EOCT Review

Math 1

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Unit 3

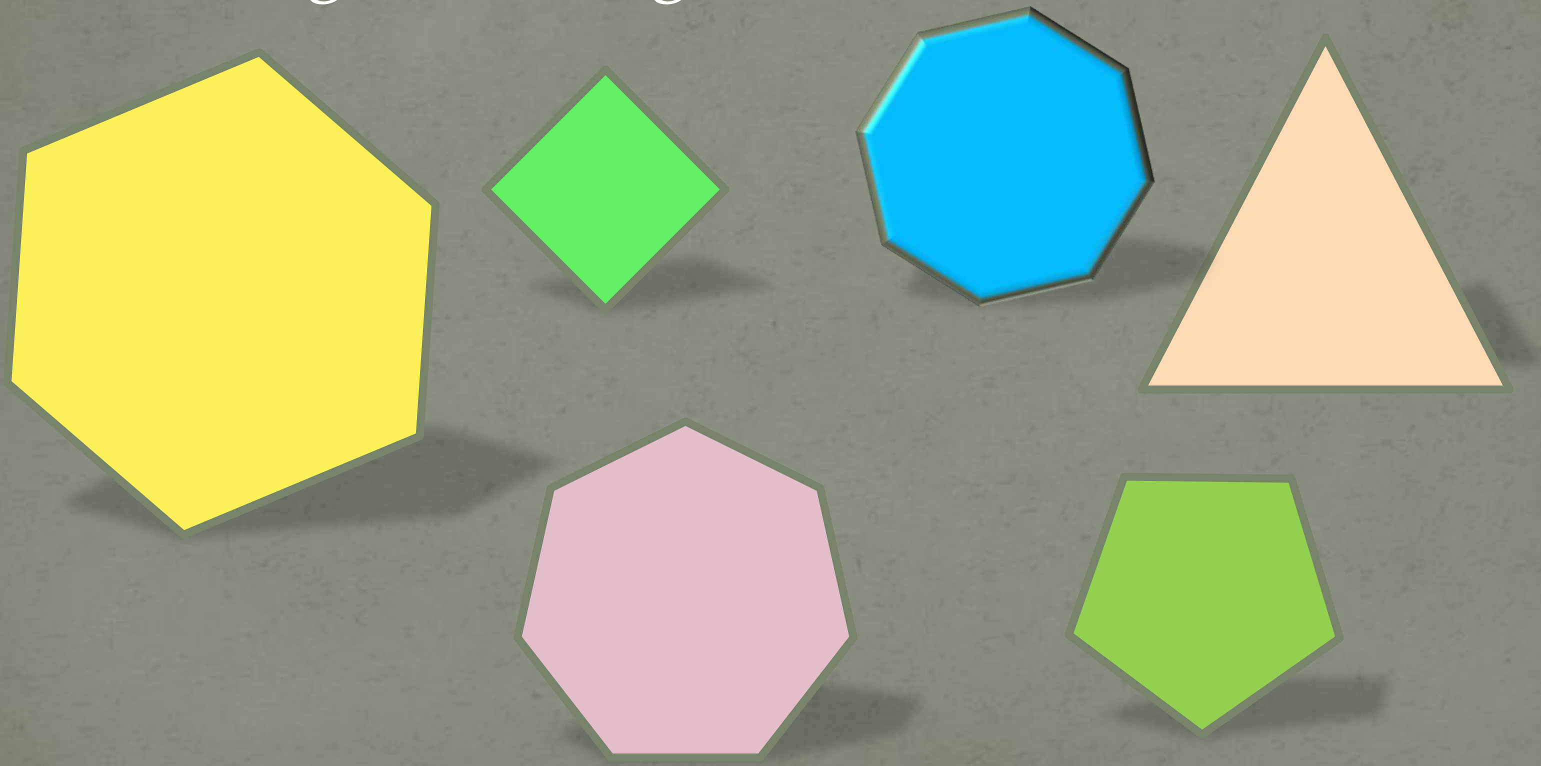
Geometry Gallery

Unit 3: Geometry Gallery

- Polygons
- Interior Sum Theorem
- Exterior Angle Inequality
- Exterior Angle Sum Theorem (polygons and triangles)
- Triangle Inequality Theorem
- Congruence (SSS, SAS, ASA, AAS, HL)
- Points of Concurrency
- Quadrilaterals
- Slope, Distance, Midpoint Formulas

Polygons

- In a **regular polygon**, all side lengths are congruent, and all angles are congruent.



MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.

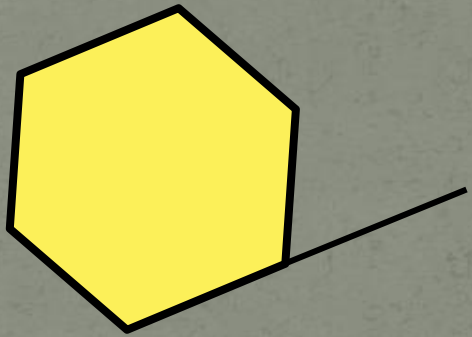
Angles of Polygons

- **Sum of the Measures of the Interior Angles** of a convex polygon is found by solving $180^0(n\text{-}2)$.
- **Measure of each interior angle** of a regular *n*-gon is found by $\dfrac{180°(n-2)}{n}$.

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
a. Determine the sum of interior and exterior angles in a polygon.

Angles of Polygons

- **Exterior angle of a polygon** is an angle that forms a linear pair with one of the angles of the polygon.

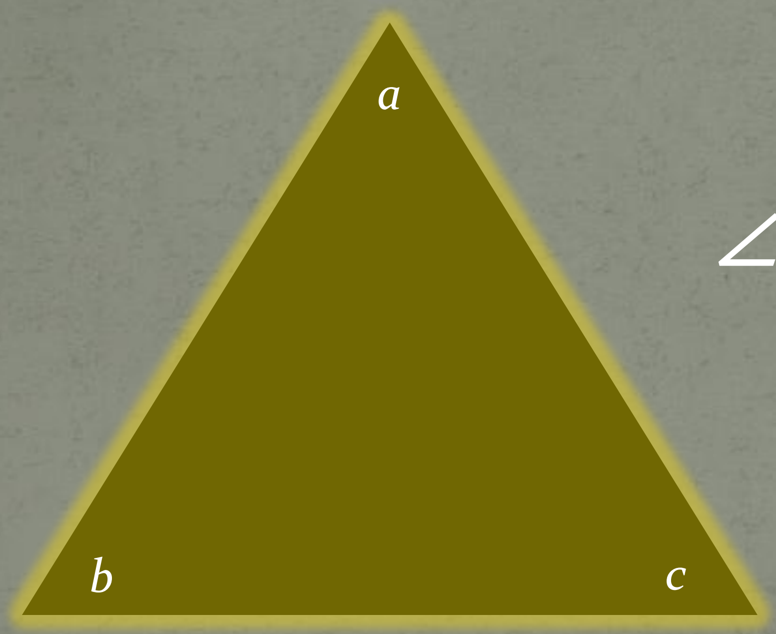


- The **Exterior Angle Sum Theorem** states that if a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360°. The measure of *each exterior angle* is $\frac{360°}{n}$.

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
a. Determine the sum of interior and exterior angles in a polygon.

Triangles

- **Interior Sum Theorem**: the sum of the measures of the three interior angles of a triangle always equal 180⁰.

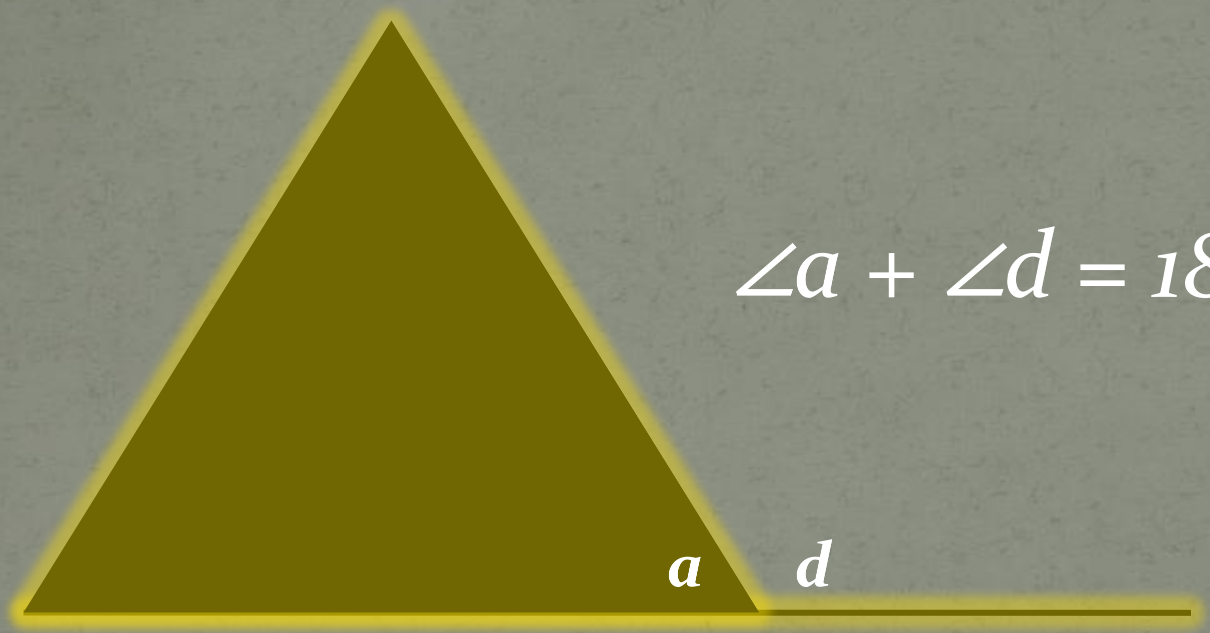


$$\angle a + \angle b + \angle c = 180^{o}$$

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
a. Determine the sum of interior and exterior angles in a polygon.

Triangles

- Interior angles and their adjacent exterior angles are always **supplementary**.

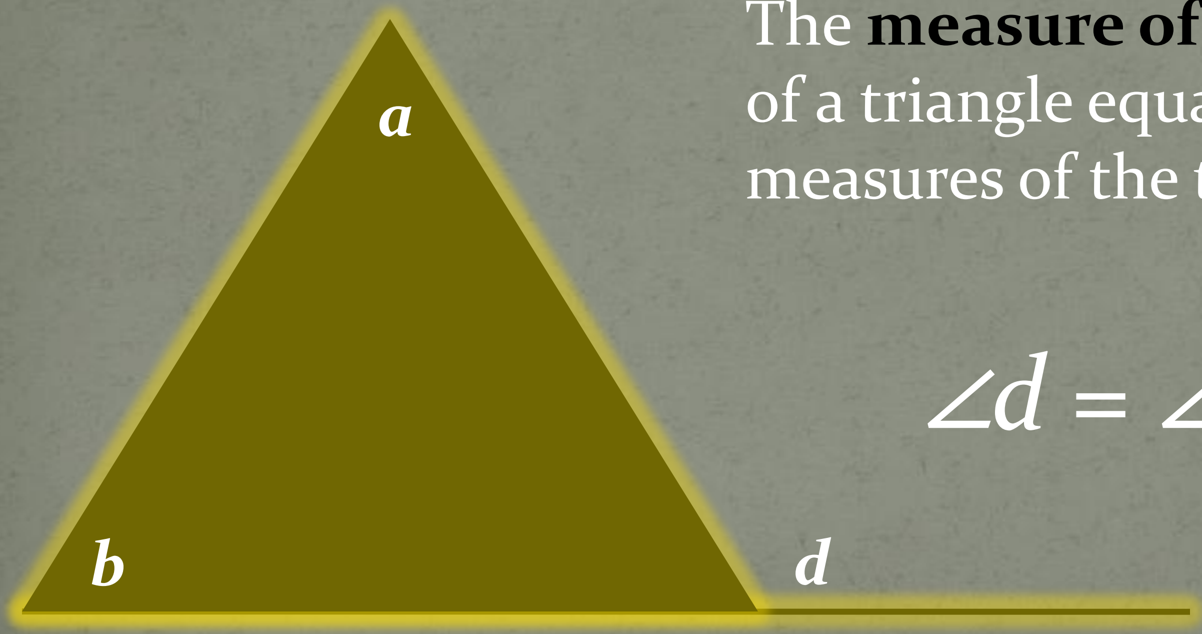


$$\angle a + \angle d = 180^{o}$$

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
a. Determine the sum of interior and exterior angles in a polygon.

Triangles

- **Remote interior angles** of a triangle are the two angles non-adjacent to the exterior angle.



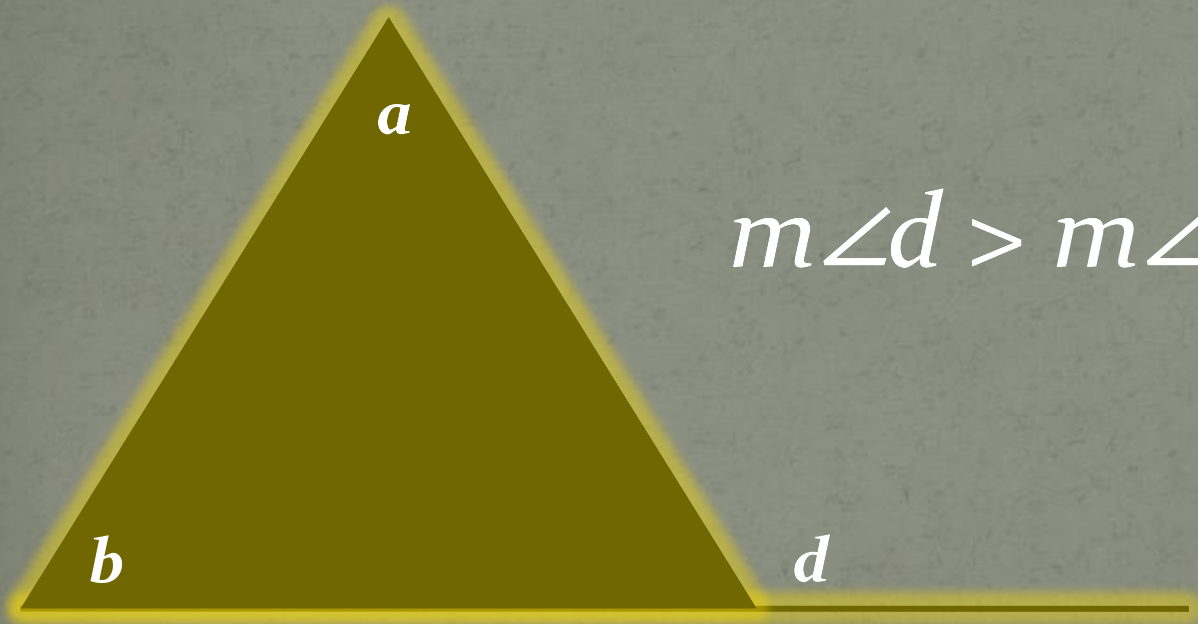
The **measure of the exterior angle** of a triangle equals the sum of the measures of the two remote angles.

$$\angle d = \angle a + \angle b$$

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
a. Determine the sum of interior and exterior angles in a polygon.

Triangles

- The **Exterior Angle Inequality** states that an exterior angle of a triangle is greater than either of the two remote interior angles.



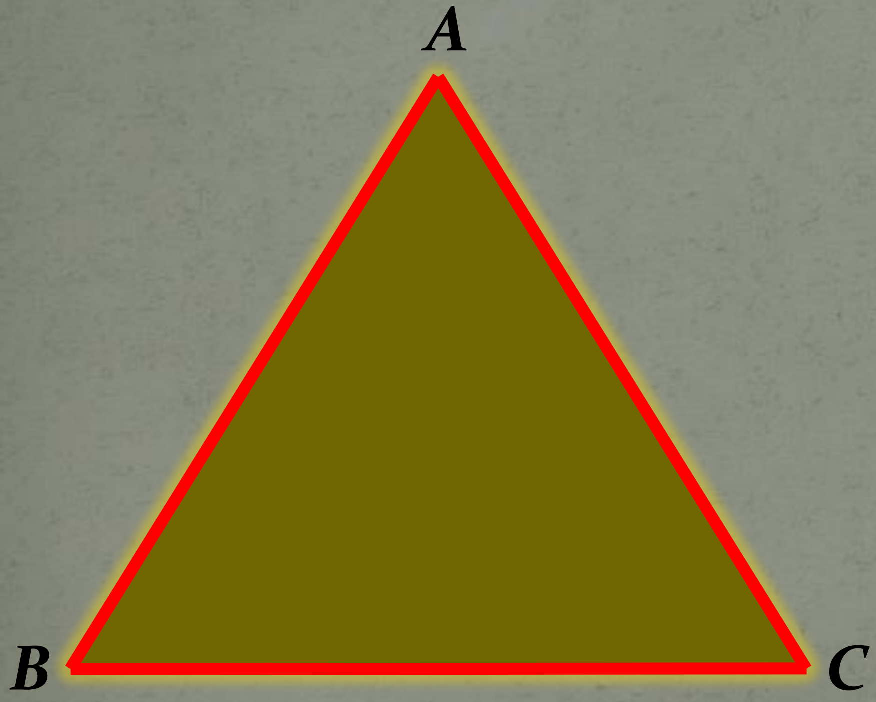
$$m\angle d > m\angle a \text{ or } m\angle b$$

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.

a. Determine the sum of interior and exterior angles in a polygon.
b. Understand and use the triangle inequality, the side-angle inequality, and the exterior-angle inequality.

Triangles

- **Triangle Inequality Theorem**: the sum of the lengths of any two sides of a triangle is greater than the length of the third side.



$$\overline{AB} + \overline{BC} > \overline{AC}$$

$$\overline{BC} + \overline{AC} > \overline{AB}$$

$$\overline{AB} + \overline{AC} > \overline{BC}$$

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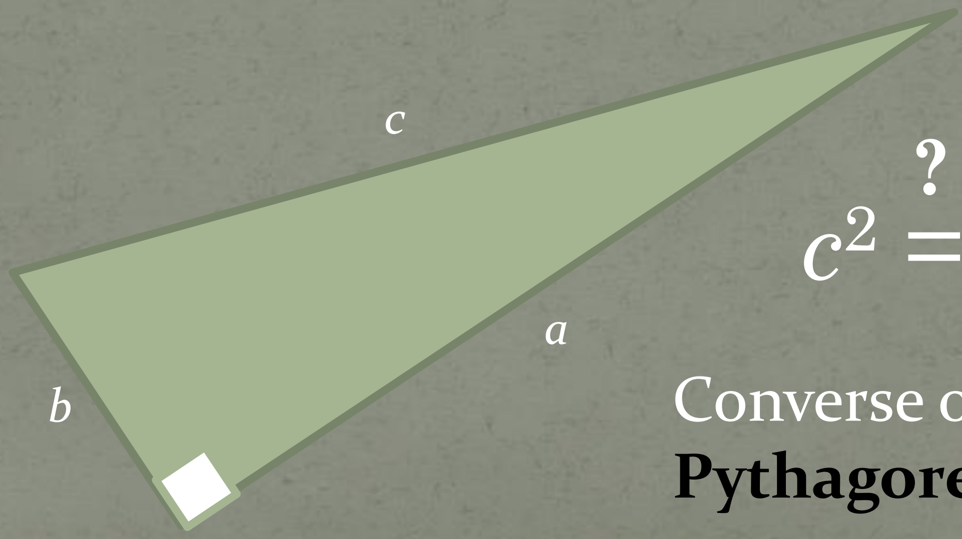
a. Determine the sum of interior and exterior angles in a polygon.

b. Understand and use the triangle inequality, the side-angle inequality, and the exterior-angle inequality.

Triangles

- If c is the measure of the longest side of a triangle, a and b are the lengths of the other two sides, and $c^2 = a^2 + b^2$, then the triangle is a right triangle.



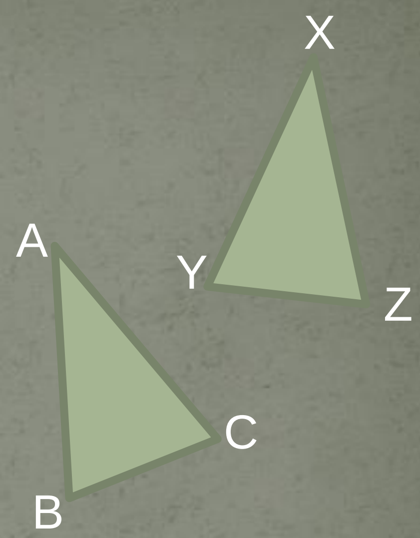
$$c^2 \stackrel{?}{=} a^2 + b^2$$

Converse of the **Pythagorean Theorem**

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.

Congruence

- The symbol ≅ means "is congruent to."



- If $\Delta ABC \cong \Delta XYZ$, then we know

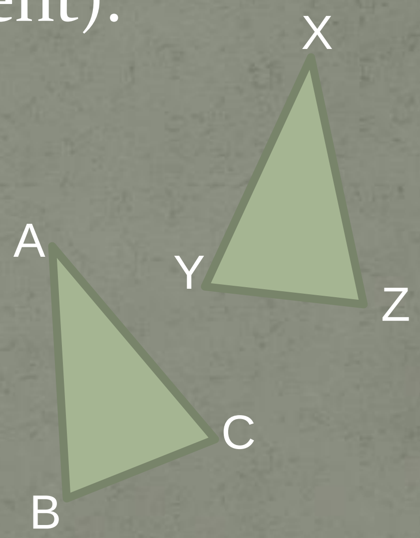
$$\overline{AB} \cong \overline{XY}, \overline{BC} \cong \overline{YZ}, \overline{AC} \cong \overline{XZ}$$

$$\angle A \cong \angle X, \angle B \cong \angle Y, \angle C \cong \angle Z$$

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
c. Understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, HL).

Congruence

- A more convenient way to say that is **CPCTC**: If a two triangles are congruent, then all of their corresponding parts are congruent (**C**orresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent).



MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
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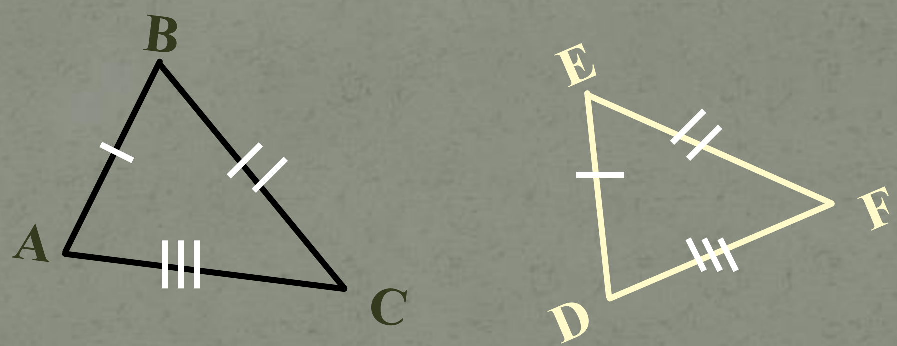
SSS Congruence Theorem



1. $\overline{AB} \cong \overline{DE}$
2. $\overline{BC} \cong \overline{EF}$
3. $\overline{AC} \cong \overline{DF}$

$\Rightarrow$ $\Delta ABC \cong \Delta DEF$

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
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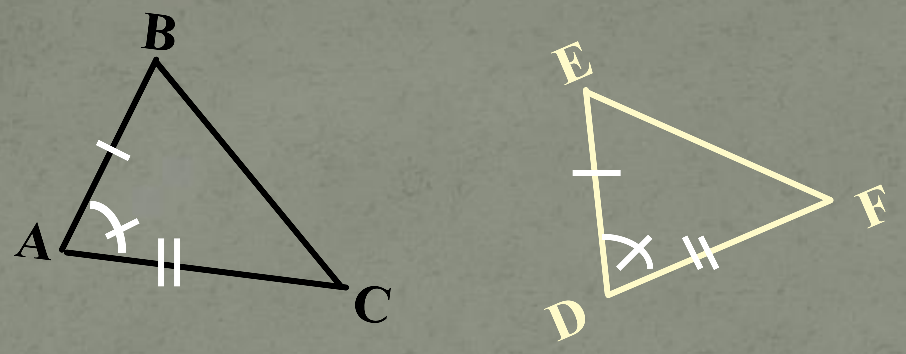
SAS Congruence Theorem



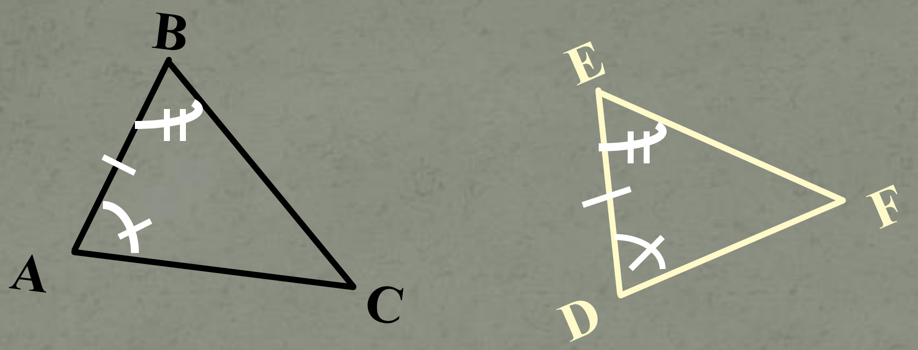
1. $\overline{AB} \cong \overline{DE}$
2. $\angle A \cong \angle D$
3. $\overline{AC} \cong \overline{DF}$

$\Delta ABC \cong \Delta DEF$

included angle

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
c. Understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, HL).

ASA Congruence Theorem



1. $\angle A \cong \angle D$
2. $\overline{AB} \cong \overline{DE}$
3. $\angle B \cong \angle E$

$\Delta ABC \cong \Delta DEF$

included side

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
c. Understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, HL).

AAS Congruence Theorem

1. $\angle A \cong \angle D$
2. $\angle B \cong \angle E$
3. $\overline{BC} \cong \overline{EF}$

$\Delta ABC \cong \Delta\, DEF$

Non-included side

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
c. Understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, HL).

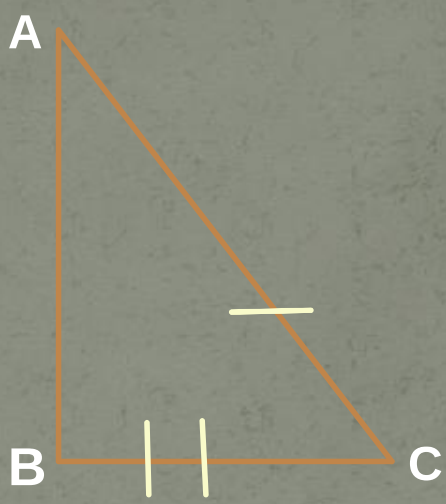
Remember…

- SSS correspondence
- ASA correspondence
- SAS correspondence
- AAS correspondence
- ~~SSA correspondence~~
- ~~AAA correspondence~~

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
c. Understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, HL).

Hypotenuse-Leg (HL) ≅ Theorem

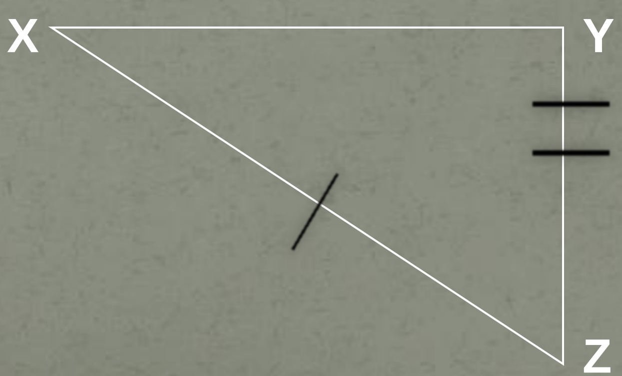
If the hypotenuse and a leg of one right Δ are ≅ to the hypotenuse and leg of another right Δ, then the Δs are ≅.



If AC ≅ XZ and

BC ≅ YZ, then

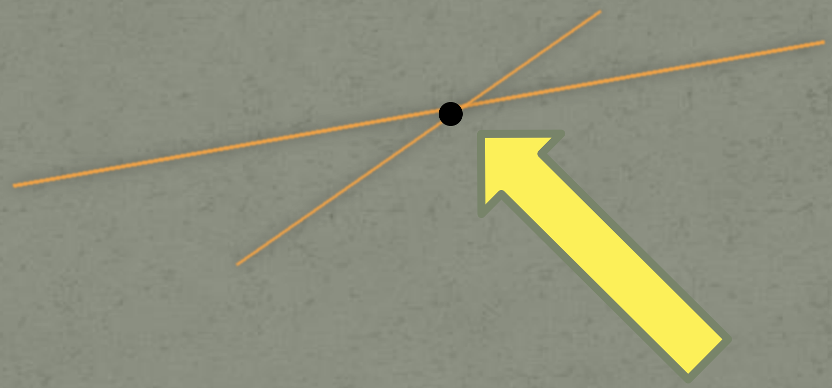
Δ ABC ≅ Δ XYZ



MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
c. Understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, HL).

Points of Concurrency

- Two or more lines that intersect in one point are ***concurrent lines***.

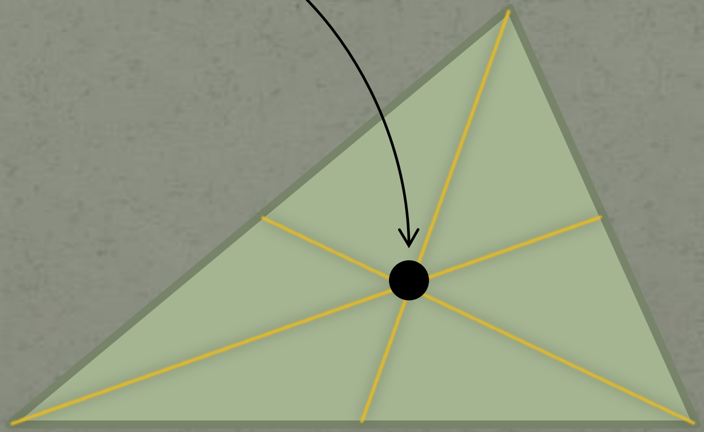


- This intersection point is known as the **point of concurrency**.

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
e. Find and use points of concurrency in triangles: incenter, orthocenter, circumcenter, and centroid.

Centroid

- The **centroid** is the point of concurrency of the *medians* of a triangle.
- A **median** of a triangle is the segment that joins a vertex of a triangle to the opposite side.



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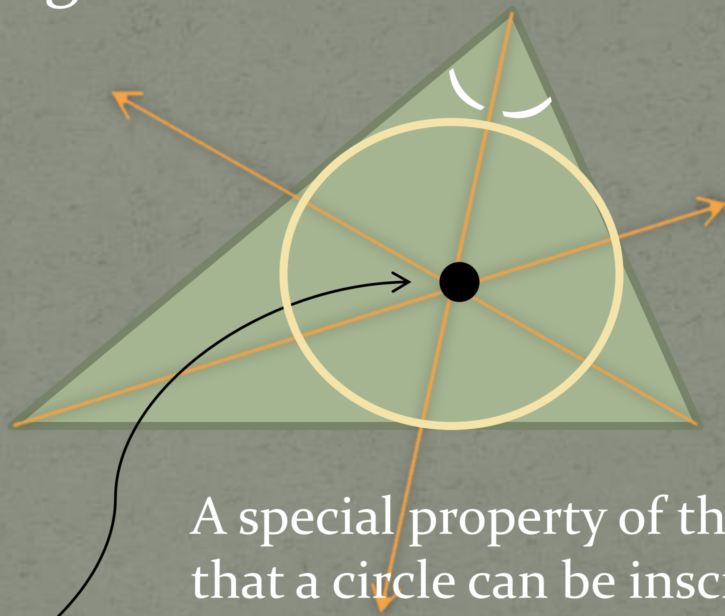
Incenter

- The **incenter** is the point of concurrency of the *angle bisectors* of the triangle.

Remember: angle bisectors cut the angle in HALF!



INCENTER

A special property of the incenter is that a circle can be inscribed in the triangle. The incenter of the triangle forms the center of the circle.

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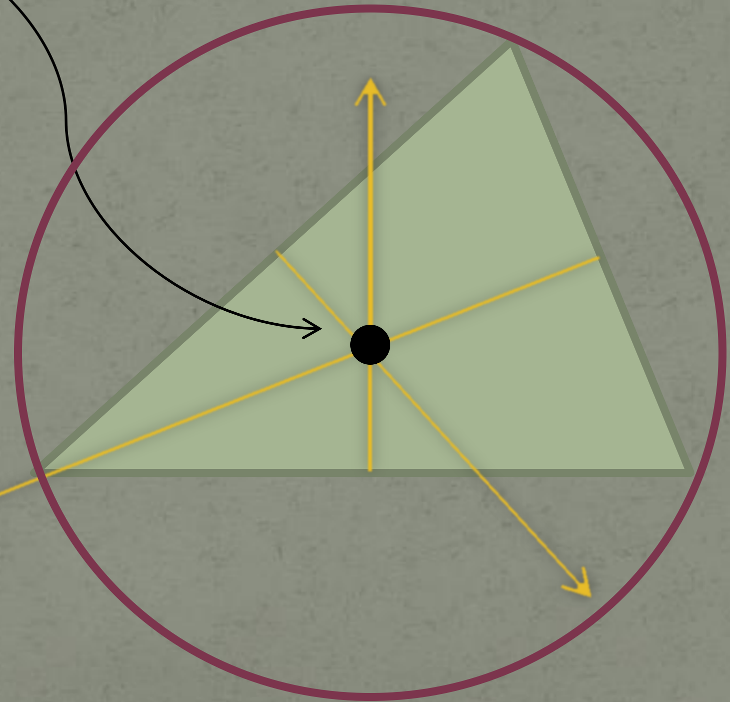
Circumcenter

- The **circumcenter** is the point of concurrency of the *perpendicular bisectors* of the a triangle.

A special property of the circumcenter is that when a circle is circumscribed about the triangle, the *circumcenter of the triangle is the **center*** of that circle.



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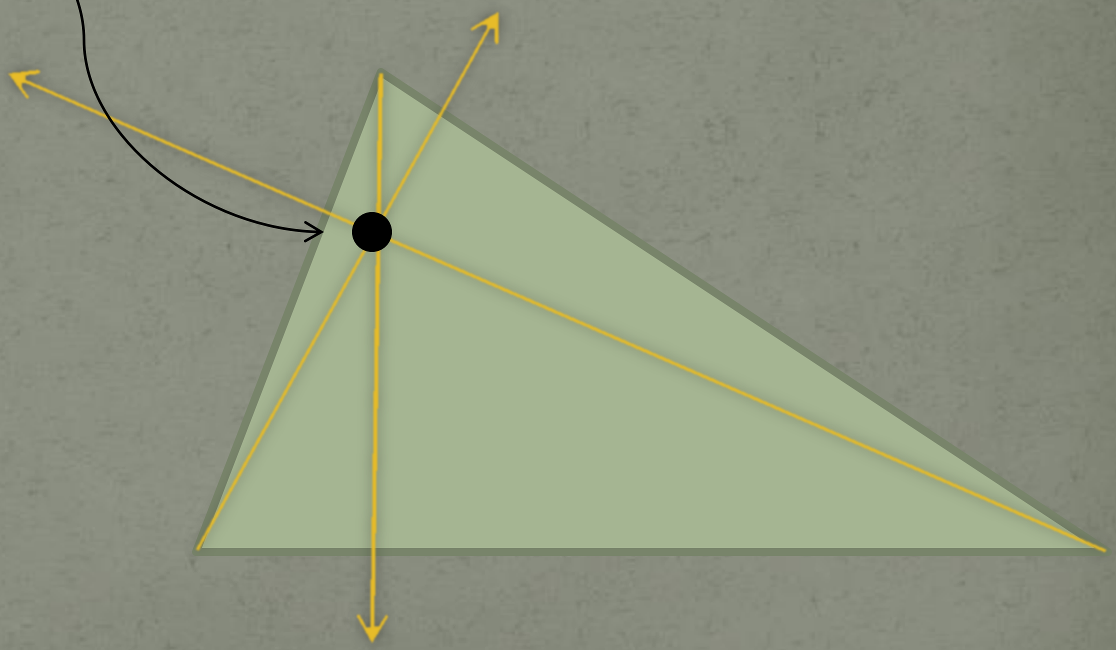
Orthocenter

- The **orthocenter** is the point of concurrency of the *altitudes* of a triangle.

Remember: the altitude is formed by dropping a *perpendicular line* from a vertex to the opposite side.



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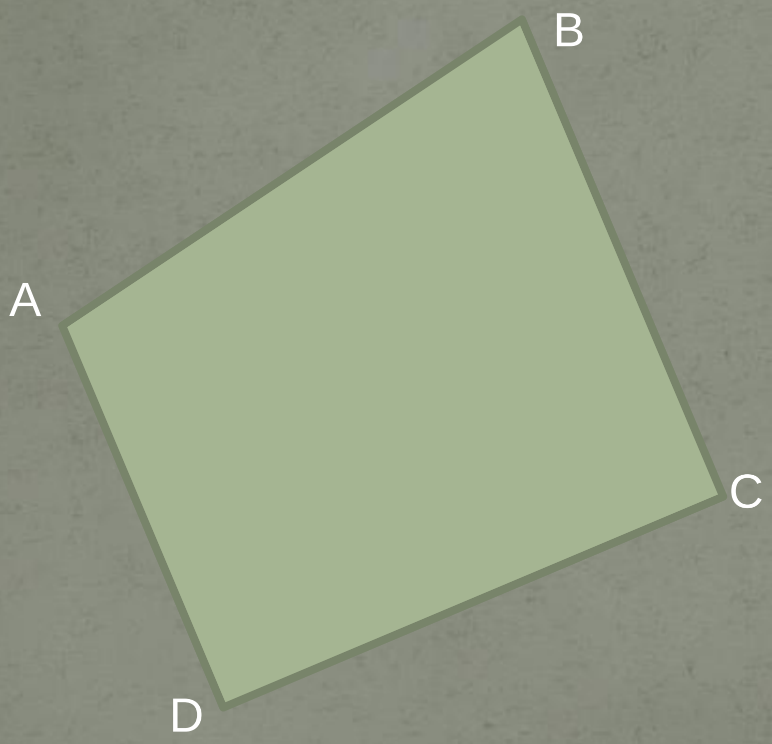
Quadrilaterals

- A **quadrilateral** is a four sided polygon. The interior angles of all convex quadrilaterals sum to 360⁰.



Named quadrilateral ABCD

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons
d. Understand, use, and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite.
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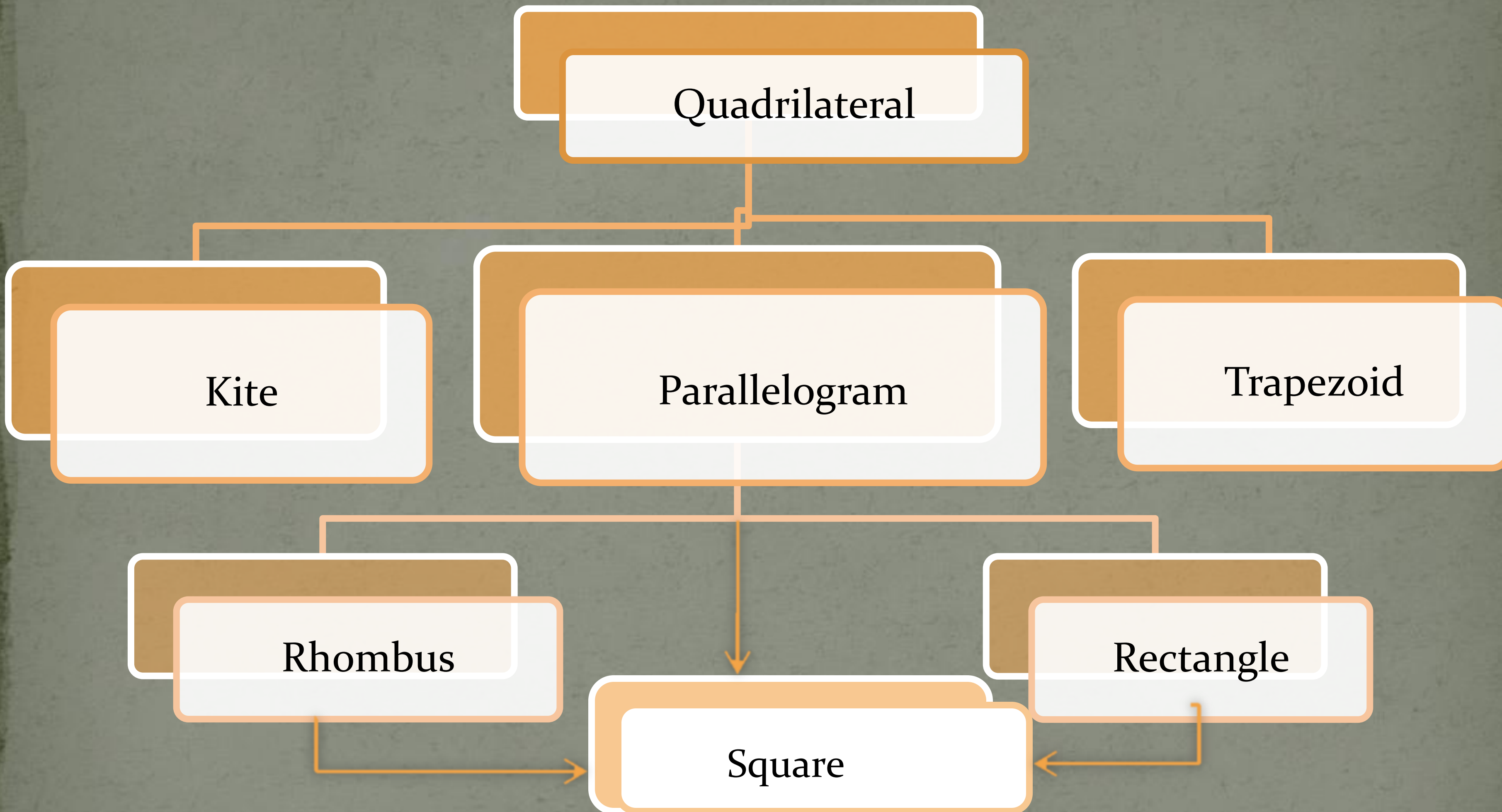
Quadrilaterals



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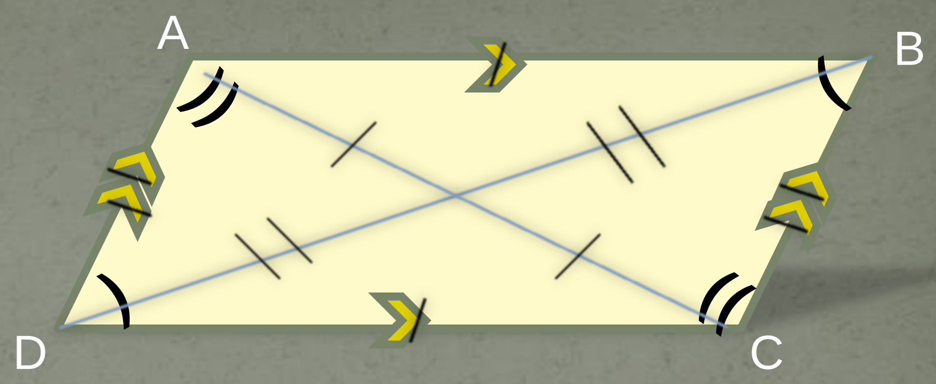
Parallelogram



- Opposite sides are parallel
- Opposite sides are congruent
- Opposite angles are congruent
- Diagonals bisect each other
- Consecutive angles are supplementary

$\angle A + \angle B = 180^\circ$, $\angle B + \angle C = 180^\circ$, etc

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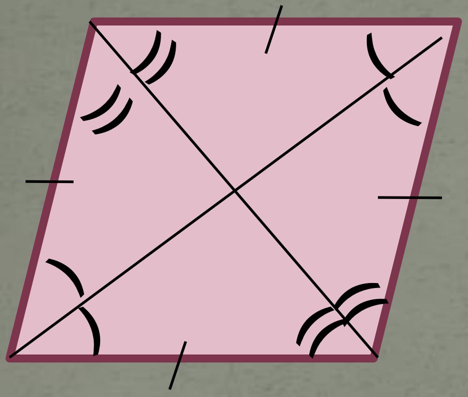
Rhombus



- Has all properties of a parallelogram

PLUS

- Four sides are equal in length
- Diagonals are perpendicular
- Diagonals bisect each pair of opposite angles

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d. Understand, use, and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite.
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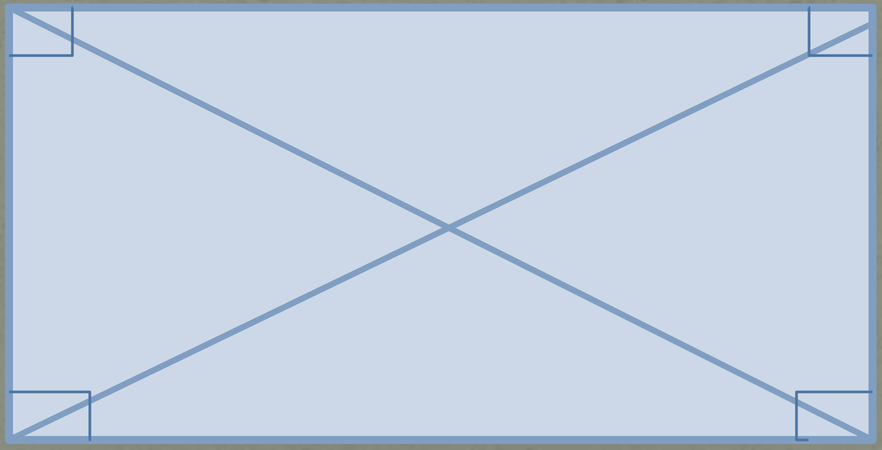
Rectangle

- Has all the properties of the parallelogram

PLUS

- Diagonals are congruent
- Contains four right angles



MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons
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Square

- Has all the properties of a parallelogram

PLUS

- Diagonals are congruent AND perpendicular
- Is a rectangle with all sides congruent.
- Is a rhombus with four right angles.

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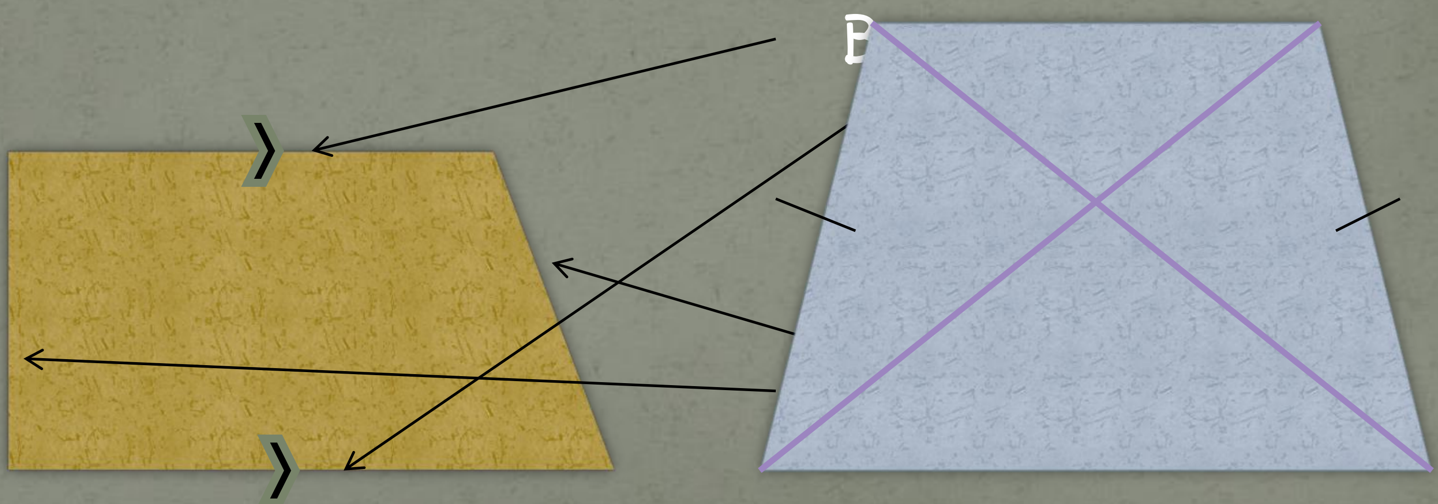
Trapezoid

- One pair of opposite sides that are parallel
- Two parallel sides are called bases and the non-parallel side are the legs.
- Isosceles trapezoids have one pair of congruent sides and congruent diagonals.



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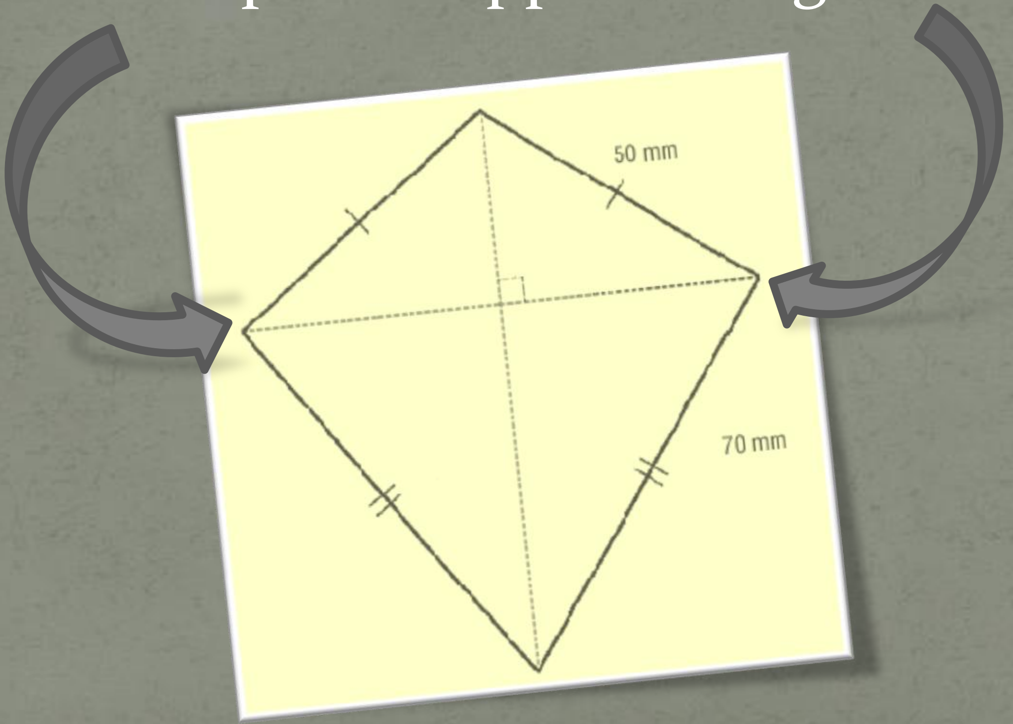
Kite

- A kite is a quadrilateral that has exactly two distinct pairs of adjacent congruent sides.
- A kite has one pair of opposite angles congruent.



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Quadrilateral Theorems

- If one pair of opposite sides of a quadrilateral is congruent and parallel, then the quadrilateral is a parallelogram.
- If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons
d. Understand, use, and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite.

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More Quad Theorems

- If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.
- If each diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.
- If the diagonals of a parallelogram are congruent, then the parallelogram is a

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Don't forget…

- Slopes of *parallel lines* are **equal**.
- Slopes of *perpendicular lines* are **opposite reciprocals**.
- **Distance formula** can be used to determine congruence in a coordinate plane.

$$d = \sqrt{\;x_2 - x_1\;{}^2 + \;y_2 - y_1\;{}^2}$$

MM1G1. Students will investigate properties of geometric figures in the coordinate plane.
d. Understand the distance formula as an application of the Pythagorean theorem.
e. Use the coordinate plane to investigate properties of and verify conjectures related to triangles and quadrilaterals.