Math 115 — Practice for Exam 2

Generated November 11, 2014

Name: ____

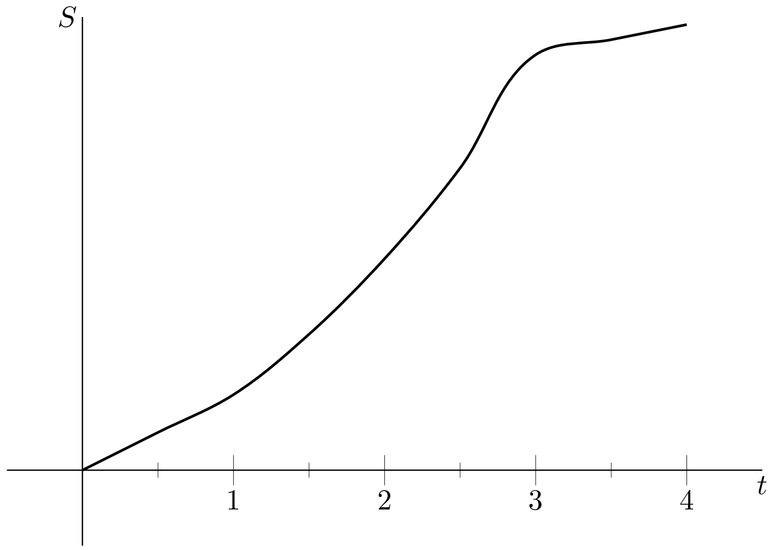
Instructor: ____________________ Section Number: ________

1. This exam has 16 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2009	2	7	drive to Chicago	10	
Winter 2013	2	1		12	
Fall 2011	3	7	atomic force	14	
Fall 2006	2	7	Poiseuille's Law	12	
Winter 2011	2	6		15	
Winter 2013	2	8		12	
Fall 2004	2	4	balloon animal	6	
Fall 2011	2	8	rose curve	12	
Winter 2013	2	7		10	
Winter 2005	3	7	string	17	
Winter 2008	2	5	microphone	16	
Fall 2013	2	1	swimming pool	7	
Winter 2007	2	7	Octopus	14	
Winter 2014	2	3		12	
Winter 2011	2	7		12	
Winter 2014	2	2	triangle	9	
Total				190	

Recommended time (based on points): 180 minutes

7. You decide to take a weekend off and drive down to Chicago. The graph below represents your distance S from Ann Arbor, measured in miles, t hours after you set out.



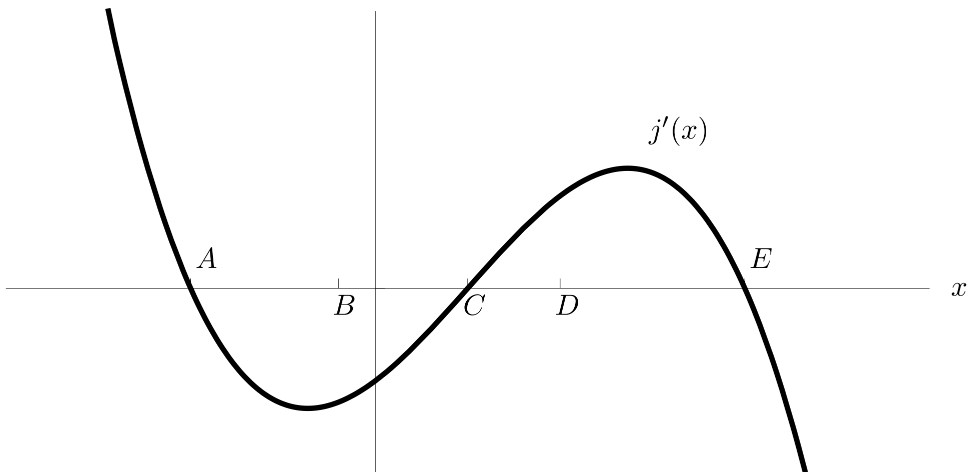
Let $A(t)$ be the slope of the line connecting the origin $(0,0)$ to the point $(t, S(t))$.

(a) (3 points) What does $A(t)$ represent in everyday language?

(b) (3 points) Estimate the time t at which $A(t)$ is maximized. Write a one sentence explanation and use the graph above to justify your estimate.

(c) (4 points) Use calculus to explain why $A(t)$ has a critical point when the line connecting the origin to the point $(t, S(t))$ is tangent to the curve $S(t)$.

1. [12 points] Consider the graph of $j'(x)$ given here. Note that this is not the graph of $j(x)$.



For each of (a)-(f) below, list **all** x-values labeled on the graph which satisfy the given statement in the blank provided. If the statement is not true at any of the labeled x-values, write "NP". You do not need to show your work. No partial credit will be given on each part of this problem.

(a) The function $j(x)$ has a local minimum at $x =$______________.

(b) The function $j(x)$ has a local maximum at $x =$______________.

(c) The function $j(x)$ is concave up at $x =$______________.

(d) The function $j(x)$ is concave down at $x =$______________.

(e) The function $j'(x)$ has a critical point at $x =$______________.

(f) The function $j''(x)$ is greatest at $x =$______________.

7. [14 points] For positive A and B, the force between two atoms is a function of the distance, r, between them:

$$f(r) = -\frac{A}{r^2} + \frac{B}{r^3} \qquad r > 0.$$

a. [2 points] Find the zeroes of f (in terms of A and B).

b. [7 points] Find the coordinates of the critical points and inflection points of f in terms of A and B.

c. [5 points] If f has a local minimum at $(1, -2)$ find the values of A and B. Using your values for A and B, justify that $(1, -2)$ is a local minimum.

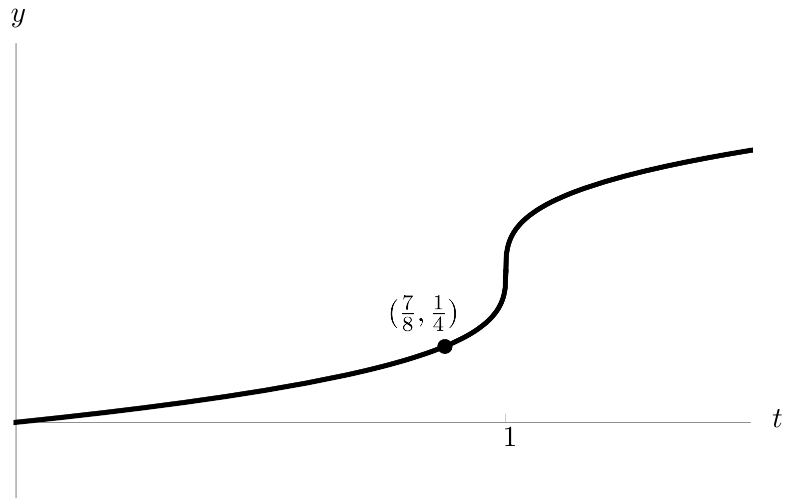
7. (12 points) The flux F, in millilitres per second, measures how fast blood flows along a blood vessel. Poiseuille's Law states that the flux is proportional to the fourth power of the radius, R, of the blood vessel, measured in millimeters. In other words $F = kR^4$ for some positive constant k.

   (a) Find a linear approximation for F as a function of R near $R = 0.5$. (Leave your answer in terms of k).

   (b) A partially clogged artery can be expanded by an operation called an angioplasty, which widens the artery to increase the flow of blood. If the initial radius of the artery was 0.5mm, use your approximation from part (a) to approximate the flux when the radius is increased by 0.1mm.

   (c) Is the answer found in part (b) an under- or over-approximation? Justify your answer.

6. [15 points] Given below is the graph of a function $h(t)$. Suppose $j(t)$ is the local linearization of $h(t)$ at $t = \frac{7}{8}$.



a. [5 points] Given that $h'(\frac{7}{8}) = \frac{2}{3}$, find an expression for $j(t)$.

b. [4 points] Use your answer from (a) to approximate $h(1)$.

c. [3 points] Is the approximation from (b) an over- or under-estimate? Explain.

d. [3 points] Using $j(t)$ to estimate values of $h(t)$, will the estimate be more accurate at $t = 1$ or at $t = \frac{3}{4}$? Explain.

8. [12 points] In the following table, both f and g are differentiable functions of x. In addition, $g(x)$ is an invertible function. Write your answers in the blanks provided. You do not need to show your work.

x	2	3	4	5
$f(x)$	7	6	2	9
$f'(x)$	-2	1	3	2
$g(x)$	1	4	7	11
$g'(x)$	1	2	3	2

a. [3 points] If $h(x) = \frac{g(x)}{f(x)}$, find $h'(4)$.

$h'(4) =$ ________________

b. [3 points] If $k(x) = f(x)g(x)$, find $k'(2)$.

$k'(2) =$ ________________

c. [3 points] If $m(x) = g^{-1}(x)$, find $m'(4)$.

$m'(4) =$ ________________

d. [3 points] If $n(x) = f(g(x))$, find $n'(3)$.

$n'(3) =$ ________________

4. (6 points) The shape of a balloon used by a clown for making a balloon animal can be approximated by a cylinder. As the balloon is inflated, assume that the radius is increasing by 2 cm/sec and the height is given by $h = 2r$. At what rate is air being blown into the balloon at the moment when the radius is 3 cm?

5. (8 points) In introductory physics one learns the formula $F = ma$, connecting the force on an object, F, with the mass of the object and the acceleration that the object experiences under the force. One also learns the formula $p = mv$ where p is the momentum of an object, m is the mass, and v is the velocity.

(a) Derive the formula $F = ma$ given that $\frac{dp}{dt} = F$, assuming that the mass is constant and that $p = mv$. Explain your answer.

(b) Derive a formula for the force F if the mass is not assumed to be constant.

8. [12 points] The equation $(x^2 + y^2)^2 = 4x^2y$ describes a two-petaled rose curve.

a. [2 points] Verify that the point $(x, y) = (1, 1)$ is on the curve.

b. [7 points] Calculate dy/dx at $(x, y) = (1, 1)$.

c. [3 points] Find the equation of the tangent line to the rose curve at the point $(x, y) = (1, 1)$.

7. [10 points] For each real number k, there is a curve in the plane given by the equation

$$e^{y^2} = x^3 + k.$$

a. [4 points] Find $\frac{dy}{dx}$.

b. [3 points] Suppose that $k = 9$. There are two points on the curve where the tangent line is horizontal. Find the x and y coordinates of each one.

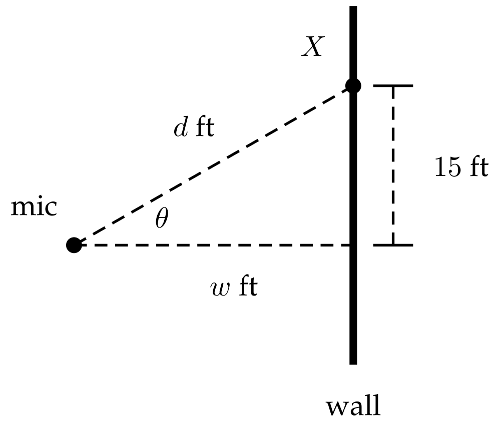
c. [3 points] Now suppose that $k = \frac{1}{2}$. How many points are there where the curve has a horizontal tangent line?

7. (17 points) At the Wizard Fair, there is a booth where wizards win Bertie Bott's Every Flavor Beans. To determine how many beans one gets, a contestant is given a string 50 inches long. From this string, contestants can cut lengths to form an *equilateral triangle* and a *rectangle whose length is twice its width.* The number of Bertie Bott's beans one wins depends on the combined areas of the triangle and rectangle. Harry, knowing calculus, goes immediately to work setting up a function, finding critical points, etc.

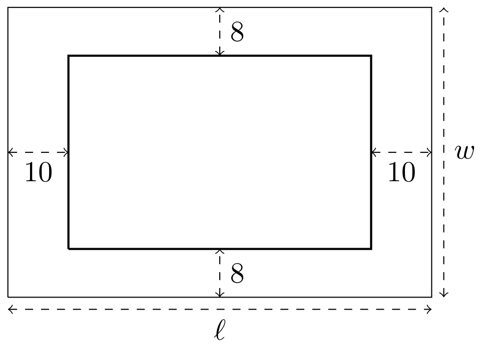
(a) Use your knowledge of calculus to determine the areas of the triangle and rectangle that will maximize the number of beans that Harry can win. Show your work.

(b) If the number of beans won is 9 times the combined area, what is the greatest number of beans a contestant can win?

5. (16 points) A directional microphone is mounted on a stand facing a wall. The sensitivity S of the microphone to sounds at point X on the wall is inversely proportional to the square of the distance d from the point X to the mic, and directly proportional to the cosine of the angle θ. That is, $S = K\frac{\cos\theta}{d^2}$ for some constant K. (See the diagram below.) How far from the wall should the mic be placed to maximize sensitivity to sounds at X?



1. [7 points] Liam wants to build a rectangular swimming pool behind his new house. The pool will have an area of 1600 square feet. He will have 8-foot wide decks on two sides of the pool and 10-foot wide decks on the other two sides of the pool (see the diagram below).



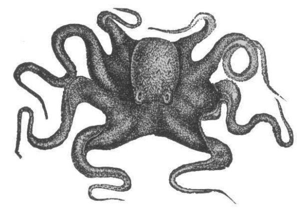
a. [4 points] Let ℓ and w be the length and width (in feet) of the pool area including the decks as shown in the diagram. Write a formula for ℓ in terms of w.

$\ell =$ ______________________________

b. [3 points] Write a formula for the function $A(w)$ which gives the total area (in square feet) of the pool **and** the decks in terms of only the width w. Your formula should not include the variable ℓ. (This is the function Liam would minimize in order to find the minimum area that his pool and deck will take up in his yard. You do not need to do the optimization in this case.)

$A(w) =$ ______________________________

7. (14 points) No matter what is done with the other exhibits, the octopus tank at the zoo must be rebuilt. (The current tank has safety issues, and there are fears that the giant octopus might escape!) The new tank will be 10 feet high and box-shaped. It will have a front made out of glass. The back, floor, and two sides will be made out of concrete, and there will be no top. The tank must contain at least 1000 cubic feet of water. If concrete walls cost \$2 per square foot and glass costs \$10 per square foot, use calculus to find the dimensions and cost of the least-expensive new tank. [Be sure to show all work.]



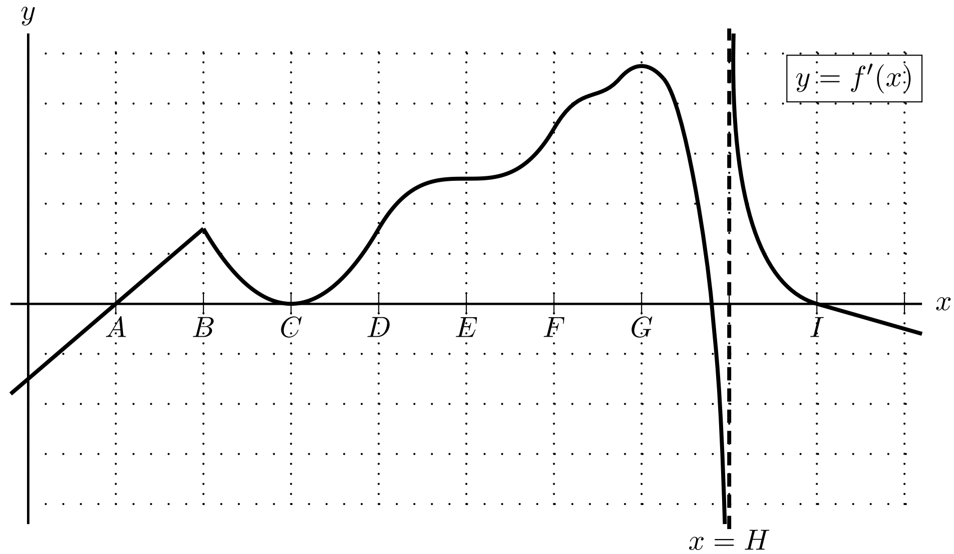
GIANT OCTOPUS (*Enteroctopus*)[2]

Dimensions: ______________________

Minimum Cost: ______________________

[2]See `http://www.cephbase.utmb.edu/Tcp/pdf/anderson-wood.pdf`. (They really DO escape....)

3. [12 points] The graph of a portion of $y = f'(x)$, the derivative of $f(x)$ is shown below. Note that there is a sharp corner at $x = B$ and that $x = H$ is a vertical asymptote.
The function $f(x)$ is continuous with domain $(-\infty, \infty)$.



For each of the questions below, circle all of the available correct answers.
(Circle NONE if none of the available choices are correct.)

a. [2 points] At which of the following six values of x is the function $f(x)$ not differentiable?

B C E F H I NONE

b. [2 points] At which of the following six values of x does the function $f'(x)$ appear to be not differentiable?

A B C D E F NONE

c. [2 points] At which of the following nine values of x does $f(x)$ have a critical point?

A B C D E F G H I NONE

d. [2 points] At which of the following nine values of x does $f(x)$ have a local minimum?

A B C D E F G H I NONE

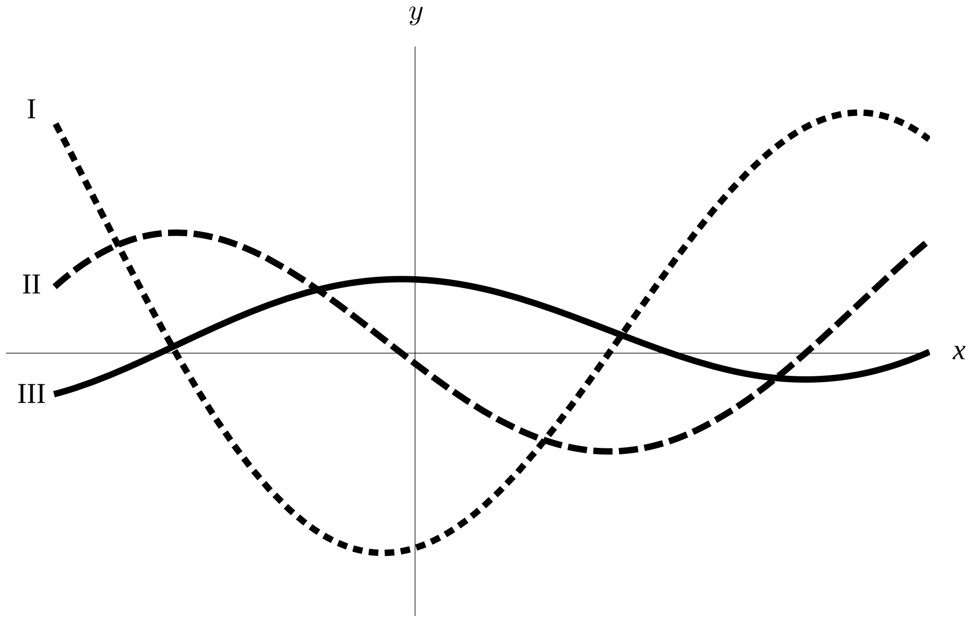
e. [2 points] At which of the following nine values of x is $f''(x) = 0$?

A B C D E F G H I NONE

f. [2 points] At which of the following nine values of x does $f(x)$ have an inflection point?

A B C D E F G H I NONE

7. [12 points] On the axes below are graphed f, f', and f''. Determine which is which, and justify your response with a brief explanation.

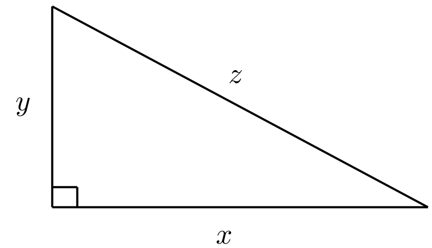


f: ______

f': ______

f'': ______

2. [9 points] Consider a right triangle with legs of length x ft and y ft and hypotenuse of length z ft, as in the following picture:



a. [2 points] Suppose that the perimeter of the triangle is 8 ft. Let $A(x)$ give the area of the triangle, in ft^2, as a function of the side length x. In the context of this problem, what is the domain of $A(x)$? *Note that you do* <u>*not*</u> *need to find a formula for* $A(x)$.

Answer: ______________________

b. [7 points] Suppose instead that the perimeter of the triangle is allowed to vary, but the area of the triangle is fixed at 3 ft^2. Let $P(x)$ give the perimeter of the triangle, in ft, as a function of the side length x.

(i) In the context of this problem, what is the domain of $P(x)$?

Answer: ______________________

(ii) Find a formula for $P(x)$. The variables y and z should <u>not</u> appear in your answer. (This is the equation one would use to find the value(s) of x minimizing the perimeter. You should <u>not</u> do the optimization in this case.)

Answer: $P(x) =$ ______________________