# Math 112
# Final Exam Practice Problems

The practice problem listed in this document serve as a survey for what is covered in MST 112. These problems are meant to guide you in your preparation for the final exam. Simply completing these problems is not sufficient preparation for the exam. You should also review all past exams, quizzes, classworks, homeworks, lecture notes, and the course textbook. Also, do not wait until the week of and especially the night before to begin preparing for the exam.

1. Use the table and the fact that

$$\int_0^{10} f(t)dt = 350$$

to evaluate the definite integrals below exactly.

| $t$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|---|
| $f(t)$ | 0 | 70 | $e^5$ | $e^3$ | 0 | $\pi/2$ | $\pi$ |

- $\int_0^{10} tf'(t)dt.$
- $\int_{20}^{30} \frac{f'(t)}{f(t)}dt.$
- $\int_{50}^{60} f(t)f'(t)\sin(f(t))dt.$

2. Consider the function $F$ defined for all $x$ by the formula

$$F(x) = \int_7^{x^2} e^{-t^2}dt.$$

- Solve the equation $F(x) = 0$.
- Calculate $F'(x)$.
- Is $F(x)$ increasing on the interval $[1, 8]$?

3. Let $h(x)$ be a differentiable function and define $H(x) = \int_0^x h(t)dt$. If $H(x)$ is always concave up, determine whether $g(x) = h(e^{-x})$ is an increasing function.

4. Calculate the following limit:

$$\lim_{x\to\infty} \left(1 + \frac{a}{x}\right)^{bx},$$

where $a, b > 0$ are constants.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $g(x)$ | 0 | 2 | 3 | 4 | 5 |
| $G(x)$ | $-7$ | $-4$ | 0 | 5 | 9 |

5. Let $g(x)$ be a differentiable, odd function and let $G(x)$ be an anti-derivative of $g(x)$. A table of values for $g(x)$ and $G(x)$ is provided below.
   Calculate the following:

   - $\int_0^1 g(x)dx.$
   - $\int_{-4}^2 g(x)dx.$
   - $\int_1^3 xg'(x)dx.$
   - $\int_0^1 g(3x)dx.$

6. Let $f(x)$ and $g(x)$ be two functions that are differentiable on $(0, \infty)$ with continuous derivatives and which satisfy the following inequalities for all $x \geq 1$:

$$\frac{1}{x} \leq f(x) \leq \frac{1}{x^{\frac{1}{2}}} \text{ and } \frac{1}{x^2} \leq g(x) \leq \frac{1}{x^{\frac{3}{4}}}.$$

   For each of the following, determine whether the integral always, sometimes, or never converges.

   - $\int_1^\infty \sqrt{f(z)}dz.$
   - $\int_3^\infty 4000g(x)dx.$
   - $\int_1^\infty f(x)g(x)dx.$
   - $\int_1^\infty g'(x)e^{g(x)}dx.$
   - $\int_1^\infty f'(x)\ln(f(x))dx.$

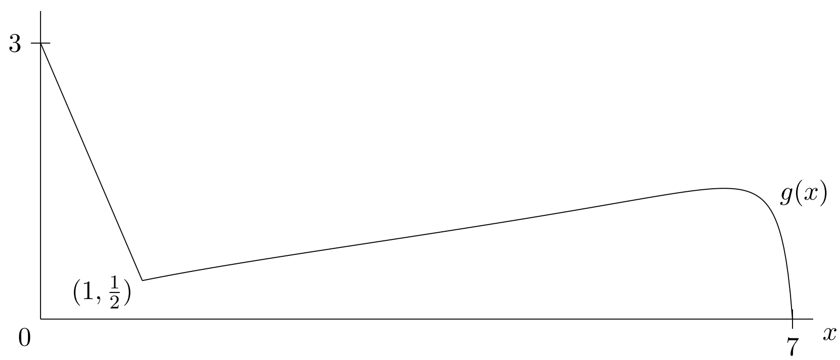
7. Determine if the following integrals converge or diverge. If an integral diverges, explain why. If it converges, find the value to which it converges. Mathematical precision is important.

   - $\int_{-1}^2 \frac{1}{2-x}dx.$
   - $\int_{10}^\infty \frac{5+2\sin(4x)}{x}dx.$
   - $\int_1^\infty \frac{x}{1+x}dx.$

- $\displaystyle\int_e^\infty \frac{1}{x\ln(x)^2}\,dx.$

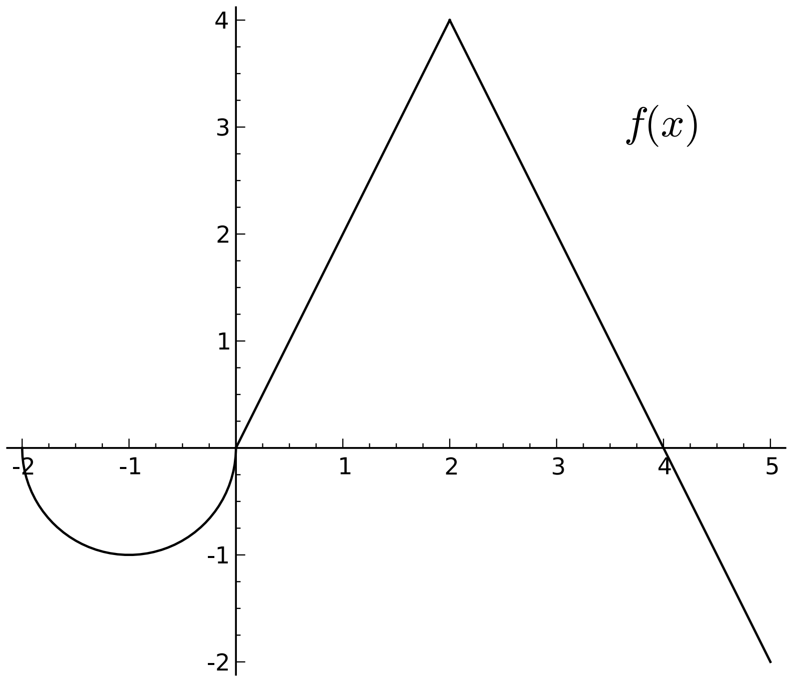
8. Suppose $G(x) = \displaystyle\int_{2x^3}^{\frac{1}{4}} \cos(t^2)\,dt$.
   - Calculate $G'(x)$.
   - Find a constant $a$ and a function $h$ so that $G(x) = \displaystyle\int_a^x h(t)\,dt$.

9. The graph of part of a function $g(x)$ is pictured below.



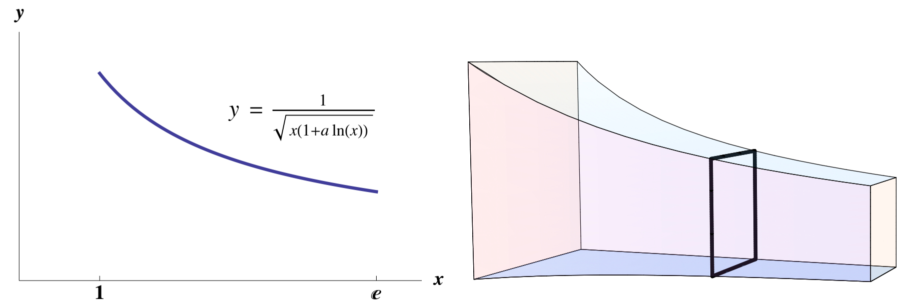

   - A thumbtack has the shape of the solid obtained by rotating the region bounded by $y = g(x)$, the $x$-axis and $y$-axis, about the $y$-axis. Find an expression involving integrals that gives the volume of the thumbtack. Do not evaluate the integrals.
   - A door knob has the shape of the solid obtained by rotating the region bounded by $y = g(x)$, the $x$-axis and $y$-axis, about the $x$-axis. Find an expression involving integrals that gives the volume of the door knob. Do not evaluate any integrals.

10. The graph of the function $f(x)$, shown below, consists of line segments and a semicircle. Compute each of the following quantities:



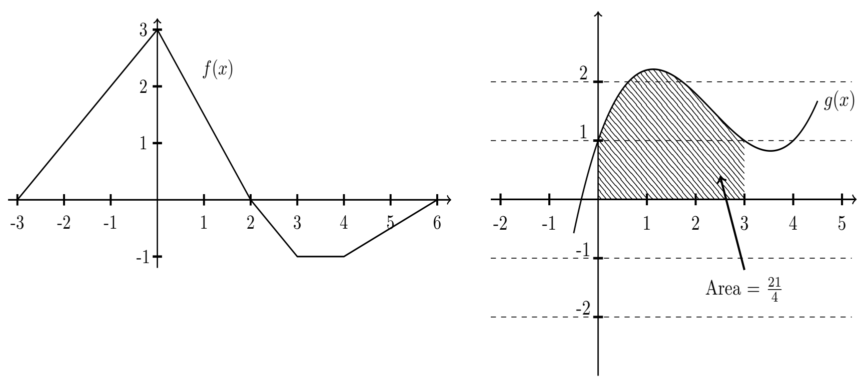

- $\int_0^2 f(x)dx$
- $\int_{-2}^2 |f(x)|dx$
- $\int_0^5 f(x)dx$
- $\int_{-2}^2 2f(x)dx + \int_5^2 3f(x)dx$
- $\int_0^1 f(5x)dx$

11. Let $S$ be the solid whose base is the region bounded by the graph of the curve $y = \dfrac{1}{x(1 + a\ln(x))}$ (for some positive constant $a > 0$), the $x$-axis, the lines $x = 1$ and $x = e$. The cross-sections of $S$ perpendicular to the $x$-axis are squares. Find the exact volume of $S$.




12. Let $S$ be a solid whose base is the region bounded by the curves $y = x^2$, $y = 6 - x$, and $x = 0$ and whose cross sections parallel to the $x$-axis are squares. Find a formula involving definite integrals that computes the volume of $S$.

13. Use the graphs of $f(x)$ and $g(x)$ to find the exact values of $A$, $B$, and $C$.




- $A = \displaystyle\int_{-3}^{6} |f(x)|dx$
- $B = \displaystyle\int_{0}^{2} xg'(x^2)dx$
- $C = \displaystyle\int_{0}^{3} 2xg'(x)dx$

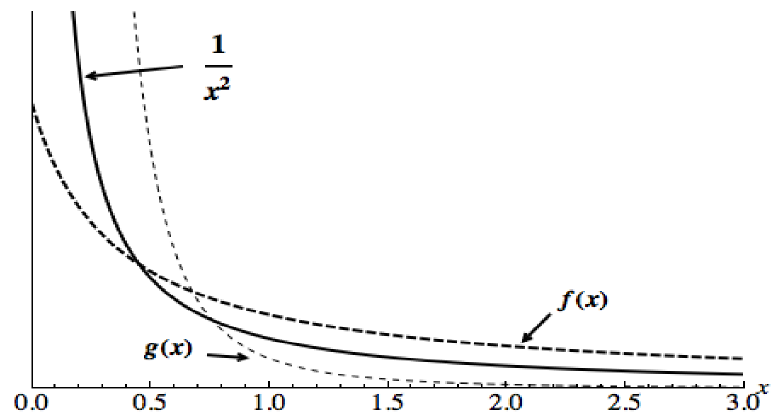
14. Consider functions $f(x)$ and $g(x)$ satisfying:

- $g(x)$ is an odd function.
- $\int_2^7 g(x)dx = 3.$
- $\int_2^7 f(x)dx = 17.$
- $f(2) = 1.$
- $\int_1^6 f'(x)dx = 12$
- $\int_2^7 f'(x)dx = 3$

Compute the value of the following quantities.

- $\int_{-2}^7 g(x)dx$
- $\int_2^7 (f(x) - 8g(x))dx$
- $f(7)$
- $\int_1^6 f'(x+1)dx.$
- $\int_2^7 xf'(x)dx.$
- $\int_2^3 xf(x^2-2)dx.$

15. Consider the functions $f(x)$ and $g(x)$ plotted below:




**Note:** These functions satisfy

- $f(x) > \dfrac{1}{x^2}$ for $x > 1$.
- $g(x) > \dfrac{1}{x^2}$ for $0 < x < \frac{1}{2}$.
- $g(x) < \dfrac{1}{x^2}$ for $x > 1$.

Using the information about $f(x)$ and $g(x)$ provided above, determine which of the following integrals is convergent or divergent. Circle your answers. If there is not enough information given to determine the convergence or divergence of the integral circle NI.

i) $\displaystyle\int_1^{\infty} f(x)dx$ **Converges** **Diverges** **NI**

ii) $\displaystyle\int_0^{1} f(x)dx$ **Converges** **Diverges** **NI**

iii) $\displaystyle\int_1^{\infty} g(x)dx$ **Converges** **Diverges** **NI**

iv) $\displaystyle\int_0^{1} g(x)dx$ **Converges** **Diverges** **NI**

16. Compute the following indefinite integral:

$$\int \frac{Ax}{B + Cx^2} dx,$$

where $A, B, C > 0$ are constants.

17. Compute the following indefinite integral:

$$\int \frac{A}{B^2 + x^2} dx,$$

where $A, B > 0$ are constants.

18. Consider the region bounded by the curve $y = x$ and the lines $x = 0$ and $x = 1$. Find the volume of the following solids:

    - The solid obtained by rotating the region around the $x$-axis.
    - The solid obtained by rotating the region around the $y$-axis.

19. Calculate the following limit:

$$\lim_{x \to 1^+} \left[ \ln(x^7 - 1) - \ln(x^5 - 1) \right].$$

20. Calculate the following limit:

$$\lim_{x \to 0} (1 - 2x)^{1/x}$$

21. Calculate the following limit:

$$\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x.$$

22. Compute the following:

    - $\displaystyle\int \frac{\sin(x)}{9 + \cos^2(x)} dx$
    - $\displaystyle\int e^x \sin(x) dx$
    - $\displaystyle\int \frac{z^2}{\sqrt{1 - z^6}} dz$
    - $\displaystyle\int x \sin^2(x^2) dx$
    - $\displaystyle\int \sin^3(t) \cos^2(t) dt$
    - $\displaystyle\int x^3 \sqrt{1 - x^2} dx$
    - $\displaystyle\int \frac{3x}{x^2 - 3x - 4} dx$
    - $\displaystyle\int \frac{1}{x \ln(x)}$
    - $\displaystyle\int \frac{1}{x^2 + x} dx$
    - $\displaystyle\int x^3 \sin(x^2) dx$

- $\lim_{x \to 0^+} (1+x)^{1/x}$
- $\int x \ln(x) dx$
- $\int \cos^4(x) dx$
- $\int_0^{\sqrt{2}} \frac{x^3}{\sqrt{4-x^2}} dx$
- $\int \frac{1}{x+x^3} dx$

23. If $A > 0$ is a constant, compute the following:
    - $\int \sqrt{A^2 - x^2} dx.$
    - $\int x\sqrt{A^2 - x^2} dx.$

24. Determine whether the following sequence is increasing decreasing or neither:

$$a_n = \frac{10^n}{n!}.$$

25. Determine whether the following sequence is bounded:

$$a_n = \frac{\sin(n^2)}{n+1}.$$

26. Suppose that $g(x)$ and $h(x)$ are positive continuous functions on the interval $(0, \infty)$ with the following properties:
    - $\int_1^\infty g(x) dx$ converges.
    - $\int_0^1 g(x) dx$ diverges.
    - $e^{-x} \leq h(x) \leq x^{-1}$ for all $x$ in $(0, \infty)$.

    Determine whether the following integrals converge or diverge or if you do not have enough information to make a conclusion.
    - $\int_1^\infty h(x)^2 dx$
    - $\int_0^1 h(x) dx$
    - $\int_1^\infty h(1/x) dx$
    - $\int_0^1 g(x)h(x) dx$
    - $\int_1^\infty g(x)h(x) dx$

- $\displaystyle\int_1^\infty e^x g(e^x)dx$

27. Calculate the following limit:

$$\lim_{x\to\infty}\left(\frac{x+2}{x}\right)^{\frac{x}{2}}.$$

28. Determine the divergence or convergence of the following improper integral:

$$\int_2^\infty \frac{5-3\sin(2x)}{x^2}dx.$$

29. Determine the divergence or convergence of the following improper integral:

$$\int_1^\infty \sqrt{a^2+\frac{1}{\sqrt{x}}}dx,$$

where $a$ is a positive constant.

30. Determine if each of the following sequences is increasing, decreasing or neither, and whether it converges or diverges. If the sequence converges, identify the limit.

- $\displaystyle a_n=\int_1^{n^3}\frac{1}{(x^2+1)^{\frac{1}{5}}}.$
- $\displaystyle b_n=\sum_{k=0}^{n}\frac{(-1)^k}{(2k+1)!}.$
- $c_n=\cos(a^n)$, where $0<a<1$.

31. Determine if the following series converge or diverge:

- $\displaystyle\sum_{n=1}^{\infty}\frac{4}{n(\ln(n))^2}.$
- $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{1+2\sqrt{n}}.$
- $\displaystyle\sum_{n=1}^{\infty}\frac{n}{\sqrt{n^3+2}}.$
- $\displaystyle\sum_{n=1}^{\infty}\frac{8^9+10^n}{9^n}.$
- $\displaystyle\sum_{n=4}^{\infty}\frac{1}{n^3+n^2\cos(n)}$

32. Let $r$ be a real number. For which values of $r$ is the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n^2}{n^r+4}$ absolutely convergent? Conditionally convergent? Divergent?

33. Find the interval and radius of convergence for the following power series:

$$\sum_{n=1}^{\infty}\frac{2^n}{3n}(x-5)^n.$$

34. Find the value of $p$ for which the following integrals converge:

- $\displaystyle\int_{-\infty}^{\infty} \frac{1}{(x^2+4)^p} dx.$
- $\displaystyle\int_{e}^{\infty} \frac{e^{px}}{x^3} dx.$

35. Find the radius of convergence for the following Taylor series:

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n2^n}.$$

36. Determine the exact value of the following series:

- $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{n!}$
- $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n}}{(2n+1)!}$

37. What is the Taylor series of $2xe^{x^2}$ centered at $x = 0$?.

38. Find the radius of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} x^{2n}$.

39. Let the sequence $a_n$ be given by

$$a_1 = -1, \qquad a_2 = \frac{\sqrt{2}}{3}, \qquad a_3 = -\frac{\sqrt{3}}{5}, \qquad a_4 = \frac{\sqrt{4}}{7}, \qquad a_5 = -\frac{\sqrt{5}}{9}.$$

- Find $a_7$.
- Write a formula for $a_n$.
- Does the sequence converge?
- Does the series $\displaystyle\sum_{n=1}^{\infty} |a_n|$ converge or diverge?
- Does the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converge or diverge?

40. Evaluate the following:

- $\displaystyle\frac{d}{dx} \int_0^1 \sqrt{1+t^3}\, dt.$
- $\displaystyle\int_0^1 \frac{d}{dt} \sqrt{1+t^3}\, dt.$
- $\displaystyle\frac{d}{dx} \int_{x^2}^{x^4} \sqrt{1+t^3}\, dt.$

41. If $\displaystyle\int_a^b f(x)\, dx = k$, evaluate the following integrals in terms of $k$.

- $\displaystyle\int_{a+5}^{b+5} f(x-5)\, dx.$

- $\int_a^b (f(x)+5)\, dx$.
- $\int_{a/5}^{b/5} f(5x)\, dx$.

42. If $\int_3^6 f(z)\, dz = 4$, evaluate the following integrals.
    - $\int_1^2 f(3z)\, dz$
    - $\int_{1/2}^2 f(7-2z)\, dz$
    - $\int_4^7 (f(z-1)+5)\, dz$

43. Let $f(x) = \int_{2x}^{3x-3} \sqrt{1+t^4}\, dt$.
    - What is $f'(0)$?
    - What is $(f^{-1})'(0)$?

44. Find the value of $C$ for which the following integral is convergent:
$$\int_0^\infty \left( \frac{1}{\sqrt{x^2+4}} - \frac{C}{x+2} \right) dx.$$

45. Determine whether the following integral is convergent or divergent:
$$\int_0^\infty \frac{1+\sin^4(x)}{x+e^x} dx.$$

46. Find the area bounded between the curves $f(x) = x$ and $f(x) = \frac{4x}{3+x^2}$.

47. If $f$ is a continuous function such that $\int_0^x f(t)\, dt = xe^{2x} + \int_0^x e^{-t} f(t)\, dt$ find an explicit formula for $f(x)$.

48. Find the value of $c$ if:
$$\sum_{n=0}^{\infty} (1+c)^{-n} = 2.$$

49. Suppose you know that for all $n$ the sequences $a_n$, $b_n$, $c_n$, $d_n$ satisfy:
$$0 \le b_n \le \frac{1}{n} \le a_n \text{ and } 0 \le c_n \le \frac{1}{n^2} \le d_n.$$
    - Which of the series $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$, $\sum_{n=1}^{\infty} c_n$, $\sum_{n=1}^{\infty} d_n$ definitely converge.
    - Which of the series $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$, $\sum_{n=1}^{\infty} c_n$, $\sum_{n=1}^{\infty} d_n$ definitely diverge.

50. Solve for $x$:

$$x - \frac{x^3}{6} + \frac{x^5}{125} - \frac{x^7}{7!} + \ldots = 1.$$

51. Find the exact value of

$$1 + 2 + \frac{4}{2!} + \frac{8}{3!} + \frac{16}{4!} + \ldots.$$

52. Find the Taylor series representation for the following function:

$$f(x) = \int_0^x te^t dt.$$

53. Find the first two **nonzero** terms in the Taylor series expansion for the following function:

$$f(x) = \int_0^x e^{-t^2} dt.$$

54. Suppose the power series $\sum_{n=0}^{\infty} c_n(x-2)^n$ converges for $x = 4$ and diverges for $x = 6$. Which of the following are true, false, or not possible to determine?

- The power series diverges for $x = 7$.
- The power series diverges for $x = \frac{1}{2}$.
- The power series diverges for $x = 5$.
- The power series diverges for $x = -3$.
- The power series diverges for $x = 1$.