Math 10

<u>Lesson 1-5</u> Mixed and Entire Radicals

I. Entire and mixed radicals

An **<u>entire radical</u>** is a number in a radical with **<u>no</u>** coefficient or multiplying number in front of the radical.

$\sqrt{23}$ $\qquad$ $\sqrt[3]{2000}$ $\qquad$ $\sqrt[4]{162}$

are all examples of **entire radicals**.

A **<u>mixed radical</u>** is a number in a radical with a coefficient or multiplying number in front of the radical.

$4\sqrt{23}$ $\qquad$ $164\sqrt[3]{2000}$ $\qquad$ $87\sqrt[4]{162}$

are all examples of **mixed radicals**.

II. Multiplication of radicals

If the radicand of a radical (i.e. – the number inside the radical) can be factored, we can express the original radical as a multiplication of the two factors. For example, consider $\sqrt{35}$. 35 can be factored into 5·7. Therefore we can write

$$\sqrt{35} \quad = \sqrt{5 \cdot 7} \qquad = \sqrt{5}\sqrt{7}$$

In the opposite sense, if the index is the same for both radicals, we can combine two radicals into one radical

$$\sqrt[4]{9}\sqrt[4]{11} \qquad = \sqrt[4]{9 \cdot 11} \qquad = \sqrt[4]{99}$$

In general, if a and b are real numbers and n is a natural number,

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a}\sqrt[n]{b}$$

III. Converting/simplifying an entire radical into a mixed radical

To convert an entire radical into a mixed radical we need to find perfect squares or perfect cubes or perfect quartics, depending on the index, that are factors of our original radicand. If there are such factors we can then remove them from the radicand.

Consider, for example, $\sqrt{20}$. First, we make a list of perfect squares: $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, and so on. The largest perfect square factor of 20 is 4. Therefore

$$\sqrt{20} = \sqrt{4}\sqrt{5}$$

but note that $\sqrt{4} = 2$. Therefore we can write

$$\sqrt{20} = 2\sqrt{5}$$

This is how we convert an entire radical into a mixed radical and $2\sqrt{5}$ is referred to as the simplified form of $\sqrt{20}$.

As demonstrated in the example above, **<u>the basic process for simplifying a radical</u>** ($\sqrt[n]{x}$) is to look for factors of *x* that are perfect squares (if n = 2), perfect cubes (if n = 3), a perfect quartic (if n = 4), etc. and then remove them from the radical. Perhaps a few examples may help.

Example 1 Simplify $\sqrt{80}$.

For square roots, we look for the largest factor that is perfect square (i.e. 4, 9, 16, 25 …). The numbers 4 and 16 are factors of 80, so we choose the largest:

$$\begin{aligned}\sqrt{80} &= \sqrt{16\cdot 5}\\ &= \sqrt{16}\sqrt{5}\\ &= 4\sqrt{5}\end{aligned}$$

Example 2 Simplify $\sqrt[3]{144}$.

For cube roots (n = 3), we look for factors of 144 that are perfect cubes (i.e. $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$ …). We try 8, 27 and 64. $144 \div 8 = 18$, $\cancel{144 \div 27 = 5.33}$ and $\cancel{144 \div 64 = 2.25}$. Therefore we use 8 and 18.

$$\begin{aligned}\sqrt[3]{144} &= \sqrt[3]{8\cdot 18}\\ &= \sqrt[3]{8}\sqrt[3]{18}\\ &= 2\sqrt[3]{18}\end{aligned}$$

Example 3 Simplify $\sqrt[4]{162}$.

In general, the larger the index the harder it is to spot appropriate factors to try. A different strategy from the one used in Examples 1 and 2 is to factor the radicand into its prime factors. Since we are dealing with the 4th root, we are looking for prime factors that repeat 4 times. The prime factorization of 162 is 2·3·3·3·3 or $2\cdot3^4 = 2\cdot81$. Thus,

$$\begin{aligned}\sqrt[4]{162} &= \sqrt[4]{2\cdot 81}\\ &= \sqrt[4]{2}\sqrt[4]{81}\\ &= 3\sqrt[4]{2}\end{aligned}$$

Example 4 Simplify $\sqrt{162}+\sqrt{98}$

Note that we cannot add 162 and 98 together: $\sqrt{162}+\sqrt{98} \neq \sqrt{162+98}$

There is no obvious way to simplify this expression. Thus, we try to simply each radical and then see what we can do from there.

$$\begin{aligned}&\sqrt{162}+\sqrt{98}\\ &= \sqrt{81\cdot 2}+\sqrt{49\cdot 2}\\ &= 9\sqrt{2}+7\sqrt{2}\\ &= 16\sqrt{2}\end{aligned}$$

Note, the Pearson text book uses prime factorization as its prime strategy. While it is a good strategy to use at times, it is not wise to use it as your only strategy when solving problems. It is wise to approach problem solving with a more playful and open attitude that is capable of seeing new and creative ways to solve problems. A playful, creative approach also has the benefit of removing some of the boredom and tedium of some math assignments.

Question 1

Simplify the following:

$\sqrt{75}$ $\quad$ $\sqrt[4]{288}$ $\quad$ $\sqrt[3]{1715}$ $\quad$ $\sqrt{30}$ $\quad$ $\sqrt[4]{32}$ $\quad$ $\sqrt[3]{40}$

IV. Expanding mixed radicals into entire radicals

Expanding a mixed radical into an entire radical is the reverse process of simplifying radicals. Generally speaking it is an easier process.

Example 5 Write $4\sqrt{3}$ as an entire radical.

Since the index of the radical is 2, we raise 4 to the 2nd power (4^2) and then multiply this by the radicand.

$$\begin{aligned} 4\sqrt{3} &= \sqrt{4^2}\sqrt{3} \\ &= \sqrt{16}\sqrt{3} \\ &= \sqrt{48} \end{aligned}$$

Example 6 Write $2\sqrt[5]{6}$ as an entire radical.

Since the index of the radical is 5, we raise 2 to the 5th power (2^5) and then multiply this into the radical.

$$\begin{aligned} 2\sqrt[5]{6} &= \sqrt[5]{2^5}\sqrt[5]{6} \\ &= \sqrt[5]{32}\sqrt[5]{6} \\ &= \sqrt[5]{192} \end{aligned}$$

Example 7 Write $3\sqrt[3]{2}$ as an entire radical.

$3^3 = 27$

$$\begin{aligned} \therefore 3\sqrt[3]{2} &= \sqrt[3]{27}\sqrt[3]{2} \\ &= \boldsymbol{\sqrt[3]{54}} \end{aligned}$$

Question 2

Write the following as entire radicals.

$3\sqrt{7}$

$3\sqrt[6]{7}$

$2\sqrt[5]{8}$

V. Nasty question of the day

Example 8 Given that $\sqrt{3} \doteq 1.732$ and <u>without using a calculator</u> determine a decimal approximation for $\sqrt{12}$.

Perhaps we could change $\sqrt{12}$ into a mixed radical and then see where that leads us.

$$\begin{aligned}\sqrt{12} &= \sqrt{4 \cdot 3} \\ &= 2\sqrt{3}\end{aligned}$$

Since $\sqrt{3} \doteq 1.732$

$$\begin{aligned}\sqrt{12} &= 2\sqrt{3} \\ &= 2 \cdot 1.732 \\ &= 3.464\end{aligned}$$

Therefore $\sqrt{12} \doteq 3.464$

Given that $\sqrt{2} \doteq 1.4142$ and <u>without using a calculator</u> determine a decimal approximation for each radical.

(a) $\sqrt{200}$

(b) $\sqrt{20000}$

(c) $\sqrt{8}$

(d) $\sqrt{18}$

(e) $\sqrt{32}$

(f) $\sqrt{50}$

VI. Assignment

1. Write each radical in simplest form.

a) $\sqrt{8}$ b) $\sqrt{12}$ c) $\sqrt{32}$ d) $\sqrt{50}$
e) $\sqrt{18}$ f) $\sqrt{27}$ g) $\sqrt{48}$ h) $\sqrt{75}$

2. Write each mixed radical as an entire radical.

a) $5\sqrt{2}$ b) $6\sqrt{2}$
e) $5\sqrt{3}$ f) $6\sqrt{3}$

3. Write each radical in simplest form, if possible.

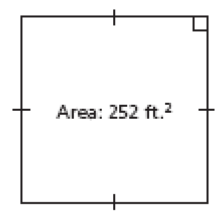
d) $\sqrt{600}$ e) $\sqrt{54}$ f) $\sqrt{91}$
g) $\sqrt{28}$ h) $\sqrt{33}$ i) $\sqrt{112}$

4. Write each radical in simplest form, if possible.

a) $\sqrt[3]{16}$ c) $\sqrt[3]{256}$ e) $\sqrt[3]{60}$ g) $\sqrt[3]{135}$ i) $\sqrt[3]{500}$

5. Write each mixed radical as an entire radical.

a) $3\sqrt{2}$ c) $6\sqrt{5}$ e) $7\sqrt{7}$ g) $3\sqrt[3]{3}$ i) $5\sqrt[3]{2}$



6. Express the side length of this 252 ft^2 square as a radical in simplest form.

7. A cube has a volume of 200 cm^3. Write the edge length of the cube as a radical in simplest form.

8. A square has an area of 54 square inches. Determine the perimeter of the square. Write the answer as a radical in simplest form.

9. Write each radical in simplest form.

a) $\sqrt[4]{48}$
c) $\sqrt[4]{1250}$

10. Write each mixed radical as an entire radical.

a) $6\sqrt[4]{3}$
c) $3\sqrt[5]{4}$

11. Here is a student's solution for writing as an entire radical.

$$\begin{aligned} 8\sqrt[3]{2} &= 8\cdot\sqrt[3]{2} \\ &= \sqrt[3]{2}\cdot\sqrt[3]{2} \\ &= \sqrt[3]{2\cdot 2} \\ &= \sqrt[3]{4} \end{aligned}$$

Identify an error the student made, then write the correct solution.

12. A student simplified a radical as shown:

$$\begin{aligned} \sqrt{96} &= \sqrt{4}\cdot\sqrt{48} \\ &= 2\cdot\sqrt{48} \\ &= 2\cdot\sqrt{8}\cdot\sqrt{6} \\ &= 2\cdot 4\cdot\sqrt{6} \\ &= 8\sqrt{6} \end{aligned}$$

Identify the errors the student made, then write a correct solution.

13. Simplify the radicals in each list. What patterns do you see in the results? Write the next 2 radicals in each list.

a)	$\sqrt{4}$	c)	$\sqrt{8}$
	$\sqrt{400}$		$\sqrt{800}$
	$\sqrt{40\,000}$		$\sqrt{80\,000}$

14. The largest square in this diagram has side length 8 cm. Calculate the side length and area of each of the two smaller squares. Write the radicals in simplest form.