MATH 1314 College Algebra Notes Spring 2012
Chapter 4: Exponential and Logarithmic Functions

Chapter 4.1: Exponential Functions

Exponential Functions are of the form $f(x)=b^x$, where the base *b* is a number $b>0$ but not equal to 1 and where x is any real number. The exponential function $f(x)=b^x$ is read as the **exponential function f with base b**. Exponential Functions are very useful in real-world applications. They are used to model situations involving Inflation of Cost, Financial Investments, Population Growth, Growth of Epidemics, Radioactive Decay, and more.

❖ Objective 1, 2: Evaluating and Graphing Exponential Functions(p413)

- Evaluating exponential functions means to "plug in" a known x-value into the function and calculate the result. You will need to use the [^] key on your TI-83/84 calculator. Use _____ when evaluating functions.
- Characteristics of Exponential Functions $f(x)=b^x$ and their graphs:
  - Domain is $(-\infty,\infty)$ Range is $(0,\infty)$
  - y-intercept is 1 because $f(0)=b^0=1$ (where $b\neq 0$)
  - if base $b>1$, then $f(x)=b^x$ increases _____
    if base $0<b<1$, then $f(x)=b^x$ decreases _____
  - _____ is the equation for the Horizontal Asymptote.

(From the graph, $f(x)=b^x$ is a one-to-one function, so it has an _____.)

❖ Objective 3, 4: Natural base *e*; Compound Interest Formulas(p417,419)

- The irrational number e is a value that $\left(1+\frac{1}{n}\right)^n$ approaches as $n\to\infty$.

  Use a TABLE and enter large values of *n*,(10, 100, 1000, 10000,...) and you can see that the value $\left(1+\frac{1}{n}\right)^n$ is approximately _____.

  We call this approximate value the natural base *e*. It is used to model natural exponential behavior that increases or decreases.
- There are 2 Compound Interest Formulas used to calculate Total Investment amounts in this section:
  - For *n* compounding periods per year use $A=P\left(1+\frac{r}{n}\right)^{(n)\cdot(t)}$ where *A* is Total amount, *P* is beginning amount, *r* is interest rate, *t* is time in years, and *n* is number of compounding periods per year.
  - For continuous compounding use $A=P\cdot e^{(r)\cdot(t)}$ where *A* is Total amount, *P* is beginning amount, *r* is interest rate, *t* is time in years. You will need to use [2nd] [LN] when entering $e^{(r)\cdot(t)}$.

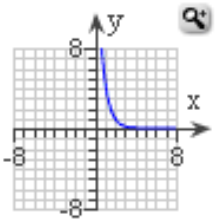
1 2 3 4 5 6 7 8 9 10 4.1.25

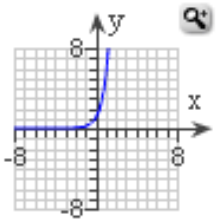
Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 12 pts) **0 of 12** complete

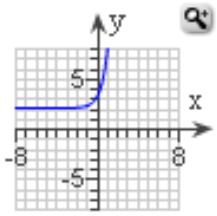
Use transformations of the graph of $f(x) = 5^x$ to identify the graph of the function given below. Use the graph to determine its domain, range, and asymptote.

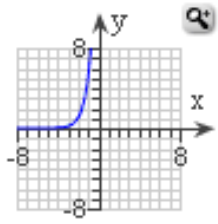
$g(x) = 5^{x+2}$

Identify the graph of $g(x) = 5^{x+2}$.

A. 

B. 

C. 

D. 

What is the domain of $g(x) = 5^{x+2}$?

(Type your answer in interval notation.)

What is the range of $g(x) = 5^{x+2}$?

(Type your answer in interval notation.)

What line is the horizontal asymptote of $g(x) = 5^{x+2}$?

$y =$

Enter any number or expression in the edit field, then click Check Answer.

1 2 3 4 5 6 7 8 9 10 4.1.53

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 12 pts) **0 of 12** complete

Use the compound interest formulas $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$ to solve the problem given.

Round answers to the nearest cent.

Find the accumulated value of an investment of $10,000 for 3 years at an interest rate of 7% if the money is **a.** compounded semiannually; **b.** compounded quarterly; **c.** compounded monthly **d.** compounded continuously.

a. What is the accumulated value if the money is compounded semiannually?

$ _____ (Round your answer to the nearest cent.)

b. What is the accumulated value if the money is compounded quarterly?

$ _____ (Round your answer to the nearest cent.)

c. What is the accumulated value if the money is compounded monthly?

$ _____ (Round your answer to the nearest cent.)

d. What is the accumulated value if the money is compounded continuously?

$ _____ (Round your answer to the nearest cent.)

Enter any number or expression in the edit field, then click Check Answer.

1 2 3 4 5 6 7 8 9 10 4.1.65

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 12 pts) **0 of 12** complete

Use a calculator with a y^x key or a $\wedge$ key to solve the following.

The exponential function $f(x) = 567(1.032)^x$ models the population of a country, f(x), in millions, x years after 1975. Complete parts (a) – (e).

a. Substitute 0 for x and, without using a calculator, find the country's population in 1975.

The country's population in 1975 was _____ million.

b. Substitute 22 for x and use your calculator to find the country's population, to the nearest million, in the year 1997 as modeled by this function.

The country's population in 1997 was _____ million.

c. Find the country's population, to the nearest million, in the year 2019 as predicted by this function.

The country's population in 2019 will be _____ million.

d. Find the country's population, to the nearest million, in the year 2041 as predicted by this function.

The country's population in 2041 will be _____ million.

e. What appears to be happening to the country's population every 22 years?

- A. It appears that the population is growing by a factor of 3 every 22 years.
- B. It appears that the population is growing by a factor of 2 every 22 years.
- C. There does not appear to be a pattern.
- D. It appears that the population is decreasing by a factor of $\frac{1}{2}$ every 22 years.

Click to select your answer, then click Check Answer.

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Chapter 4.2: Logarithmic Functions

Logarithmic Functions are of the form $f(x)=\log_b(x)$, where the base *b* is a number ____ but not equal to 1 and where $x>0$. The function $f(x)=\log_b(x)$ is read as the **logarithmic function f with base b**. Logarithms are merely an exponent for an indicated base. Logarithmic Functions are very useful in real-world applications and are used to model Earthquake and Sound Intensity, Acidity of Aqueous Solutions, Human Memory, and more.

❖ Objective 1,2 3: Change between Logarithmic and Exponential form; Evaluate logarithms(p425)

- To change between exponential and logarithmic forms using the **crossing method**:
  - line up the equal signs
  - identify base *b* and write as exponential or logarithmic form
  - switch x and y expressions

$y=\log_b(x)$ logarithmic form ↓ $x=b^y$ exponential form

$x=b^y$ exponential form ↓ $y=\log_b(x)$ logarithmic form

- To evaluate logarithms by inspection, remember that the value of a logarithm is merely an _____ for an indicated base.

  Example: Evaluate $\log_2(32)$.

  The value of $\log_2(32)$ is 5 because $2^{(\ )}=32$.

  In logarithms such as $\log(100)$, the blank base is understood to be ____.

  This type of logarithm is called a _____ Logarithm and can be rewritten as $\log_{10}(100)$. You can use the [LOG] key to evaluate it.

  In logarithms such as $\ln(4)$, the notation ln represents $\log_e$ and is called a _____ Logarithm. You must use the [LN] key to evaluate it.

❖ Objective 4: Basic Properties of Logarithms(p426)

Recall that the value of a logarithm is merely an _____ for the indicated base. The following properties are based on this fact.

- $\log_b(b)=1$ because $b^1=b$
- $\log_b(1)=0$ because $b^0=1$
- $\log_b(b^x)=x$ $\log_b$ and base *b* in ( ) cancel leaving only x.
- $b^{\log_b(x)}=x$ base *b* and $\log_b$ cancel out leaving only x.

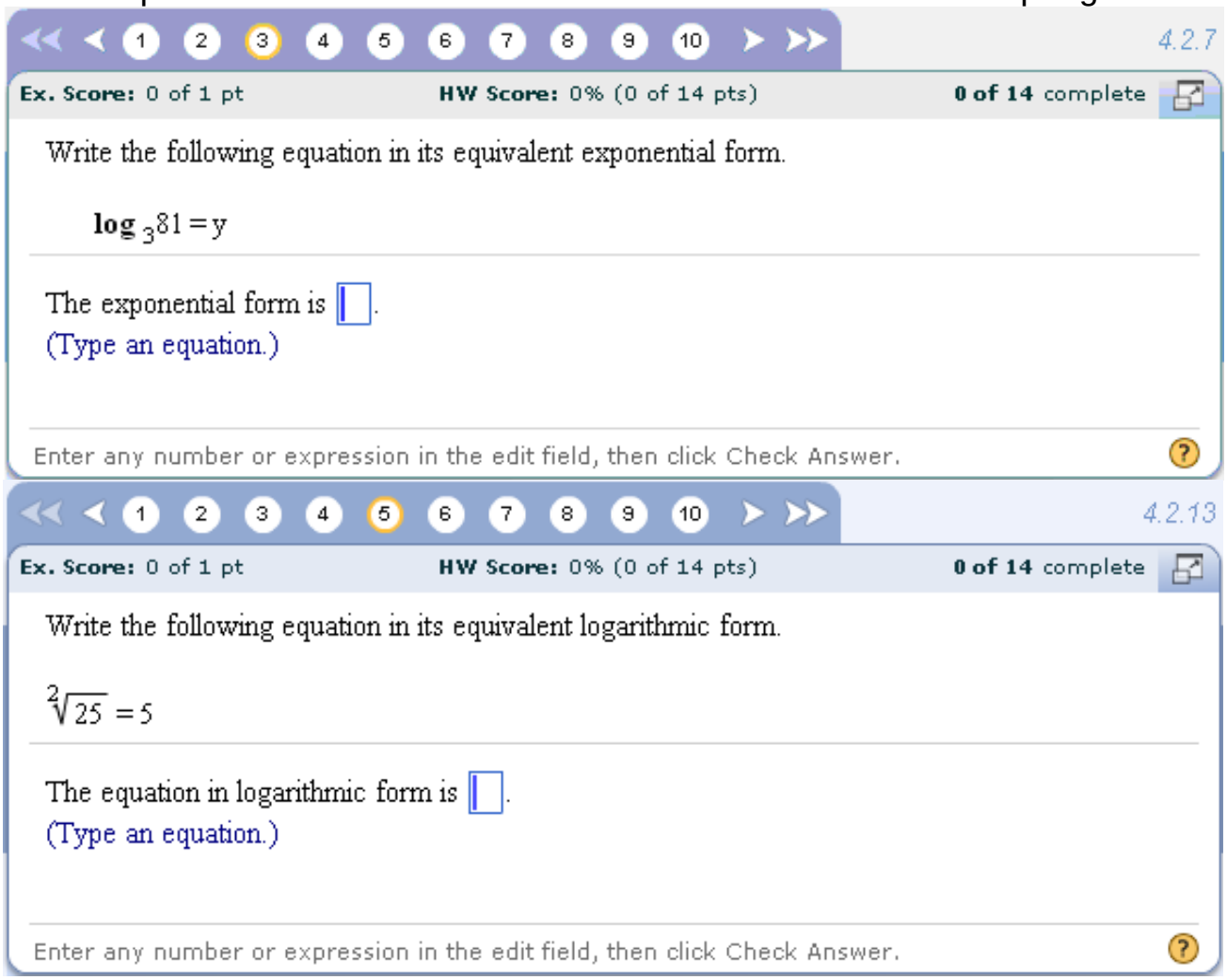
Chapter 4.2: Logarithmic Functions

❖ Objective 5,6: Graphing Logarithmic Functions(p427)

- Characteristics of Logarithmic Functions $f(x)=\log_b(x)$ and their graphs:
  - Domain is _____ Range is _____
  - x-intercept is _____
  - if base $b>1$, then $f(x)=\log_b(x)$ increases _____
    if base $0<b<1$, then $f(x)=\log_b(x)$ decreases _____
  - _____ is the equation for the _____ Asymptote.

Note: Since $f(x)=\log_b(x)$ is one-to-one and is the inverse of $f(x)=b^x$, the points $\left(-1,\frac{1}{b}\right),(0,1),(1,b)$ that are on the graph of $f(x)=b^x$ will be reflected as the points $\left(\frac{1}{b},-1\right),(1,0),(b,1)$ on the graph of $f(x)=\log_b(x)$.

Practice problems 4.2 Spring 2012



1 2 3 4 5 6 7 8 9 10 4.2.21

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 14 pts) **0 of 14** complete

Find the exact value of the logarithm without using a calculator.

$\log_6 36$

$\log_6 36 =$ ☐

Enter any number or expression in the edit field, then click Check Answer.

1 2 3 4 5 6 7 8 9 10 4.2.25

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 14 pts) **0 of 14** complete

Evaluate the expression without using a calculator.

$\log_{11} \frac{1}{11}$

$\log_{11} \frac{1}{11} =$ ☐

Enter any number or expression in the edit field, then click Check Answer.

1 2 3 4 5 6 7 8 9 10 4.2.37

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 14 pts) **0 of 14** complete

Evaluate the expression without using a calculator.

$\log_2 1$

$\log_2 1 =$ ☐

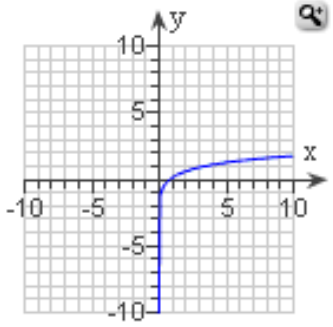
Enter any number or expression in the edit field, then click Check Answer.

11 12 13 14 4.2.53

Ex. Score: 1 of 1 pt **HW Score:** 21.43% (3 of 14 pts) **3 of 14** complete

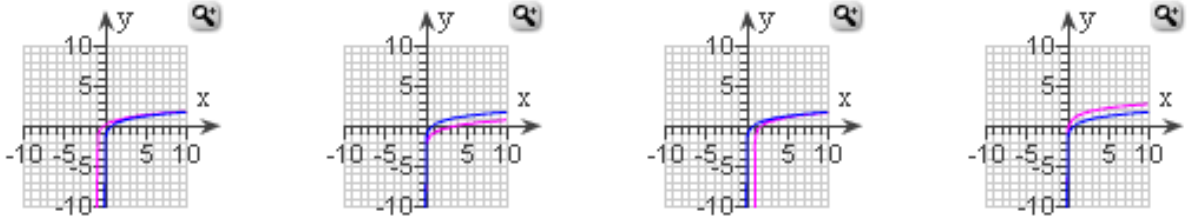
The figure on the right shows the graph of $f(x) = \log_4 x$. Use transformations of this graph to graph the given function. Find the graph's vertical asymptote, domain, and range.

$$g(x) = \log_4 (x + 1)$$



a.) What is the graph of the two functions?

A. B. C. D.



b.) What is the vertical asymptote of the shifted function, g(x)?

$x =$ _____

c.) What is the domain of g(x)?

The domain is _____.

(Type your answer in interval notation.)

d.) What is the range of g(x)?

The range is _____.

(Type your answer in interval notation.)

Question is complete.

Chapter 4.3: Properties of Logarithms

This section introduces properties of logarithms which will be used to rewrite (_____ or _____) logarithmic expressions. When solving logarithmic equations or evaluating logarithmic expressions, it is sometimes necessary to rewrite logarithms using their properties. Certain characteristics of the logarithm properties will seem familiar because the properties of _____ discussed previously correspond to properties of logarithms.

❖ Objective 1,2,3,4,5,6: Properties of Logarithms; Change of Base Formula(p438-443)

To expand logarithmic expressions, use the following order:

_____ Rule	The expression $\log_b(M\cdot N)$ expands to $\log_b(M)+\log_b(N)$
_____ Rule	The expression $\log_b\left(\frac{M}{N}\right)$ expands to $\log_b(M)-\log_b(N)$
_____ Rule	The expression $\log_b(M^p)$ expands to $p\cdot\log_b(M)$

To condense logarithmic expressions, use the following order:

_____ Rule	The expression $p\cdot\log_b(M)$ condenses to $\log_b(M^p)$
_____ Rule	The expression $\log_b(M)+\log_b(N)$ condenses to $\log_b(M\cdot N)$
_____ Rule	The expression $\log_b(M)-\log_b(N)$ condenses to $\log_b\left(\frac{M}{N}\right)$

The **Change of Base Formula** is used to evaluate a logarithm expression that has an indicated base *b* other than 10 or e.

To evaluate $\log_b(n)\Rightarrow\frac{\log(n)}{\log(b)}$ or $\frac{\ln(n)}{\ln(b)}$.

Example: Evaluate $\log_6(32)$. Round your answer to the nearest tenth.

This logarithm uses base 6. If your calculator does not have a LOG key for base 6, then the Change of Base Formula is needed.

Enter $\log_6(32)$ as $\frac{\log(32)}{\log(6)}$ or as $\frac{\ln(32)}{\ln(6)}$.

Either entry gives _____…. The answer is therefore _____.

1 2 3 4 5 6 7 8 9 10 4.3.3

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 12 pts) **0 of 12** complete

Use properties of logarithms to expand the logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.

$$\log_2(4x)$$

$$\log_2(4x) = \square$$

Enter any number or expression in the edit field, then click Check Answer.

1 2 3 4 5 6 7 8 9 10 4.3.11

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 12 pts) **0 of 12** complete

Use properties of logarithms to expand the logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.

$$\log_2\left(\frac{16}{y}\right)$$

$$\log_2\left(\frac{16}{y}\right) = \square$$

Enter any number or expression in the edit field, then click Check Answer.

1 2 3 4 5 6 7 8 9 10 4.3.35

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 12 pts) **0 of 12** complete

Use properties of logarithms to expand the logarithmic expression as much as possible. Evaluate logarithmic expressions without using a calculator if possible.

$$\log_2\sqrt[7]{\frac{s^8t}{4}}$$

$$\log_2\sqrt[7]{\frac{s^8t}{4}} = \square$$

(Use integers or fractions for any numbers in the expression.)

Enter any number or expression in the edit field, then click Check Answer.

1 2 3 4 5 6 7 8 9 10 4.3.45

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 12 pts) **0 of 12** complete

Use properties of logarithms to find the exact value of the expression. Do not use a calculator.

$\log_4 20 - \log_4 5$

$\log_4 20 - \log_4 5 =$ ☐ (Type an integer or a simplified fraction.)

Enter any number or expression in the edit field, then click Check Answer.

1 2 3 4 5 6 7 8 9 10 4.3.53

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 12 pts) **0 of 12** complete

Use properties of logarithms to condense the logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Evaluate logarithmic expressions if possible.

$9 \log_b x + 5 \log_b z$

$9 \log_b x + 5 \log_b z =$ ☐

Enter any number or expression in the edit field, then click Check Answer.

11 12 4.3.69

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 12 pts) **0 of 12** complete

Use properties of logarithms to condense the logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions.

$\log x + \log (x^2 - 49) - \log 2 - \log (x + 7)$

$\log x + \log (x^2 - 49) - \log 2 - \log (x + 7) =$ ☐

(Simplify your answer.)

Enter any number or expression in the edit field, then click Check Answer.

Chapter 4.4: Solving Exponential and Logarithmic Equations
This section will now introduce methods for solving Exponential and Logarithmic Equations, including the TI-83/84 calculator.

❖ Objective 1: Solving Exponential Equations(p448)

- **To solve exponential equations** algebraically **using like bases**:
  - Make sure both sides of equation have the _____. Rewrite if necessary. If bases cannot be made the same, use logarithms to solve.
  - Once the bases are the same, _____ and set the exponents equal to each other.
  - Solve for x.

➢ **To solve exponential equations** $b^M = b^N$ **using a TABLE**:
→ Enter left side of equation as Y1 and right side of equation as Y2.
→ Press [2nd] [GRAPH] to find the solution for x in a TABLE.
The solution will be the x-value with the same Y1 and Y2 value.

➢ **To solve exponential equations** $b^M = b^N$ **using GRAPH**:
→ Enter left side of equation as Y1 and right side of equation as Y2.
→ Press [2nd] [TRACE] to select the **5: intersect** command.
→ For First Curve? move cursor on first graph close to intersection and press [ENTER].
For Second Curve? move cursor on second graph close to intersection and press [ENTER].
Press [ENTER] again for Guess?

IMPORTANT: If exact answers are needed, you may need to use the _____ method above.

- **To solve exponential equations** algebraically with **logarithms**:
  - _____ the exponential expression.
  - Apply the _____ logarithm or the _____logarithm to each side.
  - Use the _____ Rule for logarithms to bring the variable x expression down from the exponent and solve for x.

Chapter 4.4: Solving Exponential and Logarithmic Equations

❖ Objective 2,3,4: Solving Logarithmic Equations(p449)

For equations having logarithms on one side of equation only:

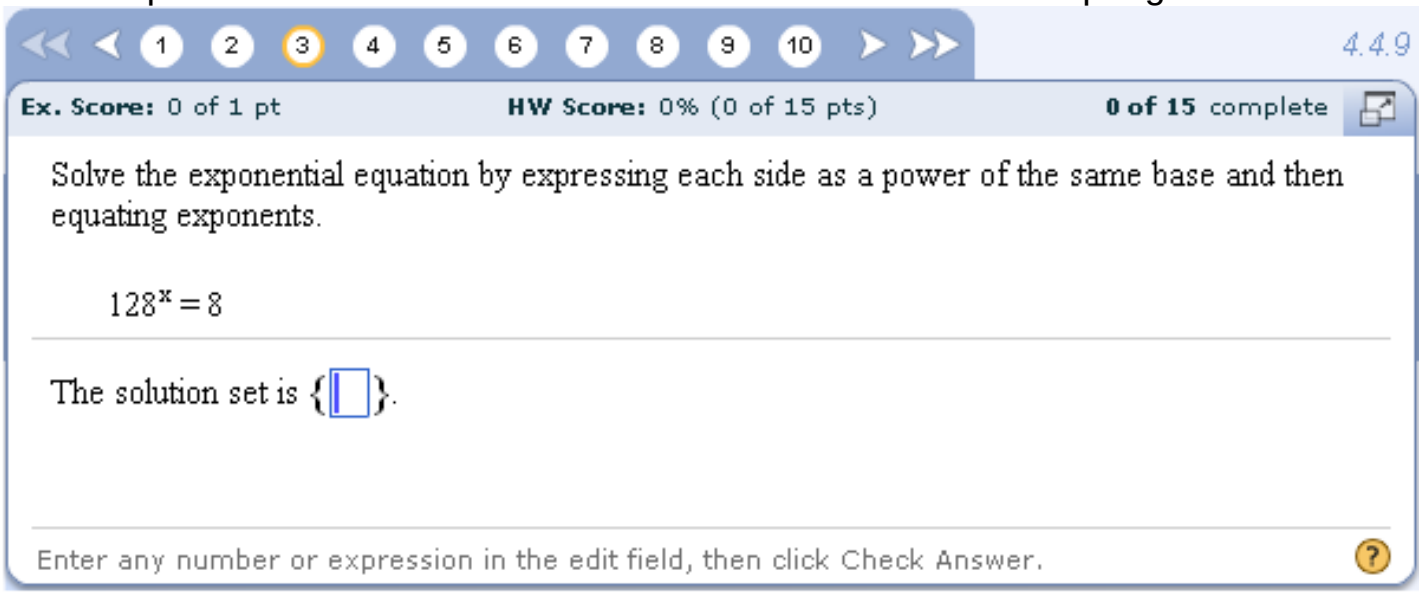
- _____ the left side of equation to form a single logarithm. If the left side of equation only has a single logarithm, then you are ready for next step.
- Change the logarithm equation to _____ form using the crossing method from section 4.2.
- Solve the new exponential equation using the like bases method.

For equations containing logarithms on both sides with the same base:

- _____ both sides of equation until it is of the form $\log_b(M)=\log_b(N)$ with each side having a coefficient of 1.
- Using the one-to-one property, you can _____ the $\log_b$ notation on both sides and set the (M and N expressions) equal to each other.
- Solve for the variable in the new equation.

To solve logarithmic equations with the TI-83/84 calculator, use a TABLE or GRAPH by following the same steps given earlier for exponential equations.

Practice problems 4.4 Spring 2012



Practice problems 4.4 Spring 2012

1 2 3 4 5 6 7 8 9 10 4.4.15

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 15 pts) **0 of 15** complete

Solve the exponential equation by expressing each side as a power of the same base and then equating exponents.

$$9^{\frac{x-2}{4}} = \sqrt{9}$$

The solution set is $\{\ \}$.

Enter any number or expression in the edit field, then click Check Answer.

1 2 3 4 5 6 7 8 9 10 4.4.33

Ex. Score: 0 of 1 pt **HW Score:** 0% (0 of 15 pts) **0 of 15** complete

Solve the following exponential equation by taking the natural logarithm on both sides. Express the solution in terms of natural logarithms. Then, use a calculator to obtain a decimal approximation for the solution.

$$e^{2-4x} = 1594$$

What is the solution in terms of natural logarithms?

The solution set is $\{\ \}$.

(Type an exact solution, using radicals and log functions as needed. Use the change of base formula to write the answer in either base 10 or base e.)

What is the decimal approximation for the solution?

The solution set is $\{\ \}$.

(Round to two decimal places.)

Enter any number or expression in the edit field, then click Check Answer.

Practice problems 4.4 Spring 2012

11 12 13 14 15 4.4.37

Ex. Score: 0 of 1 pt **HW Score:** 3.33% (0.5 of 15 pts) **1 of 15** complete

Solve the exponential equation. Express the solution in terms of natural logarithms. Then use a calculator to obtain a decimal approximation for the solution.

$$6^{(x-3)} = 390$$

What is the solution in terms of natural logarithms?

The solution set is $\{\ \}$.

Enter any number or expression in the edit field, then click Check Answer.

11 12 13 14 15 4.4.67

Ex. Score: 0 of 1 pt **HW Score:** 3.33% (0.5 of 15 pts) **1 of 15** complete

Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer.

$$\log_5(x+121) + \log_5(x+1) = 4$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The solution set is $\{\ \}$.
(Simplify your answer. Use a comma to separate answers as needed.)

B. There is no solution.

Click to select and enter your answer(s), then click Check Answer.

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Chapter 4: Exponential and Logarithmic Functions

Chapter 4.5: Exponential Growth and Decay

In this section, you will learn how to create functions to model exponential growth and exponential decay and use them to make future predictions.

❖ Objective 1: Exponential Growth and Decay(p460)

- Exponential Growth and Exponential Decay use the same mathematical model(you will use this model to make a formula):

$$A = A_0 \cdot e^{(k)\cdot(t)}$$

where A is the final amount of a sample at time t

A_0 is the beginning amount of a sample when time t = 0

k is the growth rate if $k > 0$ or is the decay rate if $k < 0$

(This constant represents the percentage of increase or decrease in the population or sample)

t is time

Important terms to know for Growth:

increase $\rightarrow$ beginning amount grows

double $\rightarrow$ 2 times beginning amount

triple $\rightarrow$ 3 times beginning amount…

Important terms to know for Decay:

decrease $\rightarrow$ beginning amount breaks down, decomposes

half-life $\rightarrow$ time needed for $\frac{1}{2}$ of a substance to decay.

❖ Objective 2: Logistic Growth(p464)

In real life, exponential growth is limited by conditions set by nature, therefore, population growth will be limited by things like the surrounding resources available and the environment. An epidemic will grow and begin to spread exponentially within a confined population, but will eventually slow down as the number of people affected approaches the population size.

- Limited Logistic Growth for populations uses the mathematical model:

$$A = \frac{c}{\left(1 + a \cdot e^{(-b)\cdot(t)}\right)}$$

where A is the size of the population affected at time t

c is the limiting(maximum) size of A as time $t \rightarrow \infty$

a and b are constants

t is time

Practice problems 4.5 Spring 2012

1 2 3 4 5 6 7 8 9 10 4.5.5

Ex. Score: 0 of 1 pt **HW Score:** 4.17% (0.5 of 12 pts) **1 of 12** complete

The exponential model $A = 373.2\,e^{0.006t}$ describes the population, A, of a country in millions, t years after 2003. Use the model to determine when the population of the country will be 380 million.

The population of the country will be 380 million in _____.
(Round to the nearest year as needed.)

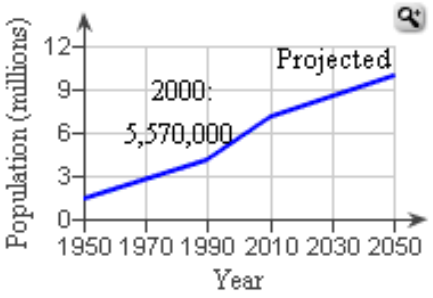
Enter any number or expression in the edit field, then click Check Answer.

1 2 3 4 5 6 7 8 9 10 4.5.7

Ex. Score: 0 of 1 pt **HW Score:** 4.17% (0.5 of 12 pts) **1 of 12** complete

a. In 2000, the population of a country was approximately 5.57 million and by 2091 it is projected to grow to 15 million. Use the exponential growth model $A = A_0\,e^{kt}$, in which t is the number of years after 2000, to find an exponential growth function that models the data.

b. By which year will the population be 11 million?



a. The exponential growth function that models the data is $A =$ _____.
(Simplify your answer. Use integers or decimals for any numbers in the expression. Round to two decimal places as needed.)

b. The country's population will be 11 million in the year _____.
(Use the answer from part a to find this answer. Round to the nearest year as needed.)

Enter any number or expression in the edit field, then click Check Answer.

Practice problems 4.5 Spring 2012

1 2 3 4 5 6 7 8 9 10 4.5.27

Ex. Score: 0 of 1 pt **HW Score:** 8.33% (1 of 12 pts) **2 of 12** complete

In 1964, paleontologists discovered the bones of a new species of dinosaur. The age of the dinosaur was estimated using potassium-40 dating of rocks surrounding the bones. Analysis of these rocks indicated that 62.5% of the original amount of the potassium-40 was still present. The decay model for potassium-40 is $A = A_0 e^{-0.52912t}$, where t is in billions of years. Let $A = 0.625A_0$ in this decay model and estimate the age of the bones of the dinosaur.

The bones are approximately _____ billion years old.

(Type an integer or decimal rounded to four decimal places as needed.)

Enter any number or expression in the edit field, then click Check Answer.

1 2 3 4 5 6 7 8 9 10 4.5.31

Ex. Score: 0 of 1 pt **HW Score:** 8.33% (1 of 12 pts) **2 of 12** complete

The half-life of a certain tranquilizer in the bloodstream is 28 hours. How long will it take for the drug to decay to 89% of the original dosage? Use the exponential decay model, $A = A_0 e^{kt}$, to solve.

_____ hours

(Round to one decimal place as needed.)

Enter any number or expression in the edit field, then click Check Answer.

Practice problems 4.5 Spring 2012

Ex. Score: 0 of 1 pt **HW Score:** 8.33% (1 of 12 pts) **2 of 12** complete

The logistic growth function at right describes the number of people, $f(t)$, who have become ill with influenza t weeks after its initial outbreak in a particular community.

$$f(t) = \frac{116{,}000}{1 + 4300\,e^{-t}}$$

a. How many people became ill with the flu when the epidemic began?
b. How many people were ill by the end of the fourth week?
c. What is the limiting size of the population that becomes ill?

a. The number of people initially infected is _____
(Round to the nearest number of people.)

b. The number of people infected after 4 weeks is _____
(Round to the nearest number of people.)

c. The limiting size of the infected popluation is _____
(Round to the nearest number of people.)

Enter any number or expression in the edit field, then click Check Answer.