfrac{EI}{m_0 L^4}}$. Here the cross-section area is $A = b^2$, the mass per unit length is $m_0 = m/L = \rho b^2$ and the second moment of area is $I = b^4/12$.

Putting this all together: $$f = \frac{C_2}{2\pi}\sqrt{\frac{Eb^4}{12\rho b^2 L^4}} = \frac{C_2}{2\pi}\sqrt{\frac{b^2}{12L^4}}\sqrt{\frac{E}{\rho}} \qquad (1)$$

Now the stiffness, $S = F/d$ is specified and dimension b is a free variable. So we need to choose b to satisfy the stiffness constraint.

From equation 5.5: $$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3} = \frac{C_1 E b^4}{12L^3} \qquad (2)$$

Rearranging for b gives $b = \left[\frac{12L^3S}{C_1E}\right]^{1/4}$ (3)

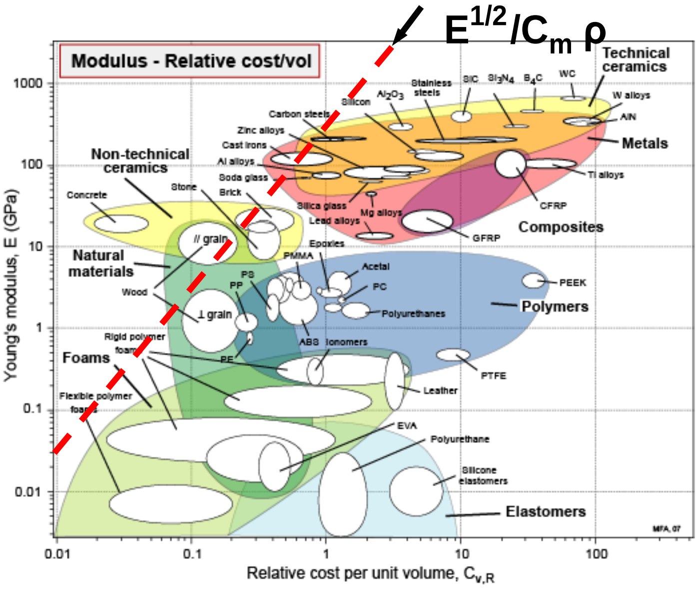
Substituting (3) into (1) gives: $f = \frac{C_2}{2\pi}\sqrt{\frac{E}{12\rho L^4}\left(\frac{12L^3S}{C_1E}\right)^{1/2}} = \frac{C_2}{2\pi}\left(\frac{S}{(12C_1)L^5}\right)^{1/4}\left(\frac{E^{1/2}}{\rho}\right)^{1/2}$

So the material index to be maximized is $\frac{E^{1/2}}{\rho}$, the same as for a 'light, stiff beam' with the same free cross section. So the statement in the text that "frequencies scale with $\sqrt{E/\rho}$" is only true if the design context (constraint and free variable) is ignored.

Exercise E5.11 A material is required for a cheap, column with a solid circular cross section that must support a load F^* without buckling. It is to have a height L. Write down an equation for the material cost of the column in terms of its dimensions, the price per kg of the material, C_m, and the material density ρ. The cross-section area A is a free variable – eliminate it by using the constraint that the buckling load must not be less than F_{crit} (equation 5.9). Hence read off the index for finding the cheapest tie. Plot the index on a copy of the appropriate chart and identify three possible candidates.

Answer.

The chart shows a copy of Figure 4.7: modulus E plotted against material cost per unit volume $C_m\rho$. A line corresponding to the material index $= E^{1/2}/C_m\rho$ is plotted on it, positioned so that only a few materials remain above it. These are the materials with the most attractive value of the index: concrete, brick, stone, cast iron and wood.



Exercise E5.12 The following table shows a selection of candidate materials for lightweight, stiffness-limited design of a beam. The shape is initially constrained to be square, but the cross sectional area can vary. Evaluate the relevant index for this problem, $E^{1/2}/\rho$, for each material. Which will be the lightest? The text showed that if the shape is allowed to vary, the mass can be improved by a factor of $\Phi^{-1/2}$, which is equivalent to increasing the material index by $\Phi^{1/2}$. Evaluate the modified index $(\Phi E)^{1/2}/\rho$, using the maximum shape factor for each material (as listed in Table 5.3). Does the ranking of the materials change?

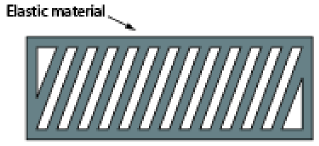
Answer.

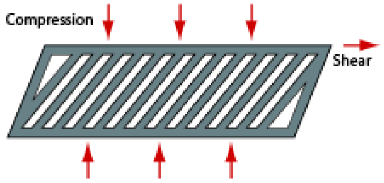
	Young's Modulus E (GPa)	Density ρ (kg/m^3)	Maximum shape factor Φ	Material index without shape $10^3 \times E^{1/2}/\rho$	Material index with shape $10^3 \times (\Phi E)^{1/2}/\rho$
Steel	210	7800	64	1.86	14.9
Al alloy	70	2700	49	3.10	**21.7**
GFRP	25	1900	36	2.63	15.8
Wood	10	650	9	**4.87**	14.6

Without shape, wood provides the lightest beam; but with maximum shape efficiency, aluminium is the best – and the relative performance of the other three become very similar.

Exercise E5.13 Devise an elastic mechanism that, when compressed, shears in a direction at right angles to the axis of compression.

Answer. The molding shown here has the required response.





Exercise E5.14 Universal joints usually have sliding bearings. Devise a universal joint that could be moulded as a single elastic unit, using a polymer.

Answer. The double loop coupling uses a moulded plastic element permanently swaged to steel or stainless steel hubs to give a compact design. It forms an effective two-stage coupling with exceptional flexibility in all three modes. Ideal for transmitting rotation in small drives, this type of coupling works without any friction, wear or noise, although its low torsional stiffness makes it less suitable for high precision positioning applications. (Courtesy of www.huco.com)

Exploring design with CES.

Exercise E5.15 Use a 'Limit' stage to find materials with modulus E > 180 GPa and price C_m < 3 \$/kg.

Answer. All are ferrous alloys:

- Cast iron, ductile (nodular)
- High carbon steel
- Low alloy steel
- Low carbon steel
- Medium carbon steel
- Stainless steel

Exercise E5.16 Use a 'Limit' stage to find materials with modulus E > 2 GPa, density ρ < 1000 kg/m^3 and price C_m < 3 \$/kg.

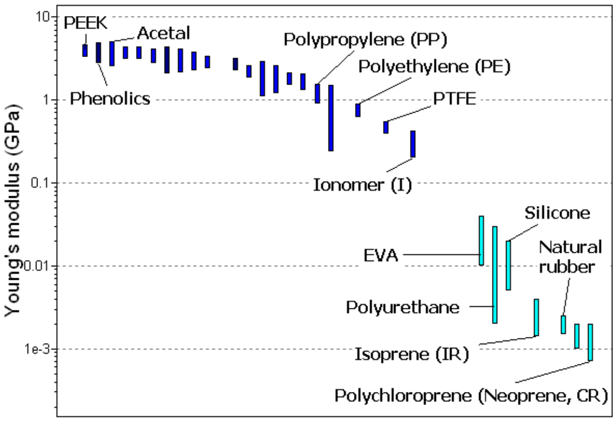
Answer. The survivors, meeting all three constraints, are:

- Bamboo
- Plywood
- Softwood: pine, along grain

Exercise E5.17 Make a bar chart of modulus, E. Add a tree stage to limit the selection to polymers alone. Which three polymers have the highest modulus?

Answer.

Several polymers have moduli of about 4 GPa. Based on the mean of the range of modulus, which is the way the software ranks materials to make bar charts, PEEK (Polyetheretherketyone) has the highest modulus (3.85 GPa). After that come Phenolics and Acetal (Polyhydroxyalkanoates).



Exercise E5.18 Repeat the material selection in Exercise E5.2 using CES. Examine the environmental resistance of the candidate materials and comment on any possible weaknesses for this application. Use the information for material applications to see if the materials identified are used in practice.

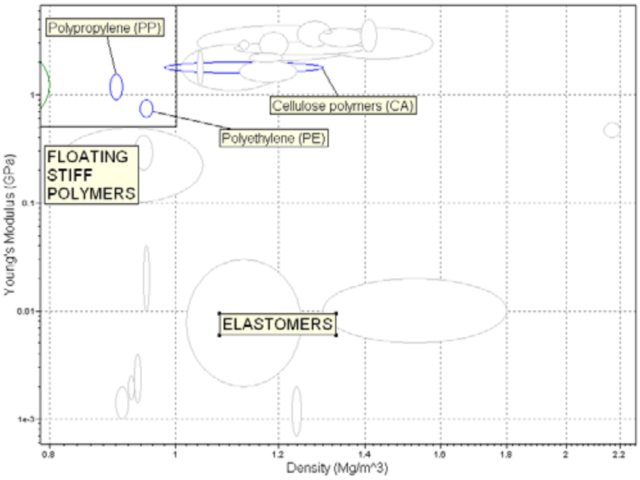
Answer.

In CES: Plot a modulus - density (restricted to Polymers), with a box selection set to a maximum of 1 Mg/m^3 (density of water), and E > 0.5 GPa.

Polymers which float ($\rho < 1$ Mgm^{-3}) and E > 0.5 GPa are:

- polypropylene (PP), polyethylene (PE);

[- also cellulose polymers (CA), but only at the very lowest densities, most would not pass this selection stage.]



Environmental resistance to Fresh Water and Salt Water are 'Very Good' for both, but 'Poor' resistance to Sunlight (UV radiation) may give problems, leading to a need for frequent replacement.

CES Results (both stages):

- [Cellulose polymers (CA)]
- Polyethylene (PE)
- Polypropylene (PP)

A CES search for 'ropes' indicates the polymers PA, PP, PE (as well as bamboo). Nylon (PA) doesn't float, but PP and PE are both used.

Exercise E5.19

(a) A component is made from brass (a copper alloy). Identify three alternative classes of alloy which offer higher Young's modulus for this component.

(b) Find materials with $E > 40$ GPa and $\rho < 2$ Mg/m^3, and identify the cheapest.

(c) Find metals and composites that are both stiffer *and* lighter than: (i) steels; (ii) Ti alloys; (iii) Al alloys.

(d) Compare the specific stiffness, E/ρ, of steels, Ti alloys, Al alloys, Mg alloys, GFRP, CFRP and wood (parallel to the grain).

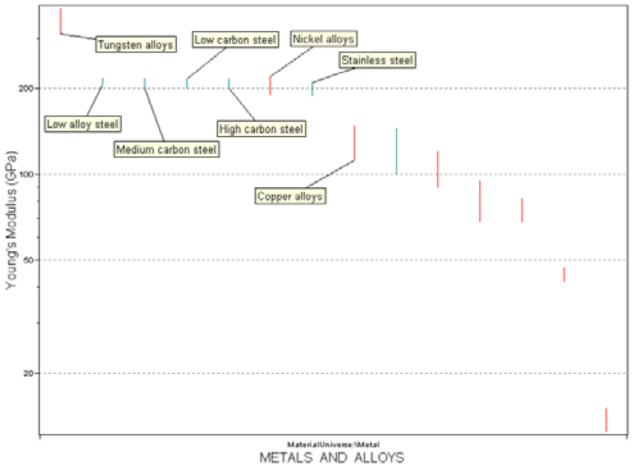
(e) Comment on the usefulness of the approaches in (c) and (d) for seeking improved performance in lightweight, stiffness-limited design.

Answer.

(a) *In CES*: plot bar chart for Young's modulus, restricted to metals.

Alloys with higher Young's modulus than Cu alloys:

- tungsten alloys
- steels (low alloy, carbon and stainless)
- nickel alloys

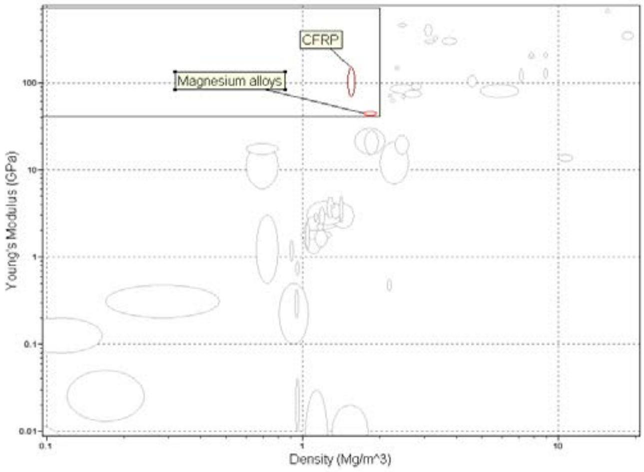


(b) *In CES*: plot Young's modulus - density chart and use a selection box. As only 2 materials pass this stage it is not worth plotting a second chart to consider cost – just browse the two records for the price/kg.

Materials with E > 40 GPa and ρ < 2 Mgm^{-3} are:

- CFRP
- Mg alloys

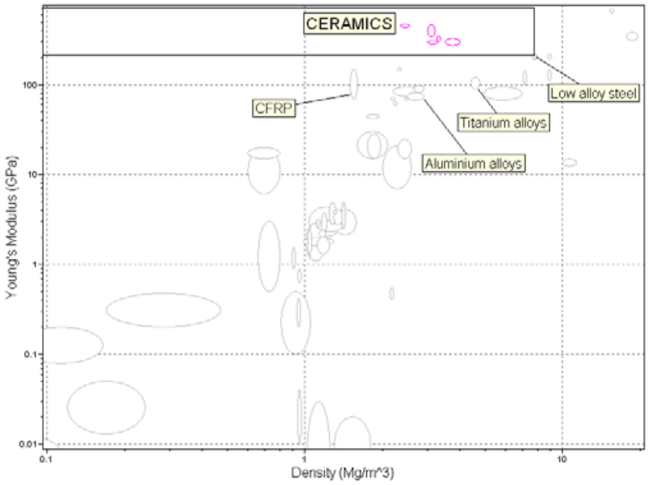
Mg is the cheaper material.



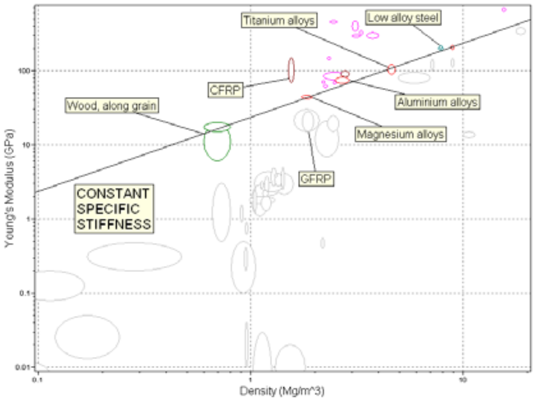
(c) *In CES*: plot Young's modulus – density chart and use a selection box with the bottom RH corner located on the material of interest (example shown is for steel).

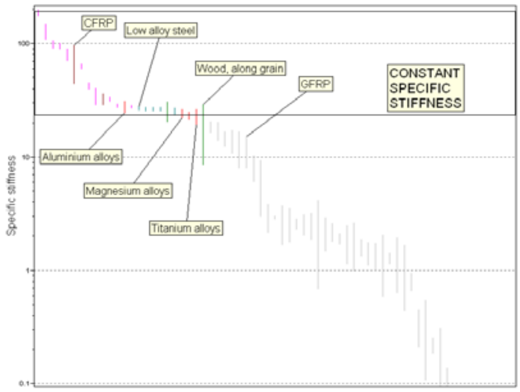
Metals and composites with higher E and lower ρ are:

- steels: there are none (only ceramics, which are already excluded: see figure)
- Ti, Al alloys: only CFRP



(d) *In CES*: either use line of slope unity on E-ρ plot, or bar chart of E/ρ.





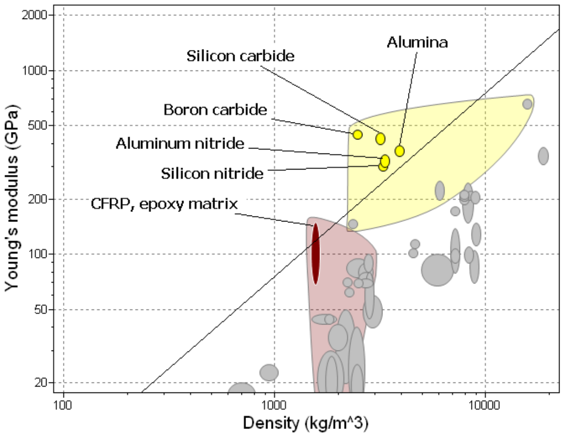
Both charts show that E/ρ is remarkably similar for steels, Ti, Al, and Mg alloys, and wood (along the grain). CFRP has a higher specific stiffness; GFRP a little lower.

(e) The approach in (c) is weak, since there is no sense of a trade-off between stiffness and density. Nothing is stiffer and lighter than steels, but clearly the "light alloys" (Mg, Ti, Al) and CFRP compete strongly for light, stiff design. The performance index approach in (d) captures this competition, revealing that higher density is compensated in proportion with increased stiffness for a wide range of materials. Only CFRP emerges as superior to all other materials – on the basis of specific stiffness.

Exercise E5.20 Make a chart showing modulus E and density ρ. Apply a selection line of slope 1, corresponding to the index E / ρ positioning the line such that six materials are left above it. Which are they and to which families do they belong?

Answer. The six that have the highest E / ρ are listed below. They come from just two families: technical ceramics and composites.

- Alumina
- Aluminum nitride
- Boron carbide
- CFRP, epoxy matrix (isotropic)
- Silicon carbide
- Silicon nitride



Exercise E5.21 A material is required for a tensile tie to link the front and back walls of a barn to stabilize both. It must meet a constraint on stiffness and be as cheap as possible. To be safe the material of the tie must have a fracture toughness K_{1c} > 18 MPa.m$^{1/2}$ (defined in Chapter 8). The relevant index is

$$M = E / C_m \rho$$

Construct a chart of E plotted against $C_m \rho$. Add the constraint of adequate fracture toughness, meaning K_{1c} > 18 MPa.m$^{1/2}$, using a "Limit" stage. Then plot an appropriate selection line on the chart and report the three materials that are the best choices for the tie.

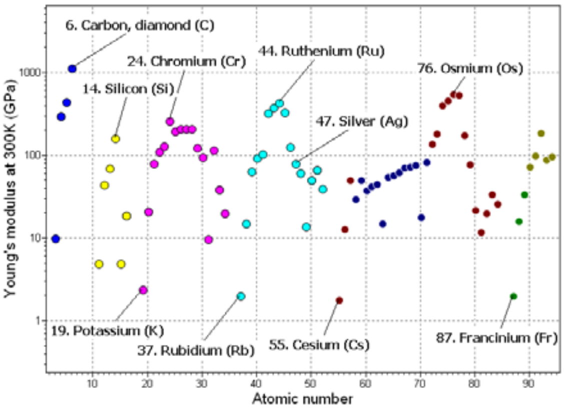
Answer. All three are plain-carbon steels:

- High carbon steel
- Low carbon steel
- Medium carbon steel

Exploring the science with CES Elements.

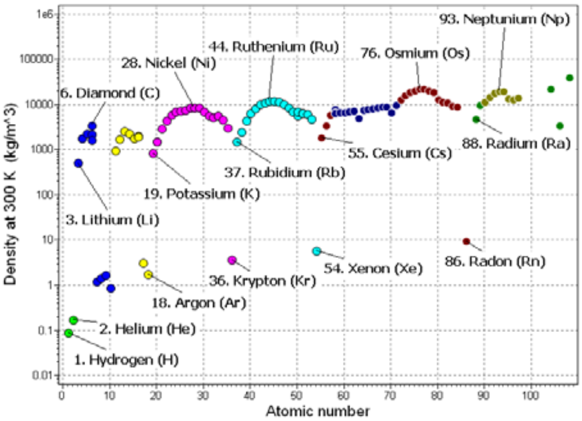
Exercise E5.22 There is nothing that we can do to change the modulus or the density of the building blocks of all materials: the elements. We have to live with the ones we have got. Make a chart of modulus E plotted against the atomic number A_n to explore the modulus across the periodic table. (Use a linear scale for A_n. To do so, change the default log scale to linear by double clicking on the axis name to reveal the axis-choice dialog box and choose ‘Linear’.) Which element has the highest modulus? Which has the lowest?

Answer. The figure shows the chart. Note how the modulus peaks in the middle of each row of the Periodic table (color coded) and falls to a minimum at either end. The element with the highest modulus is carbon in the form of diamond. That with the lowest is Cesium.



Exercise E5.23 Repeat Exercise E5.22, exploring instead the density ρ. Which solid element has the lowest density? Which has the highest?

Answer. As with modulus, the density peaks in the middle of each row of the Periodic table. The elements that are gases at STP (basically room temperature and pressure) have densities around 1 kg/m^3. The solid element with the lowest density is Lithium. That with the highest is the unstable element Hs, atomic number 108. Of the more familiar elements, Osmium and Iridium have the highest densities. Notice how the density slowly climbs across the Periodic table.

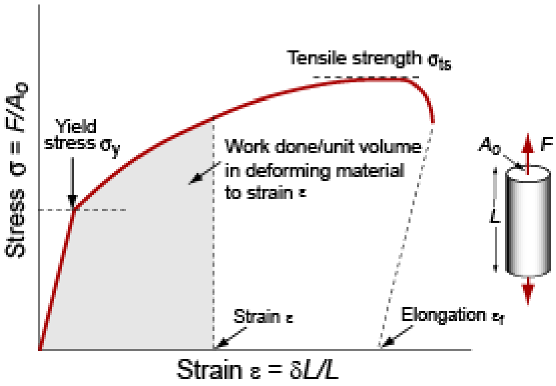


Materials: engineering, science, properties, and design
3e Solution manual

Chapter 6: Exercises with worked solutions

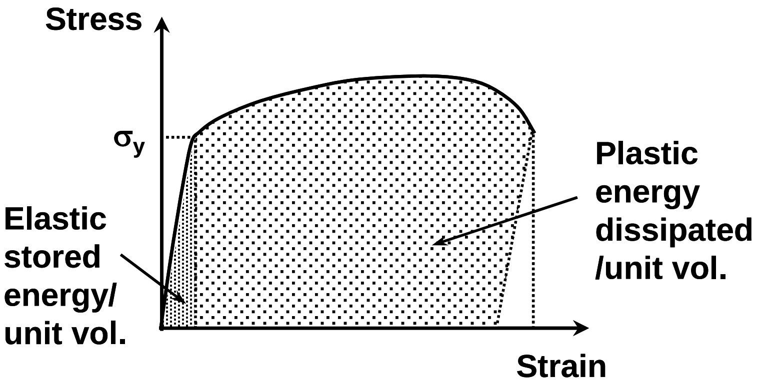
Exercise E6.1 Sketch a stress-strain curve for a typical metal. Mark on it the yield strength, σ_y, the tensile strength σ_{ts} and the ductility ε_f. Indicate on it the work done per unit volume in deforming the material up to a strain of $\varepsilon < \varepsilon_f$ (pick your own strain ε).

Answer. The figure shows the essential features, including the work of deformation.



Exercise E6.2 On a sketch of a typical stress-strain curve for a ductile metal, distinguish between the maximum elastic energy stored per unit volume and the total energy dissipated per unit volume in plastic deformation. Give examples of engineering applications in which a key material characteristic is: (i) maximum stored elastic energy per unit volume; (ii) maximum plastic energy dissipated per unit volume.

Answer.



(i) Applications in which maximising elastic stored energy per unit volume is important: springs (vaulting poles, golf clubs, suspension parts such as leaf springs etc).

(ii) Applications in which maximising plastic energy per unit volume is important: crash resistance for cars (motorway crash barriers, crash boxes behind car bumpers etc).

Exercise E6.3 (a) Write down expressions for the nominal strain and the true strain for the following situations:

(i) in a tensile test where the length of the specimen increases from L_o to L;

(ii) in a compression test where the height of the specimen decreases from h_o to h.

(b) In what circumstances can the true strain be approximated by the nominal strain?

Answer. (a) Using equations (6.8) and (6.10) in the text:

(i) In tension:

$$\text{nominal strain: } \varepsilon_n = \left(\frac{L-L_o}{L_o}\right) = \left(\frac{L}{L_o}\right) - 1, \quad \text{true strain: } \varepsilon_t = \ln\left(\frac{L}{L_o}\right) = \ln(1+\varepsilon_n)$$

(ii) In compression (both negative):

$$\text{nominal strain: } \varepsilon_n = \frac{h-h_o}{h_o}, \quad \text{true strain: } \varepsilon_t = \ln\left(\frac{h}{h_o}\right)$$

(b) $\varepsilon_t = \ln(1+\varepsilon_n) = \varepsilon_n$ + (higher order terms in ε_n). Hence $\varepsilon_t \approx \varepsilon_n$ when $\varepsilon_n \ll 1$, i.e. in most elastic problems (except for rubber, which deforms to large elastic strains), and for modest plastic strains of a few %.

Exercise E6.4 In a two-stage elongation of a specimen, the length is first increased from ℓ_0 to ℓ_1 and then from ℓ_1 to ℓ_2. Write down the true strains ε_1 and ε_2 respectively for each of the two elongation processes considered separately. Show that the true strain for the overall elongation ℓ_0 to ℓ_2 is given by the sum of the true strains for each of the two separate processes. Is the same true for the nominal strains?

Answer. True strains: $\varepsilon_{t1} = \ln\left(\frac{\ell_1}{\ell_o}\right)$, $\varepsilon_{t2} = \ln\left(\frac{\ell_2}{\ell_1}\right)$

Sum of true strains:

$$\varepsilon_{t1} + \varepsilon_{t2} = \ln\left(\frac{\ell_1}{\ell_o}\right) + \ln\left(\frac{\ell_2}{\ell_1}\right) = \ln\left(\frac{\ell_1}{\ell_o}\times\frac{\ell_2}{\ell_1}\right) = \ln\left(\frac{\ell_2}{\ell_o}\right) = (\varepsilon_t)_{total} \quad \text{(correct)}$$

Overall nominal strain = $\dfrac{\ell_2-\ell_0}{\ell_0}$

Sum of nominal strains = $\dfrac{\ell_1-\ell_0}{\ell_0} + \dfrac{\ell_2-\ell_1}{\ell_1}$, but this does *not* equal $\dfrac{\ell_2-\ell_0}{\ell_0}$.

Exercise E6.5 In a 'tandem' rolling process, a wide metal strip passes continuously between four sets of rolls, each of which apply a reduction in thickness. The strip elongates without getting wider, and therefore accelerates. For an incoming thickness h_{in} = 25mm moving at speed v_{in} = 5cms^{-1}, and an exit thickness h_{out} = 3.2mm, calculate: (i) the overall true strain applied to the material; (ii) the exit speed v_{out}. State any assumptions made.

Answer.
The additive property for true strains (Exercise 6.4) is a main reason why they are useful. Hence in multi-pass tandem rolling, the total true strain is the sum of the true strains in each reduction – we don't need to know the strain for each pass.

(i) The overall true strain (in compression) is calculated directly from the inlet and outlet thickness:

$$\varepsilon_t = \ln\left(\frac{h_{out}}{h_{in}}\right) = \ln\left(\frac{3.2}{25}\right) = -2.06$$

(ii) Plastic deformation takes place at constant volume, so as the width of the strip remains constant, continuity of volume flow gives:

$$h_{in} \times v_{in} = h_{out} \times v_{out}$$

Hence: $v_{out} = \dfrac{25\times 5}{3.2} = 39.1\ \text{cm.s}^{-1}$

Note that continuity also means that a given *length* at inlet will exit with a length increased by the same factor of 25/3.2. Hence the true strain can also be expressed as an *extension* of the material: $\varepsilon_t = \ln\left(\frac{25}{3.2}\right) = +\ 2.06$, i.e. numerically the same as the compressive true strain, but positive).

Exercise E6.6 Explain briefly why the material property bubbles for metals are very elongated on the Young's Modulus – Strength chart (Figure 6.8). Comment on whether the same is true for the polymers. Why is this?

Answer.
The modulus of a solid is a well-defined quantity with a narrow range of values. The strength is not. The strength range for a given class of metals, such as stainless steels, can span a factor of 10 or more, while the spread in stiffness is at most 10%.

The bubbles for polymers are much less elongated – a given polymer has a wider range of Young's modulus than a metal, and a more narrowly defined strength.

Variation in the microstructure of a polymer (e.g. degree of crystallinity) affects both properties more or less equally. In metals, microstructure has a strong effect on yield (e.g. precipitates, dislocation density) but little influence on stiffness.

Exercise E6.7 Use the yield strength–density chart or the yield strength–modulus chart (Figures 6.7 and 6.8) to find:

- the metal with the lowest strength
- the approximate range of strength of the composite GFRP
- whether there are any polymers that are stronger than wood measured parallel to the grain
- how the strength of GFRP compares with that of wood
- whether elastomers, that have moduli that are far lower than polymers, are also far lower in strength.

Answer.

- The metal with the lowest strength: Lead and its alloys.
- The range of strength of the composite GFRP is approximately 100–200 MPa.
- Several polymers are stronger than wood measured parallel to the grain, notably PEEK and PA (Polyamide, Nylon)
- GFRP is substantially stronger than wood
- Elastomers have moduli that are far lower than other polymers, but they are only a little less strong.

Exercise E6.8 What is meant by the ideal strength of a solid? Which material class most closely approaches it?

Answer. The ideal strength is equal to the stress needed to pull atoms of a solid apart. Typically, it is about $E/15$, where E is Young's modulus. No real engineering materials have strengths as high as this because of the defects they contain; the real strengths of most metals is nearer $E/300$. The material class of Polymers comes closest: many have strengths of about $E/40$.

Exercise E6.9

(a) Explain briefly the three main microstructural mechanisms by which metals are hardened. Identify which mechanisms account for the following increases in yield stress:

(i) Pure annealed aluminium: 25 MPa; cold rolled non-heat-treatable Al-Mn-Mg alloy: 200 MPa
(ii) Pure annealed copper: 20 MPa; cast 60-40 Brass (60% Cu, 40% Zn): 105 MPa
(iii) Pure annealed iron: 140 MPa; quenched & tempered medium carbon steel: 550 MPa.

(b) For each of the hardening mechanisms in part (a), give other examples of important engineering alloys which exploit these mechanisms.

Answer.

(a) The three dominant hardening mechanisms are: work hardening, solid solution hardening and precipitation hardening. All of them work by providing “pinning points” – obstacles which hold the dislocation back (temporarily) requiring it to bow out in a curve. This requires an increase in the imposed shear stress.

Work hardening uses plastic deformation to increase the dislocation density. Dislocations obstruct one another’s movement where they intersect, increasing the yield stress.

Solid solution hardening uses individual atoms distributed in the lattice to impede dislocation motion. Both substitutional and interstitial solute atoms perturb the atomic bonds in the lattice around them, “roughening” the slip plane, i.e. making it more difficult for the atomic bonding defect around a dislocation to move over the slip plane at this point.

Precipitation hardening involves embedding obstacles such as small crystals of compounds in the lattice. Dislocations encounter them on many slip planes, and force bowing of the dislocation between them, increasing the applied stress for yielding. Usually the precipitates are bypassed when the dislocation loop reaches a semi-circle. However, if there is some crystallographic continuity between the lattice and the precipitate, the precipitate may be sheared when the dislocation applies enough force to the precipitate – this still gives a significant increase in yield stress.

(i) Cold-rolled Al-Mn-Mg alloys are work-hardened by the rolling process, and the Mn and Mg atoms give solid solution hardening.

(ii) Brass uses strong solid-solution hardening, with the high levels of Zn added to the Cu. As the material is cast, no work hardening is used in this case (but brass is often “wrought”, i.e. shaped by deformation, and thus work hardened).

(iii) Quenching and tempering is the primary processing route to form finely dispersed precipitates of iron carbide in steels, giving very effective precipitation hardening. This dominates hardening in medium carbon steels – alloy steels also exploit solid solution hardening, and the additional effect of alloy carbide precipitates.

(b) Other important examples:

Work hardening: stainless steels, low carbon (“mild”) steels

Solid solution hardening: bronze (Cu-Sn alloys), stainless steels, most casting alloys (Mg, Zn, Al, Ti)

Precipitation hardening: alloy steels, tool steels, high temperature Ni alloys, “heat-treatable” Al alloys.

Exercise E6.10 An aluminium alloy used for making cans is cold rolled into a strip of thickness 0.3mm and width 1m. It is coiled round a drum of diameter 15cm, and the outer diameter of the coil is 1m. In the cold rolled condition, the dislocation density is approximately 10^{15} m^{-2}. Estimate:

(i) The mass of aluminium on the coil;
(ii) The total length of strip on the coil;
(iii) The total length of dislocation in the coiled strip.

Answer.

(i) Cross-sectional area of coiled metal: $\frac{\pi}{4}\left(d_{coil}^2 - d_{drum}^2\right) = \frac{\pi}{4}\left(1^2 - 0.15^2\right) = 0.768\ m^2$

Volume of metal = $0.768 \times 1 = 0.768$ m^3

Density of aluminium (from Appendix A.5) ≈ 2.7 Mg/m^3

Hence mass of aluminium on coil ≈ $2.7 \times 0.768 = 2.074$ Mg = 2074 kg (approx. 2.1 tonnes).

(ii) Cross-sectional area of strip: $1 \times 0.3 \times 10^{-3} = 0.3 \times 10^{-3}$ m^2

Length of coiled strip = Volume/cross-sectional area = $(0.768/0.3 \times 10^{-3})$ = 2560 m = 2.56 km

(iii) Length of dislocation per unit volume = 10^{15} m/m^3. Total length of dislocation = Volume × dislocation density = 0.768×10^{15} m (about 1/12th of a light year!).

Exercise E6.11 The lattice resistance of copper, like that of most FCC metals, is small. When 10% of nickel is dissolved in copper to make a solid solution, the strength of the alloy is 150 MPa. What would you expect the strength of an alloy with 20% nickel to be?

Answer. The contribution of solid solution to the yield strength (equation (6.18) et seq.) is:

$$\sigma_y \approx 3\tau_{ss} = 3\alpha\, E\, c^{1/2}$$

that is, it scales as $c^{1/2}$ where c is the concentration of solute. Thus if 10% gives a strength of 150 MPa, 20% will give a strength of 212 MPa.

Exercise E6.12 A metal matrix composite consists of aluminum containing hard particles of silicon carbide SiC with a mean spacing of 3 microns. The composite has a strength of 180 MPa. If a new grade of the composite with a particle spacing of 2 microns were developed, what would you expect its strength to be?

Answer. The contribution of particles to the yield strength (equation (6.11) et seq.) is:

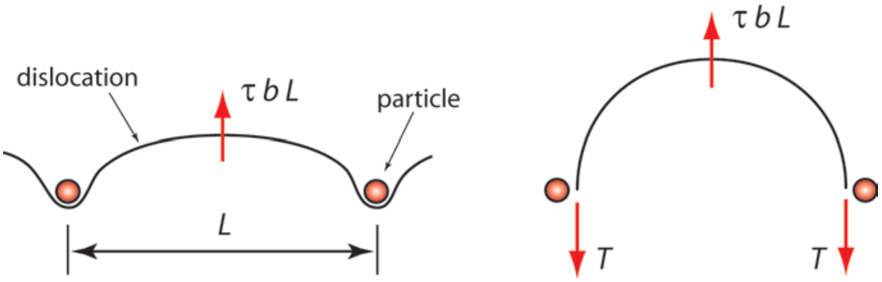
$$\sigma_y \approx 3\tau_{ppt} = \frac{6T}{bL} \approx \frac{3Eb}{L}$$

that is, it scales as $1/L$ where L is the particle spacing. If particles of spacing 3 microns gives a strength of 180 MPa, particles of spacing 2 microns will give a strength of 270 MPa.

Exercise E6.13

(a) Explain briefly how impeding the motion of dislocations with dispersed precipitates in a metal can raise the yield stress.

(b) A straight segment of dislocation line of length L is pinned at two particles in a crystal. The application of a shear stress τ causes the dislocation to bow outwards as shown in the figure, decreasing its radius of curvature as it does so. The force per unit length acting on the dislocation is $\tau b L$, where b is the Burgers vector. Given that the dislocation line tension $T = Gb^2/2$, show that the stress just large enough to fully bow out the dislocation segment into a semi-circle is given by $\tau = Gb/L$.



(c) Estimate the tensile yield stress of aluminium containing an array of equally spaced second phase precipitates, where the volume fraction of the second phase is 5% and the precipitate diameter is 0.1μm. The Burgers vector for aluminium is 0.29nm, and the shear modulus G = 27 GPa.

(d) If incorrect heat treatment of the aluminium alloy results in a non-uniform distribution of precipitates, explain the effect this would have on the yield stress.

Answer.

(a) Precipitation strengthened material derives its additional strength from the dislocation bowing mechanism. The figure in (b) shows schematically the bowing of a dislocation between precipitates acting as 'pinning points'. An additional increment of applied shear stress is needed to curve the dislocation between these two precipitates, as this is resisted by the 'line tension' of the dislocation (which is actually an energy per unit length). Once a large enough stress is applied, the dislocation escapes – either by shearing the particles (if their lattice is compatible with the surrounding crystal), or by reaching a semi-circular geometry and escaping (leaving a loop of dislocation line round the particle). For this bypassing mechanism, the additional shear stress needed increases in inverse proportion to the spacing.

(b) The force in the forward direction due to the applied stress is balanced by the line tension until it reaches the minimum (critical) radius of the dislocation loop, corresponding to a maximum of the force due to line tension:

$$\tau\, b\, l = 2T$$

Since $T = Gb^2/2$, it follows that $\tau = 2Gb^2 / 2bl$. Therefore $\tau = Gb/l$.

(c) Assuming a cubic array of equally spaced particles, radius R and centre-to-centre spacing d, the volume fraction is given by:

$$V_f = \frac{\frac{4}{3}\pi R^3}{d^3}$$

So for $V_f = 0.05$, $R = 0.05\mu\text{m}$: $0.05 = \dfrac{\frac{4}{3}\pi (0.05)^3}{d^3}$ giving $d = 0.219\mu\text{m}$.

The gap between the particles, through which the dislocations must pass, is $l = d - 2R = 0.119\mu\text{m}$.

The additional shear stress to push dislocations through this gap is given by:

$$\tau \approx Gb/l \approx \frac{(27\times 10^3)\times(0.29\times 10^{-3})}{0.119} \approx 66 \text{ MPa}.$$

This corresponds to a yield stress increment of approximately 3 × τ ≈ 200 MPa. Note that this will be an over-estimate, as statistically the spacing between particles intersected by dislocations will on average be greater than the minimum value used in the idealised geometry. To this value we need to add the intrinsic yield stress of pure aluminium (without hardening particles) – of order 25 MPa.

(d) A non-uniform distribution of precipitates will reduce the yield stress, since there will be regions in which the spacing is greater than given by a uniform dispersion, so yielding occurs more easily in these regions.

Exercise E6.14 Nano-crystalline materials have grain sizes in a range 0.01 to 0.1 μm. If the contribution of grain boundary strengthening in an alloy with grains of 0.1 μm is 20 MPa, what would you expect it to be if the grain size were reduced to 0.01 μm?

Answer. The contribution of grain boundaries to the yield strength (equation 6.21 et seq) is:

$$\sigma_y \approx 3\tau_{gb} = \frac{3k}{\sqrt{D}}$$

that is, it scales as $1/\sqrt{D}$ where D is the grain size. If 0.1 micron grains give a contribution to strength of 20 MPa, 0.01 micron grains will give a contribution of 63 MPa.

Exercise E6.15 The yield strength σ_y of plain carbon steel is moderately dependent on the grain size d and the relation can be described by the equation:

$$\sigma_y = \sigma_o + k\sqrt{\frac{1}{d}}$$

Where σ_o and k are material constants. The yield strength is 622 MPa for a grain size of 180μm and 663 MPa for a grain size of 22μm.

(a) Calculate the yield strength of the steel for a grain size of 11μm.
(b) Explain briefly the physical significance of the constant σ_o.

Answer.

(a) Set up simultaneous equations:

$$663 = \sigma_o + k\left(\frac{1}{\sqrt{22}}\right) \qquad (1)$$

$$622 = \sigma_o + k\left(\frac{1}{\sqrt{180}}\right) \qquad (2)$$

Subtracting (2) from (1):

$$41 = k\left\{\left(\frac{1}{\sqrt{22}}\right) - \left(\frac{1}{\sqrt{180}}\right)\right\}$$

Solving for *k* gives 296 MPa.$\sqrt{\mu m}$

Substituting for *k* in (1) and solving for σ_o gives:

$$\sigma_o = 663 - 296 \times \left(\frac{1}{\sqrt{22}}\right) = 600 \text{ MPa}$$

Substituting for d = 11μm:

$$\sigma_y = 600 + 296\left(\frac{1}{\sqrt{11}}\right) = 689 \text{ MPa.}$$

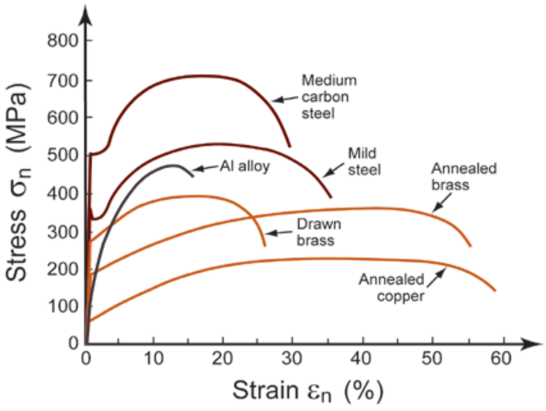
(b) When d is large, $\sigma_y \to \sigma_o$. Therefore, σ_o is the strength for large grain sizes where the strengthening effect of small grains is no longer significant, that is, an indication of the intrinsic strength of the steel.

Exercise E6.16 From the uniaxial stress-strain responses for various metals in Figure 6.23:

(a) Find the yield stress (or if appropriate a 0.2% proof stress), the tensile strengths and the ductility of each alloy.

(b) Account for the differences in strength, comparing annealed copper with each of the other alloys in turn.

Answer.

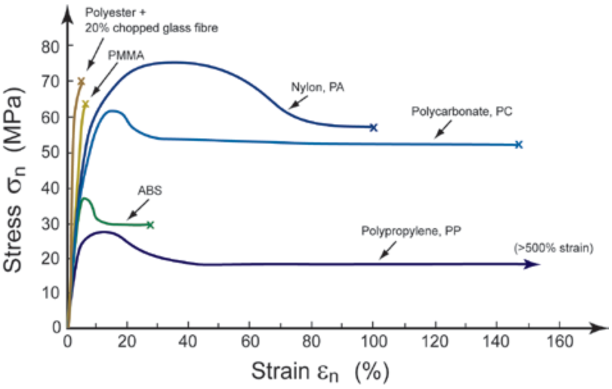


Material	Yield stress σ_y (MPa)	Tensile strength σ_{TS} (MPa)	Ductility ε_f (%)	Comparison with annealed copper
Annealed Copper	60	230	58	
Annealed Brass	190	370	55	Zinc atoms (5-40%) increase the strength of the alloy through solid solution hardening.
Drawn Brass	280	400	26	Previous cold work increases strength through increasing dislocation density.
Aluminium Alloy	110	470	16	Either solid solution or precipitation hardening.
Mild Steel	370	530	35	Iron has high intrinsic strength, increased by carbon in solution.
Medium C Steel	505	720	30	Higher carbon content than mild steel, in the normalized condition, contains around 50% of the grains hardened by Fe_3C.

Exercise E6.17 From the uniaxial stress-strain responses for various polymers in the figure:

(a) Estimate the tensile strengths and ductility of each material.

(b) Account for the main differences between the shapes of the graphs for PMMA, ABS and Polypropylene.



Answer.

Material	Tensile strength σ_{TS} (MPa)	Ductility ε_f (%)	Comparison of the shapes of the graphs
Polypropylene, PP	27	>500	Polypropylene shows peak strength followed by cold drawing (stable necking, aligning molecular chains). After large extension, the load will rise again until failure of molecular chains.
ABS	37	27	ABS shows peak strength followed by modest plastic strain prior to failure.
Polycarbonate, PC	62	145	
Nylon PA	75	100	
Polyester + 20% chopped glass fib.	70	5	
PMMA	64	6	PMMA breaks by brittle fracture from pre-existing flaws, before it can yield, so close to linear elastic up to failure.

Exercise E6.18 Polycarbonate, PC (yield strength 70 MPa) is blended with polyester PET (yield strength 50 MPa) in the ratio 30%/70%. If the strength of blends follows a rule of mixtures, what would you expect the yield strength of this blend to be?

Answer. Using a rule of mixtures the strength of the blend is

$$\sigma_y \approx 0.3 \times \text{strength of PC} + 0.7 \times \text{strength of PET} = 56 \text{ MPa}$$

Exploring design with CES (use Level 2 Materials unless otherwise suggested).

Exercise E6.19 Find, by opening the records, the yield strengths of copper, brass (a solid solution of zinc in copper) and bronze (a solid solution of tin in copper). Report the mean values of the ranges that appear in the records. What explains the range within each record, since the composition is not a variable? What explains the differences in the mean values, when composition is a variable?

Answer.

Material	Range of yield strength (MPa)	Mean yield strength (MPa)
Copper	30 - 350	190
Brass	95 - 500	298
Bronze	100 - 500	300

The range of strengths of copper (for example) can only be due to work hardening; the same is true of the ranges for brass and bronze. The differences between the mean strengths of copper, brass and bronze must be due to solid solution hardening, since that is the main difference between them.

Exercise E6.20 Use a 'Limit' stage to find materials with a yield strength σ_y greater than 100 MPa and density ρ less than 2000 kg/m^3. List the results.

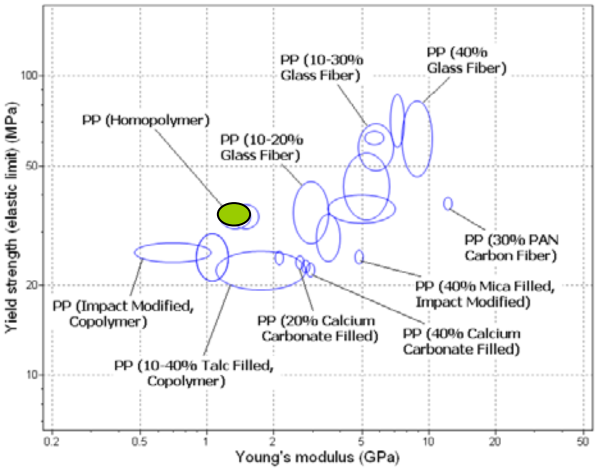
Answer. The Limit stage gives two classes of materials, alloys of magnesium, and composites:

- CFRP, epoxy matrix (isotropic)
- GFRP, epoxy matrix (isotropic)
- Cast magnesium alloys
- Wrought magnesium alloys

Exercise E6.21 Add two further constraints to the selection of the previous exercise. Require now that the material price be less than 5 $/kg and the elongation be greater than 5%.

Answer. Only magnesium alloys survive.

Exercise E6.22 Use the CES level 3 database to select Polypropylene and its blended, filled and reinforced grades. To do so, open CES Edu Level 3, apply a 'Tree' stage selecting Polymers – Thermoplastics – Polypropylene (folder). Make a chart with Young's modulus E on the x-axis and yield strength σ_y on the y-axis. Label the records on the chart by clicking on them. Explain, as far as you can, the trends you see.



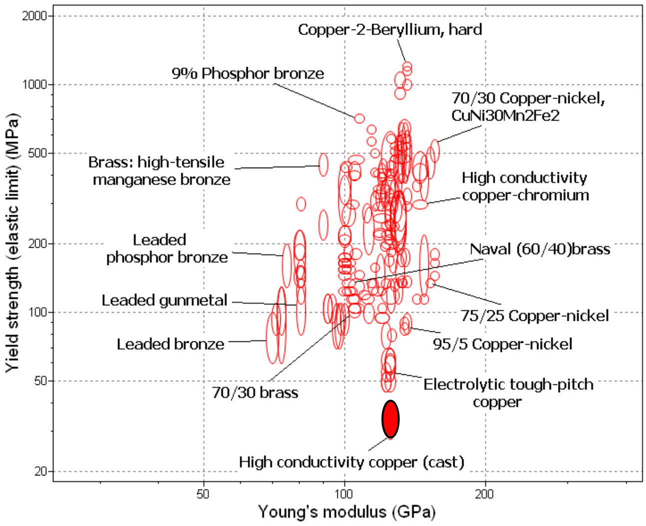
Answer. The chart shows that:

- Impact modifiers (increasing toughness) reduce both modulus and strength
- Fillers like talk, calcium carbonate and mica increase the modulus but not the strength.
- Glass and carbon fibers increase both modulus and strength.

Exercise E6.23 Apply the same procedure as that of the last exercise to explore copper and its alloys. Again, use your current knowledge to comment on the origins of the trends.

Answer. The chart shows that

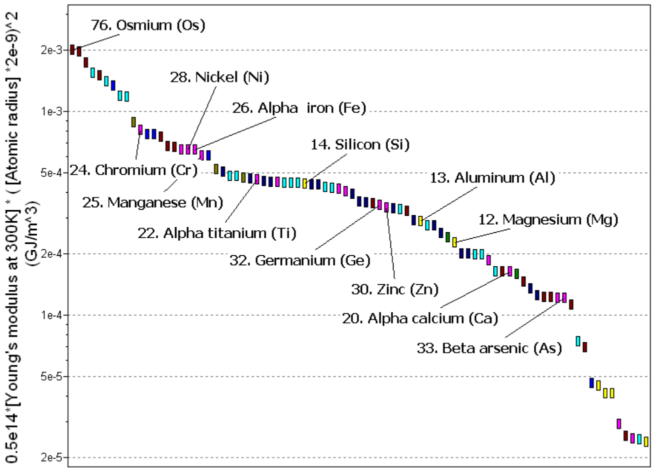
- All alloying increases the strength but most has little effect on the modulus.
- The exception is the lead-containing copper alloys that do have a significantly lower modulus
- The strongest copper alloys are the phosphor bronzes and the beryllium coppers.



Exploring the science with CES Elements.

Exercise E6.24 The elastic (potential) energy per unit length of a dislocation is $0.5\,E\,b^2$ J/m. Make a bar-chart of the energy stored in the form of dislocations, for a dislocation density of 10^{14} m/m^3. Assume that the magnitude of the Burger's vector, b, is the same as the atomic diameter. (You will need to use the 'Advanced' facility in the axis-choice dialog box to make the function.) How do the energies compare with the cohesive energy, typically 5×10^4 MJ/m^3?

Answer. The chart shows that the energy associated with this modest density of dislocations is between 0.002 and 2 MJ/m^3. This is tiny compared with the cohesive energy, of general order 5×10^4 MJ/m^3.

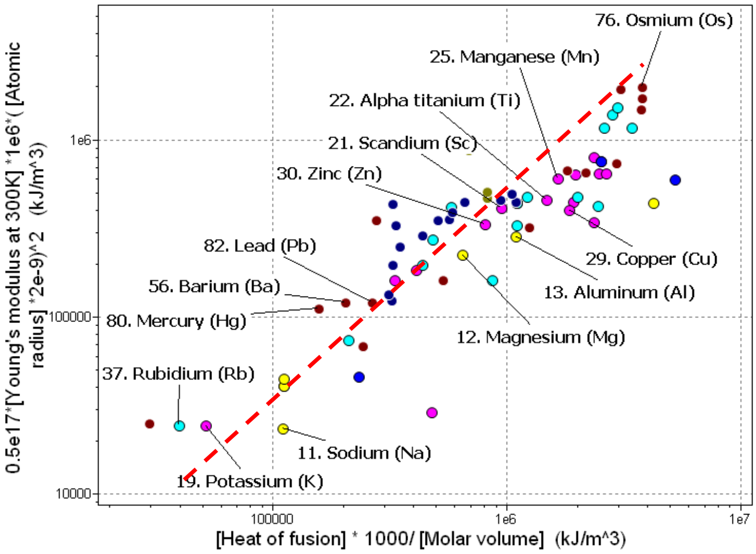


Exercise E6.25 Work hardening causes dislocations to be stored. Dislocations disrupt the crystal, and have potential energy associated with them. It has been suggested that sufficient work hardening might disrupt the crystal so much that it became amorphous. To do this, the energy associated with the dislocations would have to be about equal to the heat of fusion, since this is the difference in energy between the ordered crystal and the disordered liquid. Explore this in the following way:

(a) Calculate and plot the energy associated with a very high dislocation density of 10^{17} m/m^3 for the elements. i.e. plot a bar chart of $0.5 \times 10^{17} E\, b^2$ on the y-axis using twice atomic radius as equal to the Burger's vector b. Remember that you must convert GPa into kPa and atomic radius from nm to m to get the energy in kJ/m^3.

(b) Now add, on the x-axis, the heat of fusion energy. Convert it from kJ/mol to kJ/m^3 by multiplying H_c by 1000/molar volume, with molar volume in m^3/kmol (as it is in the database). What, approximately, is the ratio of the dislocation energy to the energy of fusion? Would you expect this very high dislocation density to be enough to make the material turn amorphous?

Answer. The plot show that the ratio is about 0.8, very high dislocation densities might just be capable of disorganizing the crystal structure sufficiently to create an amorphous (non-crystalline) state.



Materials: engineering, science, properties, and design
3e Solution manual

Chapter 7: Exercises with worked solutions

Exercise E7.1 What is meant by the *elastic section modulus*, Z_e ? A beam carries a bending moment M . In terms of Z_e, what is the maximum value that M can take without initiating plasticity in the beam?

Answer. The basic equation of elastic beam-theory is

$$\frac{\sigma}{y} = \frac{M}{I} = E\,\kappa$$

where σ is the stress in the beam, M the moment acting on it, I it second moment of area, E Young's modulus, κ the curvature and y the distance from the neutral axis. Thus the maximum stress in the beam is

$$\sigma_{max} = \frac{M\,y_m}{I}$$

where y_m is the maximum distance of the beam surface from the neutral axis. The quantity $\frac{I}{y_m}$ is called the section modulus and given the symbol Z_e. The beam will start to yield if the maximum stress exceeds the yield strength, σ_y, limiting the maximum moment to

$$M_{max} = Z_e\,\sigma_y$$

Exercise E7.2

(a) Calculate the elastic bending moment M_e that causes first yield in a beam of square cross section b × b with tensile yield strength σ_y. Calculate the fully plastic moment M_p in the same beam. What is the ratio of M_p/M_e ?

(b) Calculate the elastic torque T_e that causes first yield in a shaft of radius R with shear yield strength $\sigma_y/2$. Calculate the fully plastic torque T_p in the same shaft. What is the ratio of T_p/T_e ?

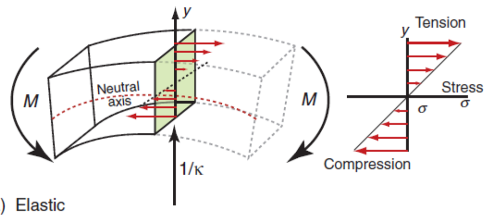
(c) Comment on the safety margin between initial yield and full plasticity in each case.

Answer.

(a) Elastic moment: From equation (7.1), the elastic stress is: $\sigma = \frac{My}{I}$.

For a beam with square cross section b × b, initial yield, $\sigma = \sigma_y$ occurs at $y = b/2$, with $I = b^4/12$, so the elastic moment is given by:

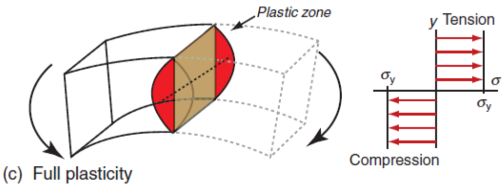
$$M_e = \frac{\sigma_y\, b^4/12}{b/2} = \frac{b^3}{6}\sigma_y .$$



(a) Elastic

Fully-plastic moment: From equation (7.3), the fully plastic moment is given by:

$$M_p = \int_{-b/2}^{b/2} b|y|\sigma_y \, dy \;=\; 2\int_0^{b/2} b\, y\, \sigma_y \, dy \;=\; \frac{b^3}{4}\sigma_y$$



So the ratio of the two is:

$$\frac{M_p}{M_e} = \frac{1/4}{1/6} = 1.5.$$

(b) Elastic torque: For a shaft with circular cross section of radius R, equation (7.4) gives the elastic stress:

$$\tau = \frac{Tr}{K}\,.$$

At initial yield, $\tau = \sigma_y/2$ occurs at $r = R$, with $K = \pi R^4/2$, so the elastic torque is:

$$T_e = \frac{(\sigma_y/2)\,(\pi R^4/2)}{R} = \frac{\pi R^3}{4}\sigma_y\,.$$

Fully plastic torque: as for the plastic bending analysis above, the fully plastic torque is given by:

$$T_p = \int_0^R r\, 2\pi r\, \tau_y \, dr = \frac{2\pi R^3}{3}\tau_y = \frac{\pi R^3}{3}\sigma_y$$

So: $$\frac{T_p}{T_e} = \frac{1/3}{1/4} = 1.33.$$

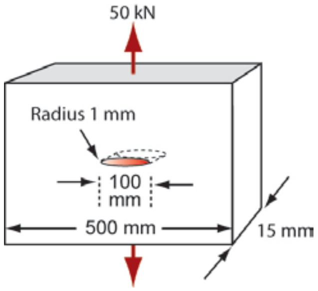
(c) There is a 50% safety margin between initial yield and plastic collapse for the beam, but only 33% for the shaft. You have to be more accurate in your design of a highly-stressed shaft than a highly stressed beam.

Exercise E7.3 A plate with a rectangular section 500mm by 15 mm carries a tensile load of 50kN. It is made of a ductile metal with a yield strength of 50 MPa. The plate contains an elliptical hole of length 100 mm and a minimum radius of 1 mm, oriented as shown in the diagram. What is:

(a) the nominal stress?

(b) the maximum stress in the plate?

Will the plate start to yield? Will it collapse completely?



Answer. The nominal stress is the load divided by the nominal area, 500 × 15 mm, giving

$$\sigma_{nom} = 6.67\ MPa$$

The maximum stress is the nominal stress times the stress concentration factor of the elliptical hole, K_{sc} (Figure 7.7 and equation (7.12) of the text):

$$K_{sc} = \frac{\sigma_{max}}{\sigma_{nom}} = 1 + \alpha\left(\frac{c}{\rho_{sc}}\right)^{1/2}$$

where α takes the value 2 for tension. Inserting the dimensions from the diagram gives

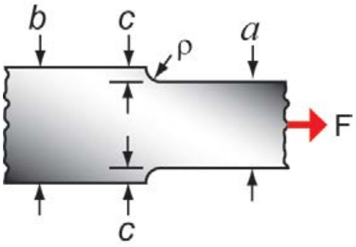
$$K_{sc} = 15$$

meaning that the maximum stress at the root of the hole is

$$\sigma_{\max} = 101\ MPa\,.$$

The plate will therefore start to yield near the hole. It will not collapse completely because this requires that the yield strength is exceeded across the entire section. Even allowing for the loss of 20% of the section, the nominal stress is far below the yield strength.

Exercise E7.4 A tie rod made of a strip of metal of thickness t has width b that reduces to width a with a radius of ρ, as shown in the figure. The material has a yield stress σ_y.



(a) Find an expression for the value of a that would be needed to avoid yield at the nominal stress level in the thinner section, with a safety factor of S.

(b) The applied load is expected to reach the stress level in (a). Find an expression for the value of the fillet radius ρ that would ensure that the maximum stress in the tie rod is less than the yield stress.

(c) Determine the value of ρ if a = 30mm, b = 40mm, and S = 3.

(d) What is the real safety factor of the part with this value of ρ?

Answer.

(a) The stress in the narrower section is $\sigma = \dfrac{F}{at}$

Yield would occur in the thinner section when $\dfrac{F}{at} = \dfrac{\sigma_y}{S}$, so $a = \dfrac{FS}{t\sigma_y}$

(b) The stress concentration factor is: $K_{SC} = \dfrac{\sigma_{\max}}{\sigma_{nom}} = 1 + \alpha \left(\dfrac{c}{\rho}\right)^{1/2}$, with $\alpha = 2$

We have $c = \dfrac{b-a}{2}$ and $\sigma_{nom} = \dfrac{F}{at} = \dfrac{\sigma_y}{S}$.

So for $\sigma_{\max} = \sigma_y$, we need $\dfrac{\sigma_{\max}}{\sigma_{nom}} = \dfrac{\sigma_y}{\sigma_y / S} = 1 + 2\left(\dfrac{b-a}{2\rho}\right)^{1/2}$, giving $\rho = \dfrac{2(b-a)}{(S-1)^2}$

(c) For b = 40mm, a = 30mm, S = 3, this gives ρ = 5mm.

(d) At this value of ρ the peak stress in the component reaches σ_y. The stress concentration factor equals 3, which exactly balances the safety factor of 3 on the nominal stress. Hence the real safety factor with respect to first yield is eliminated – it equals 1.

Exercise E7.5 Your task is to design a light-weight tie of length L with a circular cross-section of radius R. It has to carry an axial force F, without tensile failure. You will need to choose the material (with Young's modulus E and density ρ) and the corresponding cross section radius to suit your choice of material (i.e. R is a 'free variable'). Use a safety factor of 2 throughout.

(a) Find an expression for the radius R of the tie that will carry the load without failure at stress σ_f.

(b) If the length of the tie is L=0.3m and the load is F=1000N, what value of R is needed if the tie is made of: (i) PEEK; (ii) Butyl Rubber; (iii) Ti alloy, (iv) Cu alloy? Use the material properties in the table below.

(c) Write an expression for the mass m of the tie and determine the mass for each of the materials in (b). Which one would you choose?

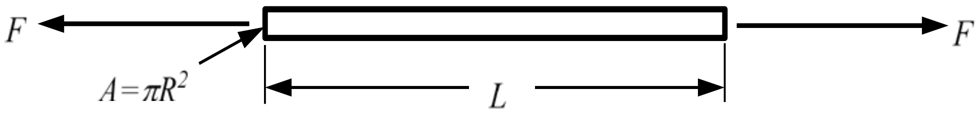
(d) Substitute the expression for radius R from (a) into the expression for mass m from (c) to show that the material index to be *minimised* is ρ/σ_f. Determine the value of the material index for the four ties.

(e) Comment on the relationship between the mass of each tie and its material index ρ/σ_f.

(f) Examine Figure 7.8 to see how these three materials compare in terms of the reciprocal material index σ_f/ρ.

	Failure stress σ_f (MPa)	Density ρ (kg/m^3)	R (mm)	m (kg)	ρ/σ_f
PEEK	80	1300			
Butyl Rubber	2.5	2400			
Ti alloy	1200	4600			
Cu alloy	150	8900			

Answer.



(a) The axial stress in the tie is: $\sigma = \dfrac{F}{A} = \dfrac{F}{\pi R^2}$.

The stress can't exceed σ_f / S, where S is the safety factor. So $R = \sqrt{\dfrac{SF}{\pi \sigma_f}}$. (1)

(b) Using the values for PEEK: $R = \sqrt{\dfrac{2(1000)}{\pi(80 \times 10^6)}} = 2.8\,\text{mm}$.

(c) The mass of the tie is: $m = \rho A L = \rho \pi R^2 L$. (2)

For PEEK: $m = 1300\,\pi (0.89 \times 10^{-3})^2 (0.3) = 9.7$ g.

(d) Eliminating the free variable, R, from (2) using (1)

$m = \rho \pi \left(\dfrac{SF}{\pi \sigma_f}\right) L = SF\, L \left(\dfrac{\rho}{\sigma_f}\right)$. So the material index to be minimized is ρ/σ_f.

(e) The material indices are proportional to the mass of the tie and consequently they rank the materials in the same order. For example, compare copper and titanium in the table: $m_{Cu}/m_{Ti} = 15.5$; $(\rho/\sigma_f)_{Cu} / (\rho/\sigma_f)_{Ti} = 15.5$, etc.

(f) Selecting materials to <u>minimize</u> ρ/σ_f is the same as selecting materials to <u>maximize</u> σ_f/ρ. Materials with high values of σ_f/ρ are the furthest to the top left of Figure 7.8, above a line of slope 1. Ti alloy is the best of this bunch.

Material	Failure stress σ_f (MPa)	Density ρ (kg/m^3)	R (mm)	m (kg)	ρ/σ_f
PEEK	80	1300	2.8	0.0097	16.25
Butyl Rubber	2.5	2400	16.0	0.576	960
Titanium alloy	1200	4600	0.73	0.0023	3.83
Copper alloy	150	8900	2.1	0.0356	59.3

Exercise E7.6 You are asked to design a light-weight cantilever beam of length L with a square cross-section of width b and depth b. The beam is to be built-in to a wall at one end and loaded at the free end with a point load F without exceeding the elastic limit. When designing the beam, you are free to specify the value of b to suit your choice of material (i.e. b is a 'free variable').

(a) Find an expression for the size of the beam b that can carry F without exceeding the elastic limit (failure stress) σ_f.

(b) If the length of the cantilever is L=1m, the load is F=1000N and the safety factor is 2, what value of b is needed if the beam is made of: (i) low alloy steel; (ii) Al alloy; (iii) CFRP? Use the material properties in the table below.

(c) Write an expression for the mass m of the beam and determine the mass for each of the materials in (b). Which one would you choose?

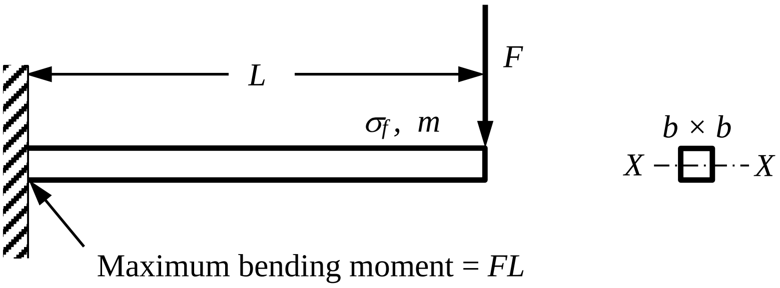
(d) Substitute the expression for beam dimension b from (a) into the expression for mass m from (c) to show that the material index to be *minimised* is $\rho/\sigma_f^{2/3}$. Determine the value of the material index for the three beams.

(e) Comment on the relationship between the mass of each beam and its material index $\rho/\sigma_f^{2/3}$.

(f) Examine Fig. 7.8 to see how these three materials compare in terms of the reciprocal material index $\sigma_f^{2/3}/\rho$.

	Failure stress σ_f (MPa)	Density ρ (kg/m^3)	b (mm)	m (kg)	$\rho/\sigma_f^{2/3}$
Low alloy steel	900	7800			
Al Alloy	300	2600			
CFRP	700	1500			

Answer.



(a) The extreme fibre stress at the root of the cantilever is: $\sigma = \dfrac{My}{I_{xx}}$. With $y_{max} = b/2$ and $I_{xx} = b^4/12$, this gives. $\sigma = \dfrac{FL\,b/2}{b^4/12} = \dfrac{6FL}{b^3}$. The stress can't exceed σ_f/S, where S is the safety factor,

so $b = \left(\dfrac{6SFL}{\sigma_f}\right)^{1/3}$ (1)

(b) For Aluminium, $b = \left(\dfrac{6\times 2\times 1000\times 1}{300\times 10^6}\right)^{1/3} = 0.0354\text{ m} = 35.4\text{ mm}.$

(c) The mass of the beam is: $m = \rho b^2 L.$ (2)

For Al, this is: $m = 2600(0.0354)^2(1) = 3.25$ kg.

(d) Substituting the free variable b from the constraint equation (1) into the objective function for the mass (2), gives: $m = \rho\left(\dfrac{6\,SF\,L}{\sigma_f}\right)^{2/3} L = (6\,SF)^{2/3} L^{5/3}\left(\dfrac{\rho}{\sigma_f^{2/3}}\right).$

So the material index to be minimized is $\dfrac{\rho}{\sigma_f^{2/3}}$

For Al: $\dfrac{\rho}{\sigma_f^{2/3}} = \dfrac{2600}{\left(300\times 10^6\right)^{2/3}} = 0.0058$, etc.

(e) The material indices are proportional to the mass of the beam and consequently they rank the materials in the same order as the mass. For example, compare Al and CFRP in the table: $m_{Al}/m_{CFRP} = 3.03 = \left(\rho/\sigma_f^{2/3}\right)_{Al} / \left(\rho/\sigma_f^{2/3}\right)_{CFRP}$. No need to re-solve the design problem (i.e. calculate *b* and *m* for each material) – just compare the values of $\rho/\sigma_f^{2/3}$.

(f) Selecting materials to <u>minimize</u> $\rho/\sigma_f^{2/3}$ is the same as selecting materials to <u>maximize</u> $\sigma_f^{2/3}/\rho$. Materials with high values of this index are furthest to the top left of Figure 7.8 above a line of slope 3/2. CFRP is significantly better than Al alloy or steel.

Material	Failure stress σ_f (MPa)	Density ρ (kg/m^3)	b (mm)	m (kg)	$\rho/\sigma_f^{2/3}$
Low Alloy Steel	900	7800	24.6	4.73	0.0083
Al Alloy	300	2600	35.4	3.25	0.0058
CFRP	700	1500	26.7	1.07	0.0019

Exercise E7.7 Derive the index for selecting materials for a light, strong panel, equation (7.17), following the steps outlined above the equation in the text.

Answer. The objective function is an equation for the mass of the panel

$$m = h\,b\,L\,\rho$$

where h, b and L are the thickness, width and length of the panel, and ρ is the density of the material from which it is made.

The constraint is that of strength: the panel must support a moment M without failing:

$$M \le Z_e\,\sigma_y = \frac{I}{y_m}\sigma_y = \frac{b\,h^2}{6}\sigma_y$$

Substituting for h from this equation into the objective function gives:

$$m = b\,L\,\rho\left(\frac{6M}{b\,\sigma_y}\right)^{1/2}$$

The mass is minimized, while meeting the constraint on strength, by choosing the material with the largest value of the index $\dfrac{\sigma_y^{1/2}}{\rho}$.

Exercise E7.8 Derive the index for selecting materials for a light, strong beam with a square cross section, equation (7.18).

Answer.

The constraint is again that of strength: the beam must be sufficient to support *F* without failing. The maximum moment will be a multiple of *F*×*L*, with a factor depending on the loading geometry. Hence, from equation (7.2)

$$F_f = C_2\frac{Z\,\sigma_f}{L}$$

where F_f is the design load at failure, C_2 is a constant, σ_f is the failure strength of the material of the beam, and Z is the elastic section modulus. Assume the proportions of the beam are fixed so that $h = b$. Use this and the relationships for Z and *A* in Figure 7.3 to first eliminate *A* in equation (7.13), and then to substitute for Z from the constraint equation above. This gives the mass of the beam that will just support the load F_f :

$$m = \left(\frac{6\sqrt{\alpha}\, F_f}{C_2\, L^2} \right)^{2/3} L^3 \left[\frac{\rho}{\sigma_y^{2/3}} \right]$$

The mass is minimized by selecting materials with the largest values of the index $\dfrac{\sigma_f^{2/3}}{\rho}$.

Exercise E7.9 Derive an index for selecting materials for a panel that meets a constraint on bending strength and is as thin as possible.

Answer. Here we wish to minimize the panel thickness h subject to the constraint that the panel must support a moment M without failing:

$$M \le Z_e\, \sigma_y = \frac{I}{y_m}\sigma_y = \frac{b\, h^2}{6}\sigma_y$$

Solving for h gives

$$h \ge \left(\frac{6M}{b\,\sigma_y} \right)^{1/2}$$

The panel thickness is minimized by selecting materials with the largest possible σ_y.

Exercise E7.10 Derive an index for selecting materials for a panel that meets a constraint on bending strength and is as cheap as possible.

Answer. The objective function is an equation for the cost C of the panel

$$C = h\, b\, L\, C_m\, \rho$$

The derivation follows the same steps as that for minimizing weight (Exercise E7.2). The resulting index is the same as that for minimizing mass with density ρ replaced by $C_m\, \rho$.

Exercise E7.11 A low temperature furnace operating at 250ºC uses solid shelves of rectangular cross-section for supporting components during heat treatment. The shelves are simply supported at the edges with the components located towards the centre, where the temperature is uniform. The designer wishes to minimise the mass, for ease of automated removal of the shelves, but failure of the shelves in bending must be avoided. The width *b* and length *L* of the shelves are fixed, but the thickness *d* can vary, up to a specified limit.

(a) For a specified load W applied at mid-span of the shelves, the maximum stress in bending is given by:

$$\sigma_{\max} = \frac{3WL}{2bd^2}$$

Show that the performance index to be maximised for minimum mass is $M = \sigma_f^{1/2} / \rho$ where σ_f is the material strength, and ρ is the density. Explain why there is also a lower limit on the material strength.

(b) Use a Strength-Density property chart to identify a short-list of candidate materials (excluding ceramics and glasses – why is this?). Eliminate materials which cannot operate at the required temperature – to do this, look ahead at the property chart in Figure 13.7, which indicates the usual upper limit on the service temperature of materials. Check whether the remaining materials would have a problem with the thickness limit, for which a minimum strength of 200 MPa is suggested.

Answer.

(a) Objective: mass of panel: $m = b\,l\,d\,\rho$

Constraint: failure when maximum stress = material strength: $\sigma_f = \dfrac{3Wl}{2bd^2}$

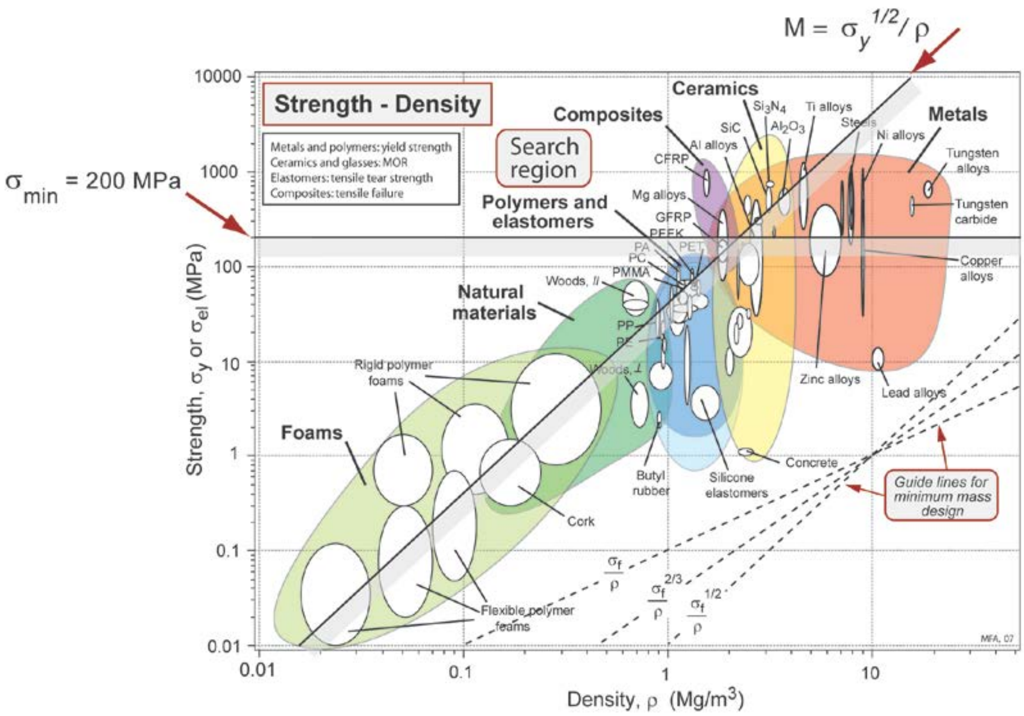
Hence free variable, depth, given by: $d = \left(\dfrac{3Wl}{2b\sigma_f}\right)^{1/2}$

Substituting into equation for mass: $m = bl\rho\left(\dfrac{3Wl}{2b\sigma_f}\right)^{1/2} = \text{constant} \times \left(\dfrac{\rho}{\sqrt{\sigma_f}}\right)$

Hence minimise mass by maximising performance index $M = \sigma_f^{1/2}/\rho$.

Thickness and strength related by: $\sigma_f = \dfrac{3Wl}{2bd^2}$. Hence a maximum allowable thickness leads to a minimum strength (depending on values for load, length, width and max. thickness).

(b)



On chart of Elastic Limit (Strength) against density, line of slope 2 positioned to give a working short-list of materials. Horizontal line at 200MPa sets strength limit (for acceptable thickness).

Results, noting those that cannot be used at 250°C (from Figure 13.7):

Material	Comments
Ceramics (alumina etc)	Disregard – data for compressive strength, and too brittle
CFRP	Good on strength at low weight, but excluded by operating temperature
Ti alloys (+ some Mg alloys)	**Pass all criteria.**
Al alloys	Excluded by operating temperature
Wood, GFRP, polymers	Below strength limit, and excluded by operating temp.

Exercise E7.12 A cable of cross-sectional area A and material density ρ is suspended over a fixed span L. The maximum allowable sag δ is specified, and the tensile stress in the cable must not exceed 0.8 of the yield stress, σ_y. It may be assumed that the sag is small compared to the span, in which case the tension in the cable is given by $T = \dfrac{wL^2}{8\delta}$ where w is the weight per unit length. Show that this design specification places a lower limit on the specific strength of the material used for the cable. Does this result depend on the cross-sectional area?

Answer.

Cable tension = $T = \dfrac{wL^2}{8\delta}$, therefore stress in cable, σ = tension/area = $\dfrac{wL^2}{8\delta A}$

Now weight per unit length of cable, $w = \rho A g$, where ρ is the density.

Hence stress in cable $= \dfrac{\rho A g L^2}{8\delta A} = \dfrac{\rho g L^2}{8\delta}$

Design specifies that maximum stress < 0.8 σ_y

Hence $\dfrac{\rho g L^2}{8\delta} < 0.8\sigma_y$, and thus $\dfrac{\sigma_y}{\rho} > 0.157\dfrac{gL^2}{\delta}$ is the lower limit for specific strength.

This result is independent of the cross-sectional area – both self-weight and stress increase with cross-sectional area, so it cancels out.

Exercise E7.13 You are to design a set of ‘rising bollards’ (that is, columns that rise up from beneath the road to provide a barrier to prevent cars from passing). The bollards have a specified height l and may be assumed to be built-in at the ground (i.e. cantilevered). A solid cylindrical cross-section has been chosen, but the radius R may be varied. In service the bollards are vertical and must support a given maximum load W, applied horizontally at mid-height, without failure. To minimise the power requirements for lifting the bollards, they must be as light as possible.

(a) Show that the maximum stress in a bollard is given by $\sigma = \dfrac{2Wl}{\pi R^3}$

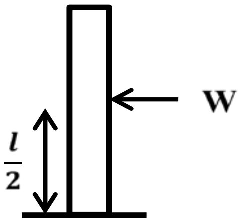
(b) Derive a performance index, including only material properties, which should be minimized for the given design specification. Hence select the best two materials for the bollards from those given in the Table below.

(c) A bollard of height 0.6m is required to resist a load of 40kN at mid-height without failure. An upper limit of 50mm is imposed on the radius of the bollard. Show that this sets a lower limit on the failure stress of the material, and if necessary, revise your material selection from the Table.

(d) Briefly outline two factors, other than strength, which might eliminate either of your chosen materials.

Material	Density (Mg/m^3)	Failure stress (MPa)
CFRP	1.6	300
Wood	0.6	100
Mild steel	7.6	350
Al alloy	2.7	350
Concrete	2.1	80

Answer.



(a) Maximum moment: $M_{\max} = \frac{W\,l}{2}$

$\sigma_{\max} = \frac{M\,R}{I}$ where $I = \frac{\pi\,R^4}{4}$, hence $\sigma_{\max} = \frac{2W\,l}{\pi\,R^3}$

(b) Mass $m = \pi\,R^2\,l\,\rho$ (objective)

From (a), at failure: $\sigma_f = \frac{2W\,l}{\pi\,R^3}$ (constraint)

Substitute for R (free variable), giving mass $m = \pi^{1/3}\,l^{5/3}\,(2W)^{2/3}\,\frac{\rho}{\sigma_f^{2/3}}$

Performance index: $M = \frac{\sigma_f^{2/3}}{\rho}$ (maximize for minimum mass)

Material	M
CFRP	28.0
Wood	35.9
Steel	6.5
Al	18.4
Concrete	8.8

So CFRP and wood appear the best.

(c) $\sigma_f = \frac{2W\,l}{\pi\,R^3}$, so for $R \le R_{\max}$, $\sigma_f \ge \frac{2W\,l}{\pi\,R_{\max}^3}$

For $W = 40$ kN, $l = 0.5$m, $R \le 0.05\text{m}$, $\sigma_f \ge 122\,\text{MPa}$.

This rules out wood (and concrete), so the best two materials are now CFRP and Al alloy.

(d) Other factors:

Cost – CFRP or Al alloy
Corrosion – Al alloy
Difficulty with fabrication – CFRP
Fracture toughness – CFRP (and perhaps Al alloy)
(Stiffness should not be an issue for either CFRP or Al alloy).

Exercise E7.14 A centrifuge has a rotor that can be idealized as a uniform disk. It has a diameter of 200 mm and is made of a material of strength 450 MPa and density 7900 kg/m^3. Use equation (7.8) to see how fast it can be spun (in radians per second) before the stresses it carries exceed its yield strength.

Answer. A disk of density ρ, radius R and thickness t, rotating at an angular velocity ω radians/sec, is subjected to a maximum stress (when Poisson's ratio is taken as 1/3) given by equation (7.8) of the text:

$$\sigma_{\max} = 0.42\,\rho\,\omega^2\,R^2$$

The disk yields when σ_{max} exceeds σ_y, and this defines the maximum allowable ω. Using the data given in the question the maximum angular velocity is 3682 radians/sec (about 35,000 rpm).

Exercise E7.15 There is a plan to build an energy storage system for the London Underground rail system. The objective is to slow trains down using 'regenerative braking' by running their electric motors 'in reverse' as electricity generators. The electricity generated by the trains will be injected back into the rail system's power grid. A flywheel (located in a hole in the ground in central London) will be used to store the surplus electrical energy and then supply it back to the next accelerating train. The goal is to store at least the kinetic energy of a single train of mass 100 tonnes, decelerated to rest from a speed of 60 km/h. The flywheel is a spinning disc with thickness t = 1m and radius R = 4m.

(a) How much energy must be stored in the flywheel (assuming that all energy conversions are 100% efficient)?
(b) Find the maximum speed ω for bursting of flywheels made of low alloy steel, GFRP and CFRP, using the property values in the table below and a safety factor of S = 2.
(c) Determine the corresponding stored energy U in each case, and compare with the energy recovered from stopping the train.
(d) Calculate the mass and the material cost of the flywheels. Which material would you choose?

	Failure stress σ_f (MPa)	Density ρ (kg/m^3)	Material cost C_m (GBP/kg)	ω (rad/sec)	U (MJ)	Mass m (kg)	Cost (GBP)
Low alloy steel	900	7800	0.6				
GFRP	190	1850	12				
CFRP	700	1600	25				

Answer.

(a) The kinetic energy of the train, which must be stored in the flywheel is $KE = \frac{1}{2}mv^2$.

With v = 60 km/h = 16.7 m/s, and m = 100 × 10^6 kg, this gives:

$$KE = \frac{1}{2}\left(100 \times 10^6\right)\left(16.7^2\right) = 1.4 \times 10^{10}\ \mathrm{J} = 14\ \mathrm{GJ}.$$

(b) From equation (7.8), $\sigma_{\max} = 0.42\,\rho\,\omega^2 R^2 = \sigma_f / S$

So the maximum speed is: $\omega_{\max} = \sqrt{\dfrac{\sigma_f / S}{0.42 \rho R^2}}$.

For steel with R = 4 m and the material properties in the table:

$$\omega_{\max} = \sqrt{\frac{900 \times 10^6 / 2}{0.42(7800)\left(4^2\right)}}. = 92.7\ \mathrm{rad/s} = 885\ \mathrm{RPM}.$$

(c) From equation (7.8), the energy stored is $U = \frac{\pi}{4}\rho\, t\, \omega^2 R^4$

For steel this is $U = \frac{\pi}{4}(7800)\,(1.0)\,92.7^2\,4^4 = 1.35 \times 10^{10}\ \mathrm{J} = 13.5\ \mathrm{GJ}$. (which is approximately right for the KE of the train). Values for other materials in table below.

(d) The mass of the steel flywheel is $m = \pi R^2 t \rho = \pi\left(4^2\right)(1.0)(7800) = 392$ tonnes. This is four times the mass of the train! The material cost of the steel flywheel alone (not including the cost of the bearings, drive system, the housing, and all the construction, etc) would be $C = mC_m = 392{,}000 \times 0.6 = 235{,}000$ GBP. Values for other materials in table below.

	Failure stress σ_f (MPa)	Density ρ (kg/m^3)	Material cost C_m (GBP/kg)	ω (rad/sec)	U (GJ)	Mass m (Mg)	Cost (MGBP)
Low alloy steel	900	7800	0.6	92.7	13.5	392	0.235
GFRP	190	1850	12	87.4	0.28	92	1.1
CFRP	700	1600	25	180	10.5	80	2.0

The steel flywheel is very heavy and runs relatively slowly, but is by far the cheapest. Of course the bearings to support such a heavy flywheel might be very expensive. The GFRP isn't up to the job – it can't store enough energy at this size – you would need to increase the size significantly. The CFRP is much lighter and faster than the steel, but much more expensive. It can almost store all the energy at this size – so some adjustments to the geometry would probably get it up to specification. CFRP would be significantly better for use on a moving vehicle, where mass would be important, but would not be a good choice here.

Exercise E7.16 A material is sought for a high-speed centrifuge. The objective is to achieve as high an angular velocity ω of the centrifuge disk as possible. The constraint is that the stress created by the centripetal force must not exceed the yield strength of the material of which it is made. Derive an index to guide the choice of material to allow the maximum ω.

Answer. A disk of density ρ, radius R and thickness t, rotating at an angular velocity ω radians/sec, is subjected to a maximum stress (when Poisson's ratio is taken as 1/3) given by equation (7.8) of the text:

$$\sigma_{max} = 0.42\,\rho\,\omega^2\,R^2$$

The disk yields when σ_{max} exceeds σ_y. Equating the two and solving for ω gives

$$\omega = \left(0.42\frac{\rho}{\sigma_y}\right)^{1/2} R$$

The best choice is that of materials with the largest values of the index σ_y / ρ.

Exercise E7.17 The engine of a car is mounted on four shear bolts designed to fail in shear in a front-end collision, detaching the engine from the car, if the deceleration exceeds 10g. Assume that all four bolts carry the same load. The mass of the engine is 80 kg. Because of space limitations, the maximum diameter of each shear bolt cannot exceed 5 mm.

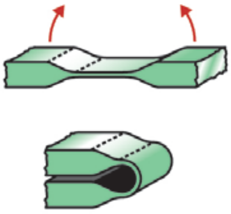
(a) Calculate the constraint on the yield strength σ_y of the material of the shear bolt, assuming that the shear strength is $\sigma_y/2$ and that the mounting is done in such a way that the shear can take place without friction.

(b) If, to save weight, the shear bolts have to be as light as possible, what metal (of those identified on the chart of Figure 7.8) would you choose to make them?

Answer. (a) $\sigma_y > 200$ MPa; (b) magnesium or titanium.

Exercise E7.18 Valve springs for high performance automobile engines must be light to minimise inertial loads, since part of their mass moves with the valves. At high engine speeds the valves, if heavy, bounce out of contact with the valve itself ('valve bounce'), impeding the flow of gas into and out of the combustion chamber. Derive the index for light springs to guide material choice for the springs, noting also that engine temperatures can reach 200°C.

Answer. The stored energy per unit volume in a material carrying a stress σ was derived in Chapter 4 – it is $\sigma^2 / 2E$. If the spring yields it deforms permanently and ceases to fulfill its function, so the maximum value of σ must not exceed σ_y, when the stored energy is $\sigma_y^2 / 2E$ per unit volume. The mass per unit volume of the material is its density, ρ, so the stored energy per unit weight is $\sigma_y^2 / 2E\,\rho$. The best choice is that of materials with the largest values of the index $\sigma_y^2 / E\rho$.



Exercise E7.19 When the new hospital in Cambridge was first opened, all the ward doors had elastic hinges, as in the figure, so that they could be pushed both ways and close automatically. Consider such a hinge as a thin ligament of material which flexes elastically.

(a) Explain briefly why the best material for the elastic hinge (of a given design) would be one that bends to the smallest radius without yielding.

(b) Consider a section of beam of thickness *t*, bent elastically into an arc of radius R (as in Figure 7.1a). Show that the maximum strain ε at the surface is given by $\varepsilon = t/2R$. Hence write down an expression for the maximum stress σ at the surface in terms of the Young's modulus E, t and R.

(c) Deduce the minimum radius R of the ligament that can be formed without the material yielding, in terms of E, t and the yield stress σ_y. Hence propose a material performance index for minimum radius.

(d) Using the Material Selection chart in Figure 7.9, identify the best choices of material for the elastic hinge. Make a short list of materials and from it select the best material, giving reasons for discarding others.

(e) Some 6 months following the opening of the hospital, many of the elastic hinges had failed. Suggest other factors that should have been taken into account (Chapters 9 and 10 will explore issues that could be relevant here).

Answer.

(a) The hinge is a thin ligament of material which flexes elastically as the door is closed, as in the figure, but it carries no significant axial loads. The best material is the one which (for given ligament dimensions) bends to the smallest radius R without fracturing or failing by fatigue.

(b) When a ligament of thickness t is bent elastically to a radius R, the surface strain can be found from equation (7.1), by setting $y = t/2$, $\sigma = E\varepsilon$ and $\kappa = 1/R$. Then:

$$\frac{E\varepsilon}{t/2} = E\frac{1}{R}, \text{ from which } \quad \varepsilon = \frac{t}{2R}$$

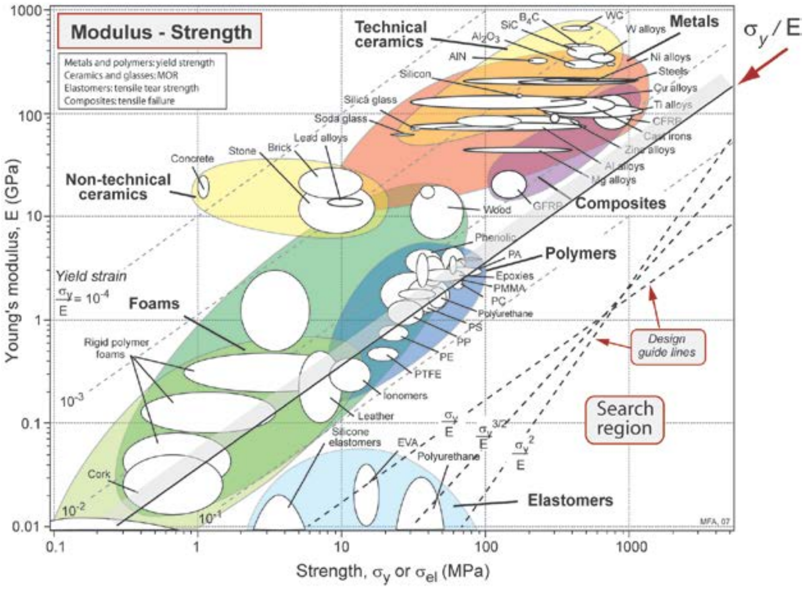
(Alternatively, this expression can be derived from the geometry in Figure 7.1a)

Since the hinge is elastic - the maximum stress is $\sigma \geq E\dfrac{t}{2R}$.

(c) This stress must not exceed the yield strength of the material σ_y. Then the radius to which the ligament can be bend without damage is $R \leq \dfrac{t}{2}\left[\dfrac{E}{\sigma_y}\right]$.

The best material is the one that can be bent to the smallest radius, that is, the one with the greatest value of the index $M = \left[\dfrac{\sigma_y}{E}\right]$.

(d) Suitable materials lie below a line of slope 1 on the selection chart below. The best choices for the hinge are all polymers. The short list includes polyethylenes (HDPE), polypropylene and nylon (PA) – cheap products with this sort of elastic hinge are generally moulded from one of these materials. Elastomers are best according to the index *M*, but they are probably too flexible for this application. This means that too much material would be needed: the hinge would be too thick and probably wouldn't fit in the space available. Another selection criterion, based on a thickness constraint would show this. Spring steel and other metallic spring materials (like phosphor bronze) are less good by the criterion *M*.



(e) The hinges are loaded repeatedly – every time the door is opened, and so the material is subject to 'fatigue' loading. An elastic hinge for this application would need to be designed for an appropriate fatigue life of many thousands or tens of thousands of cycles. In practice, the loading is not just pure bending. The hinge has to carry the weight of the door, and the material may also be stretched axially. In retrospect, plastic hinges were a very poor choice here – they are excellent for low cost applications on plastic moulded parts. Their low cost is not an advantage for high performance important hinges subject to complex loading. Perhaps the architects involved should have learned some engineering!

Exercise E7.20 You have been asked to design the pressure hull for a deep-sea submersible vehicle capable of descending to the bottom of the Pacific Ocean. The hull is to be a thin-walled sphere with a specified radius r (equal to 1m) and a uniform wall thickness t (which must be chosen). The design pressure at the bottom of the ocean is 200MPa. The sphere can fail by one of the following two mechanisms:

(i) External-pressure buckling at a pressure given by $p_b = 0.3E\left(\frac{t}{r}\right)^2$

(ii) Compressive failure at a pressure given by $p_f = 2\,\sigma_f\left(\frac{t}{r}\right)$.

E is the Young's modulus and σ_f is the failure stress.

(a) Derive material performance indices which must be maximized in order to achieve the miniumum mass:

(i) in the case of external-pressure buckling;
(ii) in the case of compressive failure.

(b) For each material listed in the Table below, calculate the mass and the wall thickness of the hull for each failure mechanism at the design pressure.

(c) Hence determine the limiting failure mechanism for each material.

(d) Which is the best material for the hull?

Material	Modulus E (GPa)	Strength σ_f (MPa)	Density ρ (kg/m^3)	Thickness t and mass m (buckling)	Thickness t and mass m (compression)	Failure mechanism
Alumina	390	5000	3900			
Glass	70	2000	2600			
Alloy steel	210	2000	7800			
Ti alloy	120	1200	4700			
Al alloy	70	500	2700			

Answer.

(a) Follow the normal procedure of (i) defining the objective function (mass); (ii) using the constraint to derive an equation for the free variable (thickness); (iii) substituting the free variable into the objective function, and separating the material properties to find the materials performance index.

(i) External-pressure buckling: $p_b = 0.3E\left(\frac{t}{r}\right)^2$

Mass $m_b = 4\pi r^2 t \rho$

Constraint gives free variable: $t = r\left(\frac{p_b}{0.3E}\right)^{1/2}$

Hence $m_b = 4\pi r^3 \rho \left(\frac{p_b}{0.3E}\right)^{1/2} = 22.9 r^3 p_b^{1/2}\left(\frac{\rho}{E^{1/2}}\right)$

Material performance index: $M_b = \left(\frac{E^{1/2}}{\rho}\right)$

(ii) Compressive failure: $p_f = 2\,\sigma_f\left(\frac{t}{r}\right)$.

Mass $m_f = 4\pi r^2 t \rho$

Constraint gives free variable: $t = r\left(\frac{p_f}{2\sigma_f}\right)$

Hence $m_f = \frac{4\pi r^3 p_f \rho}{2\sigma_f} = 2\pi r^3 p_f \left(\frac{\rho}{\sigma_f}\right)$

Material performance index: $M_f = \left(\frac{\sigma_f}{\rho}\right)$

(b, c)

(i) Mass from result above; thickness from $t_b = \frac{m_b}{4\pi r^2 \rho}$

(ii) Mass from result above; thickness from $t_f = \frac{m_b}{4\pi r^2 \rho}$

Set r = 1m and $p_b = p_f = 200$ MPa, and use values for ρ, E and σ_f from the Table.

Material	m_b (tonne)	t_b (mm)	m_f (tonne)	t_f (mm)	Mechanism
Alumina	2.02	**41**	0.98	20	Buckling
Glass	3.18	**97**	1.63	50	Buckling
Steel	5.51	**56**	4.90	50	Buckling
Titanium	4.39	74	4.92	**83**	Compressive
Aluminium	3.30	97	6.79	**200**	Compressive

The limiting failure mechanism is that which gives the *larger* of the two values of thickness for each material, in order to satisfy both design criteria. (Note that for minimum mass design it is tempting to choose the lower of the two values, but this is incorrect as it will not meet the other constraint).

(d) Optimum material is Alumina, giving the least mass of the five materials, however toughness needs to be considered before making a final choice.

Exercise E7.21 The Table shows a selection of candidate materials for lightweight, strength-limited design of a beam. The shape is initially constrained to be square, but the cross sectional area can vary. Evaluate the relevant index for this problem, $\sigma_f^{2/3}/\rho$, for each material. Which will be the lightest? The text showed that if the shape is allowed to vary, the mass can be improved by a factor of $\Phi^{-2/3}$, which is equivalent to increasing the material index by $\Phi^{2/3}$. Evaluate the modified index $(\Phi\sigma_f)^{2/3}/\rho$, using the maximum shape factor for each material (as listed in Table 7.4). Does the ranking of the materials change?

	Elastic limit σ_f (MPa)	Density ρ (kg/m^3)	Material index without shape $\sigma_f^{2/3}/\rho$	Material index with shape $(\Phi\sigma_f)^{2/3}/\rho$
Steel	600	7800		
Al alloy	400	2700		
GFRP	150	1900		
Wood	70	650		

Answer.

	Elastic limit σ_f (MPa)	Density ρ (kg/m^3)	Max. shape factor Φ	Material index without shape $\sigma_f^{2/3}/\rho$	Material index with shape $(\Phi\sigma_f)^{2/3}/\rho$
Steel	600	7800	13	0.0091	0.051
Al alloy	400	2700	10	0.020	**0.094**
GFRP	150	1900	9	0.015	0.064
Wood	70	650	3	**0.026**	0.055

Without accounting for shape, the lightest beam is the one with the highest value of $\sigma_f^{2/3}/\rho$. (Remember that the mass is proportional to $\rho/\sigma_f^{2/3}$). The winner is Wood, closely followed by Aluminium alloy.

Accounting for the ability to shape the material into thin cross sections significantly hampers wood. It is difficult to make really thin wooden sections for beams. The material with the highest value of $(\Phi\sigma_f)^{2/3}/\rho$ is Al alloy by a long way. Its advantages in strength and density are enhanced by its relatively high maximum shape factor. A shaped Aluminium beam would be about half the mass of a shaped steel or wooden beams: $0.051 / 0.094 \approx 0.5$.

***Exploring design with CES**. (Use Level 2 unless otherwise indicated).*

Exercise E7.22 Use CES Level 2 to answer the following:

(a) Find materials for which the failure strength (elastic limit) $\sigma_f > 300$ MPa, and the density $\rho < 3$ Mg/m^3. Eliminate materials with poor ductility (e.g. apply a minimum elongation of 5%) and identify the cheapest.

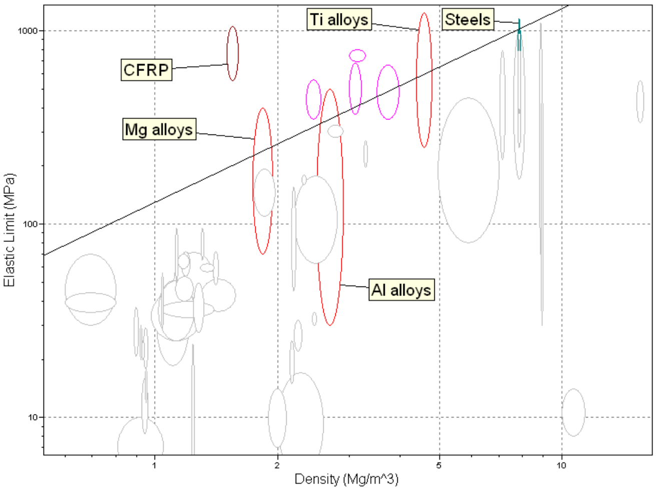
(b) Compare the specific strength, σ_f/ρ, of steels, Ti alloys, Al alloys, Mg alloys, and CFRP (taking the highest strength in each range for each material).

(c) Find metals and composites that are both stiffer and stronger than the highest strength Al alloys.

(d) State the performance index for maximum elastic energy storage per unit volume. Use this index to find materials which are suitable for efficient springs. Can you identify spring applications using these materials?

Answer.

(a) In CES, elastic limit (strength)-density chart; box selection with prescribed values (or use limit stage). Ceramics excluded by a tree stage including all other classes.



Structural materials with strength > 300 MPa and density < 3 Mg/m^3:

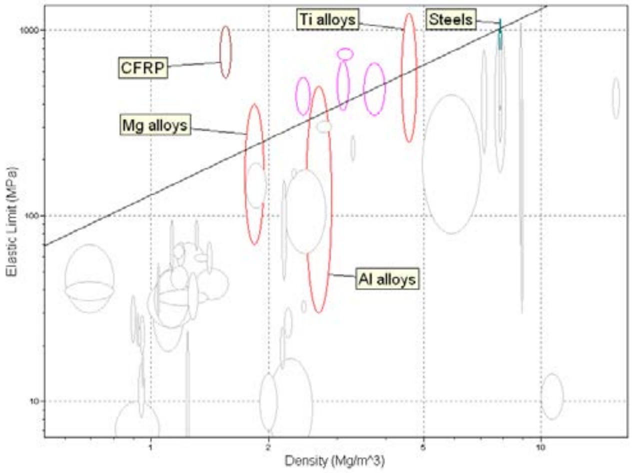
- higher strength Al and Mg alloys
- Al/SiC (just)
- CFRP: but poor elongation (< 5%)

Boron carbide also meets the strength and density requirements, but has very low elongation (ceramic).

Cost/kg of candidate materials:

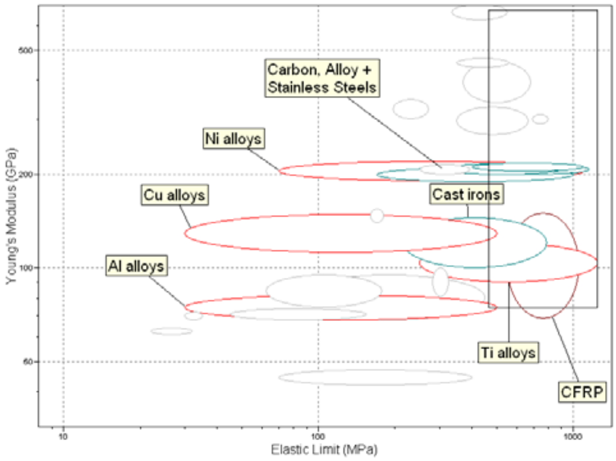
Material	Cost/kg	Comments
Al alloys	0.75 – 1.22	The cheapest
Mg alloys	2.8 – 3.2	Both more expensive than Al alloys
Al-SiC	2.5 - 5	

(b) In CES, elastic limit (strength)-density chart; line of slope = 1 (constant specific strength σ_f/ρ) located through top of the steels bubbles. Alternatively, use Advanced graph stage to plot σ_f/ρ for the required materials.



Specific strength of steels comparable to strongest Al alloys; Mg and Ti alloys extend to rather higher values; CFRP much higher value than all the alloys.

(c) In CES, Young's modulus – elastic limit (strength) chart; box selection with bottom LH corner at high strength end of Al alloy bubble. Metals and composites selected via Tree Stage.



Metals and composites which are stiffer and stronger than strongest Al alloys:

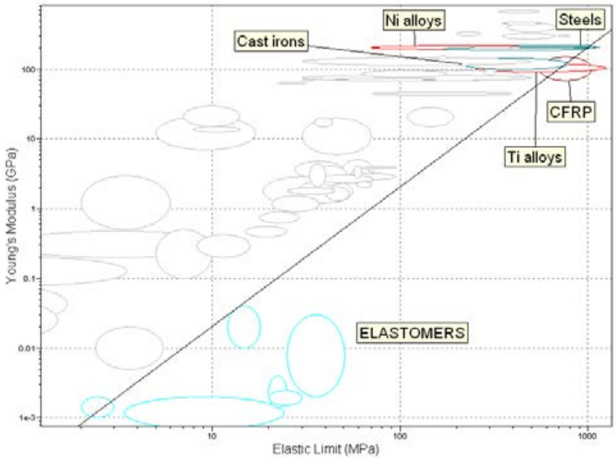
- higher strength steels, cast irons, Ni and Ti alloys

- the very strongest Cu alloys

- CFRP

(d) Performance index for maximum elastic stored energy per unit volume = σ_y^2 / E.

In CES, Young's modulus - elastic limit (strength) chart; line of slope 2 for constant value of σ_y^2 / E, move towards bottom right to optimise.



Elastomers (including rubbers) stand out as good spring materials (climbing ropes, bungee ropes)!

At the other end of the stiffness and strength spectrum, Ti alloys and CFRP are prominent (used extensively in sports products which store elastic energy).

The strongest steels (actually called "spring steels"), cast irons, and Ni alloys all join the list. Vehicle springs (coil and leaf) are mostly steel, but composites have also been trialled.

Exercise E7.23 Use the Search facility in CES to search for materials that are used for *springs*. Report what you find.

Answer. The search gives a large number of returns. All are used, in one way or another, as springs.

- CFRP, epoxy matrix (isotropic)
- High carbon steel
- Low alloy steel
- Copper alloys
- Nickel alloys
- Titanium alloys
- Polyoxymethylene (Acetal, POM)
- Bronze
- Brass
- Nickel-based superalloys
- Nickel-chromium alloys

Exercise E7.24 Use the Search facility in CES to search for materials that are used for *light springs*. Report what you find.

Answer. Here the search gives a smaller number of returns than in Exercise E7.23. They are:

- CFRP, epoxy matrix (isotropic)
- Titanium alloys
- Polyoxymethylene (Acetal, POM)
- Nickel-based superalloys

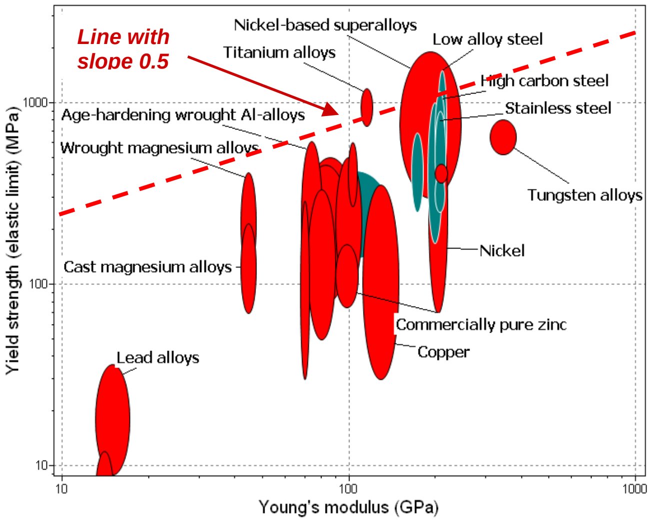
Exercise E7.25 Make a property chart with σ_y on one axis and E on the other. Use it to select materials for springs, using the index σ_y^2 / E derived in the text. Which three metals emerge as the best metallic choices?

Answer. See the chart. A Tree stage has been used to limit the plot to metals only. The index plots as a line of slope 0.5. Moving it until only the three best remain gives the selection:

- Low alloy steel
- Nickel-based superalloys
- Titanium alloys

The three materials differ enormously in price:

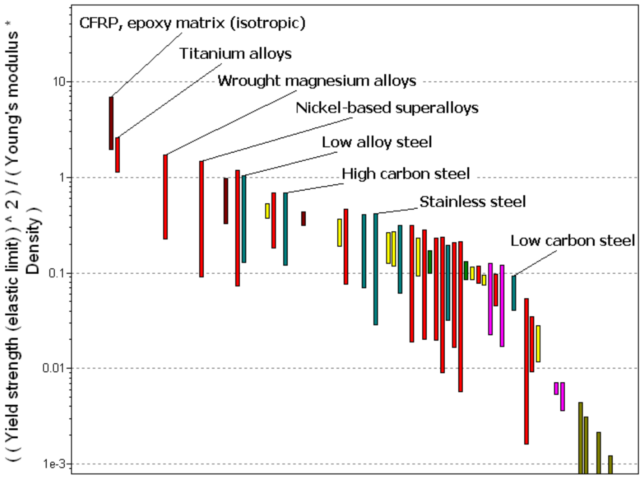
- Low alloy steel
  0.44 – 0.75 $/kg
- Nickel-based superalloys
  20 – 30 $/kg
- Titanium alloys
  45 – 60 $/kg



The least expensive, by far, is the low alloy steel.

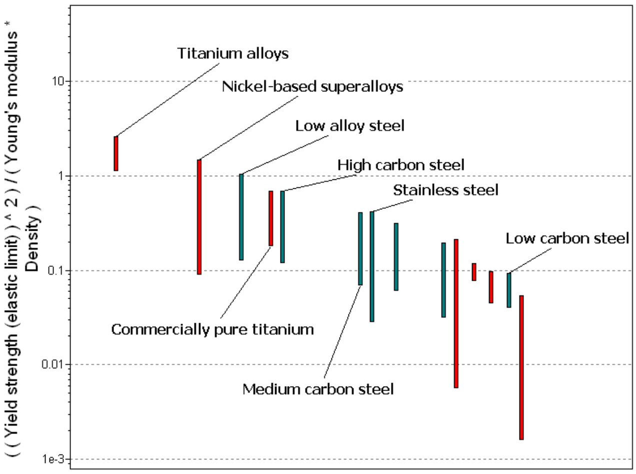
Exercise E7.26 A material is required for a spring that must be as light as possible. To be stiff enough it must also have a Young's modulus $E > 20$ GPa. Make a bar chart with the index $\sigma_y^2 / E\rho$ for selecting light springs (you will need to use the 'Advanced' facility in the axis-choice dialog box to do this). Add a Limit stage to apply the constraint $E > 20$ GPa. Hence find the two materials that are the best choices for this application.

Answer. The chart shows the value of the index. CFRP and titanium alloys emerge as the best choices.



Exercise E7.27 Exercise E7.18 describes the requirements for valve springs for high performance engines. Apply the index derived there, $\sigma_y^2 / \rho E$, by making an appropriate chart and plotting an appropriate selection line on it. Engines are hot: add a Limit stage on maximum service temperature of 250°C. Hence select metals for this application.

Answer. The chart shows the value of the index. The constraint of a maximum service temperature > 250°C, and the restriction to metals, have been applied. The best choices are titanium alloys and nickel-based superalloys.



Exercise E7.28 The designer in Exercise E7.11 is concerned about the cost, and asks for an alternative short-list based on minimum material cost. Modify your material performance index accordingly, and use a suitable material property chart in *CES Level 2*, to identify a short-list of materials (applying the same secondary constraints as before).

Answer.

Modify performance index from Exercise E7.11, for minimum cost:

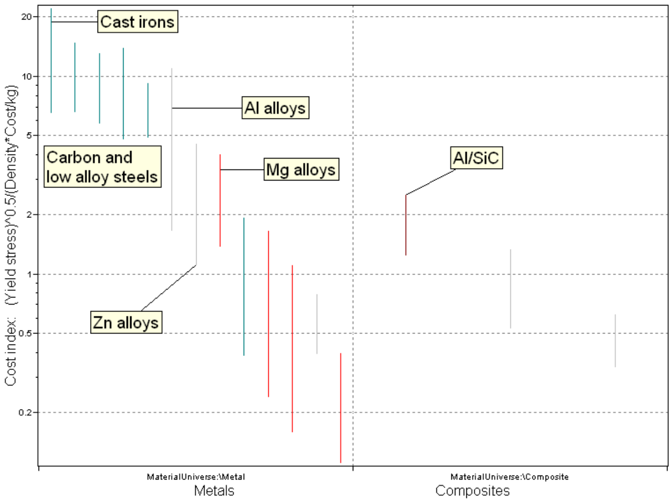
Objective: cost of panel: $C = b\,l\,d\;\rho C_m$

Applying same constraint on failure stress and eliminating free variable *d* as before:

Cost: $C = b\,l\,\rho C_m \left(\dfrac{3W\,l}{2b\sigma_f}\right)^{1/2} = \text{constant} \times \left(\dfrac{\rho\,C_m}{\sqrt{\sigma_f}}\right)$

Hence minimise cost by maximising performance index $M = \sigma_f^{1/2} \big/ (\rho C_m)$.

In CES, use "Advanced" feature in graph stage to plot this index as a bar chart (for metals and composites only – other classes eliminated by operating temp. and/or strength limit).

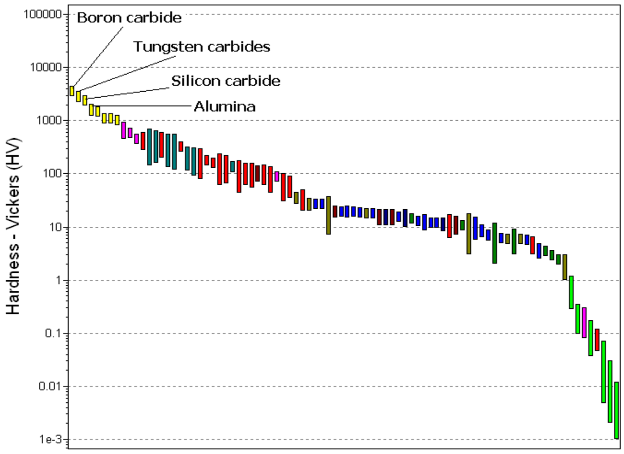


Selection results:

Material	Comments
Cast irons, carbon & alloy steels	Wide range of ferrous alloys give strength at low cost
Al alloys	Competes on cost, excluded by operating temperature
Mg alloys, Al-SiC	Less competitive on cost
Zn alloys	Close to Mg and Al-SiC, excluded by operating temp.

Exercise E7.29 Abrasives have high hardness, H . Make a bar-chart of hardness and identify the four materials with the highest values. They are prime choices for abrasive wheels and pastes.

Answer. The chart shows the hardness of the materials in the Level 2 database. The hardest are Boron carbide, Tungsten carbide, Silicon carbide and Alumina. All four are, in fact, used as abrasives.

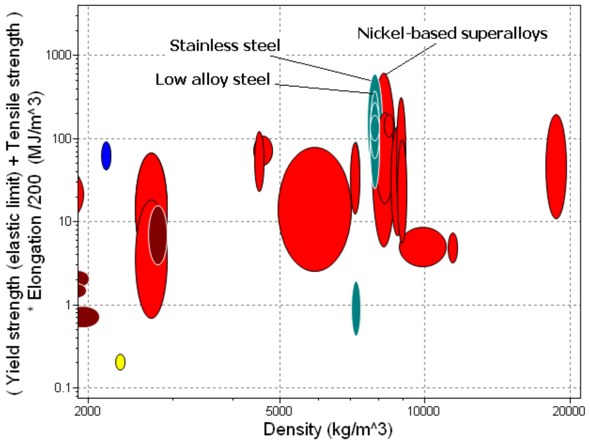


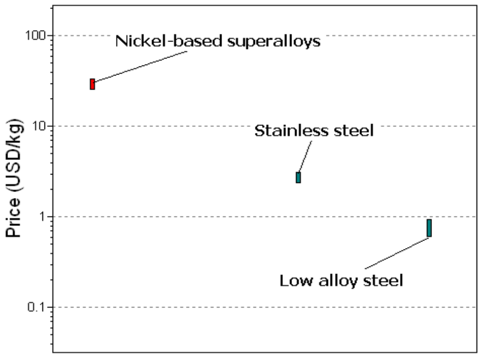
Exercise E7.30 Crash barriers, auto fenders and other protective structures rely on absorbing kinetic energy by plastic deformation. The energy W_{pl} absorbed in deforming a material to fracture (the area under the stress-strain curve) can be estimated approximately in CES as

$$W_{pl} = \frac{1}{2}\left(\sigma_y + \sigma_{ts}\right)\varepsilon_f$$

Make a chart with density on the x-axis and W_{pl} on the y-axis by using the 'Advanced' facility in the axis-selection dialog box. Use a box selection to find the three materials that absorb the most energy. Rank them by price, using a Graph stage to plot price.

Answer. The first chart shows the quantity W_{pl} , with units of MJ/m^3, showing the selection of the best three. The second shows a bar chart of price/kg (the price/m^3 ranks in almost exactly the same way since all three have nearly the same density). Low alloy steel is cheapest by far, Stainless steel is 3.5 times more expensive and Nickel-based superalloys are 37 times more expensive.



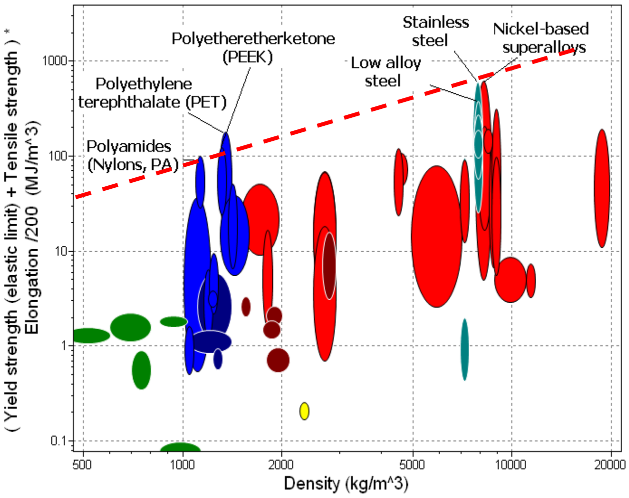


Exercise E7.31 If the crash barrier of Exercise E7.30 is part of a vehicle, fuel is saved if it is light. We then want the materials with the largest value of W_{pl} / ρ, where ρ is the density. These are found by using a selection line of slope 1 on the chart made in the last exercise and selecting the materials above the line. In this application the material must also have adequate stiffness so that they do bend elastically too much requiring a Young's modulus of at least 2.5 GPa. Apply this requirement using a Limit stage, return to the chart and move the selection line until only three materials remain. Comment on the choice.

Answer. The best three are all polymers:

- Polyamides (Nylons, PA)
- Polyetheretherketone (PEEK)
- Polyethylene terephthalate (PET)

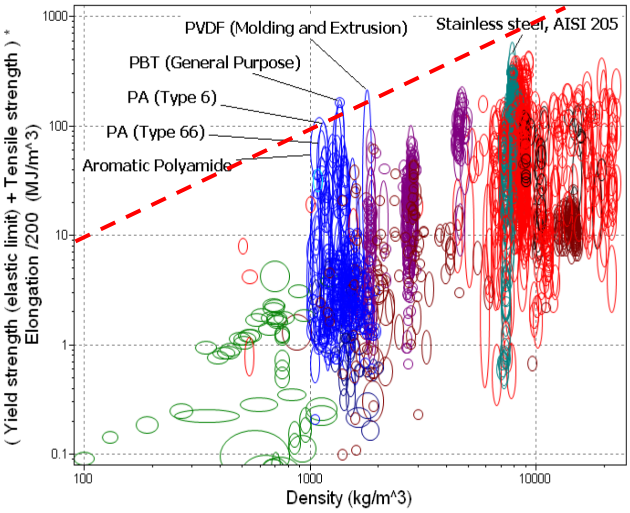
This suggests that polymers might make good crash barriers – but it would require ensuring that the materials are taken to their full failure strain everywhere to absorb the energy. The logical next step is to explore the materials more fully by using Level 3 data. The next example does so.



Exercise E7.32 Now explore what Level 3 of the database can do. Leave all the selection lines and criteria in Exercise E7.31. Go to File – Change Database – CES Edu Level 3 – Open. A dialog box asks if you wish apply this to your current project. Click Yes. Then list the materials that now appear in the results window. Comment on the choice

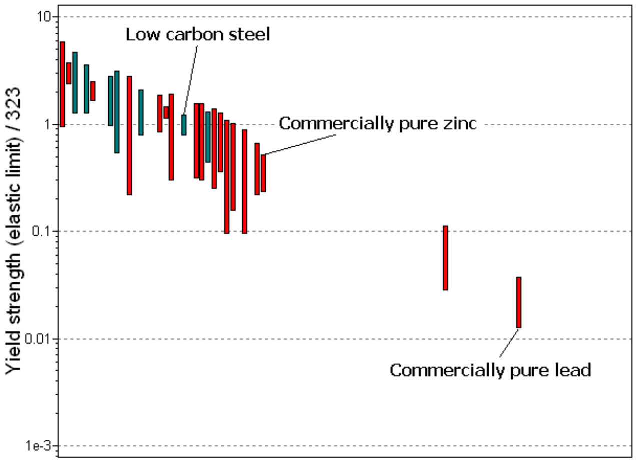
Answer. The selection is now much more specific. The top candidates are:

- Aromatic Polyamide
- PA (Type 6, Molding and Extrusion)
- PA (Type 66, Lubricated, Copolymer)
- PBT (General Purpose)
- PEEK (Unfilled)
- PET (unfilled, amorphous)
- POM (Impact Modified Copolymer)
- PVDF (Molding and Extrusion)



Exercise E7.33 The text showed that the power required to roll a metal is proportional to its yield strength. Make a bar-chart of yield strength, σ_y for metals. Open the record for low-carbon steel, find the range of its yield strength and take the average. Normalize all the yield strengths in the database by dividing them by this value, using the 'Advanced' option in the dialog box for selecting the axes, so that low-carbon steel now lies at the value 1 on the bar chart. Use the chart to read off how much less power is required to roll (a) commercially pure zinc, and (b) commercially pure lead.

Answer. The mean yield strength of low carbon steel is 323 MPa. The bar chart shows the yield strengths of other metals normalized by this value. Zinc, a factor of 0.35; (b) Lead, a factor of 0.02.



Materials: engineering, science, properties, and design
3e Solution manual

Chapter 8: Exercises with worked solutions

Exercise E8.1 What is meant by stress intensity factor? How does this differ from the stress concentration factor?

Answer. *Stress intensity factor* is a measure of the intensity of the local stress in the vicinity of a crack tip. Although the (elastic) stress at a crack tip is theoretically infinite, the stress in the vicinity of the crack tip depends on the distance r from the crack tip according to $1/\sqrt{r}$. The scale factor is $\sigma\sqrt{\pi c}$, which is known as the Mode I stress intensity factor.

Stress concentration factor is the ratio of the maximum stress in the vicinity of a geometric feature (like a hole) to the remote (applied) stress. It typically takes values in the range 1-3, but may be higher if the feature is particularly sharp.

Exercise E8.2 What is meant by toughness? How does it differ from strength, and from fracture toughness?

Answer. *Strength* is resistance to plastic flow and thus is related to the stress required to move dislocations through the solid. The initial strength is called the *yield strength*. Strength generally increases with plastic strain because of work hardening, reaching a maximum at the *tensile strength*.

Toughness is the resistance of a material to the propagation of a crack. A material with low fracture toughness, if it contains a crack, may fail before it yields. A tough material will yield, work harden even when cracked – the crack makes no significant difference.

Exercise E8.3 Why does a plastic zone form at the tip of a crack when the cracked body is loaded in tension?

Answer. The intense stress field at the tip of a crack in a ductile material generates a *process-zone*: a region in which plastic flow takes place. The stress σ_{local} rises as as the crack tip is approached

$$\sigma_{local} = Y \frac{\sigma\sqrt{\pi c}}{\sqrt{2\pi r}}$$

as the crack tip is approached (r is the radial distance from the crack tip, c the crack length and σ the remote stress). At the point where it reaches the yield strength σ_y the material yields and – except for some work hardening – the stress cannot climb higher than this. The size of the plastic zone is found by setting $\sigma_{local} = \sigma_y$ and solving for r giving

$$r_y = \left(\frac{\sigma^2 c}{2\pi\sigma_y^2}\right) = \frac{K_1^2}{\pi\sigma_y^2}$$

(taking $Y = 1$). In reality the truncated part of the elastic stress field is re-distributed, making the plastic zone larger by a factor of 2.

Exercise E8.4 Why is there a transition from ductile to brittle behavior at a transition crack length, c_{crit}?

Answer. When cracks are small, materials yield before they fracture; when they are large, the opposite is true. When the crack is small, this stress is equal to the yield stress; when large, it falls off according to equation (taking $Y = 1$ again)

$$\sigma_f = \frac{K_{1c}}{\sqrt{\pi c}}$$

The transition from yield to fracture occurs when $\sigma_f = \sigma_y$, giving the transition crack length

$$c_{crit} = \frac{K_{1c}^2}{\pi \sigma_y^2}$$

Exercise E8.5 Compare the critical crack length c_{crit} of a Titanium alloy with $\sigma_y = 550\text{MPa}$ and $K_{1c} = 40\ \text{MPam}^{1/2}$ with that of Silicon Carbide, with $\sigma_y = 490\text{MPa}$ and $K_{1c} = 4\ \text{MPam}^{1/2}$. Comment on the ductility of the two materials.

Answer.

The critical crack length is given by equation (8.13) $c_{crit} = \frac{K_{1c}^2}{\pi \sigma_y^2}$

For Titanium this is $c_{crit} = \frac{40^2}{\pi\ 550^2} = 0.0017\ \text{m} = 1.7\ \text{mm}.$

For Silicon Carbide, the same calculation gives $c_{crit} = 0.02$ mm.

A small scratch of 0.02 mm in the surface of a SiC specimen is sufficient to initiate fast fracture, whereas a Ti specimen will not fracture unless it has a deep crack or flaw of 1.7mm. The SiC will normally fracture in tension, whereas the Ti won't. The Ti is ductile, whereas the SiC is very brittle.

Exercise E8.6 A tensile sample of width 10 mm contains an internal crack of length 0.3 mm. When loaded in tension the crack suddenly propagates when the stress reaches 450 MPa. What is the fracture toughness K_{1c} of the material of the sample? If the material has a modulus E of 200 GPa, what is its toughness G_c? (Assume the geometric factor Y = 1).

Answer. 9.77 $\text{MPa.m}^{1/2}$

Exercise E8.7 The aircraft designer in Example 7.5 changes the design so that the window has a corner radius of 20mm instead of 5mm.

(a) What is the new stress concentration factor at the corner of the window and what is the maximum stress?

(b) Now suppose there is a small sharp defect near the corner of the window, such that the effective length of the crack in the fuselage is the size of the window itself. If the fracture toughness of the fuselage material is 25 $\text{MPam}^{1/2}$ and the window is of length 160mm, what stress would cause fast fracture? Is the design safe?

Answer.

(a) Following on from Example 7.5, the new stress concentration factor is

$$K_{SC} \approx 1 + \alpha \left(\frac{c}{\rho_{SC}} \right)^{1/2} = 1 + 2 \left(\frac{80}{20} \right)^{1/2} = 5,$$ and the new value of the maximum stress is

$\sigma_{max} = 5 \times 12.5 = 62.5$ MPa. This seems reasonable compared to the allowable stress of 100 MPa in Example 7.5.

(b) Assuming a small defect at the corner of the window and a crack of length 0.08 m, equation (8.4) gives the stress at fast fracture to be:

$$\sigma_c \approx \frac{K_{1c}}{\sqrt{\pi c}} = \frac{25}{\sqrt{\pi \times 0.08}} = 50\ \text{MPa}.$$

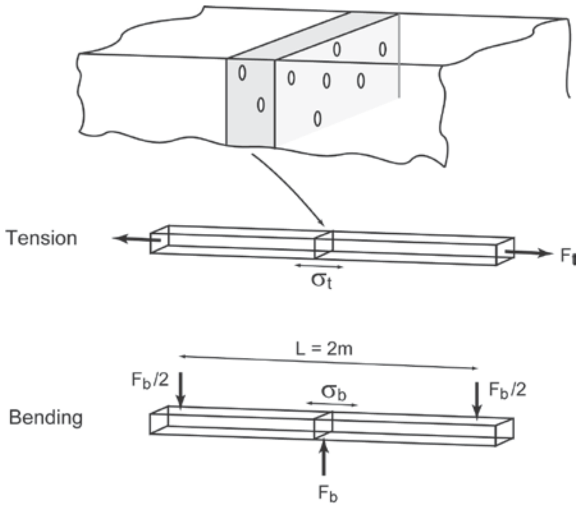
This is too close to the 62.5 MPa concentrated stress at the corner for comfort (a safety factor of 62.5/50 = 1.25). Taking account of the approximations in the analysis, the uncertainties in the material properties, and the fact that any fatigue loading would grow the crack, the design is not sufficiently safe. An even larger corner radius is needed.

Exercise E8.8 Two long wooden beams of square cross section $t \times t$ = 0.1m are butt-jointed using an epoxy adhesive. The adhesive was stirred before application, trapping air bubbles which, under pressure in forming the joint, deform to flat, penny-shaped cracks of diameter 2mm (see figure). For these cracks assume a geometry factor $Y = 1$ in the expression for stress intensity factor. The fracture toughness of the epoxy K_{1c} = 1.3 MPa.m$^{1/2}$.

(a) If the beams are loaded in axial tension, what is the load F_t that leads to fast fracture?

(b) The beams are now loaded in 3 point bending, with a central load F_b and a total span L = 2m. Show that the maximum tensile stress at the surface of the beam is given by $\sigma_b = 3F_b L / 2t^3$.

Assuming that there is a bubble close to the beam surface, find the maximum load that the beam can support without fast fracture.



Answer.

(a) The nominal tensile stress is: $\sigma = \dfrac{F_t}{t^2}$. Here we will assume that $K = \sigma\sqrt{\pi c}$, where c is the radius of the crack (though from Figure 10.2, the geometry factor for a contained penny-shaped crack is actually Y=0.7).

Hence $F_t = \dfrac{t^2 K_{1c}}{\sqrt{\pi c}}$

Taking K_{1c} = 1.3 MPa m$^{1/2}$ for epoxy, and c = 1mm: $F_t = \dfrac{0.1^2 \times 1.3 \times 10^6}{\sqrt{\pi\, 0.001}} = 232\,\text{kN}$

(b) Maximum bending moment $M = F_b L/4$ at the mid-span of the beam. Using $\dfrac{\sigma}{y} = \dfrac{M}{I}$ for the extreme fibre stress with $y = t/2$ and $I = t^4/12$ gives the maximum tensile stress at the surface of a beam as:

$$\sigma = \frac{3F_b L}{2t^3}$$

For the configuration shown, with cracks much smaller than the beam section, we can treat the crack as if it is a contained crack exposed (locally) to a uniform stress equal to the bending stress, i.e.

$$\frac{3F_b L}{2t^3}\sqrt{\pi c} = K_{1c}$$

Hence, the maximum load which can be sustained by the adhesive joint is

$$F_b = \frac{2t^3 K_{1c}}{3L\sqrt{\pi c}}$$

Taking K_{1c} as 1.3 MPa $m^{1/2}$ for epoxy, and c = 1mm: $F_b = \dfrac{2\times(0.1)^3\times 1.3\times 10^6}{3\times 2\sqrt{\pi 0.001}}$ = 7.7 kN

Exercise E8.9 Use the $K_{1c} - E$ chart of Figure 8.8 to establish whether:

- CFRP has a higher fracture toughness K_{1c} than aluminum alloys?
- Polypropylene (PP) has a higher toughness G_c than aluminum alloys
- Polycarbonate (PC) is has a higher fracture toughness K_{1c} than glass.

Answer.

- CFRP has lower fracture toughness K_{1c} than aluminum alloys.
- Polypropylene (PP) has a higher toughness G_c than aluminum alloys.
- Polycarbonate (PC) is has a much higher fracture toughness K_{1c} than glass

Exercise E8.10 Find epoxy, soda glass and GFRP (epoxy reinforced with glass fibers) on the chart of Figure 8.8 and read off an approximate mean value for the toughness G_c for each. Explain how it is that the toughness of the GFRP is so much larger than that of either of its components.

Answer. The approximate toughnesses G_c for the 3 materials are:

Epoxy	0.1 – 1	kJ/m^2
Soda glass	0.05 – 0.07	kJ/m^2
GFRP	3 – 20	kJ/m^2

CFRP is tougher than either of its components because of the energy dissipated in the process zone as broken fibers of glass are pulled out of the epoxy matrix.

Exercise E8.11 Use the chart of Figure 8.8 to compare the fracture toughness, K_{1c}, of the two composites GFRP and CFRP. Do the same for their toughness, G_c. What do the values suggest about applications they might best fill?

Answer. GFRP and CFRP have almost the same value-ranges of fracture toughness, K_{1c}, but differ in their values of toughness, G_c. GFRP, with the higher toughness, is the better choice for energy absorbing applications. CFRP, with higher stiffness and the same fracture toughness as GFRP is the better choice for load-limited applications.

Exercise E8.12 Use the $K_{1c} - \sigma_y$ chart of Figure 8.9 to find

- the range of transition crack sizes for stainless steel.
- the range of transition crack sizes for polycarbonate (PC).
- the range of transition crack sizes for silicon nitride (Si_3N_4).

Answer. The approximate transition crack sizes c_{crit} for the 3 materials are:

Stainless steel	10– 1000	mm
Polycarbonate	5 - 12	mm
Silicon nitride	0.02 – 0.03	mm

Exercise E8.13 The cylindrical ceramic samples from Example 8.4 (50mm long and 5mm radius) were found to have a survival probability of 50% when the applied stress was 120MPa. The Weibull modulus of the ceramic is *m* = 10. Find the reference stress σ_o for this volume of material. What stress can be carried with a probability of failure of lower than 1%?

Answer. Let the reference volume V_o be the same as the sample volume = 3927mm^3 (from Example 8.4). For survival probability of 50%, equation (8.19) gives

$$0.5 = \exp\left\{-\left(\frac{120}{\sigma_o}\right)^{10}\right\}$$ so the reference stress σ_o = 124.5 MPa.

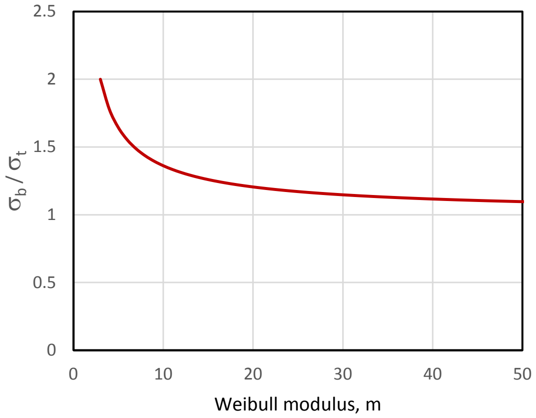
Failure probability of 1% = survival probability of 0.99, hence $0.99 = \exp\left\{-\left(\frac{\sigma}{124.5}\right)^{10}\right\}$ so the maximum allowable stress is σ = 78.6 MPa.

Exercise E8.14 The strengths of a square ceramic beam loaded in pure bending and in tension were compared in the text, equation (8.18). Plot how the strength ratio varies with the value of the Weibull modulus, *m*, and explain the shape of the curve.

Answer.

$$\frac{\sigma_b}{\sigma_t} = \left[2(m+1)\right]^{1/m}$$

The curve shows that the bending strength is always greater than the tensile strength. The Weibull modulus *m* measures the spread of the survival probability – the lower the value, the more variable the strength. So as *m* increases, the failure strength approaches a single value, and the bending strength and tensile strength converge.



Exercise E8.15 Square section silicon nitride beams were loaded in pure bending. 50% of the beams had broken when the maximum bending stress was below 500 MPa. Use equation (8.18) to find the tensile stress applied to the same samples that gives the same probability of failure. The Weibull modulus for silicon nitride is *m* = 10.

Answer.

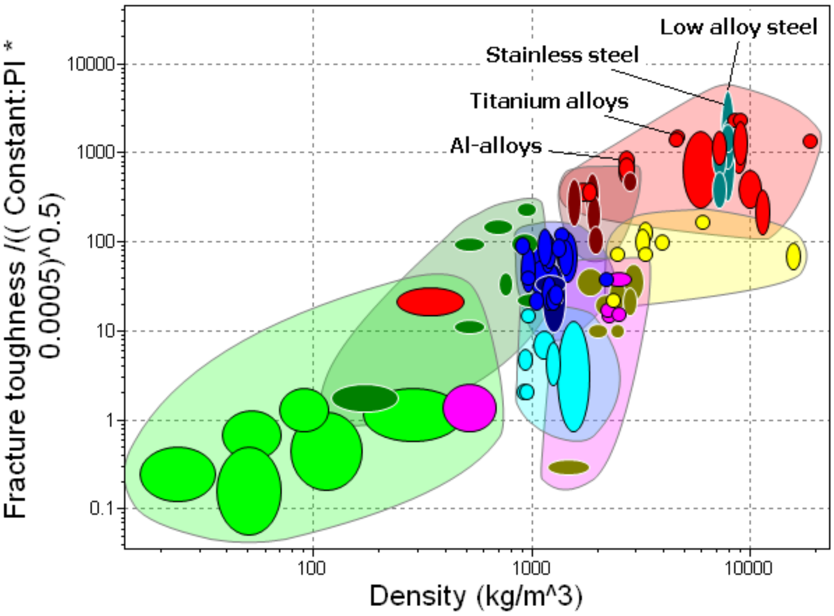
$$\frac{\sigma_b}{\sigma_t} = \left[2(m+1)\right]^{1/m}$$ For σ_b = 500MPa and *m* = 10: σ_t = 367 MPa.

Exploring design with CES. (Use Level 2 unless otherwise stated)

Exercise E8.16 Use the Advanced facility to make a bar chart showing $\sigma_f = K_{1c}/\sqrt{\pi c}$ for an internal crack of length $2c$ = 1 mm plotted on the y-axis (i.e. using the CES parameter 'Fracture toughness' for K_{1c}).

Which materials have the highest values? Add an axis of density, ρ. Use the new chart to find the two materials with highest values of σ_f / ρ.

Answer. Low alloy steels and stainless steels.

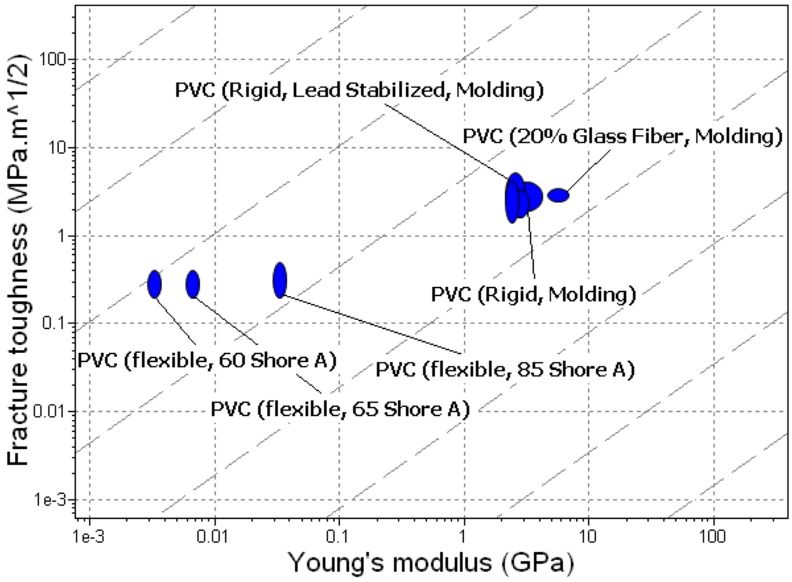


Exercise E8.17 Suppose that the resolution limit of the NDT facility available to you is 1mm, meaning that it can detect cracks of this length or larger. You are asked to explore which materials will tolerate cracks equal to or smaller than this without brittle fracture. Make a bar chart with $\sigma_f = K_{1c}/\sqrt{\pi c}$ for an internal crack of length $2c = 1$ mm plotted on the y-axis, as in the previous exercise. Add yield strength σ_y on the x-axis. The material will fracture in tension if $\sigma_f < \sigma_y$, and it will yield, despite being cracked, if $\sigma_f > \sigma_y$. Plot a line of slope 1 along which $\sigma_f = \sigma_y$, either on a printout of the chart or in CES (using the line selection tool). All the materials above the line will yield, all those below will fracture. Do age-hardening aluminum alloys lie above the line? Does CFRP?

Answer. Age-hardening aluminum alloys lie above the line, CFRP does not.

Exercise E8.18 Find data for PVCs in the Level 3 database and make a plot like that of Figure 8.19 showing how fillers, blending and fibers influence modulus and toughness.

Answer.



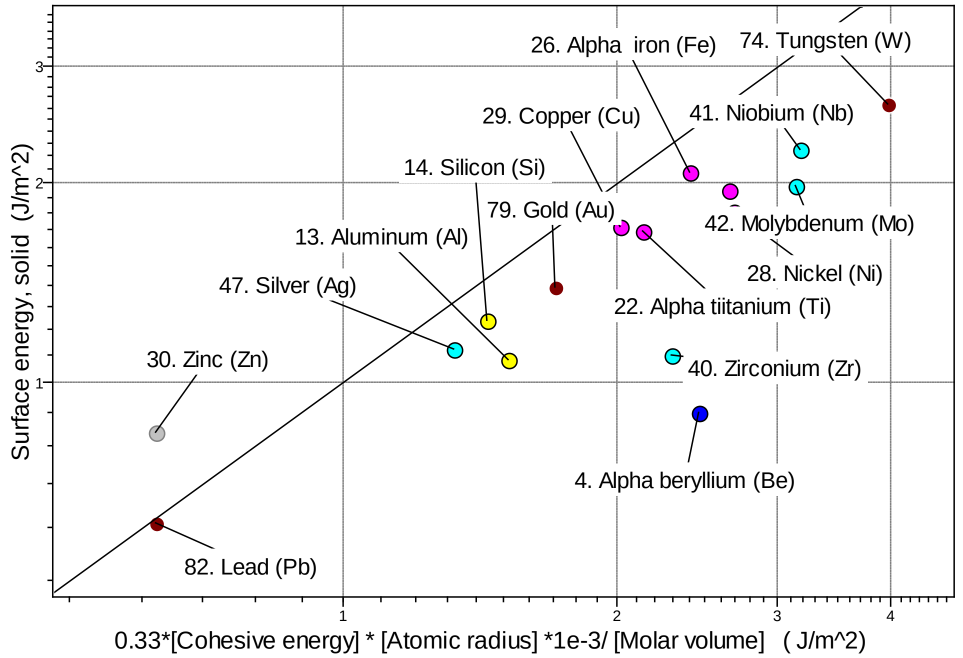
Exploring the science with CES Elements.

Exercise E8.19 Explore the origins of surface energy, γ J/m^2. The toughness, G_c cannot be less than 2γ because two new surfaces are created when a material is fractured. What determines γ and how big is it? The text explained how bonds are broken and atoms separated when a new surface is created. Its was shown in Section 8.4 that

$$\gamma \approx \frac{1}{3} H_c \; r_o$$

where H_c is the cohesive energy in J/m^3 and r_o is the atomic radius in m. To convert H_c in kJ/mol (as it is in the database) into these units, multiply it by 10^6/ molar volume and to convert r_o from nm into m multiply by 10^{-9}. All this can be done using the 'Advanced' facility in the axis-choice dialog box. Make a chart with γ calculated in this way on the x-axis and the measured value, 'Surface energy, solid' on the y-axis, and see how good the agreement is.

Answer.

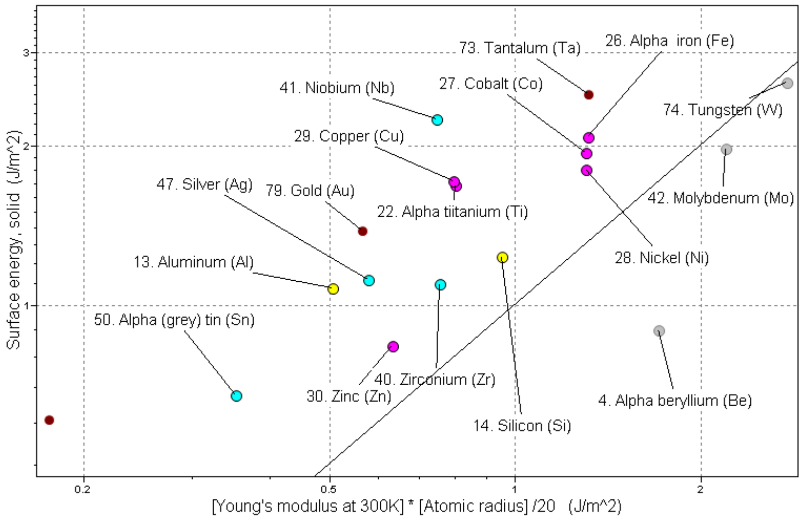


Exercise E8.20 If you pull on an atomic bond, it breaks completely at a strain of about 0.1. If the atom spacing is a_o, then breaking it requires a displacement $\delta = 0.1a_o$ and doing so creates two new surfaces, each of area a_o^2. If the bond stiffness is S, then the work done to create the new surfaces, per unit area, is

$$2\gamma \approx \frac{1}{a_o^2}\left(\frac{1}{2} S\delta\right) = \frac{E\, a_o}{20} = \frac{1}{10} E\, r_o$$

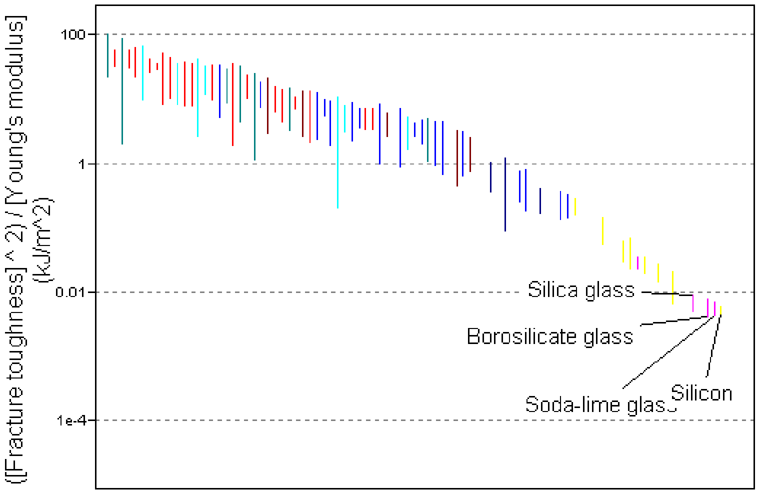
(using equation (4.17) for S), where r_o is the atomic radius. Use the CES Elements database to explore whether real surface energies can be explained in this way. (Watch out for the units.)

Answer.



Exercise E8.21 Observe the general magnitudes of surface energies γ : they are about 1.5 J/m^2. Thus the minimum value for G_c should be about 2 γ or 3 J/m^2. Return to the CES Level 3 database and find the material with the lowest value of G_c, which you can calculated as K_{1c}^2 / E. Is it comparable with 2 γ? Limit the selection to metals and alloys, polymers, technical ceramics and glasses only, using a Tree stage (materials such as foam have artificially low values of G_c because they are mostly air).

Answer.



Materials: engineering, science, properties, and design
3e Solution manual

Chapter 9: Exercises with worked solutions

Exercise E9.1 What is meant by the mechanical loss coefficient, η, of a material? Give examples of designs in which it would play a role as a design-limiting property.

Answer. The *mechanical loss-coefficient* or *damping coefficient*, η (a dimensionless quantity), measures the degree to which a material dissipates vibrational energy. If an elastic material is loaded, energy is stored. When it is unloaded some energy is returned, but not all. The difference is called the *loss coefficient*, η. It is the fraction of the stored elastic energy that is not recovered on unloading; instead it appears as heat. Applications in which resonance or fast elastic response is required (bells, high-speed relays and springs) require materials with low η. Applications in which it is desirable to damp vibration (sound isolation of buildings, suppression of vibration in machine tools) use material with high η.

Exercise E9.2 Distinguish between ‘low cycle’ and ‘high cycle’ fatigue. Find examples of engineering components which may fail by high cycle fatigue. What is meant by the endurance limit, σ_e, of a material?

Answer.
Fatigue failure in metals means crack growth under cyclic loading leading to a critical crack length which causes fast fracture. Crack growth may occur from a pre-existing flaw, or a crack may first initiate under cyclic loading.

Low cycle fatigue means failure under cyclic loading in a relatively small number of cycles (<10^4 approx.). Failure is caused by reversed plastic strain in the whole component on each cycle, leading to internal damage and failure. The number of cycles to failure is controlled by the plastic strain range in each cycle. Try bending a paperclip backwards and forwards until it breaks.

High cycle fatigue means failure under cyclic loading in a large number of cycles (>10^4 approx.). The general stress level is below yield, but failure is caused by localised reversed slip in favorably orientated grains leading to crack initiation, and propagation to failure. The number of cycles to failure is dominated by initiation, and is controlled by the applied (elastic) stress range.

Engineering components that fail by high cycle fatigue: Bicycle components: spokes, wheel axles; rotating machine components: shafts, bearings.

For many materials there exists a fatigue or endurance limit, σ_e (units: MPa). It is the stress amplitude σ_a, about zero mean stress, below which fracture does not occur, or occurs only after a very large number ($N_f > 10^7$) cycles. Design against high cycle fatigue is therefore very similar to strength-limited design, but with the maximum stresses limited by the endurance limit σ_e rather than the yield stress σ_y.

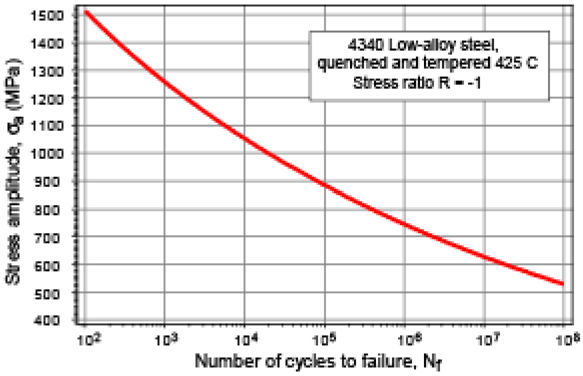
Exercise E9.3 What is the fatigue ratio? If the tensile strength σ_{ts} of an alloy is 900 MPa, what, roughly, would you expect its endurance limit σ_e to be?

Answer. The fatigue ratio R_f is the ratio of the endurance limit to the tensile stress: $R_f = \dfrac{\sigma_e}{\sigma_{ts}}$

For metals and polymers $R_f \approx 0.33$. Thus the expected value of the endurance limit for the alloy is about 300 MPa.

Exercise E9.4 The figure shows an $S - N$ curve for AISI 4340 steel, hardened to a tensile strength of 1800 MPa.

(a) What is the endurance limit?
(b) If cycled for 100 cycles at an amplitude of 1200 MPa and a zero mean stress, will it fail?
(c) If cycled for 100,000 cycles at an amplitude of 900 MPa and zero mean stress, will it fail?
(d) If cycled for 100,000 cycles at an amplitude of 800 MPa and a mean stress of 300 MPa, will it fail?



Answer.

(a) The endurance limit is about 630 MPa
(b) 100 cycles at 1200 MPa with zero mean stress: below curve – will not fail.
(c) 100,000 cycles at 900 MPa with zero mean stress: just above curve – will fail.
(d) 100,000 cycles at 800 MPa with mean stress of 300 MPa: first use Goodman's rule to find the equivalent stress amplitude at zero mean stress:

From equation (9.7): $$\Delta\sigma_{\sigma_o} = \frac{\Delta\sigma_{\sigma_m}}{\left(1 - \dfrac{\sigma_m}{\sigma_{ts}}\right)} = \frac{800}{\left(1 - \dfrac{300}{1800}\right)} = 960 \text{ MPa}$$

100,000 cycles at 960 MPa with zero mean stress: above curve – will fail.

Exercise E9.5 The high-cycle fatigue life, N_f, of an aluminum alloy is described by Basquin's law:

$$\frac{\Delta\sigma}{2} = \sigma_a = 480\left(N_f\right)^{-0.12}$$

(stress in MPa). How many cycles will the material tolerate at a stress amplitude σ_a of ± 70 MPa and zero mean stress? How will this change if the mean stress is 10 MPa? What if the mean stress is – 10 MPa? The tensile strength of the alloy is 200 MPa.

Answer.

$$N_f = \left(\frac{70}{480}\right)^{\frac{1}{-0.12}} = 9.3\times10^6 \text{ cycles to failure}$$

The effect of the mean stress can be calculated using Goodman's rule (equation (9.7)). With the addition of mean stress σ_m the equivalent stress amplitude for zero mean stress is:

$$\Delta\sigma_{\sigma_o} = \frac{\Delta\sigma_{\sigma_m}}{\left(1 - \dfrac{\sigma_m}{\sigma_{ts}}\right)} = \frac{70}{\left(1 - \dfrac{10}{200}\right)} = 73.7 \text{ MPa}.$$

The resulting fatigue life is then: $N_f = \left(\dfrac{73.7}{480}\right)^{\frac{1}{-0.12}} = 6.1\times10^6$ cycles to failure… a significant reduction. The equivalent calculation with a mean stress of $\sigma_m = -10\text{MPa}$ gives:

$$\Delta\sigma_{\sigma_o} = \frac{70}{\left(1+\dfrac{10}{200}\right)} = 66.7 \text{ MPa}, \quad \text{with } N_f = \left(\frac{66.7}{480}\right)^{\frac{1}{-0.12}} = 13.9\times 10^6 \text{ cycles.}$$

Addition of the compressive mean stress is highly beneficial to fatigue performance.

Exercise E9.6 The low-cycle fatigue of an aluminum alloy is described by Coffin's law:

$$\Delta\varepsilon^{pl} = \frac{0.2}{N_f^{0.5}}$$

How many cycles will the material tolerate at a plastic strain amplitude $\Delta\varepsilon^{pl}$ of 2%?

Answer.

$$N_f = \left(\frac{0.2}{\Delta\varepsilon^{pl}}\right)^{\frac{1}{0.5}} = \left(\frac{0.2}{0.02}\right)^2 = 100 \text{ cycles}$$

Exercise E9.7 A material with a tensile stress $\sigma_{ts} = 350$ MPa is loaded cyclically about a mean stress of 70 MPa. If the stress range that will cause fatigue fracture in 10^5 cycles under zero mean stress is ± 60 MPa, what stress range about the mean of 70 MPa will give the same life?

Answer.
For a fatigue life of 10^5 cycles, the stress amplitude for zero mean stress is $\Delta\sigma_{\sigma_o} = 60$ MPa. Using Goodman's rule, equation (9.7), stress range for failure with the mean stress σ_m is:

$$\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_o}\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right) = 60\left(1-\frac{70}{350}\right) = 48 \text{ MPa.} \quad \text{So the stress } \textit{range} \text{ is } \underline{+}\, 48\text{MPa.}$$

Exercise E9.8 A component made of the AISI 4340 steel with a tensile strength of 1800 MPa and the S-N curve shown in Exercise E9.4 is loaded cyclically between 0 and 1200 MPa. What is the *R*-value and the mean stress, σ_m? Use Goodman's rule to find the equivalent stress amplitude for an *R*-value of –1, and read off the fatigue life from the S-N curve.

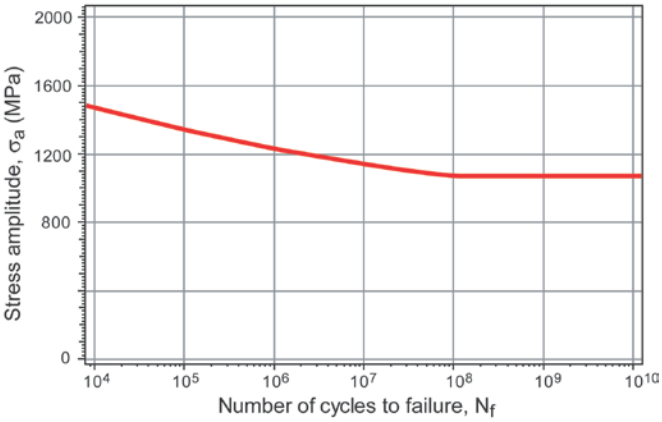
Answer.
The mean stress is 1200/2 = 600 MPa. The *R*-value, equation (9.3), is: $R = \sigma_{min}/\ \sigma_{max} = 0 / 1200 = 0$.
The equivalent stress amplitude for zero mean stress is calculated using Goodman's rule:

$$\Delta\sigma_{\sigma_o} = \frac{\Delta\sigma_{\sigma_m}}{\left(1 - \dfrac{\sigma_m}{\sigma_{ts}}\right)} = \frac{600}{\left(1-\dfrac{600}{1800}\right)} = 900\text{MPa.}$$

Reading from the S-N diagram in Exercise 9.4, the corresponding fatigue life is ≈10^5 cycles.

Exercise E9.9 Some uncracked bicycle forks are subject to fatigue loading. Approximate S-N data for the material used are given in the figure, for zero mean stress. This curve shows a 'fatigue limit': a stress amplitude below which the life is infinite.

(a) The loading cycle due to road roughness is assumed to have a constant stress range $\Delta\sigma$ of 1200 MPa and a mean stress of zero. How many loading cycles will the forks withstand before failing?
(b) Due to a constant rider load the mean stress is 100MPa. Use Goodman's rule to estimate the percentage reduction in lifetime associated with this mean stress. The tensile strength σ_{ts} of the steel is 1100 MPa.
(c) What practical changes could be made to the forks to bring the stress range below the fatigue limit and so avoid fatigue failure?



Answer.

(a) Reading directly from the graph, 2×10^6 cycles.

(b) Use Goodman's rule to convert from the stress range $\Delta\sigma_{\sigma m}$ with mean stress σ_m to an equivalent stress range $\Delta\sigma_{\sigma o}$ for zero mean stress:

$$\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_o}\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right) \Rightarrow \Delta\sigma_{\sigma_o} = \Delta\sigma_{\sigma_m}\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right)^{-1} = 1200\left(1-\frac{100}{1100}\right)^{-1} \approx 1300 \text{ MPa.}$$

From the S-N curve, the lifetime is about 2×10^5 cycles, with this stress range of 1300 MPa. The percentage reduction in lifetime is about 90%, i.e. a reduction of lifetime by a factor of 10 – an enormous effect.

(c) We can either change the design to reduce the stresses – mountain bikes have larger tubes with a larger second moment of area to reduce the stresses – or change the material to one with a higher fatigue endurance limit.

Exercise E9.10 An aluminium alloy for an airframe component was tested in the laboratory under an applied stress which varied sinusoidally with time about a mean stress of zero. The alloy failed under a stress range $\Delta\sigma$ of 280 MPa after 10^5 cycles. Under a stress range of 200 MPa, the alloy failed after 10^7 cycles. Assume that the fatigue behaviour of the alloy can be represented by:

$$\Delta\sigma \, N_f^b = C$$

where b and C are material constants.

(a) Find the number of cycles to failure for a component subject to a stress range of 150 MPa.

(b) An aircraft using the airframe components has encountered an estimated 4×10^8 cycles at a stress range of 150 MPa. It is desired to extend the life of the airframe by another 4×10^8 cycles by reducing the performance of the aircraft. Use Miner's rule to find the decrease in the stress range needed to achieve this additional life.

Answer.

(a) From $\Delta\sigma \, N_f^b = C$: $\quad 280\left(10^5\right)^b = 200\left(10^7\right)^b = C$

$$\left(\frac{10^7}{10^5}\right)^b = \frac{280}{200} = 1.4, \text{ hence } b = 0.073, \text{ and } C = 280\left(10^5\right)^{0.073} = 649 \text{ MPa.}$$

At 150 MPa, $N_f = \left(\frac{649}{150}\right)^{1/0.073} = 5.18 \times 10^8$ cycles.

(b) The fraction of life used up in the first 4×10^8 cycles is, $\dfrac{N_1}{N_{f1}} = \dfrac{4\times10^8}{5.18\times10^8} = 0.772$

Miner's rule gives: $\dfrac{N_1}{N_{f1}} + \dfrac{N_2}{N_{f2}} = 1$.

Hence the amount of life that is available for the twilight years of the aircraft is:

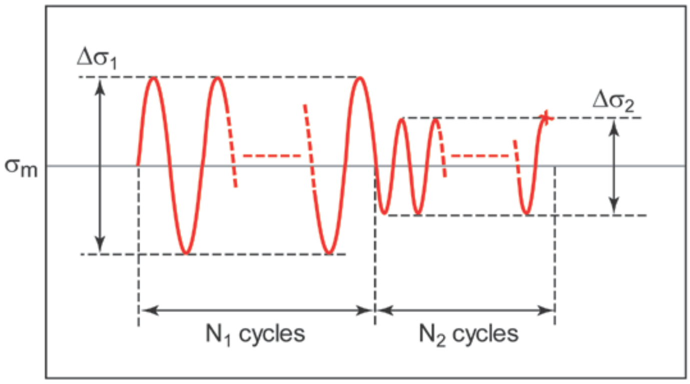
$$\frac{N_2}{N_{f2}} = 1 - 0.772 = 0.228$$

Hence: $N_{f2} = \dfrac{4\times10^8}{0.228} = 1.754\times10^9$ cycles.

For this number of cycles to failure, $\Delta\sigma = \dfrac{649}{(1.754\times10^9)^{0.073}} = 137$ MPa

The decrease in stress range is: 150 – 137 = 13.3 ≈ 13 MPa

Exercise E9.11 A medium carbon steel was tested to obtain high cycle fatigue data for the number of cycles to failure N_f in terms of the applied stress range $\Delta\sigma$ (peak-to-peak). As the test equipment was only available for a limited time, the tests had to be accelerated. This was achieved by testing the specimen using the loading history shown schematically in the figure. A first set of N_1 cycles was applied with a stress range $\Delta\sigma_1$, followed by a second set of N_2 cycles with a stress range $\Delta\sigma_2$. The table shows the test programme. In every test the mean stress σ_m was held constant at 150 MPa and each test was continued until specimen failure. A separate tensile test gave a tensile strength for the steel of 600 MPa.



Test	$\Delta\sigma_1$ (MPa)	N_1	$\Delta\sigma_2$ (MPa)	N_2
1	630	10^4	-	-
2	630	5×10^3	460	5×10^5
3	630	5×10^3	510	1.2×10^5
4	630	2.5×10^3	560	4.4×10^4

(a) Using Goodman's rule, show that the stress *range* that would give failure in 10^4 cycles with *zero* mean stress is 840MPa.

(b) Use Miner's rule to find the expected number of cycles to failure for each of the stress ranges $\Delta\sigma_2$ in the table, with the mean stress of 150 MPa.

(c) Convert the $\Delta\sigma_2 - N_f$ data obtained to the equivalent data for zero mean stress.

(d) Plot a suitable graph to show that all the fatigue life data for zero mean stress are consistent with Basquin's law for high cycle fatigue

$\Delta\sigma\, N_f^b = C_1$, and find the constant b.

Answer.

(a) Goodman's rule, applied to test 1: $\Delta\sigma_{\sigma_o} = \Delta\sigma_{\sigma_m}\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right)^{-1} = 630\left(1-\frac{150}{600}\right)^{-1} \approx 840$ MPa.

(b) Miner's rule: $\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} = 1$

For first part of each test: $\Delta\sigma_1 = 630$ MPa and $N_{f1} = 10^4$.

Hence calculate N_1 / N_{f1} in each test, and thus N_2 / N_{f2} and N_{f2} (from N_2 values in the table):

Test	N_1 / N_{f1}	N_2 / N_{f2}	N_2	N_{f2}
1	1.0	0	--	--
2	0.5	0.5	5×10^5	10^6
3	0.5	0.5	1.2×10^5	2.4×10^5
4	0.25	0.75	4.4×10^4	5.9×10^4

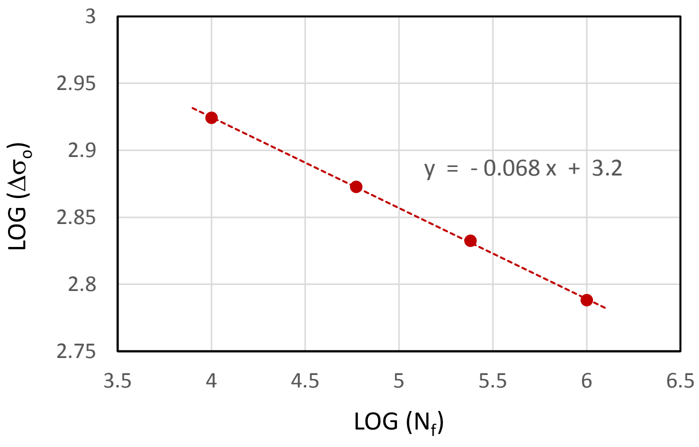
(Note that Miner's rule is applied directly, without correcting to zero mean stress. This is because the Goodman correction is applied to the stress, for fixed N_{f1} and N_{f2}, so the life fractions are unchanged).

(c) Convert $\Delta\sigma_{\sigma_m}$ to equivalent $\Delta\sigma_{\sigma_o}$ at zero mean stress, by dividing by $(1 - \sigma_m/\sigma_{ts})$ as in part (i):

Test	$\Delta\sigma_{\sigma_m}$	$\Delta\sigma_{\sigma_o}$	N_{f2}
1	630	840	10^4
2	460	614	10^6
3	510	680	2.4×10^5
4	560	746	5.9×10^4

(d) Basquin's Law: $\Delta\sigma N_f^b = C_1$, so $\log(\Delta\sigma) = -b \log(N_f) + \log C_1$

Hence plot $\log(\Delta\sigma)$ against $\log(N_f)$, gradient = – b

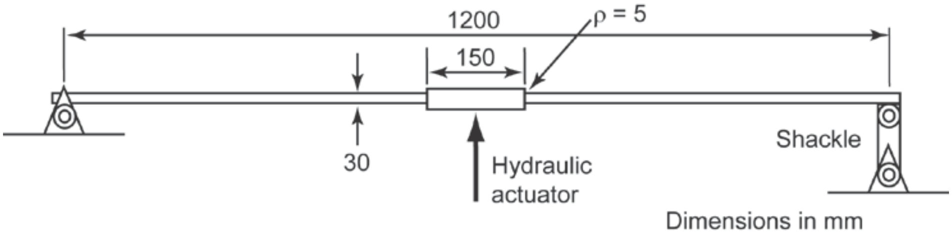


From graph:

gradient = – 0.068,

so $b = 0.068$

Exercise E9.12 The leaf spring of a heavy vehicle suspension is 1.2m long, 75mm wide and 30mm thick. A central thickened section is 150mm long and 34mm thick with a fillet of radius 5mm as shown in the figure. It is supported by a pin joint at one end and a 'shackle' at the other. This can be assumed to be equivalent to 'simple' supports. The spring is deflected sideways in a testing machine using a hydraulic actuator attached to the central section. The actuator causes the spring to oscillate cyclically with an amplitude of ± 30mm. The material is AISI 4340 steel, for which the Young's Modulus is 210GPa and the S-N curve is provided in Example E9.4. You will need to use elastic solutions for the deflection and stresses in beams under 3 point bending, from Chapter 5, and for the stress concentration factor, from Chapter 7.



(a) Find the load amplitude applied by the actuator to give the specified deflection (ignore the stiffening effect of the central thickened section).

(b) Find the bending moment and maximum surface stress on the thinner part of the spring, at the location of the fillet (initially ignoring the change in thickness). What stress concentration factor should then be applied to the stress amplitude for this location?

(c) If the spring is initially cycled about its unstressed (equilibrium) position, estimate the number of cycles of loading that it will last before it fails by fatigue at the fillet.

(d) If the test is performed with a mean load of 30kN, find the mean stress (including the stress concentration), and hence use Goodman's rule to estimate the number of cycles of loading that the spring will last before failure.

Answer.

(a) The second moment of area about the transverse (bending) axis is:

$$I_{xx} = \frac{bd^3}{12} = \frac{75 \times 30^3}{12} = 1.7 \times 10^5 \text{ mm}^4 = 1.7 \times 10^{-7} \text{ m}^4.$$

The deflection of a simply supported beam is given by equation (5.5) with C_1 = 48 (Figure 5.3). To generate 50 mm of deflection requires a force of:

$$F = \frac{48\, EI\, \delta}{L^3} = \frac{48\left(210 \times 10^9\right)\left(1.7 \times 10^{-7}\right)\left(30 \times 10^{-3}\right)}{1.2^3} = 29.8 \text{ kN} \approx 30 \text{ kN}$$

(b) The maximum bending moment at the centre of a simply supported beam is $FL/4$. Ignoring the short thickened section, the nominal stress at the fillet is given by equation (7.2):

$$\sigma = \frac{M\, y_m}{I_{xx}} = \frac{\left(\dfrac{\left(29.8 \times 10^3\right)(1.2)}{4}\right) 0.015}{1.7 \times 10^{-7}} = 790 \text{ MPa}$$

The stress concentration factor can be found using equation (7.12) with c = 2mm, ρ = 5mm and a = 1/2 for bending (see text). This gives:

$$K_{SC} = \frac{\sigma_m}{\sigma_n} = 1 + \alpha\left(\frac{c}{\rho}\right)^{1/2} = 1 + \frac{1}{2}\left(\frac{2}{5}\right)^{1/2} = 1.32. \text{ So } \sigma_{max} = 1.32 \times 790 = 1040 \text{ MPa.}$$

(c) Using the S-N curve with a stress amplitude of 1040 MPa gives N_f = 10,000 cycles.

(d) With a preload of 30 kN and the same force amplitude of 30 kN, the stress cycle now ranges from 0 MPa to 2×790 = 1580 MPa, with a mean stress of 790 MPa.

Using Goodman's rule, the equivalent stress amplitude for zero mean stress is:

$$\Delta\sigma_{\sigma_o} = \frac{\Delta\sigma_{\sigma_m}}{\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right)} = \frac{790}{\left(1 - \frac{790}{1800}\right)} = 1410\,\text{MPa.}$$

From the S-N curve, this gives a life of $N_f = 300$ cycles.

This is clearly far too low for a leaf spring that has to last for the vehicle life of 10 years or so: corresponding to millions of loading cycles. In practice, such leaf springs are 'stress peened' (see text). They are loaded statically to near yield and heavily shot peened. This puts a large compressive stress into a thin layer of material at the surface.

Suppose the compressive stress was 1000 MPa. Then Goodman would give:

$$\Delta\sigma_{\sigma_o} = \frac{\Delta\sigma_{\sigma_m}}{\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right)} = \frac{790}{\left(1 + \frac{210}{1800}\right)} = 710\text{MPa,}$$ for which the S-N curve gives a life of 2×10^6 cycles……much more reasonable.

Exercise E9.13 A material has a threshold cyclic stress intensity ΔK_{th} of 2.5 MPa.m$^{1/2}$. If it contains an internal crack of length 1 mm will it be safe (meaning, no failure) if subjected to continuous cyclic range of tensile stress $\Delta\sigma$ of 50 MPa?

Answer. Yes. A stress range of $\Delta\sigma = 62$ MPa is needed to exceed the threshold.

Exercise E9.14 A plate of width $a = 50$mm contains a sharp, transverse edge-crack of length c = 5mm. The plate is subjected to a cyclic axial stress σ that varies from 0 to 50 MPa. The Paris law constants (equation (9.10)) are $m = 4$ and $A = 5\times10^{-9}$, where σ is in MPa. The fracture toughness of the material is $K_{1c} = 10$ MPa.m$^{1/2}$.

(a) At what 'critical' crack length will fast fracture occur? (Assume the geometry factor in the expression for K is Y = 1.1)

(b) How many load cycles will it take for the crack to grow to the critical crack length?

Answer.

(a) Using equation (8.4), the critical crack length will be:

$$c^* = \frac{K_{IC}^2}{\pi Y^2 \sigma^2} = \frac{\left(10\times10^6\right)^2}{\pi(1.1)^2\left(50\times10^6\right)^2} = 10.5\text{ mm.}$$

(b) Following Example 9.4(b) in the text, the number of cycles to failure will be:

$$N_f = \frac{1}{AY^m\sigma_{\max}^m\pi^{m/2}(1-m/2)}\left[\left(c^*\right)^{1-m/2} - \left(c_i\right)^{1-m/2}\right]$$

$$= \frac{1}{5\times10^{-9}\times1.1^4\times50^4\pi^2(-1)}\left[\frac{1}{0.0105} - \frac{1}{0.005}\right] = 232\text{ cycles.}$$

Exercise E9.15 Components that are susceptible to fatigue are sometimes surface treated by 'shot peening'. Explain how the process works, and why it is beneficial to fatigue life.

Answer. Shot peening introduces a residual compressive stress in the surface of a component. Since fatigue cracks often originate at the surface and cracks only propagate during the tensile part of the stress cycle, the residual compressive stress forces the crack faces together, clamping it shut. The residual compressive stress is superimposed on the stress due to loading. So if the magnitude of the residual compressive stress is known, Goodman's rule can be used to estimate the increase in fatigue life. An example is provided at the end of the solution to Exercise 9.12, above.

Exploring design with CES.

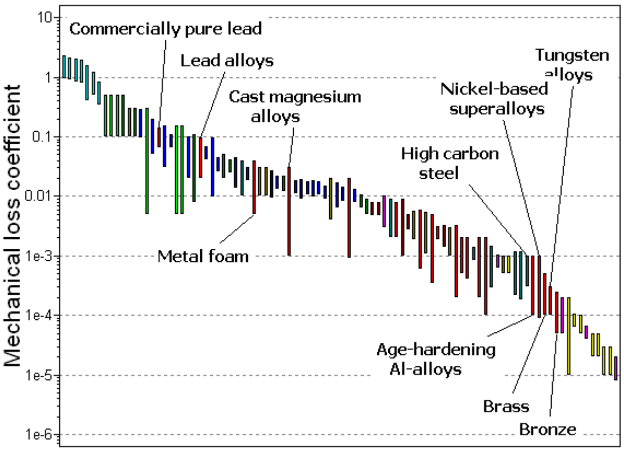
Exercise E9.16 Make a bar chart of mechanical loss coefficient, η. Low loss materials are used for vibrating systems where damping is to be minimized – bells, high-frequency relays and resonant systems. High loss materials are used when damping is desired – sound deadening cladding for buildings, cars and machinery, for instance. Use the chart to find:

(a) the metal with the lowest loss coefficient.

(b) the metal with the highest loss coefficient.

Do their applications include one or more of those listed above?

Answer. The metal with the lowest loss coefficient is bronze. Its applications include bells. The metal with the highest loss coefficient is commercially pure lead. Its applications include roof and wall cladding.



Exercise E9.17 Use the 'Search' facility to search materials that are used for:

(a) bells
(b) cladding

Answer.

Bells	Cladding
Borosilicate glass Soda-lime glass Medium carbon steel Bronze	Glass ceramic Stone GFRP, epoxy matrix (isotropic) Lead alloys Polyvinylchloride (tpPVC) Granite Bronze Age-hardening wrought Al-alloys Commercially pure zinc Commercially pure lead Lead alloys

Exercise E9.18 Use a 'Limit' stage, applied to the *Surface treatment* datatable, to find surface treatment processes that enhance fatigue resistance. To do this:

(a) Change the selection table to Process universe Level 2 Surface treatment, open a 'Limit' stage, locate Function of treatment and click on Fatigue resistance > Apply. Copy and report the results.

(b) Repeat, using the Level 3 Surface treatment datatable.

Answer.

Results at Level 2:

- Carburizing and carbonitriding
- Grinding and mechanical polishing
- Induction and flame hardening
- Laser surface hardening and melting
- Nitriding

Results at Level 3:

- Boriding
- Carburizing and Carbonitriding
- Chromizing and Plasma Chromizing
- Grinding
- Induction and flame hardening
- Ion implantation
- Laser surface hardening and melting
- Nitriding
- Shot peening and laser peening

Exercise E9.19 Explore the relationship between fatigue ratio and strength for a heat-treatable low alloy steel AISI 4340. The endurance limit σ_e is stored in the database under the heading 'Fatigue strength at 10^7 cycles'.

(a) Plot the fatigue ratio σ_e / σ_y against the yield strength σ_y.

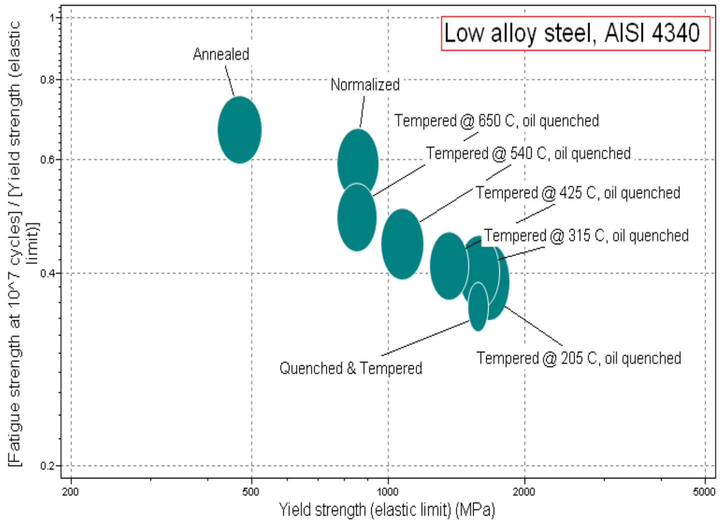
(b) Plot the fatigue ratio σ_e / σ_{ts} against the tensile strength σ_{ts}.

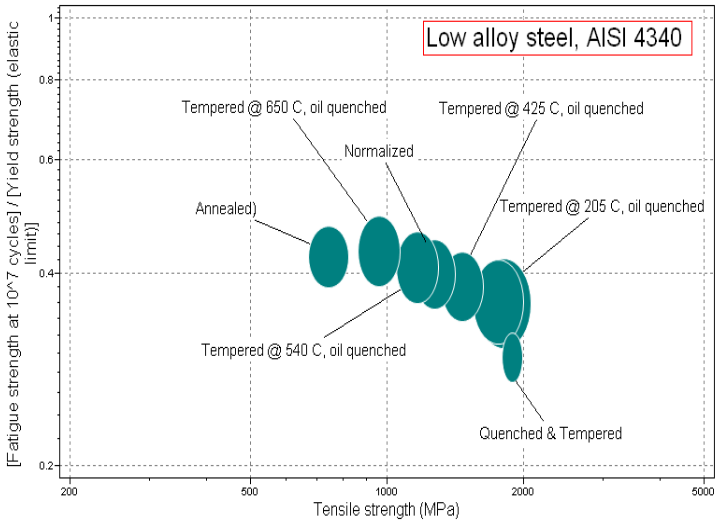
Use Level 3 of the database, apply a 'Tree' stage to isolate the folder for the low alloy steel, AISI 4340, then make the two charts, hiding all the other materials. How do you explain the trends?

Answer.

The ratio is more nearly constant using the tensile strength rather than the initial yield strength (around 0.3 – 0.4).

Tensile strength relates to the work hardened strength of the material. Fatigue crack initiation depends on cyclic plasticity in selected grains, and this too is resisted by dislocation accumulation with deformation.





Materials: engineering, science, properties, and design
3e Solution manual

Chapter 10: Exercises with worked solutions

Exercise E10.1 Supersonic wind tunnels store air under high pressure in cylindrical pressure vessels – the pressure, when released, produces hypersonic rates of flow. The pressure vessels are routinely proof-tested to ensure that they are safe. If such a cylinder, of diameter 400mm and wall thickness 20 mm, made of a steel with a fracture toughness of 42 MPa.m$^{1/2}$, survives a proof test to 40 MPa (400 atmospheres), what is the length of the largest crack it might contain?

Answer. Maximum stress in wall $\sigma = pR/t$ = 40 × 200 / 20 = 400 MPa.

Assuming a geometry factor $Y = 1$, $K_{1c} = 42 = \sigma\sqrt{\pi c} = 400\sqrt{\pi c}$. Hence c = 3.5 mm.

Exercise E10.2 A thin-walled spherical pressure vessel of radius R=1m is made of a steel with a yield stress of 600 MPa and a fracture toughness K_{1c} of 100 MPa.m$^{1/2}$. The vessel is designed for an internal working pressure p = 20 MPa.

(a) Find a suitable wall thickness if the maximum stress is to be kept below 60% of the yield stress. The stress in the wall of a uniform spherical pressure vessel is given by $\sigma = pR/2t$.

(b) The pressure vessel is inspected by a non-destructive method, and an embedded penny-shaped crack of length $2c$ = 10 mm is found in the wall (i.e. from Figure 10.2, the geometry factor Y = 0.7 in the expression for K). Will the pressure vessel survive at the working pressure, and in a proof test at 90% of the yield stress?

Answer. (a) $t = pR/2\sigma$ = (20×1000)/(2×0.6×600) = 27.8mm

(b) $K_1 = Y\sigma\sqrt{\pi c} = 0.7\times\left(\dfrac{20\times 1}{2\times 27.8\times 10^{-3}}\right)\sqrt{\pi\times 0.005}$ = 31.6 MPa.m$^{1/2}$. This is below the fracture toughness of 100 MPa.m$^{1/2}$ so the vessel is safe. Increasing the pressure to 90% of yield is a 50% increase, so K_1 also increases by 50% = 1.5 × 31.6 = 47.4 MPa.m$^{1/2}$ (still safe).

Exercise E10.3 You are asked to select a polymer to make a flexible coupling. The polymer must have a modulus greater than 2 GPa. The objective is to maximize the available flexure without fracture. Use the chart of Figure 10.4 to identify two good choices to meet these requirements. Are there any metals that are as good? If the flexible coupling is a door hinge, what other material behaviour should be considered to avoid premature failure in service?

Answer. PA (Polyamide, Nylon). No metals are as good. With reversed elastic loading, fatigue resistance should also be considered.

Exercise E10.4 Crash barriers like car fenders must absorb energy without total fracture. The most effective are those that deform plastically, absorbing energy in plastic work, but they are not reusable. Fenders that remain elastic spring back after impact. For practical reasons the material must have a modulus greater than 10 GPa. Use the chart of Figure 10.4 to find non-metallic materials for elastic fenders, assuming that the overriding consideration is that the displacement before fracture is as great as possible (the constraint on modulus ensures that it absorbs enough energy).

Answer. Best is GFRP, second best is CFRP.

Exercise E10.5 Materials with high toughness G_c generally have high modulus. Sometimes, however, the need is for high toughness with low modulus, so that the component has some flexibility. Use the chart of Figure 10.4 to find the material (from among those on the chart) that has a modulus less than 0.5 GPa and the highest toughness G_c. List applications of this material that you think exploit this combination of properties.

Answer. Leather. Shoes, belts (including, until fairly recently, belts to transmit power).

Exercise E10.6 If you want to support a precision optical system (laser metrology equipment, for instance) on a stable platform, you put it on a granite slab supported on end plinths to bring it to working height. (Granite can be ground to a flat surface and is thermally very stable and hard wearing.) The granite chosen for one such table has a fracture toughness of 0.9 MPa.m$^{1/2}$ and is known, from NDT procedures, to contain internal cracks up to 5 mm in length. If the table is 2 m long and 1 m deep, simply supported at its ends and must carry a uniformly distributed load of 2000 N on its upper surface (as in Figure 7.2 of the text), what is the minimum thickness the slab must have? Include the self-weight of the slab in the analysis. Assume that at least one of the cracks will lie in the part of the beam that carries the highest tensile stress – that is, at the lower surface. (The density of granite is 2700 kg/m^3).

Answer.

Let the table dimensions be $L \times b \times t$, where L=2m, b=1m and t is unknown.

Including the self-weight, the total load per unit length is: $w = \dfrac{2000}{L} + b\,t\,\rho\,g$.

The maximum bending moment, in the middle of the slab is: $M_{\max} = \dfrac{w\,L^2}{8}$, giving a maximum tensile stress in the middle of the base of the slab: $\sigma_{\max} = \dfrac{M_{\max}\,(t/2)}{I}$, with $I = \dfrac{b\,t^3}{12}$.

Fast fracture will occur when the stress intensity factor of a crack at the point of maximum tensile stress reaches the critical value: $1.1\,\sigma_{\max}\sqrt{\pi\,c} = K_{1c}$ (assuming a surface crack of length c – see Figure 10.2).

Combining all of these elements:

$$1.1\,\frac{(wL^2/8)\,(t/2)}{(b\,t^3/12)}\sqrt{\pi\,c} = K_{1c} \quad \text{so} \quad \frac{(2000 + b\,L\,\rho\,g\,t)}{(t^2)} = 1.212\left(\frac{b}{L}\right)\left(\frac{K_{1c}}{\sqrt{\pi\,c}}\right)$$

This may be rearranged to: $1.212\left(\dfrac{b}{L}\right)\left(\dfrac{K_{1c}}{\sqrt{\pi\,c}}\right)t^2 - (b\,L\,\rho\,g)\,t - 2000 = 0$

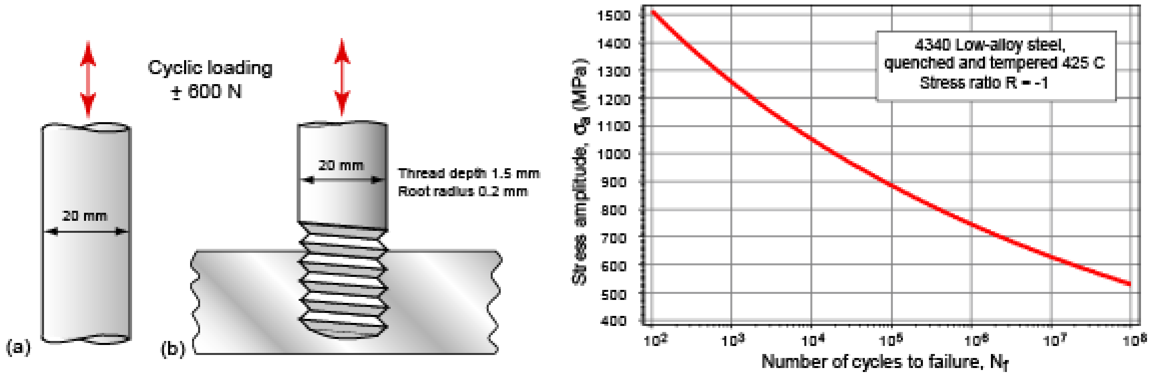
Incorporating all the numerical values (for b, L, K_{1c}, and ρ, with $g = 9.81$ ms^{-2} , and c=0.005m):

$$4.35\times10^6\ t^2 - 52974\,t - 2000 = 0\,, \quad \text{or} \quad 2175\,t^2 - 26.49\,t - 1 = 0$$

$$\text{Hence} \quad t = \frac{-(-26.49) \pm \sqrt{(26.49)^2 - 4\times2175\times(-1)}}{2\times2175} = 28.4\,\text{mm} \quad \text{(ignoring negative solution).}$$

This is only 5-6 times the crack length, so the bending stress will decrease significantly over the crack length. But as the analysis assumed the whole crack was subjected to the maximum (surface) tensile stress, the calculation should be conservative.

Exercise E10.7 The figure below shows, in (a), a cylindrical tie rod with diameter 20mm. The plan is to use it to carry a cyclic load with a stress range ± 200 kN. The figure also shows the S-N curve of the material of which it is to be made. Will it survive without failure for at least 10^5 cycles?



Answer. The stress amplitude = ± 637 MPa. The cylindrical rod will survive comfortably.

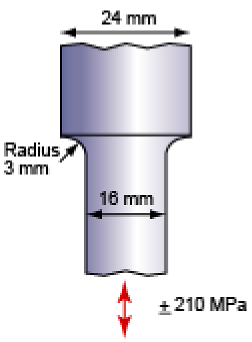
Exercise E10.8 The component of the previous exercise was made and tested. It failed in less than in less than 10^5 cycles. A post-mortem revealed that it had fractured at the root of a threaded end, shown at (b) in the figure above. The threads have a depth of 1.5mm and a root radius of 0.2 mm. Given this additional information, how many cycles would you expect it to survive?

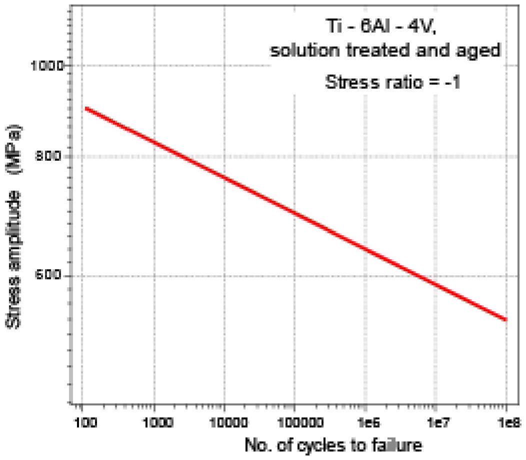
Answer. The stress concentration factor caused by a notch (Figure 7.7 and equation (7.12)) is:

$$K_{sc} = \frac{\sigma_{\max}}{\sigma_{nom}} = 1 + \alpha \left(\frac{c}{\rho_{sc}} \right)^{1/2}$$

with $\alpha \approx 2$ for tensile loading. The threads therefor create a stress concentration factor of 6.5. The local stress amplitude at the root of a thread is more than ± 4000 MPa. The component will certainly fail.

Exercise E10.9 The figure shows a component to be made from the high-strength aerospace alloy Ti-6Al-4V. It will be loaded cyclically at ± 210 MPa. How long will it last?





Answer. The stress concentration factor caused by a notch (Figure 7.7 and equation (7.12)) is:

$$K_{sc} = \frac{\sigma_{\max}}{\sigma_{nom}} = 1 + \alpha \left(\frac{c}{\rho_{sc}} \right)^{1/2}$$

with $\alpha \approx 2$ for tensile loading. The change of section of the sample introduces a stress concentration factor of 3.3. Thus the local stress range ± 695 MPa. It will last for about 10^5 cycles.

Exercise E10.10 Discuss the factors which determine whether a pressure vessel will fail gradually by yielding, leaking or by fast fracture, when subjected to repeated applications of internal pressure.

Answer. The vessel is subjected to repeated applications of internal pressure so cracks may grow by fatigue. The initial size of the crack, c_0, the wall thickness t, the radius R, and the magnitude of the internal pressure, p, will determine whether the vessel will fail by yielding, leaking or by fast fracture. The pressure maximum stress (and stress amplitude) applied is given by $\sigma = pR/t$.

A fatigue crack may reach any length between c_0 and t, with the growth rate depending on the stress amplitude (and accelerating with crack length, through ΔK). First, if the critical crack length for fracture lies between these values, then fast fracture will occur, i.e. when $K_{1c} = (pR/t)\sqrt{\pi c_{crit}}$ (for $c_0 < c_{\text{crit}} < t$). If on the other hand, the fracture toughness is sufficient to give $c_{\text{crit}} > t$, then leaking will occur as the crack penetrates the wall without fast fracture. Both cases assume that the stress level in the pressure vessel is elastic, but if the applied stress exceeds general yield, this is the failure mechanism: i.e. if $pR/t > \sigma_y$.

Exercise E10.11 Consider three cylindrical pressure vessels made of the aluminium alloys in the table below. Each has a wall thickness of t=14mm, and a radius of R = 840mm. The vessels are each periodically emptied and filled to an internal pressure of p = 2 MPa. Cracks can grow through the wall by fatigue, driven by the cycle of stress on pressurisation. Determine the failure mode of each vessel: yield / leak / fracture. Comment on which material would be best for this application.

Material	Yield stress, σ_y (MPa)	Fracture toughness, K_{1c} (MPa.m$^{1/2}$)
Aluminium 2024 T4	300	40
Aluminium 6061 T4	110	35
Aluminium 5454 H111	140	20

Answer.

The hoop stress in the vessel is $\sigma_h = \dfrac{pR}{t} = \dfrac{2(0.84)}{0.014} = 120$ MPa.

Fast fracture occurs according to equation (8.4) when $c = \dfrac{1}{\pi}\left(\dfrac{\mathrm{K}_{1c}}{\sigma}\right)^2$

For Al alloy 2024 -T4, $c = \dfrac{1}{\pi}\left(\dfrac{40}{120}\right)^2 = 0.035 \text{ m} = 35 \text{ mm}$. Since the critical crack length is greater than the wall thickness, the vessel will fail by leaking.

For 6061-T4, the same calculation gives c = 27 mm, however the hoop stress exceeds the yield stress of the material. So this vessel will yield.

For 5454-H111, c = 8.8 mm which is less than the wall thickness, so the vessel will fail by fast fracture.

Material	Yield stress, σ_y (MPa)	Fracture toughness, K_{1c} (MPa.m$^{1/2}$)	c (mm)	Outcome
Al 2024 T4	300	40	35	Leak ($c > t$; $\sigma < \sigma_y$)
Al 6061 T4	110	35	27	Yield ($\sigma > \sigma_y$)
Al 5454 H111	140	20	8.8	Fracture ($c < t$; $\sigma < \sigma_y$)

The best material is the 2024-T4 which has a good safety margin on yield strength and a critical crack length that is significantly greater than the wall thickness. Consequently if a crack did grow by fatigue, it would grow through the wall and leak out the pressure in a safe way, without bursting.

Exercise E10.12

(a) Use the Paris' law for crack growth rate (equation (10.19)) to show that the number of cycles to failure N_f of a component cycled through a uniform stress range $\Delta\sigma$, and containing an initial crack of length c_i, is:

$$N_f = \frac{1}{A\,Y^m \Delta\sigma^m \pi^{m/2}(1-m/2)}\left[\left(c^*\right)^{1-m/2} - \left(c_i\right)^{1-m/2}\right]$$

where c^* is the critical crack length, Y is a geometric factor, A and m are constants in the Paris law (with $m > 2$).

(b) A die fabricated from a low alloy steel is to withstand tensile hoop stresses that cycle between 0 and 400 MPa. Prior to use it has been determined that the length of the largest crack in the steel is 1mm. Determine the minimum number of times that the die may be used before failing by fatigue, assuming $m = 3$, $A = 1.0\times10^{-12}$ (with $\Delta\sigma$ in MPa), and $Y = 1.13$. The fracture toughness of the steel is 170 MPa.m$^{1/2}$.

Answer.

(a) The rate at which crack length c increases due to fatigue with number of stress cycles N is given by:

$$\frac{dc}{dN} = A(\Delta K)^m, \text{ where } \Delta K = Y\Delta\sigma\sqrt{\pi c} \qquad (1)$$

Rearranging (1) and integrating gives: $N_f = \int_0^{N_f} dN = \int_{c_i}^{c^*} \frac{dc}{A(\Delta K)^m} = \int_{c_i}^{c^*} \frac{dc}{A(Y\Delta\sigma\sqrt{\pi c})^m}$

So $N_f = \frac{1}{AY^m\Delta\sigma^m\pi^{m/2}}\int_{c_i}^{c^*}\frac{dc}{c^{m/2}}$, i.e. $N_f = \frac{1}{AY^m\Delta\sigma^m\pi^{m/2}(1-m/2)}\left[\left(c^*\right)^{1-m/2} - \left(c_i\right)^{1-m/2}\right]$

(b) For the die fabricated from pressure vessel steel:

First calculate critical crack length, c*, from K_{1c}:

$$c^* = \frac{1}{\pi}\left(\frac{K_{1c}}{Y\sigma}\right)^2 = \frac{1}{\pi}\left(\frac{170}{1.13\times 400}\right)^2 = 0.045\,\text{m}$$

Then with $m = 3$, $A = 10^{-12}$ MPa^{-3}m$^{-1/2}$, $c_i = 0.001$m, $Y = 1.13$, $\Delta\sigma = 400$ MPa:

$$N_f = \frac{1}{10^{-12}\times 1.13^3\times 400^3\times\pi^{3/2}\times 0.5}\left[\frac{1}{\sqrt{0.001}} - \frac{1}{\sqrt{0.045}}\right] = 104661 \approx 10^5 \text{ cycles}$$

Exercise E10.13 A cylindrical steel pressure vessel of 7.5m diameter and 40mm wall thickness is to operate at a working pressure of 5.1MPa. The design assumes that small thumb-nail shaped flaws in the inside wall will gradually extend through the wall by fatigue. Assume for this crack geometry that $K = \sigma\sqrt{\pi c}$, where c is the length of the edge-crack and σ is the hoop stress in the vessel. The fracture toughness of the steel is 200 MPa.m$^{1/2}$ and the yield stress is 1200 MPa.

(a) Would you expect the vessel to fail in service by leaking (when the crack penetrates the thickness of the wall) or by fast fracture?

(b) The vessel will be subjected to 3000 pressurisation cycles during its service life. The growth of flaws by fatigue is given by equation (10.19) with $m = 4$, $A = 2.44\times10^{-14}$ MPa^{-4}m^{-1}, and stress in MPa. Find the critical crack length for fracture, and check that the vessel will leak before fracture. Determine the initial crack length c_i that would grow to failure in 3000 cycles. Hence find the pressure to which the vessel must be subjected in a proof test to guarantee against failure in the service life. Check that this test will not yield the vessel.

Answer.

(a) $$\sigma_{\text{working}} = \frac{pR}{t} = \frac{5.1\times(7.5/2)}{0.04} = 478\text{ MPa}$$

$$K_{1c} = \sigma_{working}\sqrt{\pi c} \text{ at fracture.}$$

$$c = \frac{1}{\pi}\left\{\frac{200}{478}\right\}^2 = 5.6\times10^{-2}\,\text{m} = 56\,\text{mm}$$

This critical depth for fast fracture is greater than the wall thickness of 40 mm. The vessel will fail by leaking before the crack length becomes critical and it fails by fast fracture. [In practice we should allow for the complicated geometry of the crack, by looking up the geometry calibration factor Y in a stress intensity factor handbook. This will be particularly important as the crack approaches the free outside of the wall].

(b) The crack growth equation is: $\dfrac{dc}{dN} = A(\Delta K)^m = A(\Delta\sigma)^4 \pi^2 c^2$

This can be integrated from the initial condition after the proof test (assuming that a crack of length c_i is present), to the required end point where after 3000 cycles the crack has grown out to the wall ($c = t$), where failure would occur by leakage:

$$\int_0^{N_f} dN = \frac{1}{A\sigma_{\max}^m \pi^{m/2}} \int_{c_i}^{t} \frac{dc}{c^{m/2}}$$

giving:

$$N_f = \frac{1}{A\sigma_{\max}^m \pi^{m/2}(1-m/2)}\left[(t)^{1-m/2} - (c_i)^{1-m/2}\right]$$

Substituting values for A=2.44 x 10^{-14}, t = 40mm, N_f = 3000 cycles and m = 4, with a maximum stress of 478 MPa we can solve for c_i:

$$3000 = \frac{1}{2.44\times10^{-14}\times478^4\pi^2(-1)}\left[\frac{1}{0.04} - \frac{1}{c_i}\right], \text{ giving } c_i = 0.016\text{m}.$$

This is the initial flaw size that will penetrate the wall after 3000 loading cycles.

The proof stress p_{proof} must be sufficient to cause flaws of this size to propagate by fast fracture. This will happen if the stress intensity factor for flaws of this size exceeds the fracture toughness:

$$K = \sigma_{proof}\sqrt{\pi c_i} \geq K_{1c} \quad \text{where } \sigma_{\text{proof}} = \frac{p_{proof}\, R}{t}$$

Hence: $p_{proof} \geq \dfrac{t\,K_{1c}}{R\sqrt{\pi c_i}} = \dfrac{0.04\times200}{3.75\sqrt{\pi 0.016}} = 9.5$ MPa

The corresponding stress in the vessel is $\sigma_{\text{proof}} = \dfrac{pR}{t} = \dfrac{9.5\times(7.5/2)}{0.04} = 890$ MPa which is significantly less than the yield stress of 1200 MPa.

Exercise E10.14 Consider the design of a bicycle crank. It is assumed that the design is governed by fatigue failure, and that to avoid this the maximum stress amplitude must be kept below the endurance limit σ_e of the material. The crank is modelled as a beam in bending of square cross section $a \times a$, where a is free to vary. The length of the beam and the amplitude of the applied cyclic moment are fixed.

(a) Derive an expression for the material index which should be maximised to minimise the mass. Hence choose a shortlist of materials using the property chart, Figure 10.8, commenting on your choice. Use the most advantageous material properties within the ranges shown on the chart.

(b) An existing crank made of the best performing Al alloy weighs 0.2kg. A cyclist is prepared to pay extra for a lighter component made of titanium, but only if it saves enough weight. Use the property chart with the material index from (a) to find the ratio of the mass of the best performing Ti alloy to that of the best Al alloy. Hence estimate the mass of the corresponding titanium component.

(c) The as-manufactured costs per kg of aluminium and titanium are 10 and 40 £/kg, respectively. Estimate the costs of the two cranks. How much value (in £) would the cyclist need to associate with each kg of weight saved, in order to go ahead with the switch from aluminium to titanium?

Answer.

(a) Objective: Mass = $\rho a^2 \ell$

Constraint: $\sigma_e = \dfrac{M\,y_m}{I} = \dfrac{M\,a/2}{a^4/12} \Rightarrow a^2 = \left(\dfrac{6M}{\sigma_e}\right)^{2/3}$

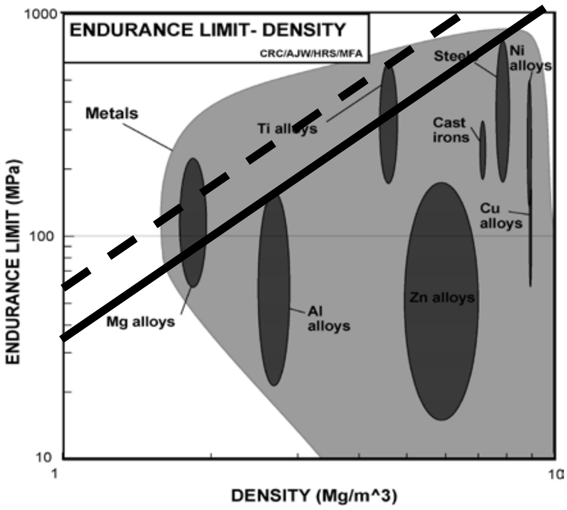
Eliminate free variable a: $\quad \text{Mass} = (6M)^{2/3} \ell \dfrac{\rho}{(\sigma_e)^{2/3}}$

Maximise $\sigma_e^{2/3} / \rho$ to minimise mass.

Draw a line of slope 3/2 on the property chart (see the lower line on the chart).

Shortlist (lightest first): Mg alloys, Ti alloys, Al alloys & Steels.

Magnesium may be difficult to machine and corrosion is a problem, and titanium is expensive. On this basis aluminium and steel look equally good.



(b) The analysis of part (a) shows that, for a fixed length and moment, the ratio of masses is in inverse proportion to the ratio of $\sigma_e^{2/3} / \rho$. The two lines of slope 3/2 shown on the graph, going through the best data points for Al alloys and Ti alloys, are lines of constant $\sigma_e^{2/3} / \rho$, with a factor of about 0.7 between them (estimated by considering the spacing at a density of 1 Mg/m^3, where the stresses differ by a factor of about 0.6). Hence the Ti alloy crank would have a mass of 0.2 × 0.7 = 0.14 kg.

(c) The manufactured cost of Ti is 0.14 × 40 = £5.6, while in Al it would be 0.2× 10 = £2.0. The additional cost is £3.6. Hence for a weight saving of 0.06 kg, it makes sense to switch if the perceived value of each kg of mass saved is greater than 3.6/0.06 = £60 /kg.

Exercise E10.15 An adhesive has a toughness G_c = 100 J/m^2 and a shear strength σ_s = 0.1 MPa. What must the dimensions of the bonded area of a lap-joint be if it is to carry an in-plane tensile F_t = 100 N, but allow peeling at a force F_p = 5 N?

Answer. From the text, the ratio of the peel force to the tensile force is:

$$\frac{F_p}{F_t} = \frac{G_c}{\sigma_s L}$$

Hence the required length is: $L = \dfrac{G_c}{\sigma_s}\left(\dfrac{F_t}{F_p}\right) = \dfrac{100}{0.1\times 10^6}\left(\dfrac{100}{5}\right) \text{ m} = 20\,\text{mm}$

The required area is given by $F_t = \sigma_s \, w L$, so the required width is:

$$w = \frac{F_t}{\sigma_s L} = \frac{100}{0.1\times 10^6 \times (20\times 10^{-3})} \text{ m} = 50\,\text{mm}$$

Exploring design with CES *(use Level 2 Materials for all selections)*

Exercise E10.16 Use the 'Search' facility in CES to find materials that are used for:

(a) pressure vessels.
(b) connecting rods.
(c) rail track.

(Search using the singular – e.g. pressure vessel – since that will find the plural too).

Answer. (a) Pressure vessels

- CFRP, epoxy matrix (isotropic)
- Low alloy steel
- Copper
- Age-hardening wrought Al-alloys
- Non age-hardening wrought Al-alloys
- Titanium alloys

(b) Connecting rods

- Low alloy steel
- Age-hardening wrought Al-alloys
- Titanium alloys

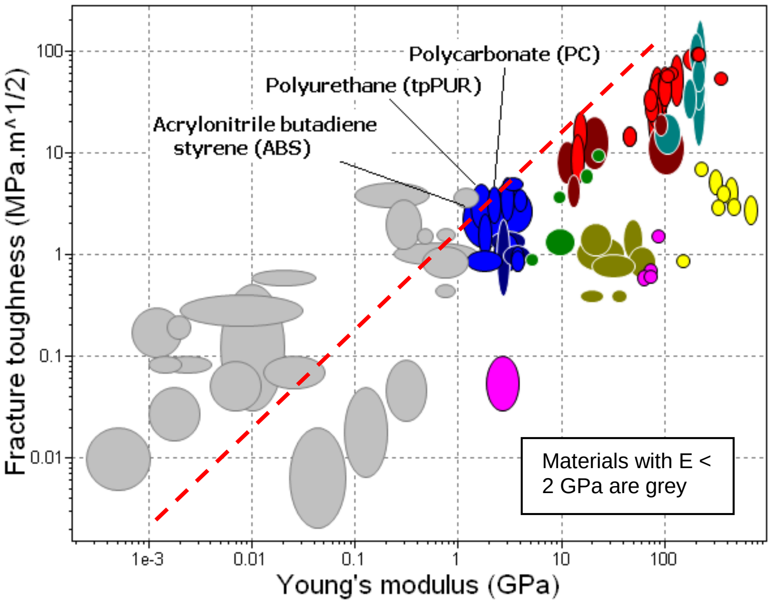
(c) Rail track

- High carbon steel
- Medium carbon steel

Exercise E10.17 You are asked, as in Exercise E10.3, to select a polymer to make a flexible coupling. The polymer must have a modulus E greater than 2 GPa. The objective is to maximize the available flexure without fracture, and that means materials with high K_{1c} / E. Use a 'Limit' stage to impose the constraint on E, then make use a 'Graph' stage to make a chart for K_{1c} and E, put on an appropriate selection line and move it until only 3 materials remain in the Results window. What are they? Rank them by price.

Answer.

- Acrylonitrile butadiene styrene (ABS)
- Polycarbonate (PC)
- Polyurethane (tpPUR)



Exercise E10.18 You are asked to recommend materials that have yield strength above 500 MPa and perform best in a design based on the leak-before-break criterion. Construct an appropriate limit stage and chart, put on the necessary selection line and list the three materials you would recommend.

Answer. Tough steels and stainless steels are the best choice. The three that score the highest are

- Low alloy steel
- Stainless steel
- Nickel

Exercise E10.19 Repeat the previous selection, applying instead a yield-before-break selection criterion.

Answer. Yield before break favours materials that have lower strength, so that they yield more easily. The three that score the highest are

- Brass
- Nickel
- Stainless steel

Exercise E10.20 A material is sought for a high performance con-rod, requiring that the index has as high a value of the index σ_e / ρ, where σ_e is the endurance limit (the fatigue strength at 10^7 cycles). It must enough toughness to tolerate stress concentrations, requiring that K_{1c} > 15 MPa.m$^{1/2}$. Make the appropriate selection stages and list the three materials that best meet the criteria.

Answer.

- CFRP, epoxy matrix (isotropic)
- Titanium alloys
- Wrought magnesium alloys

Materials: engineering, science, properties, and design
3e Solution manual

Chapter 11: Exercises with worked solutions

Exercise E11.1 Define the coefficient of friction. Explain why it is independent of the area of contact of the sliding surfaces.

Answer. The coefficient of friction μ is defined as:

$$\mu = \frac{F_s}{F_n}$$

where F_n is the normal load pressing the sliding surfaces together, and F_s is the force opposing the sliding motion. Surfaces are never perfectly flat. Thus when two surfaces are placed in contact they touch only at points where asperities meet. The load F_n is supported solely by the contacting asperities. The real contact area A_r is only a tiny fraction of the apparent, nominal, area A_n. Even small loads cause large contact stresses, enough to cause plastic deformation. The contact points flatten, forming junctions with a total area A_r – each contact resembles a local hardness indentation. Hence the total load transmitted across the surface is:

$$F_n = A_r H = A_r (3\sigma_y)$$

where H is the hardness, and σ_y is the yield strength. The limiting force opposing the sliding motion is also proportional to the real area of contact (depending also on the shear strength of the contacts). Hence both F_n and F_s are proportional to A_r, and their ratio μ is independent of the nominal area A_n.

Exercise E11.2 Woods – both resinous wood from the pine family and hard-woods like *lignum vitae* (it is so dense it sinks in water) – were, historically, used for bearings – even clocks had hard-wood bearings. Why? Think of your own ideas, then research the Web.

Answer. The resin in the wood acts as a lubricant. It is compliant enough to conform to the counter-face of the bearing and is able to trap grit particles, embedding them in the wood where they cease to be damaging.

From http://www.woodex-meco.com/woodhome.htm: Wood is arguably the oldest bearing material on the planet, in continuous use since the invention of the wheel. Gladiators in ancient Rome drove chariots with wooden bearings; Marie Antoinette rode to her untimely end in a cart whose axles were borne in wood. Bearings made of *lignum vitae* bore the rudder shafts of ships in the golden age of sail, and when the steamship rose to power, its propeller shaft spun in lignum bearings, too. Wood continues to this day to be the most "shaft-kindly" bearing material available. Even the latest engineering plastics fail to protect metal shafts as well as wood.

Exercise E11.3 Give examples, based on your experience in sport (tennis, golf, swimming, skiing, rock climbing, hang-gliding…) of instances in which friction is wanted and when it is not.

Answer. *Friction wanted:* grips of racquets; soles of running and climbing shoes; climbing gloves; tyres and brakes of cycles and cars.

Friction not wanted: the sliding surfaces of skis; the transmission and gearing of bicycles and cars, of brake cables and of archery bows.

Exercise E11.4 Now a more challenging one. Give examples, again based on your experience in sport, of instances in which wear is desirable and in which it is not.

Answer. *Wear wanted*: when used to condition the tyres of racing cars, to abrade an inner tube before applying a puncture patch, in running-in or wear-in of components.

Wear not wanted: brake pads, cycle tyres, all mechanical components with sliding surfaces.

Exercise E11.5 What are the characteristics of materials that are a good choice for use as brake pads?

Answer. Brake pad material must apply friction without vibration, wear or fade. That is not easy when you reckon that, to stop a one-tonne car from 100 kph (60 mph) requires that you find somewhere to put about 0.4 MJ. That's enough (if you don't lose any) to heat a kg of steel to 800°C. Brake materials must apply friction at temperatures that can approach 800°C, and they must do so consistently and without the stick-slip that causes 'brake squeal'. That means materials that can tolerate heat, conduct it away to limit temperature rise, and lubricate, since it is this that quenches squeal.

Brake pad materials consist of a matrix, an abrasive additive to increase friction, a reinforcement to help conduct heat and a lubricant to suppress vibration. A typical pad material has a phenolic matrix with particles of silicates, silicon carbide (sandpaper grit) to control friction, and graphite or MoS_2 as a lubricant. Those on a military jet, a 747 airliner, or a F1 car are carbon or ceramic, required to tolerate the high temperatures that are generated at the sliding surfaces.

Exercise E11.6 A prototype disk brake for a bicycle is to be mounted on the hub of the rear wheel, and consists of a pair of pads of area 5cm × 1cm which are pressed against opposing faces of the steel disk. The pads are aligned with their longer dimension in the tangential rotation direction, with the pad centres at a distance of 10cm from the centre of the disk. The brake is to be tested on a rotating machine to simulate the loading and wear in service. The disk rotates at a suitable constant speed, and the brakes are applied for 10 seconds every minute, with each pad applying a normal load of 1 kN.

(a) Find a suitable rotation speed (in revs. per minute, rpm) if the test is to simulate braking a bicycle travelling at a speed of 20km/hour, with a wheel diameter (including the tyre) of 70cm.

(b) When the brakes are applied, the machine registers an increase in torque of 30 Nm. Find the friction force that is being applied to each pad, and hence estimate the friction coefficient.

(c) Estimate the depth of material removed per hour of testing, if the Archard wear constant for the materials is $k_a = 2\times10^{-8}$ MPa^{-1}.

(d) What other factors would influence the wear rate in service?

Answer.
(a) 20km/hour = 20,000/3600 m/s = 5.56 m/s. Hence one wheel rotation takes $(\pi \times 0.7)/5.56 = 0.40$s
The rotation rate is therefore $60 \times (1/0.40)$ rpm = 150 rpm

(b) Friction force F acts at a distance of 0.1m, with half the torque coming from each side of the disk. Hence the friction force per pad, $F = 15/0.1 = 150$ N, and the friction coefficient $\mu = 150/1000 = 0.15$.

(c) In 10s of braking, the number of revolutions is 10/0.40, and the sliding distance per rev. = $\pi \times 0.1$m
Hence the sliding distance per hour = $60 \times (10/0.40) \times (\pi \times 0.1) = 471$m.

From equations (11.2)-(11.4):
Depth removed per hour = ($k_a \times F_n \times$ distance slid)/A_n = $(2\times10^{-8})\times10^{-6} \times 1000 \times 471 / (0.05 \times 0.01)$ = 19μm.

(d) Other factors: running wet will provide some lubrication, but corrosion of the disk and pad will accelerate loss of material (as will entrainment of grit and wear debris if the brakes become dirty).

Exercise E11.7 A bronze statue weighing 4 tonnes with a base of area 0.8 m^2 is placed on a granite museum floor. The yield strength of the bronze is 240 MPa. What is the true area of contact, A_r, between the base and the floor?

Answer. From equation (11.7), $A_r = \dfrac{F_n}{3\sigma_y}$. Normal force $F_n = mg = 4\times10^3\times9.81 = 39.24$ kN.

Hence $A_r = 39{,}240 / (3\times240\times10^6) = 5.45 \times 10^{-5}$ m^2 (54.5 mm^2)

Exercise E11.8 The statue of the previous example was moved to a roof courtyard with a lead floor. The yield strength of the lead is 6 MPa. What now is the true area of contact?

Answer. Now the lead asperities yield instead, and the contact area scales with the inverse of the yield stress (for the same normal load). So now A_r = 54.5 mm^2 × (240/6) = 2180 mm^2.

Exercise E11.9 How would you measure the true area of contact A_r between two surfaces pressed into contact? Use your imagination to devise ways.

Answer.

- Coat one surface with a mono-molecular layer of a fluorescent dye which is transferred to the other surface at the contact points.
- If the two materials that meet at the surface are metallic, the electrical resistance across the surface is a measure of the area of contact.
- The thermal conductivity across the surface is a measure of the area of contact.

Exercise E11.10 A 30 mm diameter plane bearing of length 20 mm is to be made of a material with the $P-v$ characteristics shown in Figure 11.11(b). If the bearing load is 300 N and the maximum rotation rate is 500 rpm, is it safe to run it dry? If not, is boundary lubrication sufficient? (Remember that the bearing pressure is the load divided by the projected area normal to the load.)

Answer. The mean bearing pressure is the load divided by the projected area normal to the load, giving 0.5 MPa. The sliding velocity in m/s at the bearing surface is the rotation speed (in revolutions per second) times the circumference of the bearing, giving 0.78 m/s. Plotting these on Figured 11.11(b) shows that in is unsafe to run the bearing dry, but that boundary lubrication is enough to protect it.

Exercise E11.11 A rotary friction weld is to be made between two solid aluminium cylinders of diameter 2cm. The rotation speed is 600 rpm, and an initial axial force of 50 kN is applied, and the increase in the measured machine torque is 60 Nm. Estimate the friction coefficient and heating rate on first contact. After a few seconds, the contact has heated up and sticking friction applies over the whole contact area, with a shear yield stress k = 15 MPa. Find the new machine torque and the power being dissipated at the contact.

Answer. From equation (11.10), the torque T developed when a cylindrical contact of radius R slips under a normal load F with a friction coefficient μ is: $T = \frac{2}{3}\,\mu F\,R$.

Hence the coefficient of friction $\mu = 60 \times (3/2) / (50{,}000 \times 0.01) = 0.18$.

The rotation speed is 600 rpm = 10 revs. per sec. , or $\omega = 10 \times 2\pi$ radians/sec.

The power is $q = T \times \omega = 60 \times 10 \times 2\pi = 3.77$ kW

With sticking friction, equation (11.11), the $T = \frac{2\pi}{3}\,k\,R^3 = (2/3)\pi \times 15\times10^6 \times (0.01)^3 = 31.4$ Nm

The power is now $q = T \times \omega = 31.4 \times 10 \times 2\pi = 1.97$ kW

11.9 Exploring design with CES

(Use Level 2 unless otherwise stated).

Exercise E11.12 Use the 'Search' facility to find materials that are used as bearings. (Search on the singular – bearing – since that picks up the plural as well.)

Answer.

- Silicon carbide
- Silicon nitride
- Low alloy steel

Exercise E11.13 Use the 'Search' facility to find materials that are used as for brake pads. Do the same for brake discs.

Answer.
Brake pads:

- Cork
- Butyl rubber
- Phenolics

Brake discs:

- Aluminum/Silicon carbide composite
- Cast iron, gray
- Cast iron, ductile (nodular)

Exercise E11.14 Use a 'Limit' selection stage applied to the Surface Treatment data-table to find processes that enhance wear resistance. (To do this, click 'Select' in the main tool bar, then on 'Selection Data', just below it. Select Level 2 Surface treatment in the dialog box. Open a 'Limit' stage, open Function of treatment, and click on Wear resistance.) Explore the record for laser-based methods. What are its typical uses?

Answer.

- Anodizing
- Carburizing and carbonitriding
- Electroless plating
- Electroplating
- Hot dip coating (Galvanizing)
- Induction and flame hardening
- Laser hardening (surface hardening and melting)
- Metal flame spray
- Nitriding
- Texturing
- Vapor metallizing (PVD)
- Vitreous enameling

Laser surface hardening is used to create hard, wear-resistant facings on tools, rolls for rolling mills, tips of rocker arms and cylinder bores in automotive and marine engines.

Exercise E11.15 Follow the same procedure as that of the previous example to search for processes used to control friction. Explore the record for grinding and mechanical polishing. What are grinding wheels made of?

Answer.

- Carburizing and carbonitriding
- Electroless plating
- Electroplating
- Etching
- Grinding and mechanical polishing
- Induction and flame hardening
- Laser surface hardening and melting
- Metal flame spray
- Nitriding
- Polymer powder coating
- Texturing

Grinding wheels are made of particles of silicon carbide ('carborundum' or 'corundum') bonded in a cement or polymer binder.

Materials: engineering, science, properties, and design
3e Solution manual

Chapter 12: Exercises with worked solutions

Exercise E12.1 Define specific heat. What are its units? How would you calculate the specific heat per unit volume from the specific heat per unit mass? If you wanted to select a material for a compact heat storing device, which of the two would you use as a criterion of choice?

Answer. The energy to heat 1 kg of a material by 1 K is called the heat capacity or specific heat. The measurement is usually made at constant pressure (atmospheric pressure) and thus is given the symbol C_p. Its units are J/kg.K. When dealing with gases, it is more usual to measure the heat capacity at constant volume (symbol C_v) and for gases this differs from C_p. For solids the difference is small. The specific heat per unit volume is that per unit mass, C_p, multiplied by the density, ρ:

$$(C_p)_{vol} = C_p \rho$$

The best choice for a heat storing device of minimum mass is the material with the greatest value of C_p; the best choice for one of minimum volume (thus compact) is that with the greatest value of $(C_p)_{vol}$ provided, in both cases, the material meets all the other constraints (e.g. an adequate maximum service temperature).

Exercise E12.2 What two metals would you choose from the $\lambda - a$ chart of Figure 12.5 to maximize the thermal displacement of a bi-metallic strip actuator? If the bimetallic strip has a thickness 2mm and an average thermal diffusivity a of 5×10^{-5} m^2/s, how long will it take to respond when the temperature suddenly changes?

Answer. Choose metals that lie far as apart on the expansion coefficient axis as possible: copper (or brass) and steel – probably not lead because it is too soft. Response time ≈ $(\text{thickness}/2)^2 / a \approx 0.02$s.

Exercise E12.3 A structural material is sought for a low-temperature device for use at –20 °C that requires high strength but low thermal conductivity. Use the $\lambda - \sigma_y$ chart of Figure 12.6 to suggest two promising candidates (reject ceramics on the grounds that they are too brittle for structural use in this application).

Answer. CFRP or GFRP.

Exercise E12.4 A new alloy has a density of 7380 kg/m^3. Its specific heat has not yet been measured. How would you make an approximate estimate of its value in the normal units of J/kg.K? What value would you then report?

Answer. Use $\rho C_p \approx 2\times10^6$ J/m^3.K for all solid materials. The result is $C_p \approx 271$ J/kg.K.

Exercise E12.5 The same new alloy of the last exercise has a melting point of 1540 K. Its thermal expansion coefficient has not yet been measured. How would you make an approximate estimate of its value? What value would you then report?

Answer. Use $\alpha T_m \approx 0.02$. The result is $\alpha \approx 1.3\times10^{-5}$ K^{-1}.

Exercise **E12.6** Interior wall insulation should insulate well, meaning low thermal conductivity, λ, but require as little heat as possible to warm up when the central heating system is turned on (if the wall absorbs heat the room stays cold). Use the $\lambda - a$ chart of Figure 12.5 to find the materials that do this best. (The contours will help.)

Answer. Polymer foams have the lowest values of λ and at the same time lie on the lowest contour of volumetric specific heat.

Exercise E12.7 An external wall of a house has an area of 15m^2 and is of single brick construction with a thickness 10cm. The houseowner keeps the internal temperature at 22^{o}C. How much heat is lost through the wall per hour, if the outside temperature is a stable 0^{o}C? What percentage reduction in heat loss through the external wall would be achieved by reducing the internal temperature to 18^{o}C? The thermal conductivity of the brick is 0.8 W/m.K.

Answer. From equation (12.2) of the text, the steady-state heat flow (W/m^2) through a thickness *d*:

$$q = \lambda \frac{(T_1 - T_2)}{d}$$

Substituting values: λ = 0.8 W/m.K, T_1 = 22^{o}C, T_2 = 0^{o}C, d = 0.1m, gives q = 176 W/m^2. For an area of 15m^2, the energy per hour = 176 × 15 × 60 × 60 = 9.50 MJ (or 2.64 kWh).

Reducing the temperature difference gives a proportionate drop in energy, so the energy falls by 4/22 or 18%.

Exercise E12.8 The houseowner in the last Exercise wishes to reduce heating costs. The wall could be thickened to a double brick construction, with a cavity filled with polyurethane foam. The foam manufacturer quotes an effective thermal conductivity of 0.25 W/m.K for this wall. How much heat would be saved per hour, compared to the single brick, for internal temperatures of 22^{o}C and 18^{o}C?

Answer. The heat loss is proportional to the conductivity, so with the cavity wall, the energy loss will become (0.25/0.8) times the previous results, i.e. (0.25/0.8) × 9.5 = 2.97 MJ = 0.82 kWh (for 22^{o}C) – a saving of (9.50 – 2.97) = 6.53 MJ (1.81 kWh). The saving will be proportionately lower for 18^{o}C, by a factor of (18/22) × 6.53 = 5.34 MJ (1.48 kWh).

Exercise E12.9 When a nuclear reactor is shut down quickly, the temperature at the surface of a thick stainless steel component falls from 600°C to 300°C in less than a second. Due to the relatively low thermal conductivity of the steel, the bulk of the component remains at the higher temperature for several seconds. The coefficient of thermal expansion of stainless steel is 1.2×10^{-5} K^{-1}, Poisson's ratio ν = 0.33, E = 200 GPa, and σ_y = 585 MPa.

(a) Find the thermal strain in the surface that is prevented from occurring by the constraint of the underlying material.

(b) The induced thermal strain will be biaxial, as constraint applies in two perpendicular directions parallel to the surface. In this case the elastic stress induced is: $\varepsilon_{elastic} = (1-\nu)\sigma / E$. Find the maximum possible elastic strain parallel to the surface, i.e. if the induced stress were to reach the yield stress, and compare this with the thermal strain imposed. What do you conclude will happen to the surface layer when the shutdown occurs?

Answer. (a) The thermal strain of the surface layer is:

$$\varepsilon_{thermal} = \alpha \Delta \mathrm{T} = 1.2 \times 10^{-5} \times (-300) = -3.6 \times 10^{-3}$$

(b) Maximum elastic strain: $\varepsilon_{elastic} = \dfrac{\sigma_y}{E}(1-\nu) = \dfrac{585}{200 \times 10^3}(1-0.33) = 1.96 \times 10^{-3}$

The total strain in the surface layer is zero, so as the thermal strain exceeds the elastic strain, the surface will yield (with plastic strain making up the difference in strain of 1.64×10^{-3} .

Exercise E12.10 Square porcelain tiles are to be manufactured with a uniform layer of glaze, which is thin compared to the thickness of the tile. They are fired at a temperature of 700°C and then cooled slowly to room temperature, 20°C. Two tile types are to be manufactured: the first is decorative, using tension in the surface to form a mosaic of cracks; the second uses compression in the surface to resist surface cracking, improving the strength. Use equation (12.11) in the text and the data below to select a suitable glaze (A, B or C) for each of these applications.

Young's modulus of glaze, E = 90 GPa, Poisson's ratio of glaze, ν = 0.2, tensile failure stress of glaze = 10 MPa.

Coefficients of thermal expansion:

	$\alpha \times 10^{-6}$ (K^{-1})
Porcelain	2.2
Glaze A	1.5
Glaze B	2.3
Glaze C	2.5

Answer. Induced thermal stress given by the tensile stress that balances the thermal contraction is:

$$\sigma = \frac{E}{(1-\nu)}(\alpha_1 - \alpha_2)\Delta T$$

The sign of the surface stress is determined purely by the difference in thermal expansion coefficients: $\alpha_1 > \alpha_2$ gives tension, $\alpha_1 < \alpha_2$ gives compression.

	$\alpha \times 10^{-6}$ (K^{-1})	σ (MPa)
Glaze A	1.5	– 53.5
Glaze B	2.3	7.6
Glaze C	2.5	23.0

For the decorative tile, choose Glaze C: first because the stress is tensile (as $\alpha_{glaze} > \alpha_{porcelain}$), but second because the stress is sufficient to cause cracking (σ > strength, 10 MPa).

For the strong tile with compression in the glaze, choose Glaze A ($\alpha_{glaze} < \alpha_{porcelain}$).

Exercise E12.11 You notice that the ceramic coffee mugs in the office get too hot to hold about 10 seconds after pouring in the hot coffee. The wall thickness of the cup is 2mm.

(a) What, approximately, is the thermal diffusivity of the ceramic of the mug?

(b) Given that the volume specific heat of solids, ρC_p, is more or less constant at 2×10^6 J/m^3.K, what approximately is the thermal conductivity of the cup material?

(c) If the cup were made of a metal with a thermal diffusivity of 2×10^{-5} m^2/s, how long could you hold it?

Answer. (a) The diffusivity $a \approx 5 \times 10^{-7}$ m^2/s.

(b) The conductivity ≈ 1 W/m.K

(c) A tenth of a second.

Exercise E12.12 A flat heat shield is designed to protect against sudden temperature surges. Its maximum allowable thickness is 10 mm. What value of thermal diffusivity is needed to ensure that, when heat is applied to one of its surfaces, the other does not start to change significantly in temperature for at least 5 minutes? Use the $\lambda - a$ chart of Figure 12.5 to identify material classes that you might use for the heat shield

Answer. The distance x moved by a thermal front in time t in a material of thermal diffusivity a is of the order of $x \approx \sqrt{at}$. For a thickness x=10mm and time $t > 300\,s$, $a < 3.3 \times 10^{-7}$ m^2/s. According to Figure 12.5 almost all polymers have a thermal diffusivity that meets this constraint, but few materials of other classes meet it.

Exercise E12.13 In the development of a new quenching facility for steel components, scale model tests were conducted on dummy samples containing a thermocouple located in the centre of the sample. The geometry of the samples was geometrically similar to the real components, but at one third scale, and the quenching process was identical. The thermocouple readout showed that on quenching a sample from a furnace into cold water, the temperature fell by 50% of the temperature interval in a time of 15 seconds. How long do you estimate it would take to cool by the same amount in the real component? In a second trial, full-sized components were tested using aluminium dummy samples, with a suitably reduced furnace temperature. What time would you expect to find is required to cool the centre to 50% of the new temperature interval? The thermal diffusivities of the steel and aluminium are 9×10^{-6} m^2/s and 7×10^{-5} m^2/s respectively.

Answer. Whatever the exact geometry of the component, the cooling history scales following the relationship between heat flow distance, and diffusivity and time: $x \approx \sqrt{a t}$. Decreasing the length-scale by a factor of 1/3 is therefore expected to decrease the time by a factor of 1/9, so for the real component the cooling time is approximately $9 \times 15 = 135$ seconds.
By changing to an aluminium dummy, at the same component size, the time scale will vary inversely with the diffusivity. Hence the time will be $(9\times10^{-6} / 7\times10^{-5}) \times 135 =$ 17 seconds.

Exercise E12.14 A material is needed for a small, super-efficient pressurized heat exchanger. The text explained that the index for this application is $M = \lambda \sigma_y$. Plot contours of this index onto a copy of the $\lambda - \sigma_y$ chart of Figure 12.6 and hence find the two class of materials that maximizes M .

Answer. Copper alloys and tungsten alloys.

Exploring design with CES

(Use Level 2, Materials, for all selections)

Exercise E12.15 Use the 'Search' facility of CES to find materials for

(a) thermal insulation

(b) heat exchangers

Answer. (a) Thermal insulation:

- Alumina
- Rigid Polymer Foam (HD)
- Rigid Polymer Foam (LD)
- Rigid Polymer Foam (MD)
- Cork
- Ionomer (I)
- Polyetheretherketone (PEEK)
- Polypropylene (PP)
- Polystyrene (PS)
- Polyurethane (tpPUR)
- Zirconia
- Ceramic foam

(b) Heat exchangers:

- Copper alloys
- Titanium alloys
- Bronze
- Brass
- Copper
- Commercially pure titanium
- Titanium alloys
- Metal foam

Exercise E12.16 The analysis of storage heaters formulated the design constraints for the heat-storage material, which can be in the form of a particle-bed or a solid block with channels to pass the air to be heated. Use the selector to find the best materials. Here is a summary of the design requirements. List the top six candidates in ranked order.

Function	• *Storage heater*
Constraints	• *Maximum service temperature > 150°C* • *Non flammable (a Durability rating)*
Objective	• *Maximize specific heat/Material cost,* C_p / C_m
Free variable	• *Choice of material*

Answer. *Method:* use a Limit stage to apply all the constraints; then make a Graph stage with Price C_m on the x-axis and Specific heat C_p on the y-axis. A line of slope +1 corresponds to the condition C_p / C_m = constant.

Results: Concrete, cement, sandstone, marble, limestone, brick

Exercise E12.17 The requirements for a material for an automobile radiator, described in the text, are summarized in the table. Use CES to find appropriate materials to make them. List the top four candidates in ranked order.

Function	• *Automobile heat exchanger*
Constraints	• *Elongation > 10%* • *Maximum use temperature > 180°C* • *Price < \$5/kg* • *Durability in fresh water = very good*
Objective	• *Maximize Thermal conductivity × Yield strength,* $\lambda \sigma_y$
Free variables	• *Wall thickness* • *Choice of material*

Answer. *Method:* use a Limit stage to apply all the constraints; then make a Graph stage with Thermal conductivity λ on one axis and Yield strength σ_y on the other. A line of slope –1 corresponds to the condition $\lambda \sigma_y$ = constant.

Results: Age hardened wrought aluminium alloys, brass, copper, non-age hardened aluminium alloys.

Exercise E12.18 A structural material is sought for a low-temperature device for use at –20 °C that requires high tensile strength σ_{ts} but low thermal conductivity λ. For reasons of damage tolerance the fracture toughness K_{1c} must be greater than 15 MPa.m$^{1/2}$. Apply the constraint on K_{1c} using a Limit stage, then make a chart with σ_{ts} on the x-axis and λ on the y-axis and observe which materials best meet the requirements.

Answer.

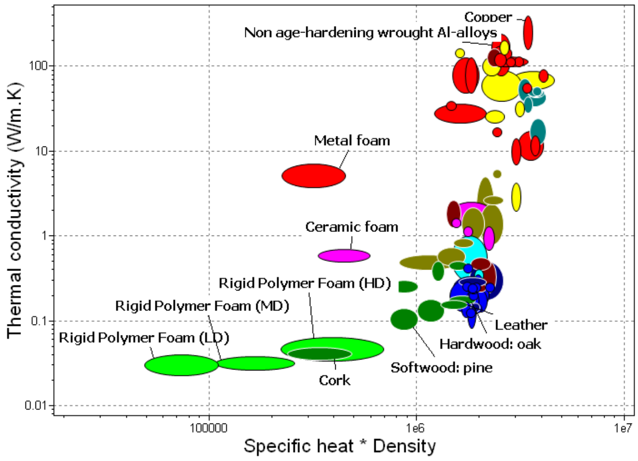
- CFRP, epoxy matrix (isotropic)
- GFRP, epoxy matrix (isotropic)

Exercise E12.19 Interior wall insulation should insulate well, meaning low thermal conductivity, λ, but require as little heat as possible to warm up to the desired room temperature when the central heating system is turned on – that means low specific heat. The thickness of the insulation is almost always limited by the cavity space between the inner and outer wall, so we need materials that it is the specific heat per unit volume, not per unit mass that is important here. To be viable the material must have enough stiffness and strength to support its own weight and be easy to install – take that to mean a modulus E > 0.01 GPa and a strength σ_y > 0.2 MPa. Make a selection based on this information.

List the materials you find that best meet the design requirements.

Answer.

- Cork
- Rigid Polymer Foam (HD)
- Rigid Polymer Foam (LD)
- Rigid Polymer Foam (MD)



Exercise E12.20 Here is the gist of an e-mail one of the authors received as this chapter was being written. 'We manufacture wood stoves and fire places that are distributed all over Europe. We want to select the best materials for our various products. The important characteristics for us are: specific heat, thermal conductivity, density…and price, of course. What can your CES software suggest?'

Form your judgement about why these properties matter to them. Rank them, deciding which you would see as constraints and which as the objective. Consider whether there are perhaps other properties they have neglected. Then – given your starting assumptions of just how these stoves are used – use CES to make a selection. Justify your choice.

There is no "right" answer to this question – it depends on the assumptions you make. The essence is in the last sentence of the last paragraph: justify your choice.

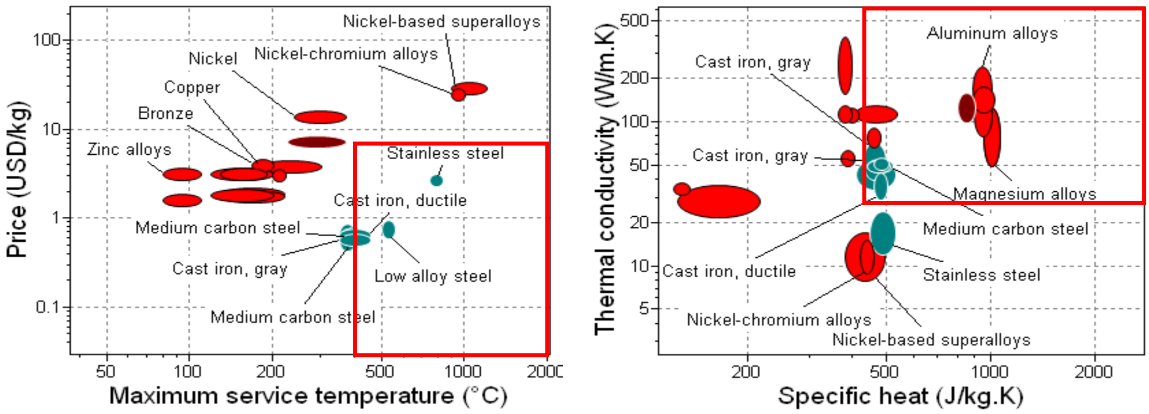
Example of an answer. First, a quick think about the constraints: what's missing? The most obvious is a lower limit on Maximum service temperature: if the material of the stove cannot tolerate the temperature at which it will be used, it is worthless. Second, the material will have to be shaped. Most wood-burning stoves have quite an intricate shape requiring casting, so we should experiment with the effect of a constraints requiring that the material can be cast. The e-mail gives no numeric limits on properties (needed for a Limit stage) so it is best to work with Graph stages.

(1) The constraint on the ability to shape by casting seems likely to be important: it is applied with a Tree stage, selecting Casting from the ProcessUniverse.

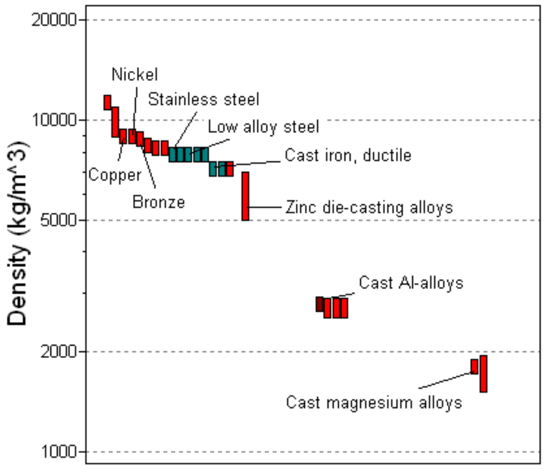
(2) I made the assumption that low material Price and adequate Maximum service temperature were important. The first figure shows these two properties with a red box surrounding those with high Maximum service temperature and low Price.

(3) It seems probable that the makers want high specific heat so that the stove stores heat and high thermal conductivity to spread it. The two are plotted in the second figure. Stainless steels and nickel alloys do not do well; cast irons are better. Aluminum alloys are best by these criteria, but their service temperature looks like a problem.

(4) Finally, density. It is plotted in the third figure. The lightest choices are aluminum and magnesium alloys. Cast irons and steels meet all the other requirements well, but are relatively dense.



Property charts showing price, maximum service temperature, thermal conductivity, and specific heat. A Tree stage selecting materials that can be cast has already been applied. The red boxes suggest where selection criteria might be placed



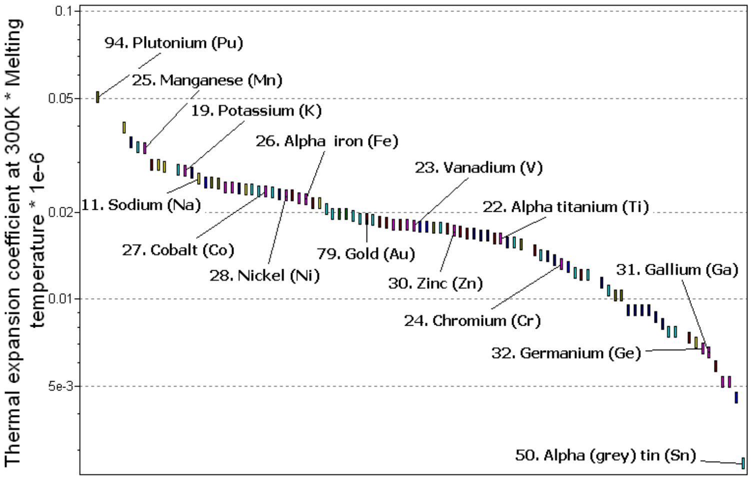
The densities of the materials that can be cast.

Conclusions. This brief survey suggests that cast irons or cast steel are the most straightforward choice if casting is the preferred shaping method. The three figures provide a starting point for more detailed discussions with the maker, establishing answers to the unanswered questions, and eliciting a more precise statement of the relative priorities the maker attaches to each property. The exercise can be repeated quickly with the constraint on casting removed and replaces by one on shaping by other methods, or none at all. Taken together this forms the starting point for an effective consultation.

Exploring the science with CES Elements

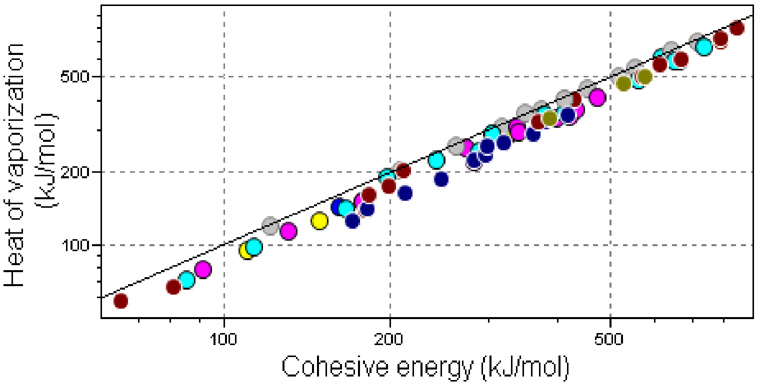
Exercise E12.21 The text says that materials expand about 2% between 0K and their melting point. Use CES Elements to explore the truth of this by making a bar-chart of the expansion at the melting point, $\alpha T_m / 10^6$ (the 10^6 is to correct for the units of α used in the database).

Answer.



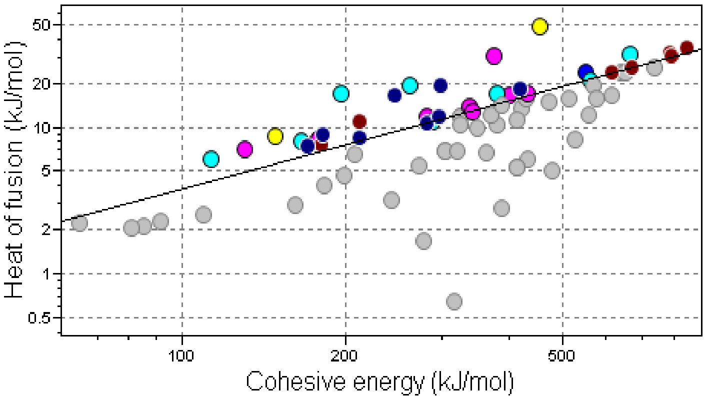
Exercise E12.22 When a solid vaporizes, the bonds between its atoms are broken. You might then expect that latent heat of vaporization, L_v, should be nearly the same as the cohesive energy H_c, since it is basic measure of the strength of the bonding. Plot one against the other, using CES Elements. How close are they?

Answer. The two quantities are almost identical.



Exercise E12.23 When a solid melts, some of the bonds between its atoms are broken but not all – liquids still have a bulk modulus, for example. You might then expect that the latent heat of melting, L_m, should be less than the cohesive energy H_c, since it is basic measure of the strength of the bonding. Plot one against the other, using CES Elements (L_m is called the Heat of fusion in the database). By what factor is L_m less than H_c? What does this tell you about cohesion in the liquid?

Answer. The latent heat of melting is about 1/25th of the cohesive energy. Bonding in the liquid is still quite strong compared to the solid.

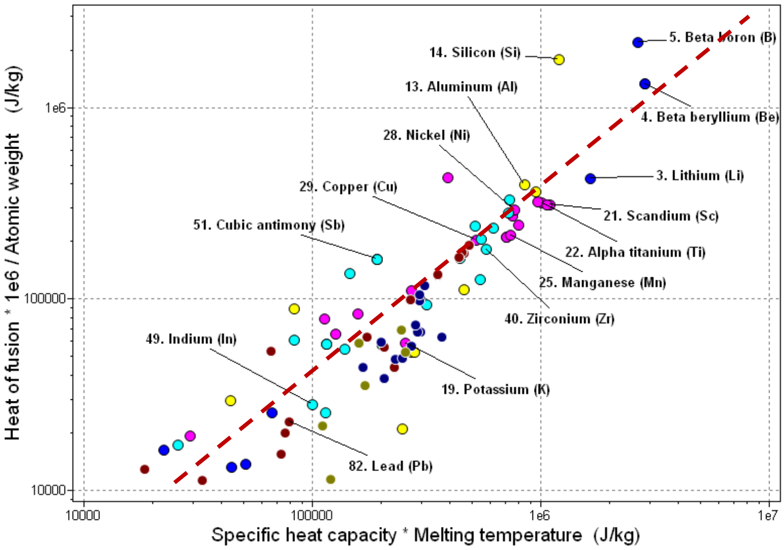


Exercise E12.24 The latent heat of melting (Heat of fusion), L_m, of a material is said to be about equal to the heat required to heat it from absolute zero to its melting point, $C_p T_m$, where C_p is the specific heat and T_m is the absolute melting point. Make a chart with H_m on one axis and $C_p T_m$ on the other. To make the comparison right we have to change the units of H_m in making the chart to J/kg instead of kJ/mol. To do this, multiply H_m by

$$\frac{10^6}{\text{Atomic weight in kg/kmol}}$$

using the 'Advanced' facility in CES. Is the statement true?

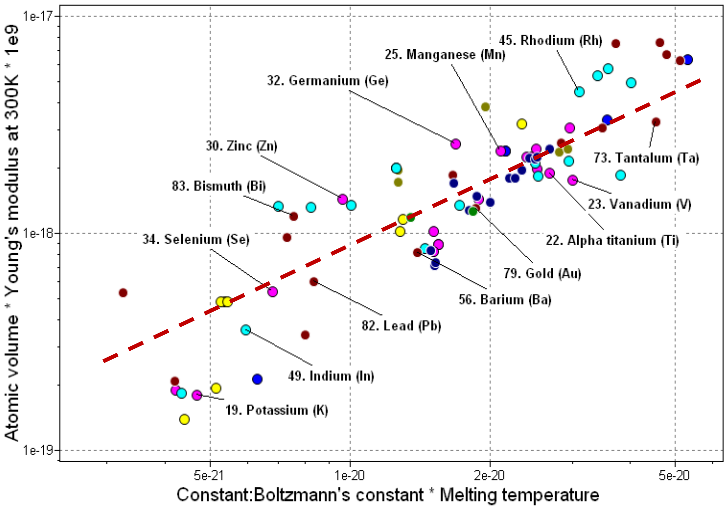
Answer. The chart shows that H_m and $C_p T_m$ are strongly correlated, but the claim that they are equal is not accurate. A more accurate statement is that $H_m \approx 0.5\, C_p\, T_m$.



Exercise E12.25 The claim was made in the text that the modulus E is roughly proportional to the absolute melting point T_m. If you use CES Elements to explore this correlation you will find that it is not, in fact, very good (try it). That is because T_m and E are measured in different units and, from a physical point of view, the comparison is meaningless. To make a proper comparison, we use instead kT_m and $E\Omega$ where k is Boltzmann's constant (1.38×10^{-23} J/K) and Ω m^3/atom is the atomic volume. These two quantities are both energies, the first proportional to the thermal energy per atom at the melting point and the second proportional to the work to elastically stretch an atomic bond. It makes better sense, from a physical standpoint, to compare these.

Make a chart for the elements with kT_m on the x-axis and $E\Omega$ on the y-axis to explore how good this correlation is. Correlations like these (if good) that apply right across the periodic table provide powerful tools for checking data, and for predicting one property (say, E) if the other (here T_m) is known. Formulate an equation relating the two energies that could be used for these purposes.

Answer. The figure shows the two quantities (E has been multiplied by 10^9, converting it to N/m^2 to make the units consistent). There is clearly a correlation between the two quantities. The line has slope 1. Its equation is: $E\,\Omega = 90\,k\,T_m$.

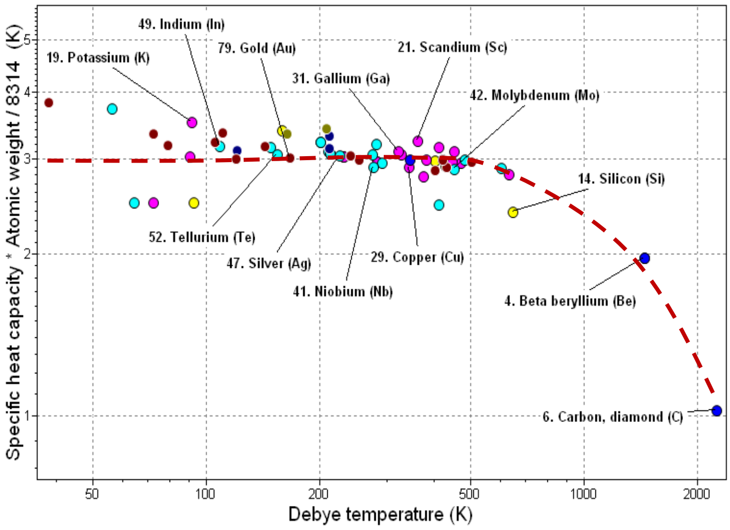


Exercise E12.26 The specific heat, once above the Debye temperature, is predicted to be 3R where R in units of kJ/per kmol.K. Make a plot for the elements with Debye temperature on the x-axis and specific heat in these units on the y-axis to explore this. You need to insert a conversion factor because of the units. Here it is, expressed in the units contained in the database:

Specific heat in kJ/kmol.K = specific heat in J/kg.K × Atomic weight in kg/kmol /1000.

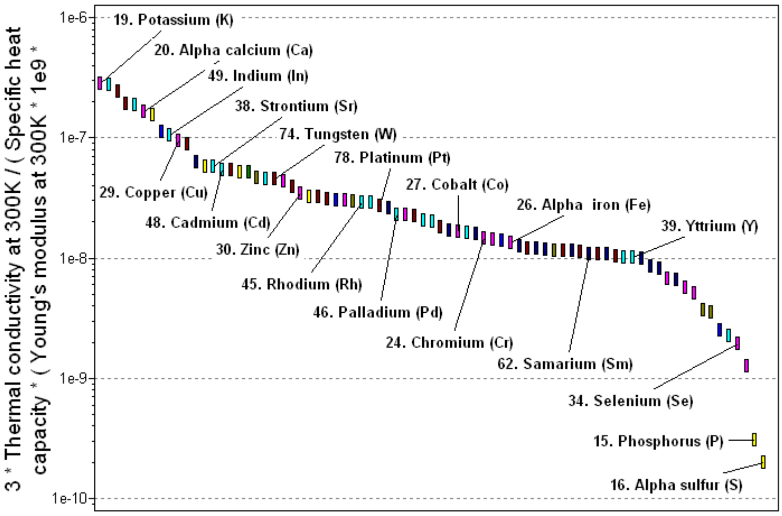
Form this quantity dividing the result by R = 8.314 kJ/kmol.K, and plot it against the Debye temperature. The result should be 3 except for materials with high Debye temperatures. Is it? Fit a curve by eye to the data. At roughly what temperature does the drop-off first begin?

Answer. The figure shows that the specific heat, in kJ/kmol.K, is close to 3R for elements with a Debye temperature below about 400 K. Those with larger Debye temperatures the lower specific heats because some of the vibration modes are not excited at room temperature (the temperature at which the specific heat is measured).



Exercise E12.27 Explore the mean free path of phonons ℓ_m in the elements using equation (12.10) of the text. Inverting it gives $\ell_m = 3\dfrac{\lambda}{\rho C_p c_o}$ in which the speed of sound $c_o = \sqrt{E/\rho}$. Use the 'Advanced' facility when defining the axes in CES to make a bar chart of ℓ_m for the elements. Which materials have the longest values? Which have the shortest?

Answer. The figure shows ℓ_m, in meters. (Young's modulus has been multiplied by 10^9, converting it to N/m^2, to make the units consistent). The shortest mean free path is that in sulphur – it is only slightly larger than the atom size. The longest is potassium – it is about 500 atom diameters.



Materials: engineering, science, properties, and design
3e Solution manual

Chapter 13: Exercises with worked solutions

Exercise E13.1 A high temperature alloy undergoes power-law creep when the applied stress is 70 MPa and the temperature is 1000°C. The stress exponent n is 5 and the activation energy Q is 300 kJ/mol. Recent modification to the manufacturing process has improved the creep resistance. The stress can be raised to 80 MPa without changing the creep rate at 1000°C. An aero-engine manufacturer is more interested in raising the operating temperature of their engine than increasing the working stresses. Estimate by how much the temperature could be raised by adopting the new manufacturing process.

Answer. From the power-law creep equation (13.5), $\dot{\varepsilon} = C'\sigma^n \exp\left(-\frac{Q_c}{RT}\right)$, retaining the same strain-rate means that: $\sigma_1^n \exp\left(-\frac{Q}{RT_1}\right) = \sigma_2^n \exp\left(-\frac{Q}{RT_2}\right)$.

Hence: $(80)^5 \exp\left(-\frac{300\times 10^3}{8.314\times(1000+273)}\right) = (70)^5 \exp\left(-\frac{300\times 10^3}{8.314\, T_2}\right)$

So the temperature can be raised to $T_2 = 1304$ K = 1031°C.

Exercise E13.2 Pipework with a radius of 20 mm and a wall thickness of 4 mm made of 2 ¼ Cr Mo steel contains a hot fluid under pressure. The pressure is 10 MPa at a temperature of 600°C. The table lists the creep constants for this steel. Calculate the creep rate of the pipe wall, assuming steady-state power-law creep.

Material	Reference strain-rate $\dot{\varepsilon}_o$ (1/s)	Reference stress σ_o (MPa)	Creep exponent n	Activation energy Q_c (kJ/mol)
2 ¼ Cr Mo steel	4.23×10^{10}	100	7.5	280

Answer. The stress in the wall $\sigma = pR/t = 10\times20/4 = 50$ MPa.

From equation (13.6), $\dot{\varepsilon} = \dot{\varepsilon}_o \left(\frac{\sigma}{\sigma_o}\right)^n \exp\left(-\frac{Q_c}{RT}\right)$, the creep rate is:

$$\dot{\varepsilon} = 4.23\times 10^{10} \times \left(\frac{50}{100}\right)^{7.5} \exp\left(-\frac{280\times 10^3}{8.314\times(600+273)}\right) = 4.12 \times 10^{-9}/\text{sec}$$

Exercise E13.3 There is concern that the pipework describe in the previous exercise might rupture in less than the design life of 1 year. If the Monkman–Grant constant for 2 ¼ Cr Mo steel is 0.06, how long will it last before it ruptures?

Answer. The rupture time is: $t_f = 0.06/4.12\times10^{-9} = 14.6\times10^6$ s = 4045 hours (≈ 169 days). So the concern is justified.

Exercise E13.4 If the creep rate of a component made of 2 ¼ Cr Mo steel must not exceed 10^{-8} / sec at 500°C, what is the greatest stress that it can safely carry? Use the data in Exercise 13.2 to find out.

Answer. $10^{-8} = 4.23\times 10^{10} \times \left(\frac{\sigma}{100}\right)^{7.5} \exp\left(-\frac{280\times 10^3}{8.314\times(500+273)}\right)$, hence the stress $\sigma = 109$ MPa.

Exercise E13.5 The designers of a chemical plant are concerned about creep failure of a critical alloy tie bar. They have carried out creep tests on specimens of the alloy under the nominal service conditions of a stress σ of 25 MPa at 620 ºC, and found a steady-state creep rate $\dot{\varepsilon}$ of 3.1×10^{-12} s^{-1}. In service it is expected that for 30% of the running time the stress and temperature may increase to 30 MPa and 650 ºC respectively, while for the remaining time the stress and temperature will be at the nominal service values. Calculate the expected *average* creep rate in service.

It may be assumed that the alloy creeps according to the equation:

$$\dot{\varepsilon} = A\,\sigma^n \exp\left(-\frac{Q}{RT}\right)$$

where A and Q are constants, R is the universal gas constant and T is the absolute temperature. For this alloy, $Q = 160$ kJ mo1^{-1} and $n = 5$.

Answer. First scale the strain-rate in stress, keeping the temperature fixed:
At 25 MPa and 620 ºC, $\dot{\varepsilon} = 3.1 \times 10^{-12}\ \text{s}^{-1}$

At 30 MPa and 620 ºC, $\dot{\varepsilon} = \left(\frac{30}{25}\right)^5 \times 3.1 \times 10^{-12} = 7.714 \times 10^{-12}\ \text{s}^{-1}$

Then scale the strain-rate with temperature, keeping the stress fixed:

$$\dot{\varepsilon} = A\sigma^5 \exp(-Q/RT)$$

$$\ln \dot{\varepsilon}_2 = \frac{Q}{R}\left\{\frac{1}{T_1} - \frac{1}{T_2}\right\} + \ln \dot{\varepsilon}_1$$

$$\ln \dot{\varepsilon}_2 = \frac{160{,}000}{8.314}\left\{\frac{1}{893} - \frac{1}{923}\right\} + \ln(7.714 \times 10^{-12}) = -24.89$$

Hence at 30 MPa and 650 ºC, $\dot{\varepsilon} = 15.5 \times 10^{-12}\ \text{s}^{-1}$

Finally, find the average strain-rate, multiplying each strain-rate by the % of the time for which the different conditions apply:

$$\dot{\varepsilon}_{average} = 3.1 \times 10^{-12} \times \frac{70}{100} + 15.5 \times 10^{-12} \times \frac{30}{100} = 6.8 \times 10^{-12}\ \text{s}^{-1}$$

Exercise E13.6 (a) The steady-state creep strain-rate at constant temperature $\dot{\varepsilon}_{ss}$ depends on the applied stress according to

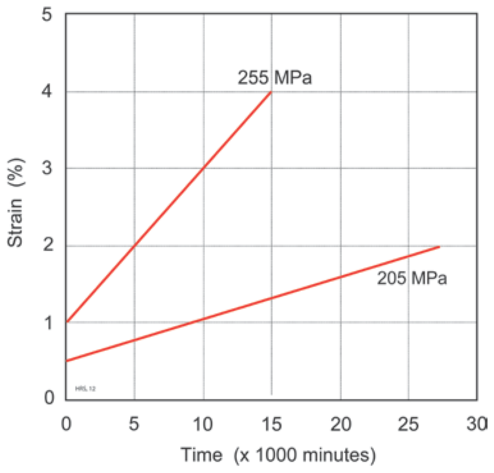
$$\dot{\varepsilon}_{ss} = B\,\sigma^n$$

where B and n are constants. State how B varies with temperature and explain why this is the case.

(b) The Figure shows the steady-state creep response of a 12% Cr steel at 500ºC. Determine B at this temperature, and the creep exponent n, for this material.

(c) A 12% Cr steel turbine blade of length 10cm runs with an initial clearance of 100μm between its tip and the housing in a generator operating at 515ºC. Determine the lifetime of the blade if it runs continuously, assuming a uniform tensile stress of 60 MPa is applied along its axis. (The creep activation energy Q for 12% Cr steel is 103 kJ/mol).

(d) Explain why in practice a more sophisticated calculation is required to determine the change in length of a rotating turbine blade.



Answer. (a) The constant B ∝ exp (-Q/RT) where Q is an activation energy for creep. The underlying mechanisms of creep are diffusion-controlled, and hence the strain-rate scales according to the exponential temperature-dependence of the diffusion process (it is 'thermally activated').

(b) From the graph:

Stress (MPa)	Strain, ε	Time (minutes)	Strain-rate (s^{-1})
255	0.01	0	3.33×10^{-8}
	0.04	15,000	
205	0.005	0	9.09×10^{-9}
	0.02	27,500	

At constant temperature, $\frac{\dot{\varepsilon}_1}{\dot{\varepsilon}_2} = \left(\frac{\sigma_1}{\sigma_2}\right)^n$, hence $\frac{3.33\times10^{-8}}{9.09\times10^{-9}} = \left(\frac{255}{205}\right)^n$ giving $n = 5.95$ (say $n \approx 6$).

Substitute data at one stress level to find B:

$$3.33\times10^{-8} = B\,(255)^6 \quad \text{giving} \quad B = 1.2 \times 10^{-22} \quad (\text{MPa}^{-6}.\text{s}^{-1})$$

(c) Now let $B = A \exp\left(-Q/RT\right)$

So at $T = 500°\text{C}$, $1.2\times10^{-22} = A \exp\left[-103{,}000/(8.314\times773)\right]$ so $A = 1.10 \times 10^{-15}$

Now at $T = 515°\text{C}$ and a stress $\sigma = 60$ MPa:

$$\dot{\varepsilon} = \left\{1.1\times10^{-15} \times \exp\left[-103{,}000/(8.314\times788)\right]\right\} \times 60^6 = 7.63 \times 10^{-12}\ \text{s}^{-1}$$

Design life given by strain to close up the clearance, divided by the strain-rate:

$$t_{life} = \frac{100\times10^{-6}/10\times10^{-2}}{7.63\times10^{-12}} = 1.31 \times 10^8\ \text{s} \approx 36{,}400 \text{ hours} \quad (\approx 4.1 \text{ years})$$

(d) A more sophisticated calculation would be needed in practice to account for the variation in stress along the blade (due to centripetal acceleration varying with radius), giving a gradient in strain-rate along the blade.

Exercise E13.7 Use the property chart in Figure 13.8 to identify materials that are suitable for carrying a tensile stress of 100 MPa at 950°C, with a creep strain-rate below 10^{-6} s^{-1}. Use the contours of specific creep strength at this temperature, $\sigma_{950°C}/\rho$, to identify the lightest material. What other materials might be worth considering for operation at this temperature, according to Figure 13.7?

Answer. For the materials with strengths above 100 MPa at 950°C, the materials with the highest values of $\sigma_{950°C}/\rho$ are:

- Carbon-carbon composite
- Silicon nitride, Si_3N_4
- Silicon carbide, SiC

Other possible materials with operating temperatures above this are stainless steels and tungsten alloys.

Exercise E13.8 The self-diffusion constants for aluminum are $D_o = 1.7 \times 10^{-4}$ m²/s and $Q_d = 142$ kJ/mol. What is the diffusion rate in aluminum at 400°C?

Answer. The diffusion rate is:

$$D = D_o \exp(-Q/RT) = 1.7\times10^{-4} \exp\left[-142000/(8.314\times673)\right] = 1.6 \times 10^{-15}\ \text{m}^2/\text{s}.$$

Exercise E13.9 Use your knowledge of diffusion to account for the following observations:

(a) The rate of diffusion of oxygen from the atmosphere into an oxide film is strongly dependent on the temperature and the concentration of oxygen in the atmosphere.

(b) Carbon diffuses far more rapidly than chromium through iron.

(c) Diffusion is more rapid in polycrystalline silver with a small grain size than in coarse-grained silver.

Answer. (a) Diffusion is a thermally activated process, i.e. the chance of oxygen molecules having enough energy to move increases with temperature according to Arrhenius law, exp $(-Q/RT)$. The concentration gradient drives the diffusion process – the greater the gradient, the higher the probability that diffusive jumps move down the gradient as opposed to up the gradient.

(b) The atomic masses are Fe=56, Cr=52, C=12, and these give an indication of the relative atomic sizes of these elements. Carbon forms an *interstitial* solid solution with iron – diffusion is by movement of the small carbon atoms between the relatively large interstices in the iron crystal structure. The energy barrier for C atoms to jump between sites is moderate, and there are always sites available for C atoms to move to. Hence C will diffuse relatively rapidly. In contrast chromium has a similar atomic radius, so it forms a *substitutional* solid solution in iron. Diffusion occurs by a chromium atom swapping position with a neighbouring vacancy. While the energy barrier to movement is again moderate, the probability of a vacancy appearing next to a chromium atom is small. Hence the diffusion of chromium is slow.

(c) Diffusion on grain boundaries is generally more rapid than in the grains themselves because of the more open crystal structure. A small grain size produces a larger contribution from grain boundary diffusion than does a large grain size, and thus increases the overall diffusion coefficient of the polycrystal.

Exercise E13.10 Molten aluminium is cast into a copper mould having a wall thickness of 5 mm. The diffusion coefficient D of aluminium in copper is 2.6×10^{-17} m^2/s at 500°C and 1.6×10^{-12} m^2/s at 1000°C.

(a) Calculate the activation energy Q for the diffusion of aluminium in copper. By considering the mechanisms of diffusion, list some factors which will affect the value of Q in diffusion-controlled processes in crystalline solids.

(b) Determine the coefficient of diffusion of aluminium in copper at 750°C.

(c) A useful "rule-of-thumb" in diffusion problems states that the characteristic diffusion distance in time t is given by $\sqrt{Dt}$, where D is the diffusion coefficient for the temperature concerned. Make an order-of-magnitude estimate of the time taken for an aluminium atom to diffuse through the wall of the copper mould if the mould temperature is 750°C.

Answer. (a) Take natural logs of $D = D_o \exp(-Q/RT)$: $\ln D = \ln D_o - \dfrac{Q}{RT}$

So $\ln 2.6\times10^{-17} = \ln D_o - \dfrac{Q}{R(773)}$ and $\ln 1.6\times10^{-12} = \ln D_o - \dfrac{Q}{R(1273)}$

Eliminating D_o and solving for Q: $Q = 180$ kJ/mol.

Diffusion mechanisms involve atoms moving through the crystal structure, either between atoms in the crystal lattice or via defects. Hence Q is affected by bond strengths, relative atomic sizes, details of the crystal structure and defects including grain boundaries, vacancies and dislocations.

(b) From $\ln D = \ln D_o - Q/RT$, at 500°C and 750°C:

$\ln 2.6\times10^{-17} = \ln D_o - 180{,}000/(8.314\times773)$ and $\ln D_{750^o C} = \ln D_o - 180{,}000/(8.314\times1023)$

Now eliminating D_o and solving for D_{750^oC}: $D_{750^oC} = 2.44\times10^{-14}$ m^2s^{-1}.

(c) $t \approx x^2/D$, hence $t \approx (5\times10^{-3})^2/2.44\times10^{-14} = 1.02\times10^9$ s (≈ 12000 days)

Exercise E13.11 A steel component is nickel plated to give corrosion protection. To increase the strength of the bond between the steel and the nickel, the component is heated for 4 hours at 1000°C. If the diffusion parameters for nickel in iron are D_o = 1.9 × 10^{-4} m^2/s and Q_d = 284 kJ/mol, how far would you expect the nickel to diffuse into the steel in this time?

Answer. $D = D_o \exp(-Q_d / RT) = 1.9 \times 10^{-4} \exp[-284{,}000 / (8.314 \times 1273)]$ = 3.77 × 10^{-16} m^2/s. The approximate depth of diffusion is $x \approx \sqrt{Dt}$ = 2.3 microns.

Exercise E13.12 The diffusion coefficient at the melting point for materials is approximately constant, with the value $D = 10^{-12}$ m^2/s. What is the diffusion distance if a material is held for 12 hours at just below its melting temperature? This distance gives an idea of the maximum distance over which concentration gradients can be smoothed by diffusion.

Answer. The distance is $x \approx \sqrt{Dt} \approx \sqrt{10^{-12} \times 12 \times 3600}$ = 0.2 mm.

Exercise E13.13 Find the diffusion coefficient D for carbon in iron at 1000°C, using the data below, and compare it with the thermal diffusivity of steel at 1000°C, $a = 9 \times 10^{-6}$ m^2s^{-1}. Estimate how much faster heat "diffuses" a distance x in steel compared to the diffusion of carbon at 1000°C.

For carbon diffusion in FCC iron: D_o = 2.3 × 10^{-5} m^2s^{-1}, and the activation energy Q = 148 kJ/mol.

Answer. The characteristic diffusion distance is given by: $x \approx \sqrt{Dt}$, and the corresponding heat flow distance $x \approx \sqrt{at}$. From the data for D_o and Q: $D = D_o \exp(-Q/RT) = 1.94 \times 10^{-11}\ m^2 s^{-1}$, and in comparison $a = 9 \times 10^{-6}\ m^2 s^{-1}$. The time to diffuse a distance x is inversely proportional to D (for diffusion) or a (for heat). Hence heat diffuses ≈ 10^6 times faster than carbon through iron. Note that D is strongly temperature-dependent, whereas thermal properties vary much less with temperature.

Exercise E13.14 In the fabrication of a silicon semiconductor device, boron is diffused into silicon in two stages: a pre-deposition stage lasting 10 minutes at a temperature of 1000°C, followed by a drive-in stage for two hours at 1100°C. Compare the characteristic diffusion distances of these two stages. For the diffusion of boron in silicon, D_o = 3.7 × 10^{-6} $m^2\ s^{-1}$, and the activation energy Q = 333 kJ/mol.

Answer. The characteristic diffusion distance is given by: $x \approx \sqrt{Dt}$. The diffusion coefficient D is given by $D = D_o \exp(-Q/RT)$ where Q is the activation energy per atom and R is the universal gas constant. Substituting D_o = 3.7 × 10^{-6} $m^2\ s^{-1}$, and Q = 333 kJ/mol:

– at 1000°C (T = 1273 K): $D = 7.63 \times 10^{-20}\ m^2\ s^{-1}$

– at 1100°C (T = 1373 K): $D = 7.57 \times 10^{-19}\ m^2\ s^{-1}$.

For t = 600s at 1000°C, $x \approx \sqrt{Dt}$ = 6.8 × 10^{-9} m.

For t = 7200s at 1100°C, $x \approx \sqrt{Dt}$ = 7.38 × 10^{-8} m, i.e. over 10 times further.

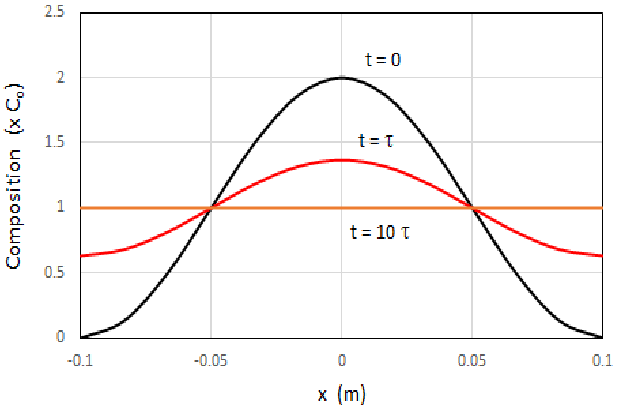
Exercise E13.15 A thick steel plate of thickness $2w = 0.2$m is continuously cast into a long sheet. The casting process gives a non-uniform concentration of a key substitutional alloying element in the steel (due to the phenomenon of segregation, discussed in Guided Learning Unit 2). The initial distribution of solute is sinusoidal with the maximum concentration on the centre-line, falling to zero at the surface. A homogenisation treatment is proposed to try and redistribute the solute to a more uniform level. Solution of the differential equation for transient diffusion predicts that the concentration profile has the following form: $C(x,t) = C_o + C_o \left(\cos\frac{\pi x}{w}\right)\exp(-t/\tau)$, where $\tau = w^2/(\pi^2 D)$, where D is the diffusion coefficient for the solute element in steel.

(a) Sketch the solute distribution for times t = 0, τ and 10τ.

(b) An attempt is made to homogenise the casting by holding at a high temperature for 12 hours. At the treatment temperature, the relevant diffusion coefficient $D = 4\times10^{-15}$ m^{2}s^{-1}. Will this treatment make any significant difference to the solute distribution?

(c) The casting process is modified to prevent the solute distribution developing across the whole plate. A micro-probe analysis then identifies a sinusoidal variation of amplitude $\pm 0.2C_o$ across the grains in the casting, about a mean of C_o. If the grain size is 50μm (i.e. the length scale of the concentration gradient $w = 25$μm), what is the maximum % deviation from the average concentration within the grain after the 12 hour heat treatment?

Answer. (a)



(b) For $D = 4\times10^{-15}$ m^{2}s^{-1}, and a diffusion length scale of $w = 0.1$m:

$$\tau = w^2/(\pi^2 D) = (0.1)^2 / (\pi^2 \times 4\times10^{-15}) = 2.5\times10^{11} \text{ s.}$$

Hence concentration on the centre-line ($x = 0$) is then:

$$C = C_o + C_o \exp(-12\times60\times60/2.5\times10^{11}) = 2C_o.$$

Diffusion is so slow for this solute, that a 12 hour treatment makes a negligible difference to the concentration distribution.

(c) For a length scale of half the grain size, $w = 25\times10^{-6}$ m, hence:

$$\tau = w^2/(\pi^2 D) = (25\times10^{-6})^2 / (\pi^2 \times 4\times10^{-15}) = 15831 \text{ s.}$$

Hence the concentration on the centre-line of the grain ($x = 0$), noting that the exponential is now multiplied by the concentration amplitude $0.2C_o$, is:

$$C = C_o + 0.2\,C_o \exp(-12\times60\times60/15831) = 1.013\,C_o.$$

The maximum deviation from the mean has been reduced from 20% to 1.3%. For practical timescales, homogenisation is only viable on the length scale of the grains in a casting.

Exercise E13.16 A carbon steel component containing C_o = 0.2 weight % C is to be carburised at a temperature of 1000°C, at which the maximum solubility of carbon is 1.6 weight % – this is the (constant) surface concentration C_s throughout the treatment. The concentration as a function of time t and depth below the surface x is given by

$$C(x,t) = C_o + (C_s - C_o)\left\{1 - erf\left(\frac{x}{2\sqrt{Dt}}\right)\right\}$$

The diffusion coefficient D for carbon diffusion in iron at 1000°C is 1.94×10^{-11} m^2s^{-1}, and values for the error function are given in the Table.

X	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$erf(X)$	0	0.112	0.223	0.329	0.428	0.520	0.604	0.678	0.742

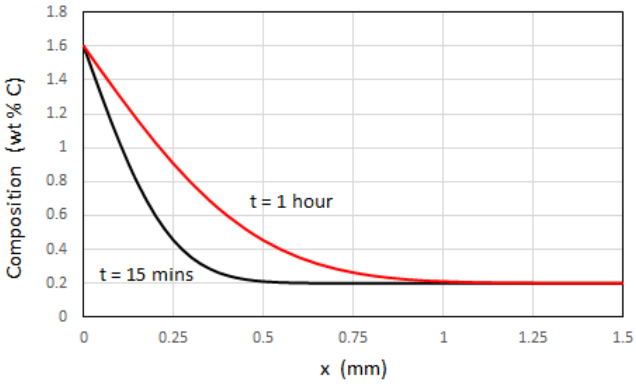
X	0.9	1.0	1.1	1.2	1.3	1.4	1.5	∞
$erf(X)$	0.797	0.843	0.880	0.910	0.934	0.952	0.966	1.0

Sketch the variation of carbon concentration C against distance x below the surface, for treatment times of 15 minutes and 1 hour. By what factor does the diffusion depth increase between these two treatment times? Find the time required for carburisation to a depth of 0.3mm, if the target concentration at this depth is C = 0.5 wt % carbon.

Answer.

The diffusion distance doubles when the time is increased by a factor of 4:

$$x \propto \sqrt{Dt}$$



For C = 0.5, and x = 0.3mm:

$$0.5 = 0.2 + 1.4 \times \left\{1 - erf\left(\frac{0.3\times10^{-3}}{2\sqrt{1.94\times10^{-11}\,t}}\right)\right\}$$

$$erf\left(\frac{0.3\times10^{-3}}{2\sqrt{1.94\times10^{-11}\,t}}\right) = 0.79$$

Interpolating from the table of values for erf(X):

$$\left(\frac{0.3\times10^{-3}}{2\sqrt{1.94\times10^{-11}\,t}}\right) = 0.88, \quad \text{giving} \quad t = 1497 \text{ s}$$

So carburizing time is 25 minutes.

Exercise E13.17 (a) Explain briefly, with sketches as appropriate, the *differences* between the following pairs of mechanisms in metal deformation:

(i) room temperature yielding and power-law creep;
(ii) power-law creep and diffusional flow.

(b) Explain the following manufacturing characteristics:

(i) superplastic forming requires a fine grain size material;
(ii) creep resistant alloys are often cast as single crystals.

Answer. (a) (i) Room temperature yielding is by dislocation motion, giving permanent deformation. Under a given stress above the initial yield stress, the deformation is finite as the material work hardens and dislocation motion stops. In power-law creep, dislocation movement again controls the permanent strain, but the dislocations are continually being released from obstacles by diffusion ('climb'), leading to a time-dependent continuation of deformation at fixed stress.

(ii) Power-law creep involves dislocation motion to enable plastic strain, with diffusion releasing dislocations from obstacles such as particles. Diffusional flow occurs by the redistribution of material between grain boundaries, by diffusion within the bulk and along grain boundaries, enabling deformation in the direction of applied stress. This requires sliding and re-arrangement of the grain boundaries to accommodate the overall shape change.

(b) (i) Superplasticity is a sheet-forming process whereby very large tensile strains are achieved without necking failure. This is done by working in the creep regime, but to obtain sufficiently fast processing times the creep rate is accelerated by using a very fine grain size. This gives rapid diffusion paths, and enables efficient grain switching to accommodate shape change.

(ii) Creep resistant alloys suppress all of the mechanisms of creep. Since diffusional flow involves redistribution of material on the grain boundaries by diffusion, it is best to eliminate grain boundaries altogether and use a single crystal. Grain size does not affect power-law creep – precipitation hardening is the way to improve resistance to power-law creep.

Exercise E13.18 (a) Briefly summarise the important microstructural characteristics of a creep-resistant material, distinguishing between features that influence diffusional flow and power-law creep.

(b) Polycrystalline copper has a steady-state creep rate of 10^{-4} s^{-1} at 560°C when subjected to a given tensile stress. The activation energy Q for self-diffusion in copper is 197 kJ/mol. Calculate the steady-state creep-rate of copper at 500°C at the same stress.

Answer. (a) high melting point alloys (as creep important for $T > T_m$) (both mechanisms); large grain size (or preferably a single crystal), to suppress diffusional flow; high alloy content, to give solid solution and precipitation hardening, to resist power-law creep.

(b) Scale the strain-rate with temperature, keeping the stress fixed:

$$\dot{\varepsilon} = A\sigma^n \exp(-Q/RT)$$

$$\ln \dot{\varepsilon}_2 = \frac{Q}{R}\left\{\frac{1}{T_1} - \frac{1}{T_2}\right\} + \ln \dot{\varepsilon}_1$$

$$\ln \dot{\varepsilon}_2 = \frac{197{,}000}{8.314}\left\{\frac{1}{833} - \frac{1}{773}\right\} + \ln(1\times 10^{-4}) = -11.42$$

Hence at 500 °C, $\dot{\varepsilon} = 1.1\times 10^{-5}\ \text{s}^{-1}$

Exercise E13.19 Creep data for a stainless steel at two constant temperatures are shown in the Figure. The steel obeys the steady-state creep equation

$$\dot{\varepsilon} = A\,\sigma^n \exp(-Q/RT)$$

where R = 8.314 J/mol K.

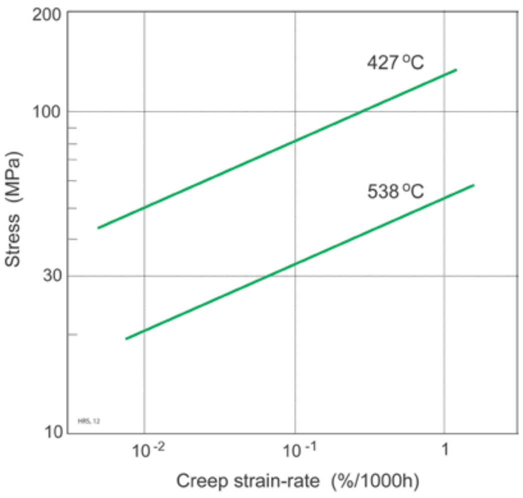
(a) A specimen of the steel of length 750mm was subjected to a tensile stress σ = 40 MPa at a temperature T = 538°C. Calculate the extension after 5000 hours.

(b) Use the creep data to evaluate the constants n and Q.

(c) A solid cylinder of the same steel was designed for operation as a support for a turbine housing under a uniform tensile load P = 6kN and a temperature T = 450°C. A routine check after 10,000 hours in service revealed that the extension of the cylinder was 50% higher than expected, due to the applied load being higher than the design specification. The extension which had occurred was half of the maximum allowable before the component would need to be replaced.

(i) Evaluate the load which had actually been applied, given that the operating temperature had been correctly been maintained.

(ii) The loading on the component could not be corrected, but a drop in operating temperature of 20°C was possible. Check whether this modification would be sufficient to ensure a further 40,000 hours of operation before replacement.



Answer. (a) Reading from the graph for σ = 40 MPa, T = 538°C:

$\dot\varepsilon$ = 0.24% / 1000 hours , so strain in 5000 hours = 1.2%

Change in length = 0.012 × 750mm = 9mm.

(b) To find n, use 2 conditions at constant T: $\dot\varepsilon = B\sigma^{\mathrm{n}}$ (where $B = A\exp[-Q/RT]$)

$$\log\sigma = \frac{1}{n}\log\dot\varepsilon - \frac{1}{n}\log B$$

So lines of constant T on graph of log σ vs. log $\dot\varepsilon$ have slope 1/n

e.g. for T = 538°C: $\dot\varepsilon = 10^{-2}$, σ ≈ 20 MPa and $\dot\varepsilon$ = 1, σ ≈ 50 MPa

$$\Delta(\log\dot\varepsilon) = 2,\ \ \Delta(\log\sigma) = 0.4:\ \text{hence}\ \ \frac{1}{n} = \frac{0.4}{2} \Rightarrow n = 5$$

To find Q, use conditions at constant $\dot\varepsilon$: $\ln\dot\varepsilon = \ln A + n\ln\sigma - \dfrac{Q}{RT}$

So $n\ln\sigma - \dfrac{Q}{RT}$ = constant, i.e. $n(\ln\sigma_1 - \ln\sigma_2) = \dfrac{Q}{R}\left(\dfrac{1}{T_1}-\dfrac{1}{T_2}\right)$

e.g. for $\dot\varepsilon = 10^{-2}$: T_1 = 538°C (811 K), σ ≈ 20 MPa and T_2 = 427°C (700 K), σ ≈ 50 MPa

$$5\times(\ln 20 - \ln 50) = \frac{Q}{8.314}\left(\frac{1}{811}-\frac{1}{700}\right)$$ giving Q = 195 kJ/mol.

(c) (i) $$\frac{\dot\varepsilon_1}{\dot\varepsilon_2} = \left(\frac{\sigma_1}{\sigma_2}\right)^n = \left(\frac{P_1}{P_2}\right)^n$$

50% higher strain in given time, so $\dfrac{\dot\varepsilon_1}{\dot\varepsilon_2} = 1.5$, hence $\left(\dfrac{P_1}{P_2}\right)^5 = 1.5$ and so P_2 = 6.51 kN.

(ii) New $\dot\varepsilon = \dot\varepsilon_3$ at temperature T_3 = 430°C (703 K).

Old $\dot\varepsilon = \dot\varepsilon_2$ at temperature T_2 = 450°C (723 K) (at higher load of 6.51 kN).

$$\frac{\dot\varepsilon_3}{\dot\varepsilon_2} = \exp -\frac{Q}{R}\left(\frac{1}{T_3}-\frac{1}{T_2}\right) = \exp -\frac{195{,}000}{8.314}\left(\frac{1}{703}-\frac{1}{723}\right) = 0.4$$

Hence the same extension will occur in 2.5 times the time that it took at the original temperature, i.e. 25,000 hours. This is less than the required 40,000 hours so the drop in temperature is not sufficient.

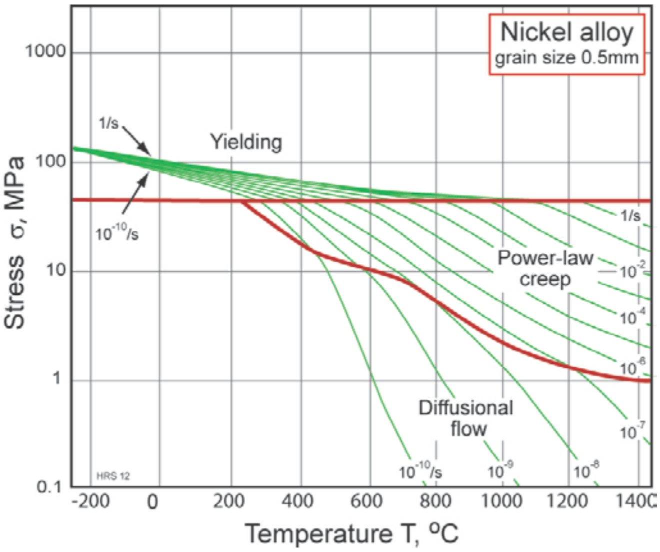
Exercise E13.20 The Figure shows a deformation-mechanism map for a nickel alloy. The contours are lines of equal strain-rate for steady-state creep in units of s^{-1}.

(a) What is the maximum allowable stress at 600°C if the maximum allowable strain rate is 10^{-10} s^{-1}? What would be the creep-rate if the stress were increased by a factor of 2, *or* the temperature raised by 100°C?

(b) The strain-rate contours on the map for both diffusional flow and power law creep are based on curve fits to the creep equation:

$$\dot{\varepsilon} = A\,\sigma^{n}\exp\left(-\frac{Q}{RT}\right)$$

Use the strain-rate contours on the map to estimate the value of the stress exponent n, for diffusional flow and for power-law creep.



Answer. (a) Measuring from the map, for a strain rate of 10^{-10} s^{-1} at a temperature of 600 °C, $\log_{10}$(stress/MPa) ≈ 0.1 [i.e. about 1/10th of the linear distance from 1 to 10 on the $\log_{10}$(stress) scale]. Allowable stress = $10^{0.1}$ ≈ 1.25 MPa.

It is most accurate to interpolate perpendicular to the contours of constant strain-rate.

- for temperature = 600 °C and stress = 2.5 MPa, this falls ≈ 1/3rd of the distance between the 10^{-10} to the 10^{-9} s^{-1} contours: i.e. $\log_{10}$(strain-rate) = (–10 + 0.33); and hence strain-rate ≈ 2.1×10^{-10} s^{-1} ≈ $2{\times}10^{-10}$ s^{-1}
- for temperature = 700 °C and stress = 2.3 MPa, this falls ≈ halfway between the 10^{-10} and 10^{-9} s^{-1} contours. i.e. $\log_{10}$(strain rate) = –9.5; and hence strain-rate ≈ 3.1×10^{-10} s^{-1} ≈ 3×10^{-10} s^{-1}

(b) Take logs of the creep equation

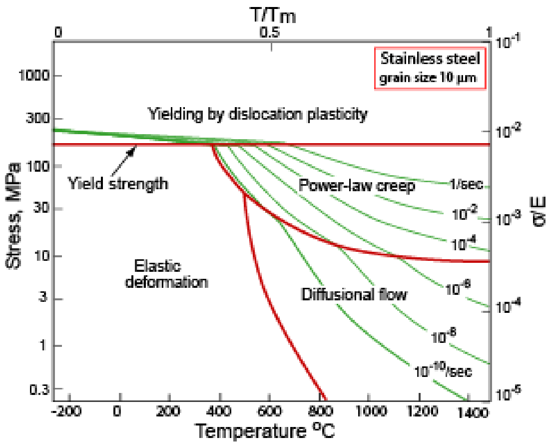
$$\log_{10}(\dot{\varepsilon}_{ss}) = \log_{10}(A) + n\log_{10}(\sigma) - \frac{1}{\ln(10)}\frac{Q}{RT}$$

Now find the change in $\log_{10}$(stress) and corresponding change in $\log_{10}$(strain-rate) over a convenient interval, keeping the temperature constant. Then:

$$\frac{\Delta\left[\log_{10}(\dot{\varepsilon}_{ss})\right]}{\Delta\left[\log_{10}(\sigma)\right]} = n$$

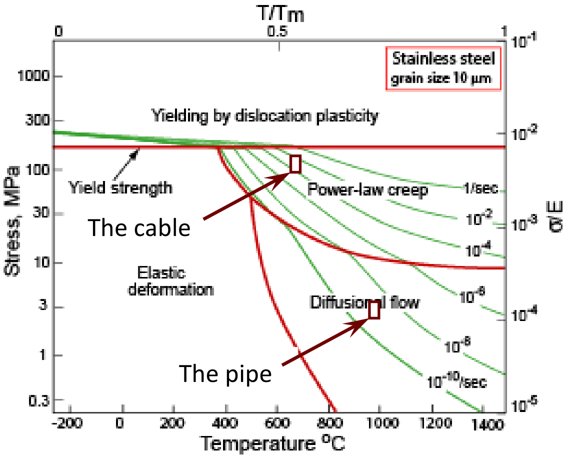
It is probably easiest to take a decade change in stress (a fixed vertical length measured between the horizontal gridlines).

- for diffusional flow, e.g. at 1000°C, a decade in stress spans one decade in strain-rate (e.g. from 10^{-9} to 10^{-8} s^{-1}), hence $n \approx 1$.
- for power-law creep, e.g. at 1000°C, a decade in stress spans ≈ 4.5 decades in strain-rate (e.g. from 10^{-7} to ≈ $10^{-2.5}$ s^{-1}), hence $n \approx 4.5$.



Exercise E13.21 A stainless steel suspension cable in a furnace is subjected to a stress of 100 MPa at 700°C. Its creep rate is found to be unacceptably high. By what mechanism is creeping? What action would you suggest to tackle the problem? The Figure shows the deformation mechanism map for the material.

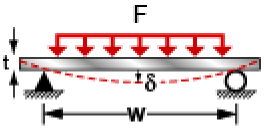
Answer. The temperature and stress on the cable cause it to deform by power-law creep. Materials that best resist power-law creep are those with high melting points, since diffusion and thus creep-rates scale as T/T_m, and a microstructure that maximizes obstruction to dislocation motion through alloying to give a solid solution and precipitate particles.



Exercise E13.22 The wall of a pipe of the same stainless steel as that of the previous exercise carries a stress of 3 MPa at the very high temperature of 1000°C. In this application it, too, creeps at a rate that is unacceptably high. By what mechanism is creeping? What action would you suggest to tackle the problem?

Answer. *See Figure in previous Answer.* The temperature and stress on the pipe wall cause it to deform by diffusional flow. Materials that best resist diffusional flow are those with a high melting temperature and a large grain size, d, so that diffusion distances are long. Single crystals are best of all; they have no grain boundaries to act as sinks and sources for vacancies, so diffusional creep is suppressed completely.

Exercise E13.23 It is proposed to make a shelf for a hot-air drying system from Acrylic sheet. The shelf is simply supported, as in the diagram, and has a width $w = 500$ mm, a thickness $t = 8$mm and a depth $b = 200$ mm. It must carry a distributed load of 50 N at 60°C with a design life of 8000 hours (about a year) of continuous use. Use the creep modulus plotted in Figure 13.19 and the solution to the appropriate elastic problem (Chapter 5) to find out how much it will sag in that time.



Answer. The deflection is $\delta = \dfrac{F\,w^3}{C_1 E_c I}$ where $C_1 = 384/5$, $I = \dfrac{bt^3}{12}$ and E_c is the creep modulus. From Figure 13.19 the creep modulus at 60°C and a time of 8000 hours (3×10^7 seconds) is just over 0.1 GPa. Inserting the values of w, b and t gives a deflection of 8 mm.

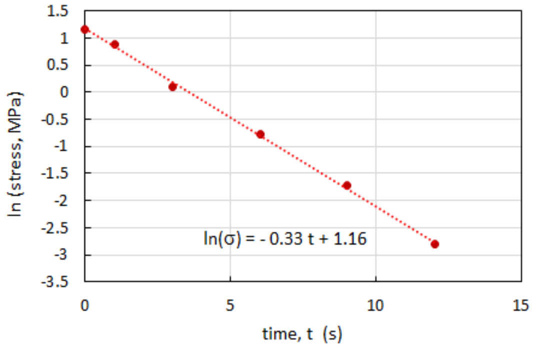
Exercise E13.24 In a series of stress relaxation experiments on PMMA conducted at 25°C, a cylindrical bar of polymer is subjected to an instantaneous step-wise compressive strain of 1%, at time *t* = 0. The stress in the bar decays with time according to the following table.

Stress (MPa)	3.2	2.4	1.1	0.46	0.18	0.06
Time (s)	0	1	3	6	9	12

The relaxation of the polymer can be represented by a Maxwell model of a spring of modulus E (for which $E = \sigma/\varepsilon$) in series with a dashpot of viscosity η, (for which $\eta = \sigma/\dot{\varepsilon}$), as in Figure 13.18. The relaxation response is given by

$$\sigma = \sigma_o \exp(-t/\tau) \quad \text{where } \tau = \eta/E \text{, and } \sigma_o \text{ is the initial stress at } t = 0.$$

Use the experimental data to estimate the values of E, η and the relaxation time $\tau = \eta/E$.



Answer. Initial strain = 0.01 with applied stress 3.2 MPa, hence E = 3.2/0.01 = 320 MPa.

Taking natural logs of the stress relaxation function: $\ln \sigma = \ln \sigma_o - t/\tau$

Hence plot ln (stress) vs. time, giving gradient of $-1/\tau = -0.33$

$\tau = 3.0$ s, hence $\eta = \tau E = 0.96$ GPa.s.

Exploring design with CES *(use Level 2 unless otherwise stated).*

Exercise E13.25 Use the Search facility in CES to find

(a) Materials for turbine blades

(b) Materials for thermal barrier coatings

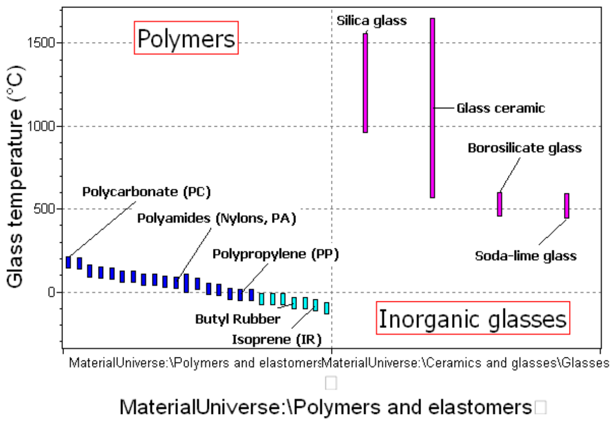
Answer.

(a) Turbine blades
- Nickel alloys
- Titanium alloys
- Nickel-based superalloys
- Zirconia
- Silicon nitride

(b) Thermal barrier coatings
- Silica glass
- Alumina
- Zirconia

Exercise E13.26 Use the CES software to make a bar-chart for the glass temperatures of polymers like that for melting point in the text. What is the range of glass temperatures for polymers?

Answer. The range is approximately –100°C to +200°C.



Exercise E13.27 The analysis of thermal barrier coatings formulated the design constraints. Use the selector to find the best material. Here is a summary of the constraints and the objective.

Function	• *Thermal barrier coating*
Constraints	• *Maximum service temperature > 1300 °C* • *Adequate strength:* $\sigma_y > 400$ *MPa*
Objective	• *Minimize thermal conductivity* λ
Free variable	• *Choice of material*

Answer. Five materials – four ceramics and one metal – meet the constraints on service temperature and strength:

- Alumina
- Boron carbide
- Silicon carbide
- Zirconia
- Tungsten alloys

A bar chart of Thermal conductivity then show that Zirconia ZrO_2 has by far the lowest thermal conductivity. It is, in fact, the principal material used for thermal barrier coatings.

Exercise E13.28 Find materials with T_{max} > 500°C and the lowest possible thermal conductivity. Switch the database to Level 3 to get more detail. Report the three materials with the lowest thermal conductivity.

Answer.

- Zirconia (Transformation toughened)(L)
- Zirconia (Y-TZP)(HIP)
- Zirconia (yttria stabilized, transformation toughened)

Exploring the science with CES Elements

Exercise E13.29 The claim is made in the text (equation (13.13a)) that the activation energy for diffusion, normalized by $\bar{R}T_m$ is approximately constant for metals:

$$\frac{Q_d}{\bar{R}T_m} \approx 18$$

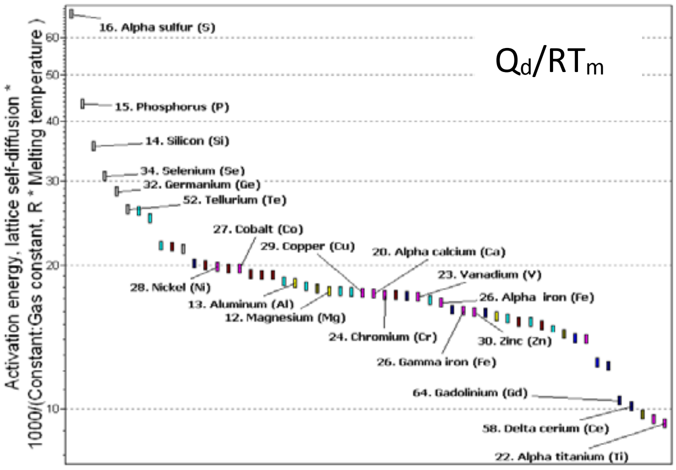
Make a bar-chart of this quantity and explore the degree to which it is true.

Answer.

The bar chart supports the claim. The normalized activation energy spans the range:

$$10 < \frac{Q_d}{\bar{R}T_m} < 60$$

but the majority lie between 16 and 20.



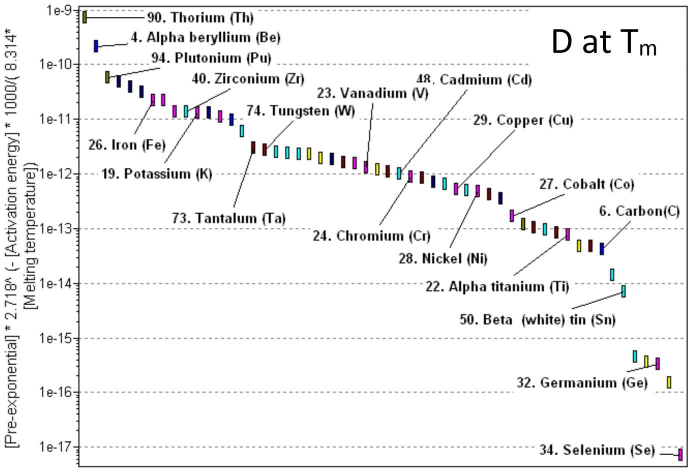
Exercise E13.30 The claim is made in the text (equation (13.13b)) that the diffusion coefficient of metals, evaluated at their melting point, is also approximately constant:

$$D_{Tm} \;=\; D_o \exp -\left(\frac{Q_d}{\bar{R}T_m}\right) \approx 10^{-12}\, m^2/s$$

Make a bar-chart of this quantity and explore the degree to which it is true.

Answer.

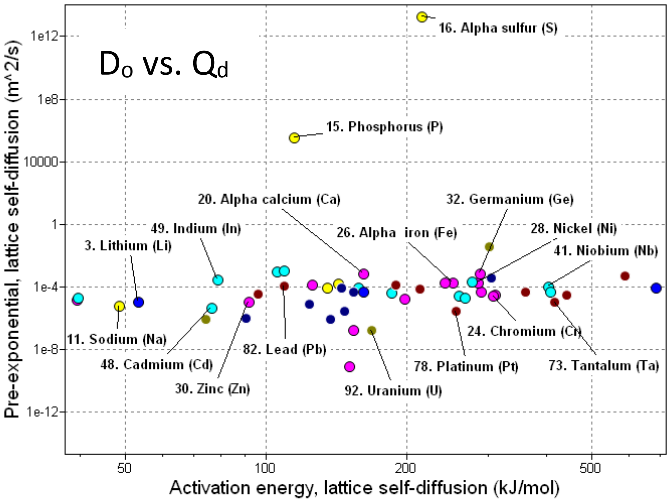
The bar-chart shows that the value $D_{Tm} \approx 10^{-12}$ m^2/s lies in the middle of the range of D_{Tm} for the elements, but the spread is very large (several orders of magnitude).



Exercise E13.31 What is a ‘typical’ value for the pre-exponential, D_o? Is it roughly constant for the elements? Make a chart with the activation energy for diffusion, Q_d, on the x-axis and the pre-exponential D_o on the y-axis to explore this.

Answer.

The great majority of the elements have pre-exponentials that lie close to $D \approx 10^{-4}$ m^2/s.



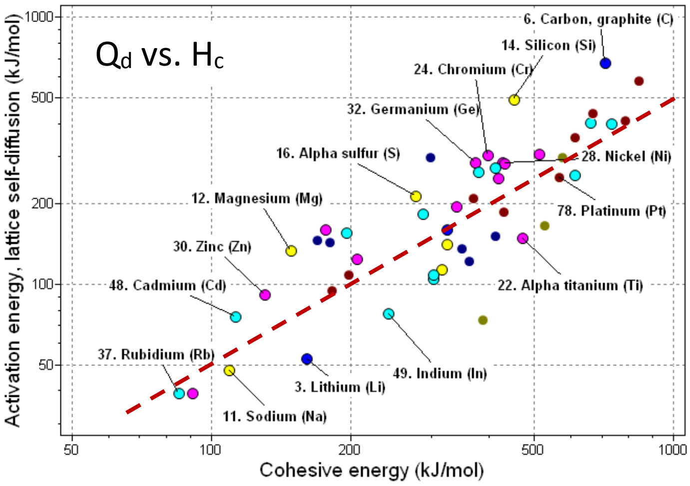
Exercise E13.32 The diffusive jump shown in Figure 13.9 requires that the diffusing atom breaks its bonds in its starting position in order to jump into is final one. You might, then, expect that there would be at least an approximate proportionality between the activation energy for diffusion Q_d and the cohesive energy H_c of the material. Make a chart with H_c on the *x*-axis and Q_d on the *y*-axis. Report what you find.

Answer.

The plot shows Q_d and H_c, both in units of kJ/mol. The line is a fit to the data. Its equation is:

$$Q_d = H_c/2$$

This is an interesting result: the activation energy for diffusion is very large, equal to half the energy needed to separate the atoms completely.



Materials: engineering, science, properties, and design
3e Solution manual

Chapter 14: Exercises with worked solutions

Exercise E14.1 Why do metals conduct electricity? Why do insulators not conduct electricity, at least when cold?

Answer. Metals are *electron-conductors*, meaning that the charge-carriers are the electrons. The electrons of an atom occupy discrete energy levels which, in a solid, broaden into bands of closely spaced levels. The electrons fill the bands from the bottom up. Whether the material is a conductor or an insulator depends on how full the bands are, and whether or not they overlap. Conductors like copper have an unfilled outer band; there are many very closely spaced levels just above the last full one, and – when accelerated by a field – electrons can use these levels to move freely through the material. In insulators the outermost band with electrons in it is full and the nearest empty band is separated from it in energy by a wide *band gap*. An electric field, E (volts/m), exerts a force Ee on an electron, where e is the charge it carries. In metals this force accelerates electrons, causing them to flow through the material and giving conduction. In insulators the force is insufficient to push electrons across the band gap, so they remain localized on the atom and are unable to move.

Exercise E14.2 A potential difference of 4 volts is applied across a tungsten wire 200 mm long and 0.1 mm in diameter. What current flows in the wire? How much power is dissipated in the wire? The resistivity ρ_e of tungsten is 5.6 μΩ.cm.

Answer. The current is given by Ohm's law: $V = i\,R$ where V is the potential difference, i is the current and R is the resistance. Converting the resistivity from $\mu\Omega$.cm to Ω.m by multiplying by 10^{-8} gives ρ_e = 5.6×10^{-8} Ω.m, so the resistance R of the wire is

$$R = \frac{\rho_e\,L}{A} = 5.6\times10^{-8}\times\frac{0.2}{\pi\left(0.5\times10^{-4}\right)^2} = 1.43\Omega$$

The current in the wire is

$$i = \frac{V}{R} = \frac{4}{1.43} = 2.8\ \text{amps}$$

The power dissipated in the wire is $P = i^2R = 11.3\ \text{Watts}$

Exercise E14.3 A gold interconnect 1mm long and with a rectangular cross-section of 10 μm × 1 μm has a potential difference of 1 mV between its ends. What is the current in the interconnect and how much power is dissipated in it? The resistivity ρ_e of gold is 2.5 μΩ.cm.

Answer. The current is given by Ohm's law: $V = i\,R$ where V is the potential difference, i is the current and R is the resistance. The resistivity of gold is ρ_e = 2.5×10^{-8} Ω.m, so the resistance R of the interconnect is

$$R = \frac{\rho_e\,L}{A} = 2.5\times10^{-8}\times\frac{1\times10^{-3}}{10\times10^{-12}} = 2.5\Omega$$

The current in the interconnect is

$$i = \frac{V}{R} = \frac{1\times10^{-3}}{2.5} = 4\times10^{-4}\ \text{amps}$$

The power dissipated in the interconnect is $P = i^2R = 4\times10^{-7}\ \text{watts}$

Exercise E14.4 The gold interconnect of the previous question (length 1mm and rectangular cross-section of 10 µm × 1 µm with a potential difference of 1 mV across its length) carries power for 10 seconds. If no heat is lost by conduction, radiation or convection, how hot will the interconnect get? The resistivity ρ_e of gold is 2.5 µΩ.cm, its specific heat is 130 J/kg.C and its density is 19300 kg/m^3.

Answer. The resistance R of the interconnect is 2.5 Ω, the current it carries is 4 x 10^{-4} amps and the power dissipated in it is $P = i^2 R = 4 \times 10^{-7}$ watts. (A watt is a joule per second.) The volume of gold in the interconnect is 1 × 10^{-14} m^3, corresponding to a mass $m =$ 1.93 × 10^{-10} kg. The energy Q required to heat this quantity of gold through a temperature interval ΔT is

$$Q = m C_p \Delta T = 1.93 \times 10^{-10} \times 130 \times \Delta T$$

Equating this to the electrical power dissipated in 10 second, 4×10^{-6} joules, gives

$$\Delta T = 159^{\circ}\text{C}$$

The heat dissipated in microcircuit interconnects can be large, requiring forced cooling.

Exercise E14.5 A 0.5mm diameter wire must carry a current $i = 10$ amps. The system containing the wire will overheat if the power dissipation, $P = i^2 R$, exceeds 5 watts/m. The table lists the resistivities ρ_e of four possible candidates for the wire. Which ones meet the design requirement?

Material	ρ_e, µΩ.cm
Aluminum	2.7
Copper	1.8
Nickel	9.6
Tungsten	5.7

Answer. If the resistivity of the material of the wire is ρ_e, the resistance of 1 meter of wire is

$$R = \frac{\rho_e L}{A} = \rho_e \times \frac{1}{\pi \left(2.5 \times 10^{-4}\right)^2} = 5.09 \times 10^6 \rho_e$$

The power dissipated in the wire

$$P = i^2 R = 25 \times 5.09 \times 10^6 \times \rho_e = 1.27 \times 10^8 \rho_e$$

Equating this to the power limit 5 watt/meter, and converting the resistivities from µΩ.cm to Ω.m by multiplying by 10^{-8} gives he condition

$$\rho_e < \frac{5}{1.27 \times 10^8} = 3.93 \times 10^{-8}\,\Omega.\text{m} = 3.93\,\mu\Omega.\text{cm}$$

Thus aluminum and copper meet the design requirement but nickel and tungsten do not.

Exercise E14.6 Which metallic superconductors have a critical temperature above the boiling point of helium? Which ceramic superconductors have a critical temperature above the boiling point of liquid nitrogen?

Answer. Figure 14.4 shows that tantalum, lead, vanadium and niobium have critical temperatures above the boiling point of helium. Four ceramic superconductors, $HgBa_2Ca_2Cu_3O_{10}$, $Tl_2Ba_2Ca_2Cu_3O_{10}$, $Bi_2Sr_2Ca_2Cu_2O_8$ and $YBa_2Cu_3O_7$, have critical temperatures above the boiling point of liquid nitrogen.

Exercise E14.7 A superconducting magnet design envisages a superconducting coil cooled in liquid nitrogen, generating a magnetic field of 30 Tesla. Is it practical to use YBCO ($YBa_2Cu_3O_7$) as the conductor? The critical temperature T_c of YBCO is 93 K and its zero-Kelvin critical field $H_{c,o}$ is 140 T. Liquid nitrogen boils at 77 K.

Answer. The critical field depends on temperature as

$$H_c = H_{c,o}\left(1-\left(\frac{T}{T_c}\right)^2\right)$$

Inserting data for YBCO we find

$$H_c = 140\left(1-\left(\frac{77}{93}\right)^2\right) = 24.1\ T$$

A 30 Tesla magnet is not possible without additional cooling below 77 K.

Exercise E14.8 What is a dielectric? What are its important dielectric properties? What is meant by polarization? Why do some dielectrics absorb microwave radiation more than others?

Answer. The electrons and protons of the atoms of dielectrics are symmetrically distributed, so the material carries no net charge or dipole moment. A field E (volts/m) exerts a force, pushing protons in the direction of the field and electrons in the opposite direction, giving the atom a dipole moment. Two charges $\pm q$ separated by a distance Δx create a dipole with dipole moment, d, given by

$$d = q\,\Delta x$$

The polarization of the material, P, is the volume-average of all the dipole moments it contains:

$$P = \frac{\sum d}{Volume}$$

The *dielectric constant* is a measure of the ease of polarization. The bigger the shift, the bigger the dielectric constant.

The other important dielectric properties are the dielectric strength (the field at which electrons are torn from the atom and cascade through the material) and the loss factor (a measure of the energy dissipated when a dielectric is placed in an oscillating electric field). Materials with a large loss factor absorb microwave radiation more than those with a small one.

Exercise E14.9 It is much easier to measure the electrical conductivity of a material than to measure its thermal conductivity. Use the Weidemann-Franz law to find the thermal conductivities of

(a) an alloy that has an electrical resistivity ρ_e of 28 μΩ.cm.

(b) tungsten of the kind used for lamp filaments that has an electrical conductivity κ_e of 9.9×10^6 S/m (a S/m is 1/Ω.m).

Answer. The Wiedemann-Franz law relates thermal conductivity λ to the resistivity ρ_e

$$\lambda \approx \frac{1400}{\rho_e}$$ (λ in W/m.K, ρ_e in μΩ.cm). Using this equation gives:

(a) the alloy has a predicted thermal conductivity of 50 W/m.K.

(b) the tungsten has a predicted thermal conductivity of 139 W/m.K

Exercise E14.10 The metal zinc has free electron concentration per unit volume, n_v of 1.3×10^{29} / m^3 and an electron mobility μ_e of 8×10^{-4} m^2/V.s. The charge carried by an electron, e, is 1.6×10^{-19} Coulomb. Based on this information, what is the electrical conductivity of zinc? Handbooks list the measured resistivity of zinc as 5.9 μΩ.cm. Is this consistent with your calculation? (Watch the units.)

Answer. The calculated conductivity is $\kappa_e = 1.66\times10^7$S/m. This translates into a resistivity of 6.0 μΩ.cm, and thus is within 2% of the measured value.

Exercise E14.11 The resistivity of pure silver is $\rho_e = 1.59\mu\Omega.\text{cm}$. Each silver atom provides one free electron. The atomic volume of silver is 1.71×10^{-29} m^3. What is the electron mobility in silver?

Answer. If each atom contributes one electron the electron concentration in copper is

$$n_v = \frac{1}{\text{Atomic volume}} = 5.85\times10^{28} \text{ per m}^3$$

The resistivity ρ_e is related to this by

$$\rho_e = \frac{1}{\kappa_e} = \frac{1}{n_v\, e\, \mu_e} \quad \text{where } e = 1.6\times10^{-19} \text{ Coulombs is the charge on an electron.}$$

Converting the resistivity from $\mu\Omega.\text{cm}$ to $\Omega.\text{m}$ by multiplying by 10^{-8} gives the electron mobility in copper as

$$\mu_e = \frac{1}{n_v\, e\, \rho_e} = 0.0067 \text{ m}^2\text{/V.s}$$

Exercise E14.12 Estimate the drift velocity of free electrons when a potential difference of 3 volts is applied between the ends of a copper wire 100 mm long. The electron mobility in copper is $\mu_e = 0.0032$ m^2/V.

Answer. The potential difference of 3 volts across 100mm of wire creates a field $E = 3/0.1 = 30$ V/m. The drift velocity is

$$v_d = \mu_e E = 0.0032\times30 = 0.096 \text{ m/s}$$

Exercise E14.13 The resistivity of brass with 60 atom% copper and 40 atom% of zinc is $\rho_e = 6.8\,\mu\Omega.\text{cm}$. Each copper atom contributes one free electron. Each zinc atom contributes two. The atomic volumes of copper and zinc are almost the same and the electron concentration in copper is $n_v = 8.45\times10^{28}/\text{m}^3$. What is the electron mobility μ_e in brass? How does it compare with that of pure copper ($\mu_e = 0.0032$ m^2/V.s)?

Answer. If each copper atom contributes one free electron and each zinc atom contributes two, the average number of free electrons per atom is (0.6 + 2×0.4) = 1.4, so the electron concentration in the brass is

$$n_v = 1.4\times8.45\times10^{28} = 1.18\times10^{29}$$

The resistivity ρ_e is related to this electron concentration by

$$\rho_e = \frac{1}{\kappa_e} = \frac{1}{n_v\, e\, \mu_e} \quad \text{where } e = 1.6\times10^{-19} \text{ Coulombs is the charge on an electron. Solving for}$$

electron mobility μ_e and converting the resistivity into $\Omega.\text{m}$ gives, for brass,

$$\mu_e = \frac{1}{n_v\, e\, \rho_e} = 0.0008 \text{ m}^2\text{/V.s}$$

This is about one quarter of the mobility of electrons in pure copper, suggesting that the mean free path of electrons in brass is much less than that in copper.

Exercise E14.14 Graphene is a semi-conductor with remarkable electrical properties. It is reported that the free-electron mobility μ_e in graphene is 1.5 m^2/V.s and that the effective free electron concentration n_v per unit volume, n_v, is $5\times10^{25}/m^3$. If these values are accepted, what would you expect the resistivity of graphene to be?

Answer. The conductivity of an electronic conductor is $\kappa_e = n_v\, e\, \mu_e$ where $e = 1.6\times10^{-19}$ Coulombs is the charge on an electron. Inserting the data in the question gives

$$\kappa_e = 5\times10^{25}\times1.6\times10^{-19}\ \times1.5 = 1.2\times10^{7}\ (\Omega.\text{m})^{-1}$$

Thus the resistivity is $\rho_e = \dfrac{1}{\kappa_e} = 8.3\times10^{-8}\ \Omega.\text{m} = 8.3\,\mu\Omega.\text{cm}$. This is about the same as a metal.

Exercise E14.15 A power line is to carry 5 kA at 11 kV using pylons 460 m apart. The dip d , in a wire of weight m_l per unit length strung between pylons L apart at a tension T is given by $d = L^2\, m_l\, / 8T$. The maximum tension allowed is 0.8 of the yield stress, σ_y . If the maximum allowable dip is 6 m, which of the materials in the table could be used?

Material	Electrical resistivity ρ_e , Ω.m	Yield stress σ_y , MPa	Density ρ , kg/m^3
Aluminum	1.7×10^{-8}	102	2700
Copper	1.5×10^{-8}	336	8900
Steel	55×10^{-8}	295	7800

Answer. The mass per unit length of cable is $m_l = \rho\, g\, A$. The tension $T = \sigma\, A$ so the required strength is $\sigma_y \geq \rho\, g\, L^2\, / 38$. Evaluating this for the materials of the table gives the results listed below.

	Aluminum	Copper	Steel
Strength, σ_y (MPa)	102	336	295
$\sigma/(0.8\times\sigma_y)$	0.64	1.05	0.587

Thus aluminum or steel can be used, but not but not copper.

Exercise E14.16 A material is required for a transmission line that gives the lowest full-life cost over a twenty year period. The total cost is the sum of the material cost and the cost of the power dissipated in Joule heating. The cost of electricity C_E is 6×10^{-3} \$/MJ. Material prices are listed in the Table below. Derive an expression for the total cost per meter of cable in terms of the cross sectional area A (which is a free parameter), the material and electrical costs and the material parameters. Show that the minimum cost occurs when the two contributions to the cost are equal. Hence derive a performance index for the material and decide on the best of the materials in the Table.

Material	Electrical resistivity ρ_e , Ωm	Density ρ , kg/m^3	Price C_m , \$/kg
Aluminum	1.7×10^{-8}	2700	1.6
Copper	1.5×10^{-8}	8900	5.2
Steel	55×10^{-8}	7800	0.5

Answer. If the material cost is C_m /kg the cost/ meter is $C_m \rho A$. The resistance/meter is $R = \rho_e / A$. The power dissipation $= i^2 \rho_e / A$. The cost of the dissipated power is $C = C_e \, i^2 \, \rho_e \, t / A$ where t is the time (20 years). Thus the total cost

$$C_{tot} = C_m \rho A + \frac{C_e \, i^2 \, \rho_e \, t}{A}$$

Minimizing C_{tot} with respect to A gives the optimum value

$$\tilde{C}_{tot} = 2\left(C_m \rho \, t C_e \, i^2 \, \rho_e\right)^{1/2}$$

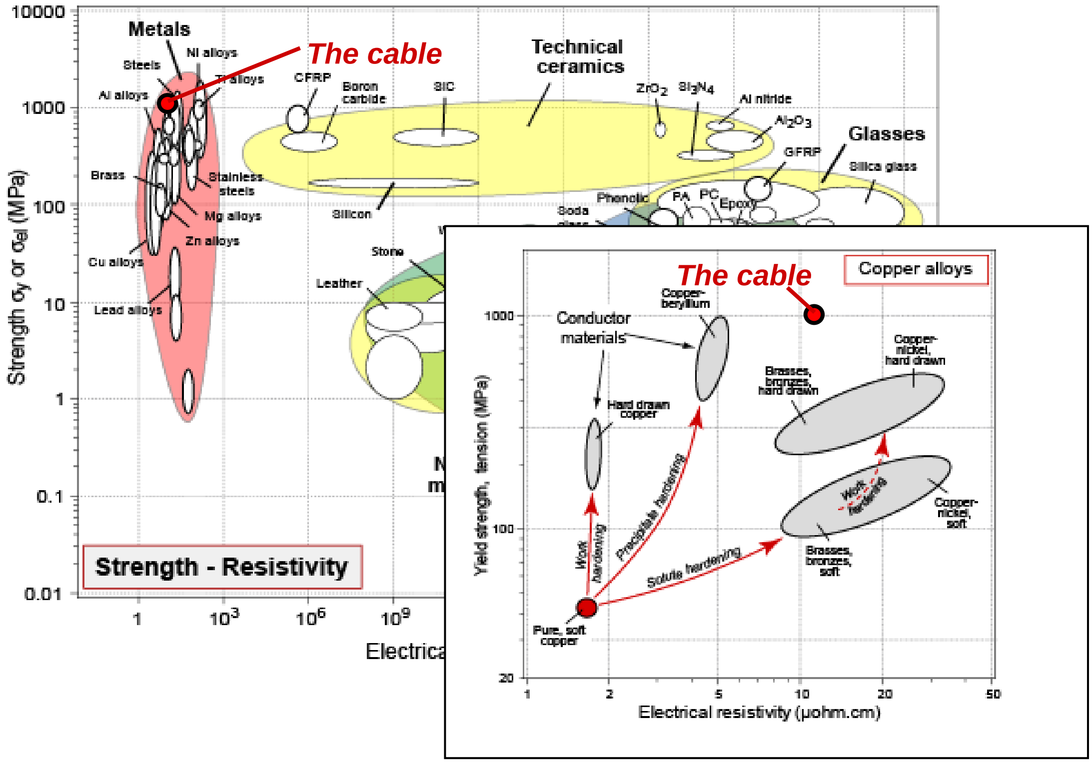
A suitable performance index is thus $C_m \, \rho \, \rho_e$. It has the value 4.6×10^{-5} for aluminum and 1.3×10^{-3} for steel. Hence choose aluminum.

Exercise E14.17 In the discussion of conductors, a 50-50 mix of copper and steel strands was suggested for transmission cables. Using the values in the table for resistivity and strength, calculate the effective values for both for the cable, assuming a rule of mixtures. Plot this on copies of the $\sigma_y - \rho_e$ chart of Figures 14.8 and 14.15a to explore its performance. Assume both strength and resistivity of cables follow a rule of mixtures.

Material	Strength, σ_y (MPa)	Resistivity, ρ_e ($\mu\Omega$.cm)
High strength steel, cold drawn	1700	22
High conductivity copper, cold drawn	300	1.7

Low density, we said, was important for long-span transmission lines because the self-weight becomes significant. Suppose you were asked to design a hybrid power-transmission cable with the lowest possible weight, what combination of materials would you choose? Use information from the charts for inspiration.

Answer. Using a rule of mixtures, the strength of the cable is 1000 MPa and its resistivity is 11.9 $\mu\Omega$.cm. The result is plotted on the two charts. The lower one is the most revealing: the cable is stronger than all but the strongest copper alloys – the copper-beryllium alloys (very expensive) and has a resistivity comparable with that of brass.



To make a cable that combines the greatest conductivity and strength with the lowest weight, we need a material with high conductivity per unit weight (λ / ρ) for the conducting strands and one with high strength per unit weight (σ_y / ρ) for the load-carrying strands. The figure above shows that aluminum has a conductivity that is only a little less than copper, and its density (2700 kg/m^3) is one third of that of copper (8.9 kg/m^3) (e.g. see Figure 6.7). Figure 6.7 also shows that the material with the highest σ_y / ρ is CFRP. A good choice might therefore be aluminum conducting strands in a carbon fiber-epoxy sheath.

Exercise E14.18 Roughly 50% of all cork that is harvested in Portugal ends up as cork dust, a worthless by-product. As a materials expert, you are approached by an entrepreneur who has the idea of making useful products out of cork dust by compacting it and heating it, using microwave heating. The loss factor L of cork is 0.21. The entrepreneur reckons he needs a power density P of at least 2 kW per m^3 for the process to be economic. If the maximum field E is limited to 10^2 V/m, what frequency f of microwaves will be needed?

Answer. When a dielectric material is placed in a cyclic electric field of amplitude E and frequency f, power P is dissipated and the field is correspondingly attenuated. The power dissipated per unit volume, in W/m^3, is

$$P \approx f\,E^2\,\varepsilon \tan\delta = f\,E^2\,\varepsilon_o\,\varepsilon_r \tan\delta$$

where ε_r is the dielectric constant of the material and $\tan\delta$ is its loss tangent (equation (14.18) of the text). This power appears as heat and is generated uniformly through the volume of the material. Thus the higher the frequency or the field strength and the greater the loss factor $\varepsilon_r \tan\delta$ the greater is the heating and energy loss. Using this equation we find that the frequency required to meet the design requirements is 4.75×10^9/s.

Exercise E14.19 Derive the expression (equation (14.15))

$$\text{Max energy density} = \frac{1}{2}\varepsilon_r\,\varepsilon_o\,E_b^2$$

for the maximum limiting electrical energy density that can be stored in a dielectric.

Answer. The energy in a dielectric film of thickness t and area A is $\frac{1}{2}CV^2$. The capacity is

$$C = \varepsilon_r\,\varepsilon_o\,\frac{A}{t}$$

And the electric field is $E = V/t$. Thus the energy stored in the dielectric is

$$\frac{1}{2}CV^2 = \frac{1}{2}\varepsilon_r\,\varepsilon_o\,\frac{A}{t}(E\,t)^2$$

The volume of the dielectric is $A\,t$ and the maximum permissible field is the breakdown field E_b. Thus the upper limiting energy density is

$$\frac{1}{2}\frac{CV^2}{At} = \frac{1}{2}\varepsilon_r\,\varepsilon_o\,E_b^2$$

Exercise E14.20 You are asked to suggest a dielectric material for a capacitor with the highest possible energy density. What material would you suggest? Use Figure 14.9 to find out.

Answer. Figure 14.9 indicates that nylon offers the largest energy density without breakdown.

Exercise E14.21 The electron mobility μ_e in germanium at 300 K is 0.36 m^2/V.s and that of holes μ_h is 0.19 m^2/V.s. The carrier density is $2.3{\times}10^{19}$ /m^3. What is the conductivity of germanium at 300 K?

Answer. The conductivity is

$$\kappa_e = n_v\, e\left(\mu_e + \mu_h\right) = 2.3\times10^{19}\times1.6\times10^{-19}\times(0.36+0.19) = 2.0\ (\Omega.\text{m})^{-1}$$

Exercise E14.22 The atomic volume of silicon is 2.0×10^{-29} m^3 and the free electron density n_e at 300K is 1.5×10^{16} /m^3. What fraction of silicon atoms have provided a conduction electron?

Answer. There are $n_{atoms} = \dfrac{1}{\text{Atomic volume}} = 5\times10^{28}$ silicon atoms per cubic meter. The fraction of silicon atoms providing a free electron is then

$$\frac{1.5\times10^{16}}{5\times10^{28}} = 3\times10^{-13}$$

This compares with a fraction of 1 for conducting electrons in copper and silver.

Exploring design with CES *(Use Level 2 unless stated otherwise)*

Exercise E14.23 Use the 'Search' facility of CES to search for

(a) electrical conductors
(b) heat sinks.

Answer.

(a) Electrical conductors
- Aluminum alloys
- Copper
- Non age-hardening wrought Al-alloys

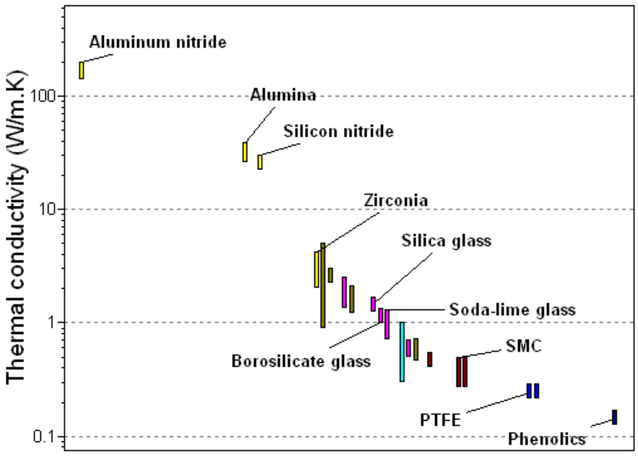
(b) Heat sinks
- Aluminum nitride
- Stainless steel
- Copper

Exercise E14.24 The analysis of heat sinks in Chapter 3 formulated the following design constraints and objective. They are summarized in the table. Use the selector to find the material that best meets them.

Function	• *Heat sink*
Constraints	• *Material must be good electrical insulator* • *Maximum operating temperature > 200 °C*
Objective	• *Maximize thermal conductivity*
Free variable	• *Choice of material*

Answer. Use a Limit stage to apply the constraints on maximum service temperature and the requirement of a good electrical insulator. Then make a graph stage of thermal conductivity, hiding the materials that failed to meet the constraints.

The result is shown here. Aluminum nitride is by far the best choice, with alumina and silicon nitride as alternatives.



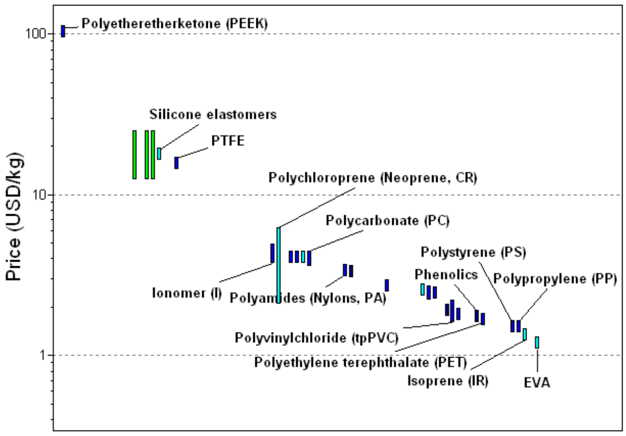
Exercise E14.25 The sheathing of the cable shown in Figure 14.24 is molded over the bundled wires. The material of the sheath must be electrically insulating, flexible, water resistant and cheap. The table translates the requirements. Apply the constraints using a 'Tree' and a 'Limit' stage, then plot a bar-chart of material price to find the three least expensive material that meets all the constraints.

Function	• *Cable sheathing*
Constraints	• *Able to be molded* • *Good electrical insulator* • *Durability in fresh water: very good* • *Young's modulus < 4 GPa*
Objective	• *Minimize material price*
Free variable	• *Choice of material*

Answer. Use a Limit stage to apply the constraints on modulus, durability and the requirement of a good electrical insulator. Use a Tree stage to require that the material can be molded. Then make a Graph stage of material price hiding the materials that failed to meet the constraints.

The four that meet the constraints and have the lowest price are

- EVA
- Isoprene (IR)
- Polypropylene (PP)
- Polystyrene (PS)

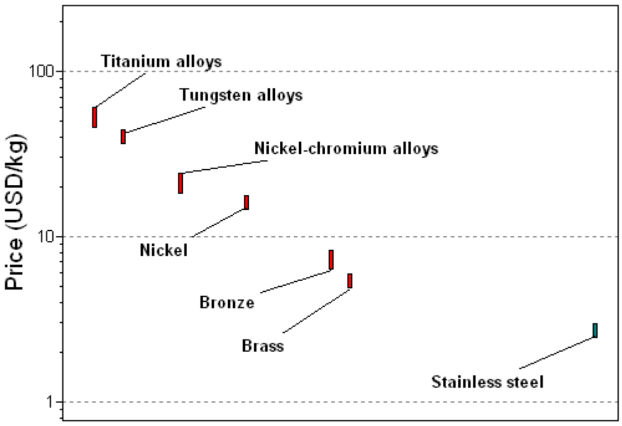


Exercise E14.26 The pins of the plug of Figure 14.25 must conduct electricity well, be corrosion resistant so that contact remains good, and be hard enough to resist abrasion and wear. The table translates the requirements. Apply these and rank the promising candidates by material price.

Function	• *Connecting pins for power plug*
Constraints	• *Good electrical conductor* • *Durability in fresh water: very good* • *Hardness > 200 HV*
Objective	• *Minimize material price*
Free variable	• *Choice of material*

Answer. Use a Limit stage to apply the constraints on durability, hardness and the requirement of a good electrical conductor. Then make a graph stage of material price, hiding the materials that failed to meet the constraints.

Stainless steel (cheapest), brass, and bronze (most expensive). Although stainless steel conducts electricity well, its resistivity is nearly 10 times that of brass, which is therefore the better choice.

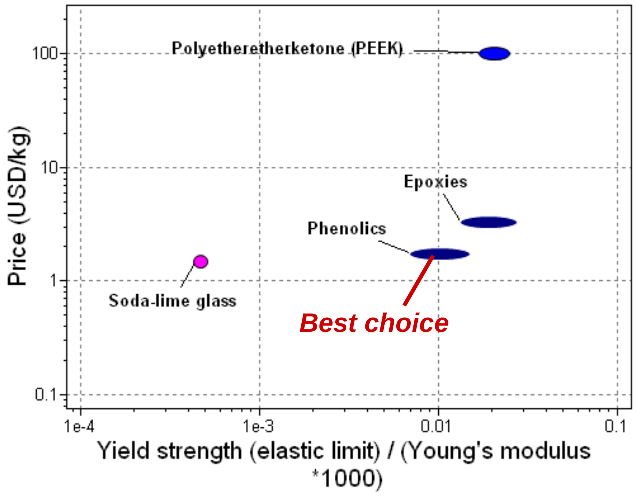


Exercise E14.27 The casing of the plug of Figure 14.25 performs both a mechanical and an electrical function. It must insulate, and it must also be rigid and resilient enough to tolerate service loads. (Resilient means it must be capable of some elastic distortion requiring a high yield strain, σ_y / E).

Function	• *Plug casing*
Constraints	• *Able to be molded* • *Good electrical insulator* • *Flammability: non-flammable or self extinguishing* • *Young's modulus > 2 GPa* • *Yield strain* $\sigma_y / E > 0.01$
Objective	• *Minimize material price*
Free variable	• *Choice of material*

Answer. Use a Limit stage to apply the constraints on modulus, flammability, modulus and the requirement of a good electrical insulator. Use a Tree stage to require that the material can be molded. Then make a Graph stage of yield strain σ_y / E (remembering to multiply material E by 1000 because it is in GPa, not MPa) plotted against price hiding the materials that failed to meet the constraints.

Soda lime glass is the cheapest material on the plot, but its yield strain is much less than 0.01. Phenolics are the best choice..

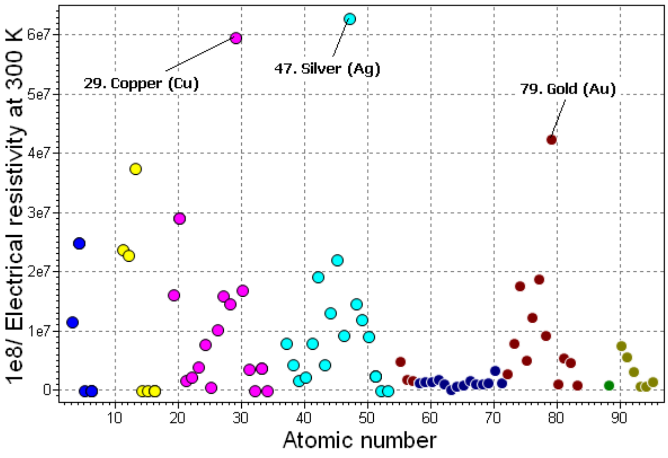


Exploring the science with CES Elements

Exercise E14.28 Make a chart with Atomic number on the x-axis (use a linear scale) and electrical conductivity κ on the y-axis. Convert the values of resistivity ρ_e (which in units of μΩ.cm in the database) to those of conductivity in Siemens/m (S/m), use $\kappa = 10^8 / \rho_e$. This can be made using the 'Advanced' facility in the axis-choice dialog box. Does conductivity vary in a periodic way across the periodic table? Which three elements have the highest conductivities?

Answer. The three with the highest conductivities are

- 29. Copper (Cu)
- 47. Silver (Ag)
- 79. Gold (Au)



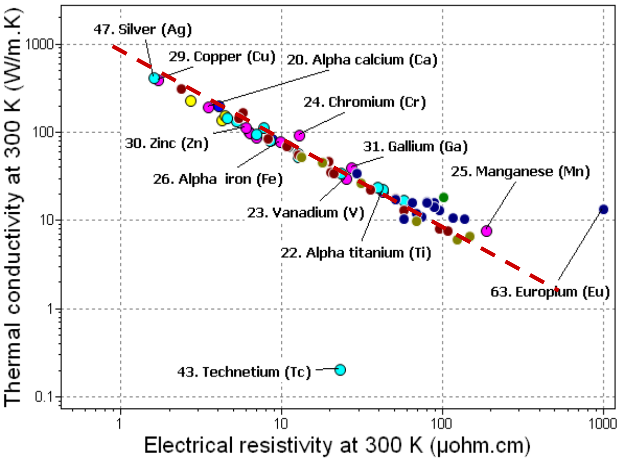
Exercise E14.29 Explore the Wiedemann-Franz relation for the elements. To do so, make a chart of with electrical conductivity, constructed as in the previous example, on the *x*-axis and thermal conductivity on the *y*-axis. Limit the selection to metals only by using a 'Limit' stage – State at 300 K. Fit an equation to the resulting plot. If you were now given a new element with an electrical conductivity of 4×10^7 S/m, what would you estimate its thermal conductivity to be? (Relations such as this one provide ways of building 'intelligent checking' of data into material property databases.)

Answer. The figure shows the plot. The line has a slope of -1, and is described by

$$\lambda \approx \frac{80}{\rho_e}$$

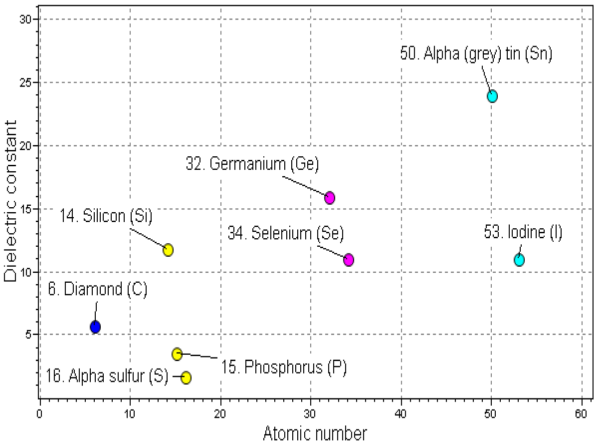
(this differs from the equation given in the text because the data for the elements derives from very pure materials, whereas engineering metals are almost all alloys.)

The new element is expected to have a thermal conductivity of 32 W/m.K.



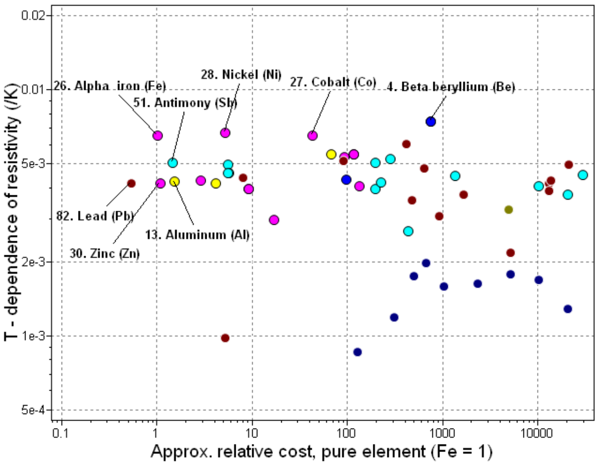
Exercise E14.30 Make a chart with atomic number on the x-axis (use a linear scale) and dielectric constant on the y-axis. Do you see any general trend? If so, how would you explain it?

Answer. There is a general increase in dielectric constant with atomic number. This is because the greater the number of electrons and protons per unit volume of material the greater is the polarization when it is placed in an electric field.



Exercise E14.31 It is proposed to use the temperature-dependence of electrical resistivity for sensing. Plot this quantity against relative cost to identify three cheap metals with high values of this temperature dependence.

Answer. The figure shows the temperature dependence of resistivity for the elements, plotted against relative cost. The materials with the greatest values of T-dependence of resistivity are iron, nickel, cobalt and beryllium. The cheapest are lead, zinc, aluminum and iron. Iron looks like the best choice.

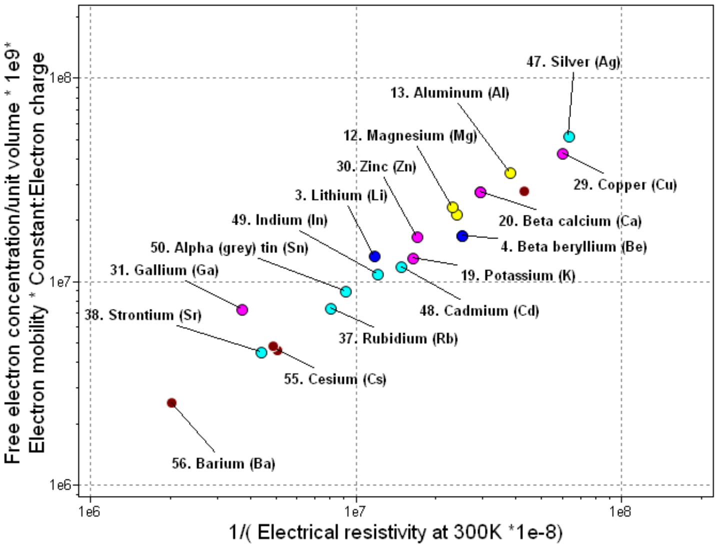


Exercise E14.32 Equation (14.21) of the text derives the electrical conductivity as

$$\kappa_e = n_v \, e \, \mu_e$$

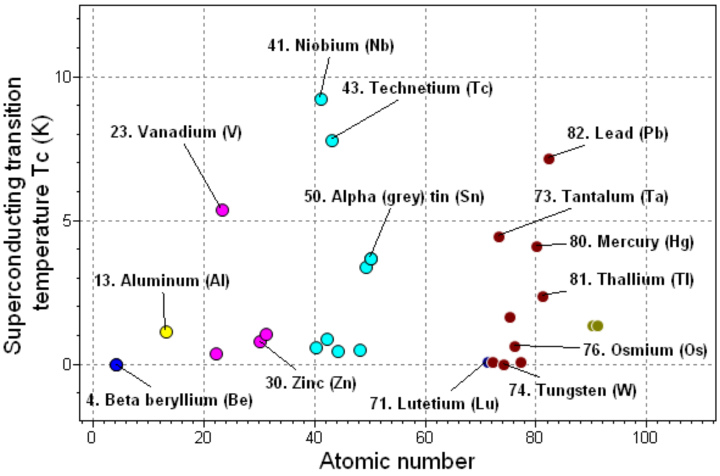
The CES Elements DB contains data for the free electron concentration n_v and the electron mobility μ_e. You will find the electronic charge under ‘Constants/Parameters’ in the ‘Advanced’ facility. Make a chart using the ‘Advanced’ facility with $\kappa_e = n_v \, e \, \mu_e$ on the y-axis in the reciprocal of the resistivity on the x-axis. Make sure you have made all the units self consistent. How well does this equation describe the measured conductivities of metals?

Answer. The plot shows the calculated and measured conductivity. (The free election concentration is stored in the Elements database as a number per mm^3; here it is multiplied by 10^9 to convert is to a number per m^3). The equation describes the measured conductivities very well.



Exercise E14.33 Explore superconductivity across the Periodic Table. Use CES Elements to plot superconducting transition temperature, T_c against the atomic number (use linear scales). Which element has the highest transition temperature? What is T_c for this element?

Answer. The element with the highest transition temperature is Niobium, $T_c = 9.25$ K.



Materials: engineering, science, properties, and design
3e Solution manual

Chapter 15: Exercises with worked solutions

Exercise E15.1 A cylindrical coil with a length L = 30 mm with n = 75 turns carries a current i = 0.1 A. What is the field in the magnet? A silicon-iron core with a relative permeability $\mu_R = 3000$ is placed in the core of the coil. What is the magnetization of the silicon-iron?

Answer. The electro-magnet produces a field

$$H = \frac{n\,i}{L} = \frac{75 \times 0.1}{30 \times 10^{-3}} = 250\ \mathrm{A/m}$$

The relative permeability of the ferrite is $\mu_R = 3000$. Thus the induction is

$$B = \mu_R\, \mu_o\, H = 3000 \times 4\pi \times 10^{-7} \times 250 = 0.94\ \text{Tesla}$$

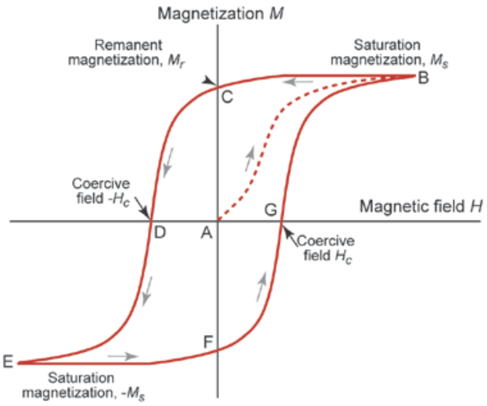
The magnetization is

$$M = \frac{B}{\mu_o} - H = \frac{0.94}{4\pi \times 10^{-7}} - 250 = 7.5 \times 10^5\ \mathrm{A/m}$$

(This is well below the saturation magnetization of silicon iron.)

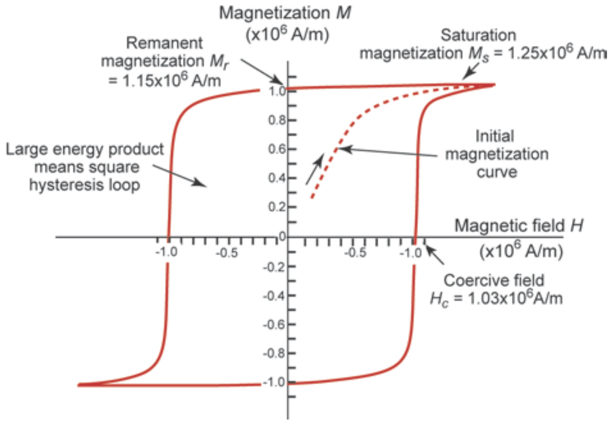
Exercise E15.2 Sketch a M - H curve for a ferro-magnetic material. Identify the important magnetic properties.

Answer. The sketch identifies the important magnetic properties.



Exercise E15.3 A neodymium-boron magnet has a coercive field H_c of 1.03 × 10^6 A/m a saturation magnetization $M_s = 1.25 \times 10^6$ A/m, a remanent magnetization $M_r = 1.1 \times 10^6$ A/m and a large energy product. Sketch approximately what its $M - H$ hysteresis curve looks like.

Answer. The hysteresis loop is drawn to scale here. The large energy product means that the loop must be nearly square. It is typical of a hard magnetic material.



Exercise E15.4 Why are some elements ferro-magnetic when others are not?

Answer. Atoms that contain unpaired electrons carry a magnetic moment. When they are assembled into crystals the atomic moments interact, but in most materials the interaction is so weak that thermal motion is enough to randomize the moments. Despite their magnetic atoms, the structure as a whole has no magnetic moment; the materials are *paramagnetic*. In a few materials the fields of neighbouring atoms interact such that their energy is reduced if their magnetic moments line up. The drop in energy – the *exchange energy* – is strong enough that it overcomes the randomizing effect of thermal energy so long as the temperature is not above the Curie temperature. The moments may line up anti–parallel, head to tail, leaving no net moment; such materials are *anti-ferromagnets*. In *ferromagnetic* elements, notably iron, nickel and cobalt, the moments spontaneously align so that – if all are parallel – the structure has a net moment that is the sum of those of all the atoms it contains.

Exercise E15.5 What is a ferrite? What are its characteristics?

Answer. Ferrites are ferri-magnetic oxides. One class of them has the formula M Fe_2O_4 where M is also a magnetic atom, such as cobalt, Co. Both the Fe and the Co atoms have dipoles but they differ in strength. They line up in the anti-parallel configuration, but because of the difference the moments the cancellation is incomplete, leaving a net moment M. The partial cancellation and the smaller number of magnetic atoms per unit volume means they have lower saturation magnetization than iron but they have other advantages, notably that, being oxides, they are electrical insulators.

Exercise E15.6 What is a Bohr magneton? A magnetic element has 2 unpaired electrons and an exchange interaction that causes them to align such that their magnetic fields are parallel. Its atomic volume, Ω, is 3.7×10^{-29} m^3. What would you expect its saturation magnetization, M_s, to be?

Answer. The saturation magnetization, then, is just the sum of all the atomic moments contained in a unit volume of material when they are all aligned in the same direction. If the magnetic dipole per atom is $n_m m_B$ (where n_m is the number of unpaired electrons per atom and m_B is the Bohr magneton) then the saturation magnetization (equation (15.10)) is

$$M_s = \frac{n_m\, m_B}{\Omega}$$

where Ω is the atomic volume. Thus the saturation magnetization is 0.50 A/m.

Exercise E15.7 Cobalt has a density ρ of 8900 kg/m^3, and atomic weight A_{wt} of 58.93 kg/kmol. The net magnetic moment of a cobalt atom, in Bohr magnetons per atom, is $n_m = 1.8$. What is the saturation magnetization M_s of cobalt?

Answer. The number of atoms per unit volume in cobalt is

$$n = \frac{N_A \rho}{A_{wt}} = 9.1 \times 10^{28}\ /\mathrm{m}^3$$

(here N_A is Avogadro's number, 6.022×10^{26} atoms/kmol). The saturation magnetization M_s is the net magnetic moment per atom $n_m\, m_B$ times the number of atoms per unit volume, n:

$$M_s = n\, n_m\, m_B$$

where $m_B = 9.27 \times 10^{-24}$ A.m^2 is the magnetic moment of one Bohr magneton. Thus the saturation magnetization of cobalt is

$$M_s = 9.1 \times 10^{28} \times 1.8 \times 9.27 \times 10^{-24} = 1.52 \times 10^{6} \ \ \mathrm{A/m}$$

Exercise E15.8 The element with the largest saturation magnetization is Holmium, with M_s = 3.0 × 10^6 A/m (although it has a miserably low Curie temperature of 20 K). The density ρ of Holmium is 8800 kg/m^3 and its atomic weight A_{wt} is 165 kg/kmol. What is the atomic magnetic moment n_m, in magnetons per atom, of Holmium?

Answer. The number of atoms per unit volume in Holmium is

$$n = \frac{N_A\ \rho}{A_{wt}} = 3.2\times 10^{28}\ /\mathrm{m}^3$$

(here N_A is Avogadro's number, 6.022 × 10^{26} atoms/kmol). The saturation magnetization M_s is the net magnetic moment per atom $n_m\, m_B$ times the number of atoms per unit volume, n:

$$M_s = n\, n_m\, m_B$$

where $m_B = 9.27 \times 10^{-24}$ A.m^2 is the magnetic moment of one Bohr magneton. Thus the atomic magnetic moment n_{BM} of Holmium is

$$n_m = \frac{M_s}{n\, m_B} = 10 \quad \text{Bohr magnetons per atom.}$$

Exercise E15.9 The soft iron laminations of transformers are made of 3wt% (6at%) silicon-iron. Silicon-iron has a density ρ of 7650 kg/m^3, and a mean atomic weight A_{wt} of 54.2 kg/kmol. The net magnetic moment of an iron atom, in Bohr magnetons per atom, is $n_m = 2.2$; that of silicon is zero. What is the saturation magnetization M_s of 3wt% silicon-iron?

Answer. The number of atoms per unit volume in silicon-iron is

$$n = \frac{N_A\, \rho}{A_{wt}} = 8.5\times 10^{28}\ /\mathrm{m}^3$$

(here N_A is Avogadro's number, 6.022 × 10^{26} atoms/kmol). The saturation magnetization M_s depends on the net magnetic moment per atom $n_m\, m_B$ as

$$M_s = n\, n_m\, m_B$$

where $m_B = 9.27 \times 10^{-24}$ A.m^2 is the magnetic moment of one Bohr magneton and n is the number of atoms carrying a magnetic moment – in this case, 96% of the total number of atoms, since the 6% of silicon atoms carry no moment. Thus the saturation magnetization of silicon iron is

$$M_s = 0.96\times 8.5\times 10^{28} \times 2.2\times\ 9.27\times 10^{-24} = 1.67\times 10^{6} \quad \mathrm{A/m}$$

Exercise E15.10 Nickel has a density ρ of 8900 kg/m^3, and atomic weight A_{wt} of 58.7 kg/kmol and a saturation magnetization M_s of 5.2 × 10^5 A/m. What is the atomic magnetic moment n_m, in Bohr magnetons per atom, of nickel?

Answer. The number of atoms per unit volume in nickel is

$$n = \frac{N_A \rho}{A_{wt}} = 9.13\times 10^{28}\ /\mathrm{m}^3$$

(here N_A is Avogadro's number, 6.022 × 10^{26} atoms/kmol). The saturation magnetization M_s depends on the net magnetic moment per atom $n_m\, m_B$ as

$$M_s = n\, n_m\, m_B$$

where $m_B = 9.27 \times 10^{-24}$ A.m^2 is the magnetic moment of one Bohr magneton. Thus

$$n_m = \frac{5.2\times 10^{5}}{9.27\times 10^{-24} \times 9.13\times 10^{28}} = 0.61 \quad \text{Bohr magnetons per nickel atom.}$$

Exercise E15.11 A coil of 50 turns and length 10 mm carries a current of 0.01 A. The core of the coil is made of a material with a susceptibility $\chi = 10^4$. What is the magnetization, M and the induction B ?

Answer. $M = 5 \times 10^5$ A/m; $B = 0.628$ T

Exercise E15.12 An inductor core is required for a low frequency harmonic filter. The requirement is for low loss and high saturation magnetization. Using the charts of Figures 15.8 and 15.9 as data sources, which class of magnetic material would you choose?

Answer. The application requires high M_s, low coercive field H_c (or, equivalently, a high susceptibility χ). The best choices are a Metglas amorphous alloy or Permendur.

Exercise E15.13 A magnetic material is required for the core of a transformer that forms part of a radar speed camera. It operates at a microwave frequency of 500 kHz. Which class of material would you choose?

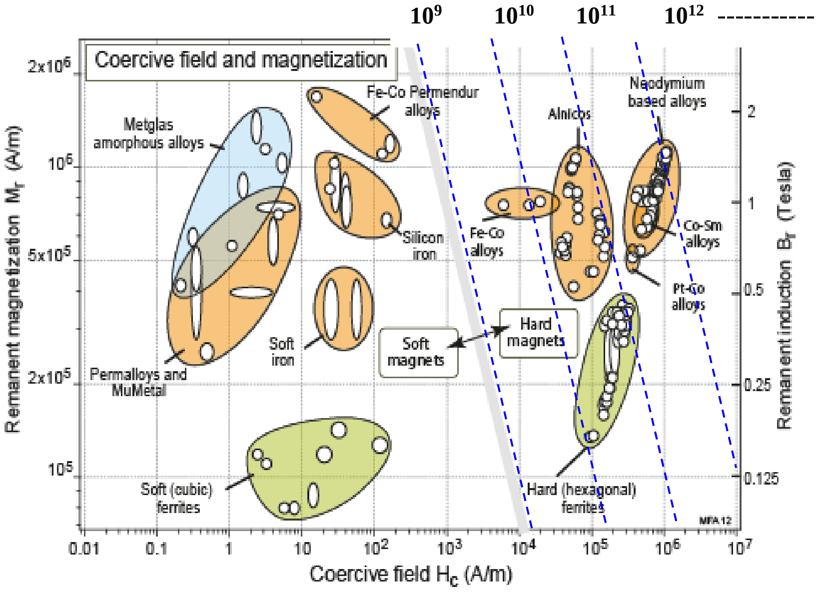
Answer. A soft magnetic cubic ferrite to give low hysteretic and eddy current loss.

Exercise E15.14 Which soft magnetic material has the greatest saturation magnetization, M_s, allowing it to concentrate magnetic field most effectively? Which has the greatest relative permeability, μ_R, allowing the highest inductance B for a given applied field H ? The Saturation magnetization – Permeability chart (Figure 15.9) can help.

Answer. The chart shows that the iron-cobalt Permadur series of alloys have the highest saturation magnetization and induction. The Metglas series of amorphous alloys have the highest permeability.

Exercise E15.15 The product $B_r H_c$ is a crude measure of the "power" of a permanent magnet (if the $B-H$ curve were a perfect rectangle, it becomes equal to what is called the energy product, a more realistic measure of the power). Draw contours of $B_r H_c$ onto a copy of the Remanent induction – Coercive field chart (Figure 15.8) and use these to identify the most powerful permanent magnet materials.

Answer. The figure shows the chart with the required contours plotted on it. Neodymium-based magnets offer by far the greatest power. After them come Cobalt-Samarium alloys Alnicos (alloys of aluminum, nickel and cobalt).



Exercise E15.16 A soft magnetic material for the core of a small high frequency power transformer. The transformer gets hot in use; forced air cooling limits the rise in temperature to 200°C. Eddy current losses are a problem if the core is electrically conducting. What material would you recommend for the core? The Coercive field – Curie temperature chart (Figure 15.10) can help.

Answer. The core must be a soft magnetic material, and electrical insulator and capable of use at 200°C. The chart of Figure 15.10 suggests that soft (cubic) ferrites can meet these constraints. The best choice is then the soft ferrite with the lowest coercive field (to reduce losses) with a Curie temperature substantially above 200°C. The alternative is to use silicon-iron or a nickel-iron Permalloy or MuMetal, laminating the core with insulating separators to limit eddy current losses.

Exercise E15.17 A material is required for a flexible magnetic seal. It must be in the form of an elastomeric sheet and must be electrically insulating. How would you propose to make such a material?

Answer. The way to make flexible magnets is to disperse ferro or ferri-magnetic particles in an elastomer such as Neoprene, Isoprene or PVC. Metallic particles of iron, nickel or cobalt can be used by if exposed to moisture or chemicals they may corrode. Ferrite particles, being oxides, are much more stable.

Exercise E15.18 What are the characteristics required of materials for magnetic information storage?

Answer. Magnetic information storage requires hard magnets with a rectangular magnetization curve (called "square"). The squareness means that a unit of the material can flip in magnetization from $+M_r$ to $-M_r$ and back when exposed to fields above $+H_c$ or $-H_c$. The unit thus stores one binary bit of information. The information is read by moving the magnetized unit past a read head where it induces an electric pulse.

Exercise E15.19 An electro-magnet is designed to pick up and move car parts – the car part becomes a temporary part of the core, completing the magnetic circuit. The field H is created by a coil 100 mm long with a cross section of 0.1 m^2 with 500 turns of conductor carrying a current of 25 A. What force can the electro-magnet exert?

Answer. The field generated by a magnet of length L of 100 mm with $n = 500$ turns carrying a current i = 25 A is

$$H = \frac{n\,i}{L} = 125{,}000 \ \text{A/m}$$

The force F exerted by an electromagnet on a section of core material is:

$$F = \frac{1}{2}\mu_o H^2 A$$

(here A is the area of the core) provided that the core does not saturate. Iron has a saturation magnetization H_s of about 1.8×10^6 A/m, plenty high enough to avoid saturation. Thus the force exerted by the electro-magnet is

$$F = \frac{1}{2} \times 4\pi \times 10^{-7} \times (125{,}000)^2 \times 0.1 = 982\,N \approx 1kN$$

Thus the magnet is capable of lifting a ferromagnetic part weighing about 98 kg.

Exercise E15.20 What, approximately, is the maximum induction B that is possible in a superconducting coil made of YBCO (the oxide superconductor $YBa_2Cu_3O_7$) at the temperature of liquid nitrogen (77K)? Use Figure 15.19 to find out. What is the maximum energy density that can be stored in such a coil? How does this compare with crude oil with an energy density of 35 MJ/liter? (not a fair comparison because the superconductor requires ancillary equipment to support it, but make the comparison anyway).

Answer. Inspection of Figure 15.19 suggests a maximum value for B of about 100 T. Thus the maximum energy-density is approximately

$$\frac{W}{V}=\frac{1}{2}\frac{B^2}{\mu_o}=\frac{1}{2}\left(\frac{100^2}{4\pi\times10^{-7}}\right)=6.37\times10^{10}\ \text{J/m}^3=3{,}979\,\text{MJ/m}^3$$

The value 35 MJ/liter for crude oil translates to 35,000 MJ/m^3.

Exploring design with CES.

Open CES at Level 3 – it opens in the 'Browse' mode. At the head of the Browse list are two pull-down menus reading 'Table' and 'Subset'. Open the 'Subset' menu and choose 'Magnetic'. Records for 233 magnetic materials are displayed, listing their magnetic properties and, where available, mechanical, thermal, electrical and other properties too.

To select, click on 'Select' in the main toolbar and choose MaterialsUniverse – Magnetic materials in the dialog box. Now we can begin.

Exercise E15.21 Use the CES Search facility to find materials for

(a) transformer cores
(b) electric motors

Answer. (a) Materials for transformer cores

- Iron-Chromium-Nickel-Base Alloy, A-286, STA
- Iron-base alloy, N-155, ST
- 45Ni-3Mo-Fe Soft Magnetic Alloy
- 45Ni-Fe Soft Magnetic Alloy, "Alloy 1", cold rolled
- 45Ni-Fe Soft Magnetic Alloy, "Alloy 1", soft (annealed)
- 49Ni-Fe Soft Magnetic Alloy, "Alloy 2B", cold rolled
- 49Ni-Fe Soft Magnetic Alloy, "Alloy 2B", soft, annealed
- 75Ni-5Cu-2Cr-Fe Soft Magnetic Alloy, "Alloy 3", soft (annealed)
- 78.5Ni-Fe Soft Magnetic Alloy
- 79Ni-4Mo-Fe Soft Magnetic Alloy, "Alloy 4", cold rolled
- 79Ni-4Mo-Fe Soft Magnetic Alloy, "Alloy 4", soft (annealed)
- Nickel-Iron Soft Magnetic Alloy, "Alloy 2A", cold rolled
- Metglas 2714A (Cobalt based)
- Metglas 2605S3A (Iron based)

(b) Materials for electric motors (some of the results are magnetic materials for the rotor and stator, others are conductors for the windings).

- Iron-Chromium-Nickel-Base Alloy, A-286, STA
- Iron-base alloy, N-155, ST
- High conductivity copper-chromium (w) (UNS C18200)
- High conductivity copper-chromium (wh) (UNS C18200)
- High conductivity copper-chromium (whp) (UNS C18200)
- High conductivity copper-chromium (wp) (UNS C18200)
- High conductivity copper-chromium (wph) (UNS C18200)
- Lead-based Babbitt Metal (Alloy 7)
- 45Ni-3Mo-Fe Soft Magnetic Alloy
- 45Ni-Fe Soft Magnetic Alloy, "Alloy 1", cold rolled
- 45Ni-Fe Soft Magnetic Alloy, "Alloy 1", soft (annealed)
- 49Ni-Fe Soft Magnetic Alloy, "Alloy 2B", cold rolled
- 49Ni-Fe Soft Magnetic Alloy, "Alloy 2B", soft, annealed
- 75Ni-5Cu-2Cr-Fe Soft Magnetic Alloy, "Alloy 3", soft (annealed)
- 78.5Ni-Fe Soft Magnetic Alloy
- 79Ni-4Mo-Fe Soft Magnetic Alloy, "Alloy 4", cold rolled
- 79Ni-4Mo-Fe Soft Magnetic Alloy, "Alloy 4", soft (annealed)
- Nickel-Iron Soft Magnetic Alloy, "Alloy 2A", cold rolled

Exercise E15.22 Find by browsing the records for:

(a) Cast Alnico 3. What is the value of its coercive force H_c?

(b) The amorphous alloy Metglas 2605-Co. What is the value of its coercive force H_c?

What do these values tell you about the potential applications of these two materials?

Answer.

(a) Cast Alnico 3 has a value of $H_c = 4730$ A/m.

(b) Metglas 20650 Co has a value of $H_c = 2.2$ A/m.

The first is suitable for permanent magnetic applications like loud-speaker magnets, the second for soft magnetic applications such as cores for transformers and chokes.

Exercise E15.23 Find by browsing the records for:

(a) Ferrite G (Ni-Zn ferrite). What the values of its coercive force H_c and resistivity ρ_e?

(b) 2.5 Si-Fe soft magnetic alloy. What the values of its coercive force H_c and resistivity ρ_e?

If you were asked to choose one of these for a transformer core, what would be your first question?

Answer.

(a) Ferrite G (Ni-Zn ferrite) has $H_c = 19.1$ A/m and $\rho_e = 1\times10^{12}$ μohm.cm.

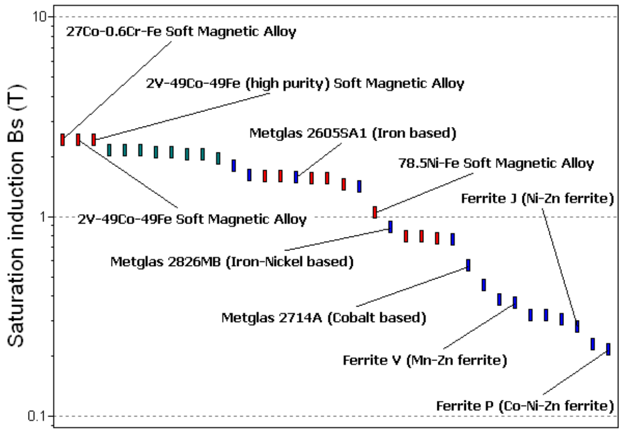
(b) 2.5 Si-Fe soft magnetic alloy has $H_c = 27.8$ A/m and $\rho_e = 40$ μohm.cm.

The question: what is the frequency of the transformer? If it is a low frequency transformer the .5 Si-Fe alloy is the best choice (it is cheap). If the frequency is above 100 kHz it will be necessary to choose the ferrite because it is an insulator, eliminating eddy-current losses.

Exercise E15.24 Make a bar-chart of saturation induction B_s (the saturation magnetization $M_s = B_s / \mu_o$, so the two are proportional). Report the three materials with the highest values.

Answer.

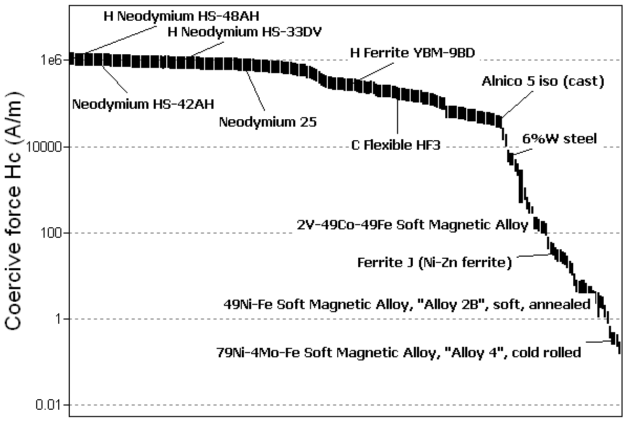
- 27Co-0.6Cr-Fe Soft Magnetic Alloy
- 2V-49Co-49Fe (high purity) Soft Magnetic Alloy
- 2V-49Co-49Fe Soft Magnetic Alloy



Exercise E15.25 Make a bar-chart of coercive force, H_c. Report the material with the lowest value.

Answer.

- 79Ni-4Mo-Fe Soft Magnetic Alloy, "Alloy 4", cold rolled
- 79Ni-4Mo-Fe Soft Magnetic Alloy, "Alloy 4", soft (annealed)
- Metglas 2714A (Cobalt based)
- Metglas 2826MB (Iron-Nickel based)

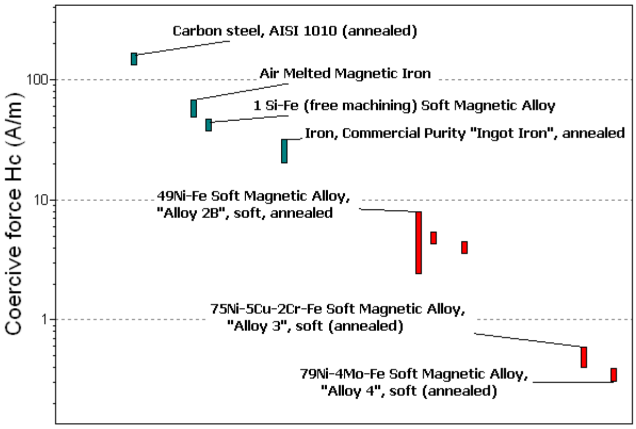


Exercise E15.26 A soft magnetic material is requires for the laminated rotor of AC induction electric motor. The material is to be rolled and further shaped by stamping, requiring an elongation of at least 40%. To keep hysteresis losses to a minimum it should have the lowest possible coercive force. Find the two materials that best meet these requirements, summarized below. Report them and their trade names.

Function	• *Motor laminations*
Constraints	• *Ferro or ferri magnetic material* • *Ductile (elongation > 40%)*
Objective	• *Minimize coercive force*
Free variable	• *Choice of material*

Answer.

- 75Ni-5Cu-2Cr-Fe Soft Magnetic Alloy, "Alloy 3", soft (annealed)
- 79Ni-4Mo-Fe Soft Magnetic Alloy, "Alloy 4", soft (annealed)



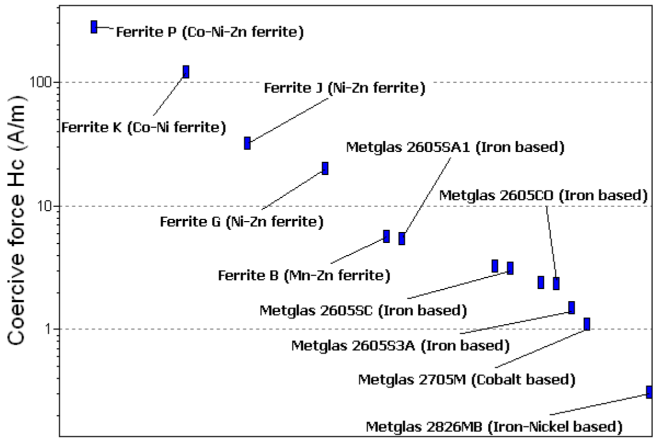
Exercise E15.27 A magnetic material is required for the read/write head of a VCR. It must have high hardness to resist wear and the lowest possible coercive force to give accurate read/write response.

(a) Use CES to identify possible candidates that meet these requirements, summarized in the table.

(b) Now add the further constraint that, for lower loss and high frequency damping, the magnetic material must have an electrical resistivity above 10^8 $\mu\Omega$.cm.

Function	• *Magnetic read-head*
Constraints	• *Ferro or ferri magnetic material* • *High hardness for wear resistance (Vickers hardness > 500 HV)*
Objective	• *Minimize coercive force*
Free variable	• *Choice of material*

Answer. The selection now is different. Adding the further constraint on electrical resistivity eliminates the Metglases and the stainless steels, leaving only the Ferrites.



- Ferrite B (Mn-Zn ferrite)
- Ferrite G (Ni-Zn ferrite)
- Ferrite J (Ni-Zn ferrite)
- Ferrite K (Co-Ni ferrite)
- Ferrite P (Co-Ni-Zn ferrite)
- Ferrite T (Mn-Zn ferrite)
- Ferrite V (Mn-Zn ferrite)

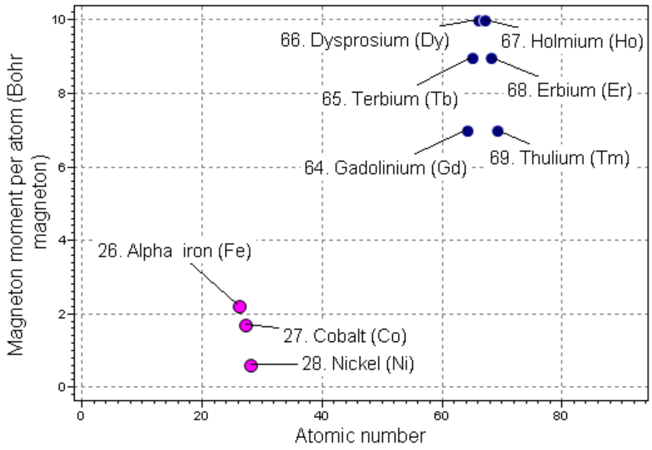
- Metglas 2605CO (Iron based)
- Metglas 2605S3A (Iron based)
- Metglas 2605SA1 (Iron based)
- Metglas 2605SC (Iron based)
- Metglas 2705M (Cobalt based)
- Metglas 2714A (Cobalt based)
- Metglas 2826MB (Iron-Nickel)

- Wrought martens. stainless steel, AISI 420, tempered @204C
- Wrought martens. stainless steel, AISI 440B, tempered @316C

Exploring the science with CES Elements

Exercise E15.28 Make a chart with Atomic number on the *x*-axis and Magneton moment per atom (Bohr magneton) on the *y*-axis (use linear scales) to identify the ferromagnetic elements. How many are there? Which have the highest magneton moment per atom?

Answer. Nine elements are ferromagnetic. The Lanthanides Dysprosium and Holmium have the highest magneton moment per atom.

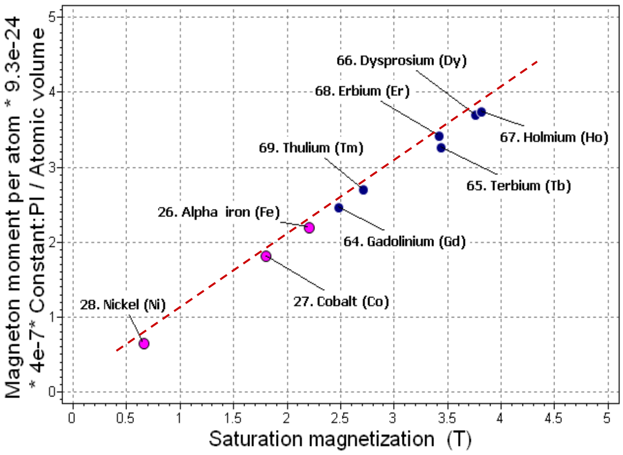


Exercise E15.29 According to equation (15.10) of the text, the saturation magnetization is

$$M_s = \frac{n_m m_B}{\Omega}$$

where n_m is the magnetic dipole per atom in units of Bohr magnetons, Ω is the atomic volume, and m_B is the value of a Bohr magneton (9.27×10^{-24} A/m^2). The saturation induction (in Tesla) $B_s = \mu_o M_s$. Make a chart showing B_s on the *y*-axis and on the *x*-axis plot the CES parameter called "saturation magnetization" (which is actually B_s, in Tesla, for the relevant elements). How accurately is this equation obeyed?

Answer. The chart shows that the equation is well obeyed. ‘Saturation magnetization’ in Tesla should strictly be called ‘Saturation induction’ (B_s).



Materials: engineering, science, properties, and design
3e Solution manual

Chapter 16: Exercises with worked solutions

Exercise E16.1 The absorption coefficient of polyethylene for optical frequencies is 86.6 cm^{-1}. How thick a slab of polyethylene is required to reduce the transmitted light intensity by absorption to one half of its initial value?

Answer. The transmitted fraction, if reflection is ignored, is

$$\frac{I}{I_o} = \exp - \beta x$$

Inverting this equation gives

$$x = -\beta \ln(\frac{I}{I_o}) = -0.866 \ln 0.5 = 0.6 \text{ m}$$

A polyethylene slab of thickness 60 cm thick will reduce the intensity by one half.

Exercise E16.2 The fraction of light that is transmitted through a 100 mm panel of PMMA is 0.97. Neglecting reflection at both surfaces, what is the absorption coefficient of PMMA?

Answer. The transmitted fraction, if reflection is ignored, is

$$\frac{I}{I_o} = \exp - \beta x$$

where β is the absorption coefficient and x is the length of the optical path in the material. Taking logs of this equation, inverting and inserting the data gives

$$\beta = -\frac{1}{x} \ln \left(\frac{I}{I_o} \right) = -\frac{1}{0.1} \ln (0.97) = 0.3 \text{m}^{-1}$$

Exercise E16.3 Derive the expression for the fraction of light that passes through a transparent panel, allowing for both reflection and absorption:

$$I = I_o (1 - \frac{I_R}{I_o})^2 \exp - \beta x$$

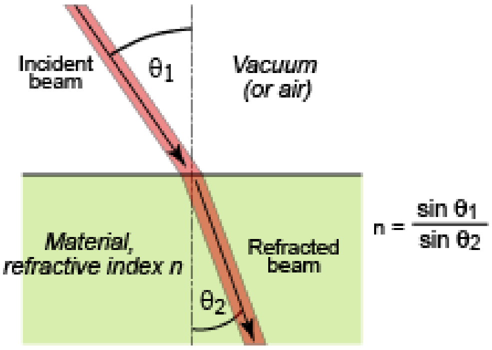
where β is the absorption coefficient and x is the length of the optical path in the material.

Answer. The trick here is to realize that two surfaces are involved – the one through which the light enters the panel and the one through which it leaves. If both surfaces of the panel are immersed in the same medium (air, for example) then a fraction $R = I_R / I_o$ of the light intensity is reflected at both surfaces. Thus the fraction entering the material through the front face of the panel is $(1 - R)$. If there were no absorption, this is the fraction reaching the back surface, where a further fraction $(1 - R)$. of the beam intensity is reflected, leaving the fraction $(1 - R)^2$ to emerge through the back face. If there is absorption (absorption coefficient β) then in passing through the material a further fraction $\exp - \beta x$ is lost, giving a final emerging fraction of

$$\frac{I}{I_o} = (1 - \frac{I_R}{I_o})^2 \exp - \beta x$$

Exercise E16.4 Define refractive index. Give examples of devices that make use of refraction.

Answer. When light enters a material from air or vacuum, it slows down. The *index of refraction*, n is the ratio of its velocity in vacuum, c_o, to that in the material, c. This retardation makes a beam of light bend or *refract* when it enters a material of different refractive index. When a beam passes from a material *1* of refractive index n_1 into a materials *2* of index n_2 with an angle of incidence θ_1, it deflects to an angle θ_2, such that



$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

The equation is known as Snell's law. The refractive index depends on wavelength, so each of the colors that make up white light is diffracted through a slightly different angle, producing a spectrum when light passes through a prism. When material 1 is vacuum or air, for which $n_1 = 1$, the equation reduces to

$$\frac{\sin\theta_1}{\sin\theta_2} = n_2$$

When light passes from a material with index $n_1 = n$ into air with $n_2 = 1$, it is bent away from the normal to the surface, as in the sketch.

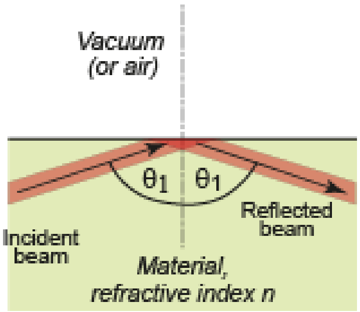
Refraction is use in prismatic glass, prismatic binoculars and periscopes, the optics of reflex cameras, in lenses and eye-glasses of all sorts, and in certain 3-D advertising panels.

Exercise E16.5 The prism for a reflex camera is made from a silica glass with a coefficient of refraction of 1.46. To present a "through the lens" image to the user of the camera, light rays entering the prism must be totally reflected. At what range of angles can they strike the internal surfaces of the prism ?

Answer. The limiting angle is given by

$$\sin\theta_1 = \frac{1}{n} = \frac{1}{1.46} = 43.2^o$$

Any incident angle θ_1 between 43.2° and 90° will be reflected.



Exercise E16.6 A unique design for an underwater camera calls for a prism viewfinder that will be immersed in water. If the prism, as in the last question, is made from a silica glass with a coefficient of refraction of 1.46 what range of incident angles will lead to total internal reflection? The refractive index of water is 1.33.

Answer. A ray in the glass (refractive index n_1) strikes the prism face at incident angle θ_1. If it is not reflected it exits into the surrounding, here water (refractive index n_2) at the angle θ_2. We know that

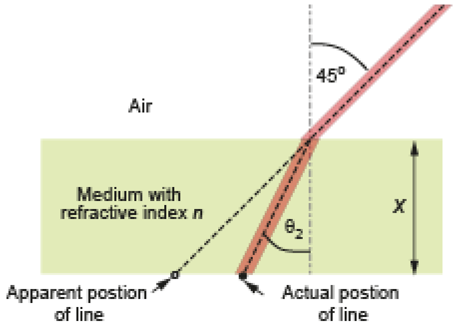
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} = \frac{1.33}{1.46} = 0.91$$

The greatest value that $\sin\theta_2$ can have is unity (1) so for the ray to leave the prism

$$\sin\theta_1 \le 0.91$$

corresponding to $\theta_1 \le 65.6^o$. An incident angle greater than this will be reflected. Thus the range of incident angles for total internal reflection, 65.6° to 90°, is smaller for the glass-water interface than for the glass-air interface.

Exercise E16.7 A 300 mm thick block of glass of refractive index 1.66 is placed over a line on a sheet of paper so that only part of the line is covered. The line is viewed from above at an angle of 45° to the glass and normal to the line. Refraction in the glass displaces the image of the part of the line it covers. By how much is the part of the line viewed through the glass displaced relative to the part that is not covered?



Answer. The sketch shows the geometry. The law of refraction is

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

here with $\theta_1 = 45^o$ and $n_1 = 1$. Thus

$$\sin\theta_2 = \sin\theta_1 \frac{n_1}{n_2} = 0.43$$

Giving $\theta_2 = 25.2^o$ If the thickness of the glass is x, the distance of the line from the vertical through the point of observation is also x. However the line appears to be a distance

$$x' = x\tan\theta_2 = 0.47\,x$$

From the vertical. If $x = 300$ mm, the line appears to have shifted a distance

$$x - x' = 159 \text{ mm}$$

Exercise E16.8 Define reflectance. Give examples of devices that make use of reflection.

Answer. When light travelling in a material of refractive index n_1 is incident normal to the surface of a second material with a refractive index n_2, the reflectivity is

$$R = \frac{I_R}{I_o} = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$

where I_o is the incident intensity and I_R that of the reflected beam. If the incident beam is in air, with refractive index 1, this becomes

$$R = \frac{I_R}{I_o} = \left(\frac{n-1}{n+1}\right)^2.$$

Thus materials with high refractive index have high reflectivity.

Reflection is used in mirrors, head-lamp and other lamp reflectors, passive reflectors on roads and obstacles, reflecting telescopes, light guides and optical fibers.

Exercise E16.9 The reflectivity of window glass is 4% per surface. What is its refractive index? Neglecting absorption, what fraction of the intensity of the light striking the window passes through?

Answer. The reflectivity n is related to the refractive index R by

$$R = \frac{I_R}{I_o} = \left(\frac{n-1}{n+1}\right)^2$$

Setting this equal to 0.04 and solving for n gives the value 1.5 for the refractive index. The fraction of the initial intensity, I_o, of the light striking the glass that enters it is

$$1 - \frac{I_R}{I_o} = 1 - R\ ,$$

A fraction R of this is reflected at the back surface. So the fraction exiting from the glass is

$$\frac{I_T}{I_o} = (1 - R)^2 = 0.922$$

So 92.2% of the light emerges from the glass.

Exercise E16.10 Compare the reflectance in air of polystyrene (refractive index 1.55) with that of PTFE (refractive index 1.36).

Answer. The reflectance is given by

$$R = \frac{I_R}{I_o} = \left(\frac{n-1}{n+1}\right)^2 .$$

Inserting the data in the question gives $R = 0.05$ for polystyrene and $R = 0.02$. The PTFE is much less reflective than the polystyrene.

Exercise E16.11 The refractive index of PMMA is 1.52. What would you estimate its dielectric constant ε_R to be?

Answer. The refractive index n is related to the dielectric constant ε_R at the same frequency by

$$n \approx \sqrt{\varepsilon_R}$$

Thus for PMMA

$$\varepsilon_R \approx n^2 = 2.31$$

Exercise E16.12 A crown glass sheet is partly coated by an acrylic film with thickness t_1 = 4 microns and refractive index n_1 = 1.49. A beam of red light strikes the plate at right angles to its surface. What is the difference in optical path length between a ray passing through the coated and the uncoated part of the glass? If the wavelength of red light is λ = 0.65 microns, is there potential for constructive or destructive interference between the two rays when they emerge from the glass?

Answer. The only difference between the paths followed by the two rays is that one passes through 4 microns of acrylic while the other passes through 4 microns of air. The optical path length through the coating is $n_1 t_1$. The optical path length of a parallel ray not passing through the coating is $n_o t_1$ where n_o is the refractive index of air, very close to 1. Thus the difference in optical path length for the two rays is

$$\Delta L_{opt} = n_1 t_1 - n_o t_1 = 0.49 \times 4 = 1.96 \text{ microns}$$

The ratio of this to the wavelength of red light is

$$\frac{\Delta L_{opt}}{\lambda} = \frac{1.96}{0.65} = 3.02$$

This is very close to an integral number of wavelengths. The emerging rays will be in phase, and thus reinforce if they interfere.

Exercise E16.13 A filter is required to screen out UV light (wavelengths λ around 0.35 microns) from crown glass windows. It is suggested that magnesium fluoride, MgF_2, (refractive index n_1 = 1.38) be used as an interference coating because it is hard wearing and easily deposited by physical vapor deposition. What thickness of MgF_2 coating is required to give maximum cancellation?

Answer. Design the coating to be most effective when the UV is travelling normal to the surface. If the thickness of the coating t_1 , then the difference in optical path length a ray that is reflected from the front surface of the coating and that reflected from the coating-glass interface is $2\, n_1 t_1$ where n_1 is the refractive index of the coating. We wish this to be one half wavelength of the UV light. Thus

$$2\, n_1 t_1 = \frac{\lambda}{2}$$

Inserting data for wavelength and refractive index gives the desired value of thickness

$$t_1 = 0.063 \text{ microns} = 63 \text{ nm.}$$

Exercise E16.14 A textured metal plate has a grid of fine parallel scratches with a regular spacing of d = 1.6 microns. Suppose white light falls normal to the surface of the plate. At what angle will the red component of the light, frequency $\nu = 5\times10^{14}$ per second, be diffracted most strongly when viewed normal to the scratches?

Answer. The illumination is a plane wave normal to the surface so that all incoming rays are in phase along the wave front. Each scratch on the surface scatters light – think of it as light source. Then difference in optical path for rays scattered from adjacent is

$$\Delta L_{opt} = n_o\, d \sin\theta\,.$$

where n_o is the refractive index of air, which we take to be 1. Reinforcement (constructive interference) occurs when this is an integral number of wavelengths. The wavelength in air of light of frequency $\nu = 5\text{x}10^{14}$ per second, is

$$\lambda = \frac{c}{\nu} = \frac{3\times10^{8}}{5\times10^{14}} = 6\times10^{-7}\,\text{m} = 0.6 \text{ microns}$$

Where c is the velocity of light in air. The angle is found by setting

$$d \sin\theta = i\lambda$$

Where i is an integer. Inserting d = 1.6 microns, $\lambda = 0.6$ and i = 1 gives

$$\sin\theta = 0.375 \quad \text{corresponding to} \quad \theta = 22^{\circ}$$

Exercise E16.15 Why are metals good reflectors of radiation?

Answer. Metals have an enormous number of very closely spaced levels in their conduction band; the electrons in the metal only fill part of this number. A photon with energy $h\nu$ can excite an electron only if there is an energy level that is exactly $h\nu$ above the Fermi level – and in metals there is. So all the photons of a light beam are captured by electrons of a metal, regardless of their wavelength. When the excited electrons drop back to their original energy level they release photons with exactly the same energy that excited it in the first place, but in a random direction. Any photons moving into the material are immediately recaptured, so none penetrate more than about 0.01µm (about 30 atom diameters) below the surface. All, ultimately, re-emerge from the metal surface – that is, they are reflected.

Exercise E16.16 It is proposed to replace soda glass windows of a green house with polycarbonate, PC. Will the PC windows reflect more? The refractive index of the glass is 1.5 and that of PC is 1.6.

Answer. The reflectivity of the glass, from equation (16.9b) of the text, is 0.04. That of the PC is 0.053. The polycarbonate windows will reflect 25% more light than the glass ones.

Exercise E16.17 Waterford glass and Steuben glass, used for expensive ornamental and cut-glass objects, are high lead glasses, meaning that they contain oxides of lead. Using the information shown on the chart of Figure 16.8, can you explain the choice of composition?

Answer. The sparkle and glitter of glass derives from refraction and reflection. The higher the refractive index and reflectance, the more brilliant the glass appears. Lead glasses have the highest values of both properties among glasses.

Exercise E16.18 An X-ray system has a beryllium window to transmit the beam. The absorption coefficient of beryllium for the wavelength of X-rays of interest here is 3.02×10^2 m^{-1}. If the window is 2 mm thick, what fraction of the incident beam intensity will pass through the window? The rest of the equipment is shielded with 4 mm of lead, with absorption coefficient for X-rays of 3.35×10^6 m^{-1}. What fraction of the intensity of the incident beam will escape through the casing?

Answer. 74% of the intensity of the radiation passes through the beryllium window. The intensity passing through the lead is essentially zero.

Exercise E16.19 What principle and material would you choose to make a light-sensing switch for a greenhouse?

Answer. The principle is that of photo-conduction: the change in resistivity of a material when exposed to light. Dielectrics are true insulators only if there are no electrons in the conduction band, since if there are any, a field will accelerate them giving an electric current. Dielectrics with a band-gap that is sufficiently narrow that the photons of visible light excite electrons across it become conducting (though with high resistance) when exposed to light. The greater the intensity of light, the greater the conductivity. This change in resistivity can be used to switch artificial light on and off in green houses.

Cadmium sulfide is an example of a photo-conductor.

Exercise E16.20 What principle and material would you choose to make a heat-sensing switch to turn the lights off in the garage when no one is moving around in it?

Answer. The principle is that of pyroelectricity – polarization caused by change of temperature. Some materials have a permanent dipole moment because their positive and negative ions balance electrically but are slightly out of line with each other. A thin disk of one of these, appropriately cut, has a dipole moment of its own. The dipole moment per unit volume of the material is the spontaneous polarization P_S. This net dipole moment exists in the absence of an applied electric field and is equivalent to a layer of bound charge on each surface. Nearby free charges such as electrons or ions are attracted to the surfaces, neutralizing the charge. If the temperature of the sample is constant, then so is P_S and no current flows through a circuit linking the faces of the disk. An increase in temperature, however, causes expansion and that changes the net dipole moment and the polarization. Redistribution of free charges to compensate for the change in bound charge induces a pyroelectric current in the circuit. Cooling rather than heating reverses the sign of the current. The pyroelectric current only flows while the temperature is changing – it is this effect that can be used for motion sensors.

Pyroelectric materials include minerals such as tourmaline, ceramics such as barium titanate, and polymers such as polyvinylidene fluoride.

Exercise E16.21 An optical fiber has a glass core with a refractive index of n_1 = 1.48, clad with a glass with refractive index n_2 = 1.45. What is the maximum angle that the incoming optical signal can deviate from the axis of the core while still remaining trapped in it?

Answer. When a beam passes from a material *1* of refractive index n_1 into a material *2* of index n_2 with an angle of incidence θ_1, it deflects to an angle θ_2, such that (equation (16.6) of the text)

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

If a beam in the core (material 1) enters the cladding (material 2) it is bent away from the normal to the interface, increasing θ_2. The largest value that θ_2 can have is 90^0, when $\sin\theta_2 = 1$; if θ_1 is increased any further the beam is totally internally reflected back into the core. The critical value of θ_1 is thus given by

$$\sin\theta_1 = \frac{n_2}{n_1} = \frac{1.45}{1.48} = 0.98.$$

This corresponds to an incident angle of 87.31^o. Thus the most that the beam can deviate from the axis of the fiber is 2.29^o.

Exploring design using CES

Use Level 2 unless otherwise stated.

Exercise E16.22 Use the ‘Search’ facility to find materials for

(a) lenses

(b) mirrors.

Answer.

(a) Lenses. Most of the materials found in the search are transparent to optical wavelengths. Silicon is transparent to infra-red and can be used as a lens for thermal imaging.

- Borosilicate glass
- Silica glass
- Soda-lime glass
- Silicon
- Silicone elastomers
- Cellulose polymers (CA)
- Polyamides (Nylons, PA)
- Polycarbonate (PC)
- Polymethyl methacrylate (Acrylic, PMMA)
- Polystyrene (PS)

(b) Mirrors. Most of the materials found in the search are used for mirror backings to support the thin silver or aluminum reflecting layer.

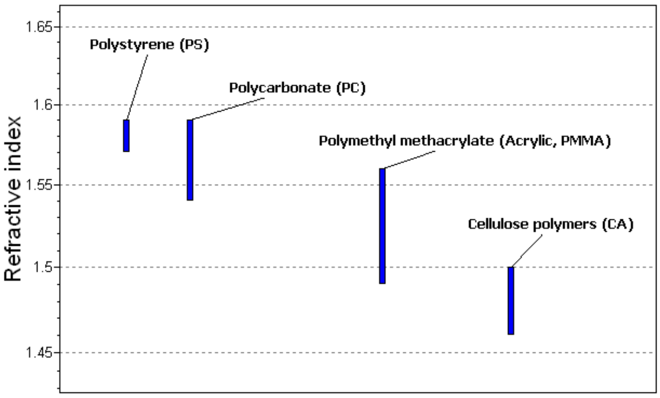
- Borosilicate glass
- Glass ceramic
- Silica glass
- Soda-lime glass
- Aluminum alloys
- Acrylonitrile butadiene styrene (ABS)
- Polystyrene (PS)
- Granite
- Non age-hardening wrought Al-alloys

Exercise E16.23 You are asked to suggest the best choice of cheap polymer for a new line of injection molded costume jewelry. Its transparency must be of Optical quality, and it should have the highest possible refractive index. The table summarizes the requirements.

Function	• *Plastic costume jewelry*
Constraints	• *Transparency of optical quality* • *Injection moldable* • *Price < $5/kg*
Objective	• *Maximize refractive index*
Free variable	• *Choice of material*

Answer. Use a Limit stage to apply the constraints on optical quality and price. Use a Tree stage to require that the material can be molded. Then make a Graph stage of refractive index, hiding the materials that failed to meet the constraints.

Four materials survive. Polystyrene, refractive index $n = 1.58$, has the highest value



Exercise E16.24 A material is required for the mirror backing of a precision reflecting telescope. It must have a modulus of at least 50 GPa so that it does not deflect under its own weight; it must be hard so that the surface (which will be silvered), once ground, does not distort; it must have the lowest possible thermal expansion to minimize thermal distortion and it must be able to be molded or cast to its initial shape before grinding. Use CES to find suitable candidates.

Function	• *Mirror backing*
Constraints	• *Young's modulus > 50 GPa* • *Processing: moldablity 5*
Objective	• *Highest hardness with lowest thermal expansion coefficient*
Free variable	• *Choice of material*

Answer. Soda lime glass or borosilicate glass.

Exercise E16.25 Find materials that are optically clear, have a moldablity rating of 5 and have excellent resistance to fresh and to salt water for use as contact lenses.

Answer.

- Borosilicate glass
- Polycarbonate (PC)
- Polymethyl methacrylate (Acrylic, PMMA)
- Polystyrene (PS)
- Soda-lime glass

Exploring the science with CES Elements

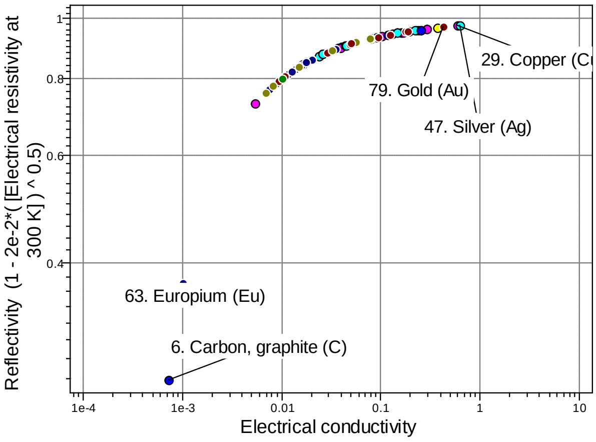
Exercise E16.26 The Hagen-Rubens law – an empirical law – says that the reflectivity of metals, R, is given by

$$R \approx 1 - 0.02\sqrt{\rho_e}$$

where the electrical resistivity ρ_e is in μΩ.cm. Make a chart with R on the *y*-axis and electrical conductivity $1/\rho_e$ on the *x*-axis. Use it to find the 3 elements with the highest reflectivities.

Answer.

The three elements with the greatest reflectivity are silver, copper and gold.

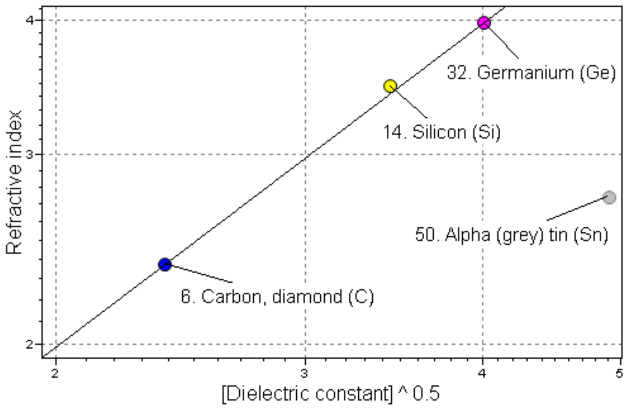


Exercise E16.27 The text explained why the refractive index n was related to the dielectric constant ε_R by the equation

$$n = \sqrt{\varepsilon_R}$$

Make a chart with $\sqrt{\varepsilon_R}$ on the *x*-axis and n on the *y*-axis to see how accurate this relationship is. Which elements best obey it?

Answer. Most elements are metals. Among those that are dielectrics, diamond (carbon), silicon, germanium and grey tin are transparent in the optical or near-optical range of wavelengths. The first three of these obey the equation very well. Tin does not.



Materials: engineering, science, properties, and design
3e Solution manual

Chapter 17: Exercises with worked solutions

Exercise E17.1 What plastic would you choose to contain hydrofluoric acid? Use Table 17.3 to find out.

Answer. The best choices (those with small green dots in Table 17.3 in the row for hydrofluoric acid) are polyethylene, polypropylene or PTFE (polytetrafluroethylene, Teflon).

Exercise E17.2 Use Table 17.3 to find a flexible polymer (that is, an elastomer) suitable to make a tube through which diesel oil will flow.

Answer. The table suggests silicone elastomers would be a good choice.

Exercise E17.3 A 600 liter (200 gallon) tank for storing rain water is made of aluminum. It is intended to for use in the garden, partly buried in top soil. The tank was extensively tested and found to survive without noticeable corrosion. The soil in which it was buried was peaty, and thus acidic. When marketed, customers complained of corrosion problems. On investigation it was found that the soil in the affected regions was clay, and strongly alkaline. Could this have been anticipated? Use Table 17.3 to find out.

Answer. The table reports that aluminum corrodes at an unacceptable rate in *Alkaline soil*.

Exercise E17.4 You rent a furnished apartment and discover, from the labels on the curtains, that those in the bedroom are made of a material with an LOI flammability rating of 17 and that those in the kitchen are made of a material with a UL94 rating of V-0. Would you be happy with these ratings?

Answer. The Underwriters Laboratory (UL) rating of V-0 is perfectly acceptable – it is self-extinguishing. The Limiting Oxygen Index (LOI) rating of 17 is not – it will burn rapidly in air.

Exercise E17.5 By what mechanisms do metals oxidize? What determines the rate of oxidation?

Answer. When is exposed to air a thin surface film of oxide forms, following the oxidation reaction

$$M(\,metal\,) + O(\,oxygen\,) = MO(\,oxide\,) + energy$$

causing the weight of the sample to increase. The weight-gain shows two different types of behavior. For some metals it is linear in time, meaning that the oxidation is progressing at a constant rate:

$$\frac{dm}{dt} = k_\ell \qquad \text{giving} \qquad \Delta m = k_\ell\, t$$

where k_ℓ is the *linear kinetic constant*. This is because the oxide film cracks (and, when thick, spalls off) and does not protecting the underlying metal. For other metals the oxide film is compact, coherent and strongly bonded to the surface. For these the weight gain is parabolic, slowing up with time, and this implies an oxidation rate with the form

$$\frac{d(\Delta m)}{dt} = \frac{k_p}{\Delta m} \qquad \text{giving} \qquad \Delta m^2 = k_p\, t$$

where k_p is the *parabolic kinetic constant.* The film, once formed, separates the metal from the oxygen. To react further either oxygen atoms must diffuse inward through the film to reach the metal or metal atoms must diffuse outward through the film to reach the oxygen. The driving force is the free energy of oxidation, but the rate of oxidation is limited by the rate of diffusion, and the thicker the film, the longer this takes.

Exercise E17.6 A 10 mm square sheet of a metal, exposed on both front and back surfaces, gains weight by 4.2 mg when heated for 20 hours in air at 300°C. If the kinetic of oxidation are linear, what is the linear rate constant k_l for this metal at 300°C?

Answer. Linear kinetics means that the weight gain per unit area, Δm (kg), after a time t (s) is

$$\Delta m = k_\ell \, t$$

Where k_ℓ is the linear kinetic constant. The mass gain is 4.2 x 10^{-6} kg, the area is 2 × 10^{-4} m^2 and the time t is 72,000 seconds, giving

$$k_\ell = 2.9 \times 10^{-7} \text{ kg/m}^2\text{.s}$$

Exercise E17.7 A 10 mm square sheet of a metal, exposed on both front and back surfaces, gains weight by 3.6 mg when heated for 10 hours in air at 700°C. If the kinetic of oxidation are parabolic, what is the parabolic rate constant k_p for this metal at 700°C?

Answer. Parabolic oxidation kinetics means that the weight gain per unit area, Δm (kg), after a time t (s) is

$$\Delta m^2 = k_p \, t$$

where k_p is the parabolic kinetic constant. The mass gain is 3.6 × 10^{-6} kg, the area is 2 × 10^{-4} m^2 and the time t is 36,000 seconds, giving

$$k_p = 9.0 \times 10^{-9} \text{ kg}^2 / \text{m}^4\text{.s}$$

Exercise E17.8 Copper oxidizes in air at 1000°C with parabolic kinetics, forming a surface film of Cu_2O. The parabolic rate constant at this temperature, k_p, is 2.1 × 10^{-6} kg^2 / m^4.s. What is the weight gain per unit area of copper surface after 1 hour? How thick will the oxide be after this time? (The atomic weight of copper is 63.5 kg/kmol, that of oxygen is 16 kg/kmol and the density of Cu_20 is 6000 kg/m^3).

Answer. Parabolic oxidation kinetics means that the weight gain per unit area, Δm (kg), after a time t (s) is

$$\Delta m^2 = k_p \, t$$

where k_p is the parabolic kinetic constant and the time t is 3,600 seconds, giving

$$\Delta m = 0.087 \text{ kg/m}^2 = 87 \text{ grams/m}^2.$$

The weight gain is due to take up of oxygen (the copper is there already). There are two atoms of copper for each atom of oxygen, the weight of the copper oxide after one hour is

$$\Delta m_{oxide} = \Delta m.\left(\frac{2x63.5+16}{16}\right) = 0.78\,\text{kg}$$

The thickness x of the oxide film is therefore

$$x = \frac{0.78}{6000} = 1.3 \times 10^{-4} \text{ m} = 0.13 \text{ mm or } 130 \text{ microns.}$$

Exercise E17.9 The oxidation kinetics of titanium to TiO_2 is limited by oxygen diffusion, with an activation energy Q_d of 275 kJ/mol. If the oxide film grows to a thickness of 0.08 microns after 1 hour at 800°C, how thick a film would you expect if it had been grown at 1000°C for 30 minutes?

Answer. Diffusion-limited oxidation has parabolic kinetics, meaning that the oxide film thickness, x, increases with time t as

$$x^2 = k_p\, t$$

where k_p is the parabolic rate constant:

$$k_p \propto \exp - \frac{Q_d}{RT}$$

If the thickness is x_1 after time t_1 at temperature T_1, then the thickness x_2 after a time t_2 at temperature T_2 is given by $\dfrac{x_2^2}{x_1^2} = \dfrac{k_2\, t_2}{k_1\, t_1} = \dfrac{t_2}{t_1} \exp - \dfrac{Q_d}{R}\left(\dfrac{1}{T_2} - \dfrac{1}{T_1}\right)$

Substituting the data (remembering to convert temperatures from C to K) gives $x_2 = 0.637$ microns.

Exercise E17.10 A soil is described as 'peaty'. What does this mean in terms of pH? The soil has an electrical resistivity of 2×10^8 μohm.cm. Would you expect it to be particularly corrosive? Use Table 17.2 to find out.

Answer. 'Peaty' means that the soil is acidic and thus has a pH < 7. According to Table 17.2 pure water has a resistivity greater than 2 x 10^{10} μohm.cm. A soil with a resistivity of 2×10^8 μohm.cm contains a high concentration of dissolved ions and thus is expected to be corrosive.

Exercise E17.11 Household vinegar has a pH of 2.8. What is the molar concentration of $[H^+]$ ions?

Answer. The pH value is defined as $pH = -\log_{10}[H^+]$ so the molar concentration of hydrogen ions $[H^+] = 10^{-2.8} = 1.58 \times 10^{-3}$.

Exercise E17.12 A solution of NaOH (caustic soda) has a pH of 10. What is the molar concentration of $[OH^-]$ ions?

Answer. The pH value is defined as $pH = -\log_{10}[H^+]$ so the molar concentration of hydrogen ions $[H^+] = 10^{-10}$. The product $[H^+].[OH^-] = 10^{-14}$ so the molar concentration of $[OH^-] = 10^{-4}$.

Exercise E17.13 A solution of copper sulfate, $CuSO_4$, contains 100 grams of $CuSO_4$ per liter of water. The atomic weight of copper is 63.5, that of sulfur is 32.1 and that of oxygen is 16.0 kg/kmol? What is the molecular concentration of copper ions, Cu^{2+}, in the solution?

Answer. One molecule of $CuSO_4$, when disoloved, creates one Cu^{2+} ion, so the molecular concentration of Cu^{2+} is that same as that of $CuSO_4$. The molecular weight of $CuSO_4$ is 159.6 kg/kmol. A one mol solution of $CuSO_4$ therefore contains 159.6 grams per liter of $CuSO_4$. Thus the molar concentration of $CuSO_4$ and thus of Cu^{2+} ions is

$$M = \frac{100}{159.6} = 0.599 \text{ mol/liter}$$

Exercise E17.14 What is meant by the standard reduction potential? A copper and a platinum electrode are immersed in a bath of dilute copper sulfate. What potential difference would you expect to measure between them? If they are connected so that a current can flow, which one will corrode?

Answer. If a metal – iron, say – is placed in a conducting solution like salt water, it dissociates into ions, releasing electrons, via a reaction of the type

$$\mathrm{Fe} \leftrightarrow \mathrm{Fe}^{2+} + 2e^-$$

The electrons accumulate on the iron giving it a negative charge that grows until the electrostatic attraction starts to pull the Fe^{2+} ions back onto the metal surface, stifling further dissociation. At this point the iron has a potential (relative to a standard, the *hydrogen standard*) of – 0.44 volts. Each metal has its own characteristic potential (called the *standard reduction potential*).

The chart of standard reduction potential (Figure 17.8 of the text) shows that the value for platinum is approximately +1.2 volts and that for copper is approximately +0.34 volts. The zero-current potential difference will therefore be approximately 0.86 volts. If the two metals are connected together in a cell, electrons will flow from the copper (the *anode*) to the platinum (the *cathode*); the copper ionizes (that is, it corrodes), following the anodic reaction

$$\mathrm{Cu} \leftrightarrow \mathrm{Cu}^{2+} + 2e^-$$

and – if the solution were one containing copper ions – copper ions, Cu^{2+}, plate out onto the platinum electrode following the cathodic reaction

$$\mathrm{Cu}^{2+} + 2e^- \leftrightarrow \mathrm{Cu}$$

Exercise E17.15 In a study of slowly propagating cracks in high strength steel plates under constant stress, it was found that in a moist air environment the crack growth rate increased with temperature as follows:

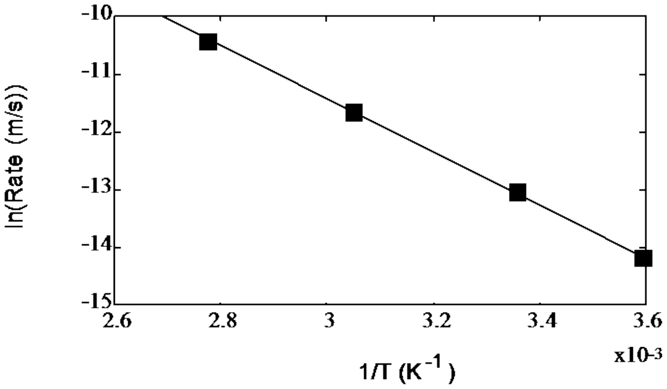
Growth rate (µm s^{-1})	0.70	2.20	8.70	29.1
Temperature (°C)	5	25	55	87

Show, using an appropriate plot, that for these conditions crack propagation is a thermally activated process, and determine the activation energy. It is believed that diffusion of one of the elements listed below is the rate-controlling mechanism. Decide which of these elements is likely to be involved.

Diffusing element in α iron	Activation energy for diffusion (kJ/mol)
hydrogen	38
nitrogen	72
carbon	84
iron	285

Answer. For a thermally activated process: $Rate = A \exp\left(-\dfrac{Q}{RT}\right)$, i.e. $\ln(Rate) = \ln(A) - \dfrac{Q}{RT}$

To check whether this applies, plot ln(Rate) against 1/T (in *Kelvin*), as in the figure. The result is a straight line, meaning that this is indeed a thermally activated process.



We get a straight line – this is indeed a thermally activated process. The slope is

$$-Q/R = -(4.2/0.92\times10^{-3})\ \mathrm{K}$$

Using R = 8.314 J/mol.K we find $Q = 8.314\times(4.2/0.92\times10^{-3}) = 38{,}000 = 38\ \mathrm{kJ/mol}$.

Comparing this with the values of Q in the table suggests that the diffusion of hydrogen is involved in the rate-controlling mechanism.

The probable cause of crack growth is *hydrogen embrittlement*. Hydrogen is formed by the reduction of water on the new metal surface produced at the growing tip. Hydrogen diffuses easily in iron, even at room temperature, and accumulates most readily in the stressed zone ahead of the tip. This makes the crack tip region more brittle, allowing the crack to grow. The rate-controlling (slowest) step in this process in the diffusion of hydrogen.

Exercise E17.16 A polymer coating is required to protect components of a microchip processing unit from attack by hydrogen fluoride (HF). Use Tables 17.3 and 17.4 to identify possible choices.

Answer. Table 17.3 ranks a number of polymers has A (Excellent durability) in HF:

Elastomers	Isoprene (IR) Neoprene (NR)
Polymers	Ionomer (I) Polycarbonate (PC) Polyethylene (PE) Polypropylene (PP) Polytetrafluorethylene (PTFE)

Table 17.4 lists PTFE and other fluorocarbon polymers as preferred materials and coatings. PTFE looks like the best choice.

Exercise E17.17 Metal pipework on an oil rig must carry hydrochloric acid solution to acidify the well. Use Tables 17.3, 17.4 and 17.6 to explore ways of providing and protecting the pipe.

Answer. Table 17.3 gives an A (Excellent durability) rating for durability in HCl to only two classes of metals:

Metals	Stainless steel Titanium alloys

Table 17.4 gives a longer list of preferred metals, many of them very expensive. It also lists

Metals	Titanium alloys Nickel alloys Molybdenum Tantalum Zirconium Precious metals
Polymers and composites	HDPE PP GFRP
Ceramics	Glass

Titanium would appear to be the best, though expensive, choice: its inherent resistance to attack by HCl is high, and inhibitors exist that give added protection. The alternative, suggested by the table, is ordinary steel pipe work that is plated with molybdenum or tantalum, or (better) lined with HDPE or enameled.

Table 17.6 reveals that a large number of inhibitors exist to slow the rate of attack of steel by HCl:

Steel (Fe)	Ethylaniline, mercaptobenzotriazole, pyridine and phenylhydrazine, ethylene oxide
Aluminum (Al)	Phenylacridine, napthoquinone, thiourea
Titanium (Ti)	Chromic acid, copper sulfate

These offer an alternative, or supplementary way of achieving acceptable durability.

Exercise E17.18 A food processing plant uses dilute acetic acid for pickling onions. The acid is piped to and from holding tanks. Select a suitable material for the pipes and tanks, given that, to have sufficient strength and toughness to tolerate external abuse they must be made of a metal.

Answer. Table 17.3 ranks three classes of metals as A (Excellent durability) in acetic acid:

Metals	Stainless steel Nickel alloys Titanium alloys

Table 17.4 confirms this choice, but also suggests aluminum as a possibility. Since this is a food processing facility, no corrosion can be tolerated because of the contamination it would cause. It is therefore wise to look at coatings. The table suggests that HDPE and PTFE both survive HCl, and both are already used in food processing equipment. The safest choice would be stainless steel lined with PTFE. Inhibitors are unacceptable in food processing fluids.

Exercise E17.19 An automaker is concerned about the consequences of the introduction of bio-methanol, CH_3OH or bio-ethanol C_2H_5OH into auto fuels. The particular concern is the corrosion of aluminum components, particularly the engine block, by methanol or ethanol. What steps could be taken to avoid this?

Answer. Table 17.3 ranks Aluminum as poor in methyl alcohol (methanol) but excellent in ethanol. Table 17.6 reveals that the addition of sodium chlorate and nitrate inhibits corrosion of aluminum by methanol, but it is not clear that this can be added to auto fuel. Best is to lobby against the use of methanol.

Exercise E17.20 The automaker of Exercise E17.19 is also concerned that spillage of methanol or ethanol bio-fuel might damage the GFRP bodywork of some models. Is the concern justified?

Answer. Table 17.3 confirms the concern: GFRP and CFRP have poor durability in both ethanol and methanol. If these are included in the fuel in any quantity it may be necessary to use an alcohol-resistant coating in the paint layers.

Exercise E17.21 The waste stream of a fertilizer plant includes dilute sulfuric acid. The dilute acid is stored in surface tanks some distance from the plant. It is suggested that the ducting to carry the acid to the tanks could, most economically, be made of wood. Is this a totally crazy suggestion?

Answer. Wood is rated B (Acceptable durability) in 10% sulfuric acid. The choice may, indeed, be the most economical solution.

Exercise E17.22 *Using the Nernst equation.* A copper electrode is put in an electrolyte that is a solution of 3 grams of copper as Cu^{2+} ions in a liter of water. What is its corrected reduction potential at 25^{o}C? The atomic weight of copper is 63.5 kg/kmol.

Answer. The zinc ions have a valence $z = 2$, a standard reduction potential E_o of – 0.76 and a molar concentration

$$C_{ion} = \frac{3}{63.5} = 0.047 \;\; M$$

Thus the corrected reduction potential is

$$E = +0.34 + \frac{0.059}{2} \log_{10}(0.047) \; = -0.76 - 0.045 = +0.30 \text{ Volts}$$

Exercise E17.23 *Using the Faraday equation.* What current and time are needed to plate a 1 mm thick layer of copper onto an object with a surface area of 0.1m^2? The density of copper is 8940 kg/m^3, its atomic weight is $M =$ 63.5 kg/kmol and copper ions have a valence of $z = 2$.

Answer. The volume of copper required per unit area is $10^{-3} \times 10^{-1} = 10^{-4}\ \text{m}^3$. This corresponds to a mass of $w =$ 0.894 kg of copper. Inverting Faraday's equation gives

$$I\,t = \frac{w\,z\,F}{M}$$

where I is the current in amps, and t is the time in seconds. Inserting values for the parameters gives

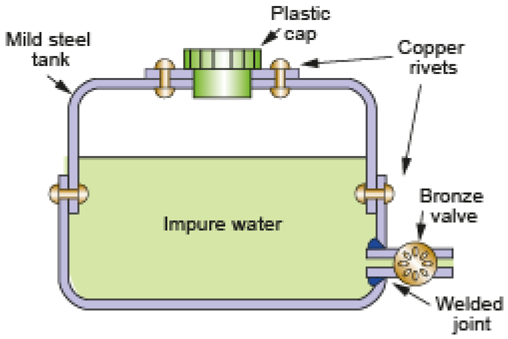
$$I\,t = 2{,}720 \text{ A.s}\,.$$

Thus the plating requires 0.1 amp for 27,200 seconds (7.5 hours) or 1 amp for 2,720 seconds (45 minutes).

Exercise E17.24 Mild sheet-steel guttering is copper-plated to protect it from corroding. The guttering acts as a drain for sea water. If the coating is damaged, exposing the steel, will the guttering corrode in a damaging way? If instead the guttering is zinc-plated, will it be better or less well protected? Use the Galvanic Series in sea water to find out.

Answer. Copper lies above iron (steel) in the scale of Galvanic Series in sea water. If the steel is exposed, the steel will be attacked. Since the area of the iron anode is small and that of the copper cathode is large, the local attack on the iron is severe. Zinc lies below iron in the same Galvanic Series. If instead the steel is zinc-coated, the zinc is the anode and the steel is the cathode. The zinc is slowly corroded but the iron is protected.

Exercise E17.25 The diagram show a proposed design for an outdoor water tank. What aspects of the design might cause concern for corrosion?



Answer. The following features are causes for concern.

- The steel tank will suffer galvanic corrosion attack because both copper and bronze lie above steel in the Galvanic Series and in the Standard Reduction Potential tables.
- The upward-facing ledges on the outside of the tank and the overlapping horizontal interfaces at and around the plastic cap will trap moisture in crevices, leading to crevice corrosion.
- The-inward facing welds are more vulnerable to attack – they should be outward-facing.
- The position of the drain prevents complete emptying of the tank, risking internal corrosion.

17.11 Exploring design with CES

Use Level 2 unless otherwise stated.

Exercise E17.26 Find, by using ‘Browse’ or ‘Search’, the record for the nickel-chromium alloys called Nichromes. What are their main applications?

Answer. Heating elements and furnace windings; bi-metallic strips; thermocouples; springs; food processing equipment; chemical engineering equipment.

Exercise E17.27 Find, by ‘Browse’ or ‘Search’, the record for Polymer powder coating (remember to search in the Process Universe, not the Materials Universe). What are the three ways of applying a polymer powder coating?

Answer. Electrostatic spray methods, methods using a fluidized bed and methods using an electrostatic bed.

Exercise E17.28 Use a ‘Limit’ stage, applied to the Level 2 Surface Treatment data-table to find surface treatment processes that impart resistance to gaseous corrosion.

Answer. Anodizing
Electroless plating
Electroplating
Vapor metallizing (PVD)
Vitreous enameling

Exercise E17.29 Use a ‘Limit’ stage, applied to the Level 2 Surface Treatment data-table to find surface treatment processes that impart resistance to aqueous corrosion.

Answer. Anodizing
Electroless plating
Electrophoretic and Autophoretic Painting
Electroplating
In-mold decoration (IMD)
Nitriding
Organic solvent-based painting
Polymer powder coating
Vapor metallizing (PVD)
Vitreous enameling
Water-based painting

Exercise E17.30 Use the CES ‘Search’ facility to find materials for food processing equipment.

Answer. Stainless steel
Silicone elastomers
Nickel-chromium alloys

Exercise E17.31 Plastic cases for electrical plugs and switch-gear should not be made of flammable materials. Use the ‘Select’ facility in CES to find polymers that are non-flammable or self-extinguishing.

Answer. Epoxies
PTFE
Phenolics
Polyetheretherketone (PEEK)
Silicone elastomers

Exercise E17.32 A vat is required to hold hot caustic soda, NaOH, a strong alkali. Use the 'Select' facility in CES to find metals that resist strong alkalis very well.

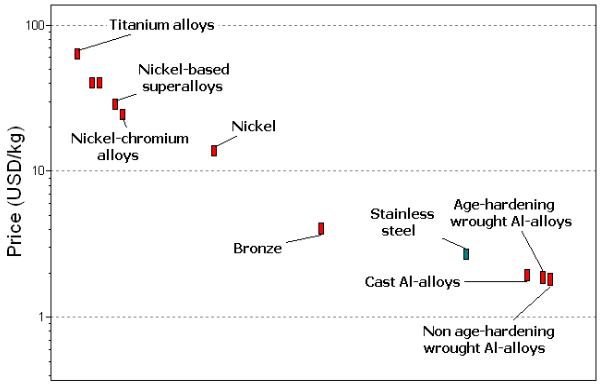
Answer. Copper
Nickel
Nickel-based superalloys
Nickel-chromium alloys
Stainless steel

Exercise E17.33 Pipework is required for a gherkin-pickling plant to carry vinegar (a dilute acid) at 100°C from one vat to another. The liquid is under pressure, requiring a material with a strength of at least 100 MPa, and for ease of installation it must be able to be bent, requiring a ductility of at least 10%. Find the four cheapest materials that meet the constraints, summarized below.

Function	• *Pipework for hot acetic acid*
Constraints	• *Durability in dilute acid = very good* • *Maximum operating temperature > 100°C* • *Yield strength > 100 MPa* • *Elongation > 10%*
Objective	• *Minimize cost*
Free variable	• *Choice of material*

Answer. A number of materials meet the constraints on durability, operating temperature, yield strength and elongation:

- Age-hardening wrought Al-alloys
- Bronze
- Cast Al-alloys
- Commercially pure titanium
- Nickel
- Nickel-based superalloys
- Nickel-chromium alloys
- Non age-hardening wrought Al-alloys
- Stainless steel
- Titanium alloys
- Tungsten alloys



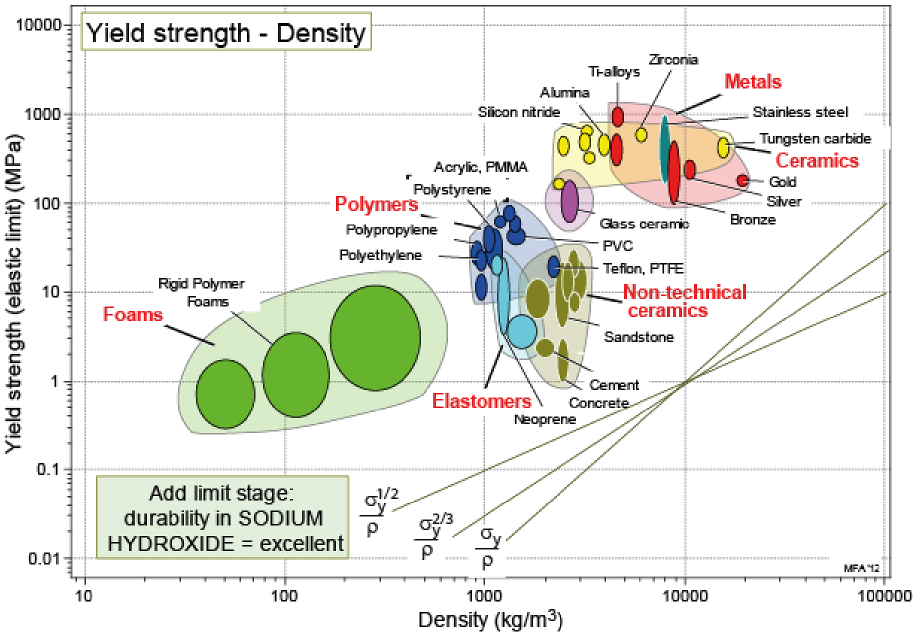
Those with the lowest price are aluminum alloys and stainless steel.

Exercise E17.34 Find materials that have excellent resistance to corrosion in 10% NaOH (sodium hydroxide) and have high strength-to-weight ratio, σ_y / ρ.

- Open CES Edu Level 2 and select the subset 'Edu Level 2 with durability properties'
- Open a LIMIT stage. Open the relevant 'Durability' folder. Find the environment and choose 'Excellent', then Apply.
- Open a GRAPH stage. Make a chart with 'Density' on the *x*-axis and 'Yield strength' on the *y*-axis.
- Put on a selection line of slope 1, click above the line and than move it upwards to leave a small subset of materials. What do you find?

Answer. The chart below shows yield strength and density for materials that are excellent in 10% NaOH. The five materials with the highest strength-to-weight ratio are:

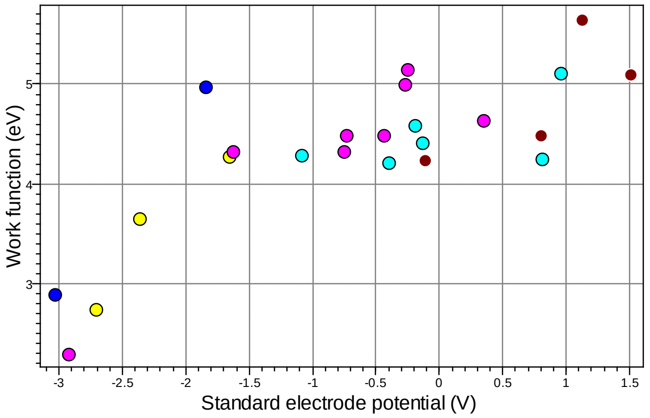
- Boron carbide
- CFRP, epoxy matrix (isotropic)
- Silicon carbide
- Silicon nitride
- Titanium alloys



17.12 Exploring the science with CES Elements

Exercise E17.35 The Work function (Chapter 14) is the energy required to pluck an electron from the top of the Fermi level of a crystal and drag it away until it is isolated in vacuum. The Standard reduction potential of Figure 17.8 involves electrons dropping into energy levels just above the Fermi level. You might suspect that the two were, in some way, related. Make a graph with Standard reduction potential on the x-axis and Work function on the y-axis to find out if they are. What is your conclusion?

Answer. The plot shows that there is a correlation between electrode potential and work function: the higher the work function the higher is the electrode potential. The relationship is not, however, a simple one. The standard reduction potential is a bulk property characteristic of the element. The work function is a surface property; it depends on the way in which the electron wave-functions (the way electrons orbit the atoms) are modified by the presence of the surface and by any foreign atoms on it.



Materials: engineering, science, properties, and design
3e Solution manual

Chapter 18: Exercises with worked solutions

Exercise E18.1 (a) Explain briefly why databases for initial process selection need to be subdivided into generic process classes, while material selection can be conducted on a single database for all materials.

(b) After initial screening of unsuitable processes, further refinement of the process selection usually requires detailed information about the materials being processed and features of the design. Explain why this is so, giving examples from the domains of shaping, joining and surface treatment.

Answer.

(a) The different process families (i.e. shaping, joining and surface treatment) fulfill different functions and do not therefore compete with one another. The attributes which describe the capabilities of the processes (technical, quality and economic) differ significantly between process families, requiring different record contents in the database. Material properties are broadly universal to all materials, so a single database suffices.

(b) Initial screening handles most technical requirements and some aspects of quality. Many design requirements are complex – the result of using a process is coupled to details of the material being processed and/or details of the design. Examples:

Shaping:

- avoiding defects: the defects formed in shaping depend on the process class (e.g. casting, forging, powder), vary from alloy to alloy, and depend on component details such as the aspect ratio of thin sections or changes in section.

- product properties, such as strength: sensitive to alloy, particularly if a heat treatment forms an integral part of the shaping process (e.g. extrusion).

Joining:

- joint properties: strength and fracture toughness of welds depend on alloy (e.g in steels, the hardenability), and the thermal history, particularly cooling rate (which depends on design features such as thickness, and process conditions such as power and speed).

Surface Treatment:

- wear resistance: depth and hardness of surface treatments for wear resistance are sensitive to process conditions and alloy used, and the consequent wear resistance depends in a complex way on service loads, environment etc.

Exercise E18.2 A manufacturing process is to be selected for an aluminum alloy piston. Its weight is between 0.8 and 1 kg, and its minimum thickness is 4-6 mm. The design specifies a precision of 0.5 mm and a surface finish in the range 2-5 μm. It is expected that the batch size will be around 1000 pistons. Use the process attribute charts earlier in the Chapter to identify a subset of possible manufacturing routes, taking account of these requirements. Would the selection change if the batch size increased to 10,000?

Answer.

Inputs used for aluminum alloy piston:

- Material: aluminum alloy
- Mass range: 0.8 – 1 kg
- Minimum thickness: 4-6 mm
- Shape: 3D solid (part hollow)
- Tolerance: < 0.5 mm
- Roughness: < 5 μm
- Economic batch size: 1000, or 10,000.

Material: all metals processes apply - sand, die, investment, and low pressure casting; forging, extrusion and sheet forming; powder forming; electro-machining and conventional machining.

Mass range: sand casting and extrusion marginal.

Thickness: sand casting marginal.

Shape: extrusion and sheet forming excluded for 3D shapes.

Tolerance and Roughness: only sand casting excluded, but both could be rectified by machining.

Combining these screening stages, many options remain: sand casting (plus machining), die, investment, and low pressure casting; forging; powder forming; machining.

Considering economic batch size, for the surviving processes:

- for 1000 units: sand casting (plus machining), investment casting; forging; machining.
- for 10,000 units: sand casting (plus machining), die and low pressure casting; forging; powder forming; machining.

In this example, only shape and economic batch size provide some discrimination. Further refinement of the selection requires a more detailed cost analysis. Experience indicates that die casting or forging are the main contenders. Note also that the choice of casting vs. forging implies a different type of aluminum alloy in each case (and hence properties).

Exercise E18.3 The choice of process for shaping a CD case was discussed in section 18.5, but what about the CDs themselves? These are made of polycarbonate (another thermoplastic), but the principal difference from the case is precision. Reading a CD involves tracking the reflections of a laser from microscopic pits at the interface between two layers in the disk. These pits are typically 0.5μm in size and 1.5μm apart, and so the tolerance (and roughness) must be rather better than 0.1 μm for the disk to work.

First estimate (or measure) the mass and thickness of a CD. Use the charts to find a short-list of processes that can meet these requirements, and will be compatible with the material class. Can these processes routinely achieve the tolerance and finish needed? See if you can find out how they are made in practice.

Answer.

Initial constraints:

Material: polycarbonate; mass of CD ≈ 15g; thickness ≈ 1.2mm.

From the charts (Figs. 18.2, 18.3, 18.4): Processes passing: injection molding, blow molding.

From the material-shape matrix (Fig. 18.9), blow molding is only applicable to dished sheet and 3D hollow. CDs could be considered to be “flat sheet”, but the exclusion of injection molding in the matrix is associated with larger working areas of flat material. Injection molding is clearly an option, via the compatibility with “3D solid”.

From Figs. 18.11 and 18.12, injection molding can achieve surface finish close to 0.1 μm, but the best tolerance is usually around 100 μm – so much too imprecise for the features on the CD.

The problem of precision is overcome in practice by placing a pre-patterned metal stamper plate on one face of the injection molding die, carrying the pattern of precise bumps needed to print the pits on the CD. There are various ways of producing the metal stampers. For example, a glass mster disc is coated with a light-sensitive material (such as a photo-resist), treated by laser pulses, and etched to produce the pattern of pits. Then the glass disk is metallised with nickel to a thickness of about 1/3 mm, to produce the metal stamper with the correct pattern of raised bumps. The much greater stiffness and hardness of glass and nickel enable the necessary precision and finish to be obtained. Injection molding of the polycarbonate discs is then conducted under normal processing conditions, taking a few seconds.

Exercise E18.4 A process is required for the production of 10,000 medium carbon steel engine crankshafts. The crankshaft is a complex three-dimensional shape about 0.3m in length with a minimum diameter of 0.06m. At the position of the bearings, the surface must be very hard, with a surface finish better than 2μm and a dimensional accuracy of ± 0.1mm. Elsewhere the surface finish can be 100μm, and the tolerance is 0.5mm. Use the process attribute charts and other information in the chapter to identify a suitable fabrication route for the crankshaft.

Answer. See annotated process attribute charts below.

Shape: complex 3D solid – so need a casting process, or forging, or powder methods.

Material: medium carbon steel – some casting processes (and extrusion).

Economic batch size: sand casting, forging and power methods all appear economic for 10,000 units.

Mass: very approximately a long cylinder 30cm × diameter 6cm.

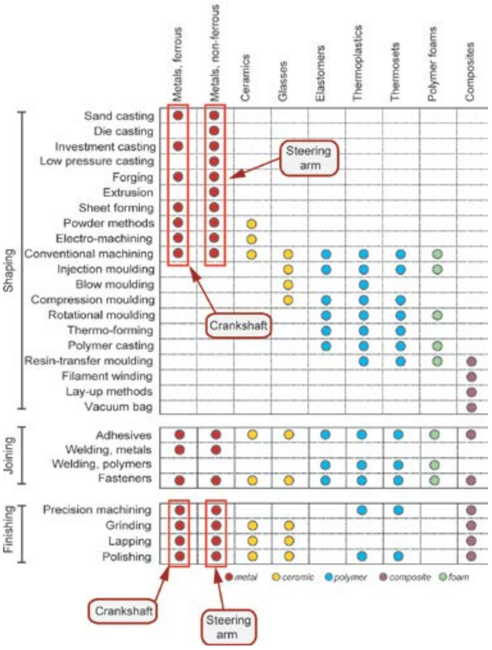
Volume = $\pi \times (0.03)^2 \times 0.3 = 8.5 \times 10^{-4}$ m^3, so for density of steel = 7.9 Mgm^{-3}, mass ≈ 6.7 kg.
All processes viable at this mass.

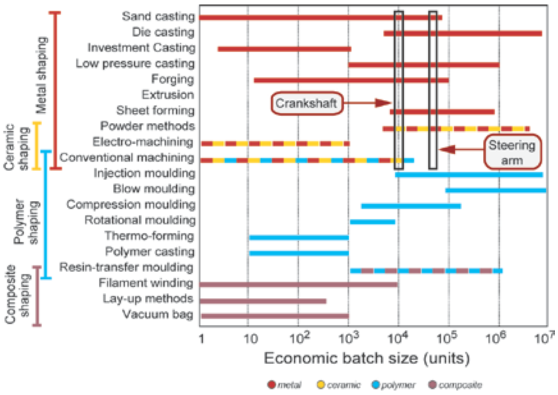
Minimum Thickness: 6cm
Sand casting and forging, are the only options that also work with steel; too large for powder methods.

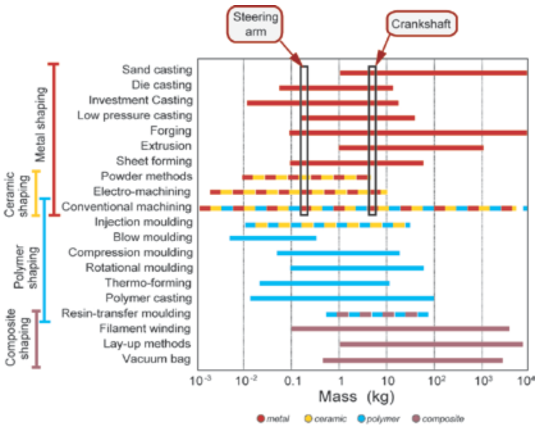
Tolerance: $\pm$ 0.1mm on critical surfaces, $\pm$ 0.5mm elsewhere.
Precision is marginal for even the best net-shape processes, but the higher quality tolerance target is not achievable by forging or sand casting – hence require post-shaping machining process to obtain target precision on the bearing surfaces.
For the rest of the component, forging can achieve the target precision – sand casting would require all-round machining to meet the specification, adding significantly to costs.

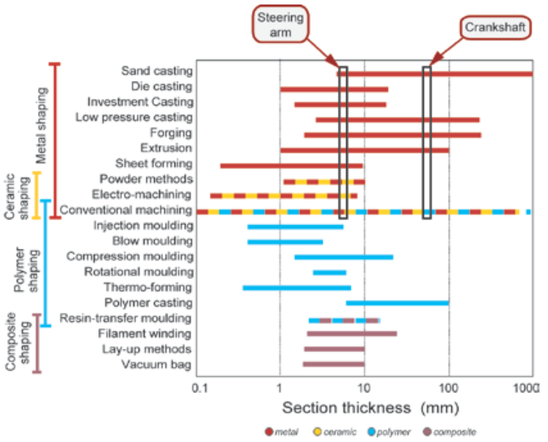
Roughness: 2µm finish beyond the capability of forging, so machining also needed to give the target finish on the bearing surface (possibly with a final grinding/polishing step). Elsewhere all processes achieve the required finish.

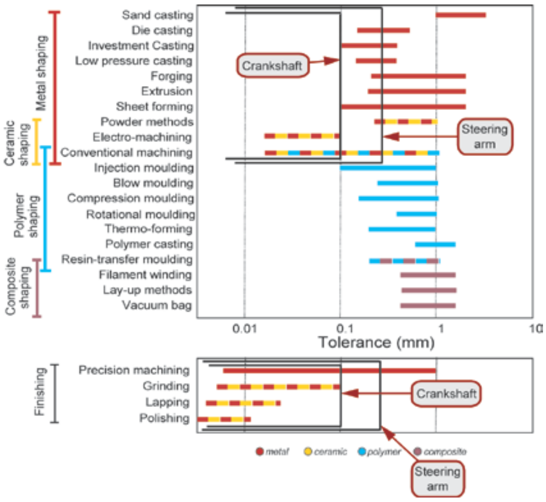
Final selection: forging offers the best option, with machining of the bearing surfaces. Note that this gives the additional benefit of a wrought microstructure (as opposed to a cast microstructure). Component is likely to be hot forged (to reduce forming loads) and to have a further heat treatment (e,g, quench and temper) to give the best combination of yield stress and fracture toughness. For the bearing surfaces, there is also the possibility of a surface hardening treatment (provided that this does not change the dimensions or the surface finish) – e.g. carburising, transformation hardening etc.

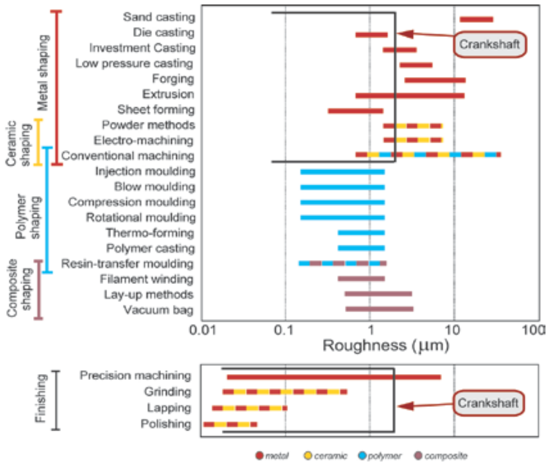












Exercise E18.5 A heat-treatable aluminium alloy automotive steering arm has a mass of 200g, a minimum thickness of 6mm and a required tolerance of 0.25mm. Production runs over 50,000 are expected. Discuss the implications for the manufacture of the component by forging, using the process attribute charts. Suggest an alternative processing route, commenting on how this might influence the choice of Al alloy and final properties.

Answer. See annotated charts above.

Shape: complex 3D solid – so need a casting process, or forging, or powder methods.

Material: aluminium alloys – no restriction on shaping process.

Economic batch size: casting (sand, die, low pressure), forging and power methods all appear economic for 50,000 units.

Mass: 0.2 kg. Too small for sand casting, other processes remain viable.

Minimum Thickness: 6mm. Marginal for sand casting, other processes OK.

Tolerance: ± 0.25mm.

Sand casting unable to achieve target precision, other processes OK. For the complex 3D shape it is unlikely to be viable to use machining over the whole surface to improve the precision.

Conclusion: forging passes all stages, as do die casting and low pressure casting. No surface finish target was provided but, as for many moving parts in an automotive application, fatigue will be limiting so a good finish is needed. The roughness chart shows that both casting options are as good as forging, with die casting the best. Choosing either casting process would require a change to an Al casting alloy. The challenge will be in matching the key material properties (strength, fatigue resistance and fracture toughness) – a heat-treatable casting alloy will be required.

Exercise E18.6 A small polyethylene bucket is to be manufactured by injection moulding or rotational moulding. The designer wishes to estimate the manufacturing cost for various batch size, using the cost equation

$$\text{Cost perpart } C \;=\; C_{material} \;+\; \frac{C_{dedicated}}{n} \;+\; \frac{\dot{C}_{capital} + \dot{C}_{overhead}}{\dot{n}}$$

Define the meaning of all of the parameters in the equation. Using the data in the Table, determine the cheapest process for batch sizes of 1000 and 50,000.

Process	$C_{material}$ (£)	$C_{dedicated}$ (£)	$\dot{C}_{capital} + \dot{C}_{overhead}$ (£/hr)	$\dot{n}$ (parts/hr)
Injection moulding	0.25	5000	20	120
Rotational moulding	0.20	1000	20	40

Answer.

$C_{material}$ is the cost of the material used per part (allowing for any losses)

$C_{dedicated}$ covers the cost of tooling (dies, moulds, fixtures) assigned wholly to this part

$\dot{C}_{capital}$ is the capital cost of equipment, spread over a suitable write-off period to give a cost/unit time

$\dot{C}_{overhead}$ covers all other background hourly operational costs (labour, energy and so on)

n is the batch size or production volume (the number of parts in the production run)

$\dot{n}$ is the production rate

Process	$C_{material}$ (£)	$\frac{\dot{C}_{capital} + \dot{C}_{overhead}}{\dot{n}}$ (£)	n	$\frac{C_{dedicated}}{n}$ (£)	Total cost per part C (£)
Injection moulding	0.25	0.17	1000	5	5.42
			50000	0.1	0.52
Rotational moulding	0.20	0.5	1000	1	1.7
			50000	0.02	0.72

Hence rotational moulding is cheapest for 1000 units, but injection moulding cheapest for 50,000.

Exercise E18.7 In a cost analysis for casting a small aluminium alloy component, costs were assigned to tooling and overheads (including capital) in the way shown in the table below. The costs are in units of the material cost of one component. Use the simple cost model presented in equation (18.7) to identify the cheapest process for a batch size of (i) 100 units and (ii) 10^6 units.

Process	Sand Casting	Investment casting	Pressure die	Gravity die
Material, $C_{material}$	1	1	1	1
Capital + Overhead, $\dot{C}_{capital} + \dot{C}_{overhead}$ (hr^{-1})	500	500	500	500
Dedicated, $C_{dedicated}$	50	11,500	25,000	7,500
Rate $\dot{n}$ (hr^{-1})	20	10	100	40

Answer.

$$\text{Cost per part } C \;=\; \overset{\text{material cost}}{\overset{\downarrow}{C_{material}}} \;+\; \overset{\text{dedicated cost}}{\overset{\downarrow}{\frac{C_{dedicated}}{\underset{\underset{\text{no. of parts}}{\uparrow}}{n}}}} \;+\; \overset{\text{capital cost}\quad\text{overhead cost}}{\overset{\downarrow\qquad\qquad\downarrow}{\frac{\dot{C}_{capital} + \dot{C}_{overhead}}{\underset{\underset{\text{production rate}}{\uparrow}}{\dot{n}}}}}$$

Sand Casting: $C = 1 + \dfrac{500}{20} + \dfrac{50}{n} = 26 + \dfrac{50}{n}$,

so for $n = 100$, $C = 26.5$; for $n = 10^6$, $C = 26$

Investment Casting: $C = 1 + \dfrac{500}{10} + \dfrac{11{,}500}{n} = 51 + \dfrac{11{,}500}{n}$,

so for $n = 100$, $C = 166$; for $n = 10^6$, $C = 50$

Pressure Die Casting: $C = 1 + \dfrac{500}{100} + \dfrac{25{,}000}{n} = 6 + \dfrac{25{,}000}{n}$,

so for $n = 100$, $C = 256$; for $n = 10^6$, $C = 6$

Gravity Die Casting: $C = 1 + \dfrac{500}{40} + \dfrac{7{,}500}{n} = 13.5 + \dfrac{7{,}500}{n}$,

so for $n = 100$, $C = 88.5$; for $n = 10^6$, $C = 13.5$

(i) for $n = 100$ units choose sand casting

(ii) for $n = 10^6$ units choose pressure die casting.

Exercise E18.8 Look at the products around your house (or your garage, or garden shed), identifying as many different types of *joints* as you can, and the process used. Select a few examples, and draw up a list of design requirements that the joining process needed to satisfy.

Sample answer.

Bicycle: welded or brazed tubes; nut-and-bolt attachments (brakes, mudguards, accessories); elastic spring clips (light fittings); rivets (mudguard stay brackets); adhesives (handlebar tape).

Examples of other types of joints: screws, adhesives, slotted press-fits (wooden furniture); screw threads (light bulb fixtures); mechanical clinching and seam welds/spot welds (radiators).

Design requirements for bicycle joints:
- materials to be joined, and whether similar/dissimilar (e.g. alloy steel tubing; Al alloy crank onto carbon steel axle)
- shape of parts (intersecting tubes, or press-fit of rectangle into a hole)
- joint geometry (inclined butt joint of welded tubes, or sleeve joint of one tube within another in brazed tubes)
- material thickness
- need to disassemble (permanent for welds, reversible for bolted fixtures/accessories – a particular issue for dissimilar alloys, which may seize due to corrosion)
- weld quality (effect on properties, e.g. welds can influence fatigue strength)

Exercise E18.9 Look at the products around your house (or your garage, or garden shed), identifying as many different types of *surface treatments* as you can. Select a few examples, and draw up a list of design requirements that the surface treatment process needed to satisfy, in particular the function of the surface treatment.

Sample answer.

Anything painted – woodwork, vehicles, bicycles….. (both decorative, and for corrosion resistance).
Polymer powder coating on metals: hi-fi and computer units, filing cabinets, kitchen racks and shelf brackets.
Chrome plate (and other metal finishes) on steel: bath and kitchen fixtures, handles, automotive trim.
More difficult to identify are surface hardening treatments – e.g. cutting edges on steel tools, clamps on vices – but most of these are 'hidden' (sliding surfaces in gears, bearings, pistons etc).

Design requirements for the edge of a steel cutting tool:
- function of treatment: hardening (for wear resistance)
- material compatibility: depends on detail of which steel, and process mechanism (e.g. whether hardenable or not)
- retention of bulk properties (in particular, avoidance of embrittlement)
- part geometry and thickness, and proportion of surface to be treated
- dimensional precision/finish: does treatment change these?

Exploring design with CES

Exercise E18.10 Use the 'Browse' facility in CES to find:

(a) the record for the shaping process Injection molding, thermoplastics. What is its economic batch size? What does this term mean?

(b) the machining process Water-jet cutting (records for machining processes are contained in the Shaping data table). What are its typical uses?

(c) the joining process Friction-stir welding. Can it be used to join dissimilar materials?

(d) the surface treatment process Laser hardening. What are the three variants of this process?

Answer.

(a) The economic batch size is the number of units of output at which the process is usually most competitive. The value for injection molding, thermoplastics is $10^4 – 10^6$.

(b) The typical uses of water-jet cutting (WJM) are cutting paper, cardboard and other packaging. In the electronics industry WJM is used to cut circuit boards. On a smaller scale it is used to cut non-reinforced polymers, rubber, leather, woven and random-mesh fabrics, polymer foams and foods. Abrasive water jet cutting (AWJM) can be used to cut almost anything. It is particularly suited for cutting non-homogeneous materials such as composites, and for cutting metals when properties are sensitive to high temperature or to the work hardening and surface damage caused by other cutting techniques.

(c) Friction-stir welding can be used to join dissimilar materials.

(d) The three variants of laser surface hardening are: surface transformation hardening, surface shocking, and surface melting and glazing.

Exercise E18.11 Use the 'Search' facility in CES to find:
(a) Processes used for boat building
(b) Processes to make bottles
(c) Processes to make tail light assemblies
(d) Processes for decoration

Answer.

(a) Processes used for boat building:

- Lay-up methods
- Resin transfer molding (RTM)
- Rotational molding
- Thermoforming
- Vacuum assisted resin transfer molding (VARTM)

(b) Blow molding is used to make bottles

(c) Processes to make tail light assemblies:

- Ultrasonic welding
- Hot plate welding

(d) Processes for decoration:

- Sewing
- Blow molding
- Injection molding
- Cubic printing
- Hot stamping
- In-mold decoration (IMD)
- Pad printing
- Vitreous enameling
- Etching
- Injection molding, thermoplastics
- Blow molding

(NB. The last two appear since the word 'decoration' is in the record as a reference to other records)

Exercise E18.12 Use CES Level 3 to explore the selection of casting process for the products listed. First check compatibility with material and shape, and make reasonable estimates for the product dimensions (to assess mass and section thickness). Then include appropriate values for tolerance, roughness, and economic batch size.

(a) large cast iron water pipes;
(b) 10,000 Zn alloy toy cars (60mm long);
(c) small Ni-Co superalloy (MAR-M432) gas turbine blades (best possible tolerance & finish);
(d) large brass ship propeller.

Answer.

(a) Inputs used for cast iron water pipe:

- Material: cast irons
- Shape: Circular prismatic
- Mass range 100 – 200 kg
- Process characteristics: Primary shaping
- Economic batch size: 1000

Output:

- CO2/silicate casting
- Green sand casting, automated
- Green sand casting, manual
- Centrifugal casting

(b) Inputs used for Zn alloy toy cars (60mm long):

- Material: Zinc alloys
- Shape: Hollow 3-D
- Mass range 0.05 – 0.1 kg
- Section thickness: 1 – 3 mm
- Process characteristics: Primary shaping
- Economic batch size: 10,000

Output:

- Centrifugally-aided casting
- High pressure die casting

(c) Inputs for small Ni-Co superalloy (MAR-M432) gas turbine blades:

- Material: MAR-M432 Ni-Co alloy
- Shape: Solid 3-D
- Mass range 0.1 – 0.2 kg
- Section thickness: 2 – 4 mm
- Tolerance < 0.2 mm
- Roughness < 2 microns
- Process characteristics: Primary shaping

Output:

- CLA/CLV casting
- Investment casting, automated

(d) Inputs used for large brass ship propeller:

- Material: Brass
- Shape: Solid 3-D
- Mass range 500 – 2000 kg
- Section thickness: 50 – 200 mm
- Process characteristics: Primary shaping
- Economic batch size: 1

Output:

- Green sand casting, manual

Exercise E18.13 A small nylon fan is to be manufactured for a vacuum cleaner. The design requirements are summarised in Table E18.1. Use CES Level 3 to identify the possible processes, making allowance if necessary for including a secondary finishing process. Suggest some aspects of the design that may merit investigation of supporting information on the selected processes.

Table E18.1: Design requirements for a nylon fan.

Function	*Vacuum cleaner fan*	
Objective	*Minimize cost*	
Constraints	*Material: nylon* *Shape: 3D solid* *Mass: 0.1 – 0.2 kg* *Minimum section: 4 mm*	*Technical constraints*
	Tolerance: < 0.5 mm *Roughness: < 1 μm*	*Quality constraints*
	Batch size: 10,000	*Economic constraint*
Free variable	*Choice of shaping process*	

Answer.

If the selection is limited to primary shaping processes (under the heading Process characteristics) one process meets all the requirements: injection molding.

Aspects of the design requiring more detailed investigation are changes in section, tapers in section, radii of curvature etc to ensure smooth removal from the mold, avoidance of hot spots and distortion.

Exercise E18.14 The selection of process for a connecting rod was discussed earlier in the chapter. Conduct the selection using CES Level 3, making reasonable estimates for any unspecified requirements. Explore the effect of changing to a 3m con rod for a ship, of approximate cross-section 10 x 10cm, in a batch size of 10.

Answer.

(a) The design requirements for the small connecting rod are summarized in Table 18.9 in the text. First, without the tolerance requirement, processes for making the small con-rods:

- Cold closed die forging
- Die pressing and sintering
- HIPing
- Hot closed die forging

Of these, the first two can meet the general tolerance requirement of 0.25mm; none can provide the high precision of 0.02mm for the bores. So any of the 4 processes could be used with some selective machining. Cold die forging and dire pressing/sintering only require finishing of the bore surfaces.

(b) The mass of the large con-rod is roughly its volume times the density of steel (7900 kg/m^3) giving a mass of about 240 kg. In a component this large the restrictions on tolerance and roughness have to be relaxed. Taking tolerance < 2 mm and roughness < 50 microns gives the processes:

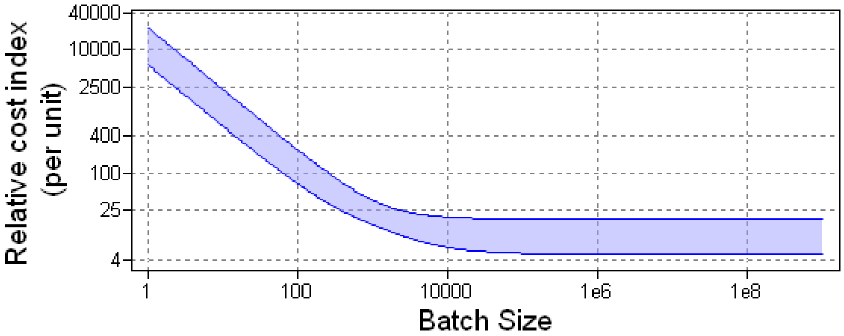
- Centrifugal casting
- Hiping, large-scale
- Green sand casting, manual

Exercise E18.15 Process selection for an aluminum piston was investigated in Exercise E18.2. Further investigation of the economics of gravity die casting and ceramic mould casting is suggested. Plot the cost against batch size for these processes, assuming a material cost of \$2kg, and a piston mass of 1 kg. The overhead rate is \$70/hour, the capital write-off time is 5 years, and the load factor is 0.5. Which process is cheaper for a batch size of 1000? Assume that as the piston is simple in shape, it will fall near the bottom of each cost-band.

Answer.

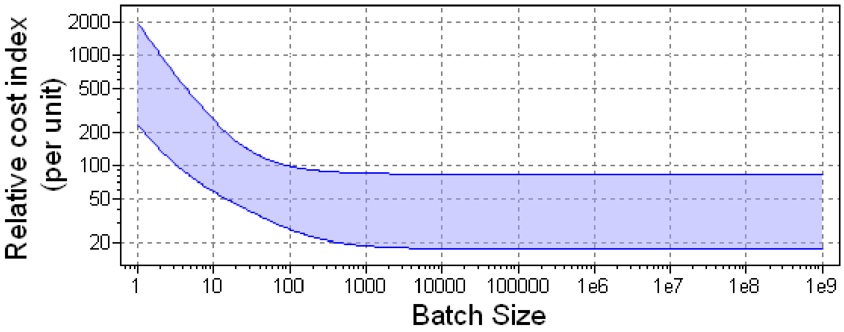
From figures below, gravity die casting ≈ \$10-15, ceramic mould casting ≈ \$20-30. Gravity die casting is the less expensive process for a batch size of 1000.

Gravity die casting



Component Mass=1kg, Material Cost=2USD/kg, Overhead Rate=70USD/hr, Capital Write-off Time=5yrs, Load Factor=0.5

Ceramic mould casting



Component Mass=1kg, Material Cost=2USD/kg, Overhead Rate=70USD/hr, Capital Write-off Time=5yrs, Load Factor=0.5

Exercise E18.16 Two examples of selection of secondary processes were discussed in the chapter. The design requirements were summarised for joining process for a radiator in Table 18.12, and for hardening a steel bearing race in Table 18.14. Use CES Level 3 to check the results obtained. For the radiator problem, use the 'Pass-Fail table' feature in CES to see if other processes could become options if the design requirements were modified.

Answer.

(a) Joining the radiator, processes are as listed in the text (excluding Diffusion bonding, ceramics):

- Brazing
- Diffusion bonding (ceramics)
- Electron beam
- Explosive welding
- Gas metal arc (MIG)
- Gas tungsten arc (TIG)
- Laser beam
- Manual metal arc (MMA)
- Oxyacetylene
- Rivet
- Soldering

Two requirements to reconsider are shear loading (as loading is light for the radiator) and watertight joints (which could be relaxed for all but the edge seals, not being required for the internal joints). This adds spot welding and a number of similar variants: projection welding, arc spot and seam welding (a continuous seam variant of spot welding).

(b) Treating the ball race, applying all constraints:

- Aluminizing
- Boriding
- Carburizing and carbonitriding
- Nitriding

(As in the text, but with the addition of Boriding and Aluminizing in this release of CES).

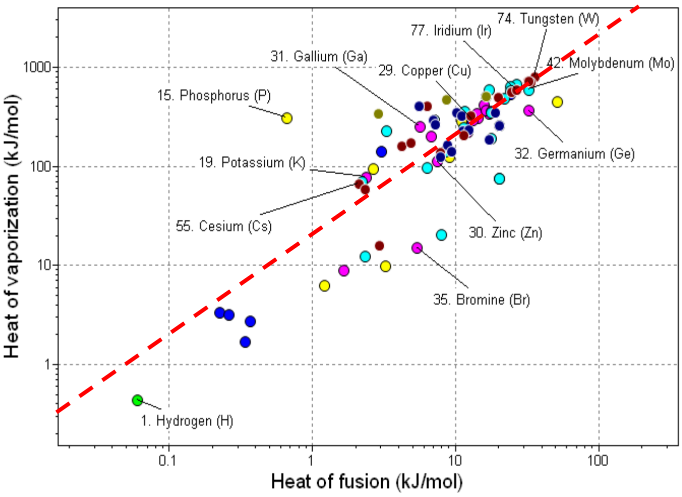
Relaxing the requirement for good/very good curved surface coverage (as the curvature of the race may not be too limiting), adds: Induction and flame hardening, Ion implantation, Laser surface hardening. The last of these has certainly been used commercially.

Exploring the science with CES Elements

Exercise E18.17 Casting processes require that the metal be melted. Vapor methods like vapor metallizing require that the metal be vaporized. Casting requires energy: the latent heat of melting is an absolute lower limit (in fact it requires more than 4 times this). Vaporization requires the latent heat of vaporization, again as an absolute lower limit. Values for both are contained in the Elements database. Make a plot of one against the other. Using these lower limits, find, approximately, how much more energy-intensive vapor methods are compared with those that simply melt.

Answer.

Vapor methods are roughly 20 times more energy intensive than those that simply melt.



Materials: engineering, science, properties, and design
3e Solution manual

Chapter 19:   Exercises with worked solutions

Exercise E19.1 Briefly explain the meaning of the following terms in relation to casting processes: homogeneous nucleation, heterogeneous nucleation, inoculants, segregation, aluminium-killed steel, poisoning.

Answer.

Homogeneous nucleation: spontaneous formation of solid crystals within the bulk of the melt.

Heterogeneous nucleation: formation of solid crystals on the surface of another solid (the mould wall, or a particle within the melt).

Inoculants: solid particles added to the melt to promote heterogeneous nucleation, as a route to a finer cast grain size with less segregation of alloying elements.

Segregation: the non-uniform distribution of solute within the solid, caused by different solubilities in liquid and solid state during solidification; occurs at the grain scale, from centre to grain boundary, and on the scale of the casting, from the mould wall to the centre.

Aluminium-killed steel: steel to which aluminium was added just before pouring, to react with residual dissolved oxygen (forming solid particles of alumina), preventing the gas from segregating during solidification, causing porosity.

Poisoning: small allying additions that change the size and shape of brittle second phases in a casting alloy. Fine-scale, more rounded particles of these phases are much less damaging to fracture toughness and ductility (e.g. adding Na to Al-Si alloys, to change the Si phase morphology).

Exercise E19.2 Briefly explain the meaning of the following terms in relation to deformation processing: annealing, recovery, recrystallisation, texture.

Answer.

Annealing: holding at high temperature in the solid state, usually followed by slow cooling. Serves various purposes – softening the alloy to enable further deformation (often via recovery and recrystallization, below), but also to relax residual stresses from prior processing (with a slow cool minimising their build-up in this process).

Recovery: dislocation rearrangement either after (static) or during (dynamic) deformation. Gives only modest softening in itself, but is a pre-cursor to subsequent recrystallization, and also eliminates work hardening in hot deformation of some metals (giving a ‘steady-state’ constant hot yield stress).

Recrystallisation: growth of new grains in deformed and recovered metals, from nuclei formed by recovery. New grain boundaries sweep through the crystal, reducing the dislocation density by orders of magnitude, with a substantial reduction in yield strength, and change in the grain size.

Texture: crystal grains contain planes of atoms, with atoms in regular arrays. The orientation of the crystal planes is rarely random in three-dimensional space, with rolling processes in particular leading to preferential orientations of the crystals with respect to the 3 axes of the rolled material – this non-random orientation distribution is referred to as ‘texture’. It strongly affects the uniformity of subsequent shaping processes, particularly in sheet metal.

Exercise E19.3 Briefly explain the meaning of the following terms in relation to bulk and surface heat treatment of metals: age hardening aluminium alloys, martensite, tempering, hardenability, transformation hardening, carburising.

Answer.

Age hardening aluminium alloys: Al alloys containing Mg and one or more of the element Si, Cu, Zn (and others), which are heat-treated for enhanced strength. This involves solutionising in the solid state close to the melting point, quenching and then ‘ageing’ at an intermediate temperature to form a fine-scale dispersion of second phases giving effective precipitation hardening.

Martensite: the phase formed in carbon steels on quenching from high temperatures (typically 900-1000ºC), when the alloy is a solid solution of C in FCC Fe (austenite). The iron lattice transforms, but the carbon remains in super-saturated solid solution, giving very high hardness but very low fracture toughness.

Tempering: the process applied at intermediate temperatures (typically 500-600ºC) to convert martensite (formed after quenching of a carbon steel) into steel that is precipitation hardened by second phase particles of iron carbide (or alloy carbides, in alloy steels). Toughness is restored, and the yield stress is higher than can be achieved by conventional slow cooling.

Hardenability: the ability of a carbon steel to form martensite on quenching – meaning the cooling rate that must be applied to form martensite, as opposed to other phases associated with slower cooling. High rates mean low hardenability – it is usually quantified instead with the size of component that will form martensite all the way through-thickness (large size – high hardenability).

Transformation hardening: the hardening of a steel surface by imposing a local transient temperature cycle to a thin surface layer, taking that region into the austenitic state (dissolving the carbon), which then transforms to martensite on quenching (usually by heat conduction into the cold substrate underneath). The hard martensite provides wear resistance, and its brittleness is not an issue for a thin layer on a tough substrate.

Carburising: the process of diffusing more carbon into the surface of a steel component held at high temperature. Carbon increases the hardness (for wear resistance) – most effectively if the carburized surface is also transformed to martensite (by quenching the surface).

Exercise E19.4 Exercises E19.1–E19.3 cover many of the reasons that alloying additions are made in metals processing. See if you can identify even more reasons for alloying, with examples.

Answer.

- alloying steels for hot strength: additions such as tungsten and vanadium increase hardenability, and lead to precipitation of alloy carbides (as above), but can also give substantial solid solution hardening when in sufficient quantities. The particular benefit is that the strength is retained at temperatures at which a plain carbon steel would soften – used for tool steels (dies and cutting tools) that run hot.

- microalloyed steels: the same alloy carbides can enhance the weldability of low carbon steels for structural applications. At the near-melting temperatures of welding, grain growth can be a problem – this enhances the steel hardenability, risking the formation of brittle martensite during welding. In microalloyed steels, the alloy carbides pin the boundaries, preventing this detrimental grain growth.

- free-machining steels: small additions of lead and other metals to steels allow very fast machining speeds, as the alloy elements concentrate at grain boundaries, giving some embrittlement (and lubrication) which ease chip formation. Only suitable for applications in which fracture toughness is not critical in the product.

- machinability in grey cast iron: the graphite flakes in grey cast iron allow unlubricated machining (as the graphite acts as a lubricant).

- second phase particles in aluminium alloys: these alloys can contain high melting point second phases of Fe, Mn, Cr and Zr. Many of these interact with processes such as recrystallization, in which they provide heterogeneous nucleation sites for new grains, providing a means to influence final grain size (and texture).

- alloying used in many ways to enhance properties directly (not via the processing directly), e.g. nickel and chromium to produce corrosion-resistance in stainless steels.

Exercise E19.5 Briefly explain the meaning of the following terms in relation to polymer processing: copolymer, polymer blend, crystallisation, fibres. Why is it possible to weld and recycle thermoplastics, but not a thermoset or an elastomer?

Answer.

Copolymer: a polymer in which the molecular chains are made of two monomers.

Polymer blend: a polymer in which two different types of chain molecule are mixed together.

Crystallisation: the formation of regions in the solid in which there is a degree of alignment and order in the chains, giving stronger van der Waals bonding between the molecules (while the remainder of the surrounding solid has an amorphous structure).

Fibres: drawn polymers in which the molecular chains have been aligned to some degree, with the alignment being retained by making a long thin strand of the polymer (either at room temperature, or by rapid cooling after drawing from a melt).

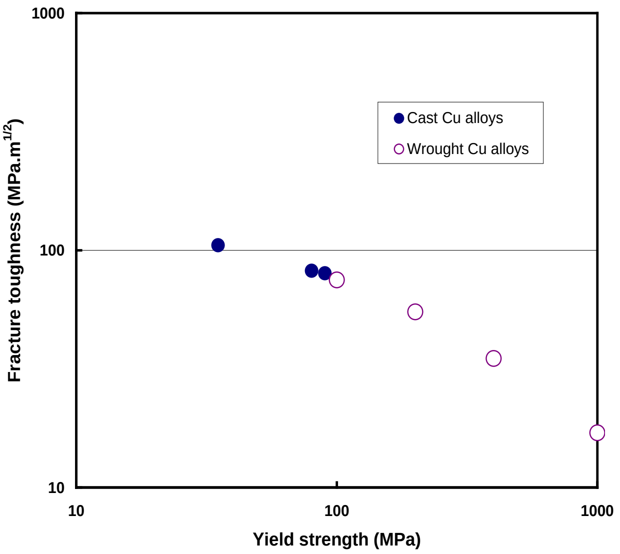
Thermoplastics can be welded and recycled because they are only bonded between the molecular chains by van der Waals bonds, which can melt and reform repeatedly. Thermosets and elastomers are cross-linked, i.e. contain covalent links between chains, and these are permanent – on heating, these polymers can only decompose (and burn), and do not melt.

Exercise E19.6 The following table shows typical data for strength and fracture toughness of a selection of copper alloys, both cast and wrought. Sketch a property chart (on log scales) and plot the data. Use the chart to answer the following:

(a) how do the cast and wrought alloys compare on fracture toughness, at comparable strength?
(b) rank the strengthening mechanisms (as indicated in the table) in order of effectiveness;
(c) do the trends observed in (a, b) follow a similar pattern to aluminum alloys (Fig. 8.18)?

Alloy	Process route	Main strengthening mechanisms	Yield strength (MPa)	Fracture toughness (MPa.m$^{1/2}$)
Pure Cu	Cast	None	35	105
Pure Cu	Hot rolled	Work	80	82
Bronze (10% Sn)	Cast	Solid solution	200	55
Brass (30% Zn)	Cast	Solid solution	90	80
Brass (30% Zn)	Wrought + annealed	Solid solution	100	75
Brass (30% Zn)	Wrought	Solid solution + Work	400	35
Cu – 2% Be	Wrought + heat treated	Precipitation	1000	17

Answer.



(a) For the data given, there is a strong correlation between strength and toughness – toughness falls steadily with increasing strength. Both cast and wrought alloys around 100MPa have comparable toughness.

(b) Solid solution is least effective, precipitation hardness is most effective; work hardening increases in effectiveness when combined with solid solution hardening.

(c) The effectiveness of the different hardening mechanisms follows the trend observed for aluminum alloys, but the strength-fracture toughness correlation is more marked in the Cu alloys shown. In aluminum the strength has little effect on fracture toughness (which is relatively low at all strengths), and the cast alloys have lower toughness than wrought alloys of comparable strength.

Exercise E19.7 The following table shows the yield strength for a number of pure metals in the annealed condition, together with typical data for the strongest wrought alloys available based on these metals. By what factor is the strength increased in each alloy system, relative to that of the pure metal on which it is based? Which shows the greatest absolute increase in strength?

Base element	Yield strength (MPa) (pure metal)	Strongest wrought alloy	Yield strength (MPa) (alloy)
Iron	120	Fe-Mo-Co-Cr-W-C quenched and tempered tool steel	2750
Aluminium	25	Al-Zn-Mg-Cu age-hardened	625
Copper	35	Cu-Be-Co-Ni age-hardened	1250
Magnesium	65	Mg-Al-Zn-Nd age-hardened	435
Nickel	70	Ni-Be cold worked	1590

Answer.

Base element	Yield strength (MPa) (pure metal)	Yield strength (MPa) (alloy)	Strength ratio	Absolute increase (MPa)
Iron	120	2750	23	2630
Aluminium	25	625	25	600
Copper	35	1250	36	1215
Magnesium	65	435	7	370
Nickel	70	1590	23	1520

Copper shows the greatest increase, relative to its initial pure state. Iron shows the greatest absolute increase.

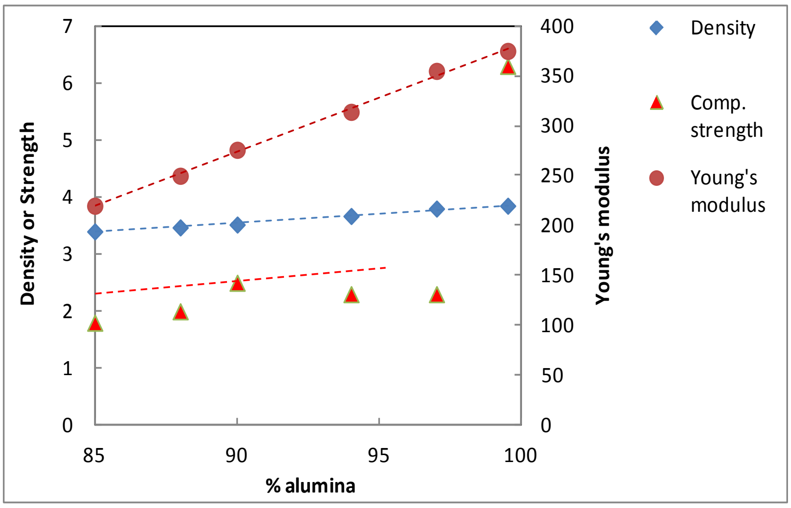
Exercise E19.8 The ceramic alumina Al_2O_3 is hot or cold pressed and sintered with a variety of binders, to facilitate densification. These binders have a much lower Young's modulus than the alumina, and there is usually some residual porosity, giving nominal compositions from 85-100% alumina. The table shows the resulting density, Young's modulus and compressive strength for a range of aluminas. Plot a graph showing the variation of these properties with alumina composition, accounting for the trends in each.

Composition (%Al_2O_3)	Density (Mg/m^3)	Young's modulus (GPa)	Compressive strength (GPa)
85.0	3.40	220	1.8
88.0	3.47	250	2.0
90.0	3.52	276	2.5
94.0	3.67	314	2.3
97.0	3.80	355	2.3
99.5 (fine grain)	3.85	375	6.3
100.0	3.96	400	2.6

Answer.

Density and Young's modulus fall approximately linearly with increasing porosity over this range, with modulus showing the greater sensitivity.

Compressive strength shows modest sensitivity to porosity, with a sudden rise in strength when nearly fully dense. This suggests that what really matters is the size of the worst defects, not their fraction. The largest defects only reduce in size when the porosity is reduced to below 1%.



Exercise E19.9 The excellent specific properties of natural and artificial fibers were highlighted on Figure 19.24. It is also of interest to explore other property combinations which measure performance – for example, the maximum elastic stored energy. The following table summarizes typical data for various fibers, together with some bulk polymers, and steel, for comparison.

Calculate the following performance indices for the materials given: (a) maximum elastic stored energy (per unit volume), σ_f^2 / E ; (b) maximum elastic stored energy (per unit mass), $\sigma_f^2 / E\rho$.

Which material appears best on each criterion? Which criterion would be more important for climbing ropes? Why is not practical for the properties of fibers to be exploited to their maximum potential?

	Hemp	Spider Web Silk	Bulk Nylon	Aramid Fiber	Polyester Fiber	PE Fiber	Nylon Fiber	Alloy Steel Wire
Young's Modulus (GPa)	8	11.0	2.5	124	13	2.85	3.9	210
Strength (MPa)	300	500	63	3930	784	1150	616	1330
Density (kg/m^3)	1490	1310	1090	1450	1390	950	1140	7800

Answer.

	Hemp	Spider Web Silk	Bulk Nylon	Aramid Fiber	Polyester Fiber	PE Fiber	Nylon Fiber	Alloy Steel Wire
Max. elastic stored energy per unit volume, σ_f^2 / E	11.3	22.7	1.6	124.6	47.3	464.0	97.3	8.4
Max. elastic stored energy per unit mass, $(\sigma_f^2 / E\rho)/1000$	7.6	17.3	1.5	85.9	34.0	488.5	85.3	1.1

PE fibers have the highest values on both criteria. The man-made fibers are superior to the natural fibers, but all the fibers are superior to steel wire.

Storing energy at minimum mass would be more important for climbing ropes, which have to be carried, and are often dragged behind or pulled up by a climber. To give sufficient elastic strain (to arrest a fall progressively), modern ropes use elastomeric cores protected by a braided sheath of nylon fibers.

To be practically useful, fibers must be converted into a usable form, such as twisted or braided rope or woven fabric. The fibers are not therefore loaded purely axially, so the properties are not fully exploited. Fiber architectures also cause bending and frictional contact between fibers, both of which can reduce their strength.

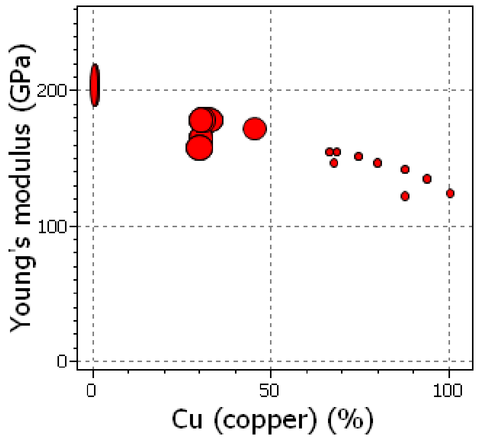
Exploring design with CES

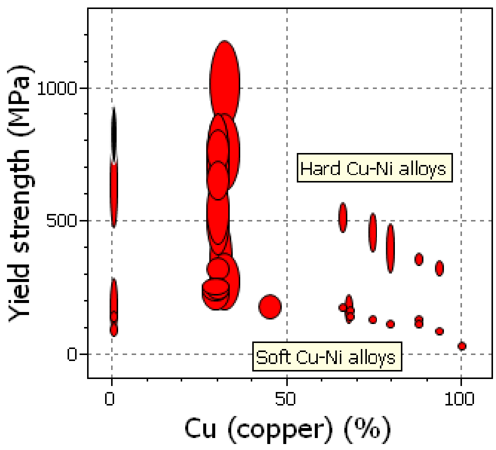
Exercise E19.10 Use CES level 3 data to explore the properties of Cu-Ni alloys. Extract data for Young's modulus, yield stress, fracture toughness, and electrical resistivity for annealed pure Cu and Ni, and a selection of alloys in between (e.g. 10%, 30%, 70% Ni). Sketch how the properties vary with Ni composition (between 0 and 100%).

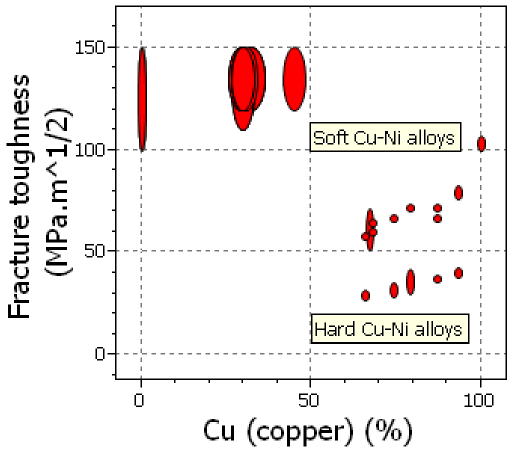
[Note that in CES you can plot the composition of the dominant element on a chart axis – try plotting each property against the %Cu, selecting both Cu-Ni and Ni-Cu alloys to cover the full range, together with pure Cu and Ni].

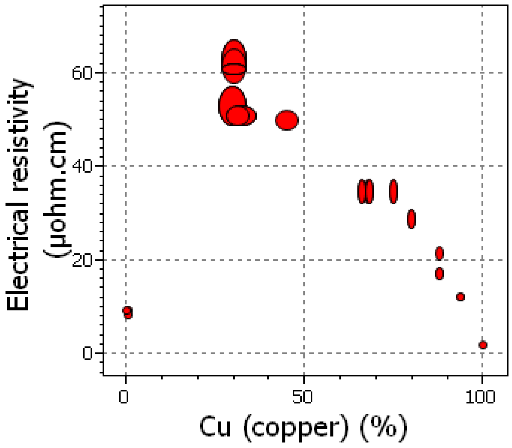
(a) Which properties follow an approximate linear rule of mixtures between the values for pure Cu and Ni?
(b) The alloys in CES are not binary Cu-Ni alloys. Which properties appear most strongly influenced by the other alloying additions?

Answer.







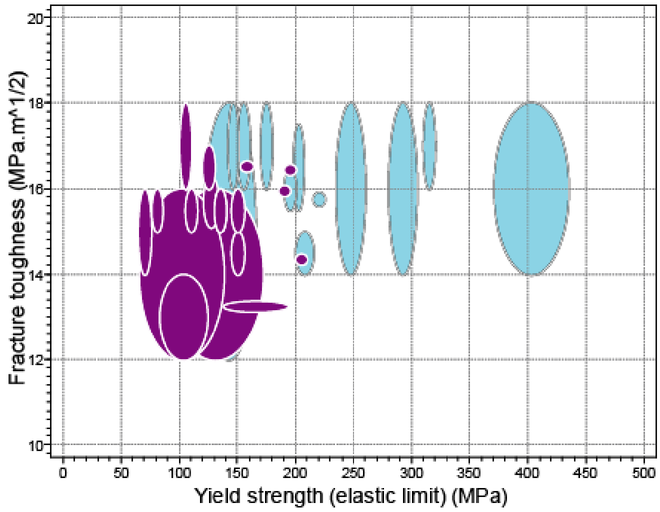


(a) Only Young's modulus follows a rule of mixtures. Yield stress and resistivity are both greater for alloys than for the pure metals; fracture toughness shows the opposite trend – alloys are generally lower toughness than pure metal. These trends are clearest for the alloys containing 50-100% Cu. Note that the strength and toughness data subdivide into two branches – soft and hard (i.e. annealed, and work hardened). Neither modulus nor resistivity show any influence of work hardening.

(b) Modulus and resistivity are relatively unaffected by the additional alloying elements. Strength is strongly affected – illustrated in particular for the alloys around 30% Cu (though part of the spread is due to work hardening). The corresponding fracture toughnesses do not vary widely – the alloys have comparable toughness to pure Ni.

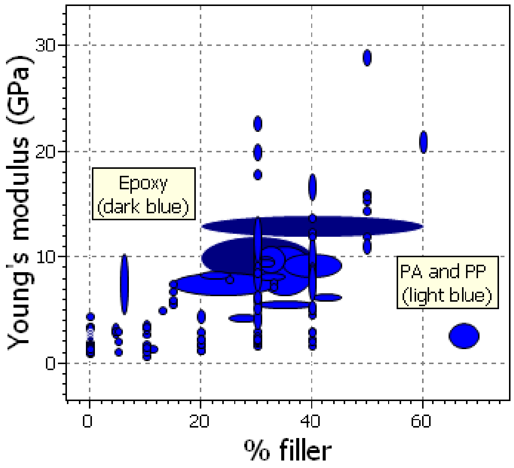
Exercise E19.11 Use CES level 3 to plot the strength and fracture toughness of Mg and its alloys. How do cast and wrought alloy variants compare?

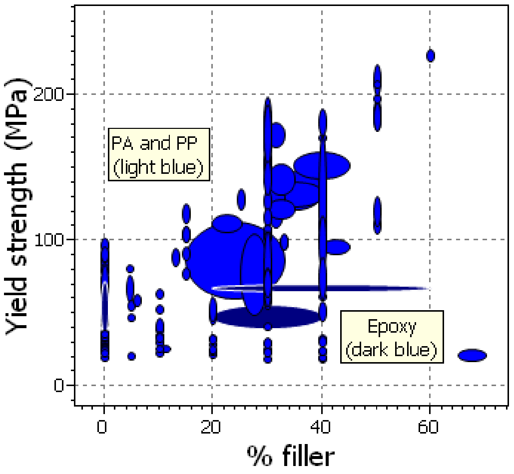
Answer.
The light blue bubbles show the wrought alloys, the purple ones the cast alloys. The cast alloys nearly have lower fracture toughness than the wrought alloys at comparable strength. All the strong alloy variants are wrought.



Exercise E19.12 The records for polymers in CES include the % filler as a parameter. Plot each of the following properties against the % filler for PA, PP and epoxy, and explore how effective fillers are at enhancing the Young's modulus and strength of these polymers. How does the addition of filler affect the fracture toughness and the price/kg?

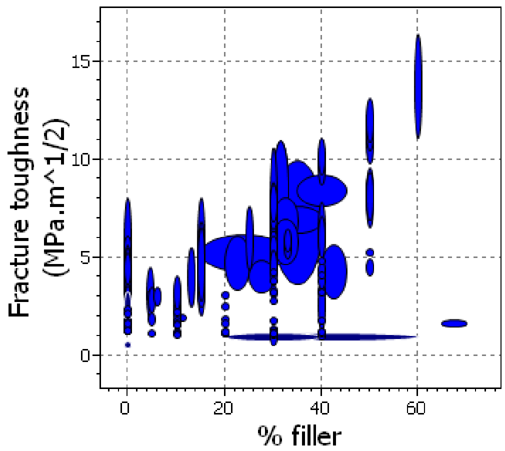
Answer.

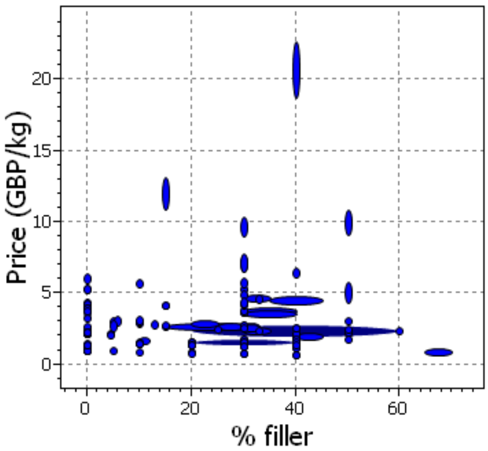




Filler can considerably stiffen and strength all three classes of polymer, though there is a large spread in resulting properties. The filled thermoplastics are generally stronger than the filled epoxies.

Fracture toughness can also be increased by adding filler, though again there is substantial spread. Since all 3 key mechanical properties can be enhanced by adding filler, it is clear why polymers are usually used with filler. The price figure shows that in general the price is not affected by the addition of filler, with one or two exceptions.



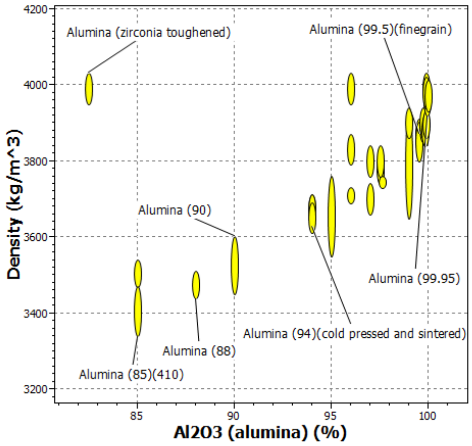


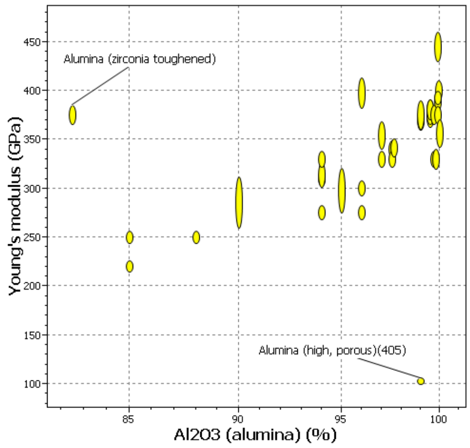
Exercise E19.13 Exercise E19.8 investigated three properties of alumina Al_2O_3, as a function of the alumina composition. Use Level 3 to plot density, Young's modulus and compressive strength against the composition parameter 'Al2O3 (alumina) (%)', for all the ceramics listed under 'Alumina'. Do these plots confirm the trends found in Exercise E19.8? Explain any outliers.

Answer.

Density:

Clear trend with the degree of porosity. The zirconia toughened outlier reflects the higher density of zirconia compared to alumina.



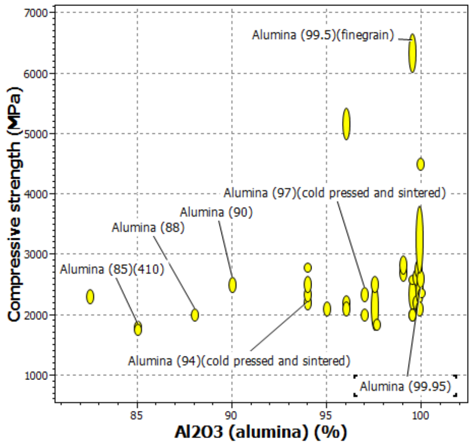


Young's modulus:

Similar to density – clear trend with porosity. Zirconia has a lower modulus than alumina (140MPa), so it is not clear why there is a zirconia-toughened outlier (as the 18% that is not alumina must be zirconia, or porosity). It is also not clear why one low porosity datapoint has such a low modulus – anomalies such as these in otherwise correlated data enable the CES developers to go back and check the integrity of their data.

Compressive strength:

As in Exercise E19.8, little dependence on porosity, but now with a wide range of strength when near fully dense. Note that the high strength variant at 99.5% dense is 'fine grain', confirming the hypothesis that it is the size of the flaws that matters for strength, more than the volume fraction of porosity.



Materials: engineering, science, properties, and design
3e Solution manual

Chapter 20: Exercises with worked solutions

Exercise E20.1 What is meant by embodied energy per kilogram of a metal? Why does it differ from the thermodynamic energy of formation of the oxide, sulphide or silicate from which it was extracted?

Answer. The *embodied energy* of a material is the energy that must be committed to create 1 kg of usable material. It is, typically, much larger than the free energy of reduction of the oxide, sulphide or other ore from which it is extracted. The thermodyamic efficiencies of processes are low, seldom reaching 50%. Only part of the output is usable – the scrap fraction ranges from a few % to more than 10%. The feedstocks used in the extraction or production themselves carry embodied energy. Transport is involved and the he production plant itself has to be lit, heated and serviced.

Exercise E20.2 What is meant by the process energy per kilogram for casting a metal? Why does it differ from the latent heat of melting of the metal?

Answer. The *process energy* H_p associated with a material is the energy, in MJ, used to shape, join and finish one 1kg of the material to create a component or product. That for the casting of metals is much larger than the latent heat of melting because the metal must first be heated to the melting point, melted and held in the molten state long enough to enable casting. The furnace itself has a heat capacity and there are heat losses. If it is electrically heated the conversion efficiency of fossil fuel to electricity (about 38%) must be included in the energy balance.

Exercise E20.3 Why is the recycling of metals more successful than that of polymers?

Answer. The answer is an economic one. Many metals (lead, iron, copper, aluminum) are relatively easy to collect, clean and remelt because they are used in large products and many of the contaminants are burnt off during remelting. The recycled metal (particularly lead and copper) command a high price, making the process economic. Polymers are used in smaller products (making collection more labor-intensive), almost always in combination with other polymers, paper, inks and other contaminants that must be removed before recycling is possible. The recycled material is less good than the virgin material, and therefore commands a lower price making the economics of polymer recycling unattractive.

Exercise E20.4 The world consumption of CFRP is rising at 8 % per year. How long does it take to double?

Answer. Equation 20.3 of the text gives the doubling time t_D as

$$t_D = \frac{100}{r}\ln(2) \approx \frac{70}{r}$$

where r is the percentage fractional rate of growth per year. Thus a growth rate of 8% means a doubling time of 8.75 years.

Exercise E20.5 The global production of platinum in 2011 stands at about 178 tonnes per year, most from South Africa. The catalytic converter of a car requires about 1 gram of precious metal catalyst, most commonly, platinum. Car manufacture in 2011 was approximately 52 million vehicles. If all have platinum catalysts, what fraction of the world production is absorbed by the auto industry? If the production rate of cars is growing at 4% per year and that of platinum is constant, how long will it be before the demand exceeds the total global supply?

Answer. If every vehicle has a platinum catalyst, the auto industry absorbs a fraction 52/178 or 29% of available platinum. If car production grows at 4% per year, its demand for platinum will reach 178 tonnes per year in 30 years.

Exercise E20.6 Global water consumption has tripled in the last 50 years. What is the growth rate, r %, in consumption C assuming exponential growth? By what factor will water consumption increase between now (2012) and 2050?

Answer. Write consumption as

$$C = C_o \exp\left\{\frac{r(t-t_o)}{100}\right\} \quad \text{which, on inversion, gives} \quad r = \frac{100}{(t-t_o)}\ln\left(\frac{C}{C_o}\right)$$

The ratio $C/C_o = 3$ for the time interval $(t-t_o) = 50$ years, giving $r = 2.2\%/\text{year}$.

If this growth rate continues, the consumption of water will increase over the next 39 years by the factor

$$\frac{C}{C_o} = \exp\left\{\frac{r(t-t_o)}{100}\right\} = 2.41$$

that is, it will more than double. Water supply in many parts of the world is already barely able to meet demand. It is not clear how this need will be met.

Exercise E20.7 Use the internet to research rare earth elements. What are they? Why are they important? Why is there concern about their availability?

Answer. The rare earth elements are the 14 members of the Lanthanide group in the 8th row of the periodic table. They are present in the Earth's crust in only very small quantities. They have unique applications in batteries and high-field permanent magnets of the sort used in electric and hybrid cars and in wind turbines. Most rare earth elements at present come from China. There is concern that demand for them within China may grow to the level at which exports become severely restricted.

Exercise E20.8 The price of cobalt, copper and nickel have fluctuated wildly in the past decade. Those of aluminum, magnesium and iron have remained much more stable. Why? Research this by examining uses (which metal are used in high value-added products?) and the localization of the producing mines. The USGS web site listed under Further Reading is a good starting point.

Answer. Take cobalt as an example. A web search (and a search of CES) gives the following information.

World production of cobalt has increased steadily year-on-year, and has almost trebled from around 20,000 tonnes in 1995 to 54,000 tonnes in 2005, an average rate of growth at times exceeding 12% per year, far higher than that of most materials. The ores of cobalt are localized in relatively few countries. Since 2002, the strongest growth in production of cobalt has come in China, where output grew at over 60% per year in 2005 as a result of expansion by domestic producers. Other recent increases have come from new projects including Voisey's Bay in Canada (Inco), Coral Bay in the Philippines (Sumitomo) and Sally Malay in Australia (Sally Malay and Sumitomo).

Cobalt costs about $30/kg – far more than common structural materials. It is used for catalysts; medical implants, cermets (tungsten carbide – cobalt) cutting tools, alloying of steels, high-temperature cobalt based superalloys, alloys for high field magnets, and as a pigment in glass and paints. These are high value-added applications, as you might anticipate from the high price of the metal.

When the supply chain for metals with unique applications, like cobalt, is unable to meet demand, the price rises steeply because there are no substitutes. When, by contrast, substitutes are readily available (plastics substituting for steel or aluminum in many applications, the existence and availability of the substituted dampens price fluctuations.

Exercise E20.9 Which phase of life would you expect to be the most energy intensive (in the sense of consuming fossil fuel) for the following products:

- A toaster
- A two car garage
- A bicycle
- A motorbike
- A wind turbine
- A ski lift

Indicate, in each case, your reasoning in one sentence.

Answer.

- A toaster. The use phase – most toasters consume about 500 Watts of electrical power.
- A two car garage. Materials, provided the garage is unheated, and lit only when taking or depositing the cars.
- A bicycle. Probably materials – though energetic cycling requires about 60 Watts of power, which has to come from somewhere.
- A motorbike. The use phase because of the fuel it consumes.
- A wind turbine. The materials. The turbine generates power when in use.
- A ski lift. The use phase through the electrical power needed to drive it.

Exercise E20.10 Car tyres create a major waste problem. Use the internet to research ways in which the materials contained in car tyres can be used, either in the form of the tyre or in some decomposition of it.

Answer. The following sites are among many that provide information about used tyres:

1. www.ct.gov/dep/
2. www.defra.gov.uk/environment/waste/topics/tyres.htm
3. www.ni-environment.gov.uk/usedtyresleaflet.pdf
4. www.tyredisposal.co.uk/cms/index.php

The first describes regulations for disposal of tyres, the second lists ways to use them, and the third provides statistics for collection, reuse and disposal. From them we learn that 3.9 million tons of used tyres are produced in the US every year, of which 18.7% are recycled.

Tyres do not compost. The main channels for reusing used tyres are these.

- ***Retreading*** almost doubles the life of tyres, but most can only be retreaded once..
- ***Rubber crumb*** is made by shredding tyres. It is used for playground and sports track surfaces.
- ***Whole or part tyres*** can be used in wall structures for earth retention.
- ***Sea defences*** can be constructed using floating arrays of tyres
- ***Fuel:*** used tyres are burnt for heat, particularly in cement making.

Exercise E20.11 What are the US CAFE rules relating to car fuel economy? Use the internet to find out and report your findings in 10 sentences or less.

Answer. The CAFE standards, penalties and credits set targets for the average fleet fuel consumption, measured in mpg, for cars sold in the US. "Fleet" means all the cars, of all sizes, sold in a given model year. The motive was to raise the fuel efficiency of new cars from an average of around 15 mpg (miles per US gallon[1]) to 27.5 mpg by 1985. The Energy Independence and Security Act, passed 22 years later (2007) raised the bar, aiming for a progressive increase to 35 mpg by 2020. Failure to meet the target incurs a penalty per car of $5.50 per tenth of a mpg below target. Exceeding the target creates a credit, calculated in the same way, which can be set against penalties in adjacent years. Electric cars and cars using bio-fuel are penalized less heavily or not at all.

---

[1] 1 $\text{mpg}_{\text{-US}}$ = 1.2 $\text{mpg}_{\text{-Imperial}}$ = 0.245 Litres/100 km

Exercise E20.12 The European Union's ELV (End of Life Vehicles) Directive on the disposal of vehicles at the end of life dictates what fraction of the weight of the vehicle must be recycled and what fraction is permitted to go to landfill. What are the current fractions? Use the internet to find out.

Answer. The European Community Directive, EC2000/53, (ELV 2000) establishes norms for recovering materials from dead cars. The initial target, a rate of reuse and recycling of 80% by weight of the vehicle and the safe disposal of hazardous materials, was established in 2006. By 2015 the target is a limit of 5% by weight to landfill and a recycling target of 85%. The motive is to encourage manufacturers to redesign their products to avoid using hazardous materials and to maximise ease of recovery and reuse.

Exercise E20.13 *Embodied energies.* Window frames are made from extruded aluminum. It is argued that making them instead from extruded PVC would be more environmentally friendly (meaning that less embodied energy is involved). If the section shape and thickness of the aluminum and the PVC windows are the same, and both are made from virgin material, is the claim justified? The table lists the data you will need.

Material	Density, kg/m^3	Embodied energy, MJ/kg
Aluminum	2700	210
PVC	1440	82

Answer. The volumes of material in the two window frames are the same.

- For aluminum: embodied energy H_m = 210 MJ/kg and density ρ = 2700 kg/m^3, so the embodied energy per unit volume. $H_m\rho$ = 5.7 × 10^5 MJ/m^3.
- For PVC: embodied energy H_m = 82 MJ/kg and density ρ = 1440 kg/m^3, so the embodied energy per unit volume is $H_m\rho$ = 0.98 × 10^5 MJ/m^3.

The aluminum window frame has an embodied energy that is larger, by a factor of 5 than the one made of PVC.

Exercise E20.14 *Recycling energies.* The aluminum window frame of Exercise E20.13 is, in reality made not of virgin aluminum but of 100% recycled aluminum with an embodied energy of 26 MJ/kg. Recycled PVC is not available, so the PVC window continues to use virgin material. Which frame now has the lower embodied energy?

Answer. For aluminum: embodied energy of recycled aluminum = 26 MJ/kg and density = 2700 kg/m^3, so the embodied energy of the recycled aluminum per unit volume is 5.4 × 10^4 MJ/m^3.

- For 100% recycled aluminum: embodied energy H_m = 26 MJ/kg and density ρ = 2700 kg/m^3, so the embodied energy per unit volume. $H_m\rho$ = 0.72 × 10^5 MJ/m^3.
- For PVC: embodied energy H_m = 82 MJ/kg and density ρ = 1440 kg/m^3, so the embodied energy per unit volume is $H_m\rho$ = 0.98 × 10^5 MJ/m^3.

The 100% recycled aluminum window frame has an embodied energy that is smaller by 26% than the one made of PVC.

Exercise E20.15 *Recycling energies.* It is found that the quality of the window frame of Exercise E20.14, made from 100% recycled aluminum, is poor because of the pick-up of impurities. It is decided to use aluminum with a "typical" recycled content of 44% instead. The PVC window is still made from virgin material. Which frame now has the lower embodied energy? The data you will need is given in the previous two exercises.

Answer. The embodied energy of the aluminum with typical recycle content is the weighted mean of that for virgin and that for 100% recycled material. Thus

- For aluminum: embodied energy of virgin material = 210 MJ/kg, embodied energy of recycled material = 26 MJ/kg and density = 2700 kg/m^3, so the embodied energy of the aluminum per unit volume of aluminum with a recycle content of 44% is

$$2700 \times (0.56 \times 210 + 0.44 \times 26) \; = \; 3.5 \times 10^5 \text{ MJ/m}^3.$$

- For PVC: embodied energy H_m = 82 MJ/kg and density ρ = 1440 kg/m^3, so the embodied energy per unit volume is $H_m \rho$ = 0.98 × 10^5 MJ/m^3.

The aluminum window frame with 44% recycled content has an embodied energy that is larger, by a factor of 3.5 than the one made of 100% virgin PVC.

Exercise E20.16 *Precious metals.* A chemical engineering reactor consists of a stainless steel chamber and associated pipe work weighing 3.5 tonnes, supported on a mild steel frame weighing 800 kg. The chamber contains 20 kg of loosely packed alumina spheres coated with 200 grams of palladium, the catalyst for the reaction. Compare the embodied energies of the components of the reactor, using data in the table.

Material	Embodied energy (MJ/kg)
Stainless steel	81
Mild steel	33
Alumina	53
Palladium	41,500

Answer. The components, masses, material embodied energies and component embodied energies are listed below.

Component	Mass (kg)	Embodied energy (MJ/kg)	Embodied energy of component (MJ)
Stainless steel reactor + pipe work	3,500	81	2.8×10^5
Mild steel frame	800	33	2.6×10^4
Alumina spheres	20	53	1.1×10^3
Palladium catalyst	0.2	41,500	8.3×10^3

The stainless steel accounts for 89% of the total embodied energy of the reactor. The palladium catalyst accounts for 2.7% of the energy, even though it contributes only 0.005% to the mass.

Exercise E20.17 The embodied energy of a mobile phone is about 400 MJ. The charger, when charging, consumes about 1 Watt. The phone is used for 2 years, during which it is charged overnight (8 hours) every night. Is the use energy larger or smaller than the energy to make the phone in the first place? What conclusions can you draw from your result? (In making the comparison, remember that electrical power is generated from primary fuel with an efficiency of about 38%).

Answer. The charger is active for 8 × 3600 seconds per day, for 2 × 365 days. Thus the electrical energy consumed by the phone over 2 years is

$$1 \times 8 \times 3600 \times 2 \times 365 = 2.1 \times 10^7 \text{ Joules} = 21 \text{ MJ}.$$

The electrical power is generated from fossil fuel with an efficiency of 38%, so the fossil-fuel equivalent of the energy to charge the phone is

$$21/0.38 = 55 \text{ MJ}$$

This is only about one tenth of the energy to make the phone in the first place. The greenest policy is to make the phone last as long as possible, since replacing it is much more energy-intensive than using it.

Exercise E20.18 The embodied energy of a mid-sized car is about 70 GJ. Assume the car is driven 250,000 km (150,000 miles) over its life, during which it averages 10 km/liter (23 miles per US gallon) of petrol (gasoline). Compare the energy involved in making the car with the energy consumed over its life. The energy content of petrol is 35 MJ/liter. What conclusions can you draw?

Answer. Over its life the car consumes 25,000 liters of petrol with an energy content of

$$25{,}000 \times 35 = 8.75 \times 10^5 \text{ MJ} = 875 \text{ GJ}$$

The energy consumed in use is thus more than ten times larger than the energy required to make the car. The fuel-efficiency of cars is improving year on year – many now achieve almost twice the efficiency of the one quoted in the question. From an energy standpoint, it makes sense to replace this car with a new, fuel efficient model.

Exercise E20.19 *Transport energies.* Cast iron scrap is collected in Europe and shipped 19,000 km to China where it is recycled. The energy to recycle cast iron is 5.2 MJ/kg. How much does the transport stage add to the total energy for recycling by this route? Is it a significant increase?

Answer. The energy for ocean shipping, from the table at the end of this Chapter, is 0.16 MJ/tonne.km. Thus the shipping adds

$$19000 \times 10^{-3} \times 0.16 = 3.0 \text{ MJ/kg}$$

to the overall procedure, an increase of 58% over recycling locally in Europe.

Exercise E20.20 *Transport energies.* Bicycles, weighing 15 kg, are manufactured in South Korea and shipped to the West Coast of the US, a distance of 10,000 km. On unloading they are transported by 32 tonne truck to the point of sale, Chicago, a distance of 2900 km. What is the transport energy per bicycle?

To meet Christmas demand, a batch of the bicycles is air-freighted from South Korea directly to Chicago, a distance by air of 10,500 km. What is the transport energy then? Transport energies are listed at the end of this Chapter.

The bikes are made almost entirely out of aluminum. How do these transport energies compare with the total embodied energy of the bike? Take the embodied energy of aluminum to be 210 MJ/kg.

Answer. Assembling the data we have:

Process step	Value	Units
Embodied energy, aluminum	210	MJ/kg
Energy for sea freight	0.16	MJ/tonne.km
Energy for 32 tonne truck	0.94	MJ/tonne.km
Energy for air freight	6.5	MJ/tonne.km

The embodied energy of the material of the bike frame is

$$15 \times 207 = 3{,}110 \text{ MJ}$$

The transport by sea and road requires energy of

$$0.015 \times (10{,}000 \times 0.16 + 2900 \times 0.94) = 64 \text{ MJ}$$

This is about 2% of the embodied energy of the aluminum. The transport by air freight requires energy of

$$0.015 \times 10{,}500 \times 6.5 = 1000 \text{ MJ}$$

This is 32% of the embodied energy of the aluminum. Air freight increases the total energy tally significantly.

Exercise E20.21 Show that the index for selecting materials for a strong panel, loaded in bending, with the minimum embodied energy content is

$$M = \frac{\sigma_y^{1/2}}{H_m \rho}$$

Answer. A panel is a flat slab, like a table top. Its length L and width is b are specified but its thickness h is free. It is loaded in bending by a central load F. The strength constraint requires that it must not yield under a bending moment M, the design load. The objective is that of minimizing the embodied energy H of the material used to make the panel. The objective function is

$$H = H_m m = H_m \, b \, h \, L \, \rho$$

where m is the mass of the panel and H_m is the embodied energy / kg of the material. The maximum stress in the panel caused by a moment M is $\sigma_{\max} = \frac{M \, y_m}{I}$, where σ_{max} is the distance of the surface from the neutral axis ($h/2$) and I is the second moment of area $\mathrm{I} = \frac{b\,h^3}{12}$

Equating σ_{max} with the yield strength σ_y and assembling these results to eliminate h gives

$$H = (6\,M\,b)^{1/2} \; L \left[\frac{H_m \, \rho}{\sigma_y^{1/2}}\right]$$

The embodied energy is minimized by selecting materials with high values of the index $\frac{\sigma_y^{1/2}}{H_m \, \rho}$.

Exercise E20.22 Use the $E - H_m \rho$ chart of Figure 20.10 to find the metal with a modulus E greater than 100 GPa and the lowest embodied energy per unit volume.

Answer. Cast iron.

Exercise E20.23 Use the $\sigma_y - H_m \rho$ chart of Figure 20.11 to find materials for strong panels with minimum embodied energy content.

Answer. Wood, stone, brick and concrete all have low values of the relevant index, $\frac{\sigma_y^{1/2}}{H_m \, \rho}$. Stone brick and concrete are brittle materials, not well suited to loading in bending, though they could be used to make a panel. The best choice is wood.

Exercise E20.24 Car bumpers used to be made of steel. Most cars now have extruded aluminium or glass-reinforced polymer bumpers. Both materials have a much higher embodied energy than steel. Take the weight of a steel bumper to be 20 kg, and that of an aluminum one to be 14 kg; a bumper-set (two bumpers) weighs twice as much. Find an equation for the energy consumption in MJ/km as a function of weight for petrol engine cars using the data plotted in Figure 20.13 of the text.

(a) Work out how much energy is saved by changing the bumper-set of a 1500 kg car from steel to aluminum.

(b) Calculate whether, over an assumed life of 200,000 km, the switch from steel to aluminum has saved energy. You will find the embodied energies of steel and aluminum in the CES 06 Level 2 database. Ignore the differences in energy in manufacturing the two bumpers – it is small. (The energy content of gasoline is 44 MJ/litre.)

(c) The switch from steel to aluminum increases the price of the car by $60. Using current pump prices for gasoline, work out whether, over the assumed life, it is cheaper to have the aluminum bumper or the steel one.

Answer. A best-fit the data petrol-engined cars gives the energy consumption E_m in units of MJ/km as a function of car weight m in units of kg as

$$E = 3.7 \times 10^{-3}\, m^{0.94}$$

The car equipped with steel bumpers weighs 1500 kg, that with aluminum bumpers weighs 12 kg less: 1488 kg.

Car weight	Energy/km MJ/km	Energy consumed over 200,00 km	Energy saved by 12 kg lower weight, MJ
1500 kg	3.58	7.16 × 10^5	6000
1488 kg	3.55	7.10 × 10^5	

There is an energy penalty because aluminum has a much higher embodied energy that aluminum. Multiplying the weights of the bumper-sets by the embodied energy gives the values, and the difference, listed in the table.

Material	Embodied energy MJ/kg	Energy content of a bumper-set, MJ	Extra energy for Al bumpers
Low carbon steel	23.6	944	4488
Age hardening Al-alloy	194	5432	

Thus there is a net energy saving over a life of 200,000 km, but it is small, just 1510 MJ, equivalent to 34 kg (about 38 litres) of fuel. The break-even point, for this car, is 150,000 km – any less, and the switch to aluminum has not saved energy.

Exercise E20.25 A total of 16 million cars were sold in China in 2010; in 2008 the sale was 6.6 million. What is the annual growth rate of car sales, expressed as % per year? If there were 16 million cars already on Chinese roads by the end of 2010 and this growth rate continues, how many cars will there be in 2020, assuming that the number that are removed from the roads in this time interval can be neglected?

Answer. Starting with equation (20.2) of the text: $P = P_o \exp\left\{\frac{r(t-t_o)}{100}\right\}$

we enter $P = 16\times10^6$ and $P_o = 6.6\times10^6$ and the time interval $(t-t_o) = 2$ years, and solve for r. The result is:

$$r = \frac{100}{2}\ \ln\left(\frac{16}{6.6}\right) = 44\%\ \text{per year.}$$

The cumulative number of cars entering use in the subsequent 10 years is found from the integral of this equation over time

$$Q_{t^*} = \int_{t_o}^{t^*} P\, dt = \frac{100\, P_o}{r}\left(\exp\left\{\frac{r(t^* - t_o)}{100}\right\} - 1\right)$$

Entering $P_o = 16\times10^6$ (the number in 2010), $r =$ 44% per year and $(t^* - t_o) = 10$ gives the additional number of cars by 2020 at $Q_{t^*} = 1.1\times10^9$. To this (if we are picky) we must add the number already there in 2010, giving a final total of 1.02×10^9, larger than the number in 2010 by a factor of 64.

Exercise E20.26 Prove the statement made in the text that, ‘at a global growth rate of just 3% per year we will mine, process and dispose of more “stuff” in the next 25 years than in the entire history of human engineering’. For the purpose of your proof, assume consumption started with the dawn of the industrial revolution, 1750.

Answer. Exponential growth has a number of alarming features, among them, the doubling time both of consumption and of the total quantity consumed. The consumption rate C of a resource grows by follows the law

$$C = C_0 \; \exp \; \alpha \, (t - t_0)$$

here C_0 is the consumption rate when $t = t_0$. The doubling time for the consumption rate is t_{DC}, meaning that the rate will have doubled to $2C_0$ in the time $t_D = t - t_0$. It is calculated by equating C in equation 1 to $2C_0$.

$$2\, C_0 = C_0 \; \exp\, (\alpha \, t_D) \quad \text{giving} \quad t_D = \frac{1}{\alpha} \ln 2$$

the quantity $\ln 2 = 0.69$, so that, at a growth rate of 3% per year $(\alpha = 0.03)$, consumption doubles in 23 years.

We are interested here not in the consumption rate but in the total quantity, Q, consumed since consumption began. It is the integral of C over time:

$$Q = \int_0^t C \, d\tau \; = \; C_0 \int_0^t \exp \, \alpha \, (\tau - t_0) \, d\tau \quad \text{giving} \quad Q = \frac{C_0}{\alpha} \left\{ \exp \, \left(\alpha (t - t_0)\right) - \exp\left(-\alpha t_0\right) \right\}$$

Q_0 is the total amount consumed prior to the present day (when $t = t_0$), thus,

$$Q_0 = \frac{C_0}{\alpha} \left\{ 1 - \exp(-\alpha \, t_0) \right\}$$

We want the time t_Q (the time to double the total quantity consumed) where $t_Q = (t - t_0)$ is the time from the present day that $Q = 2\, Q_0$, i.e. when:

$$\frac{C_0}{\alpha} \left\{ \exp\left(\alpha \, t_Q\right) - \exp(-\alpha t_0) \right\} = \frac{2\, C_0}{\alpha} \left\{ 1 - \exp(-\alpha \, t_0) \right\}$$

solving for t_Q gives: $\exp\, (\alpha \, t_Q) = 2 \; - \; \exp\, (- \alpha \, t_0)$

Now note that if $t_0 = 100$ years (meaning that consumption started 100 years ago) and $\alpha = 3\%$ per year (0.03), then $\exp - \alpha \, t_0 = 0.05$. If $t_0 = 1000$ years, it is roughly 10^{-13}. Compared with 2 both numbers are negligible. Thus, the result for the doubling time of consumption is the same as that for consumption rate, namely.

$$t_Q = \frac{1}{\alpha} \ln 2$$

That means that the total quantity of a given resource that will be consumed in the next 23 years (given the modest rate of growth of consumption of 3%/year) is equal to the total quantity consumed over all previous time. Not a happy thought.

Exploring design with CES

Exercise E20.27 Rank the three common commodity materials *Low carbon steel, Age hardening aluminum alloy* and *Polyethylene* by embodied energy / kg and embodied energy / m^3, using data drawn from Level 2 of the CES Edu database (use the means of the ranges given in the databases). Materials in products perform a primary function – providing stiffness, strength, heat transfer and the like. What is the appropriate measure of embodied energy for a given function?

Answer.

Material	Embodied energy (MJ/kg)	Density (kg/m^3)	Embodied energy (GJ/m^3)
Low carbon steel	23.6	7850	185
Age hardening aluminium alloy	194	2700	523
Polyethylene	83	950	79

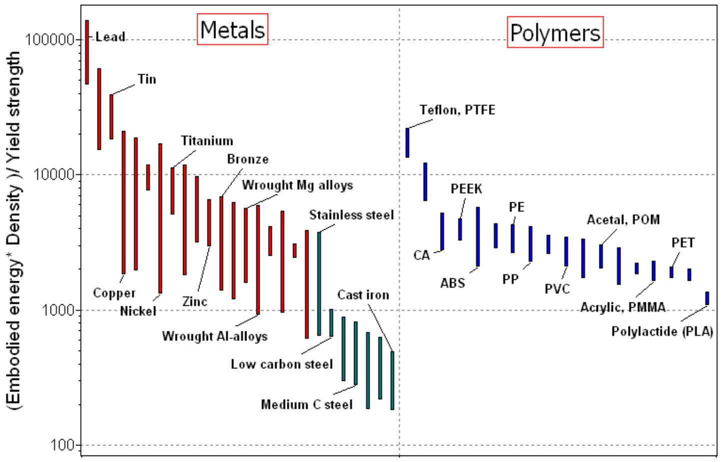
On a per kg basis, steel has the lowest energy – but materials are seldom chosen by weight. On a per m^3 basis, polyethylene is much the lowest. To compare materials properly, the measure of comparison has to be the appropriate one – embodied energy per unit function.

Exercise E20.28 Plot a bar chart for the embodied energies of metals and compare it with one for polymers, on a 'per unit yield strength' basis, using CES. You will need to use the 'Advanced' facility in the axis-selection window to make the function

$$\text{Energy per unit strength} = \frac{\text{Embodied energy} \times \text{Density}}{\text{Yield strength (elastic limit)}} = \frac{H_m \rho}{\sigma_y}$$

Which materials are attractive by this measure?

Answer. The figure shows the CES output. (The Metals and Polymers are segregated to separate columns by selecting them for the x-axis using the 'Trees' option in the 'Advanced' facility.) Carbon steels and cast irons have a lower embodied energy per unit of strength than any other metal. Among polymers, biopolymers like Polylactide (PLA) and starch-based polymers perform well.



Exercise E20.29 Drink containers co-exist that are made from a number of different materials. The masses of five competing container-types, the material of which they are made, and the specific energy content of each are listed the table. Which container-type carries the lowest overall embodied energy per unit of fluid contained?

Container type	Material	Mass, g	Embodied energy MJ/kg
PET 400 ml bottle	PET	25	84
PE 1 litre milk bottle	High density PE	38	80
Glass 750 ml bottle	Soda glass	325	14
Al 440 ml can	5000 series Al alloy	20	200
Steel 440 ml can	Plain carbon steel	45	23

Answer.

Container type	Material	Mass, g	Embodied energy, MJ/kg	Mass / litre, g/litre	Energy/litre, MJ/litre
PET 400 ml bottle	PET	25	84	62	5.4
PE 1 litre milk bottle	High density PE	38	80	38	3.2
Glass 750 ml bottle	Soda glass	325	14	433	8.2
Al 440 ml can	5000 series Al alloy	20	200	45	9.0
Steel 440 ml can	Plain carbon steel	45	23	102	**2.4**

The lightest can is the aluminum one, using the least material, but aluminum is very energy intensive, giving this container the worst rating. The material with the lowest embodied energy / kg is glass one, but it ends up with the second worst rating because glass bottles are so heavy, using a lot of material. The winner is the steel one.

Exercise E20.30 Iron is made by the reduction of iron oxide, Fe_2O_3, with carbon, aluminum by the electro-chemical reduction of Bauxite, basically Al_2O_3. The enthalpy of oxidation of iron to its oxide is 5.5 MJ/kg, that of aluminum to its oxide is 20.5 MJ/kg. Compare these with the embodied energies of cast iron and of carbon steel, and of aluminum, retrieved from the CES database (use mean values of the ranges). What conclusions do you draw?

Answer.

Material	Enthalpy of oxidation, MJ/kg	Embodied energy, MJ/kg
Cast iron	5.5	17.1
Carbon steel	5.5	23.6
Aluminum	20.5	194

The embodied energies are between 3 and 12 times larger than the thermodynamically-required energy to reduce the oxide to the metal. There are several contributions to the discrepancy. The largest is that industrial processes, at best, achieve an energy-efficiency of around 38% (the blast furnace, used to make iron, is remarkably efficient). Then there is transport, the energy to run, heat, light and maintain the plant in which the metal is made, and many other smaller but significant contributions to the total energy input per unit of usable output.

Exercise E20.31 Calculate the energy to mold PET by assuming it to be equal to the energy required to heat PET from room temperature to its melting temperature, T_m. Compare this with the actual molding energy. You will find the molding energy, the specific heat and the melting temperature in the Level 2 record for PET in CES (use mean values of the ranges). Assume that the latent heat of melting is equal to that to raise the temperature from room temperature to the melting point. What conclusions do you draw?

Answer.

Material	Specific heat C_p J/kg.K	Melting temperature, T_m, °C	Energy to heat to melting point $C_p.T_m$, MJ/kg	Molding energy, MJ/kg
PET	1450	238	0.35	11.4

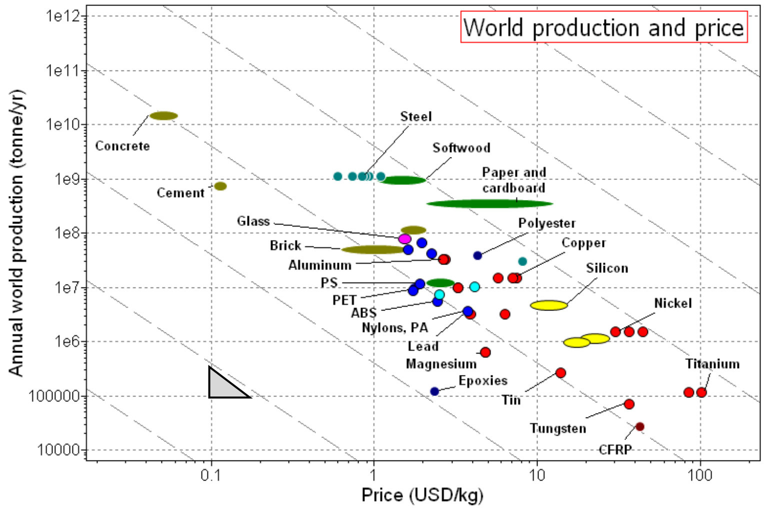
Thermodynamics tells us a little but not a lot about the energy to make and process materials. Doubling the energy to heat to include the latent heat of melting gives 0.7 MJ/kg; the reported energy to mold, based on input-output analysis is larger by a factor of 30. The message is that the energy to shape things is very largely determined by factors that do not relate directly to the material, but to the process.

Exercise E20.32 Use CES to plot the *Annual world production* of materials against their *Price*. What trend is visible?

Answer. The figure shows the plot. There is a correlation – the lower the price, the larger is the annual production. The contours have a slope -2, which, with log scales (as here) means that, very approximately,

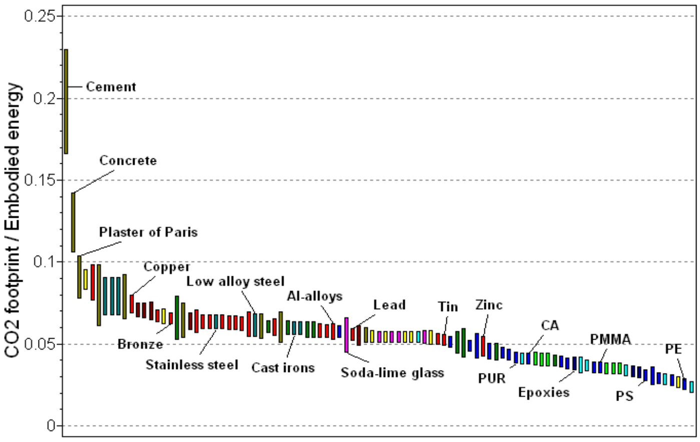
$$\text{Annual world production} \propto \frac{1}{(\text{Price})^2}$$

It is no surprise that production falls as price rises. The origin of the power of -2 is obscure.



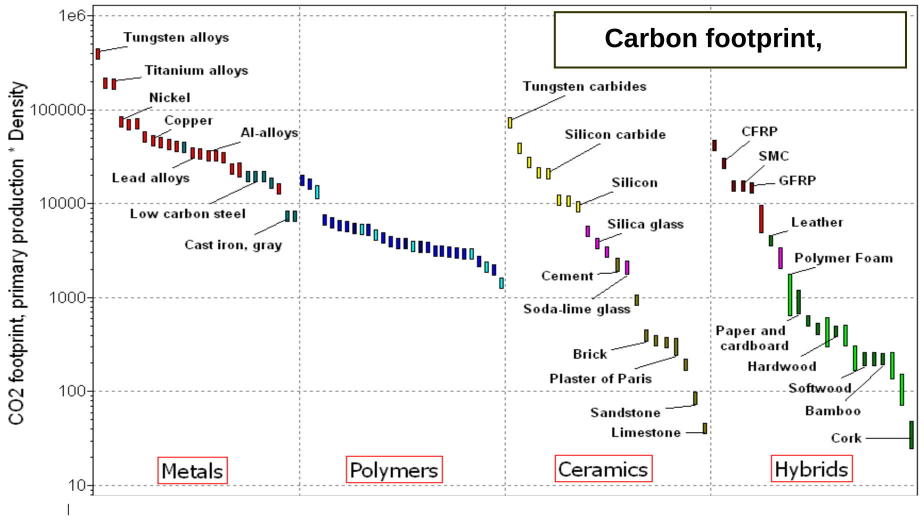
Exercise E20.33 Make a bar chart of CO_2 footprint divided by Embodied energy using the 'Advanced' facility in CES EduPack Level 2 software. Which material has the highest ratio? Why?

Answer. The Figure shows the important features. For most materials the ratio is about 0.04 - 0.07. For cement and concrete it is far higher. This is because, making cement (a component of concrete) involves "calcining" calcium carbonate, driving off carbon dioxide to leave lime. This CO_2 contribution adds to that derived from the fuel that is used to heat the kiln to give the high total.



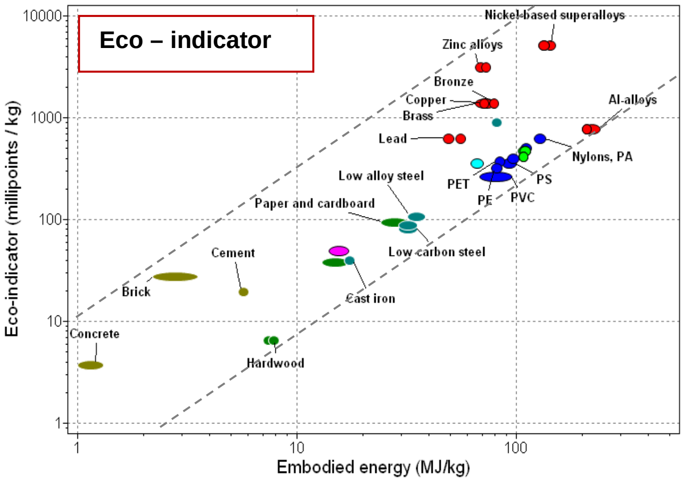
Exercise E20.34 Figure 20.9 of the text shows a plot of the embodied energy of materials per m^3. Use CES to make a similar plot for the carbon footprint per m^3 of material. Use the 'Advanced' facility in the axis selection window to plot kg CO_2/m^3 by multiplying kg CO_2/kg by the density in kg/m^3.

Answer. The chart is shown below. Not surprisingly, it looks very like the embodied energy per m^3, shown in Figure 20.9 of the text. (The Metals, Polymers, Ceramics and Hybrids are segregated to separate columns by selecting them for the *x*-axis using the 'Trees' option in the 'Advanced' facility.)



Exercise E20.35 Compare the eco-indicator values of materials with their embodied energy. To do so, make a chart with ‘Embodied energy × Density’ on the *x*-axis and ‘Eco-indicator value’ on the *y*-axis. (Ignore the data for foams since these have an artificially-inflated volume). Is there a correlation between the two? Is it linear? Given that the precision of both could be in error by 10 % are they significantly different measures? Does this give a way of estimating, approximately, eco-indicator values where none are available?

Answer. The plot of eco-indicator values against embodied energy is shown below. There is a clear correlation, but with some scatter. Is the scatter significant? Think back to the inherent uncertainty in determining embodied energy (Figure 20.4) and in the arbitrary nature of the weight factors used to calculate eco-indicators. Given these, you would expect some scatter. We conclude the embodied energy is an approximate but still useful proxy for the eco-indicator.



Exercises on Sustainability

These exercises require a looser way of thinking. There is no ‘right’ answer – instead there is a thoughtful, well-researched response that recognises the many conflicting facets of sustainable technology and seeks compromises that offer the greatest good with the least bad.

Exercise E20.36 In discussions of Sustainable Development the term Comprehensive Global Capital appears. What does it mean and what are its three main components? In your judgement, does it seem possible to adopt a life style that results in growth, or no diminution, of all three?

Answer. The three competing components of Comprehensive Capital are

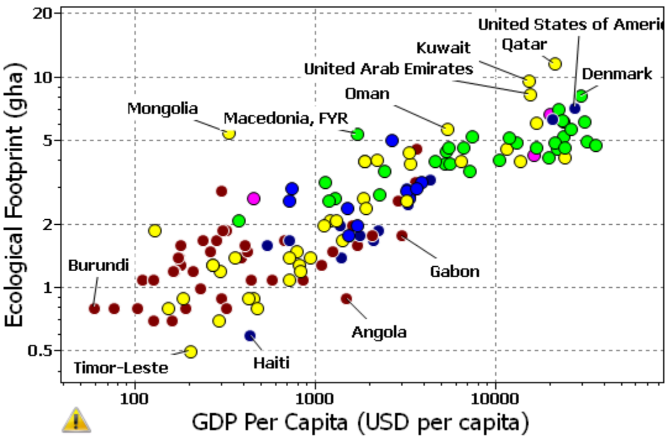
- *Manufactured capital* – Industrial capacity, institutions, roads, built environment, financial wealth (GDP)
- *Human capital* – Health, education, skills, technical expertise, accumulated knowledge, happiness
- *Natural capital* – Clean atmosphere, fresh water, fertile land, productive oceans, minerals and fossil energy.

Manufactured capital increases the capacity to provide education, health-care, and the industrial and agricultural infra-structure to create more wealth, to innovate and recover and use materials more efficiently. All of these, however, rely on draw on natural resources – oil, gas or coal, minerals, biologically productive land – some of which cannot, even with the most efficient recovery technology, be replaced. It would seem that future development must take place in a scenario of diminishing Natural Capital.

Exercise E20.37 Globally, affluence is increasing. Would you expect that Global Natural Capital would be influenced by this increase, and in which direction? The Gross Domestic Product (GDP) per capita is a measure of affluence per person per year. The Ecological Footprint is measure of the human impact on Natural Capital, per person per year. A value greater than 1 means that Natural Capital is being drained faster than it can be replaced. You may find the following URLs useful in forming a judgment.

The Ecological Footprint.
http://www.footprintnetwork.org/en/index.php/GFN/page/basics_introduction/
GDP of countries. http://en.wikipedia.org/wiki/List_of_countries_by_GDP_(PPP)

Answer. It seems likely that increasing affluence would increase demand for goods and services and thus place greater stress on Natural Capital. Using the data from the web sites listed in the question it is possible to explore the relation between Ecological Footprint per capita against GDP per capita. Only a few points are needed to establish the trend shown in the adjacent figure. It strongly suggests that increased affluence causes a greater drain on natural capital. Increased affluence, if shared and appropriately invested clearly has the potential to increase Human and Manufactured capital. But a plot of Ecological Footprint against GDP per capita strongly suggests that increased affluence causes a greater drain on natural capital.



Exercise E20.38 Use research and judgment to form a opinion about the viability of extraction of critical element from electronic waste. Critical elements are those that are rare or highly localized (leading to supply-chain constraints) or with no known effective substitutes or of strategic importance. The table lists five of these with the current grade of ore from which they are mined. Use the internet to find the approximate concentration of these in mobile phones. Does it equal or exceed that of the ores from which they are currently extracted?

Critical elements	Typically mined ore grade, wt %
Platinum	0.00025
Gold	0.0015
Silver	0.055
Cobalt	0.5
Copper	2.0

Answer. All five critical elements are found in mobile phones – see table below. A Web-based search gives the following very approximate quantities and “concentrations”. The concentrations of all five critical elements in phones is larger than the typical grade of ore from which these elements are extracted. This suggests that “mining” waste electronics might provide a viable source of critical elements. There are, however, practical problems of collection, separation and refinement to be overcome.

Critical elements in mobile phones

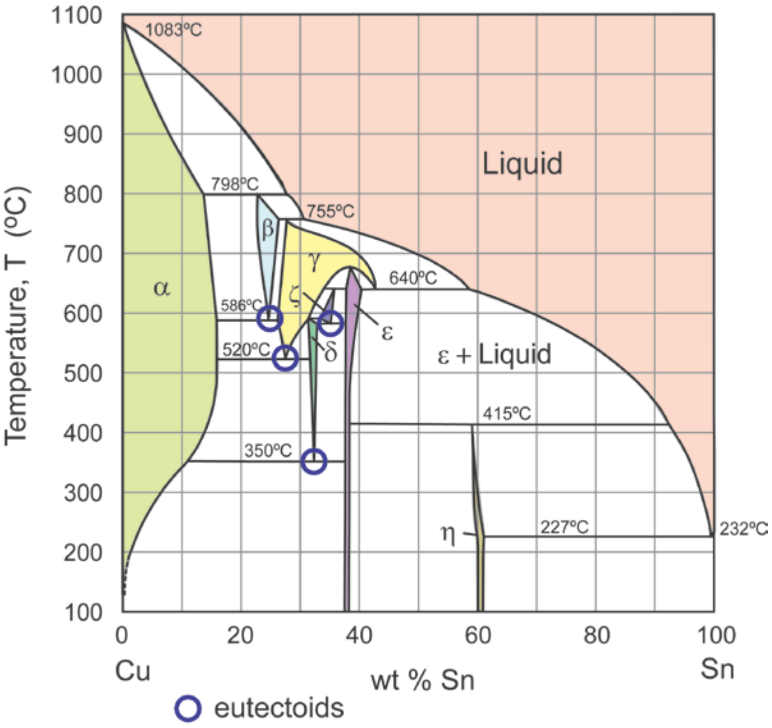
Critical elements	Grams per tonne in mobile phones	Concentration in mobile phones, %	Typically mined ore grade, %
Platinum	70	0.07	0.00025
Gold	140	0.014	0.0015
Silver	1,300	0.13	0.055
Cobalt	19,000	1.9	0.5
Copper	70,000	7	2.0

Materials: engineering, science, properties, and design
3e Solution manual

Guided Learning Unit 2:   Solutions to Further Exercises

E.22   (a) is correct. If (b) were true, 2/3 of the atoms would be the much heavier gold atoms, giving a wt% well above 66%, whereas it is 63 wt%.

E.23   (a,b)



Eutectoids:
(1) W_{Zn} = 24wt%, T = 586°C
(2) W_{Zn} = 35wt%, T = 580°C
(3) W_{Zn} = 28wt%, T = 520°C
(4) W_{Zn} = 32wt%, T = 350°C

Figure E23b (solution):   Copper-tin Cu-Sn phase diagram

(c)   Compound ε and η are at approx. 38 and 60 wt% Sn: call these $CuSn_x$ and $CuSn_y$.

(i) wt% Sn ≈ 0.38 = 118.69*x* / (63.54 + 118.69*x*).

Solving for *x*: *x* (1–0.38) × 118.69 = 0.38× 63.54.   Hence *x* ≈ 0.33. Compound therefore Cu_3Sn.

(ii) wt% Sn ≈ 0.60 = 118.69*y* / (63.54 + 118.69*y*).

Solving for *y*: *y* (1–0.60) × 118.69 = 0.60× 63.54.   Hence *y* ≈ 0.8. Compound therefore Cu_5Sn_4.

E.24   (a) (i) A: (Al) + Liquid; B: (Zn) + Liquid; C: (Al) + (Zn); D: $(Al)_1$ + $(Al)_2$; E: (Al) + (Zn).
Note that for D, it is a two phase region, but with both phases being solid solutions based on Al, with different Zn content. As the region is continuous with pure Al, the notation used is (Al), but neither are strictly Al-rich (extending to 77.7 wt% Zn).

(ii) *Eutectic* reaction at 381°C: Liquid (94 wt% Zn) → (Al) (83.1 wt% Zn)   +   (Zn) (98 wt% Zn)
Eutectoid reaction at 277°C: (Al) (77.7 wt% Zn) → (Al) (32.4 wt% Zn)   +   (Zn) (99.3 wt% Zn)

(iii) Al-Zn alloys have some of the characteristics of heat-treatable Al alloys – high solubility of Zn in Al at high temperature, and a two-phase region at low temperature – suggesting there is potential for precipitation hardening. However, the second phase is almost pure Zn, whereas effective age-hardening usually involves an intermetallic compound. Zn actually diffuses very slowly in Al, and the alloys are in fact classified as "non-heat treatable" i.e. they do not precipitate harden. Cooling Al alloys with a wide range of Zn compositions would produce a supersaturated solid solution, and thus hardening. In addition the alloys could be work hardened.

(b) 20g Al and 30g Zn, hence alloy contains 60 wt% Zn.

(i)   At 500°C: (Al) (40 wt% Zn)   +   (Liquid) (71 wt% Zn)

Proportion of (Al) = (71 – 60) / (71 – 40) × 100% ≈ 35%; and proportion of Liquid = 65%.

(ii) At 300°C: $(Al)_1$ (38 wt% Zn) + $(Al)_2$ (75 wt% Zn)

Proportion of $(Al)_1 = (75 - 60) / (75 - 38) \times 100\% \approx 41\%$; and proportion of $(Al)_2 = 59\%$.

(iii) At 250°C: (Al) (21 wt% Zn) + (Zn) (99.5 wt% Zn)

Proportion of $(Al) = (99.5 - 60) / (99.5 - 21) \times 100\% \approx 50\%$; and proportion of $(Al)_2 = 50\%$.

E.25 (a) A peritectic reaction is a three-phase reaction by which, on cooling, two phases (one of them liquid) react to give a single new solid phase:

$$\text{Liquid} + \text{solid } \alpha \rightarrow \text{solid } \beta$$

(b) A peritectoid reaction is a three-phase reaction by which, on cooling, two solid phases react to give a single new solid phase:

$$\text{Solid } \alpha + \text{solid } \beta \rightarrow \text{solid } \gamma$$

E.26 (a) Eutectic reaction at 575°C, 28 wt% Al: Liquid $\rightarrow$ α (47 wt% Al) and δ (16 wt% Al).

(b) *Above 675°C*: Liquid is stable.

From 675°C to 575°C: primary δ grains nucleate and grow as the temperature falls to 575°C. The compositions of the liquid and δ phases become richer in Al, following the liquidus and solidus lines respectively. Just above the eutectic temperature (575°C), the microstructure is about 2/3 primary δ (16 wt% Al) and 1/3 liquid of eutectic composition (28 wt% Al).

At 575°C: The eutectic reaction takes place, converting the remaining liquid into a two-phase mixture of δ (16 wt% Al) and solid solution α (47 wt% Al), with roughly 40% α, 60% δ (in the eutectic regions).

From 575°C to room temperature: The solubility of Ag in Al falls continuously to approximately zero at room temperature. This is achieved within the eutectic regions by Ag diffusing out of the α to the δ, increasing the fraction of δ. The primary δ grains remain unchanged, as the phase boundary is vertical so there is no change in the solubility of Al in δ. Final overall fraction of $\delta \approx 80/84 \approx 95\%$.

E.27

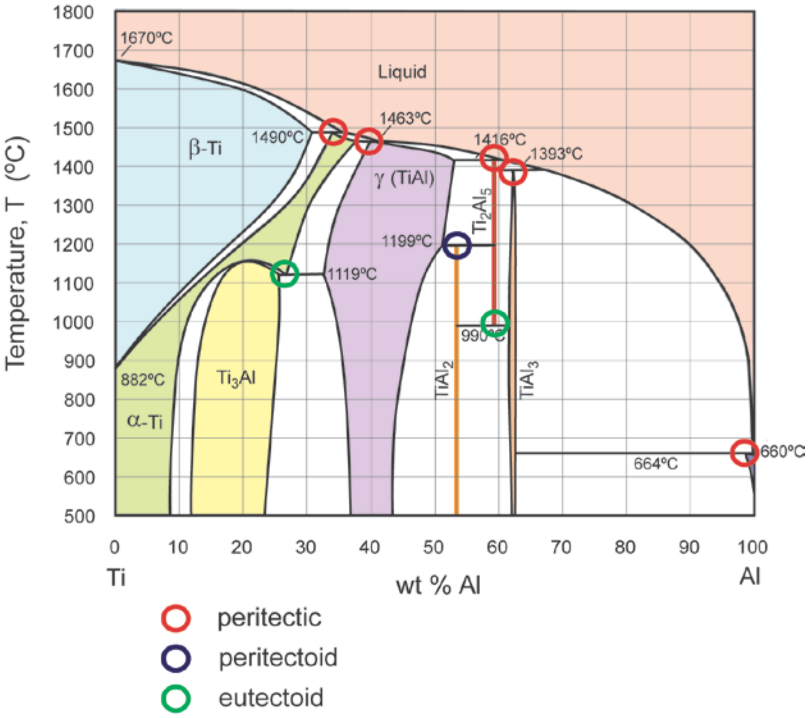


Figure E.27 (solution): Ti-Al phase diagram

(a,b) See Figure E.27 (solution). Compounds at approx. 53, 59 and 62 wt% Al: call these $TiAl_x$, $TiAl_y$, and $TiAl_z$.

(i) wt% Al $\approx 0.53 = 26.98x/(47.9+26.98x)$.

Solving for x: $x\,(1-0.53)\times 26.98 = 0.53\times 47.9$. Hence $x \approx 2.0$. Compound therefore $TiAl_2$.

(ii) wt% Al ≈ 0.59 = $26.98y/(47.9+26.98y)$.

Solving for y: $y(1–0.59)\times26.98 = 0.59\times47.9$. Hence $y \approx 2.55$. Compound therefore Ti_2Al_5.

(iii) wt% Al ≈ 0.62 = $26.98z/(47.9+26.98z)$.

Solving for z: $z(1–0.62)\times26.98 = 0.62\times47.9$ Hence $z \approx 2.9$. Compound therefore $TiAl_3$.

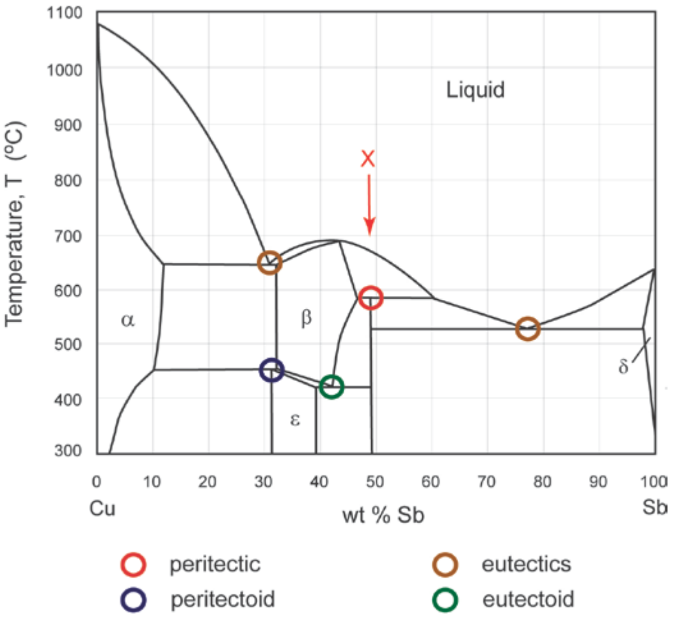
(c) Ti-6wt% Al: transformation from α to β from approx. 980 – 1010°C; melting from approx. 1650 – 1660°C.

E.28 (a) Compound at approx. 48wt% Sb: call it $CuSb_x$.

wt% Sb = 0.48 = $121.75x/(63.54+121.75x)$.

Solving for x: $x(1–0.48)\times121.75 = 0.48\times63.54$, hence $x = 0.48$. Compound therefore Cu_2Sb.

(b) See Figure E.28b (solution).



Eutectic 1: 650°C, 31wt% Sb: Liquid → α + β
Eutectic 2: 525°C, 77wt% Sb: Liquid → Cu_2Sb + δ
Eutectoid: 420°C, 42wt% Sb: β → ε + Cu_2Sb
Peritectic: 590°C, 48wt% Sb: Liquid + β → Cu_2Sb
Peritectoid: 450°C, 32wt% Sb: α + β → ε

Figure E.28b (solution): Cu-Sb phase diagram

(c) 95wt% Sb on cooling – see Figure E.28c (solution):

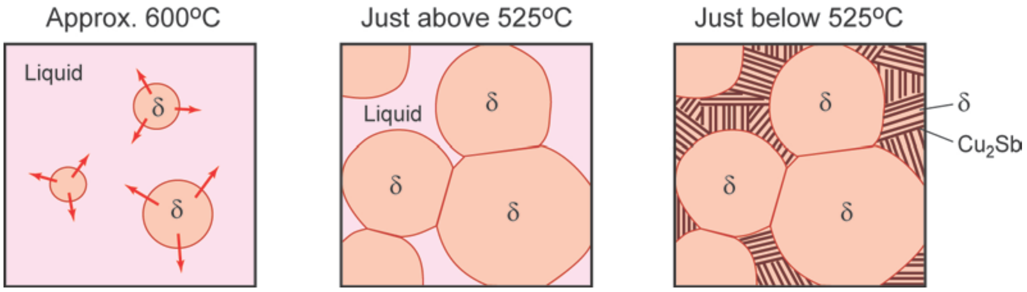


Figure E.28c (solution): Microstructure evolution on solidification of Cu-95wt% Sb alloy

- At 610°C, solidification starts with nucleation of primary δ (Sb-rich solid solution).
- Just before eutectic temperature (525°C), proportion of δ approx. (95-77)/(98-77) = 86% (by weight), and 14% remaining liquid at the eutectic composition. At this proportion, the solid grains will have impinged, leaving isolated pockets of liquid.
- On cooling through the eutectic temperature, the liquid zones form two-phase Cu_2Sb + δ with the proportion of Cu_2Sb being approximately (98-77)/(98-48) = 42% (by weight) – the phases are fairly balanced, as the eutectic V is towards the centre of its tie-line.

On further cooling, the δ phase purifies towards pure Sb, releasing Cu. In the eutectic, the phase proportions can adjust to accommodate this. In the primary δ grains, Cu_2Sb would need to precipitate throughout the grains to absorb the excess Cu – even though thermodynamics dictates that this should happen, it will depend strongly on the kinetics of nucleation as to whether it does occur in practice.

(d) Cooling curve for solidification of 95wt% Sb:

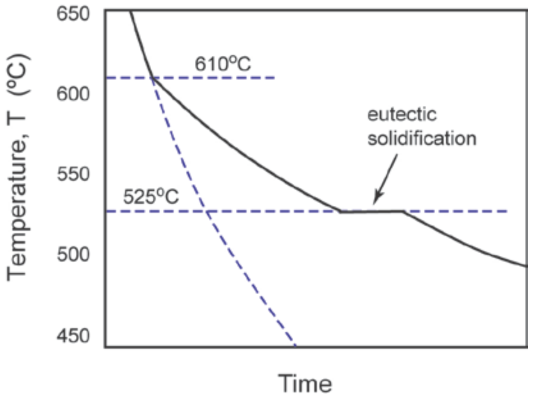


Figure E.28d (solution): Cooling curve for solidification of Cu-95wt% Sb alloy

From 610°C to 525°C, the slope of the cooling curve is reduced, as latent heat is released progressively. Then arrest of cooling occurs at 525°C, the eutectic temperature, as the remaining liquid solidifies at that temperature. The duration of the arrest is shorter, as the proportion of eutectic is only 14%.

E.29 (a) See Figure E.29a (solution)

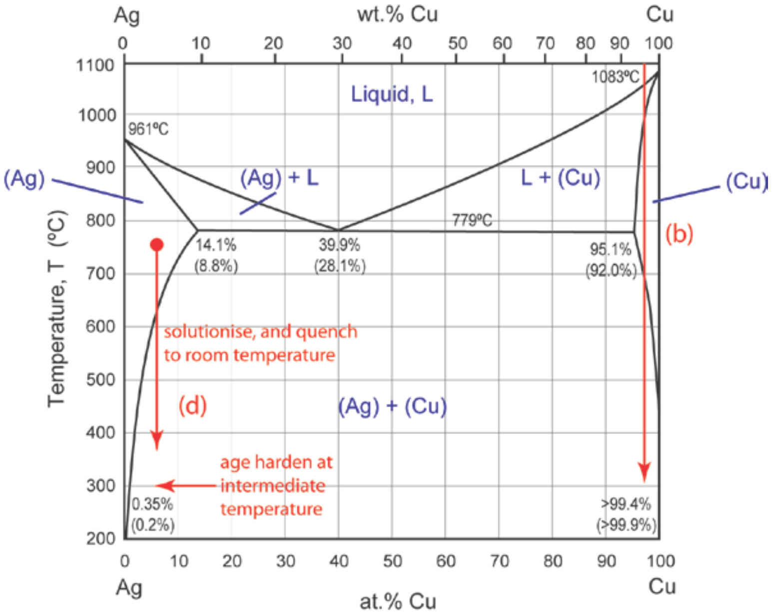


Figure E.29a (solution): Silver-copper Ag-Cu phase diagram

(b)

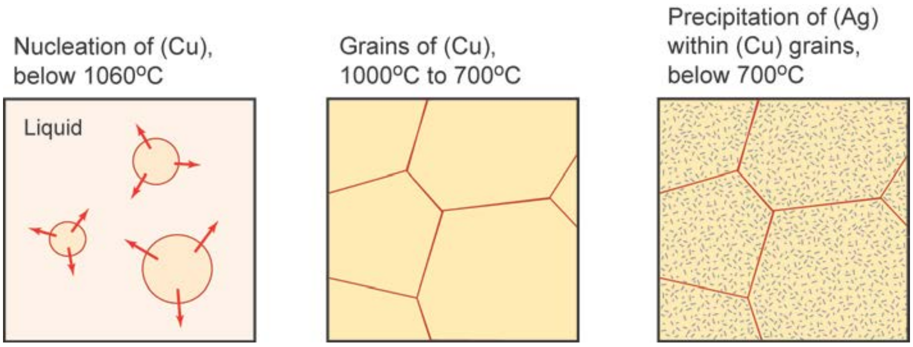


Figure E.29b (solution): Microstructure evolution in an Ag-95wt% Cu alloy

At room temperature the phases are essentially pure Ag and pure Cu. Final proportions of phases: 5% Ag, 95% Cu.

(c) Some segregation in 95 wt% Cu may occur because the first Cu-rich solid to form will be purer than 95 wt% Cu (nearer 99 wt%). As the Cu-rich grains grow, the Ag content will progressively increase, but diffusion may be too slow for Ag atoms to diffuse back to the centre of the grains to homogenise the composition to a uniform 95 wt% Cu The microstructure will form 100% Cu-rich grains, but with a gradient in Ag content from the centre to the grain boundaries.

(d) See Figure E.29a (solution). Silver will dissolve up to 8.8 wt% Cu in solid solution (at 779°C). So choose an alloy around 5-10 wt% Cu, and solutionise in the single-phase region. By quenching to room temperature, a supersaturated solid solution is formed. Re-heating to a modest temperature (say 200°C) should then precipitate fine-scale Cu, giving precipitation hardening.

E.30 (a) A: Liquid + solid solution; B: solid solution; C: (Au) + (Ni)

Au – 30 wt% Ni alloy:

At 1200°C: solid solution (53 wt% Ni) + (Liquid) (15 wt% Ni)
Proportion of liquid = (53 – 30) / (53 – 15) × 100% ≈ 61%; and proportion of solid = 39%.

At 900°C: 100% solid solution (30 wt% Ni)

At 600°C: Au-rich solid (Au) (12 wt% Ni) + Ni-rich solid (Ni) (86 wt% Ni)
Proportion of (Au) = (86 – 30) / (86 – 12) × 100% ≈ 76%; and proportion of solid = 24%.

(b) Au – 10 wt% Ni alloy:

Above 1160°C: Liquid is stable.

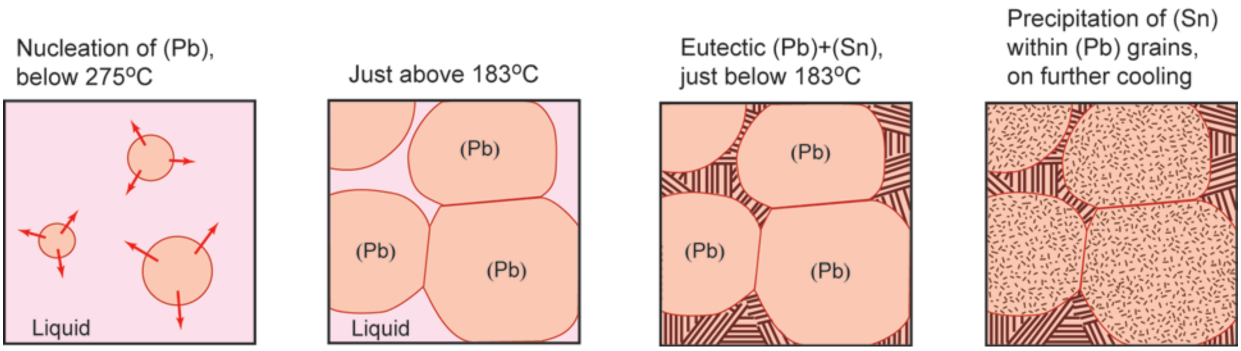
From 1160°C to 1070°C: grains of solid solution nucleate and grow, at an initial composition of 45wt% Ni, but both liquid and solid solution become purer as the temperature falls to 1070°C, at which point 100% solid solution has formed.

From 1070°C to 530°C: No change – stable structure of grains of solid solution.

From 530°C to room temperature: Solid solution transforms to two-phase mixture of Au-rich and Ni-rich solid solutions. Initial composition of Ni rich solid is around 90 wt% Ni.
At room temperature the phases are Au-rich solid (Au) (5 wt% Ni) + Ni-rich solid (Ni) (99 wt% Ni). Proportion of (Au) ≈ 89/94 ≈ 95%, and 5% (Ni).

E.31 The schematic microstructure shows primary grains containing fine precipitates, with eutectic microstructure in between. The primary grains must have formed first, as their curvature shows how they were growing out into the surrounding liquid, and had partially impinged forming grain boundaries. Since the primary grains contain extensive precipitates, these are Pb grains containing Sn precipitates (since Pb is almost insoluble in Sn at the eutectic temperature, so there is little potential for forming precipitates on cooling).

The proportion of eutectic microstructure is about 10%, so just above the eutectic temperature there would have been 90% primary (Pb) grains and 10% liquid. From a tie-line in the liquid + (Pb) region just above 183°C, the composition is therefore about Pb-23wt% Sn. On cooling from the liquid state, solidification would have started at around 275°C, the remaining 10% liquid then formed a eutectic two-phase solid at 183°C, with the precipitates finally appearing on further cooling to room temperature. These steps are illustrated below.



Nucleation of (Pb), below 275°C

Just above 183°C

Eutectic (Pb)+(Sn), just below 183°C

Precipitation of (Sn) within (Pb) grains, on further cooling

Figure E.31 (solution): Schematic microstructure evolution for solidification of Pb-23wt% Sn alloy