Math 1 Variable Manipulation Part 3
Solving Algebraic Equations

EVALUATING AN EXPRESSION BY PLUGING IN VALUE
To evaluate an algebraic expression, plug in the given values for the unknowns and calculate according to PEMDAS.

Example: Find the value of $x^2 + 5x - 6$ when $x = -2$
Solution: Plug in -2 for x and solve
$(-2)^2 + 5(-2) - 6 = 4 - 10 - 6 = -12$

Sample Questions:
1. Find the value of $5a^2 – 6a + 12$ when $a = -5$

2. Find the value of $4x – 6x + 3x – 8 + 4$ when $x = 4$

3. If $p = 2$, what is the value of $8p^2 – 5p^2 + 7 – 4p + 3$

4. If $n = 10$, then which of the following represents 552?
   a. $5n + 2$
   b. $5n^2 + 2$
   c. $5n^2 + 5n + 2$
   d. $5n^3 + 5n + 2$
   e. $5n^4 + 5n + 2$

5. When $n = ¼$, what is the value of $\frac{2n - 5}{n}$

6. What is the value of the expression $10(100x – 10{,}000) + 100$ when $x = 250$?

MULTIPLE PLUG INS

If there are more than one variable, just plug in each of the numbers into the corresponding variables.

Sample Questions:

7. If $r = 9$, $b = 5$, and $g = -6$, what does $(r + b - g)(b + g)$ equal?

8. Solve for $(9a + 7b) + (-9a + 10 - b)$ when $a = 2$ and $b = 3$

9. If $x = -4$, $y = 3$ and $z = 8$, what is the value of $2x^2 + 3y - z$?

10. What is the value of the expression $(x - y)^2$ when $x = 5$ and $y = -1$?

SOLVING A LINEAR EQUATIONS (SOLVE FOR X)

To solve an equation, isolate the variable. As long as you do the same thing to both sides of the equation, the equation is still balanced. To solve

Example: $5x - 12 = -2x + 9$

Solution: First get all the x terms on one side by adding 2x to both sides: $7x - 12 = 9$. Then add 12 to both sides: $7x = 21$, then divide both sides by 7 to get: $x = 3$.

Sample Questions:

11. For what value of a is the equation $3(a + 5) - a = 23$ true?

12. If $4(x - 2) + 5x = 3(x + 3) - 11$, then $x = ?$

13. If $5x + 5 = 25 + 3x$, then $x = ?$

14. If $9(x - 9) = -11$, then $x = ?$

15. If $7 + 3x = 22$, then $2x = ?$

16. If $4(x - 2) + 5x = 3(x + 3) - 11$, then $x = ?$

17. If $x + \frac{2}{3} = \frac{8}{21}$, then $x = ?$

18. When $\frac{1}{3}k + \frac{1}{4}k = 1$, what is the value of k?

19. The total weekly profit p, in dollars, from producing and selling x units of a certain product is given by the function $p(x) = 225x - (165x + c)$, where c is a constant. If 75 units were produced and sold last week for a profit of \$3,365, what is the value of c?

NO OR INFINATE SOLUTIONS TO EQUATIONS (SOLVE FOR X)

When you try to solve an equation, you are starting from the (unstated) assumption that there actually is a solution. When you end up with nonsense (like the nonsensical equation "4 = 5" above), this says that your initial assumption (that there was a solution) was wrong; in fact, there is no solution. Since the statement "4 = 5" is utterly false, and since there is no value of x that ever could make it true, then this equation has no solution. This is not the same as a solution where x = 0. Don't confuse these two very different situations: "the solution exists and has the value of zero" is not in any manner the same as "no solution value exists at all".

Example: Solve 11 + 3x– 7 = 6x + 5 – 3x

Solution: First, combine like terms; then solve:

11 + 3x – 7 = 6x + 5 -3x

4 + 3x = 3x + 5

-3x -3x

4 = 5 The "solution" is "no solution".

What if the answer you get is 5 = 5? Is there any value of x that would make that statement false? Since there is no "x" in the solution, the value of x is irrelevant: x can be anything. So the solution is "all x". This solution could also be stated as "all real numbers" or "all reals" or "the whole number line"; expect some variation in lingo from. Note that, if I had solved the equation by subtracting a 5 from either side of 5 + 4x = 5 + 4x to get "4x = 4x", I would have ended up with nothing other than another trivially-true statement. I could also have subtracted both 4x and 5 from both sides to get "0 = 0", but the solution would still be the same: "all x". Don't be surprised if, for "all real numbers" or "no solution" equations, you don't necessarily have the exact same steps as some of your fellow students. Since there are infinitely-many always-true equations (like "0 = 0") and infinitely-many nonsensical equations (like "3 = 4"), there will be many ways of arriving at these answers.

Example: Solve 6x + 5 – 2x = 4 + 4x + 1

Solution: First, combine like terms; then solve:

6x + 5 – 2x = 4 + 4x + 1

4x + 5 = 5 + 4x

-4x -4x

5 = 5 There are infinately many solutions as every value of x could be true.

Sample Question:

20. Which of the statements describes the solution set for -2(x + 8) = -2x + 20?
    a. x = - 2 only
    b. x = 0 only
    c. x = 20 only
    d. There are no solution for this equation.
    e. All real numbers are solutions of this equation.
21. p – 4 = -9 + p

22. 12 = -4(-6x -3)

PLUGGING IN THE ANSWERS (PITA)

"Plugging In" is a great strategy when there are variables in the question or the answers. Use Plugging ln

- when there are variables in the answer choices
- when solving word problems or plug-and-chug questions
- for questions of any difficulty level

How about when there aren't? Does that mean we have go back to algebra? Of course not! On most problems on the ACT, there are a variety of ways to solve. Plugging in the answers are very helpful when:

- answer choices are numbers in ascending or descending order
- the question asks for a specific amount. Questions will usually be "what?" or "how many?"
- you have the urge to do algebra even when there are no variables in the problem

Because many answer choices are listed in ascending order, it will be best to start with the middle choice. That way, if it's too high or too low, we'll be able to use process of elimination (POE) more efficiently.

Simple Example: Solve for 12 = -4(6x – 3)

a. 0
b. 1
c. 2
d. 3
e. 4

Solution: Since the list is in ascending order, start in the middle and plug in 2 for x. When you do you get 6*2 which is 12 -3 which is 9 times -4 which is -36. That is not it so try 1. 6 *1 is 6 – 3 is 3 times -4 = -12. That is still too low so try 0. 6*0 is 0 -3 is -3 times -4 is 12. That is the answer!

Note: This is a simple problem, so it is probably easier just to solve for x. However, this is a great skill to use when it is NOT simple or you can't figure it out another way.

More Complicated Example: In a piggy bank, there are pennies, nickels, dimes, and quarters that total $2.17 in value. If there are 3 times as many pennies as there are dimes, 1 more dime than nickels, and 2 more quarter than dimes, then how many pennies are in the piggy bank?

a. 12
b. 15
c. 18
d. 21
e. 24

Solution:

1. **Know the question.** How many pennies are in the bank?
2. **Let the answers help.** There are no variables, but the very specific question coupled with the numerical answers in ascending order gives a pretty good indication we can PITA.
3. **Break the problem into bite-sized pieces.** Make sure you take your time with this problem, because you'll need to multiply the number of each coin by its monetary value. In other words, don't forget that 1 nickel will count for 5 cents, 1 dime will count for 10 cents, and I quarter will count for 25 cents. Let's set up some columns to keep our work organized and begin with choice (C).

Since ACT has already given us the answers, we will plug those answers in and work backwards. Each of the answers listed gives a possible value for the number of pennies. Using the information in the problem, we can work backwards from that number of pennies to find the number of nickels, dimes, and quarters. When the values for the number of coins adds up to $2.17, we know we're done.

If we begin with the assumption that there are 18 pennies, then there must be 6 dimes (3 times as many pennies as there are dimes). 6 dimes means 5 nickels (1 more dime than nickels) and 8 quarters (2 more quarters than dimes).

Now multiply the number of coins by the monetary value of each and see if they total $2.17.

Pennies ($P)	Dimes ($0)	Nickels ($N)	Quarters ($Q)	Total = $2.17
c. 18 ($0.18)	6 ($0.60)	5 ($0.25)	8 ($2.00)	Total = $3.03 (NO)

That's too high, so not only is choice (C) incorrect, but also choices (D) and (E). Cross them off and try choice (B).

Pennies ($P)	Dimes ($D)	Nickels ($N)	Quarters ($Q)	Total= $2.17
b. 15 ($0.15)	5 ($0.50)	4 ($0.20)	7 ($1.75)	Total = $2.60 (NO)
a. 12 ($0.12)	4 ($0.40)	3 ($0.15)	6 ($1.50)	Total = $2.17 (YES)

Notes: Plugging In and PITA are not the only ways to solve these problems. The “best” way is the easiest way you can answer the problems. When you solve in a more complex way, there are a lot more opportunities to make careless errors.

Plugging in is not just for algebra. Remember what the main requirements are for a Plugging In problem. You need variables in the answer choices or question: that's it. It doesn't say anywhere that the problem needs to be a pure algebra problem. What is a pure algebra problem anyway? Don't forget: Part of what makes this test so hard is that ACT piles concept on cop of concept in its problems. In other words, geometry problems often are algebra problems.

Sample Questions:

23. Which of the following is NOT a solution of $(x – 5)(x – 3)(x + 3)(x + 9) = 0$?

a. 5
b. 3
c. -3
d. -5
e. -9

24. What is the value of p in the following equation: $-11 + 10(p + 10) = 4 - 5(2p + 11)$?

a. -10
b. -7
c. -5
d. 0
e. 5

25. $-12(x - 12) = -9(1 + 7x)$

a. -3
b. -1
c. 1
d. 3
e. 5

Answer Key

1. 167
2. 0
3. 14
4. C
5. -18
6. 150,100
7. -20
8. 28
9. 33
10. 36
11. 4
12. 1
13. 10
14. 70/9
15. 10
16. 1
17. -$\frac{2}{7}$
18. 12/7
19. 1,135
20. No solutions
21. No solution
22. 0
23. D (-5)
24. -7
25. -3