$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

Math 1313 Section 5.4

Section 5.4: Permutations and Combinations

Definition: n-Factorial $n!$

For any natural number n, $n(n-1)(n-2) \ldots 3 \cdot 2 \cdot 1$

0! = 1

A **permutation** is an arrangement of a specific set where the order in which the objects are arranged is important.

Rank, Title, Line up

Formula: $P(n,r) = \frac{n!}{(n-r)!}, \quad r \le n$

where n is the number of distinct objects and r is the number of distinct objects taken r at a time.

Formula: Permutations of n objects, not all distinct "WORD"

Given a set of n objects in which n_1 objects are alike and of one kind, n_2 objects are alike and of another kind,…, and, finally, n_r objects are alike and of yet another kind so that

$$n_1 + n_2 + \cdots + n_r = n$$

then the number of permutations of these n objects taken n at a time is given by

$$\frac{n!}{n_1!\, n_2! \ldots n_r!}$$

A **combination** is an arrangement of a specific set where the order in which the objects are arranged is not important.

Formula: $C(n,r) = \frac{n!}{r!(n-r)!}, \quad r \le n$

where n is the number of distinct objects and r is the number of distinct objects taken r at a time.

Example 1: You are in charge of seating 5 honored guests at the head table of a conference. How many seating arrangements are possible if the 8 chairs are on one side of the head table?

$n = 8$

$r = 5$

Line Up → Perm.

$P(8,5) = 6720$

Chairs ↑ ↑ People

$8 \quad 7 \quad 6 \quad 5 \quad 4$

Example 2: Find the number of ways 9 people can arrange themselves in a line for a group picture.

$n=9$ $r=9$ $\text{Perm}(9,9) = 362,880$

Popper 3: You are in charge of seating 4 honored guests at the head table of a conference. How many seating arrangements are possible if the 10 chairs are on one side of the head table?

a. 5040
b. 210
c. 40
d. None of the above

$n=10$ $r=4$

Example 3: An organization has 30 members. In how many ways can the positions of president, vice-president, secretary, treasurer, and historian be filled if not one person can fill more than one position?

$n=30$ $r=5$ Titles / Positions → Perm

$P(30,5) = 17,100,720$

Example 4: An organizations needs to make up a social committee. If the organization has 25 members, in how many ways can a 10 person committee be made?

$n=25$ $r=10$ Order is NOT important → Combination

$C(25,10) = 3,268,760$

Example 5: If there are 40 contestants in a beauty pageant, in how many ways can the judges award 1st prize and 2nd prize if not one person can be awarded 1st and 2nd?

$n=40$ $r=2$ Rank → Perm.

$P(40,2) = 1560$

Example 6: How many permutations can be formed from all the letters in the word MISSISSIPPI.

$n = 11$ ← number of characters

M – 1
I – 4
S – 4
P – 2

$$\frac{n!}{n_1! \cdot n_2! \cdots} = \frac{11!}{(1! \cdot 4! \cdot 4! \cdot 2!)}$$

39916800 / (1 * 24 * 24 * 2)

$= 34{,}650$

Popper 4: In a production of *West Side Story*, eight actors are considered for the male roles of Tony, Riff, and Bernardo. In how many ways can the director cast the male roles? **What type of problem is this?**

a. Combination
b. Permutation

Example 7: A museum of fine arts owns 8 paintings by a given artist. Another fine arts museum wishes to borrow 3 of these paintings for a special show. How many ways can 3 paintings be selected for shipment out of the 8 available?

$n = 8$

$r = 3$

Order does NOT matter

$C(8,3) = 56$

Example 8: A certain company has to transfer 4 of its 10 junior executives to a new location, how many ways can the 4 executives be chosen?

$n = 10$

$r = 4$

Order Not important

$C(10,4) = 210$

Example 9: A coin is tossed 5 times.
a. In how many outcomes do exactly 3 heads occur?

Order of Heads Doesn't Matter

{($H_1H_2H_3TT$), ($H_1H_2T\ H_4T$), ($H_1H_2TT\ H_5$), ($H_1TH_3T\ H_5$), ($H_1TTH_4H_5$), ($H_1T\ H_3H_4T$), ($TH_2H_3H_4T$), ($TH_2H_3T\ H_5$), ($TH_2TH_4H_5$), ($TTH_3H_4H_5$)}

$\binom{5}{3} = 10$

$n = 5$

$r = 3$

$C(5,3) = 10$

Minimum

b. In how many outcomes do at least 4 heads occur? 4 heads, 5 heads $n = 5$

{($H_1H_2H_3H_4T$), ($H_1H_2H_3T\ H_5$), ($H_1H_2TH_4H_5$), ($H_1TH_3H_4H_5$), ($TH_2H_3H_4H_5$)} (5) $r = 4$

{($H_1H_2H_3H_4H_5$)} (1) $r_2 = 5$

$C(5,4) + C(5,5) = 5 + 1 = 6$

Example 10: A coin is tossed 20 times.

a. In how many outcomes do exactly 7 tails occur?

$C(20,7) = 77,520$

b. In how many outcomes do at most 18 heads occur?

Max of 18

0 Heads
1 Head
2 Heads
⋮
+ 18 Heads
Answer

Complement : 19, 20

$C(20,19) + C(20,20) = 20 + 1 = 21$

Grand Total/Universe = $2^{\text{tosses}} = 2^{20} = 1,048,576$

Answer = Total − Complement

$= 1,048,576 - 21 = \boxed{1,048,555}$

c. In how many outcomes do at least 19 heads occur?

19 is Minimum

19, 20 Heads $C(20,19) + C(20,20) = 21$

d. In how many ways do at least 3 heads occur?

3 is Minimum 3 − 20 Heads

$C(20,3)$
$C(20,4)$
$C(20,5)$
⋮
$+ C(20,20)$
Answer

Complement : 0, 1, 2

$C(20,0) + C(20,1) + C(20,2)$

$1 + 20 + 190 = 211$

= Universe − Complement

$= 1,048,576 - 211$

$= \boxed{1,048,365}$

Example 11: A student belongs to a entertainment club. This month he must purchase 2 DVDs and 3 CDs. If there are 15 DVDs and 10 CDs to choose from, in how many ways can he choose his 5 purchases?

DVDs $n = 15$, $r = 2$: $C(15,2) = 105$

CDs $n = 10$, $r = 3$: $C(10,3) = 120$

G.M.P. $105 \cdot 120 = \boxed{12,600}$

Example 12: A committee of 16 people, 7 women and 9 men, is forming a subcommittee that is to be made up of 6 women and 6 men. In how many ways can the subcommittee be formed?

W M

$C(7,6) \cdot C(9,6)$

$7 \cdot 84 = \boxed{588}$

Example 13: A computer store receives a shipment of 35 laser printers, including 6 that are defective. Five of these printers are selected to be displayed in the store.

6 Def 29 Good

a. How many of these selections will contain no defective printers?

$\frac{6D}{0}$ $\frac{29G}{5}$ $C(6,0)\,C(29,5) = 119,755$

b. How many of these selections will contain 1 defective printer?

$\frac{6D}{1}$ $\frac{29G}{4}$ $C(6,1)\,C(29,4) = 142,506$

c. How many of these selections will contain at least 1 defective printer?

1 is Minimum 1-5

6D	29G
1	4
2	3
3	2
4	1
5	0

Complement 0 Def

$\frac{6D}{0}$ $\frac{29G}{5}$ → $C(6,0)C(29,5)$ puts a

35 printers choose 5

Answer = Total − Complement

$= C(35,5) - 119755 = 324,632 - 119,755 = \boxed{205,877}$

Example 14: A customer at a fruit stand picks a sample of 6 avocados at random from a crate containing 35 avocados of which 8 are rotten. In how many ways can the batch contain at least 2 rotten avocados?

Min of 2
2 – 6 Rotten

8R	27G
2	4
3	3
4	2
5	1
6	0

Too much work

Complement

8R	27G
0	6
1	5

$C(8,0) * C(27,6)$

$C(8,1)\ C(27,5)$

$296,010 + 645,840$

$= 941,850$

Total $= (35,6) = 1,623,160$

Answer $= 1,623,160 - 941,850$

$= \boxed{681,310}$

Popper 5: An urn contains 17 red marbles and 18 blue marbles. 16 marbles are chosen. In how many ways can 6 red marbles be chosen?

a. 102
b. 541549008
c. 8008
d. 56134
e. 12376