MATH 10C

RELATIONS AND FUNCTIONS

STUDENT NAME: Key

CHAPTER EXAM DATE: ______________________

Math 10C

Unit: **Relations and Functions**

Topic: **Representing Relations**

Objectives:

- Represent relations in different ways.

NOTES

A ***Set*** is a collection of distinct objects.

An ***element*** is one object in the set.

A ***relation*** associates the elements of one set with the elements of another set.

A set is shown as a list in braces. Eg the set of natural numbers to 5 is {1,2,3,4,5}. The order that the elements in the set are listed does not matter.

Consider the following sets:

Set1:{apples, bananas, beans, carrots, corn, oranges, pears, peas, pineapple, potatoes}

Set2:{Fruit, Vegetables}

We can associate all of the elements in set one with an element from set two. This means that the two set are a relation.

We can show a relation as:

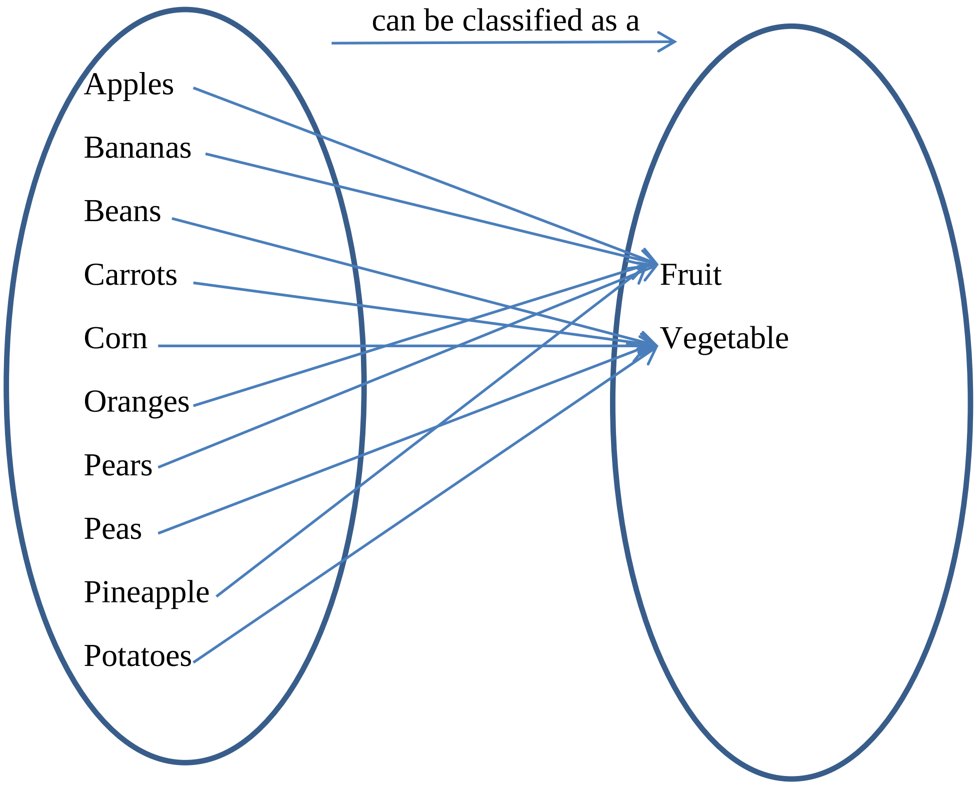
1 A set of ordered pairs,

(apples, fruit), (bananas, fruit), (beans, vegetables), (carrots, vegetables), (corn, vegetables), (oranges, fruit), (pears, fruit), (peas, vegetables), (pineapple, fruit), (potatoes, vegetable)

2 A table,

Produce	**Classification**
Apples	Fruit
Bananas	Fruit
Beans	Vegetable
Carrots	Vegetable
Corn	Vegetable
Oranges	Fruit
Pears	Fruit
Peas	Vegetable
Pineapple	Fruit
Potatoes	Vegetable

3 An arrow diagram,



4 And a graph (when one or both sets are numbers).

Example: Northern communities can be associated with the territories they are in. Consider the relation represented by the table.

Community	Territory
Hay River	NWT
Iqaluit	Nunavut
Nanisivik	Nunavut
Old Crow	Yukon
Whitehorse	Yukon
Yellowknife	NWT

- Describe the relation in words.

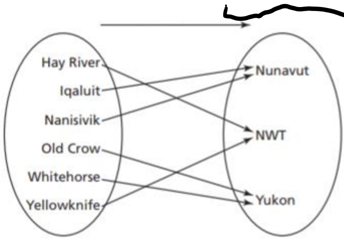
The relation shows the association "is located in"

- Represent this relation as a set of ordered pairs

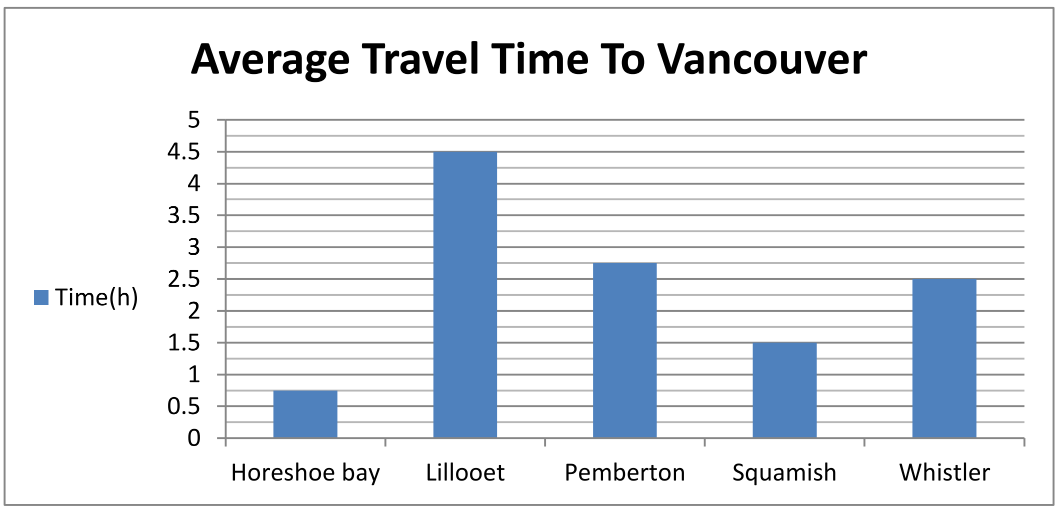
{(Hay River, NWT), (Iqualiut, Nunavut), (Nanisivik, Nunavut), (Old Crow, Yukon), (whitehorse, Yukon), (Yellowknife, NWT)}

- Represent this relation as an arrow diagram

is located in



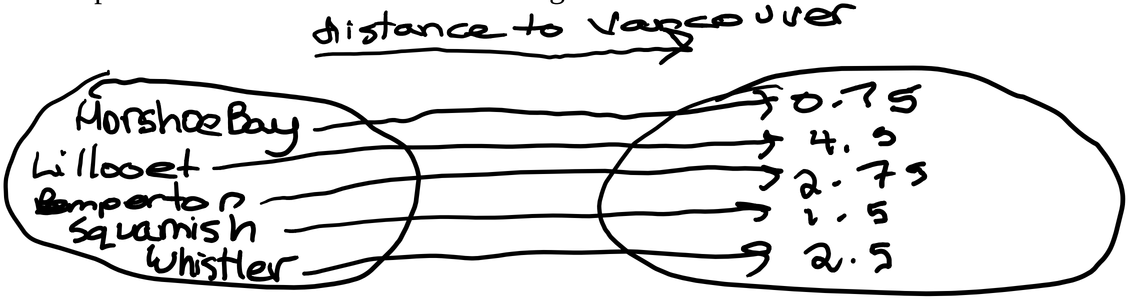
Example: Different towns in BC can be associated with the average time, in hours, that it takes to drive to Vancouver. Consider the relation represented by this graph.



- Represent this relation as a table

Town	Average Time (h)
Horseshoe Bay	0.75
Lillooet	4.5
Pemberton	2.75
Squamish	1.5
Whistler	2.5

- Represent this relation as an arrow diagram.



Sometimes a relation has so many ordered pairs that it is impossible to represent them in a list or a table. We can modify the arrow diagram to model this situation.

Example: In the diagram below, describe the relation in words and then give two ordered pairs that would belong to this relation.

$$\boxed{\text{English Words}} \xrightarrow{\text{has this number of letters}} \boxed{\text{Natural Numbers}}$$

Example (ant, 3), (school, 6)

Assignment: Textbook page 261 #2-4, 6-9, 11, 13

Math 10C

Unit: **Relations and Functions**

Topic: **Properties of Functions**

Objectives:

- Develop the concept of a function

NOTES

The *domain* is the set of elements in the first set (the x-values).

The *range* is the set of elements in the second set (the y-values).

A *Function* is a special type of relation where each element in the domain is associated with EXACTLY one element in the range.

Here is two different ways to relate vehicles and the number of wheels it has.

Diagram 1:

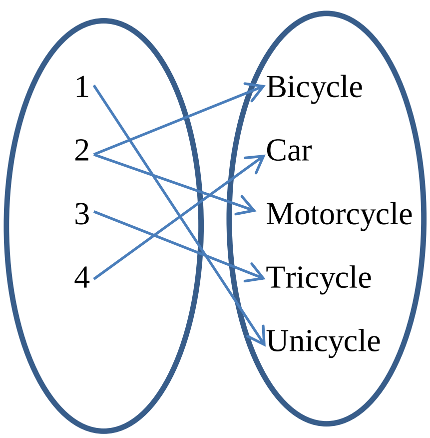


Diagram 2:

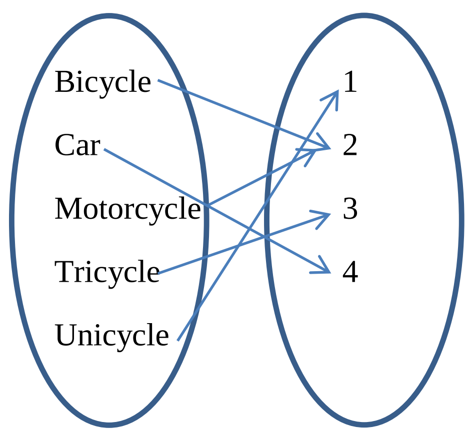


Diagram 1 is not a function because one element in the first set (2) is associated with two elements from the second set (bicycle and motorcycle). Diagram two is a function because each element in the first set is associated with only one element in the second set.

Example: Given the following relations, state if they are also functions and explain your reasoning. If it is a function, identify the domain and range.

- (1,6),(2,4),(3,5),(4,6)(5,5)

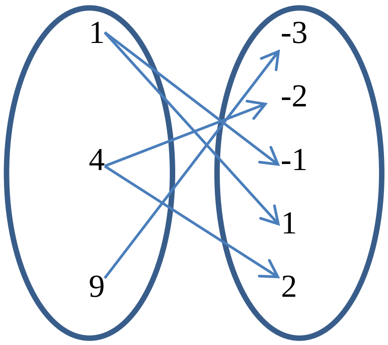
yes d: {1, 2, 3, 4, 5}

r: {6, 4, 5}

- (4,2),(6,2),(6,3),(8,2),(9,3)

no 6 maps to 2 & 3

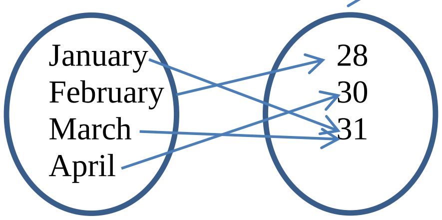
- Is the square of



no

1 maps to -1 and 1

- Has this number of days



yes

d: {January, February, March, April}

r: {28, 30, 31}

x An *Independent variable* is a variable whole value is not determined by the value of another variable and whose value helps determine the value of another (dependent) variable. In a relation between hours worked and gross pay, the hours work would be the independent variable.

y A *Dependent variable* is a variable whose value is determined by the value of another (the independent) variable. In a relation between hours worked and gross pay, the gross pay would be the dependent variable.

Example: The table shows the costs of student bus tickets, C dollars, for different numbers of tickets, n.

x y

Number of Tickets, n	Cost, C ($)
1	1.75
2	3.50
3	5.25
4	7.00
5	8.75

- Why is this relation also a function?

each x value maps to only 1 y value

- Identify the independent and dependent variables. Justify your choices.

independent: # of tickets

dependent: cost

- Write the domain and range.

d: {1, 2, 3, 4, 5}

r: {1.75, 3.50, 5.25, 7.00, 8.75}

Sometimes the independent variable is also described as the input value and the dependent variable is the output value. This means that the input value is any number in the domain and the output is any number in the range.

<u>*Function Notation*</u> is used to show the independent variable in a function for example, y
$f(x)$ means that the value of the function f depends on the value of the independent variable x.

If we want to know the value of money we have given a certain number of quarters we can write an equation to represent this like $v = 0.25q$. A function like $v = 0.25q$ can be rewritten in function notation like this $v(q) = 0.25q$. We would read this as 'v of q is equal to 0.25 times q'. This notation shows us that q is the independent variable and v is the dependent variable. When we have an equation that is not related to a context such as $y = -4x + 3$ we can write this in function notation as $f(x) = -4x + 3$. This is read as 'f of x is equal to -4 times x plus 3'.

We can also use function notation to solve equations given either the input or the output.

For example, given $v = 0.25q$ and that we have 3 quarters we can substitute 3 in for q.

$$v(q) = 0.25q$$
$$v(3) = 0.25(3)$$
$$\text{v}(3) = 0.75$$

Or the question could ask us to solve for q when $v(q) = 5.00$.

$$v(q) = 0.25q$$
$$5.00 = 0.25q$$
$$20 = q$$

Example: The equation $C = 25n + 1000$ represents the cost, C dollars for a feast following an Arctic Sports competition, where n is the number of people attending.

- Describe the function. Write in function notation.

  feast costs 1000 plus 25/person

  $C(n) = 25n + 1000$

- Determine the value of C(100). What does this value represent?

  C(100) represents 100 people attending

  $C(100) = 25(100) + 1000$
  $= \$3500$

- Determine the value of n when $C(n) = 5000$. What does this number represent?

  The total cost is \$5000.

  $5000 = 25n + 1000$

  $n = 160$

Assignment: Textbook page 270 #1-10, 13-17, 19, 22

pg 275 checkpoint

Math 10C

Unit: **Relations and Functions**

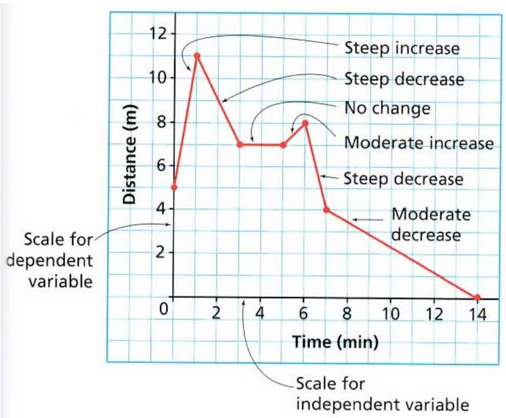
Topic: **Interpreting and Sketching Graphs**

Objectives:

- Describe a possible situation for a given graph
- Sketch a possible graph for a given situation

NOTES

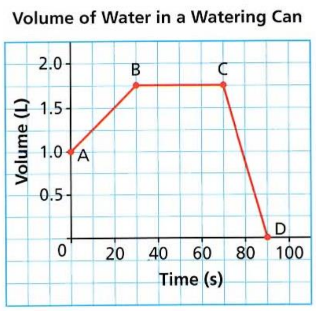
The properties of a graph can provide information about a given situation.



Example: This graph shows the depth of a scuba diver as a function of time.

- How many minutes did the dive last? 30 min
- At what times did the diver stop her descent? 10 min
- What was the greatest depth the diver reached? For how many minutes was the diver at that depth? 20 m for 4 min

Example: This graph shows how the volume of water in a watering can changes over time. Describe each segment of the graph.

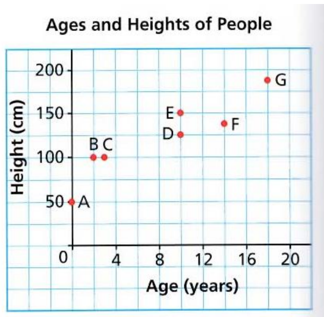


AB → starts with 1 L in can and increases to 1.75 from 0 to 30 seconds

BC → water is constant from 30 to 70 seconds

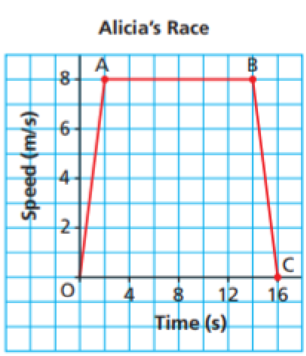
CD → volume decreases from 1.75 to 0 in 20 seconds

Example: Each point on this graph represents a person. Explain your answer to each question below.



- Which person is the oldest? What is his or her age? G, 18 years
- Which person is the youngest? What is his or her age? A, 0 years
- Which two people have the same height? What is this height? B, C, 100cm
- Which two people have the same age? What is this age? E, D, 10 years
- Does this graph represent a function? no

Example: At the beginning of a race, Alicia took 2s to reach a speed of 8 m/s. She ran at approximately 8m/s for 12s, then slowed down to a stop in 2s. Sketch a graph of speed as a function of time. Label each section of your graph, and explain what it represents.



Assignment: Textbook page 281 #1-6, 9-12, 14, 15, 17

Math 10C

Unit: **Relations and Functions**

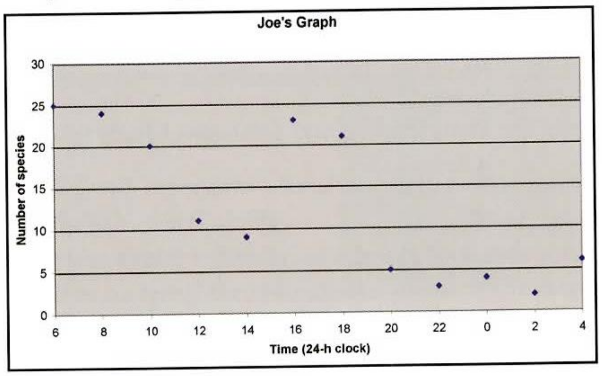
Topic: **Graphs of Relations and Functions**

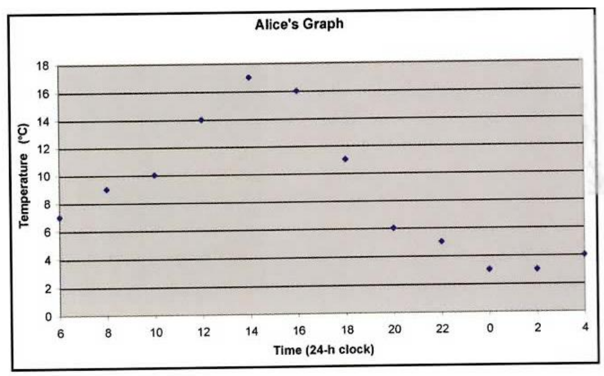
Objectives:

- Determine the properties of the graphs of relations and functions

NOTES

Example: In an environmental study in Northern Alberta, Joe collected data on the numbers of different species of birds he heard or saw in a 1-h period every 2 hours for 24 hours. Alice collected data on the temperature in the area at the end of each 1-h period. They plotted their data:





- Does each graph represent a relation? A function?

yes, and yes

- Which of these graphs should have the data points connected? Explain.

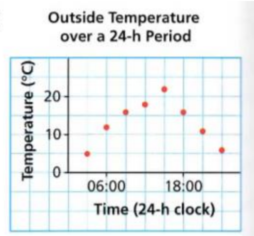
Alice graph

Remember that the domain of a function is the set of values of the independent variable (x-values) and the range of a function is the set of values of the dependent variable (y-values). When the domain is restricted to a set of discrete values, the points on the graph are not connected.

A relation that is not a function has two or more ordered pairs with the same first coordinate. So when the ordered pairs of the relation are plotted on a grid, a vertical line can be drawn to pass through more than one point. This is called the Vertical Line Test. A graph represents a function when no two points on the graph lie on the same vertical line.

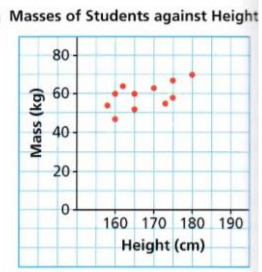
Example: Which of these graphs represents a function? Justify your answer.

a)



Yes

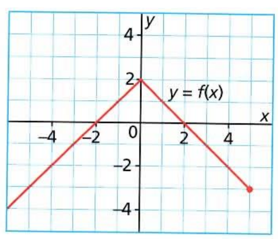
b)



No (does NOT pass vertical line test)

Example: Determine the domain and range of the graph of each function.

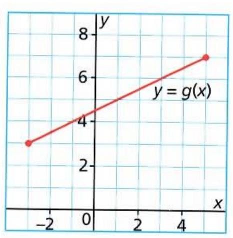
a)



d: $x \leq 5$

r: $y \leq 2$

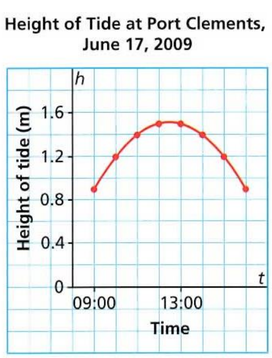
b)



d: $-3 \leq x \leq 5$

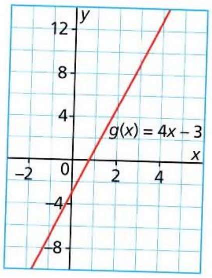
r: $3 \leq y \leq 7$

Example: This graph shows the approximate height of the tide, h meters, as a function of time, t, at Port Clements, Haida Gwaii on June 17, 2009.



- Identify the dependent (h) variable and the independent (t) variable. Justify your choices.
- Why are the points on the graph connected? Explain. continuous data
- Determine the domain and range of the graph.
  d: $9{:}00 \leq t \leq 16{:}00$ r: $0.9 \leq h \leq 1.5$

Example: Here is a graph of the function $g(x) = 4x - 3$.



$g(3) = 4(3) - 3$
$= 12 - 3$
$= 9$

$g(x) = -7 = 4x - 3$
$+3 \qquad + 3$
$-4 = 4x$
$x = -1$

- Determine the range value when the domain value is 3. $x = 3, g(x) = 9$

- Determine the domain value when the range value is -7. $g(x) = -7, x = -1$

Checkpoint 2 pg 299

Assignment: Textbook page 293 #1-12, 16, 17, 19, 20, 22

Math 10C

Unit: **Relations and Functions**

Topic: **Properties of Linear Relations**

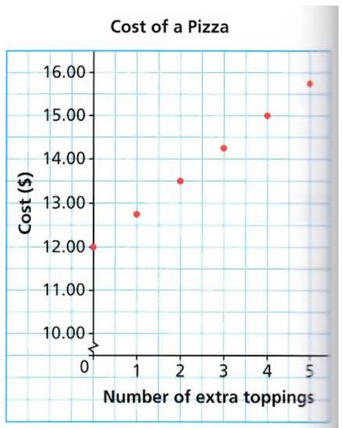
Objectives:

- Identify and represent linear relations in different ways

NOTES

The table of values and graph show the cost of a pizza with up to 5 extra toppings.

Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75



Write a rule for the pattern that relates the cost of a pizza to the number of its toppings.
A cheese pizza costs $12 plus an additional $0.75 per each topping

How can you tell from the table that the graph represents a linear relation?
the dependent variable increases by a constant amount each time.

We can identify a linear relation is different ways.

- A description in words
- A set of ordered pairs
- A table of values
- An equation
- A graph

If we had the description ‘the cost for a car rental is $60 plus $20 for every 100 km driven’, we could show this was linear in a number of ways.

A table of values.

Independent variable → Distance (km) | Cost ($) ← Dependent variable

	Distance (km)	Cost ($)	
	0	60	
+100			+20
	100	80	
+100			+20
	200	100	
+100			+20
	300	120	
+100			+20
	400	140	

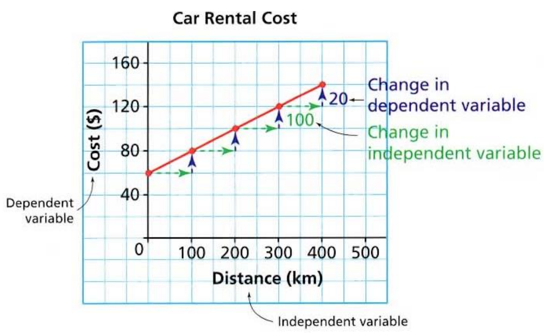
A set of ordered pairs.

+100 +100 +100 +100

{(0, 60), (100, 80), (200, 100), (300, 120), (400, 140)}

+20 +20 +20 +20

A graph.



An equation.

$C = 0.20d + 60$

- 60: initial amount
- d: independent variable
- 0.20: rate of change
- C: dependent variable

The ***rate of change*** (slope) is the change in the dependent variable divided by the change in the independent variable or rise/run or change in y divided by change in x.

Example: Which table of values represents a linear relation? Justify your answers.

The relation between the number of Bacteria in a culture, n , and time, t minutes.

t	n
0	1
20	2
40	4
60	8
80	16

not linear

The relation between the amount of goods and services tax charged, T dollars, and the amount of the purchase, A dollars.

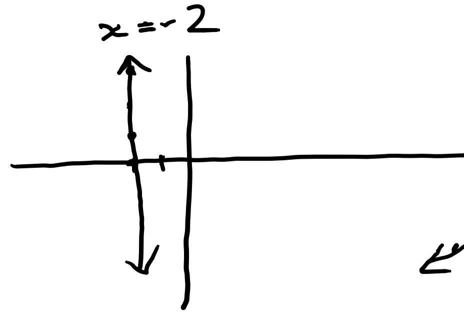
A	T
60	3
120	6
180	9
240	12
300	15

linear

Example: Graph each equation. Which equations represent linear relations? How do you know?

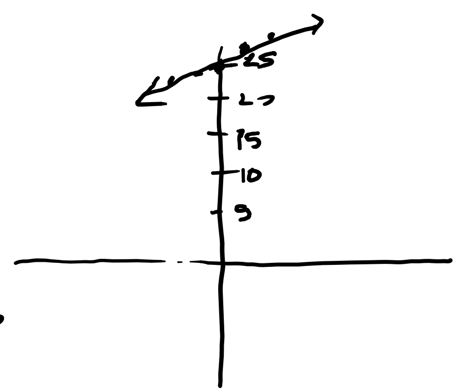
- $x = -2$

x	y
-2	0
-2	1
-2	2
-2	3



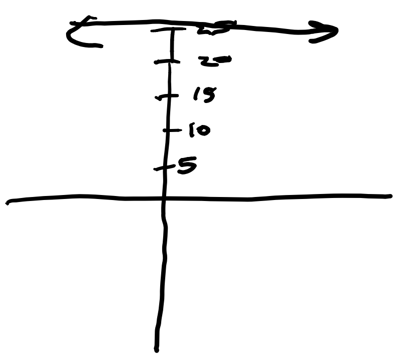
- $y = x + 25$

x	y
0	25
1	26
2	27

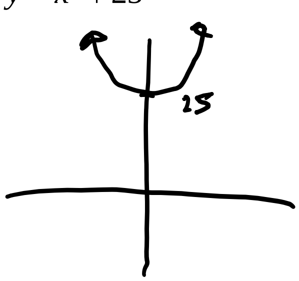


- $y = 25$

x	y
0	25
1	25
2	25



- $y = x^2 + 25$



Example: Which relation is linear? Justify your answer.

- A dogsled moves at an average speed of 10km/h along a frozen river. The distance travelled is related to time.

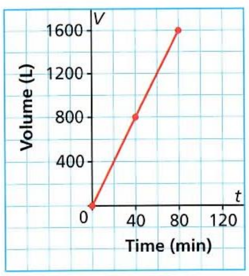
  linear as it has a constant rate of change

- The area of a square is related to the side length of the square.

  not linear; not a constant increase

Example: A hot tub contains 1600L of water. Graph A represents the hot tub being filled at a constant rate. Graph B represents the hot tub being emptied at a constant rate.

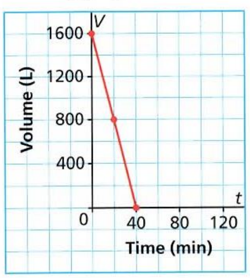
Graph A
Filling a Hot Tub



$$roc = \frac{1600\text{ L}}{80\text{ min}}$$

$$= 20\text{ L/min}$$

Graph B
Emptying a Hot Tub



$$roc = \frac{-1600}{40} = -40\text{ L/min}$$

- Identify the dependent and independent variables.

  V t

- Determine the rate of change of each relation, and describe what it represents.

Assignment: Textbook page 307 #3-8, 11-14, 16-19

Math 10C

Unit: **Relations and Functions**

Topic: **Interpreting Graphs of Linear Functions**

Objectives:

- Use intercepts, rate of change, domain and range to describe the graph of a linear function.

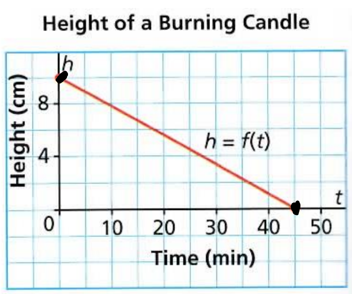
NOTES

Any graph of a line that is not vertical is a function and we call these functions linear functions.

The *vertical intercept* or y-intercept is the value of y when x=0. This means that it is the value of the graph on the y-axis. To determine the y-intercept, evaluate *f(x)* when x=0 or *f(0)*.

The *horizontal intercept* or x-intercept is the value of x when y=0. This means that it is the value of the graph on the x-axis. To determine the x-intercept, determine the value of x when *f(x)*=0.

Example: Find the y-intercept, x-intercept, rate of change, domain and range or each graph.



roc: $\frac{10\text{cm}}{45\text{min}}$

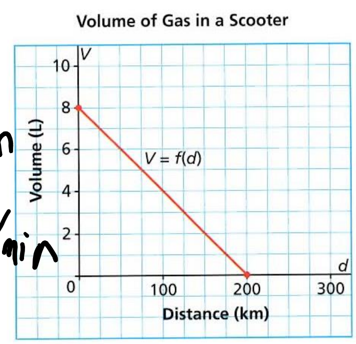
$= -0.2$ cm/min

y int = 10 cm

x int = 45 min

d: $\{t \mid 0 \le t \le 45, t \in \mathbb{R}\}$

r: $\{h \mid 0 \le h \le 10, h \in \mathbb{R}\}$



roc = $\frac{8L}{200\text{km}}$

$= -0.04$ L/km

y int = 8 L

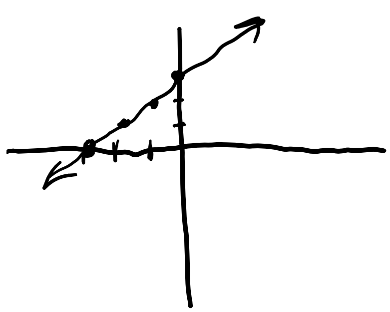
x int = 200 km

d: $\{d \mid 0 \le d \le 200, d \in \mathbb{R}\}$

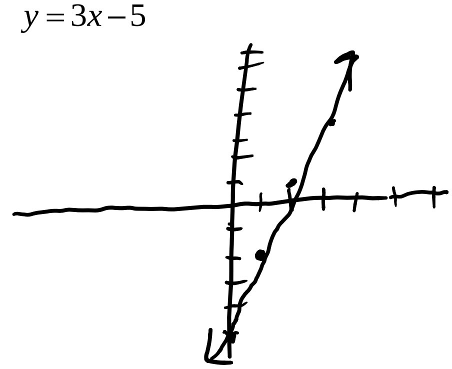
r: $\{V \mid 0 \le V \le 8, V \in \mathbb{R}\}$

Example: Sketch a graph of each of the linear functions.

$y = x + 3$



$y = 3x - 5$

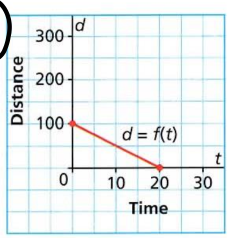


x	y
1	-2
2	1
3	4
4	7
5	10

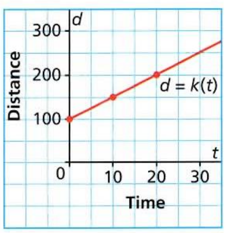
$f(x) = -2x + 1$ $\qquad\qquad$ $f(x) = 6 - x$

Example: Which graph has a rate of change of -5 and a vertical intercept of 100? Justify your answer.

a)

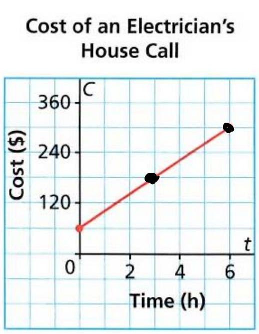


b)



- b has a positive rate of change
- $\frac{-100 \text{ km}}{20} = -5$

Example: This graph shows the total cost for a house call by an electrician for up to 6 hours of work.



$roc = \frac{300 - 180}{6 - 3} = \frac{120}{3} = \$40/hr$

The electrician charges $190 to complete a job. For how many hours did she work?

$\$60 + 40 + 40 + 10 = 3.25$ hr

Write an equation to represent this situation.

$C = 60 + 40h$

Assignment: Textbook page 319 #1-4, 6-11, 16