MATH 1330
Precalculus

Exercise Sets and
Odd-Numbered Solutions

University of Houston Department of Mathematics

MATH 1330, Precalculus: Exercise Sets and Odd-Numbered Solutions

MATH 1330 – Precalculus
Exercise Sets and Odd-Numbered Solutions
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MATH 1330 – Precalculus

Textbook and Online Resource Information

University of Houston Department of Mathematics

Math 1330 Printed Textbook:

The packet you are holding contains only the exercise sets and odd-numbered answers to the exercise sets. The entire Math 1330 printed textbook, which contains the lessons for each section *as well as these exercise sets and odd-numbered answers*, can be purchased at the University Copy Center.

Math 1330 Online:

All materials found in this textbook can also be found online at:

http://online.math.uh.edu/Math1330/

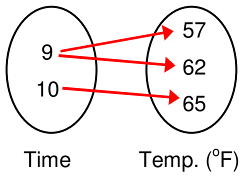
Online Lectures:

In addition to the textbook material, the online site contains flash lectures pertaining to most of the topics in the Precalculus course. These lectures simulate the classroom experience, with audio of course instructors as they present the material on prepared lesson notes. The lectures are useful in furthering understanding of the course material and can only be viewed online.

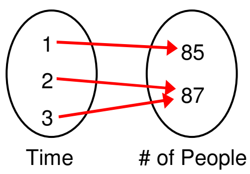
Exercise Set 1.1: An Introduction to Functions

For each of the examples below, determine whether the mapping makes sense within the context of the given situation, and then state whether or not the mapping represents a function.

1. Erik conducts a science experiment and maps the temperature outside his kitchen window at various times during the morning.

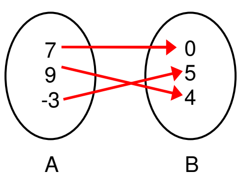


2. Dr. Kim counts the number of people in attendance at various times during his lecture this afternoon.



State whether or not each of the following mappings represents a function. If a mapping is a function, then identify its domain and range.

3.



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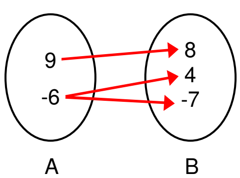

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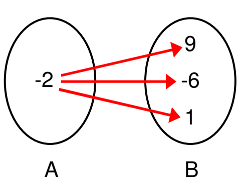

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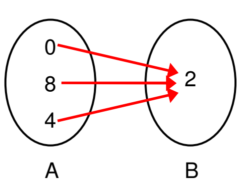


Express each of the following rules in function notation. (For example, "Subtract 3, then square" would be written as $f(x)=(x-3)^2$.)

7. Divide by 7, then add 4

8. Multiply by 2, then square

9. Take the square root, then subtract 6

10. Add 4, square, then subtract 2

Find the domain of each of the following functions. Then express your answer in interval notation.

11. $f(x)=\dfrac{5}{x-3}$

12. $f(x)=\dfrac{x-6}{x+1}$

13. $g(x)=\dfrac{x-4}{x^2-9}$

14. $f(x)=\dfrac{3x+1}{x^2+4}$

15. $f(x)=\dfrac{x^2+6x+5}{x^2-11x+28}$

16. $g(x)=\dfrac{3x+15}{x^2+8x-20}$

17. $f(t)=\sqrt{t}$

18. $h(x)=\sqrt[3]{x}$

19. $g(x)=\sqrt[5]{x}$

20. $h(t)=\sqrt[4]{t}$

21. $f(x)=\sqrt{x-5}$

22. $g(x)=\sqrt{x+7}$

23. $F(x)=\dfrac{\sqrt{3-2x}}{x+4}$

24. $G(x)=\dfrac{\sqrt{x-3}}{x-7}$

25. $f(x) = \sqrt[3]{x-5}$

26. $g(x) = \sqrt[3]{x^2+x-6}$

27. $h(t) = \sqrt[3]{\dfrac{t^2-7t-8}{t+5}}$

28. $f(x) = \sqrt[5]{\dfrac{2x-9}{4x-7}}$

29. $f(t) = \sqrt{t^2-10t+24}$

30. $g(t) = \sqrt{t^2-5t-14}$

Find the domain and range of each of the following functions. Express answers in interval notation.

31. **(a)** $f(x) = \sqrt{x}$
(b) $g(x) = \sqrt{x} - 6$
(c) $h(x) = \sqrt{x-6}$
(d) $p(x) = \sqrt{x-6} + 3$

32. **(a)** $f(t) = 3 - t$
(b) $g(t) = 3 - \sqrt{t}$
(c) $h(t) = \sqrt{3-t}$
(d) $p(t) = \sqrt{3-t} - 7$

33. **(a)** $f(x) = x^2 - 4$
(b) $g(x) = 4 - x^2$
(c) $h(x) = x^2 + 4$
(d) $p(x) = \sqrt{x^2-4}$
(e) $q(x) = \sqrt{4-x^2}$
(f) $r(x) = \sqrt{x^2+4}$

34. **(a)** $f(t) = 25 + t^2$
(b) $g(t) = t^2 - 25$
(c) $h(t) = 25 - t^2$
(d) $p(t) = \sqrt{25+t^2}$
(e) $q(t) = \sqrt{t^2-25}$
(f) $r(t) = \sqrt{25-t^2}$

35. **(a)** $f(t) = |t|$
(b) $g(t) = |9-t|$
(c) $h(t) = 9 - |t|$

36. **(a)** $f(x) = |x|$
(b) $g(x) = |x| + 1$
(c) $h(x) = |x+1|$

37. **(a)** $f(x) = |x^2+3|$
(b) $g(x) = |x^2+3| - 4$
(c) $h(x) = 2|x^2+3| + 5$

38. **(a)** $f(t) = |t^2+6|$
(b) $g(t) = |t^2+6| + 7$
(c) $h(t) = -\frac{1}{3}|t^2+6| - 8$

39. $g(t) = |2x-7| + 5$

40. $h(t) = 6 - |t-1|$

41. $f(x) = \sqrt[3]{5x+6} - 4$

42. $g(x) = \sqrt[4]{8-3x} + 2$

Find the domain and range of each of the following functions. Express answers in interval notation. ***(Hint: When finding the range, first solve for x.)***

43. **(a)** $f(x) = \dfrac{3}{x+2}$
(b) $g(x) = \dfrac{x+5}{x-2}$

44. **(a)** $f(x) = \dfrac{4}{x-3}$
(b) $g(x) = \dfrac{5x-2}{x-3}$

Exercise Set 1.1: An Introduction to Functions

Evaluate the following.

45. If $f(x) = 5x - 4$, find:

$f(3),\ f\left(-\frac{1}{2}\right),\ f(a),\ f(a+3),\ f(a)+3,\ f(a)+f(3)$

46. If $f(x) = 3x + 1$, find:

$f(5),\ f(-8),\ f\left(-\frac{4}{7}\right),\ f(t)-2,\ f(t-2),\ f(t)-f(2)$

47. If $g(x) = x^2 - 3x + 4$, find:

$g(0),\ g\left(-\frac{1}{4}\right),\ g(x+5),\ g\left(\frac{1}{a}\right),\ g(3a),\ 3g(a)$

48. If $h(t) = t^2 + 2t - 5$, find:

$h(1),\ h\left(\frac{3}{2}\right),\ h(c+6),\ -h(x),\ h(2x),\ 2h(x)$

49. If $f(x) = \dfrac{2+x}{x-3}$, find:

$f(-7),\ f(0),\ f\left(\frac{3}{5}\right),\ f(t),\ f(t^2-3)$

50. If $f(x) = \dfrac{x^2}{x+4} - x$, find:

$f(2),\ f(-5),\ f\left(-\frac{7}{4}\right),\ f(3p),\ f(p^3)+2$

51. If $f(x) = \begin{cases} 2x-5, & \text{if } x \ge 4 \\ 3-x^2, & \text{if } x < 4 \end{cases}$, find:

$f(6),\ f(2),\ f(-3),\ f(0),\ f(4),\ f\left(\frac{9}{2}\right)$

52. If $f(x) = \begin{cases} x^2+4x, & \text{if } x < -2 \\ 7-2x, & \text{if } x \ge -2 \end{cases}$, find:

$f(-5),\ f(3),\ f(0),\ f(1),\ f(-2),\ f\left(-\frac{10}{3}\right)$

53. If $f(x) = \begin{cases} 3x^2, & \text{if } x < 0 \\ 4, & \text{if } 0 \le x < 2 \\ x+5, & \text{if } x \ge 2 \end{cases}$, find:

$f(0),\ f(-6),\ f(2),\ f(1),\ f(4),\ f\left(\frac{3}{2}\right)$

54. If $f(x) = \begin{cases} -4x-7, & \text{if } x \le -1 \\ x^2+6, & \text{if } -1 < x < 3 \\ -7, & \text{if } x \ge 3 \end{cases}$, find:

$f(0),\ f(-4),\ f(-1),\ f(3),\ f(6),\ f\left(-\frac{5}{3}\right)$

Determine whether each of the following equations defines *y* as a function of *x*. (Do not graph.)

55. $3x - 5y = 8$

56. $x + 3 = y^2$

57. $x^2 + y = 3$

58. $3x^4 - 2xy = 5x$

59. $7x - y^4 = 5$

60. $3x^2 + y^2 = 16$

61. $x^3y - 2y = 6$

62. $\sqrt{x} - 3y = 5$

63. $x^3 = 5y$

64. $y^3 = -7x$

65. $|y| - 2 = x$

66. $|x| + 3y = 4$

Exercise Set 1.1: An Introduction to Functions

For each of the following problems:

(a) Find $f(x+h)$.

(b) Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$.

(Assume that $h \neq 0$.)

67. $f(x) = 7x - 4$

68. $f(x) = 5 - 3x$

69. $f(x) = x^2 + 5x - 2$

70. $f(x) = x^2 - 3x + 8$

71. $f(x) = -8$

72. $f(x) = 6$

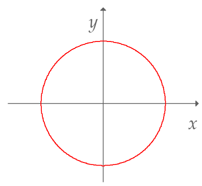
73. $f(x) = \dfrac{1}{x}$

74. $f(x) = \dfrac{1}{x-3}$

Exercise Set 1.2: Functions and Graphs

Determine whether or not each of the following graphs represents a function.

1.



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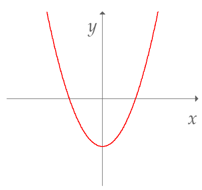

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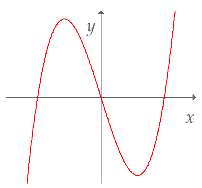

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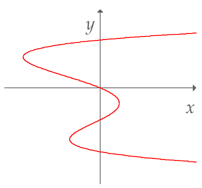

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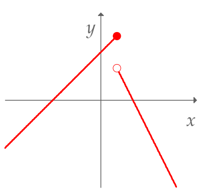

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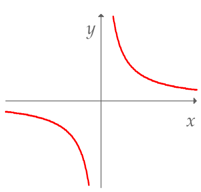

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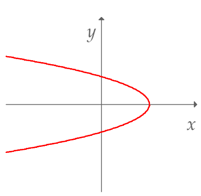

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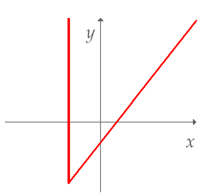

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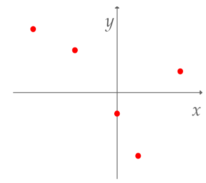

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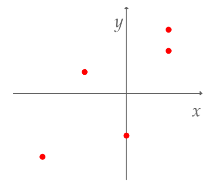

For each set of points,

(a) **Graph the set of points.**

(b) **Determine whether or not the set of points represents a function. Justify your answer.**

11. $\{(1, 5),\ (2, 4),\ (-3, 4),\ (2, -1),\ (3, 6)\}$

12. $\{(-3, 2),\ (1, 2),\ (0, -3),\ (2, 1),\ (-2, 1)\}$

Answer the following.

13. Analyze the coordinates in each of the sets above. Describe a method of determining whether or not the set of points represents a function <u>without</u> graphing the points.

14. Determine whether or not each set of points represents a function <u>without</u> graphing the points. Justify each answer.

(a) $\{(-7, 3),\ (3, -7),\ (1, 5),\ (5, 1),\ (-2, 1)\}$

(b) $\{(6, 3),\ (-4, 3),\ (2, 3),\ (-3, 3),\ (5, 3)\}$

(c) $\{(3, 6),\ (3, -4),\ (3, 2),\ (3, -3),\ (3, 5)\}$

(d) $\{(-2, -5),\ (-5, 2),\ (2, 5),\ (5, -2),\ (5, 2)\}$

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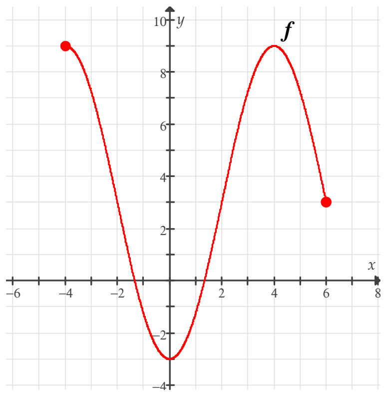
Answer the following.

15. The graph of $y = f(x)$ is shown below.



(a) Find the domain of the function. Write your answer in interval notation.

(b) Find the range of the function. Write your answer in interval notation.

(c) Find the *y*-intercept(s) of the function.

(d) Find the following function values:
$f(-2)$; $f(0)$; $f(4)$; $f(6)$

(e) For what value(s) of *x* is $f(x) = 9$?

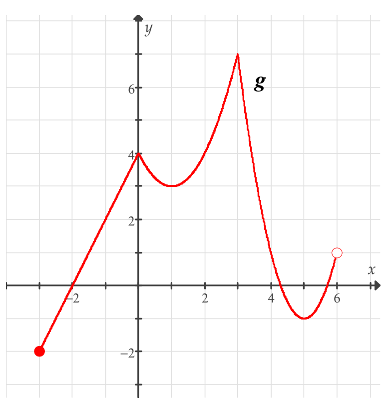
(f) On what interval(s) is *f* increasing?

(g) On what interval(s) is *f* decreasing?

(h) What is the maximum value of the function?

(i) What is the minimum value of the function?

16. The graph of $y = g(x)$ is shown below.



(a) Find the domain of the function. Write your answer in interval notation.

(b) Find the range of the function. Write your answer in interval notation.

(c) Find the *y*-intercept(s) of the function.

(d) Find the following function values:
$g(-2)$; $g(0)$; $g(1)$; $g(3)$; $g(6)$

(e) For what value(s) of *x* is $g(x) = -2$?

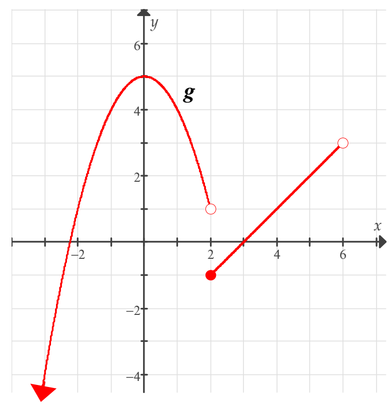
(f) On what interval(s) is *g* increasing?

(g) On what interval(s) is *g* decreasing?

(h) What is the maximum value of the function?

(i) What is the minimum value of the function?

17. The graph of $y = g(x)$ is shown below.



(a) Find the domain of the function. Write your answer in interval notation.

(b) Find the range of the function. Write your answer in interval notation.

(c) How many *x*-intercept(s) does the function have?

(d) Find the following function values:
$g(-2)$; $g(0)$; $g(2)$; $g(4)$; $g(6)$

(e) Which is greater, $g(-2)$ or $g(3)$?

(f) On what interval(s) is *g* increasing?

(g) On what interval(s) is *g* decreasing?

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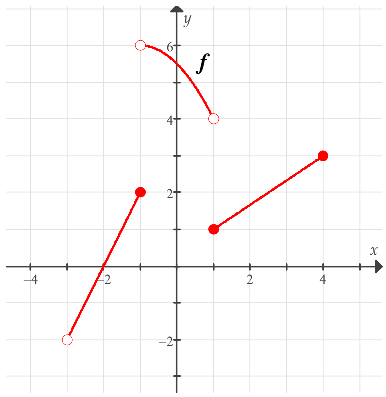
18. The graph of $y = f(x)$ is shown below.



(a) Find the domain of the function. Write your answer in interval notation.

(b) Find the range of the function. Write your answer in interval notation.

(c) Find the x-intercept(s) of the function.

(d) Find the following function values:
$f(-3)$; $f(-2)$; $f(-1)$; $f(1)$; $f(4)$

(e) Which is smaller, $f(0)$ or $f(3)$?

(f) On what interval(s) is f increasing?

(g) On what interval(s) is f decreasing?

For each of the following functions:

(c) **State the domain of the function. Write your answer in interval notation.**

(d) **Find the intercepts of the function.**

(e) **Choose x-values corresponding to the domain of the function, calculate the corresponding y-values, plot the points, and draw the graph of the function.**

19. $f(x) = -\frac{3}{2}x + 6$

20. $f(x) = \frac{2}{3}x - 4$

21. $h(x) = 3x - 5, \ -1 \le x < 3$

22. $h(x) = -2x, \ -3 < x \le 2$

23. $g(x) = |x - 3|$

24. $g(x) = |x| - 4$

25. $f(x) = \sqrt{x-3}$

26. $f(x) = \sqrt{5-x}$

27. $F(x) = x^2 - 4x$

28. $G(x) = (x-3)^2 + 1$

29. $f(x) = x^3 + 1$

30. $f(x) = x^4 - 16$

31. $g(x) = \dfrac{12}{x}$

32. $h(x) = -\dfrac{8}{x}$

For each of the following piecewise-defined functions:

(a) **State the domain of the function. Write your answer in interval notation.**

(b) **Find the y-intercept of the function.**

(c) **Choose x-values corresponding to the domain of the function, calculate the corresponding y-values, plot the points, and draw the graph of the function.**

33. $f(x) = \begin{cases} 2x + 4, & \text{if } -2 \le x < 1 \\ -x + 3, & \text{if } 1 \le x \le 5 \end{cases}$

34. $f(x) = \begin{cases} \frac{1}{3}x + 2, & \text{if } -3 \le x \le 0 \\ -4x + 3, & \text{if } x > 0 \end{cases}$

35. $f(x) = \begin{cases} 3, & \text{if } x < -2 \\ -5, & \text{if } x \ge -2 \end{cases}$

36. $f(x) = \begin{cases} -4, & \text{if } -5 \le x < 1 \\ 2, & \text{if } 1 \le x \le 3 \end{cases}$

37. $f(x) = \begin{cases} 4, & \text{if } x < 0 \\ x^2 + 1, & \text{if } x \ge 0 \end{cases}$

38. $f(x) = \begin{cases} 3 - x^2, & \text{if } x \le 1 \\ -3, & \text{if } x > 1 \end{cases}$

39. $f(x)=\begin{cases} x, \text{ if } x\le -3 \\ x^2, \text{ if } -3<x<2 \\ 4, \text{ if } x\ge 2 \end{cases}$

40. $f(x)=\begin{cases} x^2-5, \text{ if } x<0 \\ 1, \text{ if } 0\le x\le 3 \\ 2-x, \text{ if } x>3 \end{cases}$

Answer the following.

41. **(a)** If a function is odd, then it is symmetric with respect to the __________. (x-axis, y-axis, or origin?)

(b) If a function is even, then it is symmetric with respect to the __________. (x-axis, y-axis, or origin?)

42. **(a)** If a function is symmetric with respect to the y-axis, then the function is __________. (Odd, even, both, or neither?)

(b) If a function is symmetric with respect to the origin, then the function is __________. (Odd, even, both or neither?)

43. Suppose that $y=f(x)$ is an odd function and that $(-3,6)$ is a point on the graph of *f*. Find another point on the graph.

44. Suppose that $y=f(x)$ is an even function and that $(2,-7)$ is a point on the graph of *f*. Find another point on the graph.

Determine whether each of the following functions is even, odd, both or neither.

45. $f(x)=x^3-5x$

46. $f(x)=x^2+3x$

47. $f(x)=x^4+2x^2$

48. $f(x)=x^5+2x^3$

49. $f(x)=2x^3+x^2-5x+1$

50. $f(x)=3x^6+\dfrac{2}{x^2}$

Exercise Set 1.3: Transformations of Graphs

Suppose that a student looks at a transformation of $y = f(x)$ and breaks it into the following steps. State the transformation that occurs in each step below.

1. $y = 2f(-x-3)+4$

(a) From: $y = f(x)$ To: $y = f(x-3)$

(b) From: $y = f(x-3)$ To: $y = f(-x-3)$

(c) From: $y = f(-x-3)$ To: $y = 2f(-x-3)$

(d) From: $y = 2f(-x-3)$
To: $y = 2f(-x-3)+4$

2. $y = -\frac{1}{4}f(1-x)-5$

First notice that $(1-x)$ *is equivalent to* $(-x+1)$

(a) From: $y = f(x)$
To: $y = f(x+1)$

(b) From: $y = f(x+1)$
To: $y = f(-x+1) = f(1-x)$

(c) From: $y = f(1-x)$
To: $y = \frac{1}{4}f(1-x)$

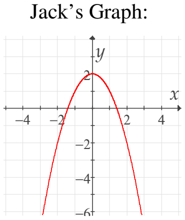
(d) From: $y = \frac{1}{4}f(1-x)$
To: $y = -\frac{1}{4}f(1-x)$

(e) From: $y = -\frac{1}{4}f(1-x)$
To: $y = -\frac{1}{4}f(1-x)-5$

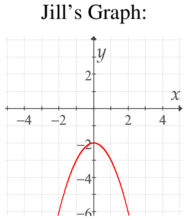
Answer the following.

3. Jack and Jill are graphing the function $f(x) = 2-x^2$. Starting with the graph of $y = x^2$, Jack first reflects the graph in the x-axis and then shifts upward two units. Jill, on the other hand, first shifts the graph $y = x^2$ upward two units and then reflects in the x-axis. Their graphs are shown below.

Jack's Graph:



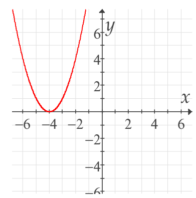
Jill's Graph:



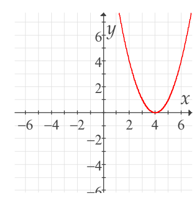
(a) Who is correct, Jack or Jill?
(b) Analyze the two methods and explain the algebraic difference between the two. Use this analysis to justify your answer in part (a).

4. Fred and Wilma are graphing the function $f(x) = (-x+4)^2$. Starting with the graph of $y = x^2$, Fred first reflects the graph in the y-axis and then shifts four units to the left. Wilma, on the other hand, first shifts the graph $y = x^2$ four units to the left and then reflects in the y-axis. Their graphs are shown below.

Fred's Graph:



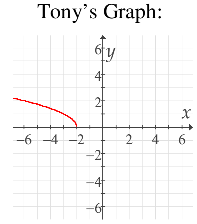
Wilma's Graph:



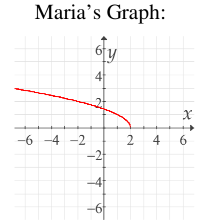
(a) Who is correct, Fred or Wilma?
(b) Analyze the two methods and explain the algebraic difference between the two. Use this analysis to justify your answer in part (a).

5. Tony and Maria are graphing the function $f(x) = \sqrt{-2-x}$. Starting with the graph of $y = \sqrt{x}$, Tony first shifts the graph two units to the right and then reflects in the y-axis. Maria, on the other hand, first reflects the graph $y = \sqrt{x}$ in the y-axis and then shifts two units to the right. Their graphs are shown below.

Tony's Graph:



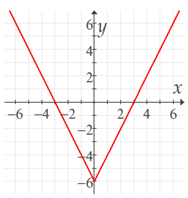
Maria's Graph:



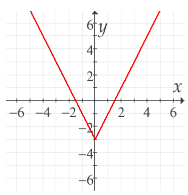
(a) Who is correct, Tony or Maria?
(b) Analyze the two methods and explain the algebraic difference between the two. Use this analysis to justify your answer in part (a).

6. Bart and Lisa are graphing the function $f(x) = 2|x| - 3$. Starting with the graph of $y = |x|$, Bart first shifts the graph downward three units and then stretches the graph vertically by a factor of 2. Lisa, on the other hand, first stretches the graph $y = |x|$ vertically by a factor of 2 and then shifts the graph downward three units.

Bart's Graph:



Lisa's Graph:



(a) Who is correct, Bart or Lisa?
(b) Analyze the two methods and explain the algebraic difference between the two. Use this analysis to justify your answer in part (a).

Matching. The left-hand column contains equations that represent transformations of $f(x) = x^2$. Match the equations on the left with the description on the right of how to obtain the graph of g from the graph of f.

7. $g(x) = (x-4)^2$

8. $g(x) = x^2 - 4$

9. $g(x) = x^2 + 4$

10. $g(x) = (x+4)^2$

11. $g(x) = -x^2$

12. $g(x) = (-x)^2$

13. $g(x) = 4x^2$

14. $g(x) = \frac{1}{4}x^2$

15. $g(x) = -x^2 - 4$

16. $g(x) = (x+4)^2 + 3$

17. $g(x) = -(x-3)^2 + 4$

18. $g(x) = (-x+4)^2$

A. Reflect in the x-axis.

B. Shift left 4 units, then reflect in the y-axis.

C. Reflect in the x-axis, then shift downward 4 units.

D. Shift right 4 units.

E. Shift right 3 units, then reflect in the x-axis, then shift upward 4 units.

F. Shift upward 4 units.

G. Reflect in the y-axis.

H. Shift left 4 units, then shift upward 3 units.

I. Shift left 4 units.

J. Shift downward 4 units.

K. Stretch vertically by a factor of 4.

L. Shrink vertically by a factor of $\frac{1}{4}$.

Write the equation that results when the following transformations are applied to the given standard function. Then state if any of the resulting functions in (a)-(e) are equivalent.

19. Standard function: $y = x^3$

(a) Shift right 7 units, then reflect in the x-axis, then stretch vertically by a factor of 5, then shift upward 1 unit.

(b) Reflect in the x-axis, then shift right 7 units, then stretch vertically by a factor of 5, then shift upward 1 unit.

(c) Stretch vertically by a factor of 5, then shift upward 1 unit, then shift right 7 units, then reflect in the x-axis.

(d) Shift right 7 units, then shift upward 1 unit, then reflect in the x-axis, then stretch vertically by a factor of 5.

(e) Reflect in the x-axis, then shift left 7 units, then stretch vertically by a factor of 5, then shift upward 1 unit.

(f) Which, if any, of the resulting functions in (a)-(e) are equivalent?

20. Standard function: $y = \sqrt{x}$

(a) Reflect in the y-axis, then shift left 2 units, then shift downward 4 units, then reflect in the x-axis

(b) Shift left 2 units, then reflect in the y-axis, then reflect in the x-axis, then shift downward 4 units.

(c) Reflect in the y-axis, then reflect in the x-axis, then shift downward 4 units, then shift right 2 units.

(d) Reflect in the x-axis, then shift left 2 units, then shift downward 4 units, then reflect in the y-axis.

(e) Shift downward 4 units, then shift left 2 units, then reflect in the y-axis, then reflect in the x-axis.

(f) Which, if any, of the resulting functions in (a)-(e) are equivalent?

Continued on the next page…

Exercise Set 1.3: Transformations of Graphs

Describe how the graph of *g* is obtained from the graph of *f*. (Do not sketch the graph.)

21. $f(x)=\sqrt{x}$, $\quad g(x)=\sqrt{-x}-2$

22. $f(x)=x^3$, $\quad g(x)=-2(x+5)^3$

23. $f(x)=|x|$, $\quad g(x)=-5|x-2|+1$

24. $f(x)=x^2$, $\quad g(x)=\frac{1}{6}(x+3)^2-7$

25. $f(x)=\dfrac{1}{x}$, $\quad g(x)=\dfrac{3}{x+8}+2$

26. $f(x)=\sqrt[3]{x}$, $\quad g(x)=\sqrt[3]{-x}+4$

Describe how the graphs of each of the following functions can be obtained from the graph of *y = f(x)*.

27. $y=f(x)+1$

28. $y=f(x-7)$

29. $y=f(-x)+3$

30. $y=-f(x+3)-8$

31. $y=-\frac{1}{4}f(x-2)-5$

32. $y=-5f(-x)+1$

33. $y=f(7-x)+2$

34. $y=f(-x-5)-7$

Sketch the graph of each of the following functions. Do not plot points, but instead apply transformations to the graph of a standard function.

35. $f(x)=x-3$

36. $f(x)=5-x$

37. $f(x)=-3x+1$

38. $f(x)=2x-7$

39. $f(x)=x^2+3$

40. $f(x)=(x-5)^2$

41. $f(x)=6-x^2$

42. $f(x)=2-(x-1)^2$

43. $f(x)=-3(x-4)^2-2$

44. $f(x)=(x+5)^2+3$

45. $f(x)=6-\sqrt{x+2}$

46. $f(x)=\frac{1}{2}\sqrt{-x}+1$

47. $f(x)=\sqrt{-x+4}+2$

48. $f(x)=\sqrt{5-x}-1$

49. $f(x)=2|x+5|-3$

50. $f(x)=-|x-2|+4$

51. $f(x)=-(x-4)^3+1$

52. $f(x)=-x^3-5$

53. $f(x)=\dfrac{1}{x-3}+6$

54. $f(x)=-\dfrac{2}{x+4}$

55. $f(x)=-\dfrac{4}{x}+3$

56. $f(x)=\sqrt[3]{x-6}$

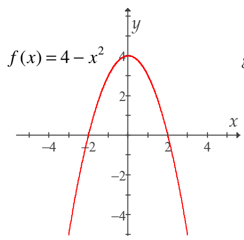
57. $f(x)=\sqrt[3]{-x}+2$

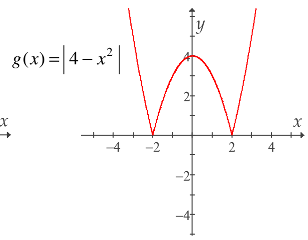
58. $f(x)=-\sqrt[3]{x+1}-5$

Exercise Set 1.3: Transformations of Graphs

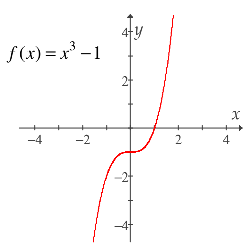
Answer the following.

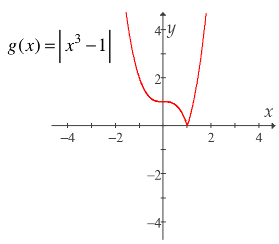
59. The graphs of $f(x)=4-x^2$ and $g(x)=\left|4-x^2\right|$ are shown below. Describe how the graph of g was obtained from the graph of *f*.





60. The graphs of $f(x)=x^3-1$ and $g(x)=\left|x^3-1\right|$ are shown below. Describe how the graph of g was obtained from the graph of *f*.





Sketch the graphs of the following functions:

61. (a) $f(x)=x^2-9$ **(b)** $g(x)=\left|x^2-9\right|$

62. (a) $f(x)=\dfrac{1}{x}$ **(b)** $g(x)=\left|\dfrac{1}{x}\right|$

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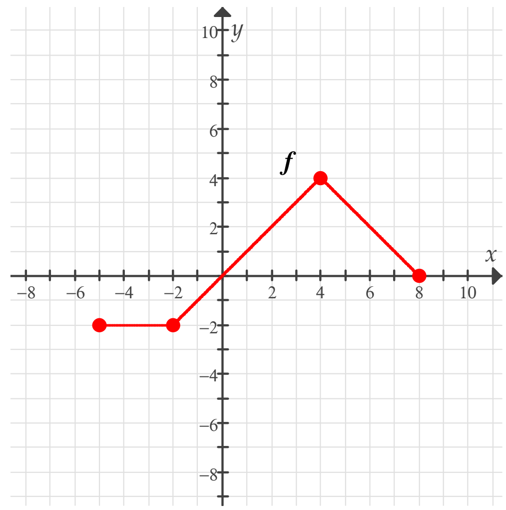
The graph of $y=f(x)$ is given below. Sketch the graph of each of the following functions.



63. $y=f(x+2)$

64. $y=f(x)-3$

65. $y=f(x-2)-1$

66. $y=f(x+1)+5$

67. $y=f(-x)$

68. $y=-f(x)$

69. $y=2f(x)$

70. $y=\frac{1}{2}f(x)$

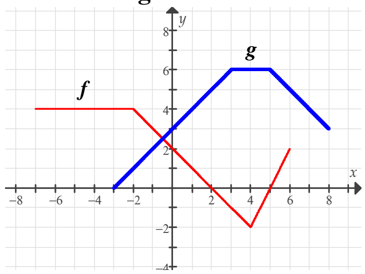
71. $y=-2f(x+1)$

72. $y=f(-x)-4$

Exercise Set 1.4: Operations on Functions

Answer the following.

1.



(a) Find $f(-3)+g(-3)$.

(b) Find $f(0)+g(0)$.

(c) Find $f(-6)+g(-6)$.

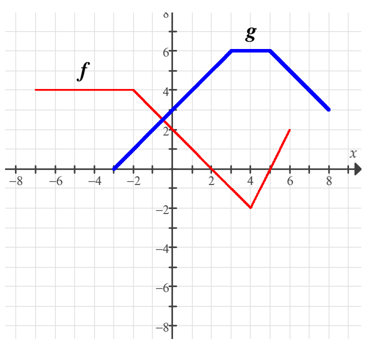
(d) Find $f(5)+g(5)$.

(e) Find $f(7)+g(7)$.

(f) Sketch the graph of $f+g$. (Hint: For any x value, add the y values of f and g.)

(g) What is the domain of $f+g$? Explain how you obtained your answer.

2.



(a) Find $f(-2)-g(-2)$.

(b) Find $f(0)-g(0)$.

(c) Find $f(-4)-g(-4)$.

(d) Find $f(2)-g(2)$.

(e) Find $f(4)-g(4)$.

(f) Sketch the graph of $f-g$. (Hint: For any x value, subtract the y values of f and g.)

(g) What is the domain of $f-g$? Explain how you obtained your answer.

For each of the following problems:

(f) **Find $f+g$ and its domain.**

(g) **Find $f-g$ and its domain.**

(h) **Find fg and its domain.**

(i) **Find $\dfrac{f}{g}$ and its domain.**

Note for (a)-(d): Do not sketch any graphs.

3. $f(x)=2x+3;\ g(x)=x^2-4x-12$

4. $f(x)=2x^3-5x;\ g(x)=x^2+8x+15$

5. $f(x)=\dfrac{3}{x-1};\ g(x)=\dfrac{x}{x+2}$

6. $f(x)=\dfrac{4}{x-5};\ g(x)=\dfrac{2x}{x-5}$

7. $f(x)=\sqrt{x-6};\ g(x)=\sqrt{10-x}$

8. $f(x)=\sqrt{2x-3};\ g(x)=\sqrt{x+4}$

9. $f(x)=\sqrt{x^2-9};\ g(x)=\sqrt{x^2+4}$

10. $f(x)=\sqrt{49-x^2};\ g(x)=x-3$

Find the domain of each of the following functions.

11. $f(x)=\dfrac{2}{x-3}+\sqrt{x-1}$

12. $h(x)=\sqrt{x+2}-\dfrac{3}{x}$

13. $g(x)=\dfrac{3}{x-7}-\dfrac{x+1}{x-2}$

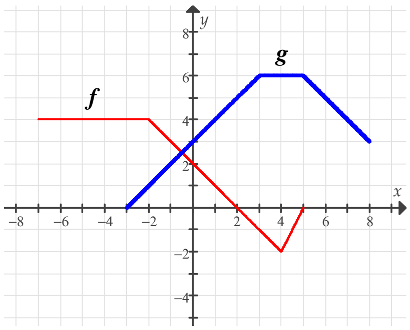
14. $f(x)=\dfrac{x-2}{x+6}+\dfrac{5}{x-1}+7$

15. $f(x)=\dfrac{\sqrt{x+2}}{x-5}$

16. $g(x)=\dfrac{x-3}{\sqrt{x-1}}$

Exercise Set 1.4: Operations on Functions

Answer the following, using the graph below.



17. **(a)** $g(2)$ **(b)** $f(g(2))$
(c) $f(2)$ **(d)** $g(f(2))$

18. **(a)** $g(0)$ **(b)** $f(g(0))$
(c) $f(0)$ **(d)** $g(f(0))$

19. **(a)** $(f \circ g)(-3)$ **(b)** $(g \circ f)(-3)$

20. **(a)** $(f \circ g)(-1)$ **(b)** $(g \circ f)(-1)$

21. **(a)** $(f \circ f)(3)$ **(b)** $(g \circ g)(-2)$

22. **(a)** $(f \circ f)(5)$ **(b)** $(g \circ g)(-3)$

23. **(a)** $(f \circ g)(4)$ **(b)** $(g \circ f)(4)$

24. **(a)** $(f \circ g)(-5)$ **(b)** $(f \circ g)(2)$

Use the functions *f* and *g* given below to evaluate the following expressions:

$f(x) = 3 - 2x$ and $g(x) = x^2 - 5x + 4$

25. **(a)** $g(0)$ **(b)** $f(g(0))$
(c) $f(0)$ **(d)** $g(f(0))$

26. **(a)** $g(-1)$ **(b)** $f(g(-1))$
(c) $f(-1)$ **(d)** $g(f(-1))$

27. **(a)** $(f \circ g)(-2)$ **(b)** $(g \circ f)(-2)$

28. **(a)** $(f \circ g)(4)$ **(b)** $(g \circ f)(4)$

29. **(a)** $(f \circ f)(6)$ **(b)** $(g \circ g)(6)$

30. **(a)** $(f \circ f)(-4)$ **(b)** $(g \circ g)(-4)$

31. **(a)** $(f \circ g)(x)$ **(b)** $(g \circ f)(x)$

32. **(a)** $(f \circ f)(x)$ **(b)** $(g \circ g)(x)$

The following method can be used to find the domain of $f \circ g$:

(a) **Find the domain of g.**
(b) **Find $f \circ g$.**
(c) **Look at the answer from part (b) as a stand-alone function (ignoring the fact that it is a composition of functions) and find its domain.**
(d) **Take the intersection of the domains found in steps (a) and (c). This is the domain of $f \circ g$.**

Note:
We check the domain of g because it is the inner function of $f \circ g$, i.e. $f(g(x))$. If an x-value is not in the domain of g, then it also can not be an input value for $f \circ g$.

Use the above steps to find the domain of $f \circ g$ for the following problems:

33. $f(x) = \dfrac{1}{x^2}$; $g(x) = \sqrt{x-5}$

34. $f(x) = \dfrac{1}{x^2}$; $g(x) = \sqrt{x+2}$

35. $f(x) = \dfrac{3}{x^2 - 4}$; $g(x) = \sqrt{x-6}$

36. $f(x) = \dfrac{5}{x^2 - 2}$; $g(x) = \sqrt{3-x}$

For each of the following problems:

(a) **Find $f \circ g$ and its domain.**
(b) **Find $g \circ f$ and its domain.**

37. $f(x) = x^2 + 3x$; $g(x) = 2x - 7$

38. $f(x) = 6x + 2$; $g(x) = 7 - x^2$

39. $f(x) = x^2$; $g(x) = \dfrac{1}{\sqrt{x-4}}$

40. $f(x) = \dfrac{3}{\sqrt{x+5}}$; $g(x) = x^2$

Exercise Set 1.4: Operations on Functions

41. $f(x)=\sqrt{x+7};\ g(x)=-5-x$

42. $f(x)=\sqrt{3-x};\ g(x)=9-2x$

Answer the following.

43. Given the functions $f(x)=x^2+2$ and $g(x)=5x-8$, find:

(a) $f(g(1))$ **(b)** $g(f(1))$
(c) $f(g(x))$ **(d)** $g(f(x))$
(e) $f(f(1))$ **(f)** $g(g(1))$
(g) $f(f(x))$ **(h)** $g(g(x))$

44. Given the functions $f(x)=x+1$ and $g(x)=3x-2x^2$, find:

(a) $f(g(-3))$ **(b)** $g(f(-3))$
(c) $f(g(x))$ **(d)** $g(f(x))$
(e) $f(f(-3))$ **(f)** $g(g(-3))$
(g) $f(f(x))$ **(h)** $g(g(x))$

45. Given the functions $f(x)=\dfrac{x+1}{x-2}$ and $g(x)=\dfrac{3}{x-5}$, find:

(a) $f(g(-2))$ **(b)** $g(f(-2))$
(c) $f(g(x))$ **(d)** $g(f(x))$

46. Given the functions $f(x)=\dfrac{2x}{x+5}$ and $g(x)=\dfrac{7-x}{x-1}$, find:

(a) $f(g(3))$ **(b)** $g(f(3))$
(c) $f(g(x))$ **(d)** $g(f(x))$

47. Given the functions
$f(x)=x^2-1,\ g(x)=3x-5,$ and $h(x)=1-2x,$ find:

(a) $f(g(h(2)))$ **(b)** $g(h(f(3)))$
(c) $f\circ g\circ h$ **(d)** $g\circ h\circ f$

48. Given the functions
$f(x)=2x^2+3,\ g(x)=x+4,$ and $h(x)=3x-2,$ find:

(a) $f(g(h(1)))$ **(b)** $g(h(f(1)))$
(c) $f\circ g\circ h$ **(d)** $g\circ h\circ f$

49. Given the functions
$f(x)=x^2+4,\ g(x)=\sqrt{x+3},$ and $h(x)=2x+1,$ find:

(a) $h(f(g(4)))$ **(b)** $f(g(h(0)))$
(c) $f\circ g\circ h$ **(d)** $h\circ f\circ g$

50. Given the functions
$f(x)=\dfrac{1}{x^2},\ g(x)=\sqrt{x-2},$ and $h(x)=3-4x,$ find:

(a) $h(f(g(5)))$ **(b)** $f(g(h(-2)))$
(c) $f\circ g\circ h$ **(d)** $h\circ f\circ g$

Functions *f* and *g* are defined as shown in the table below.

x	0	1	2	4
$f(x)$	2	4	4	7
$g(x)$	4	5	0	1

Use the information above to complete the following tables. (Some answers may be undefined.)

51.

x	0	1	2	4
$f(g(x))$				

52.

x	0	1	2	4
$g(f(x))$				

53.

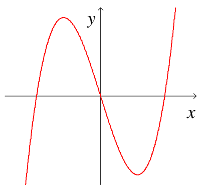
x	0	1	2	4
$f(f(x))$				

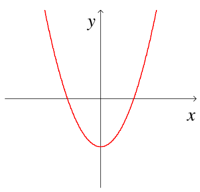
54.

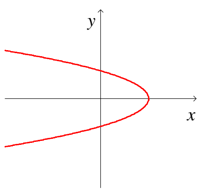
x	0	1	2	4
$g(g(x))$				

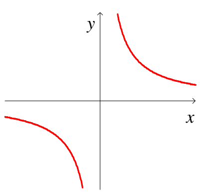
Exercise Set 1.5: Inverse Functions

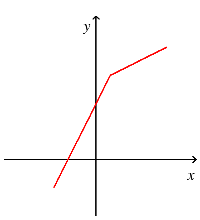
Determine whether each of the following graphs represents a one-to-one function. Explain your answer.

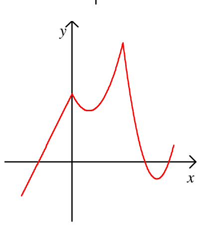
1. 

2. 

3. 

4. 

5. 

6. 

For each of the following functions, sketch a graph and then determine whether the function is one-to-one.

7. $f(x)=2x-3$

8. $g(x)=x^2+5$

9. $h(x)=(x-2)^3$

10. $f(x)=x^3-2$

11. $g(x)=|x|+4$

12. $h(x)=\frac{1}{x}-3$

13. $f(x)=-(x-2)^2+1$

14. $g(x)=|x-6|$

Answer the following.

15. If a function f is one-to-one, then the inverse function, f^{-1}, can be graphed by either of the following methods:

 (a) Interchange the ____ and ____ values.

 (b) Reflect the graph of f over the line $y=$ ____.

16. The domain of f is equal to the __________ of f^{-1}, and the range of f is equal to the __________ of f^{-1}.

A table of values for a one-to-one function $y=f(x)$ is given. Complete the table for $y=f^{-1}(x)$.

17.

x	$f(x)$
3	–4
2	7
–4	5
5	0
0	3

x	$f^{-1}(x)$
–4	
	2
5	
0	

18.

x	$f(x)$
5	9
4	5
6	–3
–8	2
2	6

x	$f^{-1}(x)$
	5
5	
	6
	–8

Exercise Set 1.5: Inverse Functions

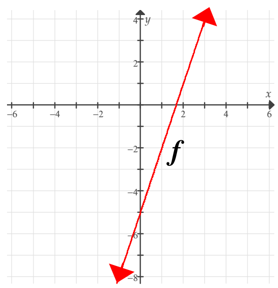
For each of the following graphs:

(a) State the domain and range of f .

(b) Sketch f^{-1} .

(c) State the domain and range of f^{-1} .

19.



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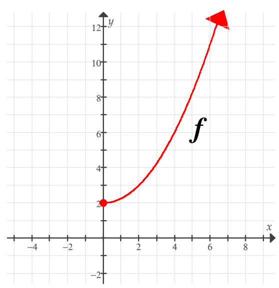

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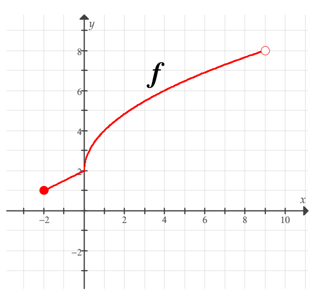

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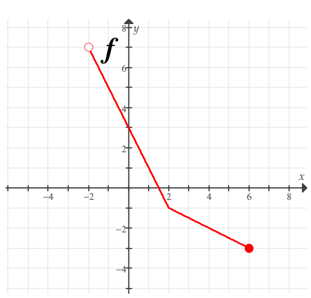


Answer the following. Assume that f is a one-to-one function.

23. If $f(4)=5$, find $f^{-1}(5)$.

24. If $f(6)=-2$, find $f^{-1}(-2)$.

25. If $f^{-1}(-3)=7$, find $f(7)$.

26. If $f^{-1}(6)=-8$, find $f(-8)$.

27. If $f(3)=9$ and $f(9)=5$, find $f^{-1}(9)$.

28. If $f(5)=-4$ and $f(2)=5$, find $f^{-1}(5)$.

29. If $f(-4)=2$, find $f\left(f^{-1}(2)\right)$.

30. If $f^{-1}(-5)=3$, find $f^{-1}\left(f(3)\right)$.

Answer the following. Assume that f and g are defined for all real numbers.

31. If f and g are inverse functions, $f(-2)=3$ and $f(4)=-2$, find $g(-2)$.

32. If f and g are inverse functions, $f(7)=10$ and $f(10)=-1$, find $g(10)$.

33. If f and g are inverse functions, $f(5)=8$ and $f(9)=3$, find $g\left(f(3)\right)$.

34. If f and g are inverse functions, $f(-1)=6$ and $f(7)=8$, find $f\left(g(6)\right)$.

For each of the following functions, write an equation for the inverse function $y=f^{-1}(x)$.

35. $f(x)=5x-3$

36. $f(x)=-4x+7$

37. $f(x)=\dfrac{3-2x}{8}$

38. $f(x)=\dfrac{6x-5}{4}$

Exercise Set 1.5: Inverse Functions

39. $f(x) = x^2 + 1$, where $x \geq 0$

40. $f(x) = 5 - x^2$, where $x \geq 0$

41. $f(x) = 4x^3 - 7$

42. $f(x) = 2x^3 + 1$

43. $f(x) = \dfrac{3}{x+2}$

44. $f(x) = \dfrac{5}{7-x}$

45. $f(x) = \dfrac{2x+3}{x-4}$

46. $f(x) = \dfrac{3-8x}{x+5}$

47. $f(x) = \sqrt{7-2x}$

48. $f(x) = 2 + \sqrt{6x+5}$

Use the Property of Inverse Functions to determine whether each of the following pairs of functions are inverses of each other. Explain your answer.

49. $f(x) = 4x - 1$; $\quad g(x) = \frac{1}{4}x + 1$

50. $f(x) = 2 + 3x$; $\quad g(x) = \dfrac{x-2}{3}$

51. $f(x) = \dfrac{4-x}{5}$; $\quad g(x) = 4 - 5x$

52. $f(x) = 2x + 5$; $\quad g(x) = \dfrac{1}{2x+5}$

53. $f(x) = x^3 - 2$; $\quad g(x) = \sqrt[3]{x+2}$

54. $f(x) = \sqrt[5]{x} - 7$; $\quad g(x) = (x+7)^5$

55. $f(x) = \dfrac{5}{x}$; $\quad g(x) = \dfrac{5}{x}$

56. $f(x) = x^2 + 9$, where $x \geq 0$;
$g(x) = \sqrt{x-9}$

Answer the following.

57. If $f(x)$ is a function that represents the amount of revenue (in dollars) by selling x tickets, then what does $f^{-1}(500)$ represent?

58. If $f(x)$ is a function that represents the area of a circle with radius x, then what does $f^{-1}(80)$ represent?

A function is said to be one-to-one provided that the following holds for all x_1 and x_2 in the domain of f:

If $f(x_1) = f(x_2)$, then $x_1 = x_2$.

Use the above definition to determine whether or not the following functions are one-to-one. If f is not one-to-one, then give a specific example showing that the condition $f(x_1) = f(x_2)$ fails to imply that $x_1 = x_2$.

59. $f(x) = 5x - 3$

60. $f(x) = x^3 + 5$

61. $f(x) = \sqrt{x} - 4$

62. $f(x) = |x| - 4$

63. $f(x) = |x - 4|$

64. $f(x) = \dfrac{1}{x} + 4$

65. $f(x) = x^2 + 3$

66. $f(x) = (x+3)^2$

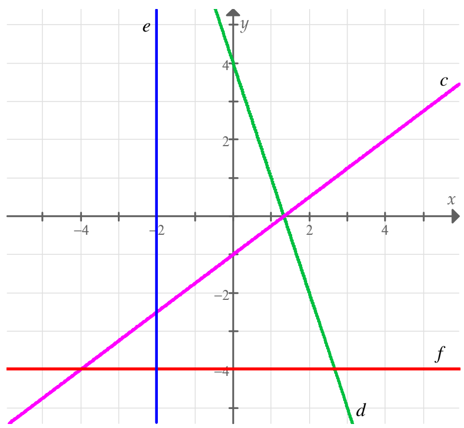
Exercise Set 2.1: Linear and Quadratic Functions

Find the slope of the line that passes through the following points. If it is undefined, state 'Undefined.'

1. $(-2, 3)$ and $(6, -7)$
2. $(-1, -6)$ and $(-5, 10)$
3. $(8, -7)$ and $(-1, -7)$
4. $(3, -8)$ and $(3, -4)$

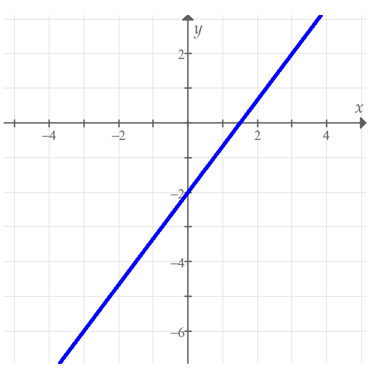
Find the slope of each of the following lines.

5. c
6. d
7. e
8. f



Find the linear function f which corresponds to each graph shown below.

9.



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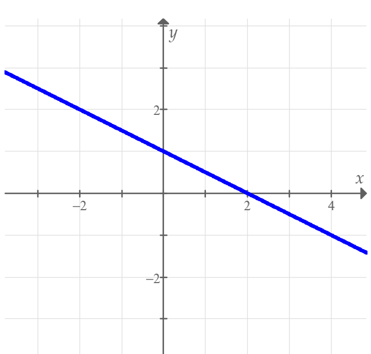


For each of the following equations,

(a) Write the equation in slope-intercept form.
(b) Write the equation as a linear function.
(c) Identify the slope.
(d) Identify the *y*-intercept.
(e) Draw the graph.

11. $2x + y = 5$
12. $3x - y = -6$
13. $x + 4y = 0$
14. $2x + 5y = 10$
15. $4x - 3y + 9 = 0$
16. $-\frac{2}{3}x + \frac{1}{2}y = -1$

Find the linear function f that satisfies the given conditions.

17. Slope $-\dfrac{4}{7}$; y-intercept 3
18. Slope -4; y-intercept 5
19. Slope $-\dfrac{2}{9}$; passes through (-3, 2)
20. Slope $\dfrac{1}{5}$; passes through (-4, -2)
21. Passes through (2, 11) and (-3, 1)
22. Passes through (-4, 5) and (1, -2)
23. x-intercept 7; y-intercept -5
24. x-intercept -2; y-intercept 6
25. Slope $-\dfrac{3}{2}$; x-intercept 4
26. Slope $\dfrac{1}{3}$; x-intercept -6
27. Passes through (-3, 5); parallel to the line $y = -1$
28. Passes through (2, -6); parallel to the line $y = 4$
29. Passes through (5, -7); parallel to the line $y = -5x + 3$

30. Passes through (5, -7); perpendicular to the line $y = -5x + 3$

31. Passes through (2, 3); parallel to the line $5x - 2y = 6$

32. Passes through (-1, 5); parallel to the line $4x + 3y = 8$

33. Passes through (2, 3); perpendicular to the line $5x - 2y = 6$

34. Passes through (-1, 5); perpendicular to the line $4x + 3y = 8$

35. Passes through (4, -6); parallel to the line containing (3, -5) and (2, 1)

36. Passes through $(8, 3)$; parallel to the line containing $(-2, -3)$ and $(-4, 6)$

37. Perpendicular to the line containing (4, -2) and (10, 4); passes through the midpoint of the line segment connecting these points.

38. Perpendicular to the line containing $(-3, 5)$ and $(7, -1)$; passes through the midpoint of the line segment connecting these points.

39. f passes through $(-3, -6)$ and f^{-1} passes through $(-8, -9)$.

40. f passes through $(2, -1)$ and f^{-1} passes through $(9, 4)$.

41. The x-intercept for f is 3 and the x-intercept for f^{-1} is -8.

42. The y-intercept for f is 4 and the y-intercept for f^{-1} is -6.

Answer the following, assuming that each situation can be modeled by a linear function.

43. If a company can make 21 computers for \$23,000, and can make 40 computers for \$38,200, write an equation that represents the cost of x computers.

44. A certain electrician charges a \$40 traveling fee, and then charges \$55 per hour of labor. Write an equation that represents the cost of a job that takes x hours.

For each of the quadratic functions given below:

(a) Complete the square to write the equation in the standard form $f(x) = a(x - h)^2 + k$.

(b) State the coordinates of the vertex of the parabola.

(c) Sketch the graph of the parabola.

(d) State the maximum or minimum value of the function, and state whether it is a maximum or a minimum.

(e) Find the axis of symmetry. (Be sure to write your answer as an equation of a line.)

45. $f(x) = x^2 + 6x + 7$

46. $f(x) = x^2 - 8x + 21$

47. $f(x) = x^2 - 2x$

48. $f(x) = x^2 + 10x$

49. $f(x) = 2x^2 - 8x + 11$

50. $f(x) = 3x^2 + 18x + 15$

51. $f(x) = -x^2 - 8x - 9$

52. $f(x) = -x^2 + 4x - 7$

53. $f(x) = -4x^2 + 24x - 27$

54. $f(x) = -2x^2 - 8x - 14$

55. $f(x) = x^2 - 5x + 3$

56. $f(x) = x^2 + 7x - 1$

57. $f(x) = 2 - 3x - 4x^2$

58. $f(x) = 7 - x - 3x^2$

Exercise Set 2.1: Linear and Quadratic Functions

Each of the quadratic functions below is written in the form $f(x) = ax^2 + bx + c$. For each function:

(a) **Find the vertex (h, k) of the parabola by using the formulas $h = -\frac{b}{2a}$ and $k = f\left(-\frac{b}{2a}\right)$.**
(Note: When only the vertex is needed, this method can be used instead of completing the square.)

(b) **State the maximum or minimum value of the function, and state whether it is a maximum or a minimum.**

59. $f(x) = x^2 - 12x + 50$

60. $f(x) = -x^2 + 14x - 10$

61. $f(x) = -2x^2 + 16x - 9$

62. $f(x) = 3x^2 - 12x + 29$

63. $f(x) = -2x^2 + 9x + 3$

64. $f(x) = -6x^2 + x - 5$

The following method can be used to sketch a reasonably accurate graph of a parabola without plotting points. Each of the quadratic functions below is written in the form $f(x) = ax^2 + bx + c$. The graph of a quadratic function is a parabola with vertex, where $h = -\frac{b}{2a}$ and $k = f\left(-\frac{b}{2a}\right)$.

(a) **Find all x-intercept(s) of the parabola by setting $f(x) = 0$ and solving for x.**

(b) **Find the y-intercept of the parabola.**

(c) **Give the coordinates of the vertex (h, k) of the parabola, using the formulas $h = -\frac{b}{2a}$ and $k = f\left(-\frac{b}{2a}\right)$.**

(d) **State the maximum or minimum value of the function, and state whether it is a maximum or a minimum.**

(e) **Find the axis of symmetry. (Be sure to write your answer as an equation of a line.)**

(f) **Draw a graph of the parabola that includes the features from parts (a) through (d).**

65. $f(x) = x^2 - 2x - 15$

66. $f(x) = x^2 - 8x + 16$

67. $f(x) = 3x^2 + 12x - 36$

68. $f(x) = -2x^2 + 16x + 40$

69. $f(x) = -4x^2 - 8x + 5$

70. $f(x) = 4x^2 - 16x - 9$

71. $f(x) = x^2 - 6x + 3$

72. $f(x) = x^2 + 10x + 5$

73. $f(x) = x^2 - 2x + 5$

74. $f(x) = x^2 + 4$

75. $f(x) = 9 - 4x^2$

76. $f(x) = 9x^2 - 100$

For each of the following problems, find a quadratic function satisfying the given conditions.

77. Vertex $(2, -5)$; passes through $(7, 70)$

78. Vertex $(-1, -8)$; passes through $(2, 10)$

79. Vertex $(5, 7)$; passes through $(3, 4)$

80. Vertex $(-4, 3)$; passes through $(1, 13)$

Answer the following.

81. Two numbers have a sum of 10. Find the largest possible value of their product.

82. Jim is beginning to create a garden in his back yard. He has 60 feet of fence to enclose the rectangular garden, and he wants to maximize the area of the garden. Find the dimensions Jim should use for the length and width of the garden. Then state the area of the garden.

83. A rocket is fired directly upwards with a velocity of 80 ft/sec. The equation for its height, H, as a function of time, t, is given by the function

$$H(t) = -16t^2 + 80t\ .$$

(a) Find the time at which the rocket reaches its maximum height.

(b) Find the maximum height of the rocket.

84. A manufacturer has determined that their daily profit in dollars from selling x machines is given by the function

$$P(x) = -200 + 50x - 0.1x^2 .$$

Using this model, what is the maximum daily profit that the manufacturer can expect?

Exercise Set 2.2: Polynomial Functions

Answer the following.

(a) State whether or not each of the following expressions is a polynomial. (Yes or No.)

(b) If the answer to part (a) is yes, then state the degree of the polynomial.

(c) If the answer to part (a) is yes, then classify the polynomial as a monomial, binomial, trinomial, or none of these. (Polynomials of four or more terms are not generally given specific names.)

1. $4+3x^3$

2. $6x^5+3x^3+\dfrac{8}{x}$

3. $3x-5$

4. $2x^3+4x^2-7x-4$

5. $\dfrac{5x^3-6x^2+7}{x^2-4x+5}$

6. 8

7. $\frac{7}{2}x^2-\frac{5}{3}x+9$

8. $\dfrac{7}{x^3}+\dfrac{5}{x^2}-\dfrac{3}{x}-2$

9. $3^{-1}x^4-7^{-1}x+2$

10. $-9x^{1/4}+2x^{1/3}-4x^{1/2}$

11. $\left|x^2-3x+1\right|$

12. $-\frac{3}{2}x^6$

13. $\dfrac{6x^3+8x^2}{x}$

14. $-3+5x^2+6x^4-3x^9$

15. $3a^3b^4-2a^2b^2$

16. $-4x^5y^{-2}-3x^{-4}y^9$

17. $4x^5y^3+\dfrac{3}{xy^2}$

18. $\frac{2}{5}x^2y^9z+3xy-\frac{1}{4}x^3y^4z^2$

19. $-4xyz^3-\frac{2}{5}y^7-\frac{3}{7}x^4y^3z^2$

20. $-a^7+2a^3b^5+b^6-3a^2b^4$

Answer True or False.

21. **(a)** $7x-2x^3$ is a trinomial.
(b) $7x-2x^3$ is a third degree polynomial.
(c) $7x-2x^3$ is a binomial.
(d) $7x-2x^3$ is a first degree polynomial

22. **(a)** $x^2-4x+7x^3$ is a second degree polynomial.
(b) $x^2-4x+7x^3$ is a binomial.
(c) $x^2-4x+7x^3$ is a third degree polynomial.
(d) $x^2-4x+7x^3$ is a trinomial.

23. **(a)** $3x^7-2x^4y^6-3y^8$ is a tenth degree polynomial.
(b) $3x^7-2x^4y^6-3y^8$ is a binomial.
(c) $3x^7-2x^4y^6-3y^8$ is an eighth degree polynomial.
(d) $3x^7-2x^4y^6-3y^8$ is a trinomial.

24. **(a)** $-3a^4b^5$ is a fifth degree polynomial.
(b) $-3a^4b^5$ is a trinomial.
(c) $-3a^4b^5$ is a ninth degree polynomial.
(d) $-3a^4b^5$ is a monomial.

Sketch a graph of each of the following functions.

25. $P(x)=x^3$

26. $P(x)=x^4$

27. $P(x)=x^6$

28. $P(x)=x^5$

29. $P(x)=x^n$, where n is odd and $n>0$.

30. $P(x)=x^n$, where n is even and $n>0$.

Answer the following.

31. The graph of $P(x)=(x-1)(x-2)^3(x+4)^2$ has x-intercepts at $x=1$, $x=2$, and $x=-4$.

(a) At and immediately surrounding the point $x=2$, the graph resembles the graph of what familiar function? (Choose one.)

$y=x$ $\qquad$ $y=x^2$ $\qquad$ $y=x^3$

Continued on the next page…

(b) At and immediately surrounding the point $x = -4$, the graph resembles the graph of what familiar function? (Choose one.)

$y = x \qquad y = x^2 \qquad y = x^3$

(c) If $P(x)$ were to be multiplied out completely, the leading term of the polynomial would be: (Choose one; do not actually multiply out the polynomial.)

$x^3;\ -x^3;\ x^4;\ -x^4;\ x^5;\ -x^5;\ x^6;\ -x^6$

32. The graph of $Q(x) = -(x+3)^2(x-5)^3$ has x-intercepts at $x = -3$ and $x = 5$.

(a) At and immediately surrounding the point $x = -3$, the graph resembles the graph of what familiar function? (Choose one.)

$y = x \qquad y = x^2 \qquad y = x^3$

(b) At and immediately surrounding the point $x = 5$, the graph resembles the graph of what familiar function? (Choose one.)

$y = x \qquad y = x^2 \qquad y = x^3$

(c) If $P(x)$ were to be multiplied out completely, the leading term of the polynomial would be: (Choose one; do not actually multiply out the polynomial.)

$x^3;\ -x^3;\ x^4;\ -x^4;\ x^5;\ -x^5;\ x^6;\ -x^6$

Match each of the polynomial functions below with its graph. (The graphs are shown in the next column.)

33. $P(x) = (x-2)(x+1)(x+4)$

34. $Q(x) = -(x+2)(x-1)(x-4)$

35. $R(x) = -(x-2)(x+1)^2(x+4)^2$

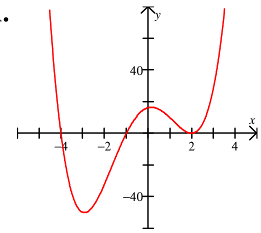
36. $S(x) = (x-2)^2(x+1)(x+4)$

37. $U(x) = (x+2)^2(x-1)^3(x-4)$

38. $V(x) = -(x+2)^3(x-1)^3(x-4)^2$

Choices for 33-38:

A.



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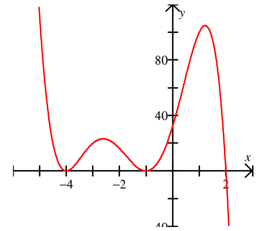

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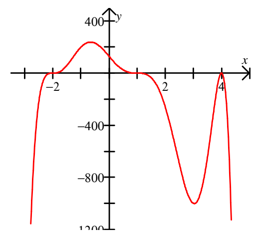

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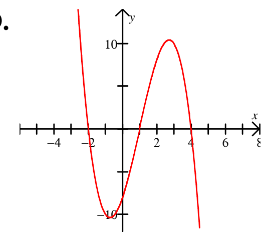

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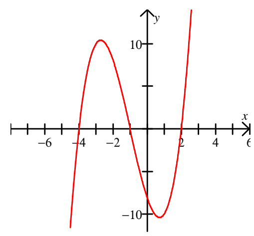

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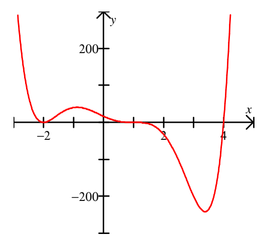


For each of the functions below:

(f) **Find the x- and y-intercepts.**

(g) **Sketch the graph of the function. Be sure to show all x- and y-intercepts, along with the proper behavior at each x-intercept, as well as the proper end behavior.**

39. $P(x) = (x-5)(x+3)$

40. $P(x) = -(x-3)(x+1)$

41. $P(x) = -(x-6)^2$

42. $P(x) = (x+3)^2$

43. $P(x) = (x-5)(x+2)(x+6)$

44. $P(x) = 3x(x-4)(x-7)$

45. $P(x) = -\frac{1}{2}(x-4)(x-1)(x+3)$

46. $P(x) = -(x+6)(x-2)(x-5)$

Exercise Set 2.2: Polynomial Functions

47. $P(x)=(x+2)^2(x-4)$

48. $P(x)=(5-x)(x+3)^2$

49. $P(x)=(3x-2)(x+4)(x-5)(x+1)$

50. $P(x)=-\frac{1}{3}(x+5)(x+1)(x+3)(x-2)$

51. $P(x)=x(x+2)(4-x)(x+6)$

52. $P(x)=(x-1)(x-3)(x+2)(x+5)$

53. $P(x)=(x-3)^2(x+4)^2$

54. $P(x)=-x(2x-5)^3$

55. $P(x)=(x+5)^3(x-4)$

56. $P(x)=x^2(x-6)^2$

57. $P(x)=(x+3)^2(x-4)^3$

58. $P(x)=-2x(3-x)^3(x+1)$

59. $P(x)=-x(x-2)^2(x+3)^2(x-4)$

60. $P(x)=(x-5)^3(x-2)^2(x+1)$

61. $P(x)=x^8(x-1)^6(x+1)^7$

62. $P(x)=-x^3(x+1)^4(x-1)^7$

63. $P(x)=x^3-6x^2+8x$

64. $P(x)=x^3-2x^2-15x$

65. $P(x)=25x-x^3$

66. $P(x)=-3x^3-5x^2+2x$

67. $P(x)=-x^4+x^3+12x^2$

68. $P(x)=x^4-16x^2$

69. $P(x)=x^5-9x^3$

70. $P(x)=-x^5-3x^4+18x^3$

71. $P(x)=x^3+4x^2-x-4$

72. $P(x)=x^3-5x^2-4x+20$

73. $P(x)=x^4-13x^2+36$

74. $P(x)=x^4-17x^2+16$

Polynomial functions can be classified according to their degree, as shown below. (Linear and quadratic functions have been covered in previous sections.)

Degree	Name
0 or 1	Linear
2	Quadratic
3	Cubic
4	Quartic
5	Quintic
n	n^{th} degree polynomial

Answer the following.

75. Write the equation of the cubic polynomial $P(x)$ that satisfies the following conditions: $P(-4)=P(1)=P(3)=0$, and $P(0)=-6$.

76. Write an equation for a cubic polynomial $P(x)$ with leading coefficient -1 whose graph passes through the point $(2, 8)$ and is tangent to the x-axis at the origin.

77. Write the equation of the quartic polynomial with y-intercept 12 whose graph is tangent to the x-axis at $(-2, 0)$ and $(1, 0)$.

78. Write the equation of the sixth degree polynomial with y-intercept -3 whose graph is tangent to the x-axis at $(-2, 0)$, $(-1, 0)$, and $(3, 0)$.

Use transformations (the concepts of shifting, reflecting, stretching, and shrinking) to sketch each of the following graphs.

79. $P(x)=x^3+5$

80. $P(x)=-x^3-2$

81. $P(x)=-(x-2)^3+4$

82. $P(x)=(x+5)^3-1$

83. $P(x)=2x^4-3$

84. $P(x)=-(x-2)^4+5$

85. $P(x)=-(x+1)^5-4$

86. $P(x)=(x+3)^5+2$

Exercise Set 2.3: Rational Functions

Recall from Section 1.2 that an even function is symmetric with respect to the *y*-axis, and an odd function is symmetric with respect to the origin. This can sometimes save time in graphing rational functions. If a function is even or odd, then half of the function can be graphed, and the rest can be graphed using symmetry.

Determine if the functions below are even, odd, or neither.

1. $f(x)=\dfrac{5}{x}$

2. $f(x)=-\dfrac{3}{x-1}$

3. $f(x)=\dfrac{4}{x^2-9}$

4. $f(x)=\dfrac{9x^2-1}{x^4}$

5. $f(x)=\dfrac{x-1}{x^2-4}$

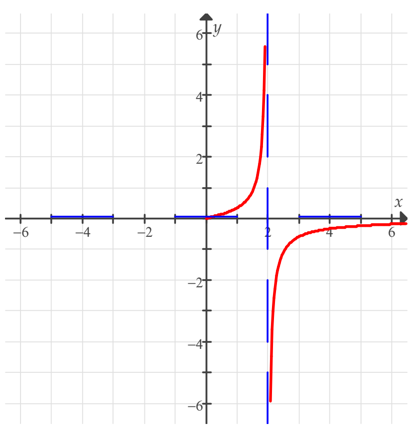
6. $f(x)=\dfrac{7}{x^3}$

In each of the graphs below, only half of the graph is given. Sketch the remainder of the graph, given that the function is:

(a) Even

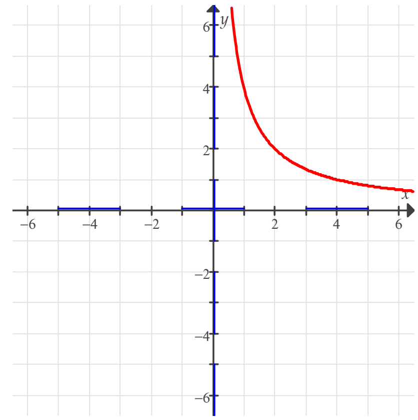
(b) Odd

7.



(Notice the asymptotes at $x=2$ and $y=0$.)

8.



(Notice the asymptotes at $x=0$ and $y=0$.)

For each of the following graphs:

(h) Identify the location of any hole(s) (i.e. removable discontinuities)

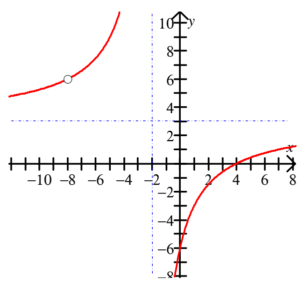
(i) Identify any *x*-intercept(s)

(j) Identify any *y*-intercept(s)

(k) Identify any vertical asymptote(s)

(l) Identify any horizontal asymptote(s)

9.



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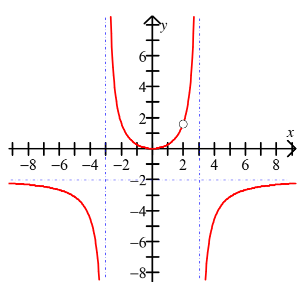


Exercise Set 2.3: Rational Functions

For each of the following rational functions:

- **(a) Find the domain of the function**
- **(b) Identify the location of any hole(s) (i.e. removable discontinuities)**
- **(c) Identify any *x*-intercept(s)**
- **(d) Identify any *y*-intercept(s)**
- **(e) Identify any vertical asymptote(s)**
- **(f) Identify any horizontal asymptote(s)**
- **(g) Identify any slant asymptote(s)**
- **(h) Sketch the graph of the function. Be sure to include all of the above features on your graph.**

11. $f(x)=\dfrac{3}{x-5}$

12. $f(x)=\dfrac{-4}{x+7}$

13. $f(x)=\dfrac{2x+3}{x}$

14. $f(x)=\dfrac{9-4x}{x}$

15. $f(x)=\dfrac{x-6}{x+3}$

16. $f(x)=\dfrac{x+5}{x-2}$

17. $f(x)=\dfrac{-4x+8}{2x+3}$

18. $f(x)=\dfrac{3x+6}{2x-1}$

19. $f(x)=\dfrac{(x-2)(x+3)}{(x-2)(x-4)}$

20. $f(x)=\dfrac{(x+3)(6-x)}{(x-2)(x+3)}$

21. $f(x)=\dfrac{x^2+x-20}{x-4}$

22. $f(x)=\dfrac{x^2-3x-10}{x-5}$

23. $f(x)=\dfrac{4x^3}{x^2-1}$

24. $f(x)=\dfrac{x^3}{2x^2-18}$

25. $f(x)=\dfrac{-(3x+5)(x-2)}{x(x-2)}$

26. $f(x)=\dfrac{-(x+4)(5x-7)}{(x-3)(x+4)}$

27. $f(x)=\dfrac{2x^2-18}{x^2+4x+3}$

28. $f(x)=\dfrac{8x^2-16x}{5x^2-20}$

29. $\dfrac{16-x^4}{2x^3}$

30. $f(x)=\dfrac{x^3-2x^2-x+2}{x^2-4}$

31. $f(x)=\dfrac{8}{x^2-4}$

32. $f(x)=\dfrac{-12}{x^2+x-6}$

33. $f(x)=\dfrac{6x-6}{x^2-x-12}$

34. $f(x)=\dfrac{-8x-16}{x^2+2x-15}$

35. $f(x)=\dfrac{(x-3)(x+2)(x-4)}{(x+1)(x-4)(x-2)}$

36. $f(x)=\dfrac{2x^3+10x^2}{x^3+5x^2-9x-45}$

37. $f(x)=\dfrac{x(x-5)(x+1)(x-3)}{(x+1)(x-3)}$

38. $f(x)=-\dfrac{(x-4)(x-3)(x-2)(x+1)}{(x-4)(x-2)}$

39. $f(x)=\dfrac{x^3+2x^2-9x-18}{x^2}$

40. $f(x)=\dfrac{x^4-10x^2+9}{x^3}$

Exercise Set 2.3: Rational Functions

Answer the following.

41. In the function $f(x)=\dfrac{x^2-5x+3}{3x^2+2x-3}$

(a) Use the quadratic formula to find the x-intercepts of the function, and then use a calculator to round these answers to the nearest tenth.

(b) Use the quadratic formula to find the vertical asymptotes of the function, and then use a calculator to round these answers to the nearest tenth.

42. In the function $f(x)=\dfrac{2x^2+7x-1}{x^2-6x+4}$

(a) Use the quadratic formula to find the x-intercepts of the function, and then use a calculator to round these answers to the nearest tenth.

(b) Use the quadratic formula to find the vertical asymptotes of the function, and then use a calculator to round these answers to the nearest tenth.

The graph of a rational function never intersects a vertical asymptote, but at times the graph intersects a horizontal asymptote. For each function $f(x)$ below,

(a) **Find the equation for the horizontal asymptote of the function.**

(b) **Find the x-value where $f(x)$ intersects the horizontal asymptote.**

(c) **Find the point of intersection of $f(x)$ and the horizontal asymptote.**

43. $f(x)=\dfrac{x^2+2x+3}{x^2-x-3}$

44. $f(x)=\dfrac{x^2+4x-2}{x^2-x-7}$

45. $f(x)=\dfrac{x^2+2x+3}{2x^2+6x-1}$

46. $f(x)=\dfrac{3x^2+5x-1}{x^2-x-3}$

47. $f(x)=\dfrac{4x^2+12x+9}{x^2-x+7}$

48. $f(x)=\dfrac{x^2+5x-1}{5x^2-10x-3}$

Answer the following.

49. The function $f(x)=\dfrac{6x-6}{x^2-x-12}$ was graphed in Exercise 33.

(a) Find the point of intersection of $f(x)$ and the horizontal asymptote.

(b) Sketch the graph of $f(x)$ as directed in Exercise 33, but also label the intersection of $f(x)$ and the horizontal asymptote.

50. The function $f(x)=\dfrac{-8x-16}{x^2+2x-15}$ was graphed in Exercise 34.

(a) Find the point of intersection of $f(x)$ and the horizontal asymptote.

(b) Sketch the graph of $f(x)$ as directed in Exercise 34, but also label the intersection of $f(x)$ and the horizontal asymptote.

Exercise Set 2.4: Applications and Writing Functions

Answer the following. If an example contains units of measurement, assume that any resulting function reflects those units. (Note: Refer to Appendix A.3 if necessary for a list of Geometric Formulas.)

1. The perimeter of a rectangle is 54 feet.

(a) Express its area, A, as a function of its width, w.

(b) For what value of w is A the greatest?

(c) What is the maximum area of the rectangle?

2. The perimeter of a rectangle is 62 feet.

(a) Express its area, A, as a function of its length, ℓ.

(b) For what value of ℓ is A the greatest?

(c) What is the maximum area of the rectangle?

3. Two cars leave an intersection at the same time. One is headed south at a constant speed of 50 miles per hour. The other is headed east at a constant speed of 120 miles per hour. Express the distance, d, between the two cars as a function of the time, t.

4. Two cars leave an intersection at the same time. One is headed north at a constant speed of 30 miles per hour. The other is headed west at a constant speed of 40 miles per hour. Express the distance, d, between the two cars as a function of the time, t.

5. If the sum of two numbers is 20, find the smallest possible value of the sum of their squares.

6. If the sum of two numbers is 16, find the smallest possible value of the sum of their squares.

7. If the sum of two numbers is 8, find the largest possible value of their product.

8. If the sum of two numbers is 14, find the largest possible value of their product.

9. A farmer has 1500 feet of fencing. He wants to fence off a rectangular field that borders a straight river (needing no fence along the river). What are the dimensions of the field that has the largest area?

10. A farmer has 2400 feet of fencing. He wants to fence off a rectangular field that borders a straight river (needing no fence along the river). What are the dimensions of the field that has the largest area?

11. A farmer with 800 feet of fencing wants to enclose a rectangular area and divide it into 3 pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the 3 pens?

12. A farmer with 1800 feet of fencing wants to enclose a rectangular area and divide it into 5 pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the 5 pens?

13. The hypotenuse of a right triangle is 6 m. Express the area, A, of the triangle as a function of the length x of one of the legs.

14. The hypotenuse of a right triangle is 11 m. Express the area, A, of the triangle as a function of the length x of one of the legs.

15. The area of a rectangle is 22 ft^2. Express its perimeter, P, as a function of its length, ℓ.

16. The area of a rectangle is 36 ft^2. Express its perimeter, P, as a function of its width, w.

17. A rectangle has a base on the x-axis and its upper two vertices on the parabola $y = 9 - x^2$.

(a) Express the area, A, of the rectangle as a function of x.

(b) Express the perimeter, P, of the rectangle as a function of x.

18. A rectangle has a base on the x-axis and its lower two vertices on the parabola $y = x^2 - 16$.

(a) Express the area, A, of the rectangle as a function of x.

(b) Express the perimeter, P, of the rectangle as a function of x.

19. In a right circular cylinder of radius r, if the height is twice the radius, express the volume, V, as a function of r.

Exercise Set 2.4: Applications and Writing Functions

20. In a right circular cylinder of radius r, if the height is half the radius, express the volume, V, as a function of r.

21. A right circular cylinder of radius r has a volume of 300 cm^3.

(a) Express the lateral area, L, in terms of r.

(b) Express the total surface area, S, as a function of r.

22. A right circular cylinder of radius r has a volume of 750 cm^3.

(a) Express the lateral area, L, in terms of r.

(b) Express the total surface area, S, as a function of r.

23. In a right circular cone of radius r, if the height is four times the radius, express the volume, V, as a function of r.

24. In a right circular cone of radius r, if the height is five times the radius, express the volume, V, as a function of r.

25. An open-top box with a square base has a volume of 20 cm^3. Express the surface area, S, of the box as a function of x, where x is the length of a side of the square base.

26. An open-top box with a square base has a volume of 12 cm^3. Express the surface area, S, of the box as a function of x, where x is the length of a side of the square base.

27. A piece of wire 120 cm long is cut into several pieces and used to construct the skeleton of a rectangular box with a square base.

(a) Express the surface area, S, of the box in terms of x, where x is the length of a side of the square base.

(b) What are the dimensions of the box with the largest surface area?

(c) What is the maximum surface area of the box?

28. A piece of wire 96 in long is cut into several pieces and used to construct the skeleton of a rectangular box with a square base.

(a) Express the surface area, S, of the box in terms of x, where x is the length of a side of the square base.

(b) What are the dimensions of the box with the largest surface area?

(c) What is the maximum surface area of the box?

29. A wire of length x is bent into the shape of a circle.

(a) Express the circumference, C, in terms of x.

(b) Express the area of the circle, A, as a function of x.

30. A wire of length x is bent into the shape of a square.

(a) Express the area, A, of the square as a function of x.

(b) Express the diagonal, d, of the square as a function of x.

31. Let $P(x, y)$ be a point on the graph of $y = x^2 - 10$.

(a) Express the distance, d, from P to the origin as a function of x.

(b) Express the distance, d, from P to the point $(0, -2)$ as a function of x.

32. Let $P(x, y)$ be a point on the graph of $y = x^2 - 7$.

(a) Express the distance, d, from P to the origin as a function of x.

(b) Express the distance, d, from P to the point $(0, 5)$ as a function of x.

33. A circle of radius r is inscribed in a square. Express the area, A, of the square as a function of r.

34. A square is inscribed in a circle of radius r. Express the area, A, of the square as a function of r.

35. A rectangle is inscribed in a circle of radius 4 cm.

(a) Express the perimeter, P, of the rectangle in terms of its width, w.

(b) Express the area, A, of the rectangle in terms of its width, w.

Exercise Set 2.4: Applications and Writing Functions

36. A rectangle is inscribed in a circle of diameter 20 cm.

(a) Express the perimeter, P, of the rectangle in terms of its length, ℓ.

(b) Express the area, A, of the rectangle in terms of its length, ℓ.

37. An isosceles triangle has fixed perimeter P (so P is a constant).

(a) If x is the length of one of the two equal sides, express the area, A, as a function of x.

(b) What is the domain of A ?

38. Express the volume, V, of a sphere of radius r as a function of its surface area, S.

39. Two cars are approaching an intersection. One is 2 miles south of the intersection and is moving at a constant speed of 30 miles per hour. At the same time, the other car is 3 miles east of the intersection and is moving at a constant speed of 40 miles per hour.

(a) Express the distance, d, between the cars as a function of the time, t.

(b) At what time t is the value of d the smallest?

40. Two cars are approaching an intersection. One is 5 miles north of the intersection and is moving at a constant speed of 40 miles per hour. At the same time, the other car is 6 miles west of the intersection and is moving at a constant speed of 30 miles per hour.

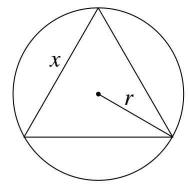
(a) Express the distance, d, between the cars as a function of the time, t.

(b) At what time t is the value of d the smallest?

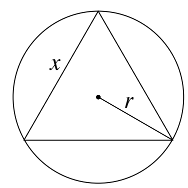
41. A straight wire 40 cm long is bent into an L shape. What is the shortest possible distance between the two ends of the bent wire?

42. A straight wire 24 cm long is bent into an L shape. What is the shortest possible distance between the two ends of the bent wire?

43. An equilateral triangle is inscribed in a circle of radius r, as shown below. Express the circumference, C, of the circle as a function of the length, x, of a side of the triangle.



44. An equilateral triangle is inscribed in a circle of radius r, as shown below. Express the area, A, within the circle, but outside the triangle, as a function of the length, x, of a side of the triangle.



Exercise Set 3.1: Exponential Functions

Solve for *x*. (If no solution exists, state 'No solution.')

1. **(a)** $3^{x+2}=0$ **(b)** $3^{2x-7}=1$

2. **(a)** $7^{x+3}=1$ **(b)** $7^{x}=0$

3. **(a)** $5^{3x-1}=125$ **(b)** $25^{3x-1}=125$

4. **(a)** $2^{5x-2}=32$ **(b)** $8^{5x-2}=32$

5. **(a)** $2^{4x-1}-8=0$ **(b)** $2^{x-3}+8=0$

6. **(a)** $7^{x+3}-\dfrac{1}{7}=0$ **(b)** $7^{2x}+3=52$

7. **(a)** $27^{x+5}=\sqrt{3}$ **(b)** $36^{\,3x+0.5}=\dfrac{1}{\sqrt{6}}$

8. **(a)** $25^{2x-1}=\sqrt[3]{5}$ **(b)** $\dfrac{\sqrt{7}}{49}=7^{2x-1}$

Graph each of the following functions by plotting points. Show any asymptote(s) clearly on your graph.

9. $f(x)=3^{x}$

10. $f(x)=5^{x}$

11. $f(x)=\left(\dfrac{1}{2}\right)^{x}$

12. $f(x)=\left(\dfrac{4}{3}\right)^{x}$

Answer the following.

13. Based on your answers to 9-12 above:
- **(a)** Draw a sketch of $f(x)=a^{x}$, where $a>1$.
- **(b)** Draw a sketch of $f(x)=a^{x}$, where $0<a<1$.
- **(c)** What point on the graph do parts (a) and (b) have in common?
- **(d)** Name any asymptote(s) for parts (a) and (b).

14. Based on your answers to 9-12 above:
- **(a)** Draw a sketch of $f(x)=2.7^{x}$ without plotting points.
- **(b)** Draw a sketch of $f(x)=0.73^{x}$ without plotting points.
- **(c)** What point on the graph do parts (a) and (b) have in common?
- **(d)** Name any asymptote(s) for parts (a) and (b).

Answer the following.Graph each of the following pairs of functions on the same set of axes.

15. **(a)** Graph the functions $f(x)=6^{x}$ and $g(x)=\left(\dfrac{1}{6}\right)^{x}$ on the same set of axes.
- **(b)** Compare the graphs drawn in part (a). What is the relationship between the graphs?
- **(c)** Explain the result from part (b) algebraically.

16. **(a)** Graph the functions $f(x)=\left(\dfrac{5}{2}\right)^{x}$ and $g(x)=\left(\dfrac{2}{5}\right)^{x}$ on the same set of axes.
- **(b)** Compare the graphs drawn in part (a). What is the relationship between the graphs?
- **(c)** Explain the result from part (b) algebraically.

Sketch a graph of each of the following functions. (Note: You do not need to plot points.) Be sure to label at least one key point on your graph, and show any asymptotes.

17. $f(x)=8^{x}$

18. $f(x)=12^{x}$

19. $f(x)=0.2^{x}$

20. $f(x)=1.4^{x}$

21. $f(x)=\left(\dfrac{7}{2}\right)^{x}$

22. $f(x)=\left(\dfrac{3}{8}\right)^{x}$

Exercise Set 3.1: Exponential Functions

For each of the following examples,

(a) Find any intercept(s) of the function.

(b) Use transformations (the concepts of reflecting, shifting, stretching, and shrinking) to sketch the graph of the function. Be sure to label the transformation of the point $(0, 1)$.

Clearly label any intercepts and/or asymptotes.

(c) State the domain and range of the function.

(d) State whether the graph is increasing or decreasing.

23. $f(x) = 2^{x-1}$

24. $f(x) = 3^{x+2}$

25. $f(x) = -5^{x}$

26. $f(x) = -\left(\dfrac{3}{4}\right)^{x}$

27. $f(x) = 8^{-x}$

28. $f(x) = 10^{-x}$

29. $f(x) = \left(\dfrac{1}{2}\right)^{x} - 4$

30. $f(x) = 4^{x} - 8$

31. $f(x) = 9^{x+2} - 27$

32. $f(x) = 7^{x-3} + 7$

33. $f(x) = -4^{x+1} + 2$

34. $f(x) = -8^{x-2} + 32$

35. $f(x) = 9 - 3^{x+4}$

36. $f(x) = -25 - 5^{x+3}$

37. $f(x) = 3^{2-x}$

38. $f(x) = 4^{-3-x}$

39. $f(x) = 8^{-1-x} - 32$

40. $f(x) = 5^{1-x} - 1$

41. $f(x) = -6^{-x} - 36$

42. $f(x) = -2^{-x} + 16$

43. $f(x) = 2 \cdot 3^{x}$

44. $f(x) = \frac{1}{2} \cdot 5^{x}$

Find the exponential function of the form $f(x) = C \cdot a^{x}$ which satisfies the given conditions.

45. Passes through the points $(0, 4)$ and $(3, 32)$.

46. Passes through the points $(0, 3)$ and $(2, 108)$.

47. Passes through the points $(0, 2)$ and $\left(-1, \frac{2}{7}\right)$.

48. Passes through the points $(0, 7)$ and $\left(-2, \frac{7}{16}\right)$.

True or False? (Answer these without using a calculator.)

49. $e^{2} < 9$

50. $e^{2} > 4$

51. $\sqrt{e} < 2$

52. $\sqrt{e} > 1$

53. $e^{-1} < 0$

54. $e^{-1} > 0$

55. $e^{3} > 27$

56. $e < \frac{5}{2}$

57. $e^{0} = 1$

58. $e^{1} = 0$

Exercise Set 3.1: Exponential Functions

Answer the following.

59. Graph $f(x) = 2^x$ and $g(x) = e^x$ on the same set of axes.

60. Graph $f(x) = e^x$ and $g(x) = 3^x$ on the same set of axes.

61. Sketch the graph of $f(x) = e^x$ and then find the following:

(a) Domain
(b) Range
(c) Horizontal Asymptote
(d) y-intercept

62. Sketch the graph of $f(x) = 3^x$ and then find the following:

(a) Domain
(b) Range
(c) Horizontal Asymptote
(d) y-intercept
(e) How do your answers compare with those in the previous question?

Graph the following functions, not by plotting points, but by applying transformations to the graph of $y = e^x$. Be sure to label at least one key point on your graph (the transformation of the point (0, 1)) and show any asymptotes. Then state the domain and range of the function.

63. $f(x) = -3e^x$

64. $f(x) = 2e^{-x}$

65. $f(x) = e^{x-1} - 6$

66. $f(x) = e^{x+2} + 1$

67. $f(x) = e^{-x} + 3$

68. $f(x) = -e^x - 2$

69. $f(x) = -4e^x - 5$

70. $f(x) = 2e^{x+6} - 4$

71. $f(x) = e^{-x-4} - 3$

72. $f(x) = e^{-x+3} + 2$

Exercise Set 3.2: Logarithmic Functions

Write each of the following equations in exponential form.

1. **(a)** $\log_3(81) = 4$ **(b)** $\log_4(8) = \frac{3}{2}$

2. **(a)** $\log_5(1) = 0$ **(b)** $\log_3\left(\frac{1}{9}\right) = -2$

3. **(a)** $\log_5(7) = x$ **(b)** $\ln(x) = 8$

4. **(a)** $\log_x(2) = 3$ **(b)** $\ln(x) = 2$

Write each of the following equations in logarithmic form.

5. **(a)** $6^0 = 1$ **(b)** $10^{-4} = \frac{1}{10{,}000}$

6. **(a)** $5^3 = 125$ **(b)** $25^{-\frac{3}{2}} = \frac{1}{125}$

7. **(a)** $7^1 = 7$ **(b)** $e^4 = x$

8. **(a)** $10^{0.5} = \sqrt{10}$ **(b)** $e^{-5} = y$

9. **(a)** $e^x = 7$ **(b)** $e^{x-2} = y$

10. **(a)** $e^x = 12$ **(b)** $e^{5x} = y - 6$

Simplify each of the following expressions.

11. **(a)** $\log_2(8)$ **(b)** $\log_5(1)$

12. **(a)** $\log_3(81)$ **(b)** $\log_8(8)$

13. **(a)** $\log_7(7)$ **(b)** $\log_6\left(6^2\right)$

14. **(a)** $\log_9(1)$ **(b)** $\log_2\left(2^5\right)$

15. **(a)** $\log_{16}(4)$ **(b)** $\log_7\left(\frac{1}{7}\right)$

16. **(a)** $\log\left(\frac{1}{10}\right)$ **(b)** $\log_{32}(2)$

17. **(a)** $\log_{27}(3)$ **(b)** $\log_9\left(\frac{1}{81}\right)$

18. **(a)** $\log_{49}(7)$ **(b)** $\log_2\left(\frac{1}{8}\right)$

19. **(a)** $\log_4(0.25)$ **(b)** $\log\left(\sqrt[4]{10}\right)$

20. **(a)** $\log_7\left(\sqrt[6]{7}\right)$ **(b)** $\log_5(0.2)$

21. **(a)** $\log_4(0.5)$ **(b)** $\log_8(32)$

22. **(a)** $\log_{25}\left(\sqrt{5}\right)$ **(b)** $\log_{32}\left(\frac{1}{16}\right)$

23. **(a)** $\ln(e)$ **(b)** $\ln\left(e^4\right)$

24. **(a)** $\ln\left(e^5\right)$ **(b)** $\ln\left(e^{-2}\right)$

25. **(a)** $\ln\left(e^{-6}\right)$ **(b)** $\ln\left(\sqrt[3]{e}\right)$

26. **(a)** $\ln\left(\sqrt[4]{e}\right)$ **(b)** $\ln(1)$

27. **(a)** $\ln\left(\frac{1}{e^4}\right)$ **(b)** $\ln\left(e^x\right)$

28. **(a)** $\ln\left(\frac{1}{\sqrt{e}}\right)$ **(b)** $\ln\left(e^{2x}\right)$

Simplify each of the following expressions.

29. **(a)** $7^{\log_7(5)}$ **(b)** $10^{\log\left(\sqrt{3}\right)}$

30. **(a)** $10^{\log(4)}$ **(b)** $5^{\log_5(0.71)}$

31. **(a)** $e^{\ln(6)}$ **(b)** $e^{\ln\left(x^4\right)}$

32. **(a)** $e^{\ln(2)}$ **(b)** $e^{\ln(x)}$

Find the value of *x* in each of the following equations. Write all answers in simplest form. (Some answers may contain *e*.)

33. **(a)** $\log_2(32) = x$ **(b)** $\log_x(9) = 2$

34. **(a)** $\log_5(125) = x$ **(b)** $\log_x(64) = 3$

35. **(a)** $\log_7(x) = 2$ **(b)** $\log_x(3) = \frac{1}{2}$

36. **(a)** $\log_3(x) = 4$ **(b)** $\log_x\left(\frac{1}{8}\right) = -1$

37. **(a)** $\log_{25}(125) = x$ **(b)** $\log_{16}(x) = \frac{1}{4}$

38. **(a)** $\log_{16}(32) = x$ **(b)** $\log_{36}(x) = 0.5$

39. **(a)** $\log|x-3| = 2$ **(b)** $\log_3\left(\log_4(x)\right) = 0$

40. **(a)** $\log\left(x^2 - 9\right) = 3$ **(b)** $\log_5\left(\log_2(x)\right) = 1$

41. **(a)** $\ln(x) = 4$ **(b)** $\ln(x+5) = 2$

42. **(a)** $\ln(x) = 0$ **(b)** $\ln|x-2| = 5$

Exercise Set 3.2: Logarithmic Functions

Solve for x. Give an exact answer, and then use a calculator to round that answer to the nearest thousandth. (Hint: Write the equations in logarithmic form first.)

43. **(a)** $10^x = 20$ **(b)** $e^x = 20$

44. **(a)** $10^x = 45$ **(b)** $e^x = 45$

45. **(a)** $10^x = \frac{2}{5}$ **(b)** $e^x = \frac{2}{5}$

46. **(a)** $10^x = \frac{4}{7}$ **(b)** $e^x = \frac{4}{7}$

47. $10^{3x-2} = 6$

48. $e^{5x+1} = 7$

49. $e^{4x^2} = 40$

50. $10^{x^2-1} = 7$

Answer the following.

51. **(a)** Sketch the graph of $f(x) = 2^x$.
(b) Sketch the graph of $f^{-1}(x)$ on the same set of axes.
(c) The inverse of $y = 2^x$ is $y =$ ________ .

52. **(a)** Sketch the graph of $f(x) = e^x$.
(b) Sketch the graph of $f^{-1}(x)$ on the same set of axes.
(c) The inverse of $y = e^x$ is $y =$ ________ .

53. **(a)** Sketch the graph of $y = \log_b(x)$, where $b > 1$.
(b) Name any intercept(s) for the graph of $y = \log_b(x)$.
(c) Name any asymptote(s) for the graph of $y = \log_b(x)$.

54. **(a)** Sketch the graph of $y = \ln(x)$.
(b) $y = \ln(x)$ can be written as $y = \log_b(x)$, where $b =$ ___ .
(c) Name any intercept(s) for the graph of $y = \ln(x)$.
(d) Name any asymptote(s) for the graph of $y = \ln(x)$.

For each of the following examples,

(a) **Use transformations (the concepts of reflecting, shifting, stretching, and shrinking) to sketch the graph of $y = \log_b(x)$. Be sure to label the transformation of the point $(1, 0)$. Clearly label any asymptotes.**
(b) **State the domain and range of the function.**
(c) **State whether the graph is increasing or decreasing.**

55. $f(x) = \log_2(x) + 3$

56. $f(x) = \log_5(x) - 4$

57. $f(x) = -\log_3(x)$

58. $f(x) = \log_6(-x)$

59. $f(x) = \log_7(x-5)$

60. $f(x) = \log_8(x+2)$

61. $f(x) = \log_5(x+1) - 4$

62. $f(x) = \log_6(x-4) + 2$

63. $f(x) = \ln(-x)$

64. $f(x) = -\ln(x)$

65. $f(x) = -\ln(x+4)$

66. $f(x) = \ln(-x) - 1$

67. $f(x) = 5 - \ln(-x)$

68. $f(x) = -3 - \ln(x+2)$

Find the domain of each of the following functions. (Note: You should be able to do this algebraically, without sketching the graph.)

69. $f(x) = \log_8(x)$

70. $f(x) = -\log_5(x)$

71. $f(x) = 2\ln(x) + 3$

72. $f(x) = -3\ln(2x)$

73. $f(x) = -\log_6(x+2)$

74. $f(x) = \log_{10}(x-3)$

75. $f(x) = \log_{10}(-x) + 3$

76. $f(x) = \log_3(3-4x)+5$

77. $f(x) = \ln(5-2x)-1$

78. $f(x) = \ln(-4x)-8$

79. $f(x) = \log_4\left(x^2+1\right)$

80. $f(x) = \log_7\left(x^2-9\right)$

81. $f(x) = \log_2\left(x^2-x-6\right)$

82. $f(x) = \log_9\left(x^2+4\right)$

Exercise Set 3.3: Laws of Logarithms

True or False?
(Note: Assume that $x > 0$, $y > 0$, and $x > y$, so that each logarithm below is defined.)

1. $\log(xy) = \log(x) + \log(y)$

2. $\log(x - y) = \log(x) - \log(y)$

3. $\log(x + y) = \log(x) + \log(y)$

4. $\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$

5. $\log(x) - \log(y) = \frac{\log(x)}{\log(y)}$

6. $\log(x) + \log(y) = (\log x)(\log y)$

7. $\left[\ln(x)\right]^3 = 3\ln(x)$

8. $\ln\left(x^5\right) = 5\ln(x)$

9. $\ln\left(x^4\right) = 4\ln(x)$

10. $\left[\ln(x)\right]^7 = 7\ln(x)$

11. $\log_7\left(\frac{x}{y}\right) = \frac{\log_7(x)}{\log_7(y)}$

12. $\log_5(xy) = \left[\log_5(x)\right] \cdot \left[\log_5(y)\right]$

13. $\log_8(0) = 1$

14. $\log_8(1) = 0$

15. $\ln(1) = 0$

16. $\ln(0) = 1$

17. $\log_7(2) = \frac{\ln(2)}{\ln(7)}$

18. $\log_7(2) = \frac{\ln(7)}{\ln(2)}$

19. $\log_5(4) = \frac{\log(5)}{\log(4)}$

20. $\log_5(4) = \frac{\log(4)}{\log(5)}$

Rewrite each of the following expressions so that your answer contains sums, differences, and/or multiples of logarithms. Your answer should not contain logarithms of any product, quotient, or power.

21. $\log_5(9CD)$

22. $\log_3(FGH)$

23. $\log_7\left(\frac{KP}{L}\right)$

24. $\log_8\left(\frac{7RT}{M}\right)$

25. $\ln\left(B^2 P^3 \sqrt[6]{K}\right)$

26. $\ln\left(Y^5 X^4 \sqrt{Z}\right)$

27. $\log\left(\frac{9k^2 m^3}{p^4 w}\right)$

28. $\log\left(\frac{7r^5 p^4}{xyz^3}\right)$

29. $\ln\left[\sqrt{x}\,(y+3)\right]$

30. $\ln\left[\sqrt[4]{C}\,(J - M)\right]$

31. $\log_3\left(\sqrt[4]{7x}\right)$

32. $\log_9 \sqrt[5]{\frac{P}{Q}}$

33. $\ln\left[\frac{x^4(x-5)}{\sqrt{x+3}}\right]$

34. $\ln\left[\frac{x^7\left(x^3+2\right)^5}{\sqrt[3]{x-6}}\right]$

35. $\log\left[\sqrt{\frac{x^2-7}{\left(x^2+3\right)(x-4)^2}}\right]$

Exercise Set 3.3: Laws of Logarithms

36. $\log\left[\sqrt{\dfrac{(x+2)^3}{(x^3-5)(x+7)^4}}\right]$

Answer the following.

37. **(a)** Rewrite the following expression as a single logarithm: $\log_5(A)+\log_5(B)-\log_5(C)$.

(b) Rewrite the number 1 as a logarithm with base 5.

(c) Use the result from part (b) to rewrite the following expression as a single logarithm: $\log_5(A)+\log_5(B)-\log_5(C)+1$

(d) Use the result from part (b) to rewrite the following expression as a single logarithm: $\log_5(A)+\log_5(B)-\log_5(C)-1$

38. **(a)** Rewrite the following expression as a single logarithm: $\ln(x)-\ln(y)-\ln(z)$.

(b) Rewrite the number 1 as a natural logarithm.

(c) Use the result from part (b) to rewrite the following expression as a single logarithm: $\ln(x)-\ln(y)-\ln(z)+1$

(d) Use the result from part (b) to rewrite the following expression as a single logarithm: $\ln(x)-\ln(y)-\ln(z)-1$

Rewrite each of the following expressions as a single logarithm.

39. $\ln(Y)-\ln(Z)+\ln(W)$

40. $\ln(B)-\ln(C)-\ln(D)-1$

41. $4\log(K)+3\log(P)-\frac{1}{2}\log(Q)$

42. $7\log(F)-\frac{1}{4}\log(V)+\log(R)$

43. $3\log_7(x-2)-\frac{1}{5}\log_7(x^2-3)-4\log_7(x+5)+1$

44. $\frac{1}{2}\log_6(x^4+2)+8\log_6(7-x)-2\log_6(x)$

45. $\log_7(5)+3\log_7(2)$

46. $4\log_2(3)-2\log_2(5)+1$

47. $\ln\left(\dfrac{x^2-9}{x-3}\right)+\ln\left(\dfrac{x^2+6x+8}{x+4}\right)$

48. $\ln\left(\dfrac{x^2-5x-35}{x-7}\right)-\ln\left(\dfrac{x^2-4x}{x}\right)$

49. $\frac{1}{6}\left[5\ln x+2\ln(x+3)\right]-4\ln(x^2+5)-1$

50. $\frac{1}{8}\left[2\ln(x-3)+7\ln(x)\right]-\frac{1}{2}\left[9\ln(x-2)+\ln(x^2-5)\right]$

Evaluate the following using the laws of logarithms. Simplify your answer as much as possible. (Note: These should be done *without* a calculator.)

51. $\log_2(24)-\log_2(3)$

52. $\log_{12}(3)+\log_{12}(48)$

53. $\log_3(45)-\log_3(5)+\log_3\left(\sqrt[4]{3}\right)$

54. $\log_6(9)+\log_6(4)+\log_6\left(\sqrt{6}\right)$

55. $\log_4(10)+\log_4(6.4)$

56. $\dfrac{\left[\log_2(160)-\log_2(10)\right]}{\log_2(8)}$

57. $\dfrac{\left[\log(5)+\log(2)\right]}{\log\left(\sqrt[3]{10}\right)}$

58. $\log(3{,}000)-\log 3$

59. $\log_3\left(\sqrt{27}\right)$

60. $\log_4\left(\sqrt[3]{16}\right)$

61. $\dfrac{\left[\log_5\left(\sqrt{5}\right)+\log_5 1\right]}{\log_2\left(\sqrt[3]{2}\right)}$

62. $\log_5\left(\sqrt{5}\right)+\log_5(1)-\log_2\left(\sqrt[3]{2}\right)$

63. $e^{2\ln(5)}$

64. $e^{3\ln(2)}$

65. $10^{4\log(3)}$

66. $10^{2\log(7)}$

67. $\log_3\left(9^{17}\right)$

68. $\log_5\left(125^{90}\right)$

69. $\dfrac{e^{3\ln(4)}}{\ln\left(\sqrt[5]{e}\right)}$

70. $\dfrac{\left[10^{3\log(2)}\right]}{\log_2\left(\sqrt[3]{16}\right)}$

71. $7^{\log_7(6)+\log_7(5)}$

72. $3^{\log_3(5)-\log_3(7)}$

Use the laws of logarithms to express *y* as a function of *x*.

73. $\ln(y)=3\ln(x)$

74. $\log(y)-\frac{1}{3}\log(x)=0$

75. $\log(y)+5\log(x)=\log(7)$

76. $\ln(y)-4\ln(x)=\ln(5)$

77. $\ln(y)-3\ln(x+2)=4\ln(x)-\ln(7)$

78. $\log(y)+5\log(4)=7\log(x+3)+8\log(x)$

Use the Change of Base Formula to write the following in terms of natural logarithms. Then use a calculator to write your answer as a decimal, correct to the nearest thousandth.

79. **(a)** $\log_5(2)$ **(b)** $\log_6(17.2)$

80. **(a)** $\log_3(7)$ **(b)** $\log_8(23.5)$

Use the Change of Base Formula to write the following in terms of common (base 10) logarithms. Then use a calculator to write your answer as a decimal, correct to the nearest thousandth.

81. **(a)** $\log_9(4)$ **(b)** $\log_4(8.9)$

82. **(a)** $\log_7(8)$ **(b)** $\log_2(0.7)$

Evaluate the following. (Note: These should be done *without* a calculator.)

83. Find the value of the following:

$$\left[\log_2(3)\right]\left[\log_3(5)\right]\left[\log_5(8)\right]$$

84. Find the value of the following:

$$\left[\log_{27}(36)\right]\left[\log_{36}(49)\right]\left[\log_{49}(81)\right]$$

85. Find the value of the following:

$$\left[\log_3(3)\right]\left[\log_3(9)\right]\left[\log_3(27)\right]\left[\log_3(81)\right]$$

86. Find the value of the following:

$$\left[\log_5(5)\right]\left[\log_5(25)\right]\left[\log_5(125)\right]$$

Rewrite the following as sums so that each logarithm contains a prime number.

87. **(a)** $\ln(6)$ **(b)** $\log(28)$

88. **(a)** $\ln(10)$ **(b)** $\log(45)$

89. **(a)** $\log(60)$ **(b)** $\ln(270)$

90. **(a)** $\log(168)$ **(b)** $\ln(480)$

Exercise Set 3.3: Laws of Logarithms

If $\log_b 2 = A$, $\log_b 3 = B$, and $\log_b 5 = C$ ($b > 1$), write each of the following logarithms in terms of A, B, and C.

91. **(a)** $\log_b(10)$ **(b)** $\log_b\left(\frac{1}{3}\right)$

92. **(a)** $\log_b(6)$ **(b)** $\log_b\left(\frac{3}{2}\right)$

93. **(a)** $\log_b\left(\frac{2}{5}\right)$ **(b)** $\log_b(25)$

94. **(a)** $\log_b(8)$ **(b)** $\log_b(0.25)$

95. **(a)** $\log_b(45)$ **(b)** $\log_b\left(\frac{27}{20}\right)$

96. **(a)** $\log_b\left(\frac{1}{90}\right)$ **(b)** $\log_b(150)$

97. **(a)** $\log_b\left(\sqrt[3]{12}\right)$ **(b)** $\log_b(0.09)$

98. **(a)** $\log_b(0.3)$ **(b)** $\log_b\left(\sqrt{30}\right)$

99. **(a)** $\log_b(100b)$ **(b)** $\log_3(5)$

100. **(a)** $\log_5(2)$ **(b)** $\log_b\left(\frac{18}{5b}\right)$

Exercise Set 3.4: Exponential and Logarithmic Equations and Inequalities

Solve each of the following equations.

(a) **Solve for x. (Note: Further simplification may occur in step (b).) Write the answer in terms of natural logarithms, unless no logarithms are involved.**

(b) **Rewrite the answer so that each individual logarithm from part (a) is written as a sum or difference of logarithms of prime numbers,**

e.g., $\ln(40) = \ln(2^3 \cdot 5) = \ln(2^3) + \ln(5)$

$= 3\ln(2) + \ln(5)$.

Write the answer in simplest form.

(c) **If the simplified answer from part (b) contains logarithms, rewrite the answer as a decimal, correct to the nearest thousandth; otherwise, simply rewrite the answer.**

1. $7^x = 12$

2. $8^x = 3$

3. $e^x = 22$

4. $e^x = 45$

5. $3 \cdot 6^x = 42$

6. $2 \cdot 9^x = 48$

7. $e^x + 7 = 3$

8. $e^x - 9 = -3$

9. $19^{-3x} = 7$

10. $5^{-2x} = 3$

11. $6e^x = 13$

12. $8e^x = 18$

13. $e^{5x+4} - 5 = 31$

14. $2e^{7-3x} = -20$

15. $7^{x+4} = 10$

16. $8^{3x-2} = 5$

17. $32^{\frac{x}{3}} = 20$

18. $12^{\frac{x}{6}} = 18$

19. $5^{4x} = 6^{7x-1}$

20. $7^{x+3} = 15^{8x}$

21. $2^{3x-1} = 8^{2x+5}$

22. $12^{4x-3} = 45^{x+7}$

23. $\dfrac{5}{3 - e^{-x}} = 7$

24. $\dfrac{3}{2 - e^{-x}} = 4$

25. $e^{2x} - 3e^x - 40 = 0$

26. $e^{2x} + 5e^x - 14 = 0$

27. $3^{2x} + 2(3^x) - 15 = 0$

28. $5^{2x} - 4(5^x) - 21 = 0$

29. $49^x - 7^x - 6 = 0$

30. $4^x + 8 \cdot 2^x + 7 = 0$

31. $x^2 \cdot 7^x - 9 \cdot 7^x = 0$

32. $x^2 \cdot 2^x - 16 \cdot 2^x = 0$

33. $x^2 e^x - 4xe^x - 5e^x = 0$

34. $x^2 e^x - 8xe^x + 12e^x = 0$

35. $2^{\log_2(3x-7)} = 3x - 7$

36. $6^{\log_6(5x+9)} = 5x + 9$

Exercise Set 3.4: Exponential and Logarithmic Equations and Inequalities

Solve each of the following equations. Write all answers in simplest form.

37. $\ln x = 8$

38. $\ln x = 12$

39. $\ln x = -7$

40. $\ln x = -10$

41. $\log x = 3$

42. $\log x = -2$

43. $\log_5 (x+1) = 2$

44. $\log_2 (3x+7) = 5$

45. $\log(4x-1)+3 = 5$

46. $\log(2x+1)-7 = -4$

47. $\ln(3x+8)-6 = 3$

48. $\ln(5x-4)+2 = 9$

49. $\ln x + \ln(x-5) = \ln 6$

50. $\ln x + \ln(x-7) = \ln 18$

51. $\log_2 x + \log_2 (x-2) = 3$

52. $\log_6 x + \log_6 (x+1) = 1$

53. $\log_2 (x+3) + \log_2 (x+2) = 1$

54. $\log_6 (x+2) + \log_6 (x+3) = 1$

55. $\log_7 (x+8) - \log_7 (x-5) = \log_7 4$

56. $\log_2 (x+7) - \log_2 (x-9) = \log_2 11$

57. $\log_5 (3x+5) - \log_5 (x+11) = 0$

58. $\log_6 (2x+1) - \log_6 (x-5) = 0$

59. $\log_2 (x+4) = 3 + \log_2 (x+5)$

60. $\log_3 (15x+8) = 2 + \log_3 (x+1)$

61. $\log 3 + \log(x+4) = \log 5 + \log(x-3)$

62. $\ln 6 + \ln(x-8) = \ln 9 + \ln(x+3)$

63. $2\log x = \log 2 + \log(x+12)$

64. $2\log x = \log 4 + \log(x+8)$

65. $\ln\left[\ln(x)\right] = -5$

66. $\ln\left[\ln(x)\right] = 5$

67. $e^{\ln(x)} = 5$

68. $e^{\ln(x)} = -5$

69. $\ln\left(x^4\right) = 4\ln(x)$

70. $\left[\ln(x)\right]^4 = 4\ln(x)$

71. $\log_7\left(3x^2\right) = 2\log_7(3x)$

72. $\log_7 (3x)^2 = 2\log_7(3x)$

73. $2\log(x) = \log 25$

74. $2\ln(x) = \ln(100)$

Exercise Set 3.4: Exponential and Logarithmic Equations and Inequalities

For each of the following inequalities,

(a) Solve for x. (Note: Further simplification may occur in step (b).) Write the answer in terms of natural logarithms, unless no logarithms are involved.

(b) Rewrite the answer so that each individual logarithm from part (a) is written as a sum or difference of logarithms of prime numbers,

$$\text{e.g., } \ln(40) = \ln(2^3 \cdot 5) = \ln(2^3) + \ln(5)$$
$$= 3\ln(2) + \ln(5).$$

Simplify further if possible, and then write the answer in interval notation.

(c) If the simplified answer from part (b) contains logarithms, rewrite the answer as a decimal, correct to the nearest thousandth; otherwise, simply rewrite the answer. Then write this answer in interval notation.

75. $8^{x+3} > 15$

76. $7^{x-4} \le 2$

77. $7^{x} > 0$

78. $7^{x} < 0$

79. $e^{2-x} \le 9$

80. $e^{x+4} > 10$

81. $0.2^{x} < 7$

82. $0.8^{x} \ge 6$

83. $8\left(0.4^{x} - 2\right) > 109$

84. $9\left(3 - 0.75^{x}\right) \le 11$

85. $5\left(8 + e^{x}\right) \le -6$

86. $\left(9 - e^{x}\right) \le 4$

87. $\log_2(x) \ge 0$

88. $\log_2(x) < 0$

89. $\ln(7 - 3x) \le 0$

90. $\ln(5x + 2) < 0$

Exercise Set 3.5: Applications of Exponential Functions

Answer the following.

1. Compound interest is calculated by the following formula:

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

State the meaning of each variable in the formula above.

2. Continuously compounded interest is calculated by the following formula:

$$A(t) = Pe^{rt}$$

State the meaning of each variable in the formula above.

3. The *present value* of a sum of money is the amount of money that must be invested now (in the *present*) in order to obtain a certain amount of money after t years.

In the following formula:

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

(a) Which variable represents the present value?
(b) Which variable represents the amount of money after t years?

4. Exponential growth is calculated according to the following formula:

$$N(t) = N_0\, e^{rt}$$

State the meaning of each variable in the formula above.

Answer the following. In general, round answers to the nearest hundredth. When giving answers for population size, round to the nearest whole number.

5. If \$3,000 is invested at an interest rate of 7% per year, find the value of the investment after 10 years if the interest is compounded:

(a) annually **(b)** semiannually
(c) quarterly **(d)** monthly
(e) continuously

6. If \$18,000 is invested at an interest rate of 4% per year, find the value of the investment after 15 years if the interest is compounded:

(a) annually **(b)** semiannually
(c) quarterly **(d)** monthly
(e) continuously

7. The *present value* of a sum of money is the amount of money that must be invested now (in the *present*) in order to obtain a certain amount of money after t years. Find the present value of \$15,000 if interest is paid a rate of 8% per year, compounded semiannually, for 5 years.

8. The *present value* of a sum of money is the amount of money that must be invested now (in the *present*) in order to obtain a certain amount of money after t years. Find the present value of \$20,000 if interest is paid a rate of 5% per year, compounded quarterly, for 14 years.

9. Find the annual percentage rate for an investment that earns 6% per year, compounded quarterly.

10. Find the annual percentage rate for an investment that earns 4% per year, compounded monthly.

11. Which of the following scenarios would be the better investment?
(a) 7.5% per year, compounded quarterly
(b) 7.3% per year, compounded continuously

12. Which of the following scenarios would be the better investment?
(a) 8% per year, compounded semiannually
(b) 7.97% per year, compounded continuously

13. A population of rabbits grows in such a way that the population t days from now is given by $A(t) = 250e^{0.02t}$.

(a) How many rabbits are present now?
(b) What is the relative growth rate of the rabbits? (Write your answer as a percentage.)
(c) How many rabbits will there be after 4 days?
(d) How many rabbits will there be after three weeks?
(e) How many days will it take for the rabbit population to double?

Exercise Set 3.5: Applications of Exponential Functions

14. A population of mosquitoes grows in such a way that the population t hours from now is given by $A(t) = 1{,}350e^{0.001t}$.

(a) How many mosquitoes are present now?
(b) What is the relative growth rate of the mosquitoes? (Write your answer as a percentage.)
(c) How many mosquitoes will there be after 6 hours?
(d) How many mosquitoes will there be after two days?
(e) How many hours will it take for the mosquito population to triple?

15. There are approximately 18,000 bacteria in a culture, and the bacteria have a relative growth rate of 75% per hour.

(a) Write a function that models the bacteria population t hours from now.
(b) Use the function found in part (a) to estimate the bacteria population 8 hours from now.
(c) In how many hours will the bacteria population reach 99,000?

16. In the year 2000, there were approximately 35,000 people living in Exponentia, and the population of Exponentia has a relative growth rate of 4.1% per year.

(a) Write a function that models the population of Exponentia t years from the year 2000.
(b) Use the function found in part (a) to estimate the population of Exponentia in the year 2007.
(c) In how many years will the population reach 42,000?

17. The population of Smallville has a relative growth rate of 1.2% per year. If the population of Smallville in 1995 was 2,761, find the projected population of the town in 2005.

18. The ladybug population in a certain area is currently estimated to be 4,000, with a relative growth rate of 1.5% per day. Estimate the number of ladybugs 9 days from now.

19. An environmental group is studying a particular type of buffalo, and they move a certain number of these buffalo into a wildlife preserve. The relative growth rate for this population of buffalo is 2% per year. Five years after the buffalos are moved into the preserve, the group finds that there are 350 buffalo.

(a) How many buffalo were initially moved into the preserve?
(b) Estimate the buffalo population after 10 years of being in the preserve.
(c) How long from the time of the initial move will it take for the buffalo population to grow to 1,000?

20. A certain culture of bacteria has a relative growth rate of 135% per hour. Two hours after the culture is formed, the count shows approximately 10,000 bacteria.

(a) Find the initial number of bacteria in the culture.
(b) Estimate the number of bacteria in the culture 9 hours after the culture was started. Round your answer to the nearest million.
(c) How long after the culture is formed will it take for the bacteria population to grow to 1,000,000?

21. A certain radioactive substance decays according to the formula $m(t) = 70e^{-0.062t}$, where m represents the mass in grams that remains after t days.

(a) Find the initial mass of the radioactive substance.
(b) How much of the mass remains after 15 days?
(c) What percent of mass remains after 15 days?
(d) Find the half-life of this substance.

22. A certain radioactive substance decays according to the formula $m(t) = 12e^{-0.089t}$, where m represents the mass in grams that remains after t days.

(a) Find the initial mass of the radioactive substance.
(b) How much of the mass remains after 20 days?
(c) What percent of mass remains after 20 days?
(d) Find the half-life of this substance.

23. If \$20,000 is invested at an interest rate of 5% per year, compounded quarterly,

(a) Find the value of the investment after 7 years.
(b) How long will it take for the investment to double in value?

24. If \$7,000 is invested at an interest rate of 6.5% per year, compounded monthly,

(a) Find the value of the investment after 4 years.
(b) How long will it take for the investment to triple in value?

25. If \$4,000 is invested at an interest rate of 10% per year, compounded continuously,

(a) Find the value of the investment after 15 years.
(b) How long will it take for the investment to grow to a value of \$7,000?

26. If \$20,000 is invested at an interest rate of 5.25% per year, compounded continuously,

(a) Find the value of the investment after 12 years.
(b) How long will it take for the investment to grow to a value of \$50,000?

27. A certain radioactive substance has a half-life of 50 years. If 75 grams are present initially,

(a) Write a function that models the mass of the substance remaining after t years. (Within the formula, round the rate to the nearest ten-thousandth.)
(b) Use the function found in part (a) to estimate how much of the mass remains after 80 years.
(c) How many years will it take for the substance to decay to a mass of 10 grams?

28. A certain radioactive substance has a half-life of 120 years. If 850 milligrams are present initially,

(a) Write a function that models the mass of the substance remaining after t years. (Within the formula, round the rate to the nearest ten-thousandth.)
(b) Use the function found in part (a) to estimate how much of the mass remains after 500 years.
(c) How many years will it take for the substance to decay to a mass of 200 milligrams?

Exercise Set 4.1: Special Right Triangles and Trigonometric Ratios

Answer the following.

1. If two sides of a triangle are congruent, then the __________ opposite those sides are also congruent.

2. If two angles of a triangle are congruent, then the __________ opposite those angles are also congruent.

3. In any triangle, the sum of the measures of its angles is _____ degrees.

4. In an isosceles right triangle, each acute angle measures _____ degrees.

5. Fill in each missing blank with one of the following: *smallest, largest*

In any triangle, the longest side is opposite the __________ angle, and the shortest side is opposite the __________ angle.

6. Fill in each missing blank with one of the following: 30^o, 60^o, 90^o

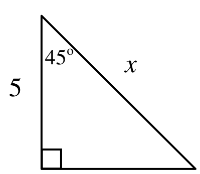
In a 30^o-60^o-90^o triangle, the hypotenuse is opposite the _____ angle, the shorter leg is opposite the _____ angle, and the longer leg is opposite the _____ angle.

For each of the following,

(a) Use the theorem for 45^o-45^o-90^o triangles to find x.

(b) Use the Pythagorean Theorem to verify the result obtained in part (a).

7.



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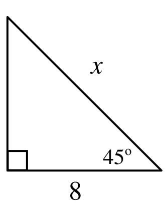

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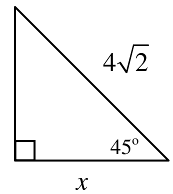

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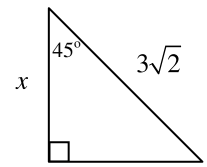

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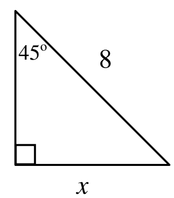

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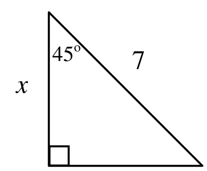

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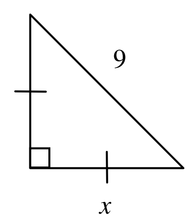

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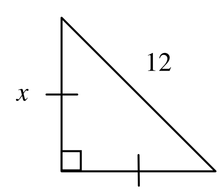

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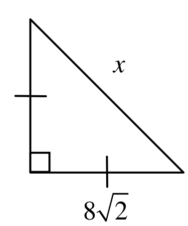

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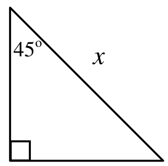

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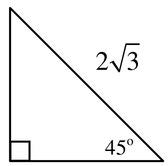

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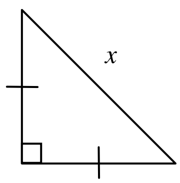
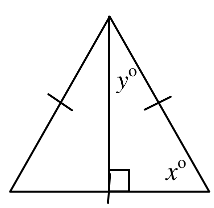


The following examples help to illustrate the theorem regarding 30°-60°-90° triangles.

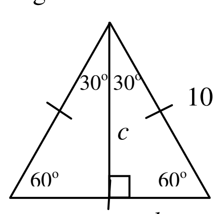
19. What is the measure of each angle of an equilateral triangle?

20. An altitude is drawn to the base of the equilateral triangle drawn below. Find the measures of x and y.



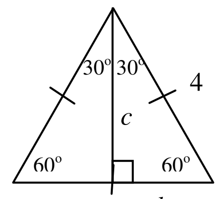
21. In the figure below, an altitude is drawn to the base of an equilateral triangle.

(a) Find a and b.
(b) Justify the answer obtained in part (a).
(c) Use the Pythagorean Theorem to find c, the length of the altitude.



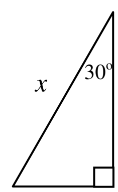
22. In the figure below, an altitude is drawn to the base of an equilateral triangle.

(a) Find a and b.
(b) Justify the answer obtained in part (a).
(c) Use the Pythagorean Theorem to find c, the length of the altitude. (Write c in simplest radical form.)



For each of the following, Use the theorem for 30°-60°-90° triangles to find x and y.

23.



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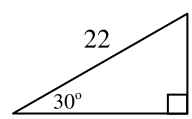

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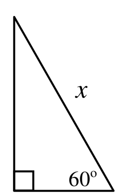

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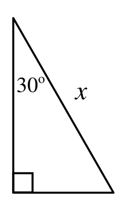

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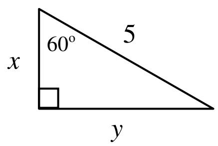

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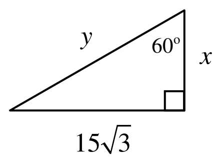

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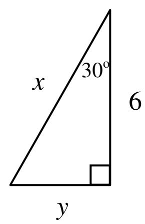

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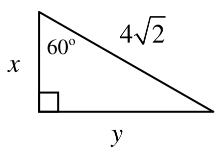

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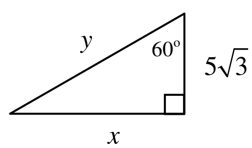

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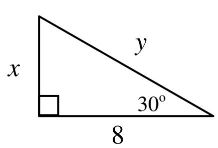
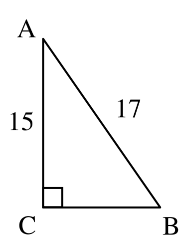


Answer the following. Write answers in simplest form.

33.



(a) Use the Pythagorean Theorem to find BC.

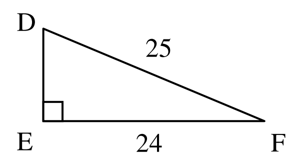
(b) Find the following:

$\sin(A) =$ _____ $\sin(B) =$ _____

$\cos(A) =$ _____ $\cos(B) =$ _____

$\tan(A) =$ _____ $\tan(B) =$ _____

34.



(a) Use the Pythagorean Theorem to find DE.

(b) Find the following:

$\sin(D) =$ _____ $\sin(F) =$ _____

$\cos(D) =$ _____ $\cos(F) =$ _____

$\tan(D) =$ _____ $\tan(F) =$ _____

35. Suppose that θ is an acute angle of a right triangle and $\sin(\theta) = \frac{5}{7}$. Find $\cos(\theta)$ and $\tan(\theta)$.

36. Suppose that θ is an acute angle of a right triangle and $\tan(\theta) = \frac{4\sqrt{2}}{7}$. Find $\sin(\theta)$ and $\cos(\theta)$.

37. The reciprocal of the sine function is the _______________ function.

38. The reciprocal of the cosine function is the _______________ function.

39. The reciprocal of the tangent function is the _______________ function.

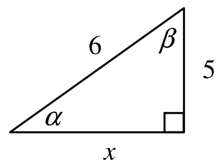
40. The reciprocal of the cosecant function is the _______________ function.

41. The reciprocal of the secant function is the _______________ function.

42. The reciprocal of the cotangent function is the _______________ function.

Exercise Set 4.1: Special Right Triangles and Trigonometric Ratios

43.

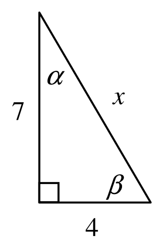


(a) Use the Pythagorean Theorem to find x.

(b) Find the six trigonometric functions of α.

(c) Find the six trigonometric functions of β.

44.

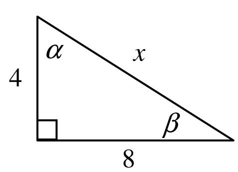


(a) Use the Pythagorean Theorem to find x.

(b) Find the six trigonometric functions of α.

(c) Find the six trigonometric functions of β.

45.

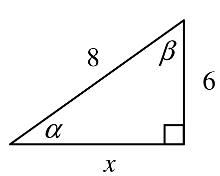


(a) Use the Pythagorean Theorem to find x.

(b) Find the six trigonometric functions of α.

(c) Find the six trigonometric functions of β.

46.



(a) Use the Pythagorean Theorem to find x.

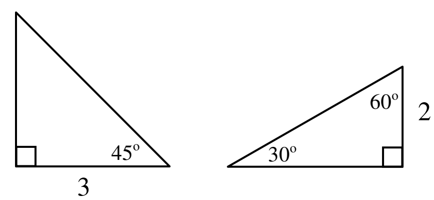
(b) Find the six trigonometric functions of α.

(c) Find the six trigonometric functions of β.

47. Suppose that θ is an acute angle of a right triangle and $\cot(\theta) = \dfrac{2\sqrt{10}}{3}$. Find the six trigonometric functions of θ.

48. Suppose that θ is an acute angle of a right triangle and $\sec(\theta) = \dfrac{5}{2}$. Find the six trigonometric functions of θ.

49.



(a) Use the theorems for special right triangles to find the missing side lengths in the triangles above.

(b) Using the triangles above, find the following:

$\sin(45^\circ) =$ _____	$\csc(45^\circ) =$ _____
$\cos(45^\circ) =$ _____	$\sec(45^\circ) =$ _____
$\tan(45^\circ) =$ _____	$\cot(45^\circ) =$ _____

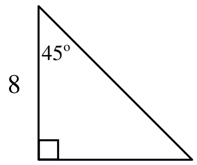
(c) Using the triangles above, find the following:

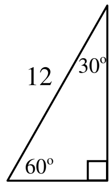
$\sin(30^\circ) =$ _____	$\csc(30^\circ) =$ _____
$\cos(30^\circ) =$ _____	$\sec(30^\circ) =$ _____
$\tan(30^\circ) =$ _____	$\cot(30^\circ) =$ _____

(d) Using the triangles above, find the following:

$\sin(60^\circ) =$ _____	$\csc(60^\circ) =$ _____
$\cos(60^\circ) =$ _____	$\sec(60^\circ) =$ _____
$\tan(60^\circ) =$ _____	$\cot(60^\circ) =$ _____

50.





(a) Use the theorems for special right triangles to find the missing side lengths in the triangles above.

(b) Using the triangles above, find the following:

$\sin(45^\circ) = \_\_\_\_\_$ $\csc(45^\circ) = \_\_\_\_\_$

$\cos(45^\circ) = \_\_\_\_\_$ $\sec(45^\circ) = \_\_\_\_\_$

$\tan(45^\circ) = \_\_\_\_\_$ $\cot(45^\circ) = \_\_\_\_\_$

(c) Using the triangles above, find the following:

$\sin(30^\circ) = \_\_\_\_\_$ $\csc(30^\circ) = \_\_\_\_\_$

$\cos(30^\circ) = \_\_\_\_\_$ $\sec(30^\circ) = \_\_\_\_\_$

$\tan(30^\circ) = \_\_\_\_\_$ $\cot(30^\circ) = \_\_\_\_\_$

(d) Using the triangles above, find the following:

$\sin(60^\circ) = \_\_\_\_\_$ $\csc(60^\circ) = \_\_\_\_\_$

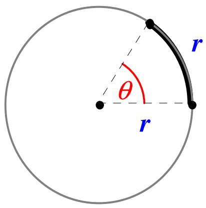
$\cos(60^\circ) = \_\_\_\_\_$ $\sec(60^\circ) = \_\_\_\_\_$

$\tan(60^\circ) = \_\_\_\_\_$ $\cot(60^\circ) = \_\_\_\_\_$

51. Compare the answers to parts (b), (c), and (d) in the previous two examples. What do you notice?

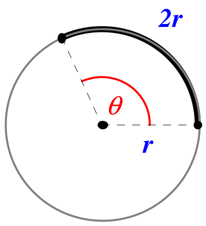
Exercise Set 4.2: Radians, Arc Length, and the Area of a Sector

If we use central angles of a circle to analyze angle measure, a *radian* is an angle for which the arc of the circle has the same length as the radius, as illustrated in the figures below.



$\theta = 1$ radian, since the arc length is the same as the length of the radius.

Note: Standard notation is to say $\theta = 1$; radians are implied when there is no angle measure.



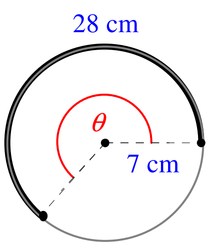
$\theta = 2$ radians, since the arc length is twice the length of the radius.

Note: Standard notation is to say $\theta = 2$.

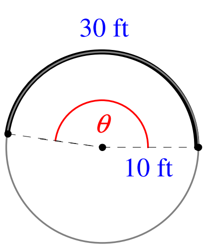
The number of radians can therefore be determined by dividing the arc length *s* by the radius *r*, i.e. $\theta = \dfrac{s}{r}$.

Find θ in the examples below.

1.



2.



3. $r = 0.3$ m; $s = 72$ cm

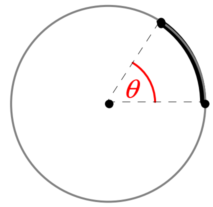
4. $r = 0.6$ in; $s = 3.06$ in

5. $r = 64$ ft; $s = 32$ ft

6. $r = 60$ in; $s = 21$ ft

Answer the following.

7. In the figure below, $\theta = 1$ radian.



(a) Use this figure as a guide to sketch and estimate the number of radians in a complete revolution.

(b) Give an exact number for the number of radians in a complete revolution. Justify your answer. Then round this answer to the nearest hundredth and compare it to the result from part (a).

8. Fill in the blanks:

(a) $360^\circ =$ _____ radians

(b) $180^\circ =$ _____ radians

Convert the following degree measures to radians. First, give an exact result. Then round each answer to the nearest hundredth.

9. **(a)** 30° **(b)** 90° **(c)** 135°

10. **(a)** 45° **(b)** 60° **(c)** 150°

11. **(a)** 120° **(b)** 225° **(c)** 330°

12. **(a)** 210° **(b)** 270° **(c)** 315°

13. **(a)** 19° **(b)** 40° **(c)** 72°

14. **(a)** 10° **(b)** 53° **(c)** 88°

Convert the following radian measures to degrees.

15. **(a)** $\dfrac{\pi}{4}$ **(b)** $\dfrac{4\pi}{3}$ **(c)** $\dfrac{5\pi}{6}$

16. **(a)** $\dfrac{\pi}{2}$ **(b)** $\dfrac{7\pi}{6}$ **(c)** $\dfrac{5\pi}{4}$

17. **(a)** π **(b)** $\dfrac{11\pi}{12}$ **(c)** $\dfrac{61\pi}{36}$

18. **(a)** $\dfrac{\pi}{9}$ **(b)** $\dfrac{7\pi}{18}$ **(c)** $\dfrac{53\pi}{30}$

Exercise Set 4.2: Radians, Arc Length, and the Area of a Sector

Convert the following radian measures to degrees. Round answers to the nearest hundredth.

19. **(a)** 2.5 **(b)** 0.506

20. **(a)** 3.8 **(b)** 0.297

Answer the following.

21. If two angles of a triangle have radian measures $\frac{\pi}{12}$ and $\frac{2\pi}{5}$, find the radian measure of the third angle.

22. If two angles of a triangle have radian measures $\frac{\pi}{9}$ and $\frac{3\pi}{8}$, find the radian measure of the third angle.

To find the length of the arc of a circle, think of the arc length as simply a fraction of the circumference of the circle. If the central angle θ defining the arc is given in degrees, then the arc length can be found using the formula:

$$s = \frac{\theta}{360^\circ}(2\pi r)$$

Use the formula above to find the arc length s.

23. $\theta = 60^\circ$; $r = 12$ cm

24. $\theta = 90^\circ$; $r = 10$ in

25. $\theta = 225^\circ$; $r = 4$ ft

26. $\theta = 150^\circ$; $r = 12$ cm

If the central angle θ defining the arc is instead given in radians, then the arc length can be found using the formula:

$$s = \frac{\theta}{2\pi}(2\pi r) = r\theta$$

Use the formula $s = r\theta$ to find the arc length s:

27. $\theta = \frac{7\pi}{6}$; $r = 9$ yd

28. $\theta = \frac{3\pi}{4}$; $r = 6$ cm

29. $\theta = \pi$; $r = 2$ ft

30. $\theta = \frac{5\pi}{3}$; $r = 30$ in

In numbers 31-34, change θ to radians and then find the arc length using the formula $s = r\theta$. Compare results with those from exercises 23-26.

31. $\theta = 60^\circ$; $r = 12$ cm

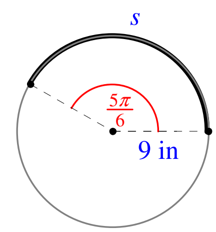
32. $\theta = 90^\circ$; $r = 10$ in

33. $\theta = 225^\circ$; $r = 4$ ft

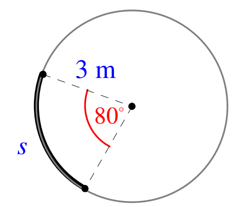
34. $\theta = 150^\circ$; $r = 12$ cm

Find the missing measure in each example below.

35.



36.



37. $r = 6$ cm; $\theta = 300^\circ$; $s =$?

38. $r = 10$ ft; $\theta = \frac{3\pi}{4}$; $s =$?

39. $s = \frac{12\pi}{5}$ m; $\theta = \frac{\pi}{2}$; $r =$?

40. $s = \frac{50\pi}{3}$ in; $\theta = \frac{5\pi}{6}$; $r =$?

41. $s = 7$ ft; $\theta = \frac{3\pi}{4}$; $r =$?

42. $s = 20$ cm; $\theta = \frac{2\pi}{3}$; $r =$?

43. $s = 12$ in; $\theta = 5$; $r =$?

44. $s = 7$ in; $\theta = 3$; $r =$?

45. $s = 15$ m; $\theta = 270^\circ$; $r =$?

46. $s = 8$ yd; $\theta = 225^\circ$; $r =$?

Exercise Set 4.2: Radians, Arc Length, and the Area of a Sector

Answer the following.

47. Find the perimeter of a sector of a circle with central angle $\frac{\pi}{6}$ and radius 8 cm.

48. Find the perimeter of a sector of a circle with central angle $\frac{7\pi}{4}$ and radius 3 ft.

To find the area of a sector of a circle, think of the sector as simply a fraction of the circle. If the central angle θ defining the sector is given in degrees, then the area of the sector can be found using the formula:

$$A = \frac{\theta}{360^\circ}\left(\pi r^2\right)$$

Use the formula above to find the area of the sector:

49. $\theta = 60^\circ$; $r = 12$ cm

50. $\theta = 90^\circ$; $r = 10$ in

51. $\theta = 225^\circ$; $r = 4$ ft

52. $\theta = 150^\circ$; $r = 12$ cm

If the central angle θ defining the sector is instead given in radians, then the area of the sector can be found using the formula:

$$A = \frac{\theta}{2\pi}\left(\pi r^2\right) = \frac{1}{2}r^2\theta$$

Use the formula $A = \frac{1}{2}r^2\theta$ to find the area of the sector:

53. $\theta = \frac{7\pi}{6}$; $r = 9$ yd

54. $\theta = \frac{3\pi}{4}$; $r = 6$ cm

55. $\theta = \pi$; $r = 2$ ft

56. $\theta = \frac{5\pi}{3}$; $r = 30$ in

In numbers 57-60, change θ to radians and then find the area of the sector using the formula $A = \frac{1}{2}r^2\theta$. Compare results with those from exercises 49-52.

57. $\theta = 60^\circ$; $r = 12$ cm

58. $\theta = 90^\circ$; $r = 10$ in

59. $\theta = 225^\circ$; $r = 4$ ft

60. $\theta = 150^\circ$; $r = 12$ cm

Answer the following.

61. A sector of a circle has central angle $\frac{\pi}{4}$ and area $\frac{49\pi}{8}$ cm^2. Find the radius of the circle.

62. A sector of a circle has central angle $\frac{5\pi}{6}$ and area $\frac{5\pi}{27}$ ft^2. Find the radius of the circle.

63. A sector of a circle has central angle 120° and area $\frac{16\pi}{75}$ in^2. Find the radius of the circle.

64. A sector of a circle has central angle 210° and area $\frac{21\pi}{4}$ m^2. Find the radius of the circle.

65. A sector of a circle has radius 6 ft and area $\frac{63\pi}{2}$ ft^2. Find the central angle of the sector (in radians).

66. A sector of a circle has radius $\frac{4}{7}$ cm and area $\frac{8\pi}{49}$ cm^2. Find the central angle of the sector (in radians).

Answer the following. SHOW ALL WORK involved in obtaining each answer. Give exact answers unless otherwise indicated.

67. A CD has a radius of 6 cm. If the CD's rate of turn is 900°/sec, find the following.

(a) The angular speed in units of radians/sec.
(b) The linear speed in units of cm/sec of a point on the outer edge of the CD.
(c) The linear speed in units of cm/sec of a point halfway between the center of the CD and its outer edge.

68. Each blade of a fan has a radius of 11 inches. If the fan's rate of turn is 1440°/sec, find the following.

(a) The angular speed in units of radians/sec.
(b) The linear speed in units of inches/sec of a point on the outer edge of the blade.
(c) The linear speed in units of inches/sec of a point on the blade 7 inches from the center.

69. A bicycle has wheels measuring 26 inches in diameter. If the bicycle is traveling at a rate of 20 miles per hour, find the wheels' rate of turn in revolutions per minute (rpm). Round the answer to the nearest hundredth.

70. A car has wheels measuring 16 inches in diameter. If the car is traveling at a rate of 55 miles per hour, find the wheels' rate of turn in revolutions per minute (rpm). Round the answer to the nearest hundredth.

71. A car has wheels with a 10 inch radius. If each wheel's rate of turn is 4 revolutions per second,
- **(a)** Find the angular speed in units of radians/second.
- **(b)** How fast is the car moving in units of inches/sec?
- **(c)** How fast is the car moving in miles per hour? Round the answer to the nearest hundredth.

72. A bicycle has wheels with a 12 inch radius. If each wheel's rate of turn is 2 revolutions per second,
- **(a)** Find the angular speed in units of radians/second.
- **(b)** How fast is the bicycle moving in units of inches/sec?
- **(c)** How fast is the bicycle moving in miles per hour? Round the answer to the nearest hundredth.

73. A clock has an hour hand, minute hand, and second hand that measure 4 inches, 5 inches, and 6 inches, respectively. Find the distance traveled by the tip of each hand in 20 minutes.

74. An outdoor clock has an hour hand, minute hand, and second hand that measure 12 inches, 14 inches, and 15 inches, respectively. Find the distance traveled by the tip of each hand in 45 minutes.

Exercise Set 4.3: Unit Circle Trigonometry

Sketch each of the following angles in standard position. (Do not use a protractor; just draw a quick sketch of each angle.)

1. **(a)** 30° **(b)** -135° **(c)** 300°

2. **(a)** 120° **(b)** -60° **(c)** 210°

3. **(a)** $\frac{\pi}{3}$ **(b)** $\frac{7\pi}{4}$ **(c)** $-\frac{5\pi}{6}$

4. **(a)** $-\frac{\pi}{4}$ **(b)** $\frac{4\pi}{3}$ **(c)** $\frac{11\pi}{6}$

5. **(a)** 90° **(b)** $-\pi$ **(c)** 450°

6. **(a)** 270° **(b)** 180° **(c)** -4π

7. **(a)** -240° **(b)** $\frac{13\pi}{6}$ **(c)** -510°

8. **(a)** -315° **(b)** $-\frac{9\pi}{4}$ **(c)** 1020°

Find three angles, one negative and two positive, that are coterminal with each angle below.

9. **(a)** 50° **(b)** -200°

10. **(a)** 300° **(b)** -830°

11. **(a)** $\frac{2\pi}{5}$ **(b)** $-\frac{7\pi}{2}$

12. **(a)** $\frac{8\pi}{3}$ **(b)** $-\frac{4\pi}{9}$

Answer the following.

13. List four quadrantal angles in degree measure, where each angle θ satisfies the condition $0^\circ \le \theta < 360^\circ$.

14. List four quadrantal angles in radian measure, where each angle θ satisfies the condition $\frac{\pi}{2} \le \theta < \frac{5\pi}{2}$.

15. List four quadrantal angles in radian measure, where each angle θ satisfies the condition $\frac{5\pi}{4} < \theta \le \frac{13\pi}{4}$.

16. List four quadrantal angles in degree measure, where each angle θ satisfies the condition $800^\circ < \theta \le 1160^\circ$.

Sketch each of the following angles in standard position and then specify the reference angle or reference number.

17. **(a)** 240° **(b)** -30° **(c)** 60°

18. **(a)** 315° **(b)** 150° **(c)** -120°

19. **(a)** $\frac{7\pi}{6}$ **(b)** $\frac{5\pi}{3}$ **(c)** $-\frac{7\pi}{4}$

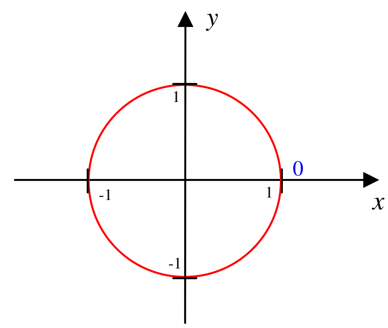
20. **(a)** $-\frac{\pi}{3}$ **(b)** $\frac{5\pi}{4}$ **(c)** $-\frac{5\pi}{6}$

21. **(a)** 840° **(b)** $-\frac{37\pi}{6}$ **(c)** -660°

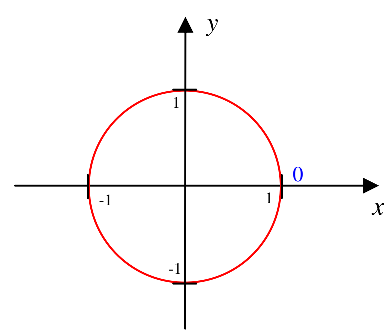
22. **(a)** $\frac{11\pi}{3}$ **(b)** -780° **(c)** $-\frac{11\pi}{4}$

The exercises below are helpful in creating a comprehensive diagram of the unit circle. Answer the following.

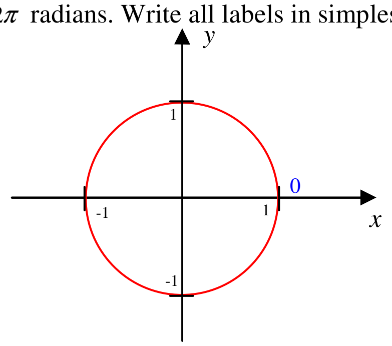
23. Using the following unit circle, draw and then label the terminal side of all multiples of $\frac{\pi}{2}$ from 0 to 2π radians. Write all labels in simplest form.



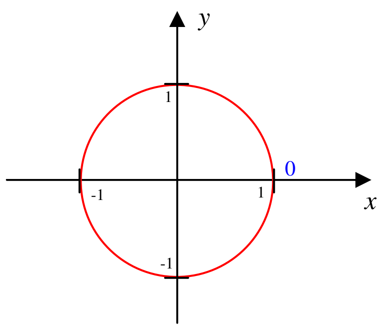
24. Using the following unit circle, draw and then label the terminal side of all multiples of $\frac{\pi}{4}$ from 0 to 2π radians. Write all labels in simplest form.



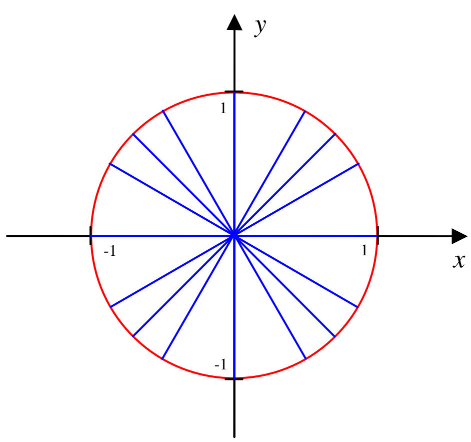
25. Using the following unit circle, draw and then label the terminal side of all multiples of $\frac{\pi}{3}$ from 0 to 2π radians. Write all labels in simplest form.



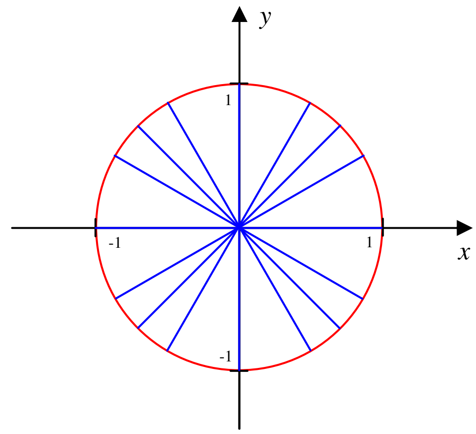
26. Using the following unit circle, draw and then label the terminal side of all multiples of $\frac{\pi}{6}$ from 0 to 2π radians. Write all labels in simplest form.



27. Use the information from numbers 23-26 to label all the special angles on the unit circle in radians.



28. Label all the special angles on the unit circle in degrees.



Name the quadrant in which the given conditions are satisfied.

29. $\sin(\theta) > 0,\ \cos(\theta) < 0$

30. $\sin(\theta) < 0,\ \sec(\theta) > 0$

31. $\cot(\theta) > 0,\ \sec(\theta) < 0$

32. $\csc(\theta) > 0,\ \cot(\theta) > 0$

33. $\tan(\theta) < 0,\ \csc(\theta) < 0$

34. $\csc(\theta) < 0,\ \tan(\theta) > 0$

Fill in each blank with $<, >,$ or $=$.

35. $\sin(40^\circ)$ _____ $\sin(140^\circ)$

36. $\cos(20^\circ)$ _____ $\cos(160^\circ)$

37. $\tan(310^\circ)$ _____ $\tan(50^\circ)$

38. $\sin(195^\circ)$ _____ $\sin(15^\circ)$

39. $\cos(355^\circ)$ _____ $\cos(185^\circ)$

40. $\tan(110^\circ)$ _____ $\tan(290^\circ)$

Exercise Set 4.3: Unit Circle Trigonometry

Let $P(x, y)$ denote the point where the terminal side of an angle θ meets the unit circle. Use the given information to evaluate the six trigonometric functions of θ.

41. P is in Quadrant I and $y = \frac{2}{3}$.

42. P is in Quadrant I and $x = \frac{3}{8}$.

43. P is in Quadrant IV and $x = \frac{4}{5}$.

44. P is in Quadrant III and $y = -\frac{24}{25}$.

45. P is in Quadrant II and $x = -\frac{1}{5}$.

46. P is in Quadrant II and $y = \frac{2}{7}$.

For each quadrantal angle below, give the coordinates of the point where the terminal side of the angle intersects the unit circle. Then give the six trigonometric functions of the angle. If a value is undefined, state "Undefined."

47. 90°

48. -180°

49. -2π

50. $\frac{3\pi}{2}$

51. $-\frac{5\pi}{2}$

52. 8π

Rewrite each expression in terms of its reference angle, deciding on the appropriate sign (positive or negative). For example,

$$\sin(240^\circ) = -\sin(60^\circ) \qquad \tan\left(\frac{4\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right)$$

$$\sec(-315^\circ) = \sec(45^\circ) \qquad \cos\left(\frac{7\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right)$$

53. (a) $\cos(300^\circ)$ **(b)** $\tan(135^\circ)$

54. (a) $\sin(-45^\circ)$ **(b)** $\cot(210^\circ)$

55. (a) $\sin(140^\circ)$ **(b)** $\sec\left(-\frac{2\pi}{3}\right)$

56. (a) $\csc(-190^\circ)$ **(b)** $\cos\left(\frac{11\pi}{6}\right)$

57. (a) $\csc\left(\frac{25\pi}{6}\right)$ **(b)** $\cot(-460^\circ)$

58. (a) $\tan(520^\circ)$ **(b)** $\sec\left(-\frac{9\pi}{4}\right)$

For each angle below, give the coordinates of the point where the terminal side of the angle intersects the unit circle. Then give the six trigonometric functions of the angle.

59. 45°

60. 60°

61. 210°

62. 135°

63. $\frac{5\pi}{3}$

64. $\frac{11\pi}{6}$

An alternate method of finding trigonometric functions of 30°, 45°, or 60° is shown below.

65.

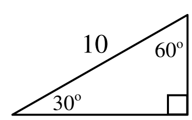


Diagram 1

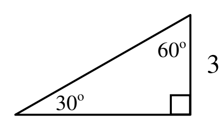


Diagram 2

(a) Find the missing side measures in each of the diagrams above.

(b) Use right triangle trigonometric ratios to find the following, using Diagram 1:

$\sin(30^\circ) =$ _____ $\sin(60^\circ) =$ _____

$\cos(30^\circ) =$ _____ $\cos(60^\circ) =$ _____

$\tan(30^\circ) =$ _____ $\tan(60^\circ) =$ _____

(c) Repeat part (b), using Diagram 2.

(d) Use the unit circle to find the trigonometric ratios listed in part (b).

(e) Examine the answers in parts (b) through (d). What do you notice?

Exercise Set 4.3: Unit Circle Trigonometry

66.

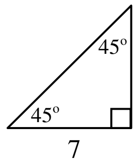


Diagram 1

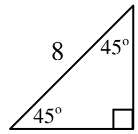


Diagram 2

(a) Find the missing side measures in each of the diagrams above.

(b) Use right triangle trigonometric ratios to find the following, using Diagram 1:

$\sin(45^\circ) =$ _____ $\qquad \csc(45^\circ) =$ _____

$\cos(45^\circ) =$ _____ $\qquad \sec(60^\circ) =$ _____

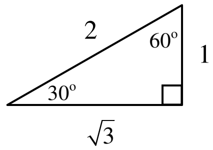
$\tan(45^\circ) =$ _____ $\qquad \cot(60^\circ) =$ _____

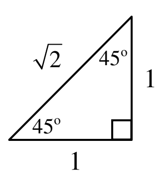
(c) Repeat part (b), using Diagram 2.

(d) Use the unit circle to find the trigonometric ratios listed in part (b).

(e) Examine the answers in parts (b) through (d). What do you notice?

The following two diagrams can be used to quickly evaluate the trigonometric functions of any angle having a reference angle of 30°, 45°, or 60°. Use right trigonometric ratios along with the concept of reference angles, to evaluate the following. (Remember that when converting degrees to radians, $30^\circ = \frac{\pi}{6}$, $45^\circ = \frac{\pi}{4}$, $60^\circ = \frac{\pi}{3}$.)





67. **(a)** $\sin(240^\circ)$ **(b)** $\tan(135^\circ)$

68. **(a)** $\cos(330^\circ)$ **(b)** $\csc(-225^\circ)$

69. **(a)** $\cos\left(-\frac{\pi}{4}\right)$ **(b)** $\sec\left(\frac{11\pi}{6}\right)$

70. **(a)** $\sin\left(\frac{2\pi}{3}\right)$ **(b)** $\cot\left(-\frac{7\pi}{6}\right)$

Use either the unit circle or the right triangle method from numbers 65-70 to evaluate the following. (Note: The right triangle method can not be used for quadrantal angles.) If a value is undefined, state "Undefined."

71. **(a)** $\tan(30^\circ)$ **(b)** $\sin(-135^\circ)$

72. **(a)** $\cos(180^\circ)$ **(b)** $\csc(-60^\circ)$

73. **(a)** $\csc(-150^\circ)$ **(b)** $\sin(270^\circ)$

74. **(a)** $\sec(225^\circ)$ **(b)** $\tan(-240^\circ)$

75. **(a)** $\cot(-450^\circ)$ **(b)** $\cos(495^\circ)$

76. **(a)** $\sin(-210^\circ)$ **(b)** $\cot(-420^\circ)$

77. **(a)** $\csc(\pi)$ **(b)** $\cos\left(\frac{2\pi}{3}\right)$

78. **(a)** $\sin\left(-\frac{\pi}{3}\right)$ **(b)** $\cot\left(\frac{5\pi}{4}\right)$

79. **(a)** $\sec\left(-\frac{\pi}{6}\right)$ **(b)** $\tan\left(\frac{3\pi}{4}\right)$

80. **(a)** $\csc\left(\frac{11\pi}{4}\right)$ **(b)** $\sec\left(-\frac{3\pi}{2}\right)$

81. **(a)** $\cot\left(-\frac{10\pi}{3}\right)$ **(b)** $\sec\left(\frac{7\pi}{4}\right)$

82. **(a)** $\tan(-5\pi)$ **(b)** $\cos\left(-\frac{11\pi}{2}\right)$

Use a calculator to evaluate the following to the nearest ten-thousandth. Make sure that your calculator is in the appropriate mode (degrees or radians).

Note: Be careful when evaluating the reciprocal trigonometric functions. For example, when evaluating $\csc(\theta)$ on your calculator, use the identity $\csc(\theta) = \frac{1}{\sin(\theta)}$. Do NOT use the calculator key labeled $\sin^{-1}(\theta)$; this represents the inverse sine function, which will be discussed in Section 5.4.

83. **(a)** $\sin(37^\circ)$ **(b)** $\tan(-218^\circ)$

84. **(a)** $\tan(350^\circ)$ **(b)** $\cos(-84^\circ)$

85. **(a)** $\csc(191^\circ)$ **(b)** $\cot(21^\circ)$

86. **(a)** $\cot\left(310^\circ\right)$ **(b)** $\sec\left(73^\circ\right)$

87. **(a)** $\cos\left(\dfrac{\pi}{5}\right)$ **(b)** $\csc\left(-\dfrac{11\pi}{7}\right)$

88. **(a)** $\sin\left(-\dfrac{10\pi}{9}\right)$ **(b)** $\cot(4.7\pi)$

89. **(a)** $\tan(-4.5)$ **(b)** $\sec(3)$

90. **(a)** $\csc(-0.457)$ **(b)** $\tan(9.4)$

Exercise Set 4.4: Trigonometric Expressions and Identities

Answer the following.

1. Beginning with the Pythagorean identity $\cos^2(\theta)+\sin^2(\theta)=1$, establish another Pythagorean identity by dividing each term by $\cos^2(\theta)$. Show all work.

2. Beginning with the Pythagorean identity $\cos^2(\theta)+\sin^2(\theta)=1$, establish another Pythagorean identity by dividing each term by $\sin^2(\theta)$. Show all work.

Solve the following algebraically, using identities from this section.

3. If $\cos(\theta)=\frac{5}{13}$ and $\frac{3\pi}{2}<\theta<2\pi$, find the exact values of $\sin(\theta)$ and $\tan(\theta)$.

4. If $\sin(\theta)=\frac{4}{5}$ and $\frac{\pi}{2}<\theta<\pi$, find the exact values of $\cos(\theta)$ and $\tan(\theta)$.

5. If $\sin(\theta)=-\frac{1}{9}$ and $180^\circ<\theta<270^\circ$, find the five remaining trigonometric functions of θ.

6. If $\cos(\theta)=\frac{1}{10}$ and $270^\circ<\theta<360^\circ$, find the five remaining trigonometric functions of θ.

7. If $\csc(\theta)=-\frac{9}{4}$ and $\frac{3\pi}{2}<\theta<2\pi$, find the exact values of $\cot(\theta)$ and $\sin(\theta)$.

8. If $\sec(\theta)=-\frac{5}{2}$ and $\pi<\theta<\frac{3\pi}{2}$, find the exact values of $\tan(\theta)$ and $\cos(\theta)$.

9. If $\cot(\theta)=-\frac{\sqrt{13}}{6}$ and $90^\circ<\theta<180^\circ$, find the exact values of $\csc(\theta)$ and $\sin(\theta)$.

10. If $\tan(\theta)=\frac{1}{3}$ and $180^\circ<\theta<270^\circ$, find the exact values of $\sec(\theta)$ and $\cot(\theta)$.

11. If $\tan(\theta)=\frac{3}{4}$ and $\pi<\theta<\frac{3\pi}{2}$, find the five remaining trigonometric functions of θ.

12. If $\cot(\theta)=-\frac{12}{5}$ and $\frac{\pi}{2}<\theta<\pi$, find the five remaining trigonometric functions of θ.

Perform the following operations and combine like terms. (Do not rewrite the terms or the solution in terms of any other trigonometric functions.)

13. $\left[\sin(\theta)-3\right]\left[\sin(\theta)+5\right]$

14. $\left[\tan(\theta)-7\right]\left[\tan(\theta)-2\right]$

15. $\left[\csc(x)-3\right]^2$

16. $\left[3+\sec(x)\right]^2$

17. $\frac{3}{\cos(\theta)}-\frac{7}{\sin(\theta)}$

18. $-\frac{6}{\sin(\theta)}+\frac{2}{\cos(\theta)}$

19. $5\tan(\theta)\sin(\theta)-8\tan(\theta)\sin(\theta)$

20. $8\cos(\theta)\cot(\theta)+9\cos(\theta)\cot(\theta)$

Factor each of the following expressions.

21. $25\sin^2(\theta)-49\cos^2(\theta)$

22. $16\cos^2(\theta)-81\sin^2(\theta)$

23. $\sec^2(\theta)-7\sec(\theta)+12$

24. $\tan^2(\theta)+9\tan(\theta)+12$

25. $10\cot^2(\theta)-13\cot(\theta)-3$

26. $6\csc^2(\theta)+7\csc(\theta)-5$

Simplify the following.

27. $\cos(\theta)\tan(\theta)$

28. $\sin(\theta)\cot(\theta)$

29. $\csc^2(x)\tan^2(x)$

30. $\sec^3(x)\cot^3(x)$

Exercise Set 4.4: Trigonometric Expressions and Identities

31. $\cot(x)\csc(x)\tan^2(x)$

32. $\sin(x)\sec(x)\cot^2(x)$

33. $1-\sin^2(x)$

34. $\cos^2(x)-1$

35. $[\sec(\theta)-1][\sec(\theta)+1]$

36. $[\csc(x)+1][\csc(x)-1]$

37. $\sin(\theta)[\csc(\theta)-\sin(\theta)]$

38. $\cos(\theta)[\sec(\theta)-\cos(\theta)]$

39. $[\sec^2(\theta)-1][\csc^2(\theta)-1]$

40. $\dfrac{1-\sin^2(x)}{\cos^2(x)-1}$

41. $\sin(x)[\cot(x)+\tan(x)]$

42. $\cos(x)[\tan(x)+\cot(x)]$

43. $[\sec(x)-\tan(x)][\sec(x)+\tan(x)]$

44. $[\csc(x)-\cot(x)][\csc(x)+\cot(x)]$

45. $[1-\cos(x)][\csc(x)+\cot(x)]$

46. $[\csc(x)-1][\sec(x)+\tan(x)]$

47. $\dfrac{\cot(x)+\tan(x)}{\csc^2(x)}$

48. $\dfrac{\sec^2(x)}{\tan(x)+\cot(x)}$

49. $\dfrac{\cos(x)}{1-\sin(x)}-\dfrac{\cos(x)}{1+\sin(x)}$

50. $\dfrac{\cot(\theta)}{\csc(\theta)+1}+\dfrac{\cot(\theta)}{\csc(\theta)-1}$

51. $\dfrac{\cos^2(\theta)-\sin^2(\theta)}{\sin^4(\theta)-\cos^4(\theta)}$

52. $\dfrac{\sec^4(x)-\tan^4(x)}{\sec^2(x)+\tan^2(x)}$

Prove each of the following identities.

53. $\sec(x)-\sin(x)\tan(x)=\cos(x)$

54. $\cos(x)\cot(x)-\csc(x)=-\sin(x)$

55. $\dfrac{\sec(\theta)\csc(\theta)}{\tan(\theta)+\cot(\theta)}=1$

56. $[\sin(\theta)-\cos(\theta)]^2+[\sin(\theta)+\cos(\theta)]^2=2$

57. $\tan(x)-\sin(x)\cos(x)=\tan(x)\sin^2(x)$

58. $\cot(x)-\cos(x)\sin(x)=\cot(x)\cos^2(x)$

59. $\cot^2(x)-\cos^2(x)=\cot^2(x)\cos^2(x)$

60. $\tan^2(x)-\sin^2(x)=\tan^2(x)\sin^2(x)$

61. $\dfrac{\cot(x)-\tan(x)}{\sin(x)\cos(x)}=\csc^2(x)-\sec^2(x)$

62. $\dfrac{\tan(x)}{1+\sec(x)}+\dfrac{1+\sec(x)}{\tan(x)}=2\csc(x)$

63. $2\cos^2(x)+5\sin^2(x)=2+3\sin^2(x)$

64. $3\sin^2(x)-4\cos^2(x)=3-7\cos^2(x)$

65. $\tan^4(\theta)+\tan^2(\theta)=\sec^4(\theta)-\sec^2(\theta)$

66. $\csc^4(x)-\csc^2(x)=\cot^4(x)+\cot^2(x)$

67. $\dfrac{\sin(x)}{1+\cos(x)}=\csc(x)-\cot(x)$

68. $\dfrac{\cos(x)}{1+\sin(x)}=\sec(x)-\tan(x)$

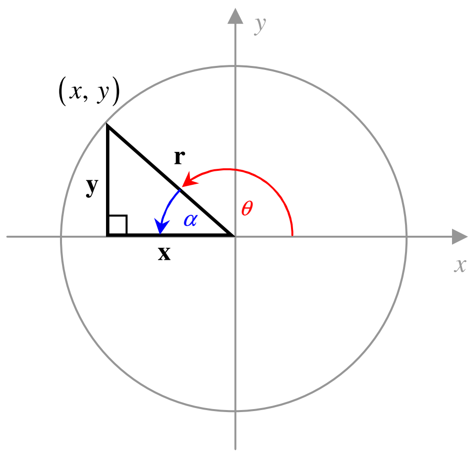
69. $\dfrac{\cos(x)}{1+\sin(x)}=\dfrac{1-\sin(x)}{\cos(x)}$

70. $\dfrac{1-\cos(\theta)}{\sin(\theta)}=\dfrac{\sin(\theta)}{1+\cos(\theta)}$

Exercise Set 4.4: Trigonometric Expressions and Identities

Another method of solving problems like exercises 3-12 is shown below.

Let the terminal side of an angle θ intersect the circle $x^2 + y^2 = r^2$. First draw the reference angle, α, for θ. Then, draw a right triangle with its hypotenuse extending from the origin to (x, y) and its leg is on the *x*-axis, as shown below. Label the legs *x* and *y* (<u>which may be negative</u>), and the hypotenuse with positive length *r*.



Then find the six trigonometric functions of α, using right triangle trigonometry ($\sin\alpha = \dfrac{\text{the leg opposite angle } \alpha}{\text{the hypotenuse}}$, etc…) The six trigonometric functions of α are the same as the six trigonometric functions of θ. (This can be compared with the ratios $\sin\theta = \dfrac{y}{r}$, etc. at the beginning of the text in Section 4.3).

In each of the following examples, use the given information to sketch and label a right triangle in the appropriate quadrant, as described above. Use the Pythagorean Theorem to find the missing side, and then use right triangle trigonometry to evaluate the five remaining trigonometric functions of θ.

71. $\sin(\theta) = -\dfrac{1}{4}$ and $270^\circ < \theta < 360^\circ$

72. $\cos(\theta) = -\dfrac{\sqrt{13}}{7}$ and $180^\circ < \theta < 270^\circ$

73. $\sec(\theta) = -\dfrac{7}{2}$ and $\dfrac{\pi}{2} < \theta < \pi$

74. $\csc(\theta) = 5$ and $\dfrac{\pi}{2} < \theta < \pi$

75. $\tan(\theta) = \dfrac{2}{5}$ and $\pi < \theta < \dfrac{3\pi}{2}$

76. $\cot(\theta) = -\dfrac{\sqrt{11}}{5}$ and $\dfrac{3\pi}{2} < \theta < 2\pi$

Answer the following.

77. If the terminal ray of an angle θ in standard position passes through the point $(12, -5)$, find the six trigonometric functions of θ.

78. If the terminal ray of an angle θ in standard position passes through the point $(-8, 15)$, find the six trigonometric functions of θ.

79. If the terminal ray of an angle θ in standard position passes through the point $(-6, -2)$, find the six trigonometric functions of θ.

80. If the terminal ray of an angle θ in standard position passes through the point $(3, -9)$, find the six trigonometric functions of θ.

Exercise Set 5.1: Trigonometric Functions of Real Numbers

Use the given information to evaluate the five remaining trigonometric functions of *t*. If a value is undefined, state "Undefined."

Note: These questions are similar to those in Chapter 4. The difference is that these are trigonometric functions of real numbers, rather than of angles. The solution process, however, is the same.

1. $\sin(t) = -\frac{2}{7}$ and $\frac{3\pi}{2} < t < 2\pi$

2. $\cos(t) = -\frac{1}{5}$ and $\pi < t < \frac{3\pi}{2}$

3. $\sec(t) = -4$ and $\frac{\pi}{2} < t < \pi$

4. $\csc(t) = -\frac{8}{5}$ and $\frac{3\pi}{2} < t < 2\pi$

5. $\cot(t) = \frac{4}{3}$ and $\pi < t < 2\pi$

6. $\tan(t) = -\frac{15}{8}$ and $\frac{\pi}{2} < t < \frac{3\pi}{2}$

7. $\tan(t) = -\frac{2\sqrt{10}}{3}$ and $\frac{3\pi}{2} < t < 2\pi$

8. $\cot(t) = 4\sqrt{3}$ and $\pi < t < \frac{3\pi}{2}$

Use the opposite-angle identities and/or the periodicity identities to evaluate the following.

Note: These questions are similar to those in Chapter 4. The identities mentioned above offer an alternative method of finding trigonometric functions of negative numbers or of numbers whose absolute value is greater than 2π.

9. **(a)** $\cos\left(-\frac{\pi}{4}\right)$ **(b)** $\csc\left(-\frac{\pi}{6}\right)$

10. **(a)** $\sin\left(-\frac{\pi}{3}\right)$ **(b)** $\sec\left(-\frac{\pi}{4}\right)$

11. **(a)** $\sin\left(-\frac{2\pi}{3}\right)$ **(b)** $\tan\left(-\frac{\pi}{2}\right)$

12. **(a)** $\cos\left(-\frac{\pi}{2}\right)$ **(b)** $\cot\left(-\frac{5\pi}{6}\right)$

13. **(a)** $\sec\left(-\frac{4\pi}{3}\right)$ **(b)** $\cot\left(-\frac{11\pi}{6}\right)$

14. **(a)** $\csc\left(-\frac{5\pi}{4}\right)$ **(b)** $\tan\left(-\frac{5\pi}{3}\right)$

15. **(a)** $\sin\left(\frac{11\pi}{4}\right)$ **(b)** $\cot\left(\frac{8\pi}{3}\right)$

16. **(a)** $\cos\left(\frac{17\pi}{6}\right)$ **(b)** $\tan\left(\frac{15\pi}{4}\right)$

17. **(a)** $\cot\left(\frac{11\pi}{2}\right)$ **(b)** $\sec\left(\frac{13\pi}{3}\right)$

18. **(a)** $\csc\left(\frac{31\pi}{6}\right)$ **(b)** $\sec\left(\frac{13\pi}{2}\right)$

19. **(a)** $\csc(9\pi)$ **(b)** $\cos\left(-\frac{19\pi}{3}\right)$

20. **(a)** $\tan\left(-\frac{35\pi}{4}\right)$ **(b)** $\sin(-11\pi)$

21. **(a)** $\dfrac{\sin\left(\frac{49\pi}{4}\right)}{\tan\left(\frac{29\pi}{4}\right)}$

(b) $\tan(6\pi) - \sin\left(\frac{14\pi}{3}\right)\cos\left(\frac{17\pi}{6}\right)$

22. **(a)** $\dfrac{\csc\left(\frac{19\pi}{3}\right)}{\sec\left(\frac{19\pi}{3}\right)}$

(b) $\cot\left(\frac{10\pi}{3}\right) + \dfrac{\cot\left(\frac{16\pi}{3}\right)}{\cos\left(\frac{17\pi}{6}\right)}$

23. **(a)** $\dfrac{\sin\left(\frac{20\pi}{3}\right)\cot\left(\frac{11\pi}{4}\right)}{\cos\left(-\frac{5\pi}{6}\right)}$

(b) $\dfrac{\sin\left(\frac{19\pi}{3}\right)\sin\left(\frac{31\pi}{6}\right)}{\cos(8\pi)\tan\left(\frac{5\pi}{4}\right)}$

Exercise Set 5.1: Trigonometric Functions of Real Numbers

24. (a) $\dfrac{\tan\left(\frac{11\pi}{4}\right)\cos\left(\frac{19\pi}{6}\right)}{\sin\left(\frac{31\pi}{3}\right)}$

(b) $\dfrac{\cos\left(\frac{3\pi}{2}\right)\sin(6\pi)}{\sin\left(\frac{19\pi}{2}\right)\tan\left(\frac{21\pi}{4}\right)}$

Simplify the following. Write all trigonometric functions in terms of *t*.

25. $\sin(-t)\csc(t)$

26. $\cos(-t)\sec(t)$

27. $\sec(-t)\cot(-t)$

28. $\csc(-t)\tan(-t)$

29. $\dfrac{\tan(t)}{\sin(-t)}$

30. $\dfrac{\cot(-t)}{\cos(-t)}$

31. $\cos(-t)\cot(-t)+\sin(-t)$

32. $\sin(-t)\tan(-t)+\cos(-t)$

33. $\sin(-t)+\sin(-t)\cot^2(-t)$

34. $\cos(-t)+\cos(-t)\tan^2(-t)$

35. $\sec(t+4\pi)\cos(t+2\pi)$

36. $\sin(t+6\pi)\csc(t-2\pi)$

37. $\dfrac{1+\tan(t-\pi)}{1+\cot(t+2\pi)}$

38. $\dfrac{\sec(t+2\pi)+\csc(t-6\pi)}{1+\tan(t+3\pi)}$

Prove each of the following identities.

39. $\dfrac{\cos(-t)}{1-\sin(-t)}=\sec(-t)+\tan(-t)$

40. $\dfrac{\sin(-t)}{1+\cos(-t)}=\cot(t)+\csc(-t)$

41. $\dfrac{\sin(t)}{\cos(-t)-1}=\dfrac{1+\cos(-t)}{\sin(-t)}$

42. $\dfrac{1-\sin(-t)}{\cos(-t)}=\dfrac{\cos(t)}{1+\sin(-t)}$

43. $\dfrac{1+\sec(-t)}{\csc(-t)}=\sin(-t)+\tan(-t)$

44. $\dfrac{1-\csc(-t)}{\sec(-t)}=\cos(-t)-\cot(-t)$

45. $\dfrac{\sin(t-4\pi)}{1+\cos(t+2\pi)}+\dfrac{1+\cos(t-2\pi)}{\sin(t+6\pi)}=2\csc(t+4\pi)$

46. $\dfrac{\sin(t+8\pi)}{1-\cot(t-2\pi)}-\dfrac{\cos(t-2\pi)}{\tan(t+\pi)-1}=\sin(t)+\cos(t)$

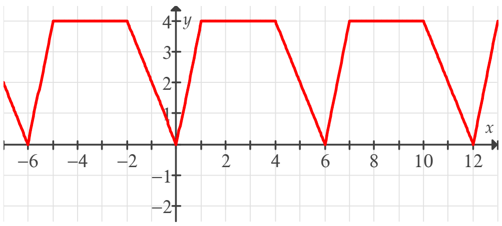
Exercise Set 5.2: Graphs of the Sine and Cosine Functions

For each of the following functions,

(a) Find the period.

(b) Find the amplitude.

1.



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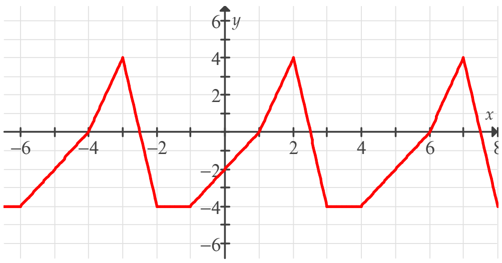

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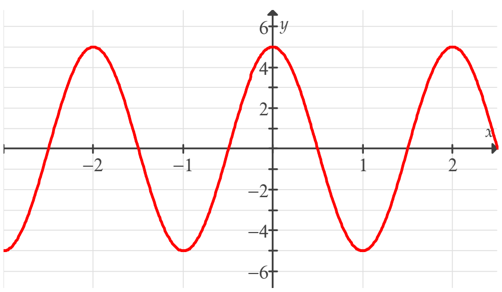

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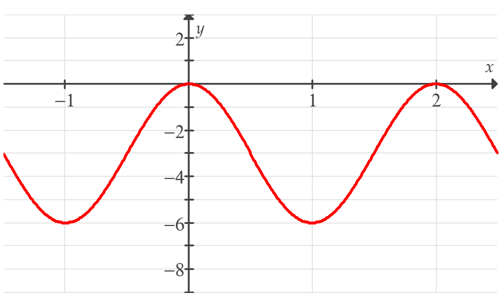

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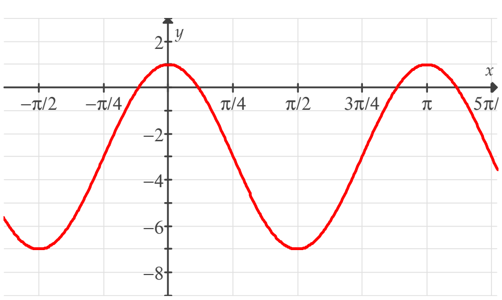

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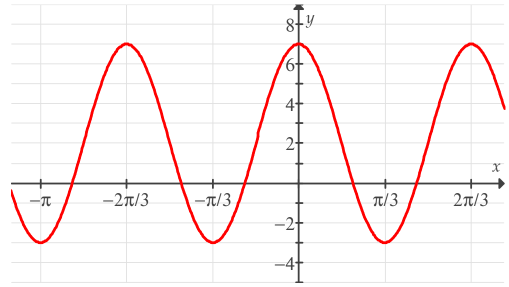
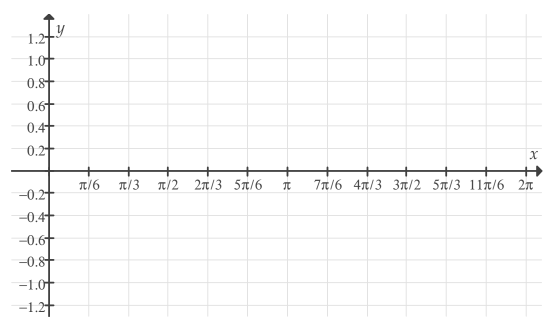


Answer the following.

7. **(a)** Use your calculator to complete the following chart. Round to the nearest hundredth.

x	$y=\sin(x)$	x	$y=\sin(x)$
0		π	
$\frac{\pi}{6}$		$\frac{7\pi}{6}$	
$\frac{\pi}{4}$		$\frac{5\pi}{4}$	
$\frac{\pi}{3}$		$\frac{4\pi}{3}$	
$\frac{\pi}{2}$		$\frac{3\pi}{2}$	
$\frac{2\pi}{3}$		$\frac{5\pi}{3}$	
$\frac{3\pi}{4}$		$\frac{7\pi}{4}$	
$\frac{5\pi}{6}$		$\frac{11\pi}{6}$	
		2π	

(b) Plot the points from part (a) to discover the graph of $f(x)=\sin(x)$.



(c) Use the graph from part (b) to sketch an extended graph of $f(x)=\sin(x)$, where $-4\pi \le x \le 4\pi$. Be sure to show the intercepts as well as the maximum and minimum values of the function.

(d) State the domain and range of $f(x) = \sin(x)$. Do not base this answer simply on the limited domain from part (c), but the entire graph of $f(x) = \sin(x)$.

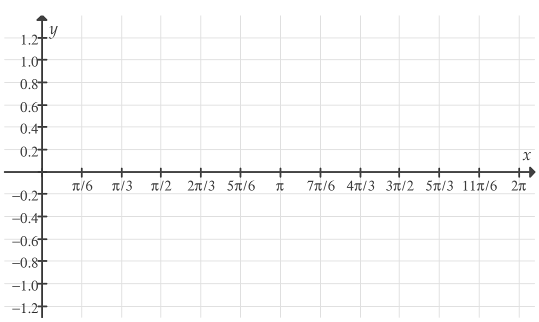
(e) State the amplitude and period of $f(x) = \sin(x)$.

(f) Give an interval on which $f(x) = \sin(x)$ is increasing.

8. **(a)** Use your calculator to complete the following chart. Round to the nearest hundredth.

x	$y = \cos(x)$	x	$y = \cos(x)$
0		π	
$\frac{\pi}{6}$		$\frac{7\pi}{6}$	
$\frac{\pi}{4}$		$\frac{5\pi}{4}$	
$\frac{\pi}{3}$		$\frac{4\pi}{3}$	
$\frac{\pi}{2}$		$\frac{3\pi}{2}$	
$\frac{2\pi}{3}$		$\frac{5\pi}{3}$	
$\frac{3\pi}{4}$		$\frac{7\pi}{4}$	
$\frac{5\pi}{6}$		$\frac{11\pi}{6}$	
		2π	

(b) Plot the points from part (a) to discover the graph of $g(x) = \cos(x)$.



(c) Use the graph from part (b) to sketch an extended graph of $g(x) = \cos(x)$, where $-4\pi \le x \le 4\pi$. Be sure to show the intercepts as well as the maximum and minimum values of the function.

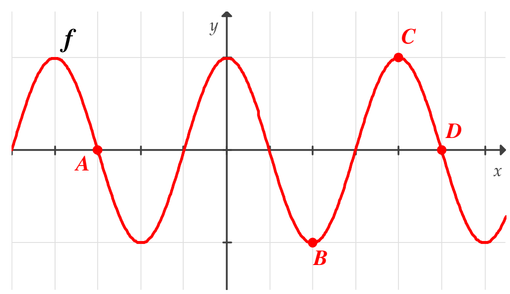
(d) State the domain and range of $g(x) = \cos(x)$. Do not base this answer simply on the limited domain from part (c), but the entire graph of $g(x) = \cos(x)$.

(e) State the amplitude and period of $g(x) = \cos(x)$.

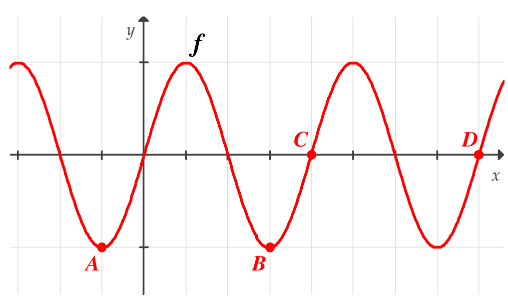
(f) Give an interval on which $g(x) = \cos(x)$ is decreasing.

Answer the following.

9. Use the graph of $f(x) = \cos(x)$ below to determine the coordinates of points A, B, C, and D.



10. Use the graph of $f(x) = \sin(x)$ below to determine the coordinates of points A, B, C, and D.



Exercise Set 5.2: Graphs of the Sine and Cosine Functions

Use the graphs of $f(x)=\sin(x)$ and $g(x)=\cos(x)$ to answer the following.

11. Use the appropriate graph to find the following function values.

(a) $\cos(0)$

(b) $\cos\left(-\dfrac{3\pi}{2}\right)$

(c) $\sin\left(\dfrac{5\pi}{2}\right)$

(d) $\sin(-3\pi)$

12. Use the appropriate graph to find the following function values.

(a) $\sin(0)$

(b) $\sin\left(-\dfrac{\pi}{2}\right)$

(c) $\cos(\pi)$

(d) $\cos\left(-\dfrac{7\pi}{2}\right)$

13. For $0 \le x < 2\pi$, use the graph of $f(x)=\sin(x)$ to find the value(s) of *x* for which:

(a) $\sin(x)=1$

(b) $\sin(x)=0$

(c) $\sin(x)=-1$

14. For $0 \le x < 2\pi$, use the graph of $g(x)=\cos(x)$ to find the value(s) of *x* for which:

(a) $\cos(x)=0$

(b) $\cos(x)=-1$

(c) $\cos(x)=1$

15. For $-2\pi \le x < 0$, use the graph of $g(x)=\cos(x)$ to find the value(s) of *x* for which:

(a) $\cos(x)=-1$

(b) $\cos(x)=1$

(c) $\cos(x)=0$

16. For $-2\pi \le x < 0$, use the graph of $f(x)=\sin(x)$ to find the value(s) of *x* for which:

(a) $\sin(x)=1$

(b) $\sin(x)=-1$

(c) $\sin(x)=0$

Graph each of the following functions over the interval $-2\pi \le x \le 2\pi$.

17. $f(x)=|\cos(x)|$

18. $g(x)=|\sin(x)|$

Matching. The left-hand column contains equations that represent transformations of $f(x)=\sin(x)$. Match the equations on the left with the description on the right of how to obtain the graph of $y=g(x)$ from the graph of f.

19. $y=\sin(x-3)$

20. $y=\sin(x)-3$

21. $y=\sin(3x)$

22. $y=\dfrac{1}{3}\sin(x)$

23. $y=3\sin(x)$

24. $y=\sin(x+3)$

25. $y=\sin(-x+3)$

26. $y=\sin\left(\dfrac{1}{3}x\right)$

27. $y=\sin(x+3)+2$

28. $y=-\sin(x-2)+3$

29. $y=-\sin(x)-3$

30. $y=\sin(x)+3$

A. Shrink horizontally by a factor of $\frac{1}{3}$.

B. Shift left 3 units, then reflect in the *y*-axis.

C. Reflect in the *x*-axis, then shift downward 3 units.

D. Shift right 3 units.

E. Shift right 2 units, then reflect in the *x*-axis, then shift upward 3 units.

F. Shift upward 3 units.

G. Stretch horizontally by a factor of 3.

H. Shift left 3 units, then shift upward 2 units.

I. Shift left 3 units.

J. Shift downward 3 units.

K. Stretch vertically by a factor of 3.

L. Shrink vertically by a factor of $\frac{1}{3}$.

Exercise Set 5.2: Graphs of the Sine and Cosine Functions

Matching. The left-hand column contains equations that represent transformations of $f(x)=\cos(x)$. Match the equations on the left with the description on the right of how to obtain the graph of $y=g(x)$ from the graph of f.

31. $y=\cos\left[2(x-8)\right]$

32. $y=\cos(2x-8)$

33. $y=\cos(2x)-8$

34. $y=\cos\left[\frac{1}{2}(x-8)\right]$

35. $y=\cos\left[\frac{1}{2}x-8\right]$

36. $y=\cos\left(\frac{1}{2}x\right)-8$

A. Stretch horizontally by a factor of 2, then shift right 16 units.

B. Shrink horizontally by a factor of $\frac{1}{2}$, then shift right 4 units.

C. Shrink horizontally by a factor of $\frac{1}{2}$, then shift downward 8 units.

D. Stretch horizontally by a factor of 2, then shift right 8 units.

E. Stretch horizontally by a factor of 2, then shift downward 8 units.

F. Shrink horizontally by a factor of $\frac{1}{2}$, then shift right 8 units.

For each of the following functions,

(a) State the period.
(b) State the amplitude.
(c) State the phase shift.
(d) State the vertical shift.
(e) Use transformations to sketch the graph of the function over one period. For consistency in solutions, perform the appropriate transformations on the function $g(x)=\sin(x)$ or $h(x)=\cos(x)$ where $0 \le x \le 2\pi$. (Note: The resulting graph may not fall within this same interval.) The x-axis should be labeled to show the transformations of the intercepts of $g(x)=\sin(x)$ or $h(x)=\cos(x)$, as well as any x-values where a maximum or minimum occurs. The y-axis should be labeled to reflect the maximum and minimum values of the function.

37. $f(x)=4\sin(x)$

38. $f(x)=3\cos(x)$

39. $f(x)=-5\cos(x)$

40. $f(x)=-2\sin(x)$

41. $f(x)=6\sin(x)-2$

42. $f(x)=4\cos(x)+3$

43. $f(x)=\cos(2x)$

44. $f(x)=-\sin(3x)$

45. $f(x)=-\cos\left(\dfrac{x}{4}\right)$

46. $f(x)=\dfrac{2}{5}\cos\left(\dfrac{x}{3}\right)$

47. $f(x)=\dfrac{4}{3}\sin(\pi x)$

48. $f(x)=-\cos\left(\dfrac{\pi x}{3}\right)+4$

49. $f(x)=\sin\left(\dfrac{\pi x}{2}\right)-1$

50. $f(x)=4\cos\left(\dfrac{\pi x}{3}\right)+2$

51. $f(x)=\cos\left(x-\dfrac{\pi}{2}\right)$

52. $f(x)=\sin(x+\pi)$

53. $f(x)=-3\cos(\pi x+\pi)$

54. $f(x)=5\sin(4\pi x+\pi)$

55. $f(x)=\sin(2x+3\pi)$

56. $f(x)=\cos(3x-\pi)$

57. $f(x)=-7\cos(4x-\pi)$

58. $f(x)=5\sin\left(\dfrac{1}{2}x+\dfrac{3\pi}{4}\right)$

59. $f(x) = -3\sin\left(2\pi x - \dfrac{2\pi}{3}\right) + 5$

60. $f(x) = 5\cos\left(\dfrac{\pi}{3}x + \pi\right) + 2$

61. $f(x) = 4\sin\left(\dfrac{x}{2} - \dfrac{\pi}{2}\right) - 2$

62. $f(x) = -3\cos\left(2x - \dfrac{\pi}{2}\right) + 4$

63. $f(x) = 7\sin(-x)$ *Omit part (c)*

64. $f(x) = 10\sin(-x)$ *Omit part (c)*

65. $f(x) = -2\sin(-2\pi x)$ *Omit part (c)*

66. $f(x) = -6\sin(-3x) - 4$ *Omit part (c)*

67. $f(x) = 4\cos\left(\dfrac{\pi}{2} - x\right) + 3$ *Omit part(c)*

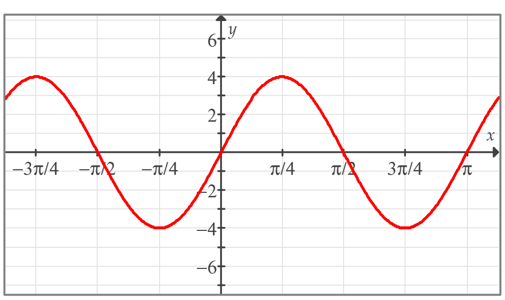
68. $f(x) = 5\sin(\pi - x)$ *Omit part(c)*

For each of the following graphs,

(a) **Give an equation of the form $f(x) = A\sin(Bx - C) + D$ which could be used to represent the graph.** ***(Note: C or D may be zero. Answers vary.)***

(b) **Give an equation of the form $f(x) = A\cos(Bx - C) + D$ which could be used to represent the graph.** ***(Note: C or D may be zero. Answers vary.)***

69.



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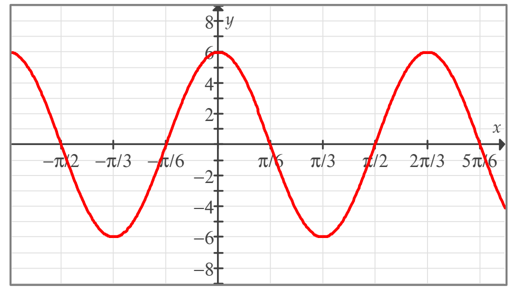

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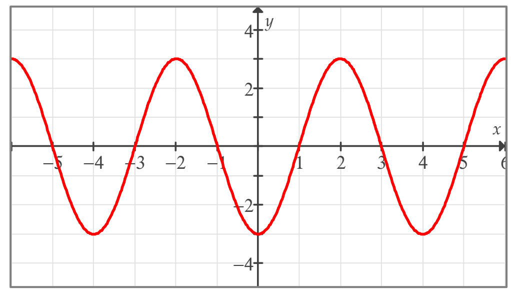

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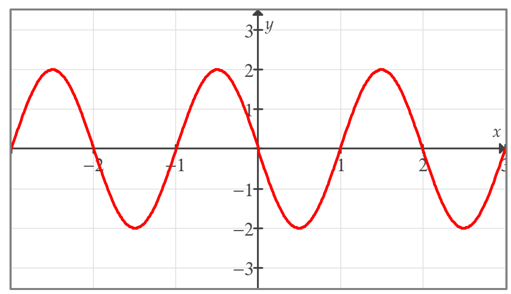

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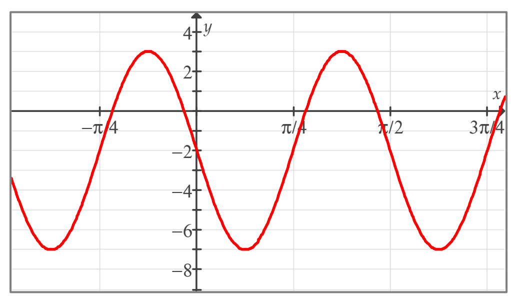

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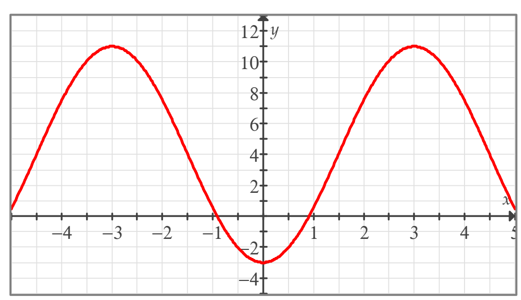
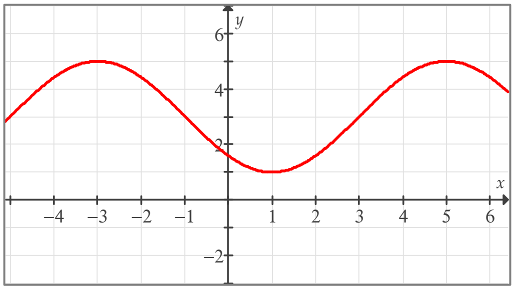


75.



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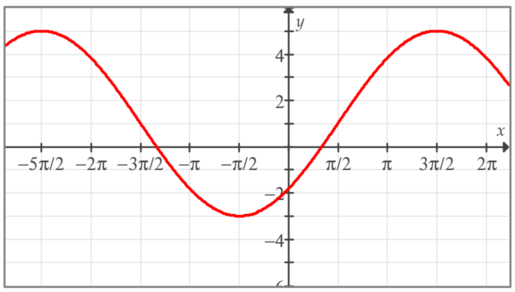


Answer the following.

77. The depth of the water at a certain point near the shore varies with the ocean tide. Suppose that the high tide occurs today at 2AM with a depth of 6 meters, and the low tide occurs at 8:30AM with a depth of 2 meters.

(a) Write a trigonometric equation that models the depth, D, of the water t hours after midnight.

(b) Use a calculator to find the depth of the water at 11:15AM. (Round to the nearest hundredth.)

78. The depth of the water at a certain point near the shore varies with the ocean tide. Suppose that the high tide occurs today at 4AM with a depth of 9 meters, and the low tide occurs at 11:45AM with a depth of 3 meters.

(a) Write a trigonometric equation that models the depth, D, of the water t hours after midnight.

(b) Use a calculator to find the depth of the water at 2:30PM. (Round to the nearest hundredth.)

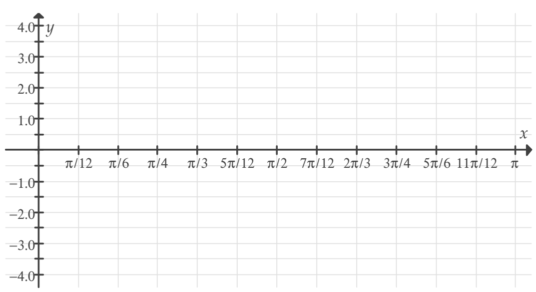
Exercise Set 5.3: Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

Answer the following.

1. **(a)** Use your calculator to complete the following chart. Round to the nearest hundredth. If a value is undefined, state "Undefined." Notice that the *x*-values on the chart increase by increments of $\frac{\pi}{12}$.

x	$y = \tan(x)$	x	$y = \tan(x)$
0		$\frac{\pi}{2}$	
$\frac{\pi}{12}$		$\frac{7\pi}{12}$	
$\frac{\pi}{6}$		$\frac{2\pi}{3}$	
$\frac{\pi}{4}$		$\frac{3\pi}{4}$	
$\frac{\pi}{3}$		$\frac{5\pi}{6}$	
$\frac{5\pi}{12}$		$\frac{11\pi}{12}$	
		π	

(b) Plot the points from part (a) to discover the graph of $f(x) = \tan(x)$.



(c) Use the graph from part (b) to sketch an extended graph of $f(x) = \tan(x)$, where $-2\pi \le x \le 2\pi$. Be sure to show the intercepts as well as any asymptotes.

(d) State the domain and range of $f(x) = \tan(x)$. Do not base this answer simply on the limited domain from part (c), but the entire graph of $f(x) = \tan(x)$.

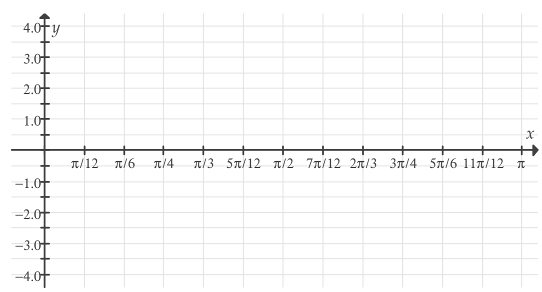
(e) State the period of $f(x) = \tan(x)$.

(f) Give an interval, if one exists, on which $f(x) = \tan(x)$ is increasing. Then give an interval , if one exists, on which $f(x) = \tan(x)$ is decreasing.

2. **(a)** Use your calculator to complete the following chart. Round to the nearest hundredth. If a value is undefined, state "Undefined." Notice that the *x*-values on the chart increase by increments of $\frac{\pi}{12}$.

x	$y = \cot(x)$	x	$y = \cot(x)$
0		$\frac{\pi}{2}$	
$\frac{\pi}{12}$		$\frac{7\pi}{12}$	
$\frac{\pi}{6}$		$\frac{2\pi}{3}$	
$\frac{\pi}{4}$		$\frac{3\pi}{4}$	
$\frac{\pi}{3}$		$\frac{5\pi}{6}$	
$\frac{5\pi}{12}$		$\frac{11\pi}{12}$	
		π	

(b) Plot the points from part (a) to discover the graph of $g(x) = \cot(x)$.



(c) Use the graph from part (b) to sketch an extended graph of $g(x) = \cot(x)$, where $-2\pi \le x \le 2\pi$. Be sure to show the intercepts as well as any asymptotes.

Continued on the next page…

Exercise Set 5.3: Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

(d) State the domain and range of $g(x) = \cot(x)$. Do not base this answer simply on the limited domain from part (c), but the entire graph of $g(x) = \cot(x)$.

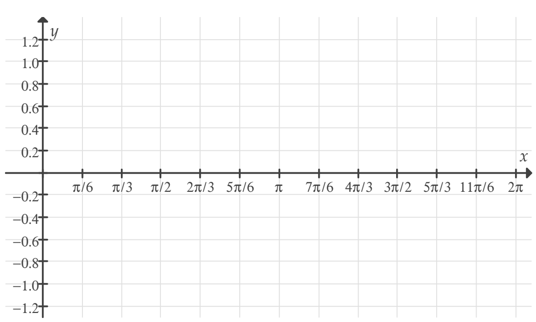
(e) State the period of $g(x) = \cot(x)$.

(f) Give an interval, if one exists, on which $g(x) = \cot(x)$ is increasing. Then give an interval , if one exists, on which $g(x) = \cot(x)$ is decreasing.

3. **(a)** Use your calculator to complete the following chart. Round to the nearest hundredth. If a value is undefined, state "Undefined."

x	$y = \sec(x)$	x	$y = \sec(x)$
0		π	
$\frac{\pi}{6}$		$\frac{7\pi}{6}$	
$\frac{\pi}{4}$		$\frac{5\pi}{4}$	
$\frac{\pi}{3}$		$\frac{4\pi}{3}$	
$\frac{\pi}{2}$		$\frac{3\pi}{2}$	
$\frac{2\pi}{3}$		$\frac{5\pi}{3}$	
$\frac{3\pi}{4}$		$\frac{7\pi}{4}$	
$\frac{5\pi}{6}$		$\frac{11\pi}{6}$	
		2π	

(b) Plot the points from part (a) to discover the graph of $f(x) = \sec(x)$.



(c) Use the graph from part (b) to sketch an extended graph of $f(x) = \sec(x)$, where $-2\pi \le x \le 2\pi$. Be sure to show the intercepts as well as any local maxima and minima.

(d) On the graph from part (c), superimpose the graph of $h(x) = \cos(x)$ lightly or in another color. What do you notice? How can this serve as an aid in sketching the graph of $f(x) = \sec(x)$?

(e) State the domain and range of $f(x) = \sec(x)$. Base this answer on the entire graph of $f(x) = \sec(x)$, not only on the partial graphs obtained in parts (b)-(d).

(f) State the period of $f(x) = \sec(x)$.

(g) Give two intervals on which $f(x) = \sec(x)$ is increasing.

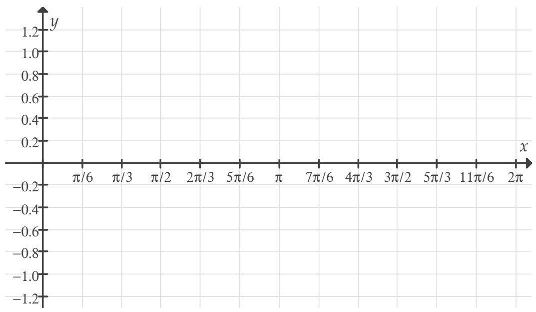
4. **(a)** Use your calculator to complete the following chart. Round to the nearest hundredth. If a value is undefined, state "Undefined."

x	$y = \csc(x)$	x	$y = \csc(x)$
0		π	
$\frac{\pi}{6}$		$\frac{7\pi}{6}$	
$\frac{\pi}{4}$		$\frac{5\pi}{4}$	
$\frac{\pi}{3}$		$\frac{4\pi}{3}$	
$\frac{\pi}{2}$		$\frac{3\pi}{2}$	
$\frac{2\pi}{3}$		$\frac{5\pi}{3}$	
$\frac{3\pi}{4}$		$\frac{7\pi}{4}$	
$\frac{5\pi}{6}$		$\frac{11\pi}{6}$	
		2π	

Continued on the next page…

Exercise Set 5.3: Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

(b) Plot the points from part (a) to discover the graph of $g(x) = \csc(x)$.



(c) Use the graph from part (b) to sketch an extended graph of $g(x) = \csc(x)$, where $-2\pi \le x \le 2\pi$. Be sure to show the intercepts as well as any local maxima and minima.

(d) On the graph from part (c), superimpose the graph of $h(x) = \sin(x)$ lightly or in another color. What do you notice? How can this serve as an aid in sketching the graph of $g(x) = \csc(x)$?

(e) State the domain and range of $g(x) = \csc(x)$. Base this answer on the entire graph of $g(x) = \csc(x)$, not only on the partial graphs obtained in parts (b)-(d).

(f) State the period of $g(x) = \csc(x)$.

(g) Give two intervals on which $g(x) = \csc(x)$ is increasing.

Matching. The left-hand column contains equations that represent transformations of $f(x) = \tan(x)$. Match the equations on the left with the description on the right of how to obtain the graph of $y = g(x)$ from the graph of f.

5. $y = \tan(x - 2)$

6. $y = \tan(x) - 2$

7. $y = \tan(2x)$

8. $y = \dfrac{1}{2}\tan(x)$

9. $y = \tan(2x - 8)$

10. $y = 2\tan(x)$

11. $y = \tan\left(\dfrac{1}{2}x\right)$

12. $y = \tan\left(\dfrac{x}{2}\right) - 4$

13. $y = -2\tan(4x - 4)$

14. $y = -\tan(x) + 2$

A. Shrink horizontally by a factor of $\frac{1}{2}$.

B. Reflect in the x-axis, then shift upward 2 units.

C. Shift right 2 units.

D. Reflect in the x-axis, stretch vertically by a factor of 2, shrink horizontally by a factor of $\frac{1}{4}$, then shift right 1 unit.

E. Stretch horizontally by a factor of 2, then shift downward 4 units.

F. Stretch horizontally by a factor of 2.

G. Shrink horizontally by a factor of $\frac{1}{2}$, then shift right 4 units.

H. Shift downward 2 units.

I. Stretch vertically by a factor of 2.

J. Shrink vertically by factor of $\frac{1}{2}$.

Continued on the next page…

Exercise Set 5.3: Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

Matching. The left-hand column contains equations that represent transformations of $f(x)=\csc(x)$. Match the equations on the left with the description on the right of how to obtain the graph of $y=g(x)$ from the graph of f.

15. $y=\frac{1}{2}\csc(3x+\pi)$

16. $y=\frac{1}{2}\csc\left[3(x+\pi)\right]$

17. $y=2\csc(3x)+\pi$

18. $y=2\csc\left[\frac{1}{3}\left(x+\frac{\pi}{3}\right)\right]$

19. $y=-\csc\left[\frac{1}{3}x+\frac{\pi}{3}\right]$

20. $y=-\csc\left(\frac{1}{3}x\right)+\pi$

A. Reflect in the x-axis, stretch horizontally by a factor of 3, then shift left π units.

B. Shrink vertically by a factor of $\frac{1}{2}$, shrink horizontally by a factor of $\frac{1}{3}$, then shift left $\frac{\pi}{3}$ units.

C. Stretch vertically by a factor of 2, shrink horizontally by a factor of $\frac{1}{3}$, then shift upward π units.

D. Stretch vertically by a factor of 2, stretch horizontally by a factor of 3, then shift left $\frac{\pi}{3}$ units.

E. Reflect in the x-axis, stretch horizontally by a factor of 3, then shift upward π units.

F. Shrink vertically by a factor of $\frac{1}{2}$, shrink horizontally by a factor of $\frac{1}{3}$, then shift left π units.

For each of the following functions,

(a) State the period.

(b) Use transformations to sketch the graph of the function over one period. Label any asymptotes clearly. Be sure to show the exact transformation of each point on the basic graphs of $g(x)=\tan(x)$ or $h(x)=\cot(x)$ whose x-value is a multiple of $\frac{\pi}{4}$. ***For consistency in solutions, perform the appropriate transformations on $g(x)=\tan(x)$ or $h(x)=\cot(x)$ within the following intervals. (Note: The resulting graph may not fall within these intervals.)***

$$g(x)=\tan(x),\ \text{where } -\frac{\pi}{2}<x<\frac{\pi}{2}$$

$$h(x)=\cot(x),\ \text{where } 0<x<\pi$$

21. $f(x)=5\tan(x)$

22. $f(x)=4\cot(x)$

23. $f(x)=4\tan(x)+1$

24. $f(x)=5\cot(x)-4$

25. $f(x)=2\cot(3x)$

26. $f(x)=-4\tan(2x)$

27. $f(x)=5\cot\left(-\frac{x}{4}\right)+2$

28. $f(x)=-3\cot\left(\frac{x}{2}\right)-5$

29. $f(x)=-7\tan(4\pi x)-3$

30. $f(x)=2\tan(-\pi x)+1$

31. $f(x)=\cot\left(x+\frac{\pi}{3}\right)$

32. $f(x)=-\tan\left(x-\frac{\pi}{2}\right)$

33. $f(x)=2\tan\left(\frac{x}{2}-\frac{\pi}{4}\right)-5$

34. $f(x)=5\tan\left(\frac{1}{4}x+\frac{\pi}{4}\right)-2$

35. $f(x)=-3\cot\left(\pi x+\frac{\pi}{2}\right)+2$

36. $f(x)=4\cot\left(\pi x-\frac{\pi}{3}\right)+3$

Exercise Set 5.3: Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

Answer the following.

37. The graph of $f(x)=\sin(x)$ can be useful in sketching transformations of the graph of $g(x)=\csc(x)$. Answer the following, using the interval $-2\pi \le x \le 2\pi$ for each graph.

- **(a)** Sketch the graphs of $f(x)=\sin(x)$ and $g(x)=\csc(x)$ on the same set of axes.
- **(b)** Where are the *x*-intercepts of $f(x)=\sin(x)$?
- **(c)** Where are the asymptotes of $g(x)=\csc(x)$? Explain the relationship between the answers in parts (b) and (c).
- **(d)** On a different set of axes, sketch the graph of $h(x)=-2\sin(3x)$.
- **(e)** Use the graph in part (d) to sketch the graph of $p(x)=-2\csc(3x)$ on the same set of axes.
- **(f)** On a different set of axes, sketch the graph of $q(x)=4\sin\left(x-\frac{\pi}{2}\right)-3$.
- **(g)** Use the graph in part (f) to sketch the graph of $r(x)=4\csc\left(x-\frac{\pi}{2}\right)-3$ on the same set of axes.

38. The graph of $f(x)=\cos(x)$ can be useful in sketching transformations of the graph of $g(x)=\sec(x)$. Answer the following, using the interval $-2\pi \le x \le 2\pi$ for each graph.

- **(a)** Sketch the graphs of $f(x)=\cos(x)$ and $g(x)=\sec(x)$ on the same set of axes.
- **(b)** Where are the *x*-intercepts of $f(x)=\cos(x)$?
- **(c)** Where are the asymptotes of $g(x)=\sec(x)$? Explain the relationship between the answers in parts (b) and (c).
- **(d)** On a different set of axes, sketch the graph of $h(x)=3\cos(2x)+1$.
- **(e)** Use the graph in part (d) to sketch the graph of $p(x)=3\sec(2x)+1$ on the same set of axes.
- **(f)** On a different set of axes, sketch the graph of $q(x)=-5\cos(2x+\pi)$.
- **(g)** Use the graph in part (f) to sketch the graph of $r(x)=-5\sec(2x+\pi)$ on the same set of axes.

For each of the following functions,

- **(a) State the period.**
- **(b) Use transformations to sketch the graph of the function over one period. The *x*-axis should be labeled to reflect the location of any *x*-values where a relative maximum or minimum occurs. The *y*-axis should be labeled to reflect any maximum and minimum values of the function. Label any asymptotes clearly.** ***For consistency in solutions, perform the appropriate transformations on $g(x)=\sec(x)$ where $0 \le x \le 2\pi$, or $h(x)=\csc(x)$, where $0 < x < 2\pi$. (Note: The resulting graph may not fall within these intervals.)***

39. $f(x)=8\sec(x)$

40. $f(x)=5\csc(x)$

41. $f(x)=-4\csc(x)+5$

42. $f(x)=2\sec(x)-3$

43. $f(x)=-\sec\left(\frac{x}{2}\right)$

44. $f(x)=-\frac{2}{3}\sec\left(\frac{x}{5}\right)$

45. $f(x)=7\sec(3x)-1$

46. $f(x)=-4\csc(2x)+3$

47. $f(x)=\frac{7}{2}\csc(2\pi x)+1$

48. $f(x)=-2\csc(-\pi x)-3$

Exercise Set 5.3: Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

49. $f(x) = 4\csc\left(-\dfrac{\pi x}{3}\right) + 5$

50. $f(x) = 2\sec\left(\dfrac{\pi x}{4}\right) - 6$

51. $f(x) = -4\sec(x - \pi) - 2$

52. $f(x) = 8\csc\left(x + \dfrac{\pi}{2}\right) + 3$

53. $f(x) = -2\csc\left(\pi x + \dfrac{\pi}{2}\right)$

54. $f(x) = 4\csc(2\pi x - \pi)$

55. $f(x) = 3\sec(\pi - 3x)$

56. $f(x) = -9\sec(3\pi - 2x)$

57. $f(x) = 2\csc\left(\dfrac{x}{3} - \dfrac{\pi}{2}\right) + 4$

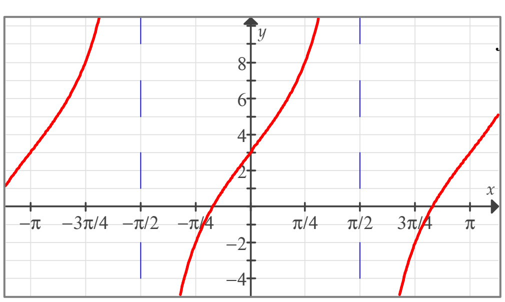
58. $f(x) = -9\sec\left(\dfrac{\pi x}{6} + \dfrac{2\pi}{3}\right) + 2$

For each of the following graphs,

(a) **Give an equation of the form $f(x) = A\tan(Bx - C) + D$ which could be used to represent the graph.** ***(Note: C or D may be zero. Answers vary.)***

(b) **Give an equation of the form $f(x) = A\cot(Bx - C) + D$ which could be used to represent the graph.** ***(Note: C or D may be zero. Answers vary.)***

59.



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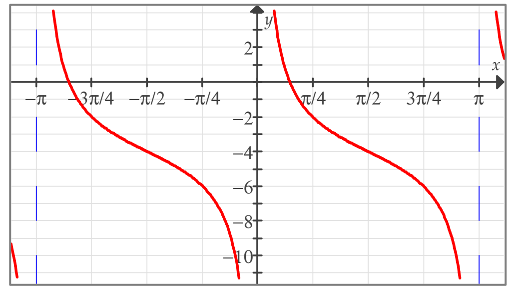

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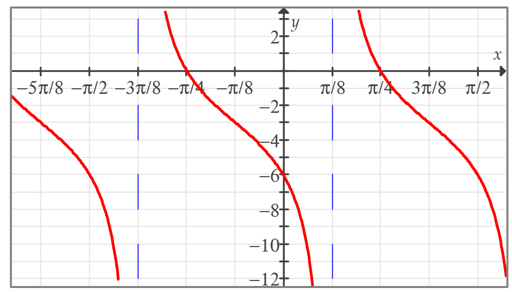

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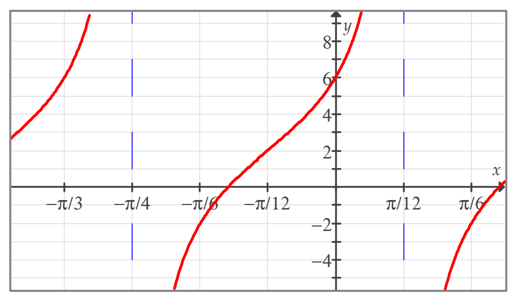
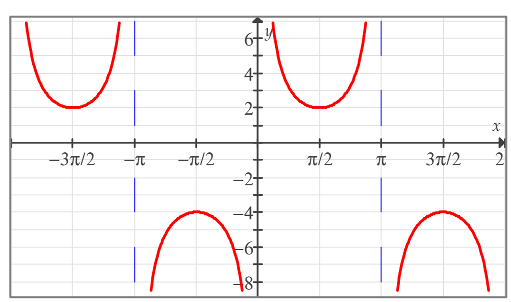


For each of the following graphs,

(a) **Give an equation of the form $f(x) = A\sec(Bx - C) + D$ which could be used to represent the graph.** ***(Note: C or D may be zero. Answers vary.)***

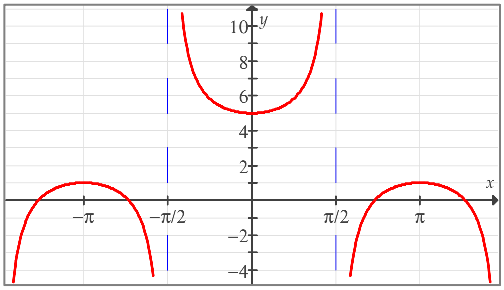
(b) **Give an equation of the form $f(x) = A\csc(Bx - C) + D$ which could be used to represent the graph.** ***(Note: C or D may be zero. Answers vary.)***

63.



Exercise Set 5.3: Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

64.



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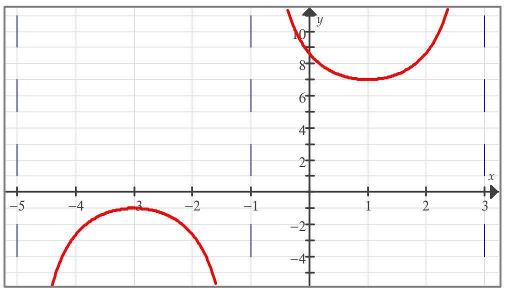

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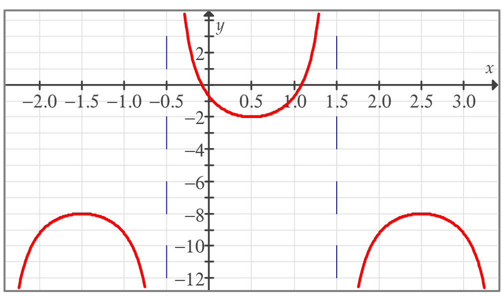


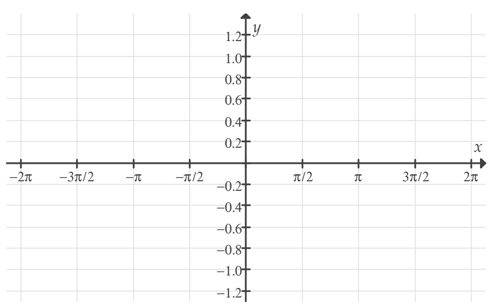
Exercise Set 5.4: Inverse Trigonometric Functions

Exercises 1-6 help to establish the graphs of the inverse trigonometric functions, and part (h) of each exercise shows a shortcut for remembering the range. Answer the following.

1. **(a)** Complete the following chart. Round to the nearest hundredth.

x	$y=\cos(x)$	x	$y=\cos(x)$
0			
$\frac{\pi}{2}$		$-\frac{\pi}{2}$	
π		$-\pi$	
$\frac{3\pi}{2}$		$-\frac{3\pi}{2}$	
2π		-2π	

(b) Plot the points from part (a) on the axes below. Then use those points along with previous knowledge of trigonometric graphs to sketch the graph of $y=\cos(x)$.

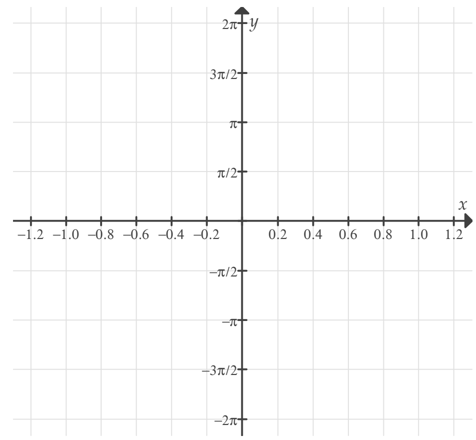


(c) Use the chart from part (a) to complete the following chart for $x=\cos(y)$.

$x=\cos(y)$	y	$x=\cos(y)$	y
	0		
	$\frac{\pi}{2}$		$-\frac{\pi}{2}$
	π		$-\pi$
	$\frac{3\pi}{2}$		$-\frac{3\pi}{2}$
	2π		-2π

Continued in the next column…

(d) Plot the points from part (c) on the axes below. Then use those points along with the graph from part (b) to sketch the graph of $x=\cos(y)$.



(e) Is the inverse relation in part (d) a function? Why or why not?

(f) Using the graph of $x=\cos(y)$ from part (d), sketch the portion of the graph with each of the following restricted <u>ranges</u>. Then state if each graph represents a function.

i. $[0,\pi]$ **ii.** $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$

iii. $[-\pi,0]$ **iv.** $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

(g) One of the functions from part (f) is the graph of $f(x)=\cos^{-1}(x)$. Which one? Explain why that particular range may be the most reasonable choice.

(h) Below is a way to remember the range of $f(x)=\cos^{-1}(x)$ without drawing a graph:

i. Draw a unit circle on the coordinate plane. In each quadrant, write "+" if the cosine of angles in that quadrant are positive, and write "–" if the cosine of angles in that quadrant are negative.

ii. Using the diagram from part i, find two consecutive quadrants where one is labeled "+" and the other is labeled "–". Write down any other pairs of consecutive quadrants where this occurs.

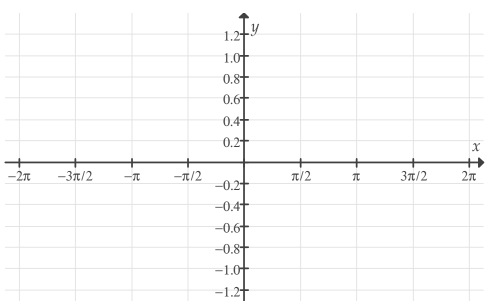
Continued on the next page…

iii. Of the answers from part ii, choose the pair of quadrants that includes Quadrant I; this illustrates the range of $f(x)=\cos^{-1}(x)$. Write the range in interval notation.

2. **(a)** Complete the following chart. Round to the nearest hundredth.

x	$y=\sin(x)$	x	$y=\sin(x)$
0			
$\frac{\pi}{2}$		$-\frac{\pi}{2}$	
π		$-\pi$	
$\frac{3\pi}{2}$		$-\frac{3\pi}{2}$	
2π		-2π	

(b) Plot the points from part (a) on the axes below. Then use those points along with previous knowledge of trigonometric graphs to sketch the graph of $y=\sin(x)$.

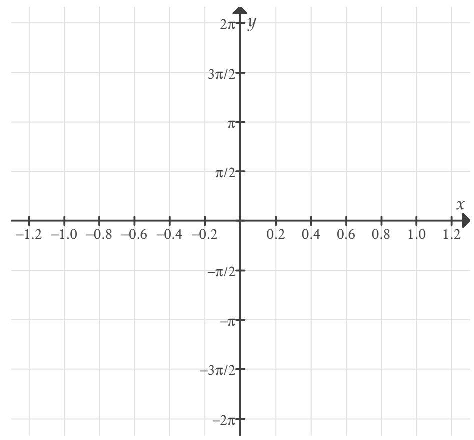


(c) Use the chart from part (a) to complete the following chart for $x=\sin(y)$.

$x=\sin(y)$	y	$x=\sin(y)$	y
	0		
	$\frac{\pi}{2}$		$-\frac{\pi}{2}$
	π		$-\pi$
	$\frac{3\pi}{2}$		$-\frac{3\pi}{2}$
	2π		-2π

Continued in the next column…

(d) Plot the points from part (c) on the axes below. Then use those points along with the graph from part (b) to sketch the graph of $x=\sin(y)$.



(e) Is the inverse relation in part (d) a function? Why or why not?

(f) Using the graph of $x=\sin(y)$ from part (d), sketch the portion of the graph with each of the following restricted ranges. Then state if each graph represents a function.

i. $[0,\pi]$ **ii.** $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$

iii. $[-\pi,0]$ **iv.** $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

(g) One of the functions from part (f) is the graph of $f(x)=\sin^{-1}(x)$. Which one? Explain why that particular range may be the most reasonable choice.

(h) Below is a way to remember the range of $f(x)=\sin^{-1}(x)$ without drawing a graph:

i. Draw a unit circle on the coordinate plane. In each quadrant, write "+" if the sine of angles in that quadrant are positive, and write "−" if the sine of angles in that quadrant are negative.

ii. Using the diagram from part i, find two consecutive quadrants where one is labeled "+" and the other is labeled "−". Write down any other pairs of consecutive quadrants where this occurs.

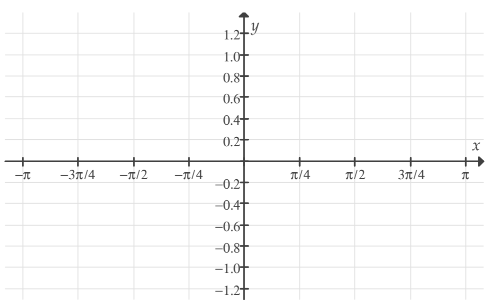
Continued on the next page…

iii. Of the answers from part ii, choose the pair of quadrants that includes Quadrant I; this illustrates the range of $f(x) = \sin^{-1}(x)$. Write the range in interval notation.

3. **(a)** Complete the following chart. Round to the nearest hundredth. If a value is undefined, state "Undefined."

x	$y = \tan(x)$	x	$y = \tan(x)$
0			
$\frac{\pi}{4}$		$-\frac{\pi}{4}$	
$\frac{\pi}{2}$		$-\frac{\pi}{2}$	
$\frac{3\pi}{4}$		$-\frac{3\pi}{4}$	
π		$-\pi$	

(b) Plot the points from part (a) on the axes below. Then use those points along with previous knowledge of trigonometric graphs to sketch the graph of $y = \tan(x)$.

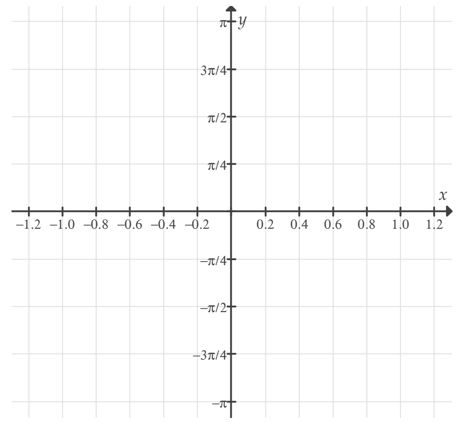


(c) Use the chart from part (a) to complete the following chart for $x = \tan(y)$.

$x = \tan(y)$	y	$x = \tan(y)$	y
	0		
	$\frac{\pi}{4}$		$-\frac{\pi}{4}$
	$\frac{\pi}{2}$		$-\frac{\pi}{2}$
	$\frac{3\pi}{4}$		$-\frac{3\pi}{4}$
	π		$-\pi$

Continued in the next column…

(d) Plot the points from part (c) on the axes below. Then use those points along with the graph from part (b) to sketch the graph of $x = \tan(y)$.



(e) Is the inverse relation in part (d) a function? Why or why not?

(f) Using the graph of $x = \tan(y)$ from part (d), sketch the portion of the graph with each of the following restricted ranges. Then state if each graph represents a function.

i. $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

ii. $\left(-\pi, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, 0\right)$

iii. $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

iv. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(g) One of the functions from part (f) is the graph of $f(x) = \tan^{-1} x$. Which one? Explain why that particular range may be the most reasonable choice.

(h) Below is a way to remember the range of $f(x) = \tan^{-1}(x)$ without drawing a graph:

i. Draw a unit circle on the coordinate plane. In each quadrant, write "+" if the tangent of angles in that quadrant are positive, and write "−" if the tangent of angles in that quadrant are negative. Also make a note next to any quadrantal angles for which the tangent function is undefined.

Continued on the next page…

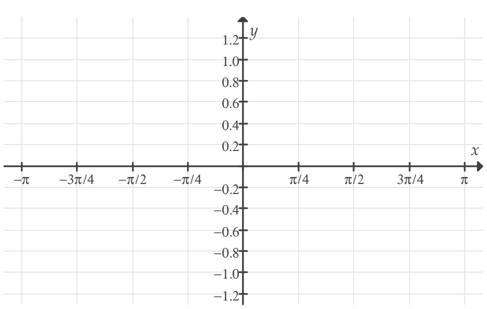
ii. Using the diagram from part i, find two consecutive quadrants where one is labeled "+" and the other is labeled "–". Write down any other pairs of consecutive quadrants where this occurs.

iii. Of the answers from part ii, write down any pairs of quadrants that include Quadrant I. (There are two pair.) Suppose for a moment that either pair could illustrate the range of $f(x)=\tan^{-1}(x)$, and write down the ranges for each in interval notation, remembering to consider where the function is undefined. Then choose the range that is represented by a single interval. This is the range of $f(x)=\tan^{-1}(x)$.

4. (a) Complete the following chart. Round to the nearest hundredth. If a value is undefined, state "Undefined."

x	$y=\cot(x)$	x	$y=\cot(x)$
0			
$\frac{\pi}{4}$		$-\frac{\pi}{4}$	
$\frac{\pi}{2}$		$-\frac{\pi}{2}$	
$\frac{3\pi}{4}$		$-\frac{3\pi}{4}$	
π		$-\pi$	

(b) Plot the points from part (a) on the axes below. Then use those points along with previous knowledge of trigonometric graphs to sketch the graph of $y=\cot(x)$.

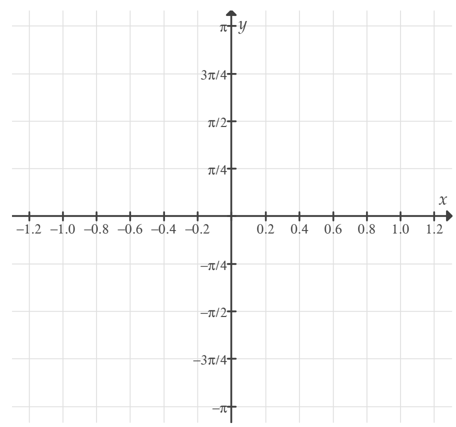


Continued in the next column…

(c) Use the chart from part (a) to complete the following chart for $x=\cot(y)$.

$x=\cot(y)$	y	$x=\cot(y)$	y
	0		
	$\frac{\pi}{4}$		$-\frac{\pi}{4}$
	$\frac{\pi}{2}$		$-\frac{\pi}{2}$
	$\frac{3\pi}{4}$		$-\frac{3\pi}{4}$
	π		$-\pi$

(d) Plot the points from part (c) on the axes below. Then use those points along with the graph from part (b) to sketch the graph of $x=\cot(y)$.



(e) Is the inverse relation in part (d) a function? Why or why not?

(f) Using the graph of $x=\cot(y)$ from part (d), sketch the portion of the graph with each of the following restricted ranges. Then state if each graph represents a function.

i. $(0,\pi)$ **ii.** $\left(-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right)$

iii. $(-\pi,0)$ **iv.** $\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$

(g) One of the functions from part (f) is the graph of $f(x)=\cot^{-1}x$. Which one? Explain why that particular range may be the most reasonable choice.

Continued on the next page…

(h) Below is a way to remember the range of $f(x) = \cot^{-1}(x)$ without drawing a graph:

i. Draw a simple coordinate plane by sketching the *x* and *y*-axes. In each quadrant, write "$+$" if the cotangent of numbers in that quadrant are positive, and write "$-$" if the cotangent of numbers in that quadrant are negative. Also make a note next to any quadrantal angles for which the cotangent function is undefined.

ii. Using the diagram from part i, find two consecutive quadrants where one is labeled "$+$" and the other is labeled "$-$". Write down any other pairs of consecutive quadrants where this occurs.

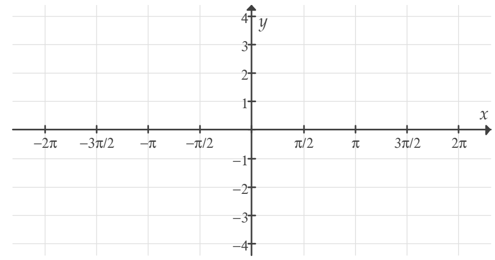
iii. Of the answers from part ii, write down any pairs of quadrants that include Quadrant I. (There are two pair.) Suppose for a moment that either pair could illustrate the range of $f(x) = \cot^{-1}(x)$, and write down the ranges for each in interval notation, remembering to consider where the function is undefined. Then choose the range that is represented by a single interval. This is the range of $f(x) = \cot^{-1}(x)$.

5. **(a)** Complete the following chart. Round to the nearest hundredth. If a value is undefined, state "Undefined."

x	$y = \csc(x)$		
0			
$\frac{\pi}{2}$		$-\frac{\pi}{2}$	
π		$-\pi$	
$\frac{3\pi}{2}$		$-\frac{3\pi}{2}$	
2π		-2π	

Continued in the next column…

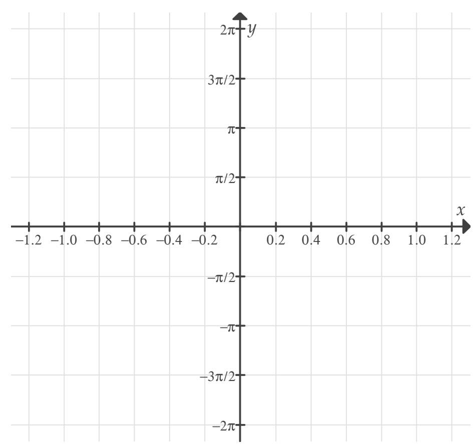
(b) Plot the points from part (a) on the axes below. Then use those points along with previous knowledge of trigonometric graphs to sketch the graph of $y = \csc(x)$.



(c) Use the chart from part (a) to complete the following chart for $x = \csc(y)$.

$x = \csc(y)$	y	$x = \csc(y)$	y
	0		
	$\frac{\pi}{2}$		$-\frac{\pi}{2}$
	π		$-\pi$
	$\frac{3\pi}{2}$		$-\frac{3\pi}{2}$
	2π		-2π

(d) Plot the points from part (c) on the axes below. Then use those points along with the graph from part (b) to sketch the graph of $x = \csc(y)$.



(e) Is the inverse relation in part (d) a function? Why or why not?

Continued on the next page…

(f) Using the graph of $x = \csc(y)$ from part (d), sketch the portion of the graph with each of the following restricted ranges. Then state if each graph represents a function.

i. $(0, \pi)$ **ii.** $\left[\frac{\pi}{2}, \pi\right) \cup \left(\pi, \frac{3\pi}{2}\right]$

iii. $(-\pi, 0)$ **iv.** $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

(g) One of the functions from part (f) is the graph of $f(x) = \csc^{-1}(x)$. Which one? Explain why that particular range may be the most reasonable choice.

(h) Below is a way to remember the range of $f(x) = \csc^{-1}(x)$ without drawing a graph:

i. Draw a unit circle on the coordinate plane. In each quadrant, write "$+$" if the cosecant of angles in that quadrant are positive, and write "$-$" if the cosecant of angles in that quadrant are negative. Also make a note next to any quadrantal angles for which the cosecant function is undefined.

ii. Using the diagram from part i, find two consecutive quadrants where one is labeled "$+$" and the other is labeled "$-$". Write down any other pairs of consecutive quadrants where this occurs.

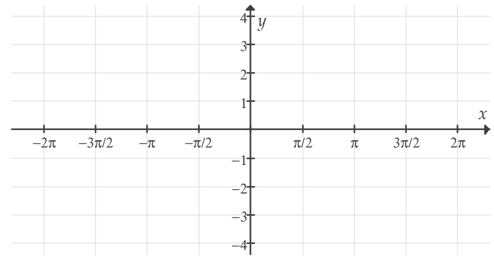
iii. Of the answers from part ii, choose the pair of quadrants that includes Quadrant I; this illustrates the range of $f(x) = \csc^{-1}(x)$. Consider where the function is undefined, and then write the range in interval notation.

6. (a) Complete the following chart.

x	$y = \sec(x)$	x	$y = \sec(x)$
0			
$\frac{\pi}{2}$		$-\frac{\pi}{2}$	
π		$-\pi$	
$\frac{3\pi}{2}$		$-\frac{3\pi}{2}$	
2π		-2π	

Continued in the next column…

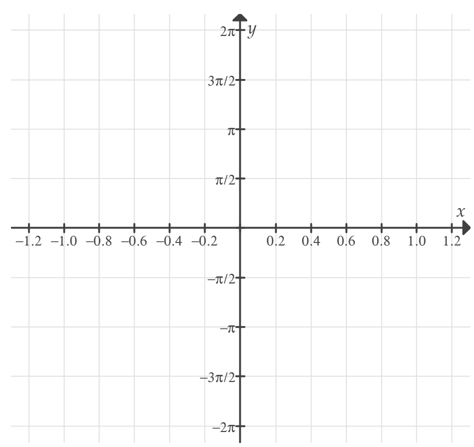
(b) Plot the points from part (a) on the axes below. Then use those points along with previous knowledge of trigonometric graphs to sketch the graph of $y = \sec(x)$.



(c) Use the chart from part (a) to complete the following chart for $x = \sec(y)$.

$x = \sec(y)$	y	$x = \sec(y)$	y
	0		
	$\frac{\pi}{2}$		$-\frac{\pi}{2}$
	π		$-\pi$
	$\frac{3\pi}{2}$		$-\frac{3\pi}{2}$
	2π		-2π

(d) Plot the points from part (c) on the axes below. Then use those points along with the graph from part (b) to sketch the graph of $x = \sec(y)$.



(e) Is the inverse relation in part (d) a function? Why or why not?

Continued on the next page…

(f) Using the graph of $x = \sec(y)$ from part (d), sketch the portion of the graph with each of the following restricted ranges. Then state if each graph represents a function.

i. $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

ii. $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

iii. $\left[-\pi, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, 0\right]$

iv. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(g) One of the functions from part (f) is the graph of $f(x) = \sec^{-1}(x)$. Which one? Explain why that particular range may be the most reasonable choice.

(h) Below is a way to remember the range of $f(x) = \sec^{-1}(x)$ without drawing a graph:

i. Draw a unit circle on the coordinate plane. In each quadrant, write "$+$" if the secant of angles in that quadrant are positive, and write "$-$" if the secant of angles in that quadrant are negative. Also make a note next to any quadrantal angles for which the secant function is undefined.

ii. Using the diagram from part i, find two consecutive quadrants where one is labeled "$+$" and the other is labeled "$-$". Write down any other pairs of consecutive quadrants where this occurs.

iii. Of the answers from part ii, choose the pair of quadrants that includes Quadrant I; this illustrates the range of $f(x) = \sec^{-1}(x)$. Consider where the function is undefined, and then write the range in interval notation.

Inverse functions may be easier to remember if they are translated into words. For example:

$\sin^{-1}(x) =$ **the number** (in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$) **whose sine is x.**

Translate each of the following expressions into words (and include the range of the inverse function in parentheses as above). Then find the exact value of each expression. Do not use a calculator.

7. **(a)** $\cos^{-1}\left(-\frac{1}{2}\right)$ **(b)** $\tan^{-1}(1)$

8. **(a)** $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ **(b)** $\cot^{-1}(-1)$

9. **(a)** $\sin^{-1}(-1)$ **(b)** $\sec^{-1}\left(\sqrt{2}\right)$

10. **(a)** $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **(b)** $\csc^{-1}(-2)$

11. $\sin\left[\sin^{-1}(0.2)\right]$

12. $\cos\left[\arccos(0.7)\right]$

13. $\tan\left[\arctan(4)\right]$

14. $\csc\left[\csc^{-1}(3.5)\right]$

Find the exact value of each of the following expressions. Do not use a calculator. If undefined, state, "Undefined."

15. **(a)** $\sin^{-1}\left(\frac{1}{2}\right)$ **(b)** $\cos^{-1}(1)$

16. **(a)** $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ **(b)** $\arcsin(1)$

17. **(a)** $\arccos\left(\sqrt{3}\right)$ **(b)** $\sin^{-1}(0)$

18. **(a)** $\sin^{-1}\left(-\sqrt{2}\right)$ **(b)** $\cos^{-1}(-1)$

19. **(a)** $\tan^{-1}(-1)$ **(b)** $\tan^{-1}\left(\sqrt{3}\right)$

20. **(a)** $\arctan(0)$ **(b)** $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

Exercise Set 5.4: Inverse Trigonometric Functions

21. **(a)** $\cot^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ **(b)** $\cot^{-1}(0)$

22. **(a)** $\cot^{-1}(1)$ **(b)** $\cot^{-1}\left(-\sqrt{3}\right)$

23. **(a)** $\sec^{-1}(-2)$ **(b)** $\operatorname{arccsc}(-1)$

24. **(a)** $\csc^{-1}\left(\frac{2\sqrt{3}}{3}\right)$ **(b)** $\sec^{-1}(0)$

25. **(a)** $\csc^{-1}\left(\frac{\sqrt{2}}{2}\right)$ **(b)** $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$

26. **(a)** $\sec^{-1}\left(-\sqrt{2}\right)$ **(b)** $\sec^{-1}\left(-\frac{1}{2}\right)$

Answer True or False. Assume that all *x*-values are in the domain of the given inverse functions.

27. **(a)** $\sec^{-1}(x)=\frac{1}{\cos^{-1}(x)}$

    (b) $\sec^{-1}(x)=\cos^{-1}\left(\frac{1}{x}\right)$

28. **(a)** $\csc^{-1}(x)=\sin^{-1}\left(\frac{1}{x}\right)$

    (b) $\csc^{-1}(x)=\frac{1}{\sin^{-1}(x)}$

29. **(a)** $\cot^{-1}(x)=\tan^{-1}\left(\frac{1}{x}\right)$

    (b) $\cot^{-1}(x)=\frac{1}{\tan^{-1}(x)}$

30. Explain why $\cot^{-1}(x)\neq\tan^{-1}\left(\frac{1}{x}\right)$. Are there any values for which $\cot^{-1}(x)=\tan^{-1}\left(\frac{1}{x}\right)$? If so, which values?

Use a calculator to find the value of each of the following expressions. Round each answer to the nearest thousandth. If undefined, state, "Undefined."

Note: To find the inverse cotangent of a negative number, such as $\cot^{-1}(-2.1)$, *first find a reference number (to be likened to a reference angle) by finding* $\tan^{-1}\left(\frac{1}{2.1}\right)\approx 0.444$. *The expression* $\cot^{-1}(-2.1)$ *can be translated as "the cotangent of a number is* -2.1.*" Since the range of the inverse cotangent function is* $(0,\pi)$, *and the cotangent of the number is* -2.1, *a negative number, the solution lies in the interval* $\left(\frac{\pi}{2},\pi\right)$. *Therefore,* $\cot^{-1}(-2.1)\approx\pi-0.444\approx 2.698$. *A more exact answer can be obtained by rounding later in the calculation:* $\pi-\tan^{-1}\left(\frac{1}{2.1}\right)\approx 2.697$. *(Either answer is acceptable.)*

31. **(a)** $\cos^{-1}(0.37)$ **(b)** $\sin^{-1}(-0.8)$

32. **(a)** $\sin^{-1}(0.9)$ **(b)** $\cos^{-1}(-0.24)$

33. **(a)** $\sin^{-1}\left(-\frac{4}{7}\right)$ **(b)** $\arccos\left(-\frac{7}{9}\right)$

34. **(a)** $\cos^{-1}(-3)$ **(b)** $\sin^{-1}\left(-\frac{1}{5}\right)$

35. **(a)** $\tan^{-1}(4)$ **(b)** $\arctan(-0.12)$

36. **(a)** $\arctan(0.28)$ **(b)** $\tan^{-1}(-30)$

37. **(a)** $\cot^{-1}\left(\frac{3}{8}\right)$ **(b)** $\cot^{-1}(-0.8)$

38. **(a)** $\cot^{-1}(6)$ **(b)** $\cot^{-1}\left(-\frac{11}{5}\right)$

39. **(a)** $\cot^{-1}\left(-\frac{1}{5}\right)$ **(b)** $\operatorname{arccot}(-10)$

40. **(a)** $\cot^{-1}(-7)$ **(b)** $\cot^{-1}(-.62)$

41. **(a)** $\csc^{-1}(2.4)$ **(b)** $\sec^{-1}\left(-\frac{10}{3}\right)$

Exercise Set 5.4: Inverse Trigonometric Functions

42. **(a)** $\sec^{-1}\left(\frac{3}{4}\right)$ **(b)** $\csc^{-1}(-7.2)$

43. **(a)** $\sec^{-1}(-0.71)$ **(b)** $\csc^{-1}\left(-\frac{31}{5}\right)$

44. **(a)** $\csc^{-1}(-9.5)$ **(b)** $\operatorname{arcsec}(8.8)$

Find the exact value of each of the following expressions. Do not use a calculator. If undefined, state, "Undefined."

45. **(a)** $\sin\left[\sin^{-1}(-0.84)\right]$ **(b)** $\cos\left[\cos^{-1}\left(\sqrt{3}\right)\right]$

46. **(a)** $\sin\left[\sin^{-1}(-5)\right]$ **(b)** $\cos\left[\cos^{-1}\left(\frac{2}{3}\right)\right]$

47. **(a)** $\csc\left[\csc^{-1}(3.5)\right]$ **(b)** $\cot\left[\cot^{-1}(-7)\right]$

48. **(a)** $\sec\left[\sec^{-1}(-0.3)\right]$ **(b)** $\tan\left[\tan^{-1}(0.5)\right]$

49. **(a)** $\sin\left[\cos^{-1}\left(\frac{3}{5}\right)\right]$ **(b)** $\tan\left[\sin^{-1}\left(\frac{8}{17}\right)\right]$

50. **(a)** $\cos\left[\tan^{-1}\left(\frac{12}{5}\right)\right]$ **(b)** $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$

51. **(a)** $\sec\left[\tan^{-1}\left(\frac{2}{7}\right)\right]$ **(b)** $\cot\left[\csc^{-1}(5)\right]$

52. **(a)** $\csc\left[\cot^{-1}(4)\right]$ **(b)** $\tan\left[\sec^{-1}\left(\frac{7}{4}\right)\right]$

53. **(a)** $\sin\left[\cos^{-1}\left(-\frac{12}{13}\right)\right]$ **(b)** $\tan\left[\cos^{-1}\left(-\frac{1}{6}\right)\right]$

54. **(a)** $\cos\left[\tan^{-1}\left(-\frac{1}{7}\right)\right]$ **(b)** $\cot\left[\sin^{-1}\left(-\frac{3}{7}\right)\right]$

55. **(a)** $\sin\left[\tan^{-1}(-1)\right]$

(b) $\cos\left[\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$

56. **(a)** $\tan\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

(b) $\cos\left[\tan^{-1}\left(-\sqrt{3}\right)\right]$

57. **(a)** $\csc\left[\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right]$

(b) $\tan\left[\sec^{-1}(-2)\right]$

58. **(a)** $\cot\left[\cos^{-1}\left(-\frac{1}{2}\right)\right]$

(b) $\sin\left[\csc^{-1}\left(-\sqrt{2}\right)\right]$

59. **(a)** $\cos^{-1}\left[\cos\left(\frac{4\pi}{3}\right)\right]$ **(b)** $\sin^{-1}\left[\sin\left(\frac{\pi}{4}\right)\right]$

60. **(a)** $\cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right]$ **(b)** $\sin^{-1}\left[\sin\left(\frac{2\pi}{3}\right)\right]$

61. **(a)** $\sec^{-1}\left[\sec\left(\frac{7\pi}{6}\right)\right]$ **(b)** $\tan^{-1}\left[\tan\left(\frac{3\pi}{4}\right)\right]$

62. **(a)** $\cot^{-1}\left[\cot\left(\frac{7\pi}{4}\right)\right]$ **(b)** $\csc^{-1}\left[\csc\left(\frac{4\pi}{3}\right)\right]$

Sketch the graph of each function.

63. $y=\cos^{-1}(x-2)$

64. $y=\arctan(x+1)$

65. $y=\cot^{-1}(x)-\frac{\pi}{2}$

66. $y=\csc^{-1}(x)-\pi$

67. $y=\arcsin(x-1)+\pi$

68. $y=\cos^{-1}(x+2)+\frac{\pi}{2}$

Exercise Set 6.1: Sum and Difference Formulas

Simplify each of the following expressions.

1. $\sin(\pi - x)$

2. $\cos\left(x + \dfrac{3\pi}{2}\right)$

3. $\cos\left(x - \dfrac{\pi}{2}\right)$

4. $\sin(\pi + x)$

5. $\sin(60^\circ - \theta) + \sin(60^\circ + \theta)$

6. $\cos(60^\circ - \theta) + \cos(60^\circ + \theta)$

7. $\cos\left(x - \dfrac{\pi}{4}\right) + \cos\left(x + \dfrac{\pi}{4}\right)$

8. $\sin\left(x + \dfrac{\pi}{6}\right) + \sin\left(x - \dfrac{\pi}{6}\right)$

9. $\sin(\theta - 180^\circ) + \sin(\theta + 180^\circ)$

10. $\cos(90^\circ + \theta) + \cos(90^\circ - \theta)$

Answer the following.

11. Given that $\tan(\alpha) = -4$, evaluate $\tan\left(\alpha + \dfrac{3\pi}{4}\right)$.

12. Given that $\tan(\beta) = 2$, evaluate $\tan\left(\dfrac{\pi}{4} - \beta\right)$.

13. Given that $\tan(x) = \dfrac{2}{3}$, evaluate $\tan\left(x - \dfrac{5\pi}{4}\right)$.

14. Given that $\tan(x) = -\dfrac{1}{5}$, evaluate $\tan\left(x + \dfrac{7\pi}{4}\right)$.

15. Given that $\tan(x) = 5$ and $\tan(y) = 6$, evaluate $\tan(x + y)$.

16. Given that $\tan(x) = 4$ and $\tan(y) = -2$, evaluate $\tan(x - y)$.

17. Given that $\tan(\alpha) = -3$ and $\tan(\beta) = \dfrac{2}{5}$, evaluate $\tan(\alpha + \beta)$.

18. Given that $\tan(\alpha) = -\dfrac{4}{3}$ and $\tan(\beta) = -\dfrac{1}{2}$, evaluate $\tan(\alpha - \beta)$.

Simplify each of the following expressions as much as possible without a calculator.

19. $\cos(55^\circ)\cos(10^\circ) + \sin(55^\circ)\sin(10^\circ)$

20. $\cos(75^\circ)\cos(15^\circ) - \sin(75^\circ)\sin(15^\circ)$

21. $\sin(45^\circ)\cos(15^\circ) - \cos(45^\circ)\sin(15^\circ)$

22. $\sin(53^\circ)\cos(7^\circ) + \cos(53^\circ)\sin(7^\circ)$

23. $\cos\left(\dfrac{\pi}{10}\right)\cos\left(\dfrac{\pi}{9}\right) - \sin\left(\dfrac{\pi}{10}\right)\sin\left(\dfrac{\pi}{9}\right)$

24. $\sin\left(\dfrac{\pi}{5}\right)\cos\left(\dfrac{\pi}{7}\right) - \cos\left(\dfrac{\pi}{5}\right)\sin\left(\dfrac{\pi}{7}\right)$

25. $\sin\left(\dfrac{13\pi}{12}\right)\cos\left(\dfrac{\pi}{12}\right) + \cos\left(\dfrac{13\pi}{12}\right)\sin\left(\dfrac{\pi}{12}\right)$

26. $\cos\left(\dfrac{5\pi}{12}\right)\cos\left(\dfrac{17\pi}{12}\right) + \sin\left(\dfrac{5\pi}{12}\right)\sin\left(\dfrac{17\pi}{12}\right)$

27. $\sin(2A)\cos(A) - \cos(2A)\sin(A)$

28. $\cos(3\alpha)\cos(\alpha) + \sin(3\alpha)\cos(\alpha)$

29. $\dfrac{\tan(32^\circ) + \tan(2^\circ)}{1 - \tan(32^\circ)\tan(2^\circ)}$

30. $\dfrac{\tan(49^\circ) - \tan(4^\circ)}{1 + \tan(49^\circ)\tan(4^\circ)}$

31. $\dfrac{\tan\left(\frac{5\pi}{12}\right)-\tan\left(\frac{\pi}{12}\right)}{1+\tan\left(\frac{5\pi}{12}\right)\tan\left(\frac{\pi}{12}\right)}$

32. $\dfrac{\tan\left(\frac{13\pi}{12}\right)+\tan\left(\frac{7\pi}{12}\right)}{1-\tan\left(\frac{13\pi}{12}\right)\tan\left(\frac{7\pi}{12}\right)}$

33. $\dfrac{\tan(a-b)+\tan(b)}{1-\tan(a-b)\tan(b)}$

34. $\dfrac{\tan(2c+3d)-\tan(d-c)}{1+\tan(2c+3d)\tan(d-c)}$

Answer the following.

35. Rewrite each special angle below so that it has a denominator of 12.

(a) $\frac{\pi}{6}$ **(b)** $\frac{\pi}{4}$ **(c)** $\frac{\pi}{3}$

(d) $\frac{2\pi}{3}$ **(e)** $\frac{3\pi}{4}$ **(f)** $\frac{5\pi}{6}$

36. Use the answers from Exercise 35 to write each fraction below as the <u>sum</u> of two special angles. *(Hint: Each of the solutions contains a multiple of* $\frac{\pi}{4}$*.)*

(a) $\frac{5\pi}{12}$ **(b)** $\frac{7\pi}{12}$

(c) $\frac{13\pi}{12}$ **(d)** $\frac{11\pi}{12}$

37. Use the answers from Exercise 35 to write each fraction below as the <u>difference</u> of two special angles. *(Hint: Each of the solutions contains a multiple of* $\frac{\pi}{4}$*.)*

(a) $\frac{5\pi}{12}$ **(b)** $\frac{7\pi}{12}$

(c) $-\frac{7\pi}{12}$ **(d)** $-\frac{5\pi}{12}$

38. For fractions with larger magnitude than those in Exercises 36 and 37, it can be helpful to use larger multiples of $\frac{\pi}{4}$. Rewrite each special angle below so that it has a denominator of 12.

(a) $\frac{5\pi}{4}$ **(b)** $\frac{7\pi}{4}$

(c) $\frac{9\pi}{4}$ **(d)** $\frac{11\pi}{4}$

39. Use the answers from Exercises 35 and 38 to write each fraction below as the <u>sum</u> of two special angles. *(Hint: Each of the solutions contains a multiple of* $\frac{\pi}{4}$*.)*

(a) $\frac{19\pi}{12}$ **(b)** $\frac{25\pi}{12}$

(c) $\frac{35\pi}{12}$ **(d)** $\frac{43\pi}{12}$

40. Use the answers from Exercises 35 and 38 to write each fraction below as the <u>difference</u> of two special angles. *(Hint: Each of the solutions contains a multiple of* $\frac{\pi}{4}$*.)*

(a) $\frac{19\pi}{12}$ **(b)** $\frac{25\pi}{12}$

(c) $-\frac{23\pi}{12}$ **(d)** $-\frac{29\pi}{12}$

41. Use the answers from Exercises 35 and 38, <u>along with their negatives</u> to write each fraction below as the difference, $x-y$, of two special angles, where x is negative and y is positive.

(a) $-\frac{7\pi}{12}$ **(b)** $-\frac{13\pi}{12}$

(c) $-\frac{29\pi}{12}$ **(d)** $-\frac{41\pi}{12}$

42. Use the answers from Exercises 35 and 38, <u>along with their negatives</u> to write each fraction below as the difference, $x-y$, of two special angles, where x is negative and y is positive.

(a) $-\frac{5\pi}{12}$ **(b)** $-\frac{19\pi}{12}$

(c) $-\frac{31\pi}{12}$ **(d)** $-\frac{43\pi}{12}$

Exercise Set 6.1: Sum and Difference Formulas

Answer the following.

43. Use a sum or difference formula to prove that $\sin(-\theta) = -\sin(\theta)$.

44. Use a sum or difference formula to prove that $\cos(-\theta) = \cos(\theta)$.

45. Use a sum or difference formula to prove that $\tan(-\theta) = -\tan(\theta)$.

46. Use a sum or difference formula to prove that $\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$.

Find the exact value of each of the following.

47. $\cos(15^\circ)$

48. $\sin(75^\circ)$

49. $\sin(195^\circ)$

50. $\cos(-255^\circ)$

51. $\tan(105^\circ)$

52. $\tan(165^\circ)$

53. $\sin\left(\frac{7\pi}{12}\right)$

54. $\cos\left(\frac{5\pi}{12}\right)$

55. $\cos\left(-\frac{17\pi}{12}\right)$

56. $\sin\left(\frac{11\pi}{12}\right)$

57. $\cos\left(\frac{31\pi}{12}\right)$

58. $\sin\left(\frac{43\pi}{12}\right)$

59. $\tan\left(-\frac{13\pi}{12}\right)$

60. $\tan\left(\frac{7\pi}{12}\right)$

61. $\tan\left(\frac{37\pi}{12}\right)$

62. $\tan\left(-\frac{29\pi}{12}\right)$

Answer the following. (Hint: It may help to first draw right triangles in the appropriate quadrants and label the side lengths.)

63. Suppose that $\sin(\alpha) = \frac{4}{5}$ and $\sin(\beta) = \frac{12}{13}$, where $0 < \alpha < \frac{\pi}{2} < \beta < \pi$. Find:

(a) $\sin(\alpha - \beta)$

(b) $\cos(\alpha + \beta)$

(c) $\tan(\alpha - \beta)$

64. Suppose that $\cos(\alpha) = \frac{8}{17}$ and $\cos(\beta) = \frac{2}{3}$, where $0 < \beta < \alpha < \frac{\pi}{2}$. Find:

(a) $\sin(\alpha + \beta)$

(b) $\cos(\alpha - \beta)$

(c) $\tan(\alpha + \beta)$

65. Suppose that $\tan(\alpha) = -\frac{5}{2}$ and $\cos(\beta) = \frac{1}{2}$, where $\frac{3\pi}{2} < \alpha < \beta < 2\pi$. Find:

(a) $\sin(\alpha + \beta)$

(b) $\cos(\alpha + \beta)$

(c) $\tan(\alpha - \beta)$

66. Suppose that $\tan(\alpha) = 3$ and $\sin(\beta) = \frac{1}{4}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$. Find:

(a) $\sin(\alpha - \beta)$

(b) $\cos(\alpha - \beta)$

(c) $\tan(\alpha + \beta)$

Exercise Set 6.1: Sum and Difference Formulas

Evaluate the following.

67. $\cos\left[\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right]$

68. $\sin\left[\tan^{-1}(4)+\tan^{-1}\left(\frac{1}{4}\right)\right]$

69. $\tan\left[\cos^{-1}\left(\frac{3}{5}\right)-\sin^{-1}\left(\frac{5}{13}\right)\right]$

70. $\tan\left[\tan^{-1}\left(\frac{7}{24}\right)+\cos^{-1}\left(\frac{4}{5}\right)\right]$

Simplify the following.

71. $\cos(A)\sin(B)\left[\cot(B)+\tan(A)\right]$

72. $\tan(A)\cos(B)\left[\cos(A)-\cos(A)\cot(A)\tan(B)\right]$

73. $\sin(A-B)\sin(A+B)$

74. $\cos(A+B)\cos(A-B)$

Prove the following.

75. $\sin(x-y)+\sin(x+y)=2\sin(x)\cos(y)$

76. $\cos(x-y)+\cos(x+y)=2\cos(x)\cos(y)$

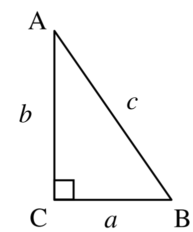
77. $\dfrac{\cos(x-y)+\cos(x+y)}{\sin(x)\sin(y)}=2\cot(x)\cot(y)$

78. $\dfrac{\sin(x-y)+\sin(x+y)}{\cos(x)\cos(y)}=2\tan(x)$

79. $\dfrac{\sin(x-y)}{\sin(x+y)}=\dfrac{\tan(x)-\tan(y)}{\tan(x)+\tan(y)}$

80. $\dfrac{\cos(x-y)}{\cos(x+y)}=\dfrac{1+\tan(x)\tan(y)}{1-\tan(x)\tan(y)}$

Use right triangle ABC below to answer the following questions regarding cofunctions, in terms of side lengths *a*, *b*, and *c*.



81. **(a)** Find $\sin(A)$.

(b) Find $\cos(B)$.

(c) Analyze the answers for (a) and (b). What do you notice?

(d) What is the relationship between angles A and B? (i.e. If you knew the measure of one angle, how would you find the other?) Write answers in terms of degrees.

(e) Complete the following cofunction relationships by filling in blank. Write your answers in terms of *A*.

$\sin(A)=\cos(\underline{\qquad\qquad})$

$\cos(A)=\sin(\underline{\qquad\qquad})$

82. **(a)** Find $\tan(A)$.

(b) Find $\cot(B)$.

(c) Analyze the answers for (a) and (b). What do you notice?

(d) What is the relationship between angles A and B? (i.e. If you knew the measure of one angle, how would you find the other?) Write answers in terms of degrees.

(e) Complete the following cofunction relationships by filling in blank. Write your answers in terms of *A*.

$\tan(A)=\cot(\underline{\qquad\qquad})$

$\cot(A)=\tan(\underline{\qquad\qquad})$

83. **(a)** Find $\sec(A)$.

(b) Find $\csc(B)$.

(c) Analyze the answers for (a) and (b). What do you notice?

Continued on the next page…

(d) What is the relationship between angles A and B? (i.e. If you knew the measure of one angle, how would you find the other?) Write answers in terms of degrees.

(e) Complete the following cofunction relationships by filling in the blank. Write your answers in terms of *A*.

$\sec(A) = \csc(\_\_\_\_\_\_\_)$

$\csc(A) = \sec(\_\_\_\_\_\_\_)$

84. Use a sum or difference formula to prove that $\sin(90^\circ - \theta) = \cos(\theta)$.

Use cofunction relationships to solve the following for acute angle x.

85. $\sin(75^\circ) = \cos(x)$

86. $\cos\left(\frac{3\pi}{10}\right) = \sin(x)$

87. $\sec\left(\frac{2\pi}{5}\right) = \csc(x)$

88. $\csc(41^\circ) = \sec(x)$

89. $\tan(72^\circ) = \cot(x)$

90. $\cot\left(\frac{\pi}{3}\right) = \tan(x)$

Simplify the following.

91. $\cos(90^\circ - x)\cdot\csc(x)$

92. $\sin\left(\frac{\pi}{2} - x\right)\cdot\sec(x)$

93. $\sin\left(\frac{\pi}{2} - x\right)\cdot\tan(x)$

94. $\csc\left(\frac{\pi}{2} - x\right)\cdot\sin(x)$

95. $\sec(90^\circ - \theta)\cdot\cos(\theta)$

96. $\tan(90^\circ - x)\cdot\csc(90^\circ - x)$

97. $-\sec(20^\circ)\sin(70^\circ)$

98. $\cos\left(\frac{\pi}{6}\right)\csc\left(\frac{\pi}{3}\right)$

99. $\cot^2\left(\frac{\pi}{12}\right) - \sec^2\left(\frac{5\pi}{12}\right)$

100. $\cos^2(72^\circ) + \cos^2(18^\circ)$

Exercise Set 6.2: Double-Angle and Half-Angle Formulas

Answer the following.

1. **(a)** Evaluate $\sin\left[2\left(\frac{\pi}{6}\right)\right]$.

   (b) Evaluate $2\sin\left(\frac{\pi}{6}\right)$.

   (c) Is $\sin\left[2\left(\frac{\pi}{6}\right)\right] = 2\sin\left(\frac{\pi}{6}\right)$?

   (d) Graph $f(x) = \sin(2x)$ and $g(x) = 2\sin(x)$ on the same set of axes.

   (e) Is $\sin(2x) = 2\sin(x)$?

2. **(a)** Evaluate $\cos\left[2\left(\frac{\pi}{6}\right)\right]$.

   (b) Evaluate $2\cos\left(\frac{\pi}{6}\right)$.

   (c) Is $\cos\left[2\left(\frac{\pi}{6}\right)\right] = 2\cos\left(\frac{\pi}{6}\right)$?

   (d) Graph $f(x) = \cos(2x)$ and $g(x) = 2\cos(x)$ on the same set of axes.

   (e) Is $\cos(x) = 2\cos(x)$?

3. **(a)** Evaluate $\tan\left[2\left(\frac{\pi}{6}\right)\right]$.

   (b) Evaluate $2\tan\left(\frac{\pi}{6}\right)$

   (c) Is $\tan\left[2\left(\frac{\pi}{6}\right)\right] = 2\tan\left(\frac{\pi}{6}\right)$?

   (d) Graph $f(x) = \tan(2x)$ and $g(x) = 2\tan(x)$ on the same set of axes.

   (e) Is $\tan(2x) = 2\tan(x)$?

4. Derive the formula for $\sin(2\theta)$ by using a sum formula on $\sin(\theta+\theta)$.

5. Derive the formula for $\cos(2\theta)$ by using a sum formula on $\cos(\theta+\theta)$.

6. Derive the formula for $\tan(2\theta)$ by using a sum formula on $\tan(\theta+\theta)$.

7. The sum formula for cosine yields the equation $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$. To write $\cos(2\theta)$ strictly in terms of the sine function,

   (a) Using the Pythagorean identity $\cos^2(\theta) + \sin^2(\theta) = 1$, solve for $\cos^2(\theta)$.

   (b) Substitute the result from part (a) into the above equation for $\cos(2\theta)$.

8. The sum formula for cosine yields the equation $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$. To write $\cos(2\theta)$ strictly in terms of the cosine function,

   (a) Using the Pythagorean identity $\cos^2(\theta) + \sin^2(\theta) = 1$, solve for $\sin^2(\theta)$.

   (b) Substitute the result from part (a) into the above equation for $\cos(2\theta)$.

Answer the following.

9. Suppose that $\cos(\alpha) = \frac{12}{13}$ and $\frac{3\pi}{2} < \alpha < 2\pi$.

   Find:

   (a) $\sin(2\alpha)$

   (b) $\cos(2\alpha)$

   (c) $\tan(2\alpha)$

10. Suppose that $\tan(\alpha) = \frac{3}{4}$ and $\pi < \alpha < \frac{3\pi}{2}$.

    Find:

    (a) $\sin(2\alpha)$

    (b) $\cos(2\alpha)$

    (c) $\tan(2\alpha)$

11. Suppose that $\sin(\alpha) = \frac{2}{5}$ and $\frac{\pi}{2} < \alpha < \pi$.

    Find:

    (a) $\sin(2\alpha)$

    (b) $\cos(2\alpha)$

    (c) $\tan(2\alpha)$

Exercise Set 6.2: Double-Angle and Half-Angle Formulas

12. Suppose that $\tan(\alpha) = -3$ and $\frac{3\pi}{2} < \alpha < 2\pi$.

Find:

(a) $\sin(2\alpha)$

(b) $\cos(2\alpha)$

(c) $\tan(2\alpha)$

Simplify each of the following expressions as much as possible without a calculator.

13. $2\sin(15^\circ)\cos(15^\circ)$

14. $\cos^2\left(\frac{\beta}{2}\right) - \sin^2\left(\frac{\beta}{2}\right)$

15. $2\cos^2(34^\circ) - 1$

16. $1 - 2\sin^2\left(\frac{5\pi}{12}\right)$

17. $\dfrac{2\tan(105^\circ)}{1-\tan^2(105^\circ)}$

18. $1 - \sin^2(x)$

19. $-2\sin\left(\frac{7\pi}{8}\right)\cos\left(\frac{7\pi}{8}\right)$

20. $2\sin(23^\circ)\cos(23^\circ)$

21. $\cos^2\left(\frac{3\pi}{8}\right) - \sin^2\left(\frac{3\pi}{8}\right)$

22. $\dfrac{2\tan\left(\frac{11\pi}{12}\right)}{1-\tan^2\left(\frac{11\pi}{12}\right)}$

23. $2\sin^2\left(\frac{7\pi}{12}\right) - 1$

24. $\sin^2(67.5^\circ) - \cos^2(67.5^\circ)$

25. $\dfrac{-2\tan(41^\circ)}{1-\tan^2(41^\circ)}$

26. $1 - 2\cos^2(22.5^\circ)$

27. $2\cos^2\left(\frac{\beta}{2}\right) - 1$

28. $\dfrac{2\tan(3\alpha)}{\tan^2(3\alpha) - 1}$

The formulas for $\sin\left(\frac{x}{2}\right)$ and $\cos\left(\frac{x}{2}\right)$ both contain a $\pm$ sign, meaning that a choice must be made as to whether or not the sign is positive or negative. For each of the following examples, first state the quadrant in which the angle lies. Then state whether the given expression is positive or negative. (Do not evaluate the expression.)

29. **(a)** $\cos(105^\circ)$

(b) $\sin(67.5^\circ)$

30. **(a)** $\sin(15^\circ)$

(b) $\cos(112.5^\circ)$

31. **(a)** $\cos\left(\frac{15\pi}{8}\right)$

(b) $\sin\left(\frac{13\pi}{12}\right)$

32. **(a)** $\sin\left(\frac{7\pi}{8}\right)$

(b) $\cos\left(\frac{19\pi}{12}\right)$

Exercise Set 6.2: Double-Angle and Half-Angle Formulas

In the text, $\tan\left(\frac{s}{2}\right)$ is defined as:

$$\tan\left(\frac{s}{2}\right)=\frac{\sin(s)}{1+\cos(s)}.$$

The following exercises can be used to derive this formula along with two additional formulas for $\tan\left(\frac{s}{2}\right)$.

33. **(a)** Write the formula for $\sin\left(\frac{s}{2}\right)$.

(b) Write the formula for $\cos\left(\frac{s}{2}\right)$.

(c) Derive a new formula for $\tan\left(\frac{s}{2}\right)$` using the identity $\tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)}$, where $\theta=\frac{s}{2}$. Leave both the numerator and denominator in radical form. Show all work.

34. **(a)** This exercise will outline the derivation for: $\tan\left(\frac{s}{2}\right)=\frac{\sin(s)}{1+\cos(s)}$. In exercise 33, it was discovered that

$$\tan\left(\frac{s}{2}\right)=\pm\sqrt{\frac{1-\cos(s)}{1+\cos(s)}}.$$

Rationalize the denominator by multiplying both the numerator and denominator by $\sqrt{1+\cos(s)}$. Simplify the expression and write the result for $\tan\left(\frac{s}{2}\right)$.

(b) A detailed analysis of the signs of the trigonometric functions of s and $\frac{s}{2}$ in various quadrants reveals that the $\pm$ symbol in part (a) is unnecessary. (This analysis is lengthy and will not be shown here.) Given this fact, rewrite the formula from part (a) without the $\pm$ symbol.

(c) How does this result from part (b) compare with the formula given in the text for $\tan\left(\frac{s}{2}\right)$?

35. **(a)** In exercise 33, it was discovered that

$$\tan\left(\frac{s}{2}\right)=\pm\sqrt{\frac{1-\cos(s)}{1+\cos(s)}}.$$

Rationalize the numerator by multiplying both the numerator and denominator by $\sqrt{1-\cos(s)}$. Simplify the expression and write the new result for $\tan\left(\frac{s}{2}\right)$.

(b) A detailed analysis of the signs of the trigonometric functions of s and $\frac{s}{2}$ in various quadrants reveals that the $\pm$ symbol in part (a) is unnecessary. (This analysis is lengthy and will not be shown here.) Given this fact, rewrite the formula from part (a) without the $\pm$ symbol. This gives yet another formula which can be used for $\tan\left(\frac{s}{2}\right)$.

(c) Use the results from Exercises 33-35 to write the three formulas for $\tan\left(\frac{s}{2}\right)$. Which formula seems easiest to use and why? Which formula seems hardest to use and why?

36. **(a)** In the text, $\tan\left(\frac{s}{2}\right)$ is defined as:

$$\tan\left(\frac{s}{2}\right)=\frac{\sin(s)}{1+\cos(s)}.$$

Multiply both the numerator and denominator of the right-hand side of the equation by $\left[1-\cos(s)\right]$. Then simplify to obtain a formula for $\tan\left(\frac{s}{2}\right)$. Show all work.

(b) How does the result from part (a) compare to the identity obtained in part (b) of Exercise 39?

Exercise Set 6.2: Double-Angle and Half-Angle Formulas

Answer the following. SHOW ALL WORK. Do not leave any radicals in the denominator, i.e. rationalize the denominator whenever appropriate.

37. **(a)** Find $\cos\left(75^\circ\right)$ by using a sum or difference formula.

(b) Find $\cos\left(75^\circ\right)$ by using a half-angle formula.

(c) Enter the results from parts (a) and (b) into a calculator and round each one to the nearest hundredth. Are they the same?

38. **(a)** Find $\sin\left(165^\circ\right)$ by using a sum or difference formula.

(b) Find $\sin\left(165^\circ\right)$ by using a half-angle formula.

(c) Enter the results from parts (a) and (b) into a calculator and round each one to the nearest hundredth. Are they the same?

39. **(a)** Find $\sin\left(112.5^\circ\right)$ by using a half-angle formula.

(b) Find $\cos\left(112.5^\circ\right)$ by using a half-angle formula.

(c) Find $\tan\left(112.5^\circ\right)$ by computing $\dfrac{\sin\left(112.5^\circ\right)}{\cos\left(112.5^\circ\right)}$.

(d) Find $\tan\left(112.5^\circ\right)$ by using a half-angle formula.

40. **(a)** Find $\sin\left(105^\circ\right)$ by using a half-angle formula.

(b) Find $\cos\left(105^\circ\right)$ by using a half-angle formula.

(c) Find $\tan\left(105^\circ\right)$ by computing $\dfrac{\sin\left(105^\circ\right)}{\cos\left(105^\circ\right)}$.

(d) Find $\tan\left(105^\circ\right)$ by using a half-angle formula.

Find the exact value of each of the following by using a half-angle formula. Do not leave any radicals in the denominator, i.e. rationalize the denominator whenever appropriate.

41. **(a)** $\sin\left(\dfrac{13\pi}{12}\right)$ **(b)** $\cos\left(\dfrac{13\pi}{12}\right)$ **(c)** $\tan\left(\dfrac{13\pi}{12}\right)$

42. **(a)** $\sin\left(\dfrac{3\pi}{8}\right)$ **(b)** $\cos\left(\dfrac{3\pi}{8}\right)$ **(c)** $\tan\left(\dfrac{3\pi}{8}\right)$

43. **(a)** $\sin\left(\dfrac{11\pi}{8}\right)$ **(b)** $\cos\left(\dfrac{11\pi}{8}\right)$ **(c)** $\tan\left(\dfrac{11\pi}{8}\right)$

44. **(a)** $\sin\left(157.5^\circ\right)$ **(b)** $\cos\left(157.5^\circ\right)$ **(c)** $\tan\left(157.5^\circ\right)$

45. **(a)** $\sin\left(285^\circ\right)$ **(b)** $\cos\left(285^\circ\right)$ **(c)** $\tan\left(285^\circ\right)$

46. **(a)** $\sin\left(\dfrac{17\pi}{12}\right)$ **(b)** $\cos\left(\dfrac{17\pi}{12}\right)$ **(c)** $\tan\left(\dfrac{17\pi}{12}\right)$

Exercise Set 6.2: Double-Angle and Half-Angle Formulas

Answer the following.

47. If $\cos(\theta)=\frac{5}{9}$ and $\frac{3\pi}{2}<\theta<2\pi$,

(a) Determine the quadrant of the terminal side of θ.

(b) Complete the following: ___ $<\frac{\theta}{2}<$ ___

(c) Determine the quadrant of the terminal side of $\frac{\theta}{2}$.

(d) Based on the answer to part (c), determine the sign of $\sin\left(\frac{\theta}{2}\right)$.

(e) Based on the answer to part (c), determine the sign of $\cos\left(\frac{\theta}{2}\right)$.

(f) Find the exact value of $\sin\left(\frac{\theta}{2}\right)$.

(g) Find the exact value of $\cos\left(\frac{\theta}{2}\right)$.

(h) Find the exact value of $\tan\left(\frac{\theta}{2}\right)$.

48. If $\sin(\theta)=-\frac{\sqrt{21}}{5}$ and $\pi<\theta<\frac{3\pi}{2}$,

(a) Determine the quadrant of the terminal side of θ.

(b) Complete the following:

_____ $<\frac{\theta}{2}<$ _____

(c) Determine the quadrant of the terminal side of $\frac{\theta}{2}$.

(d) Based on the answer to part (c), determine the sign of $\sin\left(\frac{\theta}{2}\right)$.

(e) Based on the answer to part (c), determine the sign of $\cos\left(\frac{\theta}{2}\right)$.

(f) Find the exact value of $\sin\left(\frac{\theta}{2}\right)$.

(g) Find the exact value of $\cos\left(\frac{\theta}{2}\right)$.

(h) Find the exact value of $\tan\left(\frac{\theta}{2}\right)$.

49. If $\tan(\theta)=-\frac{\sqrt{7}}{3}$ and $\frac{\pi}{2}<\theta<\pi$,

(a) Find the exact value of $\sin\left(\frac{\theta}{2}\right)$.

(b) Find the exact value of $\cos\left(\frac{\theta}{2}\right)$.

(c) Find the exact value of $\tan\left(\frac{\theta}{2}\right)$.

50. If $\cos(\theta)=\frac{5}{6}$ and $\frac{3\pi}{2}<\theta<2\pi$,

(a) Find the exact value of $\sin\left(\frac{\theta}{2}\right)$.

(b) Find the exact value of $\cos\left(\frac{\theta}{2}\right)$.

(c) Find the exact value of $\tan\left(\frac{\theta}{2}\right)$.

Prove the following.

51. $\frac{1-\cos(2x)}{\sin(2x)}=\tan(x)$

52. $\frac{\cos(3x)}{\cos(x)}-\frac{\sin(3x)}{\sin(x)}=-2$

53. $\left[1-\sqrt{2}\sin(x)\right]\left[1+\sqrt{2}\sin(x)\right]=\cos(2x)$

54. $\cos^4(x)-\sin^4(x)=\cos(2x)$

55. $\csc(x)-\cot(x)=\tan\left(\frac{x}{2}\right)$

56. $\frac{1+\tan^2\left(\frac{x}{2}\right)}{1-\tan^2\left(\frac{x}{2}\right)}=\sec(\theta)$

Exercise Set 6.3: Solving Trigonometric Equations

Does the given *x*-value represent a solution to the given trigonometric equation? Answer yes or no.

1. $x=\dfrac{\pi}{6}$; $2\cos^2(x)-7\cos(x)=4$

2. $x=\dfrac{5\pi}{4}$; $3\tan^2(x)=8\tan(x)-5$

3. $x=\dfrac{3\pi}{2}$; $\sin^3(x)-4\sin^2(x)-2\sin(x)=-3$

4. $x=-\dfrac{\pi}{3}$; $6\cos^2(x)-7\cos(x)-2=0$

For each of the following equations,

(a) Solve the equation for $0^\circ \le x < 360^\circ$

(b) Solve the equation for $0 \le x < 2\pi$.

(c) Find all solutions to the equation, in radians.

5. $\cos(x)=-\dfrac{1}{2}$

6. $\sin(x)=\dfrac{\sqrt{3}}{2}$

7. $\tan(x)=-1$

8. $\cos(x)=0$

9. $\sin(x)=1$

10. $\sqrt{3}\cot(x)=-1$

11. $\sec(x)=-\dfrac{2\sqrt{3}}{3}$

12. $\csc(x)=-2$

13. $2\cos(x)=\sqrt{2}$

14. $\sin(x)=\sqrt{2}$

15. $2\sin^2(x)-5\sin(x)-3=0$

16. $2\cos^2(x)+\cos(x)=1$

17. $\cos^2(x)=2\cos(x)-1$

18. $2\sin^2(x)=-7\sin(x)+4$

When solving an equation such as $\sin(x)=-\frac{1}{2}$ for $0^\circ \le x < 360^\circ$, a typical thought process is to first compute $\sin^{-1}\left(\frac{1}{2}\right)$ to find the reference angle (in this case, 30°) and then use that reference angle to find solutions in quadrants where the sine value is negative (in this case, 210° and 330°. In each of the the following examples of the form $\sin(x)=C$,

(a) Use a calculator to find $\sin^{-1}|C|$ to the nearest tenth of a degree. This represents the reference angle.

(b) Use the reference angle from part (a) to find the solutions to the equation for $0^\circ \le x < 360^\circ$.

19. $\sin(x)=-\dfrac{4}{5}$

20. $\sin(x)=-0.3$

21. $\sin(x)=-0.46$

22. $\sin(x)=-\dfrac{\sqrt{2}}{5}$

The method in Exercises 19-22 can be used for other trigonometric functions as well. Use a calculator to solve each of the following equations for $0^\circ \le x < 360^\circ$. Round answers to the nearest tenth of a degree. If no solution exists, state "No solution."

Note: Since calculators do not contain inverse keys for cosecant, secant, and cotangent, use reciprocal relationships to rewrite equations in terms of sine, cosine and tangent. For example, to solve the equation $\sec(x)=-4$, first rewrite the equation as $\cos(x)=-\frac{1}{4}$ and then proceed to solve the equation.

23. **(a)** $\cos(x)=-\dfrac{3}{7}$ **(b)** $\csc(x)=\dfrac{7}{5}$

24. **(a)** $\tan(x)=-6$ **(b)** $\sec(x)=-7$

25. **(a)** $\cot(x)=-2.9$ **(b)** $\sin(x)=5.6$

26. **(a)** $\csc(x)=\dfrac{1}{4}$ **(b)** $\tan(x)=2.5$

Exercise Set 6.3: Solving Trigonometric Equations

Answer the following.

27. The following example is designed to demonstrate a common error in solving trigonometric equations. Consider the equation $2\cos^2(x) = \sqrt{3}\cos(x)$ for $0 \le x < 2\pi$.

(a) Divide both sides of the equation by $\cos(x)$ and then solve for *x*.

(b) Move all terms to the left side of the equation, and then solve for *x*.

(c) Are the answers in parts (a) and (b) the same?

(d) Which method is correct, part (a) or (b)? Why is the other method incorrect?

28. The following example is designed to demonstrate a common error in solving trigonometric equations. Consider the equation $\tan^2(x) = \tan(x)$ for $0^\circ \le x < 360^\circ$.

(a) Divide both sides of the equation by $\tan(x)$ and then solve for *x*.

(b) Move all terms to the left side of the equation, and then solve for *x*.

(c) Are the answers in parts (a) and (b) the same?

(d) Which method is correct, part (a) or (b)? Why is the other method incorrect?

Solve the following equations for $0^\circ \le x < 360^\circ$. If no solution exists, state "No solution."

29. $\sqrt{2}\sin^2(x) = \sin(x)$

30. $\cos^2(x) = -\cos(x)$

31. $\sqrt{3}\tan^2(x) = -\tan(x)$

32. $2\sin^2(x) = \sin(x)$

33. $4\sin^2(x)\cos(x) - \cos(x) = 0$

34. $\sin(x)\tan^2(x) = \sin(x)$

35. $2\cos^3(x) = -3\cos^2(x) - \cos(x)$

36. $2\sin^3(x) + 9\sin^2(x) = 5\sin(x)$

The following exercises show a method of solving an equation of the form:

$$\sin(Ax + B) = C, \text{ for } 0 \le x < 2\pi.$$

(The same method can be used for the other five trigonometric functions as well, and can similarly be applied to intervals other than $0 \le x < 2\pi$.) Answer the following, using the method described below.

(a) **Write the new interval obtained by multiplying each term in the solution interval (in this case, $0 \le x < 2\pi$) by A and then adding B.**

(b) **Let $u = Ax + B$. Find all solutions to $\sin(u) = C$ within the interval obtained in part (a).**

(c) **For each solution u from part (b), set up and solve $u = Ax + B$ for x. These x-values represent all solutions to the initial equation.**

37. $\sin(2x) = \dfrac{\sqrt{2}}{2}$

38. $\sin(3x) = -\dfrac{1}{2}$

39. $\sin\left(x - \dfrac{\pi}{2}\right) = -\dfrac{\sqrt{3}}{2}$

40. $\sin\left(x + \dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

41. $\sin(3x + \pi) = 0$

42. $\sin\left(\dfrac{x}{2} - \dfrac{\pi}{3}\right) = \dfrac{1}{2}$

Solve the following, using either the method above or the method described in the text. If no solution exists, state "No solution."

43. $2\cos(2x) = \sqrt{2}$, for $0 \le x < 2\pi$

44. $\sqrt{3}\tan(2x) + 1 = 0$, for $0^\circ \le x < 360^\circ$

45. $\csc(2x) = -\sqrt{2}$, for $0^\circ \le x < 360^\circ$

46. $\sec(2x) = 2$, for $0 \le x < 2\pi$

Exercise Set 6.3: Solving Trigonometric Equations

47. $2\sin\left(\frac{x}{2}\right)=-1$, for $0\le x<2\pi$

48. $\tan(3x)=\sqrt{3}$, for $0^\circ\le x<360^\circ$

49. $2\sin(3x)+\sqrt{3}=0$, for $0^\circ\le x<360^\circ$

50. $2\cos\left(\frac{x}{2}\right)=\sqrt{3}$, for $0^\circ\le x<360^\circ$

51. $2\cos(4x)=-\sqrt{3}$, for $0\le x<2\pi$

52. $\csc(5x)=-1$, for $0\le x<2\pi$

53. $\tan\left(x+90^\circ\right)=1$, for $0^\circ\le x<360^\circ$

54. $2\cos\left(x-45^\circ\right)=-1$, for $0^\circ\le x<360^\circ$

55. $\sqrt{3}\sec\left(x-\frac{5\pi}{6}\right)+2=0$, for $0\le x<2\pi$

56. $\cot(x+\pi)=0$, for $0\le x<2\pi$

57. $\sin\left(2x-270^\circ\right)=1$, for $0^\circ\le x<360^\circ$

58. $\tan(3x+2\pi)=0$, for $0\le x<2\pi$.

59. $\tan\left(\frac{x}{2}+60^\circ\right)=1$, for $0^\circ\le x<360^\circ$

60. $\cos\left(\frac{x}{3}-\frac{\pi}{3}\right)+1=0$, for $0\le x<2\pi$.

61. $2\cos\left(45^\circ-2x\right)=\sqrt{3}$, for $0^\circ\le x<360^\circ$

62. $2\sin\left(-3x-\frac{\pi}{4}\right)=\sqrt{2}$, for $0\le x<2\pi$.

Use a calculator to solve each of the following equations for $0^\circ\le x<360^\circ$. Round answers to the nearest tenth of a degree. If no solution exists, state "No solution."

63. $15\sin^2(x)=6\sin(x)$

64. $6\tan^2(x)-\tan(x)=1$

65. $3\cot^2(x)=5\cot(x)+2$

66. $4\csc^2(x)-8\csc(x)=-3$

67. $\sec^2(x)=16$

68. $3\cos^2(x)=-2\cos(x)$

Solve the following for $0\le x<2\pi$. If no solution exists, state "No solution."

69. $2\sin^2(x)-\cos(x)=1$

70. $6\cos^2 x-13\sin(x)-11=0$

71. $\cot^2(x)+\csc(x)-1=0$

72. $\sec^2(x)+2\tan(x)=0$

73. $|\tan(x)|=\sqrt{3}$

74. $|\sec(x)|=1$

75. $\log_2\left[\cos(x)\right]=-1$

76. $\log_3\left[\cot(x)\right]=\frac{1}{2}$

77. $\log_3\left[\tan(x)\right]=0$

78. $\log_{\sqrt{2}}\left[\csc(x)\right]=1$

Exercise Set 7.1: Solving Right Triangles

For each of the following examples,

(a) Write a trigonometric equation that can be used to find *x*, and write a trigonometric equation that can be used to find *y*. Each equation should involve the given acute angle and the given side length from the diagram. (Do not solve the equations in this step.)

(b) Solve the equations from part (a) to find exact values for *x* and *y*.

(c) As an alternative solution to the method above, use properties of special right triangles (from Section 4.1) to find exact values of *x* and *y*.

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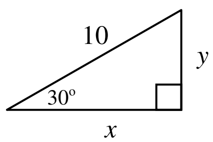

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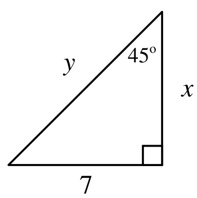

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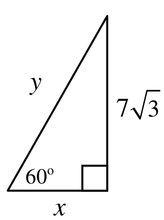

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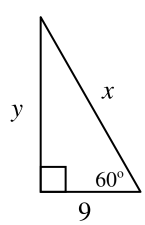

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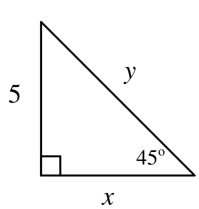

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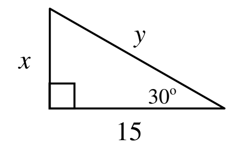

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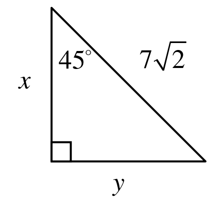

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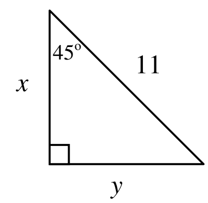


Use a calculator to evaluate the following. Round answers to the nearest ten-thousandth.

9. **(a)** $\sin\left(48^\circ\right)$ **(b)** $\cos\left(73^\circ\right)$

10. **(a)** $\cos\left(16^\circ\right)$ **(b)** $\tan\left(67^\circ\right)$

11. **(a)** $\tan\left(\dfrac{\pi}{9}\right)$ **(b)** $\sin\left(\dfrac{2\pi}{7}\right)$

12. **(a)** $\sin\left(\dfrac{\pi}{10}\right)$ **(b)** $\cos\left(\dfrac{3\pi}{11}\right)$

In the equations below, *x* represents an acute angle of a triangle. Use a calculator to solve for *x*. Round answers to the nearest hundredth of a degree.

13. $\sin(x) = 0.7481$

14. $\cos(x) = 0.0256$

15. $\tan(x) = 4$

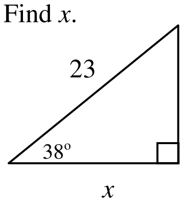
16. $\tan(x) = \dfrac{8}{15}$

17. $\cos(x) = \dfrac{16}{17}$

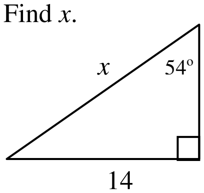
18. $\sin(x) = \dfrac{4}{7}$

Find the indicated side or angle measures (where angles are measured in degrees). Round answers to the nearest hundredth.

19. Find x.



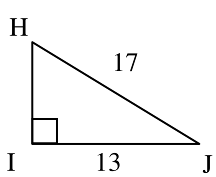
20. Find x.



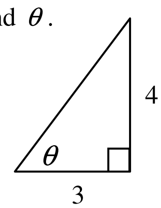
21. In ΔABC with right angle C, $\angle A = 74^\circ$ and $BC = 32\text{ in}$. Find AC.

22. In ΔQRS with right angle R, $QR = 7\text{ in}$ and $\angle Q = 12^\circ$. Find QS.

23. Find $\angle H$.



24. Find θ.

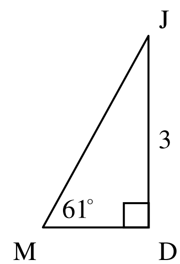


25. In ΔGEO with right angle E, $GE = 9\text{ cm}$ and $OE = 7\text{ cm}$. Find $\angle G$.

26. In ΔCAR with right angle R, $CA = 11\text{ in}$ and $AR = 8\text{ in}$. Find $\angle A$.

Solve each of the following triangles, i.e. find all missing side and angle measures (where angles are measured in degrees). Round all answers to the nearest hundredth.

27.



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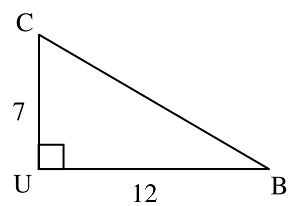

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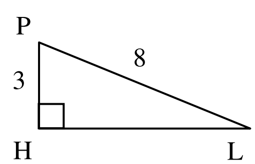

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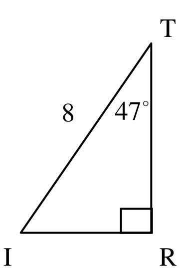

Exercise Set 7.1: Solving Right Triangles

For each of the following examples,

(a) Draw a diagram to represent the given situation.

(b) Find the indicated measure, to the nearest tenth.

31. An isosceles triangle has sides measuring 9 inches, 14 inches, and 14 inches. What are the measures of its angles?

32. An isosceles triangle has sides measuring 10 inches, 7 inches, and 7 inches. What are the measures of its angles?

33. An 8-foot ladder is leaned against the side of the house. If the ladder makes a 72° angle with the ground, how high up does the ladder reach?

34. A 14-foot ladder is leaned against the side of the house. If the ladder makes a 63° angle with the ground, how high up does the ladder reach?

35. The angle of elevation to the top of a flagpole from a point on the ground 40 ft from the base of the flagpole is 51°. Find the height of the flagpole.

36. The angle of elevation to the top of a flagpole from a point on the ground 18 meters from the base of the flagpole is 43°. Find the height of the flagpole.

37. A loading dock is 8 feet high. If a ramp makes an angle of 34° with the ground and is attached to the loading dock, how long is the ramp?

38. A loading dock is 5 feet high. If a ramp makes an angle of 25° with the ground and is attached to the loading dock, how long is the ramp?

39. Silas stands 600 feet from the base of the Sears Tower and sights the top of the tower. If the Sears Tower is 1,450 feet tall, approximate the angle of elevation from Silas' perspective as he sights the top of the tower. (Disregard Silas' height in your calculations.)

40. Mimi is sitting in her boat and sees a lighthouse which is 20 meters tall. If she sights the top of the lighthouse at a 32° angle of elevation, approximately how far is Mimi's boat from the base of the lighthouse? (Disregard Mimi's distance from the water in your calculations.)

41. Jeremy is flying his kite at Lake Michigan and lets out 200 feet of string. If his eyes are 6 feet from the ground as he sights the kite at an 55° angle of elevation, how high is the kite? (Assume that he is holding the kite near his eyes.)

42. Pam is flying a kite at the beach and lets out 120 feet of string. If her eyes are 5 feet from the ground as she sights the kite at a 64° angle of elevation, how high is the kite? (Assume that she is also holding the kite near her eyes.)

43. Doug is out hiking and walks to the edge of a cliff to enjoy the view. He looks down and sights a cougar at a 41° angle of depression. If the cliff is 60 meters high, approximately how far is the cougar from the base of the cliff? (Disregard Doug's height in your calculations.)

44. Juliette wakes up in the morning and looks out her window into the back yard. She sights a squirrel at a 54° angle of depression. If Juliette is looking out the window at a point which is 28 feet from the ground, how far is the squirrel from the base of the house?

45. From a point on the ground, the angle of elevation to the top of a mountain is 42°. Moving out a distance of 150 m (on a level plane) to another point on the ground, the angle of elevation is 33°. Find the height of the mountain.

46. From a point on the ground, the angle of elevation to the top of a mountain is 31°. Moving out a distance of 200 m (on a level plane) to another point on the ground, the angle of elevation is 18°. Find the height of the mountain.

Exercise Set 7.2: Area of a Triangle

Use the formula $A = \frac{1}{2}bh$ to find the area of each of the following triangles. (You may need to find the base and/or the height first, using trigonometric ratios or the Pythagorean Theorem.) Give exact values whenever possible. Otherwise, round answers to the nearest hundredth.

Note: Figures may not be drawn to scale.

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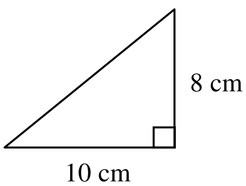

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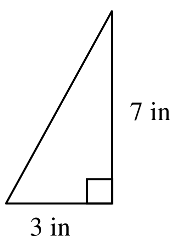

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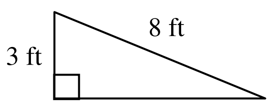

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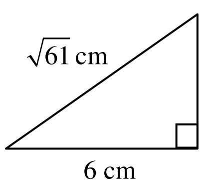

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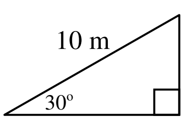

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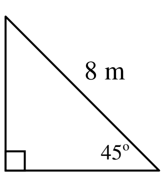

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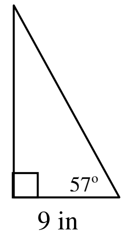

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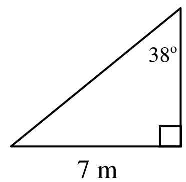
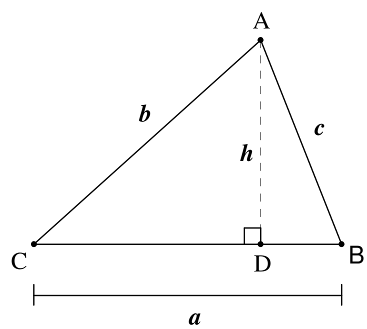


In ΔABC below, $\overline{AD} \perp \overline{BC}$. Use the diagram below to answer the following questions.



9. **(a)** Use ΔACD to write a trigonometric ratio that involves $\angle C$, b, and h.

(b) Using the equation from part (a), solve for h.

(c) If a represents the base of ΔABC, and h represents the height, then the area K of ΔABC is $K = \frac{1}{2}ah$. Substitute the equation from part (b) into this equation to obtain a formula for the area of ΔABC that no longer contains h.

10. **(a)** Use ΔABD to write a trigonometric ratio that involves $\angle B$, c, and h.

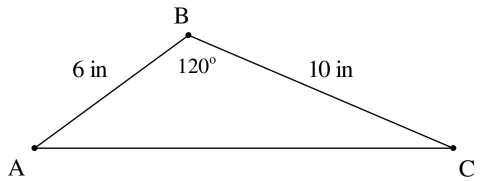
(b) Using the equation from part (a), solve for h.

(c) If a represents the base of ΔABC, and h represents the height, then the area K of ΔABC is $K = \frac{1}{2}ah$. Substitute the equation from part (b) into this equation to obtain a formula for the area of ΔABC that no longer contains h.

Exercise Set 7.2: Area of a Triangle

Find the area of each of the following triangles. Give exact values whenever possible. Otherwise, round answers to the nearest hundredth.
Note: Figures may not be drawn to scale.

11.



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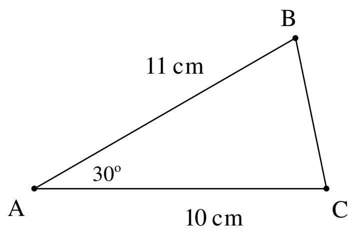

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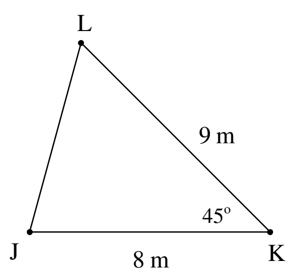

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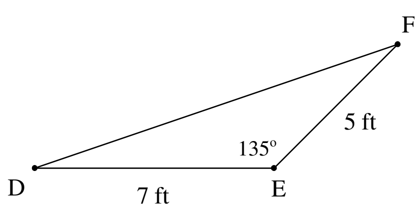

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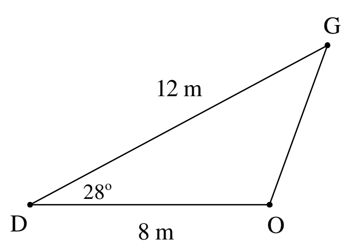

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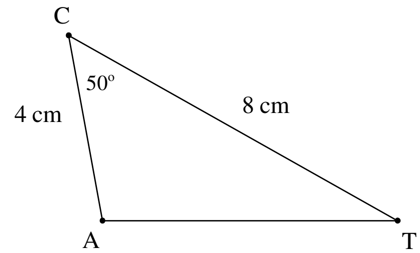

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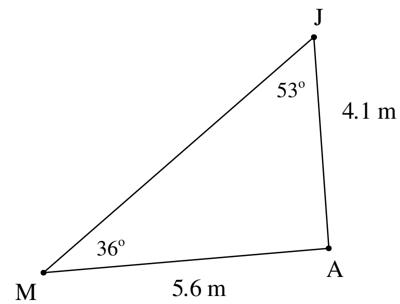

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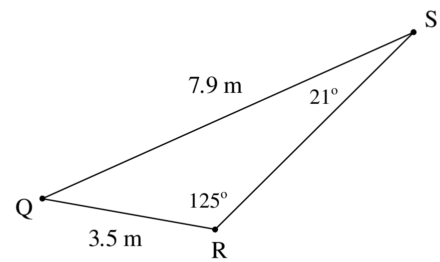


Answer the following. Give exact values whenever possible. Otherwise, round answers to the nearest hundredth.

19. Find the area of an isosceles triangle with legs measuring 7 inches and base angles measuring 22.5°.

20. Find the area of an isosceles triangle with legs measuring 10 cm and base angles measuring 75°.

21. Find the area of ΔGHJ, where $\angle G = 120^\circ$, $h = 8$ cm, and $j = 15$ cm.

22. Find the area of ΔPQR, where $\angle R = 47^\circ$, $p = 7$ in, and $q = 10$ in.

Exercise Set 7.2: Area of a Triangle

23. Find the area of ΔTUV, where $\angle T = 28^\circ$, $\angle U = 81^\circ$, $t = 12$ m, and $u = 6.5$ m.

24. Find the area of ΔFUN, where $\angle F = 92^\circ$, $\angle N = 28^\circ$, $f = 9$ cm, and $n = 8$ cm.

25. **(a)** Find all solutions to $\sin(x) = 0.5$ for $0^\circ < x < 180^\circ$.

(b) If the area of ΔRST is 20 cm^2, $r = 16$ cm and $t = 5$ cm, find all possible measures for $\angle S$.

26. **(a)** Find all solutions to $\sin(x) = 0.2$ for $0^\circ < x < 180^\circ$.

(b) If the area of ΔPHL is 12 m^2, $p = 10$ m and $h = 6$ m, find all possible measures for $\angle L$.

27. If the area of ΔTRG is 37 in^2, $t = 8$ in and $g = 12.5$ in, find all possible measures for $\angle R$.

28. If the area of ΔPSU is $20\sqrt{2}$ cm^2, $p = 4$ cm and $s = 5$ cm, find all possible measures for $\angle U$.

29. A regular octagon is inscribed in a circle of radius 12 in. Find the area of the octagon.

30. A regular hexagon is inscribed in a circle of radius 11 cm. Find the area of the hexagon.

31. A regular hexagon is circumscribed about a circle of radius 6 in. Find the area of the hexagon.

32. A regular hexagon is circumscribed about a circle of radius 10 cm. Find the area of the hexagon.

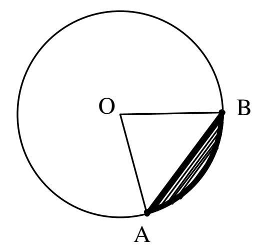
33. Find the area of an equilateral triangle with side length 4 ft.

34. Find the area of a regular hexagon with side length 10 cm.

35. Find the area of a regular decagon with side length 8 in.

36. Find the area of a regular octagon with side length 12 m.

In the following questions, the radius of circle *O* is given, as well as the measure of central angle *AOB*. Find the area of the segment of circle *O* bounded by $\overline{AB}$ and $\overset{\frown}{AB}$. Give exact values whenever possible. Otherwise, round answers to the nearest hundredth.



37. Radius: 8 cm

Central Angle: $\dfrac{3\pi}{7}$

38. Radius: 7 in

Central Angle: $\dfrac{4\pi}{5}$

39. Radius: 4 in

Central Angle: $\dfrac{\pi}{3}$

40. Radius: 6 cm

Central Angle: $\dfrac{3\pi}{4}$

Exercise Set 7.3: The Law of Sines and the Law of Cosines

Find the indicated part of each triangle. Give exact values whenever possible. Otherwise, round answers to the nearest hundredth. If no such triangle exists, state "No such triangle exists."

1. ΔABC
$a = 9$ m
$c = 5$ m
$\angle B = 71^\circ$
$b = ?$

2. ΔABC
$a = 16$ cm
$\angle A = 35^\circ$
$\angle B = 65^\circ$
$b = ?$

3. ΔABC
$a = 9$ in
$\angle A = 60^\circ$
$\angle C = 45^\circ$
$c = ?$

4. ΔRST
$s = 8$ mm
$\angle S = 135^\circ$
$\angle T = 30^\circ$
$t = ?$

5. ΔNAP
$n = 5$ in
$a = 10$ in
$p = 7$ in
$\angle N = ?$

6. ΔKLM
$k = 3$ in
$l = 4$ in
$m = 8$ in
$\angle M = ?$

7. ΔSUN
$n = 10$ cm
$\angle N = 63^\circ$
$\angle S = 48^\circ$
$s = ?$

8. ΔPSU
$s = 2.5$ in
$\angle P = 112^\circ$
$\angle U = 37^\circ$
$p = ?$

9. ΔCAN
$c = 11$ cm
$a = 9$ cm
$n = 14$ cm
$\angle C = ?$

10. ΔHAT
$a = 14$ mm
$\angle A = 45^\circ$
$\angle T = 15^\circ$
$h = ?$

11. ΔRST
$r = 9$ mm
$s = 15$ in
$t = 4$ in
$\angle R = ?$

12. ΔBOX
$o = 10$ cm
$x = 7$ cm
$\angle B = 43^\circ$
$b = ?$

13. ΔTRY
$y = 12$ cm
$\angle T = 30^\circ$
$\angle R = 105^\circ$
$t = ?$

Exercise Set 7.3: The Law of Sines and the Law of Cosines

14. ΔWIN
$w = 4.9$ ft
$\angle I = 42^\circ$
$\angle N = 100^\circ$
$n = ?$

Find all possible measures for the indicated angle of the triangle. (There may be 0, 1, or 2 triangles with the given measures.) Round answers to the nearest hundredth. If no such triangle exists, state "No such triangle exists."

15. ΔDEF
$d = 25$ mm
$e = 13$ mm
$\angle E = 21^\circ$
$\angle D = ?$

16. ΔCAR
$c = 3$ cm
$r = 7$ cm
$\angle C = 15^\circ$
$\angle R = ?$

17. ΔABC
$b = 9$ cm
$c = 6$ cm
$\angle B = 95^\circ$
$\angle C = ?$

18. ΔBUS
$b = 18$ mm
$u = 16$ mm
$\angle B = 120^\circ$
$\angle U = ?$

19. ΔGEO
$g = 11$ cm
$o = 4$ cm
$\angle O = 33^\circ$
$\angle G = ?$

20. ΔABC
$b = 10$ ft
$c = 7$ ft
$\angle B = 70^\circ$
$\angle A = ?$

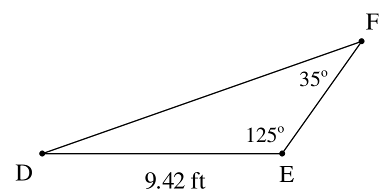
21. ΔANT
$n = 12$ cm
$t = 15$ cm
$\angle T = 129^\circ$
$\angle A = ?$

22. ΔJAY
$j = 15$ in
$y = 9$ in
$\angle Y = 160^\circ$
$\angle J = ?$

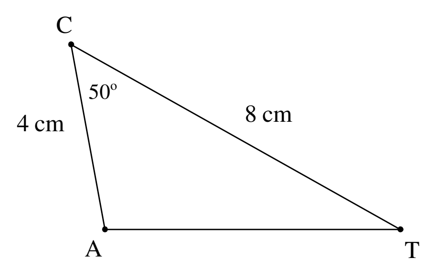
Solve each of the following triangles, i.e. find all missing side and angle measures. There may be two groups of answers for some exercises. Round answers to the nearest hundredth. If no such triangle exists, state "No such triangle exists."

Note: Figures may not be drawn to scale.

23.



24.



25. ΔWHY
$h = 8$ in
$y = 4$ in
$\angle Y = 28^\circ$

26. ΔABC
$a = 12$ in
$b = 17$ in
$c = 6$ in

27. ΔRUN
$r = 7$ cm
$u = 12$ cm
$n = 4$ cm

28. ΔJKL
$j = 6$ m
$k = 5$ m
$\angle J = 74^\circ$

29. ΔPEZ
$p = 12$ mm
$z = 10$ mm
$\angle E = 130^\circ$

30. ΔBAD
$b = 11$ ft
$a = 9$ ft
$d = 5$ ft

31. ΔPIG
$p = 6$ cm
$i = 13$ cm
$g = 11$ cm

32. ΔQIZ
$i = 12.4$ mm
$z = 11.5$ mm
$\angle Z = 66^\circ$

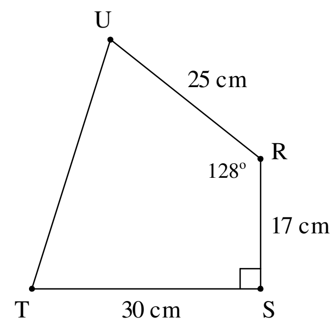
33. ΔDEF
$d = 10$ mm
$f = 3$ mm
$\angle F = 21^\circ$

34. ΔSEA
$s = 8$ cm
$e = 9$ cm
$a = 21$ cm

Find the area of each of the following quadrilaterals. Round all intermediate computations to the nearest hundredth, and then round the area of the quadrilateral to the nearest tenth.

Note: Figures may not be drawn to scale.

35.



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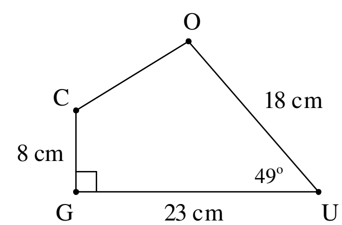

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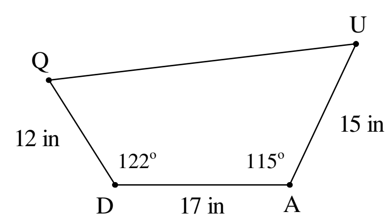

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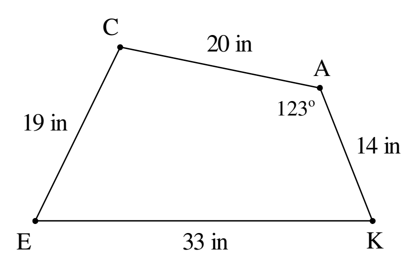


Exercise Set 7.3: The Law of Sines and the Law of Cosines

Answer the following. Round answers to the nearest hundredth.

Note: Figures may not be drawn to scale.

39. **(a)** Use the Law of Sines or the Law of Cosines to find the measures of the acute angles of a 3-4-5 right triangle.

(b) Use right triangle trigonometry to find the measures of the acute angles of a 3-4-5 right triangle.

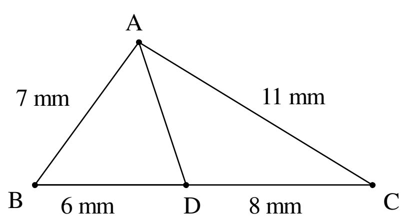
40. **(a)** Use the Law of Sines or the Law of Cosines to find the measures of the acute angles of a 7-24-25 right triangle.

(b) Use right triangle trigonometry to find the measures of the acute angles of a 7-24-25 right triangle.

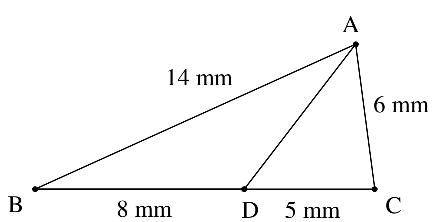
41. In ΔHEY, $\angle H = 125^\circ$ and $h = \frac{5}{4}y$. Find the measures of $\angle E$ and $\angle Y$.

42. In ΔFUN, $\angle F = 100^\circ$ and $u = \frac{2}{3}f$. Find the measures of $\angle U$ and $\angle N$.

43. Find the length of $\overline{AD}$.



44. Find the length of $\overline{AD}$.



Exercise Set 8.1: Parabolas

Write each of the following equations in the standard form for the equation of a parabola, where the standard form is represented by one of the following equations:

$$(x-h)^2 = 4p(y-k) \qquad (x-h)^2 = -4p(y-k)$$
$$(y-k)^2 = 4p(x-h) \qquad (y-k)^2 = -4p(x-h)$$

1. $y^2 - 14y - 2x + 43 = 0$

2. $x^2 + 10x - 12y = -61$

3. $-9y = x^2 - 8x - 10$

4. $-7x = y^2 - 10y + 24$

5. $x = 3y^2 - 24y + 50$

6. $y = 2x^2 + 12x + 15$

7. $3x^2 - 3x - 5 - y = 0$

8. $5y^2 + 5y = x - 6$

For each of the following parabolas,

(a) Write the given equation in the standard form for the equation of a parabola. (Some equations may already be given in standard form.)

It may be helpful to begin sketching the graph for part (h) as a visual aid to answer the questions below.

(b) State the equation of the axis.

(c) State the coordinates of the vertex.

(d) State the equation of the directrix.

(e) State the coordinates of the focus.

(f) State the focal width.

(g) State the coordinates of the endpoints of the focal chord.

(h) Sketch a graph of the parabola which includes the features from (c)-(e) and (g). Label the vertex V and the focus F.

9. $x^2 - 4y = 0$

10. $y^2 - 12x = 0$

11. $-10x = y^2$

12. $-6y = x^2$

13. $(x-2)^2 = 8(y+5)$

14. $(y-4)^2 = 16x$

15. $y^2 = 4(x-3)$

16. $(x+3)^2 = 4(y-1)$

17. $(x+5)^2 = -2(y-4)$

18. $(y+1)^2 = -10(x+3)$

19. $(y-6)^2 = -1(x-2)$

20. $(x-1)^2 = -8(y-6)$

21. $x^2 + 12x - 6y + 24 = 0$

22. $x^2 - 2y = 8x - 10$

23. $y^2 - 8x = 4y + 36$

24. $y^2 + 6y - 4x + 5 = 0$

25. $x^2 + 25 = -16y - 10x$

26. $y^2 + 10y + x + 28 = 0$

27. $y^2 - 4y + 2x - 4 = 0$

28. $12y + x^2 - 4x = -16$

29. $3y^2 + 30y - 8x + 67 = 0$

30. $5x^2 + 30x - 16y = 19$

31. $2x^2 - 8x + 7y = 34$

32. $4y^2 - 8y + 9x + 40 = 0$

Use the given features of each of the following parabolas to write an equation for the parabola in standard form.

33. Vertex: $(-2, 5)$
Focus: $(4, 5)$

34. Vertex: $(1, -3)$
Focus: $(1, 0)$

Exercise Set 8.1: Parabolas

35. Vertex: $(2, 0)$
Focus: $(2, -4)$

36. Vertex: $(-4, -2)$
Focus: $(-6, -2)$

37. Focus: $(-2, -3)$
Directrix: $y = -9$

38. Focus: $(4, 1)$
Directrix: $y = 5$

39. Focus: $(4, -1)$
Directrix: $x = 7\frac{1}{2}$

40. Focus: $(-3, 5)$
Directrix: $x = -4$

41. Focus: $(-4, -2)$
Opens downward
$p = 7$

42. Focus: $(1, 5)$
Opens to the right
$p = 3$

43. Vertex: $(5, 6)$
Opens upward
Length of focal chord: 6

44. Vertex: $\left(0, \frac{1}{2}\right)$
Opens to the left
Length of focal chord: 2

45. Vertex: $(-3, 2)$
Horizontal axis
Passes through $(6, 5)$

46. Vertex: $(2, 1)$
Vertical axis
Passes through $(8, 5)$

47. Endpoints of focal chord: $(0, 5)$ and $(0, -5)$
Opens to the left

48. Endpoints of focal chord: $(-2, 3)$ and $(6, 3)$
Opens downward

Answer the following.

49. Write an equation of the line tangent to the parabola with equation $f(x) = x^2 + 5x + 4$ at:
(a) $x = 3$
(b) $x = -2$

50. Write an equation of the line tangent to the parabola with equation $f(x) = -3x^2 + 6x + 1$ at:
(a) $x = 0$
(b) $x = -1$

51. Write an equation of the line tangent to the parabola with equation $f(x) = 2x^2 + 5x - 1$ at:
(a) $x = -1$
(b) $x = \frac{1}{2}$

52. Write an equation of the line tangent to the parabola with equation $f(x) = -x^2 + 4x - 2$ at:
(a) $x = -3$
(b) $x = \frac{3}{2}$

53. Write an equation of the line tangent to the parabola with equation $f(x) = 4x^2 - 5x - 3$ at the point $(1, -4)$.

54. Write an equation of the line tangent to the parabola with equation $f(x) = 2x^2 + 5x + 4$ at the point $(-3, 7)$.

55. Write an equation of the line tangent to the parabola with equation $f(x) = -x^2 - 6x + 1$ at the point $(-5, 6)$.

56. Write an equation of the line tangent to the parabola with equation $f(x) = -3x^2 + 4x + 9$ at the point $(2, 5)$.

Exercise Set 8.1: Parabolas

Give the point(s) of intersection of the parabola and the line whose equations are given.

57. $f(x) = x^2 - 4x + 11$
$g(x) = 5x - 3$

58. $f(x) = x^2 - 8x + 1$
$g(x) = -2x - 10$

59. $f(x) = x^2 + 10x + 10$
$g(x) = 8x + 9$

60. $f(x) = x^2 - 5x - 10$
$g(x) = -7x + 14$

61. $f(x) = -x^2 + 6x - 5$
$g(x) = 6x - 14$

62. $f(x) = -x^2 + 3x - 2$
$g(x) = -5x + 13$

63. $f(x) = -3x^2 + 6x + 1$
$g(x) = -3x + 7$

64. $f(x) = -2x^2 + 8x - 5$
$g(x) = 6x - 5$

Exercise Set 8.2: Ellipses

Write each of the following equations in the standard form for the equation of an ellipse, where the standard form is represented by one of the following equations:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \qquad \frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

1. $25x^2+4y^2-100=0$
2. $9x^2+16y^2-144=0$
3. $9x^2-36x+4y^2-32y+64=0$
4. $4x^2+24x+16y^2-32y-12=0$
5. $3x^2+2y^2-30x-12y=-87$
6. $x^2+8y^2+113=14x+48y$
7. $16x^2-16x-64=-8y^2-24y+42$
8. $18x^2+9y^2=153-24x+6y$

Answer the following.

9. **(a)** What is the equation for the eccentricity, e, of an ellipse?
   (b) As e approaches 1, the ellipse appears to become more (choose one):
   elongated circular
   (c) If $e=0$, the ellipse is a __________.

10. The sum of the focal radii of an ellipse is always equal to __________.

Answer the following for each ellipse. For answers involving radicals, give exact answers and then round to the nearest tenth.

(a) Write the given equation in the standard form for the equation of an ellipse. (Some equations may already be given in standard form.)

It may be helpful to begin sketching the graph for part (g) as a visual aid to answer the questions below.

(b) State the coordinates of the center.

(c) State the coordinates of the vertices of the major axis, and then state the length of the major axis.

(d) State the coordinates of the vertices of the minor axis, and then state the length of the minor axis.

(e) State the coordinates of the foci.

(f) State the eccentricity.

(g) Sketch a graph of the ellipse which includes the features from (b)-(e). Label the center C, and the foci F_1 and F_2.

11. $\dfrac{x^2}{9}+\dfrac{y^2}{49}=1$
12. $\dfrac{x^2}{36}+\dfrac{y^2}{4}=1$
13. $\dfrac{(x-2)^2}{16}+\dfrac{y^2}{4}=1$
14. $\dfrac{x^2}{9}+\dfrac{(y+1)^2}{5}=1$
15. $\dfrac{(x-2)^2}{25}+\dfrac{(y+3)^2}{16}=1$
16. $\dfrac{(x+5)^2}{16}+\dfrac{(y+2)^2}{25}=1$
17. $\dfrac{(x+4)^2}{9}+\dfrac{(y-3)^2}{1}=1$
18. $\dfrac{(x+2)^2}{36}+\dfrac{(y+3)^2}{16}=1$
19. $\dfrac{(x-2)^2}{11}+\dfrac{(y+4)^2}{36}=1$
20. $\dfrac{(x+3)^2}{20}+\dfrac{(y-5)^2}{4}=1$
21. $4x^2+9y^2-36=0$
22. $4x^2+y^2=1$
23. $25x^2+16y^2-311=50x-64y$
24. $16x^2+25y^2=150y+175$
25. $16x^2-32x+4y^2-40y+52=0$
26. $25x^2+9y^2-100x+54y-44=0$
27. $16x^2+7y^2+64x-42y+15=0$
28. $4x^2+3y^2-16x+6y-29=0$

Exercise Set 8.2: Ellipses

Use the given features of each of the the following ellipses to write an equation for the ellipse in standard form.

29. Center: $(0,0)$
$a=8$
$b=5$
Horizontal Major Axis

30. Center: $(0,0)$
$a=7$
$b=3$
Vertical Major Axis

31. Center: $(-4,7)$
$a=5$
$b=3$
Vertical Major Axis

32. Center: $(2,-4)$
$a=5$
$b=2$
Horizontal Major Axis

33. Center: $(-3,-5)$
Length of major axis $=6$
Length of minor axis $=4$
Horizontal Major Axis

34. Center: $(2,1)$
Length of major axis $=10$
Length of minor axis $=2$
Vertical Major Axis

35. Foci: $(2,5)$ and $(2,-5)$
$a=9$

36. Foci: $(4,-3)$ and $(-4,-3)$
$a=7$

37. Foci: $(-8,1)$ and $(2,1)$
$a=6$

38. Foci: $(-2,-3)$ and $(-2,5)$
$a=8$

39. Foci: $(-1,2)$ and $(7,2)$
Passes through the point $(3,5)$

40. Foci: $(-3,4)$ and $(7,4)$
Passes through the point $(2,1)$

41. Center: $(-5,2)$
$a=8$
$e=\frac{3}{4}$
Vertical major axis

42. Center: $(-4,-2)$
$a=6$
$e=\frac{2}{3}$
Horizontal major axis

43. Foci: $(0,4)$ and $(0,8)$
$e=\frac{1}{3}$

44. Foci: $(1,5)$ and $(1,-3)$
$e=\frac{1}{2}$

45. Foci: $(2,3)$ and $(6,3)$
$e=0.4$

46. Foci: $(2,1)$ and $(10,1)$
$e=0.8$

47. Foci: $(3,0)$ and $(-3,0)$
Sum of the focal radii $=8$

48. Foci: $(0,\sqrt{11})$ and $(0,-\sqrt{11})$
Sum of the focal radii $=12$

Exercise Set 8.2: Ellipses

A circle is a special case of an ellipse where $a = b$. It then follows that $c^2 = a^2 - b^2 = a^2 - a^2 = 0$, so $c = 0$. (Therefore, the foci are at the center of the circle, and this is simply labeled as the center and not the focus.)

The standard form for the equation of a circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

(Note: If each term in the above equation were divided by r^2, it would look like the standard form for an ellipse, with $a = b = r$.)

Using the above information, use the given features of each of the following circles to write an equation for the circle in standard form.

49. Center: $(0,0)$
Radius: 9

50. Center: $(0,0)$
Radius: $\sqrt{5}$

51. Center: $(7,-2)$
Radius: 10

52. Center: $(-2,5)$
Radius: 7

53. Center: $(-3,-4)$
Radius: $3\sqrt{2}$

54. Center: $(-8,0)$
Radius: $2\sqrt{5}$

55. Center: $(2,-5)$
Passes through the point $(7,-6)$

56. Center: $(-6,-3)$
Passes through the point $(-8,-2)$

57. Endpoints of diameter: $(-4,-6)$ and $(-2,0)$

58. Endpoints of diameter: $(3,0)$ and $(7,10)$

59. Center: $(-3,5)$
Circle is tangent to the x-axis

60. Center: $(-3,5)$
Circle is tangent to the y-axis

Answer the following for each circle. For answers involving radicals, give exact answers and then round to the nearest tenth.

(a) **Write the given equation in the standard form for the equation of a circle. (Some equations may already be given in standard form.)**

(b) **State the coordinates of the center.**

(c) **State the length of the radius.**

(d) **Sketch a graph of the circle which includes the features from (b) and (c). Label the center C and show four points on the circle itself (these four points are equivalent to the vertices of the major and minor axes for an ellipse).**

61. $x^2 + y^2 - 36 = 0$

62. $x^2 + y^2 - 8 = 0$

63. $\dfrac{(x-3)^2}{16} + \dfrac{(y+2)^2}{16} = 1$

64. $\dfrac{x^2}{9} + \dfrac{(y+2)^2}{9} = 1$

65. $(x+5)^2 + (y-2)^2 = 4$

66. $(x-1)^2 + (y-4)^2 = 36$

67. $(x+4)^2 + (y+3)^2 = 12$

68. $(x-1)^2 + (y-5)^2 = 7$

69. $x^2 + y^2 + 2x - 10y + 17 = 0$

70. $x^2 + y^2 + 6x - 2y - 6 = 0$

71. $x^2 + y^2 + 10x - 8y + 36 = 0$

72. $x^2 + y^2 - 4x - 14y + 50 = 0$

73. $3x^2 + 3y^2 + 18x - 24y + 63 = 0$

74. $2x^2 + 2y^2 + 10x + 12y + 52 = 10$

Exercise Set 8.2: Ellipses

Answer the following.

75. A circle passes through the points $(7, 6)$, $(7, -2)$ and $(1, 6)$. Write the equation of the circle in standard form.

76. A circle passes through the points $(-4, 3)$, $(-2, 3)$ and $(-2, -1)$. Write the equation of the circle in standard form.

77. A circle passes through the points $(2, 1)$, $(2, -3)$ and $(8, 1)$. Write the equation of the circle in standard form.

78. A circle passes through the points $(-7, -8)$, $(-7, 2)$ and $(-1, 2)$. Write the equation of the circle in standard form.

Exercise Set 8.3: Hyperbolas

Identify the type of conic section (parabola, ellipse, circle, or hyperbola) represented by each of the following equations. (In the case of a circle, identify the conic section as a circle rather than an ellipse.) Do NOT write the equations in standard form; these questions can instead be answered by looking at the signs of the quadratic terms.

1. $2y + x^2 + 9x = 0$
2. $14x^2 + 7x - 12y = -6y^2 + 95$
3. $7x^2 - 3y^2 = 5x - y + 40$
4. $y^2 + 9 = 9y - x$
5. $3x^2 - 7x + 3y^2 = -12y + 13$
6. $x^2 + 10x = -2y - y^2 + 5$
7. $4y^2 + 2x^2 = 8y - 6x + 9$
8. $8y^2 + 24x = 8x^2 + 30$

Write each of the following equations in the standard form for the equation of a hyperbola, where the standard form is represented by one of the following equations:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

9. $y^2 - 8x^2 - 8 = 0$
10. $3x^2 - 10y^2 - 30 = 0$
11. $x^2 - y^2 - 6x = -2y - 3$
12. $9x^2 - 3y^2 = 48y + 192$
13. $7x^2 - 5y^2 + 14x + 20y - 48 = 0$
14. $9y^2 - 2x^2 + 90y + 16x + 175 = 0$

Answer the following.

15. The length of the transverse axis of a hyperbola is __________.
16. The length of the conjugate axis of a hyperbola is __________.
17. The following questions establish the formulas for the slant asymptotes of

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1.$$

(a) State the point-slope equation for a line.

(b) Substitute the center of the hyperbola, (h, k) into the equation from part (a).

(c) Recall that the formula for slope is represented by $\frac{\text{rise}}{\text{run}}$. In the equation $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$, what is the "rise" of each slant asymptote from the center? What is the "run" of each slant asymptote from the center?

(d) Based on the answers to part (c), what is the slope of each of the asymptotes for the graph of $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$? (Remember that there are two slant asymptotes passing through the center of the hyperbola, one having positive slope and one having negative slope.)

(e) Substitute the slopes from part (d) into the equation from part (b) to obtain the equations of the slant asymptotes.

18. The following questions establish the formulas for the slant asymptotes of

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1.$$

(a) State the point-slope equation for a line.

(b) Substitute the center of the hyperbola, (h, k) into the equation from part (a).

(c) Recall that the formula for slope is represented by $\frac{\text{rise}}{\text{run}}$. In the equation $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, what is the "rise" of each slant asymptote from the center? What is the "run" of each slant asymptote from the center?

(d) Based on the answers to part (c), what is the slope of each of the asymptotes for the graph of $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$? (Remember that there are two slant asymptotes passing through the center of the hyperbola, one having positive slope and one having negative slope.)

(e) Substitute the slopes from part (d) into the equation from part (b) to obtain the equations of the slant asymptotes.

19. In the standard form for the equation of a hyperbola, a^2 represents (choose one):

the larger denominator
the denominator of the first term

20. In the standard form for the equation of a hyperbola, b^2 represents (choose one):

the smaller denominator
the denominator of the second term

Answer the following for each hyperbola. For answers involving radicals, give exact answers and then round to the nearest tenth.

(a) **Write the given equation in the standard form for the equation of a hyperbola. (Some equations may already be given in standard form.)**

It may be helpful to begin sketching the graph for part (h) as a visual aid to answer the questions below.

(b) **State the coordinates of the center.**

(c) **State the coordinates of the vertices, and then state the length of the transverse axis.**

(d) **State the coordinates of the endpoints of the conjugate axis, and then state the length of the conjugate axis.**

(e) **State the coordinates of the foci.**

(f) **State the equations of the asymptotes. (Answers may be left in point-slope form.)**

(g) **State the eccentricity.**

(h) **Sketch a graph of the hyperbola which includes the features from (b)-(f), along with the central rectangle. Label the center C, the vertices V_1 and V_2, and the foci F_1 and F_2.**

21. $\frac{y^2}{9}-\frac{x^2}{49}=1$

22. $\frac{x^2}{36}-\frac{y^2}{25}=1$

23. $9x^2-25y^2-225=0$

24. $16y^2-x^2-16=0$

25. $\frac{(x+1)^2}{16}-\frac{(y-5)^2}{9}=1$

26. $\frac{(x+5)^2}{4}-\frac{(y+2)^2}{16}=1$

27. $\frac{(y-3)^2}{25}-\frac{(x-1)^2}{36}=1$

28. $\frac{(y-6)^2}{64}-\frac{(x+4)^2}{36}=1$

29. $x^2-25y^2+8x-150y-234=0$

30. $4y^2-81x^2=-162x+405$

31. $64x^2-9y^2+18y=521-128x$

32. $16x^2-9y^2-64x-18y-89=0$

33. $5y^2-4x^2-50y-24x+69=0$

34. $7x^2-9y^2-72y=32-70x$

35. $x^2-3y^2=18x+27$

36. $4y^2-21x^2-8y-42x-89=0$

Use the given features of each of the following hyperbolas to write an equation for the hyperbola in standard form.

37. Center: $(0,0)$
$a=8$
$b=5$
Horizontal Transverse Axis

38. Center: $(0,0)$
$a=7$
$b=3$
Vertical Transverse Axis

39. Center: $(-2,-5)$
$a=2$
$b=10$
Vertical Transverse Axis

Exercise Set 8.3: Hyperbolas

40. Center: $(3,-4)$
$a=1$
$b=6$
Horizontal Transverse Axis

41. Center: $(-6,1)$
Length of transverse axis: 10
Length of conjugate axis: 8
Vertical Transverse Axis

42. Center: $(2,5)$
Length of transverse axis: 6
Length of conjugate axis: 14
Horizontal Transverse Axis

43. Foci: $(0,9)$ and $(0,-9)$
Length of transverse axis: 6

44. Foci: $(5,0)$ and $(-5,0)$
Length of conjugate axis: 4

45. Foci: $(-2,3)$ and $(10,3)$
Length of conjugate axis: 10

46. Foci: $(-3,8)$ and $(-3,-6)$
Length of transverse axis: 8

47. Vertices: $(4,-7)$ and $(4,9)$
$b=4$

48. Vertices: $(1,6)$ and $(7,6)$
$b=7$

49. Center: $(5,3)$
One focus is at $(5,8)$
One vertex is at $(5,6)$

50. Center: $(-3,-4)$
One focus is at $(7,-4)$
One vertex is at $(5,-4)$

51. Center: $(-1,2)$
Vertex: $(3,2)$
Equation of one asymptote:
$7x-4y=-15$

52. Center: $(-1,2)$
Vertex: $(3,2)$
Equation of one asymptote:
$6x+5y=-7$

53. Vertices: $(-1,4)$ and $(7,4)$
$e=3$

54. Vertices: $(2,6)$ and $(2,-1)$
$e=\frac{7}{3}$

55. Center: $(4,-3)$
One focus is at $(4,6)$
$e=\frac{3}{2}$

56. Center: $(-1,-2)$
One focus is at $(9,-2)$
$e=\frac{5}{2}$

57. Foci: $(3,0)$ and $(3,8)$
$e=\frac{4}{3}$

58. Foci: $(-6,-5)$ and $(-6,7)$
$e=2$

Exercise Set A.1: Factoring Polynomials

At times, it can be difficult to tell whether or not a quadratic of the form $ax^2 + bx + c$ can be factored into the form $(dx+e)(fx+g)$, where $a, b, c, d, e, f,$ and g are integers. If $b^2 - 4ac$ is a perfect square, then the quadratic can be factored in the above manner.

For each of the following problems,

(a) Compute $b^2 - 4ac$.

(b) Use the information from part (a) to determine whether or not the quadratic can be written as factors with integer coefficients. (Do not factor; simply answer Yes or No.)

1. $x^2 - 5x + 3$

2. $x^2 - 7x + 10$

3. $x^2 + 6x - 16$

4. $x^2 + 6x + 4$

5. $9 - x^2$

6. $7x - x^2$

7. $2x^2 - 7x - 4$

8. $6x^2 - x - 1$

9. $2x^2 + 2x + 5$

10. $5x^2 - 4x + 1$

Factor the following polynomials. If the polynomial can not be rewritten as factors with integer coefficients, then write the original polynomial as your answer.

11. $x^2 + 4x - 5$

12. $x^2 + 9x + 14$

13. $x^2 - 5x + 6$

14. $x^2 - x - 6$

15. $x^2 - 7x - 12$

16. $x^2 - 8x + 15$

17. $x^2 + 12x + 20$

18. $x^2 + 7x + 18$

19. $x^2 - 5x - 24$

20. $x^2 + 9x - 36$

21. $x^2 + 16x + 64$

22. $x^2 - 6x + 9$

23. $x^2 - 15x + 56$

24. $x^2 - 6x - 27$

25. $x^2 - 11x - 60$

26. $x^2 + 19x + 48$

27. $x^2 + 17x + 42$

28. $x^2 - 12x - 64$

29. $x^2 - 49$

30. $x^2 + 36$

31. $x^2 - 3$

32. $x^2 - 8$

33. $9x^2 + 25$

34. $16x^2 - 81$

35. $2x^2 - 5x - 3$

36. $3x^2 + 16x + 15$

37. $8x^2 - 2x - 3$

38. $4x^2 - 16x + 15$

39. $9x^2 + 9x - 4$

40. $5x^2 + 17x + 6$

41. $4x^2 - 3x - 10$

42. $9x^2 + 21x + 10$

43. $12x^2 - 17x + 6$

44. $8x^2 + 26x - 7$

Factor the following. Remember to first factor out the Greatest Common Factor (GCF) of the terms of the polynomial, and to factor out a negative if the leading coefficient is negative.

45. $x^2 + 9x$

46. $x^2 - 16x$

47. $-5x^2 + 20x$

48. $4x^2 - 28x$

49. $2x^2 - 18$

50. $-8x^2 + 8$

51. $-5x^4 + 20x^2$

52. $3x^3 - 75x$

53. $2x^2 + 10x + 8$

54. $3x^2 + 12x - 63$

55. $-10x^2 + 10x + 420$

56. $-4x^2 + 40x - 100$

57. $x^3 + 9x^2 - 22x$

58. $x^3 - 7x^2 - 6x$

59. $-x^3 - 4x^2 - 4x$

60. $x^5 + 10x^4 + 21x^3$

61. $x^4 + 6x^3 + 6x^2$

62. $-x^3 - 2x^2 + 80x$

63. $9x^5 - 100x^3$

64. $49x^{12} - 64x^{10}$

65. $50x^2 + 55x + 15$

66. $30x^2 + 24x - 72$

Factor the following polynomials. (Hint: Factor first by grouping, and then continue to factor if possible.)

67. $x^3 + 2x^2 - 25x - 50$

68. $x^3 - 3x^2 - 4x + 12$

69. $x^3 - 5x^2 + 4x - 20$

70. $9x^3 + 18x^2 - 25x - 50$

71. $4x^3 + 36x^2 - x - 9$

72. $9x^3 - 27x^2 + 4x - 12$

Exercise Set A.2: Dividing Polynomials

Use long division to find the quotient and the remainder.

73. $\dfrac{x^2-6x+11}{x-2}$

74. $\dfrac{x^2+5x+12}{x+3}$

75. $\dfrac{x^2+7x-2}{x+1}$

76. $\dfrac{x^2-6x-5}{x-4}$

77. $\dfrac{x^3-2x^2-19x-12}{x+3}$

78. $\dfrac{x^3-2x^2-22x+33}{x-5}$

79. $\dfrac{6x^3+5x^2+6x-12}{2x-1}$

80. $\dfrac{12x^3+13x^2-22x-14}{3x+4}$

81. $\dfrac{2x^3+13x^2+28x+21}{x^2+3x+1}$

82. $\dfrac{x^4-7x^3+4x^2-42x-12}{x^2-7x-2}$

83. $\dfrac{2x^5+32x^4+3x^3+44x^2-14}{4x^2+6}$

84. $\dfrac{10x^8+20x^6+x^4+2x^3+28x^2+4}{2x^4+6x^2+3}$

85. $\dfrac{3x^4-x^3-15}{x^2+5}$

86. $\dfrac{3x^5-4x^3-2x+7}{x^2-2x}$

Use synthetic division to find the quotient and the remainder.

87. $\dfrac{x^2-8x+4}{x-10}$

88. $\dfrac{x^2-4x-6}{x+3}$

89. $\dfrac{3x^3+13x^2-6x+28}{x+5}$

90. $\dfrac{2x^3-x^2-31x}{x-4}$

91. $\dfrac{x^4+3x^2-4}{x+1}$

92. $\dfrac{2x^5+3x^4-7x+8}{x-1}$

93. $\dfrac{3x^4-11x^3-27x^2+18x+10}{x-5}$

94. $\dfrac{2x^4+3x^3-18x^2+5x-12}{x-2}$

95. $\dfrac{x^3+8}{x+2}$

96. $\dfrac{x^4-81}{x+3}$

97. $\dfrac{4x^3-7x+5}{x-\frac{1}{2}}$

98. $\dfrac{6x^4+x^3-10x^2+9}{x-\frac{1}{3}}$

Evaluate *P(c)* using the following two methods:
(a) Substitute *c* into the function.
(b) Use synthetic division along with the Remainder Theorem.

99. $P(x)=x^3-4x^2+5x+2;\ c=2$

100. $P(x)=5x^3+7x^2-8x-3;\ c=-1$

101. $P(x) = 7x^3 + 8x^2 - 4x - 12;\ c = -1$

102. $P(x) = 2x^4 - 7x^3 + 6x - 14;\ c = 3$

Evaluate *P(c)* using synthetic division along with the Remainder Theorem. (Notice that substitution without a calculator would be quite tedious in these examples, so synthetic division is particularly useful.)

103. $P(x) = 3x^7 - 8x^6 - 38x^5 + 70x^3 + 21x^2 + 3;\ c = 5$

104. $P(x) = x^6 + 3x^5 + 10x^4 - 35x^2 - 2x - 11;\ c = -2$

105. $P(x) = 4x^4 - 5x^3 - 22x^2 - 1;\ c = -\frac{3}{4}$

106. $P(x) = 6x^6 - 19x^5 + x^4 - 32x^3 + 59x + 13;\ c = \frac{7}{2}$

When the remainder is zero, the dividend can be written as a product of two factors (the divisor and the quotient), as shown below.

$$\frac{30}{5} = 6, \text{ so } 30 = 5 \cdot 6.$$

$$\frac{x^2 + x - 6}{x + 3} = x - 2, \text{ so } x^2 + x - 6 = (x + 3)(x - 2)$$

In the following examples, use either long division or synthetic division to find the quotient, and then write the dividend as a product of two factors.

107. $\dfrac{x^2 - 11x + 24}{x - 8}$

108. $\dfrac{x^2 + 3x - 40}{x - 5}$

109. $\dfrac{x^2 - 7x - 18}{x + 2}$

110. $\dfrac{x^2 + 10x + 21}{x + 3}$

111. $\dfrac{4x^2 - 25x - 21}{x - 7}$

112. $\dfrac{3x^2 - 22x + 24}{x - 6}$

113. $\dfrac{2x^2 + 7x + 5}{x + 1}$

114. $\dfrac{5x^2 + 4x - 12}{x + 2}$

Odd-Numbered Answers to Exercise Set 1.1: An Introduction to Functions

1. This mapping does not make sense, since Erik could not record two different temperatures at 9AM.

No, the mapping is not a function.

3. Yes, the mapping is a function.
Domain: $\{-3, 7, 9\}$
Range: $\{0, 4, 5\}$

5. No, the mapping is not a function.

7. $f(x) = \frac{x}{7} + 4$

9. $f(x) = \sqrt{x} - 6$

11. $x \neq 3$
Interval Notation: $(-\infty, 3) \cup (3, \infty)$

13. $x \neq 3$ and $x \neq -3$
Interval Notation: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

15. $x \neq 4$ and $x \neq 7$
Interval Notation: $(-\infty, 4) \cup (4, 7) \cup (7, \infty)$

17. $t \geq 0$
Interval Notation: $[0, \infty)$

19. All real numbers
Interval Notation: $(-\infty, \infty)$

21. $x \geq 5$
Interval Notation: $[5, \infty)$

23. $x \leq \frac{3}{2}$ and $x \neq -4$
Interval Notation: $(-\infty, -4) \cup \left(-4, \frac{3}{2}\right]$

25. All real numbers
Interval Notation: $(-\infty, \infty)$

27. $t \neq -5$
Interval Notation: $(-\infty, -5) \cup (-5, \infty)$

29. $t \leq 4$ and $t \geq 6$
Interval Notation: $(-\infty, 4] \cup [6, \infty)$

31. **(a)** Domain: $[0, \infty)$
Range: $[0, \infty)$

(b) Domain: $[0, \infty)$
Range: $[-6, \infty)$

(c) Domain: $[6, \infty)$
Range: $[0, \infty)$

(d) Domain: $[6, \infty)$
Range: $[3, \infty)$

33. **(a)** Domain: $(-\infty, \infty)$
Range: $[-4, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $(-\infty, 4]$

(c) Domain: $(-\infty, \infty)$
Range: $[4, \infty)$

(d) Domain: $(-\infty, -2] \cup [2, \infty)$
Range: $[0, \infty)$

(e) Domain: $[-2, 2]$
Range: $(-\infty, 2]$

(f) Domain: $(-\infty, \infty)$
Range: $[2, \infty)$

35. **(a)** Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

(c) Domain: $(-\infty, \infty)$
Range: $(-\infty, 9]$

37. **(a)** Domain: $(-\infty, \infty)$
Range: $[3, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $[-1, \infty)$

(c) Domain: $(-\infty, \infty)$
Range: $[11, \infty)$

Odd-Numbered Answers to Exercise Set 1.1: An Introduction to Functions

39. Domain: $(-\infty, \infty)$
Range: $[5, \infty)$

41. Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

43. **(a)** Domain: $(-\infty, -2) \cup (-2, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$

(b) Domain: $(-\infty, 2) \cup (2, \infty)$
Range: $(-\infty, 1) \cup (1, \infty)$

45. $f(3) = 11$

$f\left(-\frac{1}{2}\right) = -\frac{13}{2}$

$f(a) = 5a - 4$

$f(a+3) = 5a + 11$

$f(a) + 3 = 5a - 1$

$f(a) + f(3) = 5a + 7$

47. $g(0) = 4$

$g\left(-\frac{1}{4}\right) = \frac{37}{8}$

$g(x+5) = x^2 + 7x + 14$

$g\left(\frac{1}{a}\right) = \frac{1}{a^2} - \frac{3}{a} + 4 = \frac{1 - 3a + 4a^2}{a^2}$

$g(3a) = 9a^2 - 9a + 4$

$3g(a) = 3a^2 - 9a + 12$

49. $f(-7) = \frac{1}{2}$

$f(0) = -\frac{2}{3}$

$f\left(\frac{3}{5}\right) = -\frac{13}{12}$

$f(t) = \frac{2+t}{t-3}$

$f\left(t^2 - 3\right) = \frac{t^2 - 1}{t^2 - 6}$

51. $f(6) = 7$

$f(2) = -1$

$f(-3) = -6$

$f(0) = 3$

$f(4) = 3$

$f\left(\frac{9}{2}\right) = 4$

53. $f(0) = 4$

$f(-6) = 108$

$f(2) = 7$

$f(1) = 4$

$f(4) = 9$

$f\left(\frac{3}{2}\right) = 4$

55. Yes

57. Yes

59. No

61. Yes

63. Yes

65. No

67. $f(x+h) = 7x + 7h - 4$

$$\frac{f(x+h) - f(x)}{h} = 7$$

69. $f(x+h) = x^2 + 2xh + h^2 + 5x + 5h - 2$

$$\frac{f(x+h) - f(x)}{h} = 2x + h + 5$$

71. $f(x+h) = -8$

$$\frac{f(x+h) - f(x)}{h} = 0$$

73. $f(x+h) = \dfrac{1}{x+h}$

$$\frac{f(x+h) - f(x)}{h} = \frac{-1}{x(x+h)}$$

Odd-Numbered Answers to Exercise Set 1.2: Functions and Graphs

1. No, the graph does not represent a function.

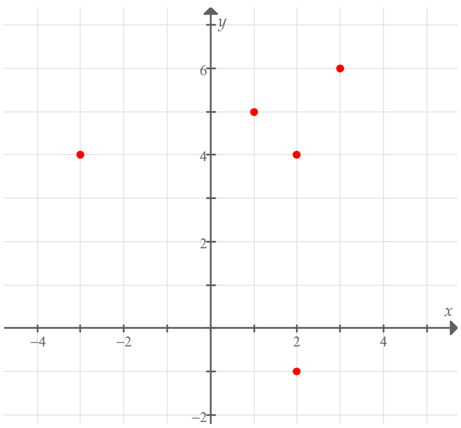
3. Yes, the graph represents a function.

5. Yes, the graph represents a function.

7. No, the graph does not represent a function.

9. Yes, the graph represents a function.

11. **(a)**



(b) No, the set of points does not represent a function. The graph does not pass the vertical line test at $x = 2$.

13. If each x value is paired with only one y value, then the set of points represents a function. If an x value is paired with more than one y value (i.e. two or more coordinates have the same x value but different y values), then the set of points does not represent a function.

15. **(a)** Domain: $[-4, 6]$

(b) Range: $[-3, 9]$

(c) y-intercept: -3

(d) $f(-2) = 3$

$f(0) = -3$

$f(4) = 9$

$f(6) = 3$

(e) $x = -4$, $x = 4$

(f) f is increasing on the interval $(0, 4)$.

(g) f is decreasing on the interval $(-4, 0) \cup (4, 6)$.

(h) The maximum value of the function is 9.

(i) The minimum value of the function is -3.

17. **(a)** Domain: $(-\infty, 6)$

(b) Range: $(-\infty, 5]$

(c) The function has two x-intercepts.

(d) $g(-2) = 1$

$g(0) = 5$

$g(2) = -1$

$g(4) = 1$

$g(6)$ is undefined

(e) $g(-2)$ is greater than $g(3)$, since $1 > 0$.

(f) g is increasing on the interval $(-\infty, 0) \cup (2, 6)$.

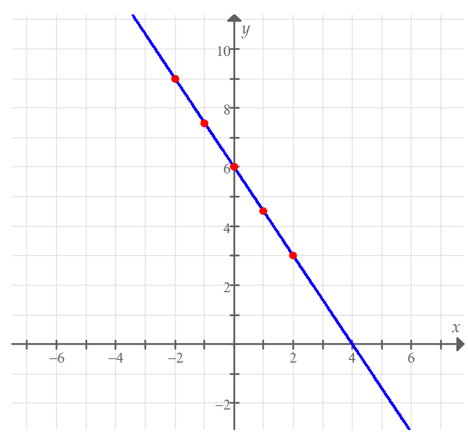
(g) g is increasing on the interval $(0, 2)$.

19. **(a)** Domain: $(-\infty, \infty)$

(b) x-intercept: 4; y-intercept: 6

(c)

x	$f(x) = -\frac{3}{2}x + 6$
-2	9
-1	$\frac{15}{2} = 7.5$
0	6
1	$\frac{9}{2} = 4.5$
2	3

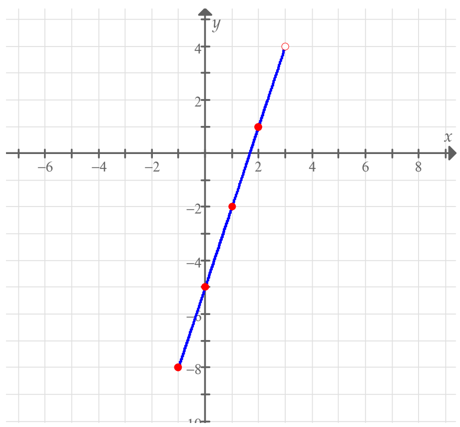


21. **(a)** Domain: $[-1, 3)$

(b) x-intercept: $\frac{5}{3}$; y-intercept: -5

21. **(c)**

x	$h(x) = 3x - 5$
-1	-8
0	-5
1	-2
2	1
3	**undefined** (open circle at $y = 4$)

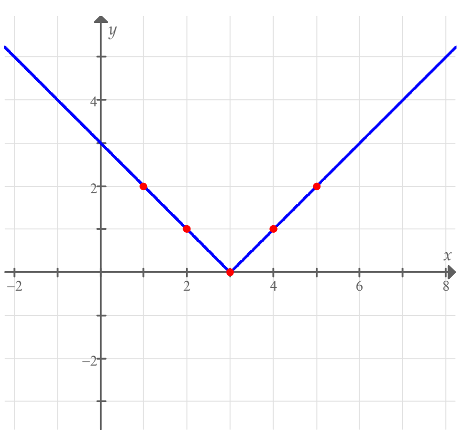


23. **(a)** Domain: $(-\infty, \infty)$

(b) x-intercept: 3; y-intercept: 3

(c)

x	$g(x) = \lvert x - 3 \rvert$
1	2
2	1
3	0
4	1
5	2

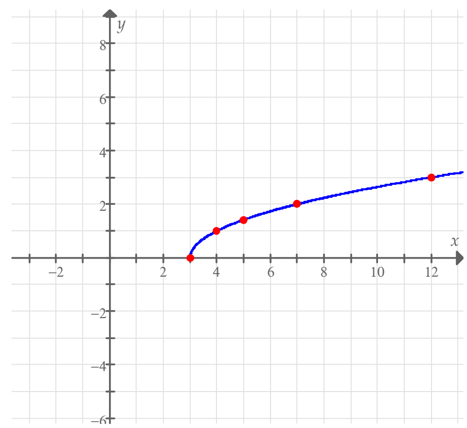


25. **(a)** Domain: $[3, \infty)$

(b) x-intercept: 3; y-intercept: None

(c)

x	$f(x) = \sqrt{x - 3}$
3	0
4	1
5	$\sqrt{2} \approx 1.4$
7	2
12	3

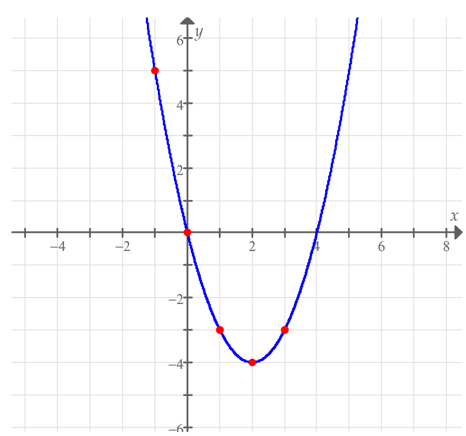


27. **(a)** Domain: $(-\infty, \infty)$

(b) x-intercepts: 0, 4 y-intercept: 0

(c)

x	$F(x) = x^2 - 4x$
-1	5
0	0
1	-3
2	-4
3	-3

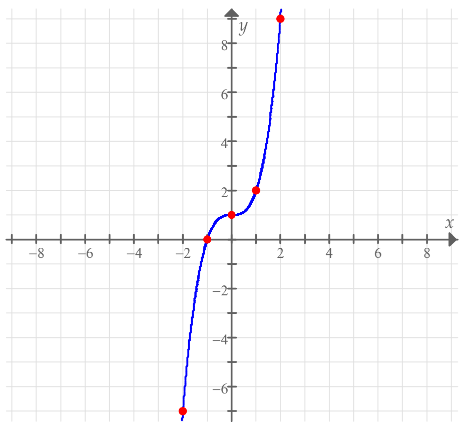


29. **(a)** Domain: $(-\infty, \infty)$

(b) x-intercept: -1; y-intercept: 1

(c)

x	$f(x) = x^3 + 1$
-2	**-7**
-1	**0**
0	**1**
1	**2**
2	**9**



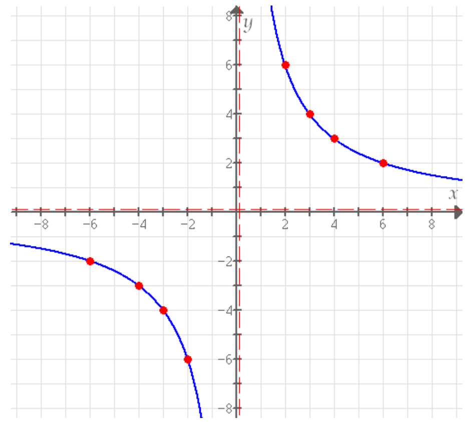
31. **(a)** Domain: $(-\infty, 0) \cup (0, \infty)$

(b) x-intercepts: None; y-intercept: None

(c)

x	$g(x) = \frac{12}{x}$
-6	**-2**
-4	**-3**
-3	**-4**
-2	**-6**
2	**6**
3	**4**
4	**3**
6	**2**

31. **(c)** Graph:

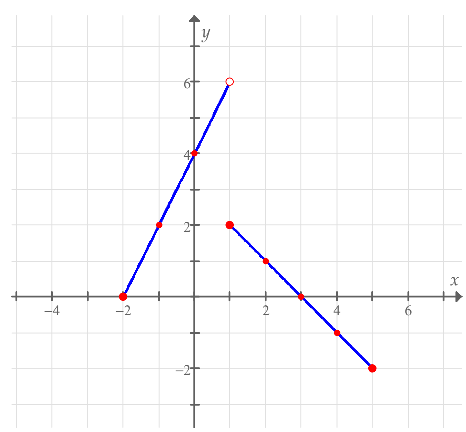


33. **(a)** Domain: $[-2, 5]$

(b) y-intercept: 4

(c)

x	$f(x)$
-2	**0**
-1	**2**
0	**4**
1	**2** (open circle at $y = 6$)
2	**1**
3	**0**
4	**-1**
5	**-2**

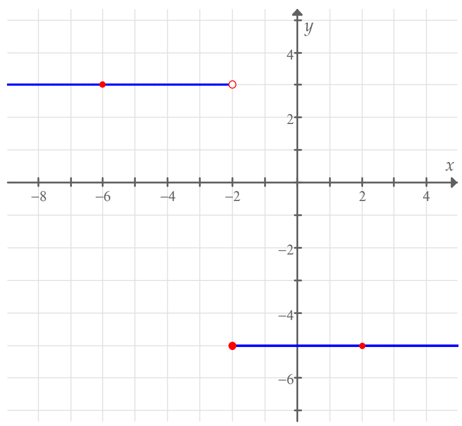


35. **(a)** Domain: $(-\infty, \infty)$

(b) y-intercept: -5

(c)

x	$f(x)$
-6	**3**
-2	**-5** (open circle at $y = 3$)
2	**-5**

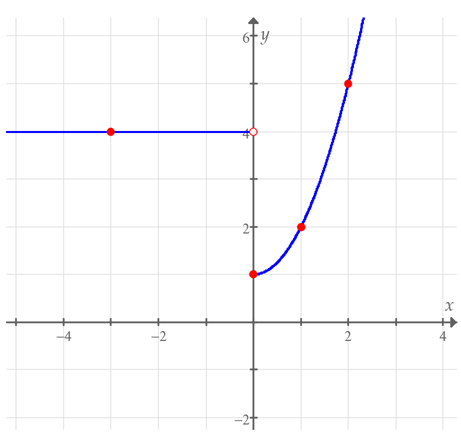


37. **(a)** Domain: $(-\infty, \infty)$

(b) y-intercept: 1

(c)

x	$f(x)$
-3	**4**
0	**1** (open circle at $y = 3$)
1	**2**
2	**5**

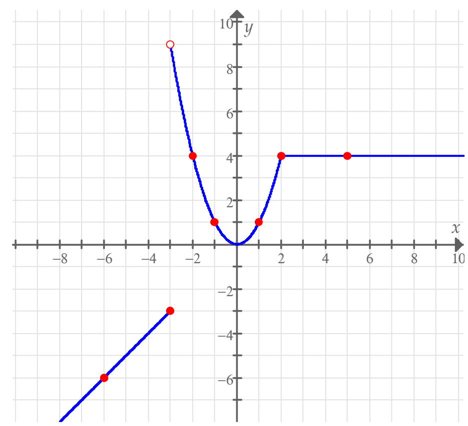


39. **(a)** Domain: $(-\infty, \infty)$

(b) y-intercept: 0

(c)

x	$f(x)$
-6	**-6**
-3	**-3** (open circle at $y = 9$)
-2	**4**
-1	**1**
0	**0**
1	**1**
2	**4**
5	**4**



41. **(a)** origin
(b) y-axis

43. $(3, -6)$

45. Odd

47. Even

49. Neither

Odd-Numbered Answers to Exercise Set 1.3: Transformations of Graphs

1. **(a)** Shift right 3 units.
(b) Reflect in the y-axis.
(c) Stretch vertically by a factor of 2.
(d) Shift upward 4 units.

3. **(a)** Jack is correct.
(b) Analysis of Jack's Method:

Base graph	$y = x^2$
Reflect in the x-axis	$y = -x^2$
Shift upward 2 units	$y = -x^2 + 2 = 2 - x^2$

Analysis of Jill's Method:

Base graph	$y = x^2$
Shift upward 2 units	$y = x^2 + 2$
Reflect in the x-axis	$y = -\left(x^2 + 2\right) = -2 - x^2$

Jack's method is correct, as the final result matches the given equation, $y = 2 - x^2$. Jill's method, however, is incorrect, as it yields the equation $y = -2 - x^2$.

5. **(a)** Tony is correct.
(b) Analysis of Tony's Method:

Base graph	$y = \sqrt{x}$
Shift right 2 units	$y = \sqrt{x-2}$
Reflect in the y-axis	$y = \sqrt{-x-2} = \sqrt{-2-x}$

Analysis of Maria's Method:

Base graph	$y = \sqrt{x}$
Reflect in the y-axis	$y = \sqrt{-x}$
Shift right 2 units	$y = \sqrt{-(x-2)} = \sqrt{2-x}$

Tony's method is correct, as the final result matches the given equation, $y = \sqrt{-2-x}$. Maria's method, however, is incorrect, as it yields the equation $y = \sqrt{-(x-2)} = \sqrt{2-x}$.

7. D

9. F

11. A

13. K

15. C

17. E

19. Note: When two equations are given, either is acceptable.

(a) $f(x) = -5(x-7)^3 + 1$

(b) $f(x) = -5(x-7)^3 + 1$

(c) $f(x) = -\left[5(x-7)^3 + 1\right] = -5(x-7)^3 - 1$

(d) $f(x) = -5\left[(x-7)^3 + 1\right] = -5(x-7)^3 - 5$

(e) $f(x) = -5(x+7)^3 + 1$

(f) The equations in parts (a) and (b) are equivalent.

21. Reflect in the y-axis, then shift downward 2 units.

23. Shift right 2 units, then stretch vertically by a factor of 5, then reflect in the x-axis, then shift upward 1 unit.

25. Shift left 8 units, then stretch vertically by a factor of 3, then shift upward 2 units.

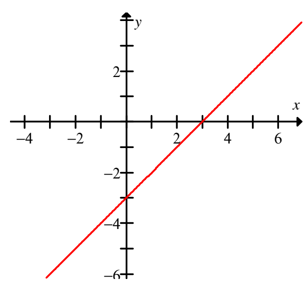
27. Shift upward 1 unit.

29. Reflect in the y-axis, then shift upward 3 units.

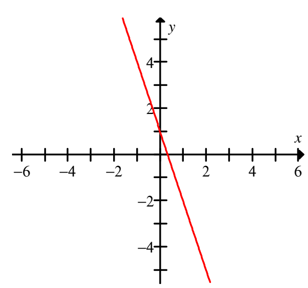
31. Shift right 2 units, then shrink vertically by a factor of $\frac{1}{4}$, then reflect in the x-axis, then shift downward 5 units.

33. Shift left 7 units, then reflect in the y-axis, then shift upward 2 units.

35.

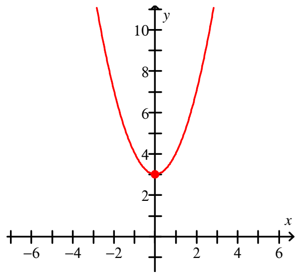


37.

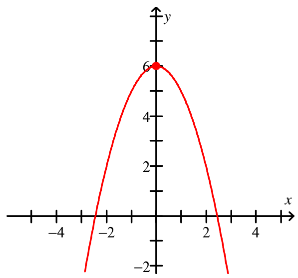


Odd-Numbered Answers to Exercise Set 1.3: Transformations of Graphs

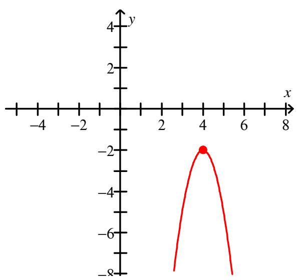
39.



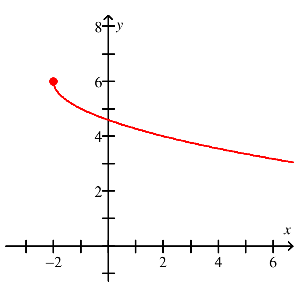
41.



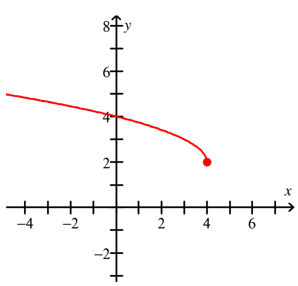
43.



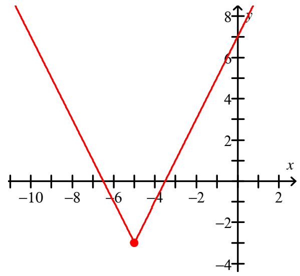
45.



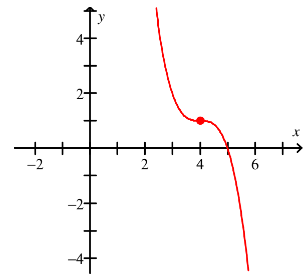
47.



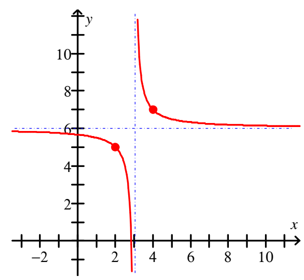
49.



51.

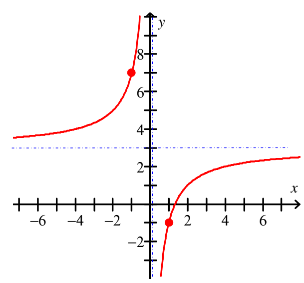


53.



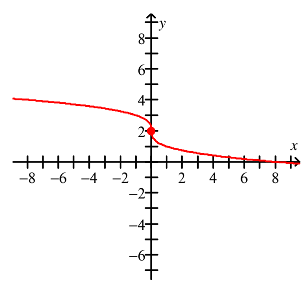
Notice the asymptotes (the blue dotted lines) at $x = 3$ and $y = 6$.

55.



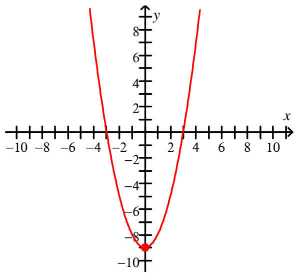
Notice the asymptotes (the blue dotted lines) at $x = 0$ and $y = 3$.

57.

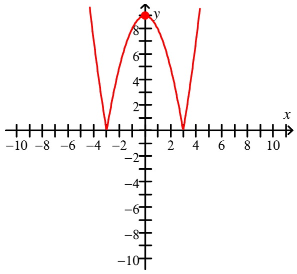


59. All of the points with negative y-values have reflected over the x-axis so that they have positive y-values.

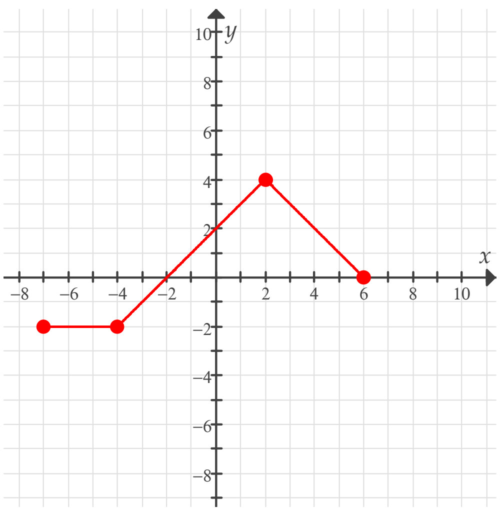
61. **(a)**



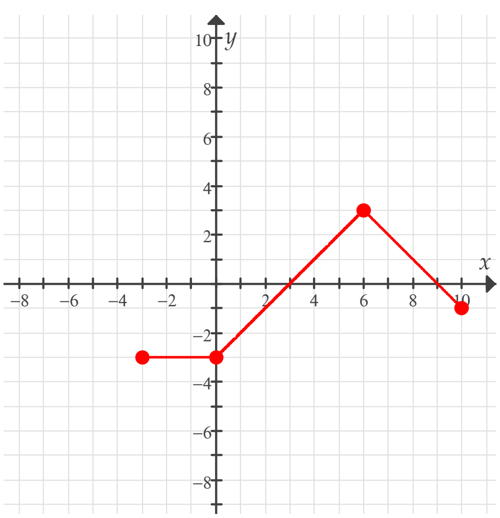
(b)



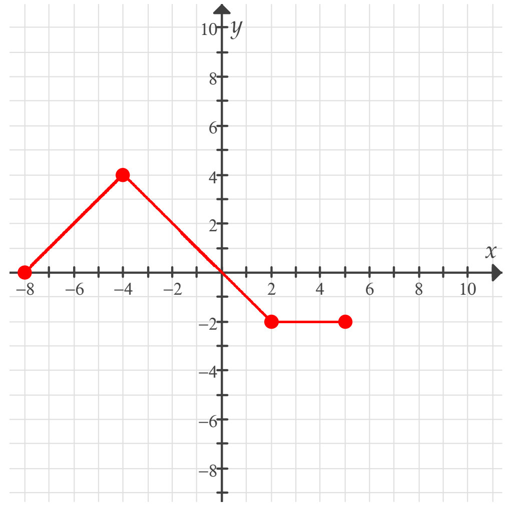
63.



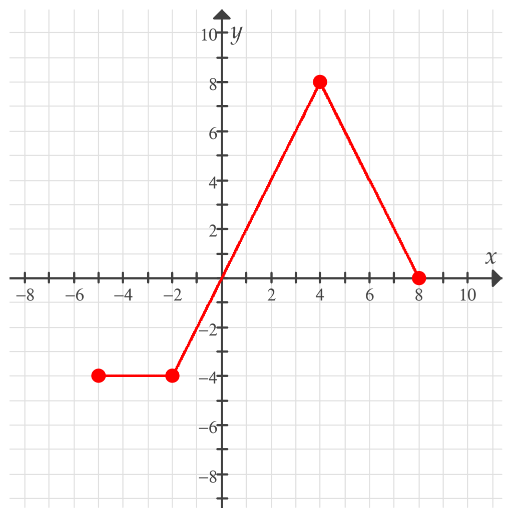
65.



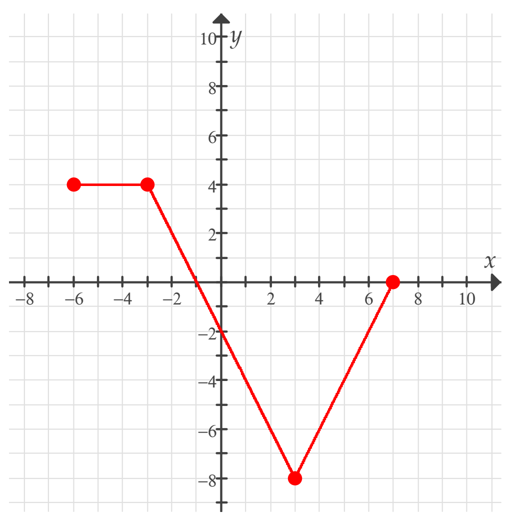
67.



69.



71.



Odd-Numbered Answers to Exercise Set 1.4: Operations on Functions

1. **(a)** $f(-3)+g(-3)=4+0=4$.

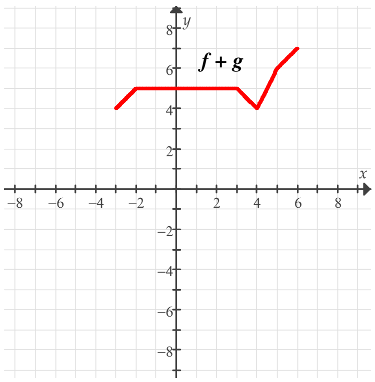
(b) $f(0)+g(0)=2+3=5$.

(c) $f(-6)+g(-6)$ is undefined, since $g(-6)$ is undefined.

(d) $f(5)+g(5)=0+6=6$.

(e) $f(7)+g(7)$ is undefined, since $f(7)$ is undefined.

(f) Graph of $f+g$:



(g) The domain of $f+g$ is $[-3,6]$, since this is where the domains of *f* and *g* intersect (overlap).

3. **(a)** $(f+g)(x)=x^2-2x-9$
Domain: $(-\infty,\infty)$

(b) $(f-g)(x)=-x^2+6x+15$
Domain: $(-\infty,\infty)$

(c) $(fg)(x)=2x^3-5x^2-36x-36$
Domain: $(-\infty,\infty)$

(d) $\left(\dfrac{f}{g}\right)(x)=\dfrac{2x+3}{x^2-4x+12}=\dfrac{2x+3}{(x+2)(x-6)}$

Domain: $(-\infty,-2)\cup(-2,6)\cup(6,\infty)$,
i.e. $x\neq-2$ and $x\neq 6$.

5. **(a)** $(f+g)(x)=\dfrac{x^2+2x+6}{(x-1)(x+2)}$

Domain: $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$,
i.e. $x\neq-2$ and $x\neq 1$.

(b) $(f-g)(x)=\dfrac{-x^2+4x+6}{(x-1)(x+2)}$

Domain: $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$,
i.e. $x\neq-2$ and $x\neq 1$.

(c) $(fg)(x)=\dfrac{3x}{(x-1)(x+2)}$

Domain: $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$,
i.e. $x\neq-2$ and $x\neq 1$.

(d) $\left(\dfrac{f}{g}\right)(x)=\dfrac{3x+6}{x(x-1)}$

Domain: $(-\infty,-2)\cup(-2,0)\cup(0,1)\cup(1,\infty)$,
i.e. $x\neq-2$ and $x\neq 0$ and $x\neq 1$.

7. **(a)** $(f+g)(x)=\sqrt{x-6}+\sqrt{10-x}$
Domain: $[6,10]$

(b) $(f-g)(x)=\sqrt{x-6}-\sqrt{10-x}$
Domain: $[6,10]$

(c) $(fg)(x)=\sqrt{x-6}\cdot\sqrt{10-x}=\sqrt{-x^2+16x-60}$
Domain: $[6,10]$

(d) $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x-6}}{\sqrt{10-x}}=\sqrt{\dfrac{x-6}{10-x}}$

Domain: $[6,10)$.

9. **(a)** $(f+g)(x)=\sqrt{x^2-9}+\sqrt{x^2+4}$
Domain: $(-\infty,-3]\cup[3,\infty)$

(b) $(f-g)(x)=\sqrt{x^2-9}-\sqrt{x^2+4}$
Domain: $(-\infty,-3]\cup[3,\infty)$

(c) $(fg)(x)=\sqrt{x^2-9}\cdot\sqrt{x^2+4}=\sqrt{x^4-5x^2-36}$
Domain: $(-\infty,-3]\cup[3,\infty)$

(d) $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x^2-9}}{\sqrt{x^2+4}}=\sqrt{\dfrac{x^2-9}{x^2+4}}$

Domain: $(-\infty,-3]\cup[3,\infty)$.

11. $[1,3)\cup(3,\infty)$

13. $(-\infty,2)\cup(2,7)\cup(7,\infty)$

15. $[-2,5)\cup(5,\infty)$

17. **(a)** $g(2)=5$ **(b)** $f(g(2))=0$
(c) $f(2)=0$ **(d)** $g(f(2))=3$

19. **(a)** $(f\circ g)(-3)=2$ **(b)** $(g\circ f)(-3)=6$

Odd-Numbered Answers to Exercise Set 1.4: Operations on Functions

21. **(a)** $(f\circ f)(3)=3$ **(b)** $(g\circ g)(-2)=4$

23. **(a)** $(f\circ g)(4)$ is undefined, since $f(6)$ is undefined.
(b) $(g\circ f)(4)=1$

25. **(a)** $g(0)=4$ **(b)** $f(g(0))=-5$
(c) $f(0)=3$ **(d)** $g(f(0))=-2$

27. **(a)** $(f\circ g)(-2)=-33$ **(b)** $(g\circ f)(-2)=18$

29. **(a)** $(f\circ f)(6)=21$ **(b)** $(g\circ g)(6)=54$

31. **(a)** $(f\circ g)(x)=-2x^2+10x-5$
(b) $(g\circ f)(x)=4x^2-2x-2$

33. **(a)** $\{x\mid x\geq 5\}$
(b) $(f\circ g)(x)=\dfrac{1}{x-5}$
(c) $\{x\mid x\neq 5\}$
(d) $\{x\mid x>5\}$; Interval Notation: $(5,\infty)$

35. **(a)** $\{x\mid x\geq 6\}$
(b) $(f\circ g)(x)=\dfrac{3}{x-10}$
(c) $\{x\mid x\neq 10\}$
(d) $\{x\mid x\geq 6 \text{ and } x\neq 10\}$;
Interval Notation: $[6,10)\cup(10,\infty)$

37. **(a)** $(f\circ g)(x)=4x^2-22x+28$
Domain of $f\circ g$: $(-\infty,\infty)$
(b) $(g\circ f)(x)=2x^2+6x-7$
Domain of $g\circ f$: $(-\infty,\infty)$

39. **(a)** $(f\circ g)(x)=\dfrac{1}{x-4}$
Domain of $f\circ g$: $(4,\infty)$
(b) $(g\circ f)(x)=\dfrac{1}{\sqrt{x^2-4}}$
Domain of $g\circ f$: $(-\infty,-2)\cup(2,\infty)$

41. **(a)** $(f\circ g)(x)=\sqrt{2-x}$
Domain of $f\circ g$: $(-\infty,2]$
(b) $(g\circ f)(x)=-5-\sqrt{x+7}$
Domain of $g\circ f$: $[-7,\infty)$

43. **(a)** $f(g(1))=11$
(b) $g(f(1))=7$
(c) $f(g(x))=25x^2-80x+66$
(d) $g(f(x))=5x^2+2$
(e) $f(f(1))=11$
(f) $g(g(1))=-23$
(g) $f(f(x))=x^4+4x^2+6$
(h) $g(g(x))=25x-48$

45. **(a)** $f(g(-2))=-\frac{4}{17}$
(b) $g(f(-2))=-\frac{12}{19}$
(c) $f(g(x))=\dfrac{x-2}{13-2x}$
(d) $g(f(x))=\dfrac{3x-6}{11-4x}$

47. **(a)** $f(g(h(2)))=195$
(b) $g(h(f(3)))=-50$
(c) $(f\circ g\circ h)(x)=36x^2+24x+3$
(d) $(g\circ h\circ f)(x)=4-6x^2$

49. **(a)** $h(f(g(4)))=23$
(b) $f(g(h(0)))=8$
(c) $(f\circ g\circ h)(x)=2x+8$
(d) $(h\circ f\circ g)(x)=2x+15$

51.

x	0	1	2	4
$f(g(x))$	**7**	**undefined**	**2**	**4**

53.

x	0	1	2	4
$f(f(x))$	**4**	**7**	**7**	**undefined**

Odd-Numbered Answers to Exercise Set 1.5: Inverse Functions

1. No, it is not a one-to-one function.

Explanation: The graph passes the vertical line test, so it represents a function; but it does not pass the horizontal line test, so it is does not represent a one-to-one function.

3. No, it is not a one-to-one function.

Explanation: The graph does not pass the vertical line test, so it does not represent a function; therefore, it cannot be a one-to-one function.

5. Yes, it is a one-to-one function.

Explanation: The graph passes the vertical line test, so it represents a function; it also passes the horizontal line test, so it represents a one-to-one function.

7. Yes, the function is one-to-one.

9. Yes, the function is one-to-one.

11. No, the function is not one-to-one.

13. No, the function is not one-to-one.

15. **(a)** Interchange the ***x*** and ***y*** values.

(b) Reflect the graph of f over the line $y = x$.

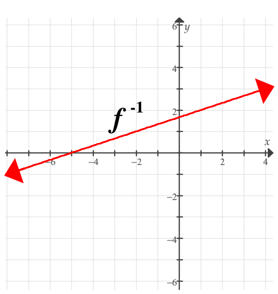
17.

x	$f^{-1}(x)$
–4	**3**
7	2
5	**–4**
0	**5**
3	**0**

19. **(a)** Domain of f: $(-\infty, \infty)$

Range of f: $(-\infty, \infty)$

(b)



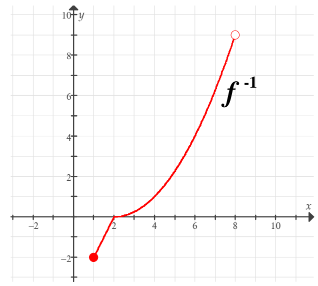
(c) Domain of f^{-1}: $(-\infty, \infty)$

Range of f^{-1}: $(-\infty, \infty)$

21. **(a)** Domain of f: $[-2, 9)$

Range of f: $[1, 8)$

(b)



(c) Domain of f^{-1}: $[1, 8)$

Range of f^{-1}: $[-2, 9)$

23. 4

25. –3

27. 3

29. 2

31. 4

33. 3 (Extraneous information is given in this problem. Note that for any inverse functions f and g, $f\left[g(x)\right] = x$ and $g\left[f(x)\right] = x$.)

35. $f^{-1}(x) = \dfrac{x+3}{5}$

37. $f^{-1}(x) = \dfrac{8x-3}{-2} = \dfrac{3-8x}{2}$

39. $f^{-1}(x) = \sqrt{x-1}$, where $x \geq 1$

41. $f^{-1}(x) = \sqrt[3]{\dfrac{x+7}{4}}$

43. $f^{-1}(x) = \dfrac{3-2x}{x}$

45. $f^{-1}(x) = \dfrac{4x+3}{x-2}$

47. $f^{-1}(x) = \dfrac{x^2-7}{-2} = \dfrac{7-x^2}{2}$

49. No, f and g are not inverses of each other, since $f\left[g(x)\right] \neq x$ and $g\left[f(x)\right] \neq x$.

51. Yes, f and g are inverses of each other, since $f\left[g(x)\right] = x$ and $g\left[f(x)\right] = x$.

53. Yes, f and g are inverses of each other, since $f\left[g(x)\right] = x$ and $g\left[f(x)\right] = x$.

55. Yes, f and g are inverses of each other, since $f\left[g(x)\right] = x$ and $g\left[f(x)\right] = x$.

57. $f^{-1}(500)$ represents the number of tickets sold when the revenue is \$500.

59. Yes, f is one-to-one.

61. Yes, f is one-to-one.

63. No, f is not one-to-one.
Using $x_1 = 3$ and $x_2 = -3$, for example, it can be shown that $f(x_1) = f(x_2)$. However, $x_1 \neq x_2$, and therefore f is not one-to-one. (Answers vary for this counterexample.)

65. No, f is not one-to-one.
Using $x_1 = 2$ and $x_2 = -2$ for example, it can be shown that $f(x_1) = f(x_2)$. However, $x_1 \neq x_2$, and therefore f is not one-to-one. (Answers vary for this counterexample.)

Odd-Numbered Answers to Exercise Set 2.1: Linear and Quadratic Functions

1. $-\dfrac{5}{4}$

3. 0

5. $\dfrac{3}{4}$

7. Undefined

9. $f(x)=\dfrac{4}{3}x-2$

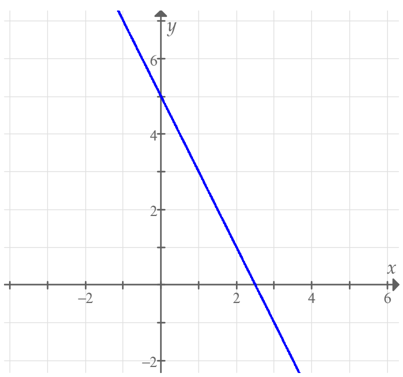
11. **(a)** $y=-2x+5$

(b) $f(x)=-2x+5$

(c) Slope: -2

(d) y-intercept: 5

(e) Graph:



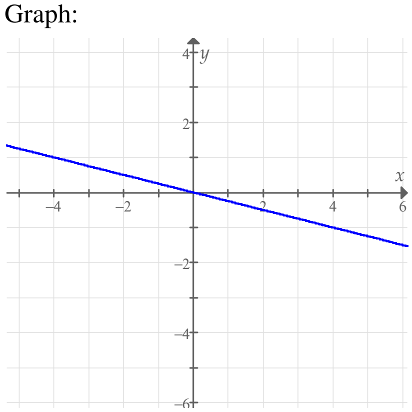
13. **(a)** $y=-\dfrac{1}{4}x$

(b) $f(x)=-\dfrac{1}{4}x$

(c) Slope: $-\dfrac{1}{4}$

(d) y-intercept: 0

(e) Graph:



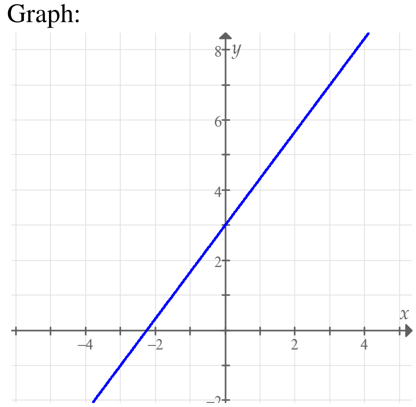
15. **(a)** $y=\dfrac{4}{3}x+3$

(b) $f(x)=\dfrac{4}{3}x+3$

(c) Slope: $\dfrac{4}{3}$

(d) y-intercept: 3

(e) Graph:



17. $f(x)=-\dfrac{4}{7}x+3$

19. $f(x)=-\dfrac{2}{9}x+\dfrac{4}{3}$

21. $f(x)=2x+7$

23. $f(x)=\dfrac{5}{7}x-5$

25. $f(x)=-\dfrac{3}{2}x+6$

27. $f(x)=5$

29. $f(x)=-5x+18$

31. $f(x)=\dfrac{5}{2}x-2$

33. $f(x)=-\dfrac{2}{5}x+\dfrac{19}{5}$

35. $f(x)=-6x+18$

37. $f(x)=x+8$

39. $f(x) = \frac{1}{3}x - 5$

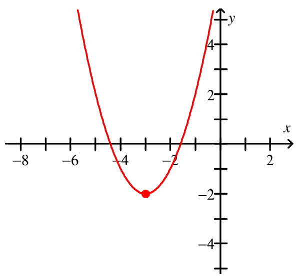
41. $f(x) = \frac{8}{3}x - 8$

43. $C(x) = 800x + 6200$

45. **(a)** $f(x) = (x+3)^2 - 2$

(b) Vertex: $(-3, -2)$

(c)



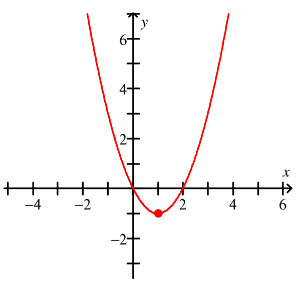
(d) Minimum value: -2

(e) Axis of symmetry: $x = -3$

47. **(a)** $f(x) = (x-1)^2 - 1$

(b) Vertex: $(1, -1)$

(c)



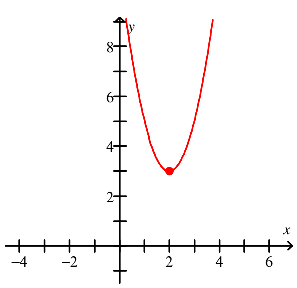
(d) Minimum value: -1

(e) Axis of symmetry: $x = 1$

49. **(a)** $f(x) = 2(x-2)^2 + 3$

(b) Vertex: $(2, 3)$

(c)



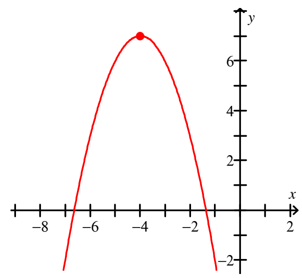
(d) Minimum value: 3

(e) Axis of symmetry: $x = 2$

51. **(a)** $f(x) = -(x+4)^2 + 7$

(b) Vertex: $(-4, 7)$

(c)



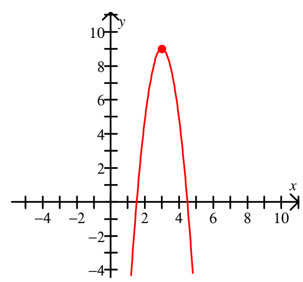
(d) Maximum value: 7

(e) Axis of symmetry: $x = -4$

53. **(a)** $f(x) = -4(x-3)^2 + 9$

(b) Vertex: $(3, 9)$

(c)



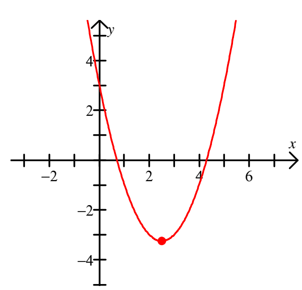
(d) Maximum value: 9

(e) Axis of symmetry: $x = 3$

55. **(a)** $f(x) = \left(x - \frac{5}{2}\right)^2 - \frac{13}{4}$

(b) Vertex: $\left(\frac{5}{2}, -\frac{13}{4}\right)$

(c)



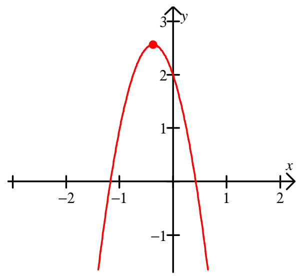
(d) Minimum value: $-\frac{13}{4}$

(e) Axis of symmetry: $x = \frac{5}{2}$

57. **(a)** $f(x) = -4\left(x + \frac{3}{8}\right)^2 + \frac{41}{16}$

(b) Vertex: $\left(-\frac{3}{8}, \frac{41}{16}\right)$

(c)



(d) Maximum value: $\frac{41}{16}$

(e) Axis of symmetry: $x = -\frac{3}{8}$

59. **(a)** Vertex: $(6, 14)$
(b) MinimumValue: 14

61. **(a)** Vertex: $(4, 23)$
(b) MaximumValue: 23

63. **(a)** Vertex: $\left(\frac{9}{4}, \frac{105}{8}\right)$

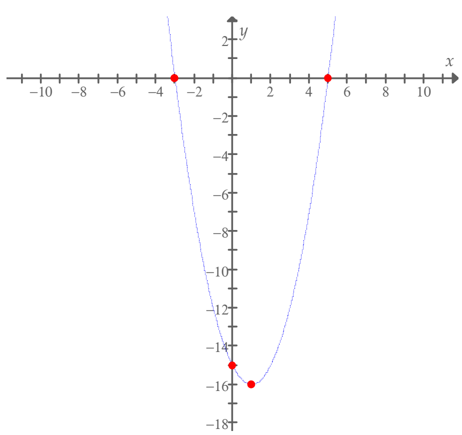
(b) MaximumValue: $\frac{105}{8}$

65. **(a)** x-intercepts: $-3, 5$
(b) y-intercept: -15
(c) Vertex: $(1, -16)$

(d) Minimum value: -16

(e) Axis of symmetry: $x = 1$

(f)



67. **(a)** x-intercepts: $-6, 2$

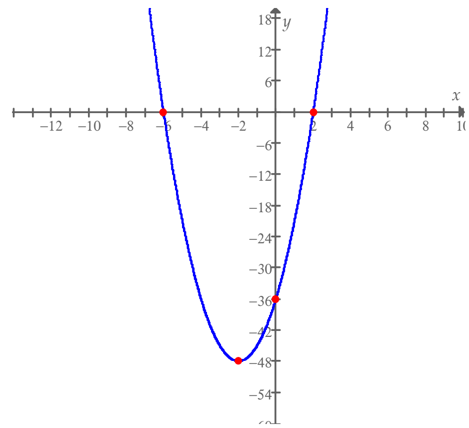
(b) y-intercept: -36

(c) Vertex: $(-2, -48)$

(d) Minimum Value: -48

(e) Axis of symmetry: $x = -2$

(f)



69. **(a)** x-intercepts: $\frac{1}{2}, -\frac{5}{2}$

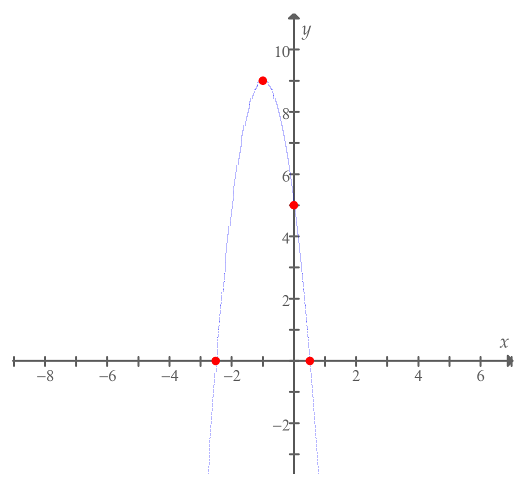
(b) y-intercept: 5

(c) Vertex: $(-1, 9)$

(d) Maximum Value: 9

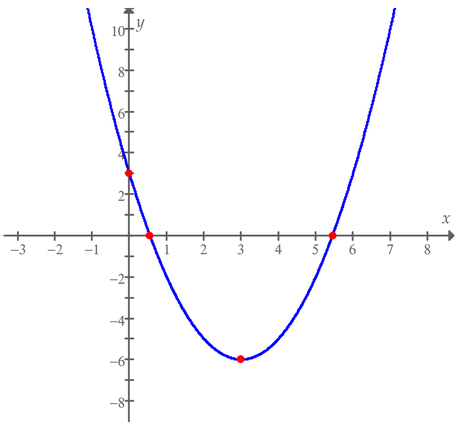
(e) Axis of symmetry: $x = -1$

(f)

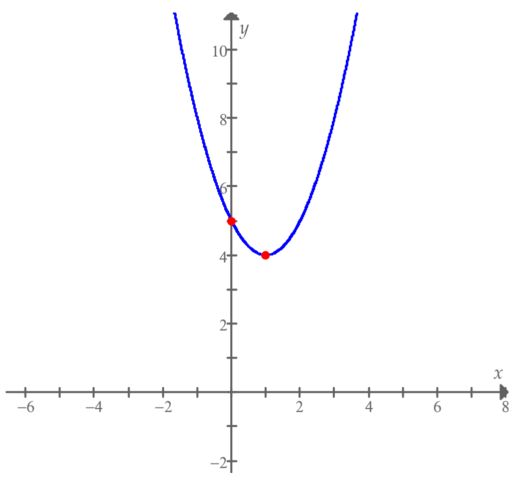


Odd-Numbered Answers to Exercise Set 2.1: Linear and Quadratic Functions

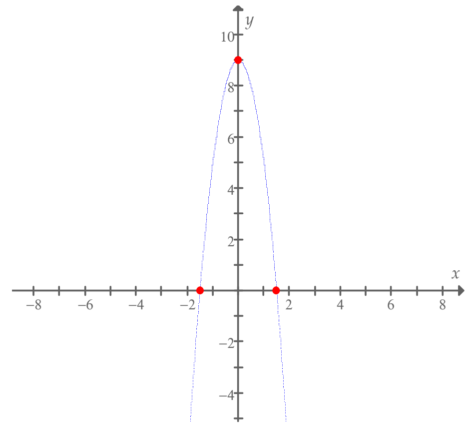
71. **(a)** x-intercepts: $3 \pm \sqrt{6}$ (approx. 0.55 and 5.45)
(b) y-intercept: 3
(c) Vertex: $(3, -6)$
(d) Minimum Value: -6
(e) Axis of symmetry: $x = 3$
(f)



73. **(a)** x-intercepts: None
(b) y-intercept: 5
(c) Vertex: $(1, 4)$
(d) Minimum Value: 4
(e) Axis of symmetry: $x = -1$
(f)



75. **(a)** x-intercepts: $\frac{3}{2}, -\frac{3}{2}$
(b) y-intercept: 9
(c) Vertex: $(0, 9)$
(d) Maximum Value: 9
(e) Axis of symmetry: $x = 0$
(f)



77. $f(x) = 3(x-2)^2 - 5$

79. $f(x) = -\frac{3}{4}(x-5)^2 + 7$

81. The largest possible product is 25.

83. **(a)** 2.5 seconds
(b) 100 feet

Odd-Numbered Answers to Exercise Set 2.2: Polynomial Functions

1. **(a)** Yes
(b) Degree: 3
(c) Binomial

3. **(a)** Yes
(b) Degree: 1
(c) Binomial

5. **(a)** No
(b) N/A
(c) N/A

7. **(a)** Yes
(b) Degree: 2
(c) Trinomial

9. **(a)** Yes
(b) Degree: 4
(c) Trinomial

11. **(a)** No
(b) N/A
(c) N/A

13. **(a)** No
(b) N/A
(c) N/A

15. **(a)** Yes
(b) Degree: 7
(c) Binomial

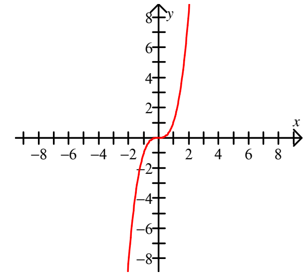
17. **(a)** No
(b) N/A
(c) N/A

19. **(a)** Yes
(b) Degree: 9
(c) Trinomial

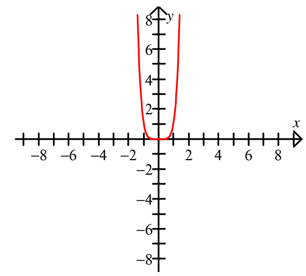
21. **(a)** False
(b) True
(c) True
(d) False

23. **(a)** True
(b) False
(c) False
(d) True

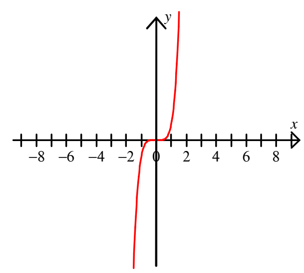
25.



27.



29.



Note: The scale on the y-axis will vary, depending on the value of n.

31. **(a)** $y = x^3$
(b) $y = x^2$
(c) x^6

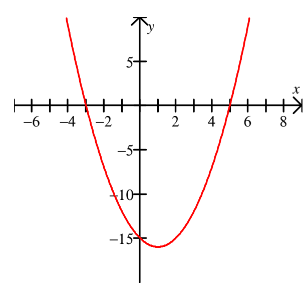
33. E

35. B

37. F

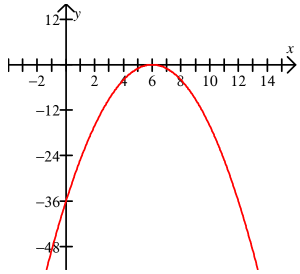
39. **(a)** x-intercepts: $5, -3$ y-intercept: -15

(b)



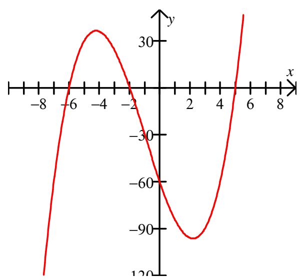
41. **(a)** x-intercept: 6 y-intercept: -36

(b)



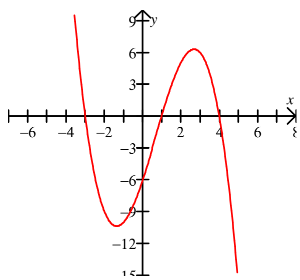
43. **(a)** x-intercepts: $5, -2, -6$ y-intercept: -60

(b)



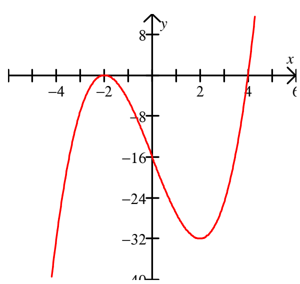
45. **(a)** x-intercepts: $4, 1, -3$ y-intercept: -6

(b)



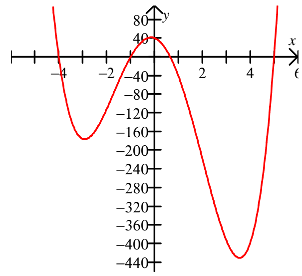
47. **(a)** x-intercepts: $-2, 4$ y-intercept: -16

(b)



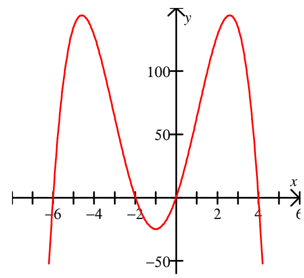
49. **(a)** x-intercepts: $\frac{2}{3}, -4, 5, -1$ y-intercept: 40

(b)



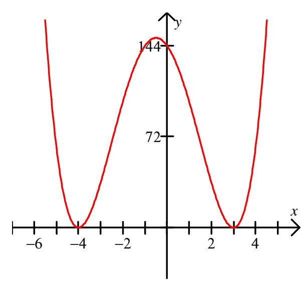
51. **(a)** x-intercepts: $0, -2, 4, -6$ y-intercept: 0

(b)



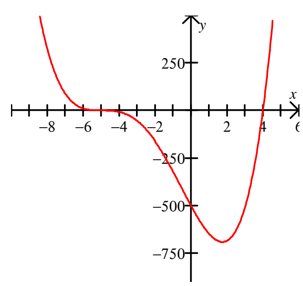
53. **(a)** x-intercepts: $3, -4$ y-intercept: 144

(b)



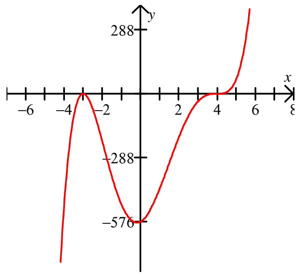
55. **(a)** x-intercepts: $-5, 4$ y-intercept: -500

(b)



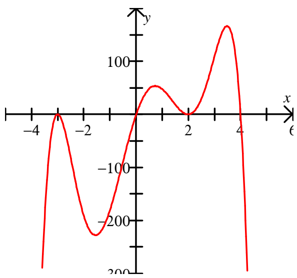
57. (a) x-intercepts: $-3, 4$ y-intercept: -576

(b)



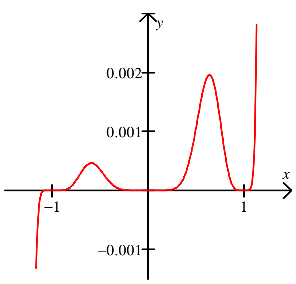
59. (a) x-intercepts: $0, 2, -3, 4$ y-intercept: 0

(b)



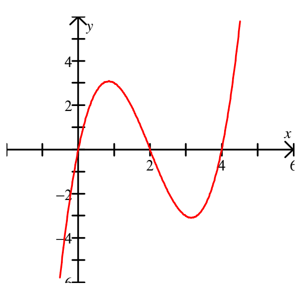
61. (a) x-intercepts: $0, 1, -1$ y-intercept: 0

(b)



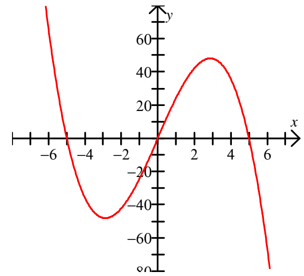
63. (a) x-intercepts: $0, 2, 4$ y-intercept: 0

(b)



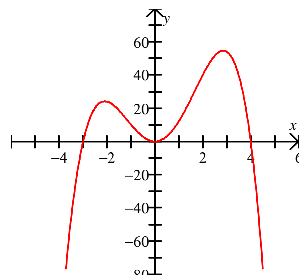
65. (a) x-intercepts: $0, -5, 5$ y-intercept: 0

(b)



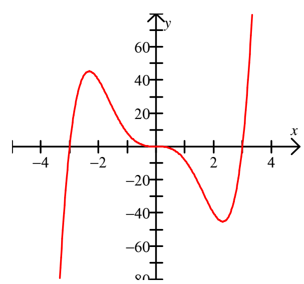
67. (a) x-intercepts: $0, 4, -3$ y-intercept: 0

(b)



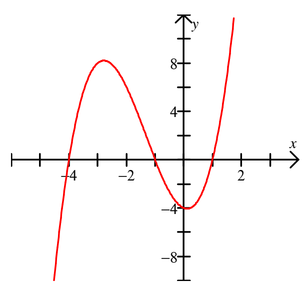
69. (a) x-intercepts: $0, -3, 3$ y-intercept: 0

(b)



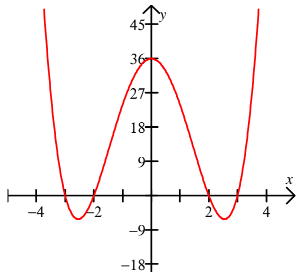
71. (a) x-intercepts: $-4, 1, -1$ y-intercept: -4

(b)



73. **(a)** x-intercepts: $3, -3, 2, -2$ y-intercept: 36

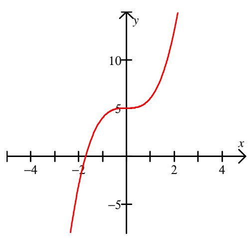
(b)



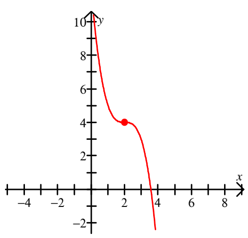
75. $P(x) = -\frac{1}{2}(x+4)(x-1)(x-3)$

77. $P(x) = 3(x+2)^2(x-1)^2$

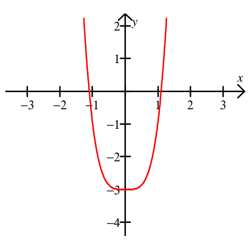
79.



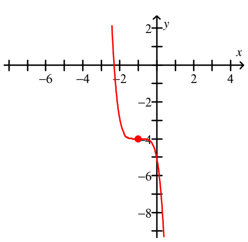
81.



83.



85.



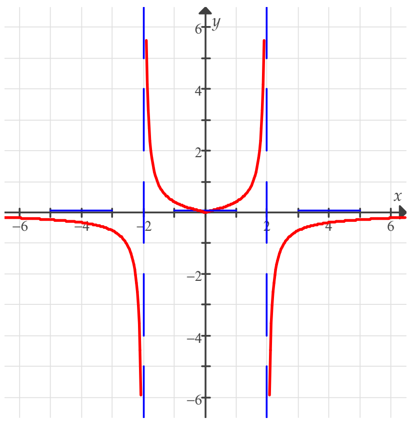
Odd-Numbered Answers to Exercise Set 2.3: Rational Functions

1. Odd

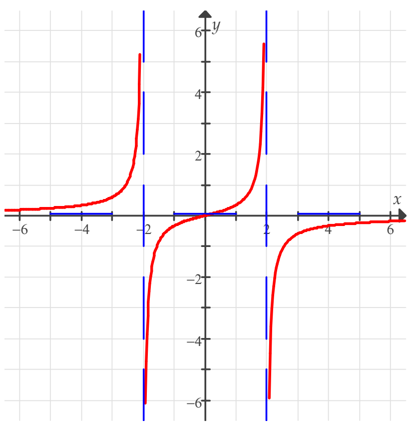
3. Even

5. Neither

7. **(a)**



(b)



9. **(a)** Hole: when $x = -8$

(b) x-intercept: 4

(c) y-intercept: −6

(d) Vertical Asymptote: $x = -2$

(e) Horizontal Asymptote: $y = 3$

11. **(a)** Domain: $(-\infty, 5) \cup (5, \infty)$

(b) Hole(s): None

(c) x-intercept(s): None

(d) y-intercept: $-\frac{3}{5}$

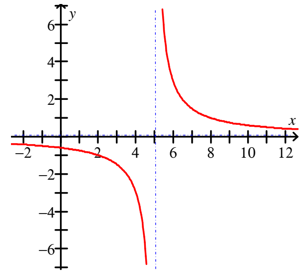
(e) Vertical Asymptote: $x = 5$

Continued in the next column…

(f) Horizontal Asymptote: $y = 0$

(g) Slant Asymptote(s): None

(h) Graph:



13. **(a)** Domain: $(-\infty, 0) \cup (0, \infty)$

(b) Hole(s): None

(c) x-intercept: $-\frac{3}{2}$

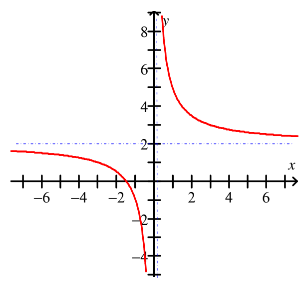
(d) y-intercept(s): None

(e) Vertical Asymptote: $x = 0$

(f) Horizontal Asymptote: $y = 2$

(g) Slant Asymptote(s): None

(h) Graph:



15. **(a)** Domain: $(-\infty, -3) \cup (-3, \infty)$

(b) Hole(s): None

(c) x-intercept: 6

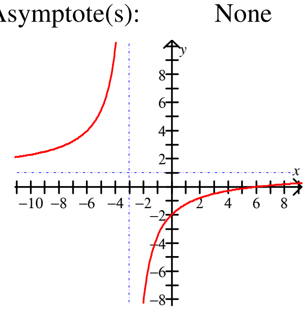
(d) y-intercept: −2

(e) Vertical Asymptote: $x = -3$

(f) Horizontal Asymptote: $y = 1$

(g) Slant Asymptote(s): None

(h) Graph:



17. **(a)** Domain: $\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, \infty\right)$

(b) Hole(s): None

(c) x-intercept: 2

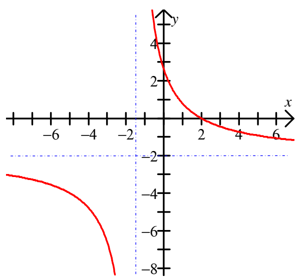
(d) y-intercept: $\frac{8}{3}$

(e) Vertical Asymptote: $x = -\frac{3}{2}$

(f) Horizontal Asymptote: $y = -2$

(g) Slant Asymptote(s): None

(h) Graph:



19. **(a)** Domain: $\left(-\infty, 2\right) \cup (2, 4) \cup (4, \infty)$

(b) Hole: when $x = 2$

(c) x-intercept: -3

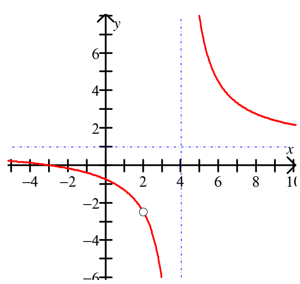
(d) y-intercept: $-\frac{3}{4}$

(e) Vertical Asymptote: $x = 4$

(f) Horizontal Asymptote: $y = 1$

(g) Slant Asymptote(s): None

(h) Graph:



21. **(a)** Domain: $\left(-\infty, 4\right) \cup (4, \infty)$

(b) Hole: when $x = 4$

(c) x-intercept: -5

(d) y-intercept: 5

(e) Vertical Asymptote(s): None

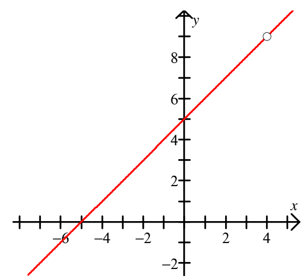
(f) Horizontal Asymptote(s): None

Continued in the next column…

(g) Slant Asymptote(s): None

(Note: There is no slant asymptote when the remainder after performing long division is zero. The graph of this function is simply a line with a removable discontinuity / hole at $x = 4$.)

(h) Graph:



23. **(a)** Domain: $\left(-\infty, -1\right) \cup (-1, 1) \cup (1, \infty)$

(b) Hole(s): None

(c) x-intercept: 0

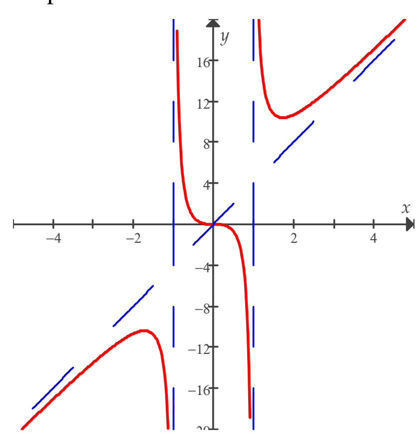
(d) y-intercept: 0

(e) Vertical Asymptotes: $x = -1,\ x = 1$

(f) Horizontal Asymptote(s): None

(g) Slant Asymptote: $y = 4x$

(h) Graph:



25. **(a)** Domain: $\left(-\infty, 0\right) \cup (0, 2) \cup (2, \infty)$

(b) Hole: when $x = 2$

(c) x-intercepts: $-\frac{5}{3}$

(d) y-intercepts: None

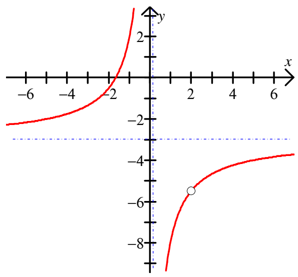
(e) Vertical Asymptote: $x = 0$

Continued on the next page…

(f) Horizontal Asymptote: $y = -3$

(g) Slant Asymptote(s): None

(h) Graph:



27. **(a)** Domain: $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$

(b) Hole: when $x = -3$

(c) x-intercept: 3

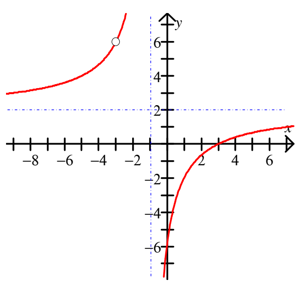
(d) y-intercept: -6

(e) Vertical Asymptote: $x = -1$

(f) Horizontal Asymptote: $y = 2$

(g) Slant Asymptote(s): None

(h) Graph:



29. **(a)** Domain: $(-\infty, 0) \cup (0, \infty)$

(b) Hole(s): None

(c) x-intercept(s): $-2, 2$

(d) y-intercept(s): None

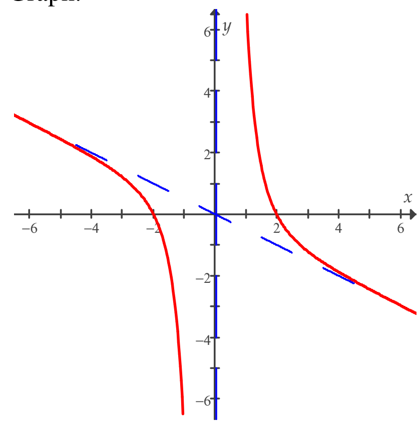
(e) Vertical Asymptote: $x = 0$

(f) Horizontal Asymptote: None

(g) Slant Asymptote(s): $y = -\frac{1}{2}x$

Continued in the next column…

(h) Graph:



31. **(a)** Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

(b) Hole(s): None

(c) x-intercept(s): None

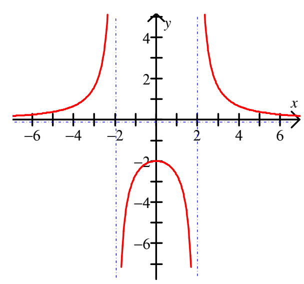
(d) y-intercept: -2

(e) Vertical Asymptotes: $x = -2,\ x = 2$

(f) Horizontal Asymptotes: $y = 0$

(g) Slant Asymptote(s): None

(h) Graph:



33. **(a)** Domain: $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$

(b) Hole(s): None

(c) x-intercept: 1

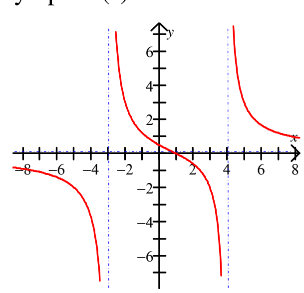
(d) y-intercept: $\frac{1}{2}$

(e) Vertical Asymptotes: $x = -3,\ x = 4$

(f) Horizontal Asymptote: $y = 0$

(g) Slant Asymptote(s): None

(h) Graph:



35. **(a)** Domain: $(-\infty,-1)\cup(1,2)\cup(2,4)\cup(4,\infty)$

(b) Hole: when $x=4$

(c) x-intercepts: $-2,\ 3$

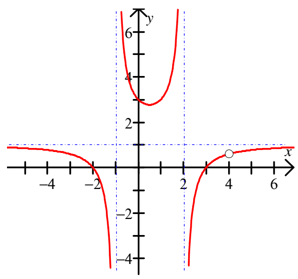
(d) y-intercept: 3

(e) Vertical Asymptotes: $x=-1,\ x=2$

(f) Horizontal Asymptote: $y=1$

(g) Slant Asymptote(s): None

(h) Graph:



37. **(a)** Domain: $(-\infty,-1)\cup(-1,3)\cup(3,\infty)$

(b) Holes: when $x=-1$ and $x=3$

(c) x-intercept: 0, 5

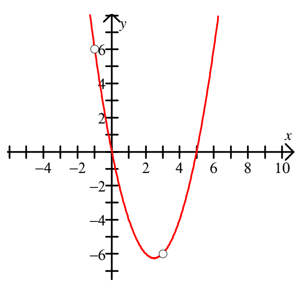
(d) y-intercept: 0

(e) Vertical Asymptote(s): None

(f) Horizontal Asymptote(s): None

(g) Slant Asymptote(s): None

(h) Graph:



39. **(a)** Domain: $(-\infty,0)\cup(0,\infty)$

(b) Hole(s): None

(c) x-intercepts: $x=-3,\ x=-2,\ x=3$

(d) y-intercept: None

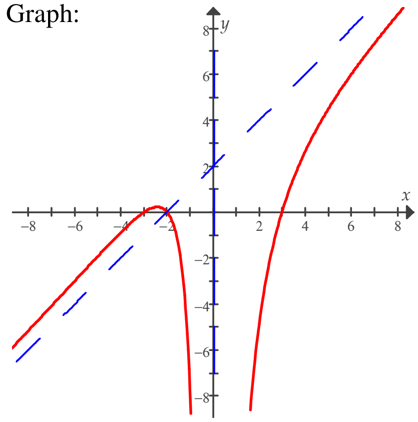
(e) Vertical Asymptote: $x=0$

(f) Horizontal Asymptote(s): None

(g) Slant Asymptote: $y=x+2$

Continued in the next column…

(h) Graph:



41. **(a)** x-intercepts: $x=\frac{5}{2}\pm\frac{\sqrt{13}}{2}$;

$x\approx 0.7$; $x\approx 4.3$

(b) Vertical Asymptotes:

$x=-\frac{1}{3}-\frac{\sqrt{10}}{3}$; $x=-\frac{1}{3}+\frac{\sqrt{10}}{3}$

$x\approx -1.4$; $x\approx 0.7$

43. **(a)** $y=1$

(b) $x=-2$

(c) $(-2,1)$

45. **(a)** $y=\frac{1}{2}$

(b) $x=\frac{7}{2}$

(c) $\left(\frac{7}{2},\frac{1}{2}\right)$

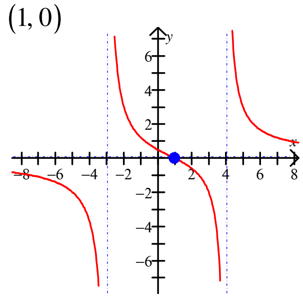
47. **(a)** $y=4$

(b) $x=\frac{19}{16}$

(c) $\left(\frac{19}{16},4\right)$

49. **(a)** $(1,0)$

(b)



Odd-Numbered Answers to Exercise Set 2.4: Applications and Writing Functions

1. **(a)** $A(w) = 27w - w^2$

(b) A is greatest when $w = \dfrac{27}{2} = 13.5$ ft.

(c) The maximum area of the rectangle is $A = \dfrac{729}{4} = 182.25 \text{ ft}^2$.

3. $d(t) = 130t$

5. 100

7. 16

9. 375 ft × 750 ft (where the 750 ft side is parallel to the river)

11. $20{,}000 \text{ ft}^2$

13. $A(x) = \dfrac{x}{2}\sqrt{36 - x^2}$

15. $P(\ell) = 2\ell + \dfrac{44}{\ell} = \dfrac{2\ell^2 + 44}{\ell}$

17. **(a)** $A(x) = \left|18x - 2x^3\right|$

(b) $P(x) = 4|x| + \left|18 - 2x^2\right|$

19. $V(r) = 2\pi r^3$

21. **(a)** $L(r) = \dfrac{600}{r}$

(b) $S(r) = \dfrac{600}{r} + 2\pi r^2 = \dfrac{2\pi r^3 + 600}{r}$

23. $V(r) = \dfrac{4\pi r^3}{3}$

25. $S(x) = x^2 + \dfrac{80}{x} = \dfrac{x^3 + 80}{x}$

27. **(a)** $S(x) = 120x - 6x^2$

(b) 10 cm × 10 cm × 10 cm

(c) 1000 cm^2

29. **(a)** $C(x) = x$

(b) $A(x) = \dfrac{x^2}{4\pi}$

31. **(a)** $d(x) = \sqrt{x^4 - 19x^2 + 100}$

(b) $d(x) = \sqrt{x^4 - 15x^2 + 64}$

33. $A(r) = 4r^2$

35. **(a)** $P(w) = 2w + 2\sqrt{64 - w^2}$

(b) $A(w) = w\sqrt{64 - w^2}$

37. **(a)** $A(x) = \dfrac{(P - 2x)\sqrt{4Px - x^2}}{4}$

(b) Domain of A: $\dfrac{P}{4} < x < \dfrac{P}{2}$

39. **(a)** $d(t) = \sqrt{2500t^2 - 360t + 13}$

(b) Note: To minimize $d(t)$, minimize the quadratic under the radical sign.
$t = \dfrac{9}{125}$ hr $= 0.072$ hr $= 4.32$ min

41. $d = 20\sqrt{2}$ cm ≈ 28.28 cm
Note: See the hint in the solutions for numbers 39 and 40 for minimizing the function.

43. $C(x) = \dfrac{2\pi x}{\sqrt{3}} = \dfrac{2\pi x\sqrt{3}}{3}$

Odd-Numbered Answers to Exercise Set 3.1: Exponential Equations and Functions

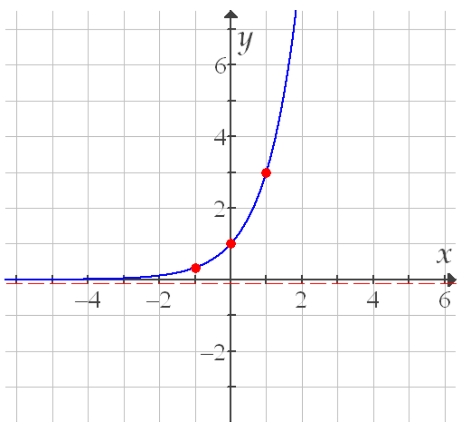
1. **(a)** No solution **(b)** $x = \frac{7}{2}$

3. **(a)** $x = \frac{4}{3}$ **(b)** $x = \frac{5}{6}$

5. **(a)** $x = 1$ **(b)** No solution

7. **(a)** $x = -\frac{29}{6}$ **(b)** $x = -\frac{1}{4}$

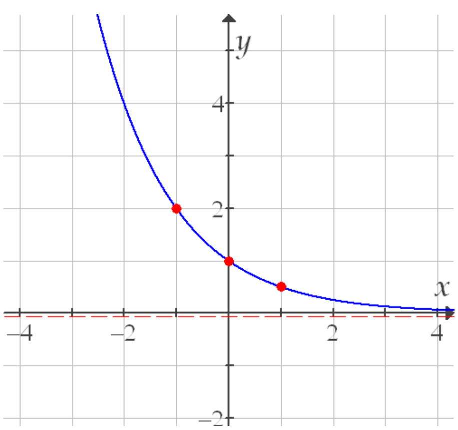
9.



x	$f(x)$
0	1
1	3
–1	$\frac{1}{3}$

Horizontal Asymptote at $y = 0$

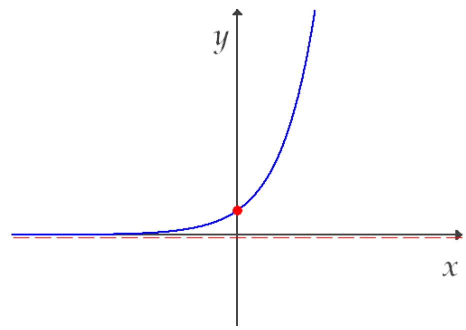
11.



x	$f(x)$
0	1
1	$\frac{1}{2}$
–1	2

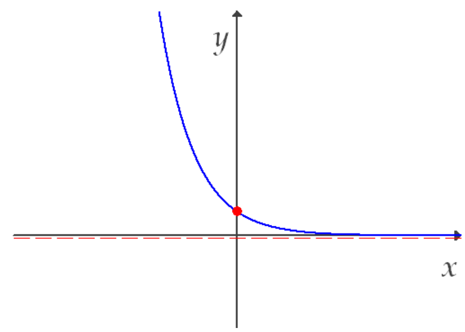
Horizontal Asymptote at $y = 0$

13. **(a)**



Horizontal Asymptote at $y = 0$
Passes through $(0, 1)$

(b)

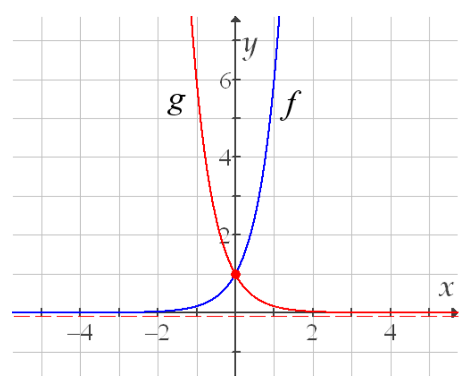


Horizontal Asymptote at $y = 0$
Passes through $(0, 1)$

(c) The two graphs have the point $(0, 1)$ in common.

(d) Both graphs have a horizontal asymptote at $y = 0$.

15. **(a)**



Horizontal Asymptote at $y = 0$
Both graphs pass through $(0, 1)$

(b) To obtain the graph of g, reflect the graph of f across the y-axis.

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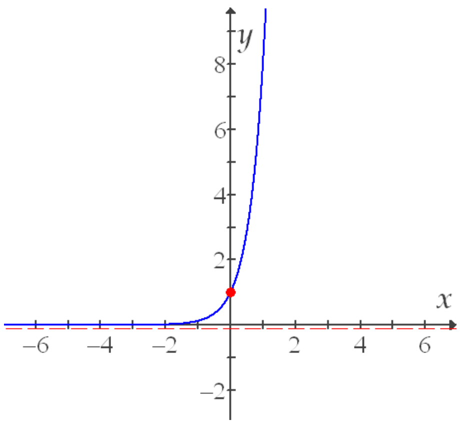
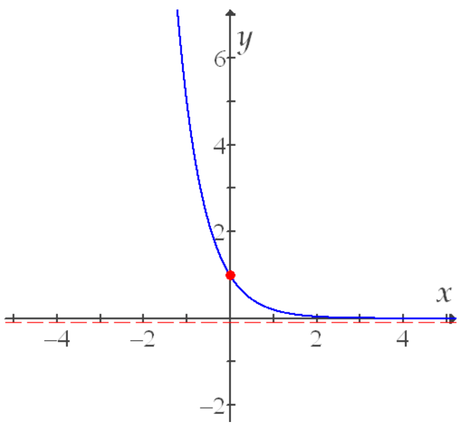
(c) $g(x)=\left(\frac{1}{6}\right)^x=\left(6^{-1}\right)^x=6^{-x}$. Therefore, $g(x)=f(-x)$. It is known from previous lessons on transformations that to obtain the graph of $g(x)=6^{-x}$, the graph of $f(x)=6^x$ is reflected in the y-axis.

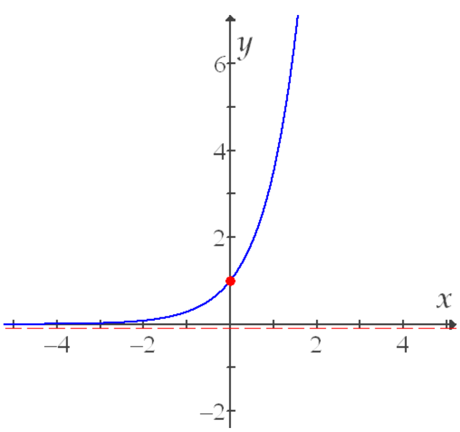
17.



19.



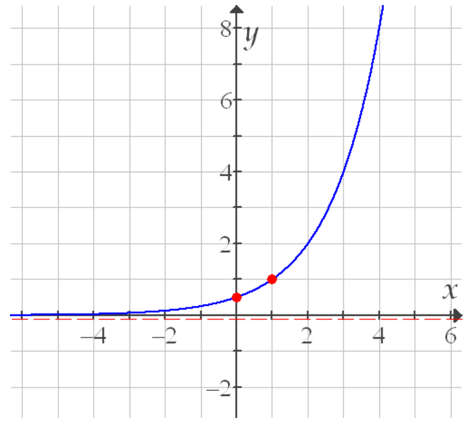
21.



23. **(a)** x-intercept(s): None

y-intercept: $\frac{1}{2}$

(b)



(c) Domain: $(-\infty,\infty)$

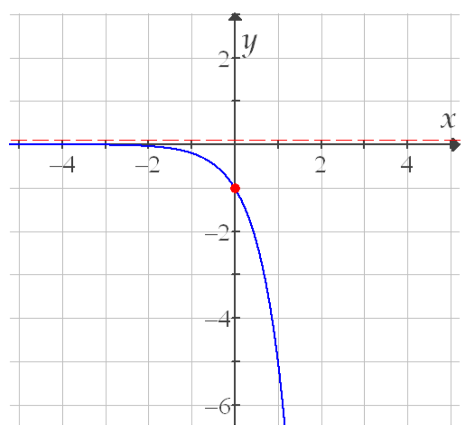
Range: $(0,\infty)$

(d) Increasing

25. **(a)** x-intercept(s): None

y-intercept: -1

(b)



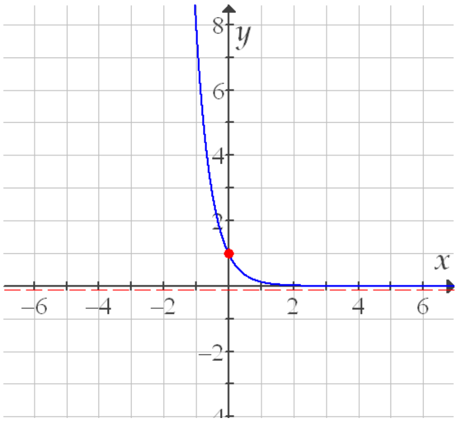
(c) Domain: $(-\infty,\infty)$

Range: $(-\infty,0)$

(d) Decreasing

27. **(a)** x-intercept(s): None
y-intercept: 1

(b)

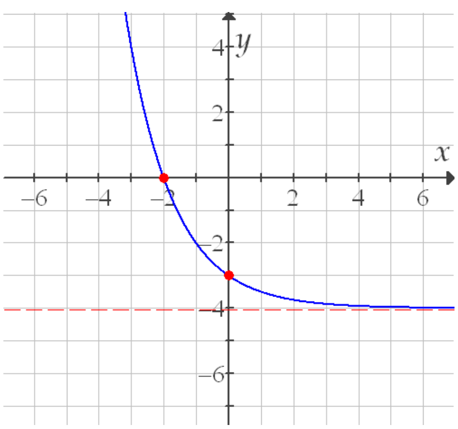


(c) Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

(d) Decreasing

29. **(a)** x-intercept: -2
y-intercept: -3

(b)



(c) Domain: $(-\infty, \infty)$
Range: $(-4, \infty)$

(d) Decreasing

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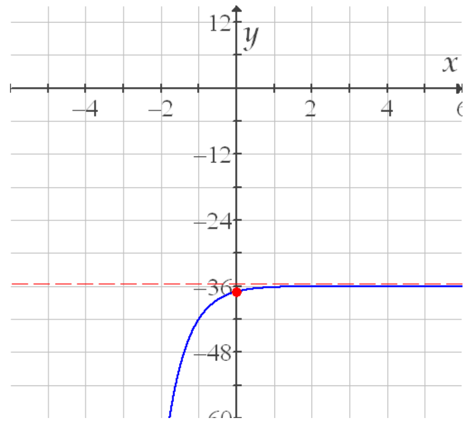
31. **(a)** x-intercept: $-\frac{1}{2}$

y-intercept: 54

(b)



Note: The graph passes through $(-2, -26)$.

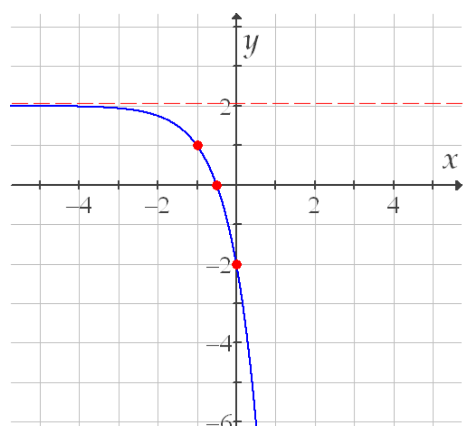
(c) Domain: $(-\infty, \infty)$
Range: $(-27, \infty)$

(d) Increasing

33. **(a)** x-intercept: $-\frac{1}{2}$

y-intercept: -2

(b)



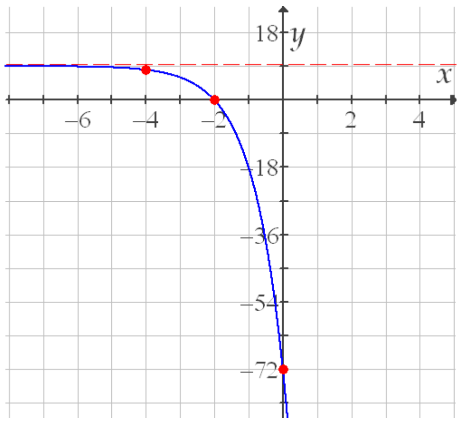
(c) Domain: $(-\infty, \infty)$
Range: $(-\infty, 2)$

(d) Decreasing

35. **(a)** x-intercept: -2

y-intercept: -72

(b)



Note: The graph passes through $(-4, 8)$.

(c) Domain: $(-\infty, \infty)$

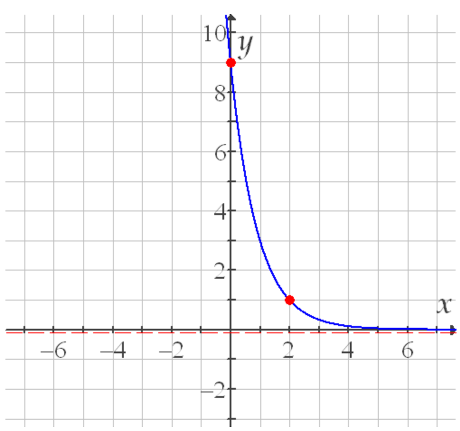
Range: $(-\infty, 9)$

(d) Decreasing

37. **(a)** x-intercept: None

y-intercept: 9

(b)



(c) Domain: $(-\infty, \infty)$

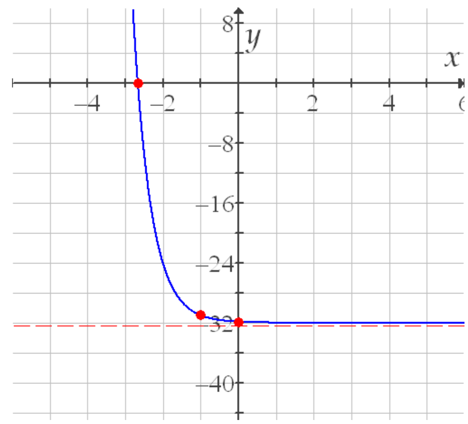
Range: $(0, \infty)$

(d) Decreasing

39. **(a)** x-intercept: $\left(-\frac{8}{3}, 0\right)$

y-intercept: $\left(0, -31\frac{7}{8}\right)$

(b)



Note: The graph passes through $(-1, -31)$.

(c) Domain: $(-\infty, \infty)$

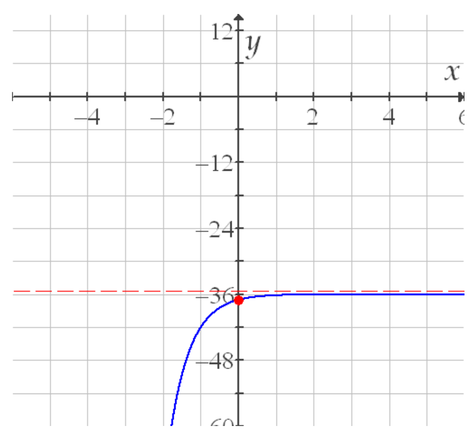
Range: $(-32, \infty)$

(d) Decreasing

41. **(a)** x-intercept(s): None

y-intercept: -37

(b)



The graph passes through the point $(0, -37)$.

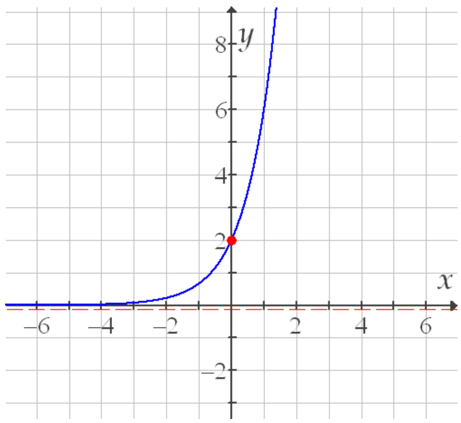
(c) Domain: $(-\infty, \infty)$

Range: $(-\infty, -36)$

(d) Increasing

43. **(a)** x-intercept: None
y-intercept: 2

(b)



(c) Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

(d) Increasing

45. $f(x) = 4 \cdot 2^x$

47. $f(x) = 2 \cdot 7^x$

49. True

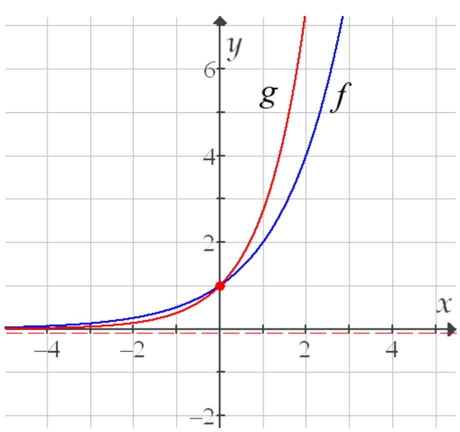
51. True

53. False

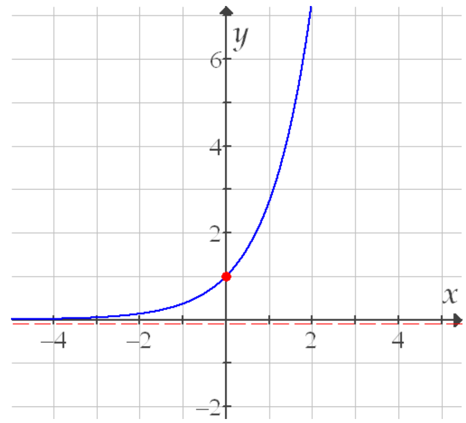
55. False

57. True

59.



61. **Graph:**



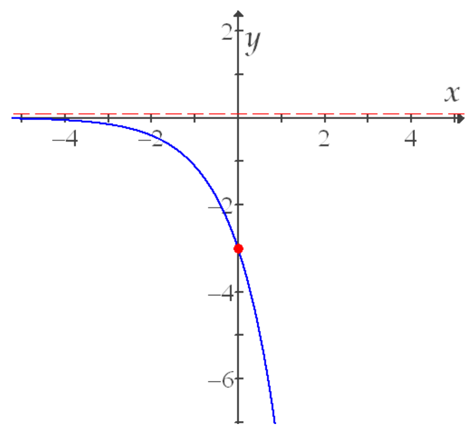
(a) Domain: $(-\infty, \infty)$

(b) Range: $(0, \infty)$

(c) Horizontal Asymptote at $y = 0$

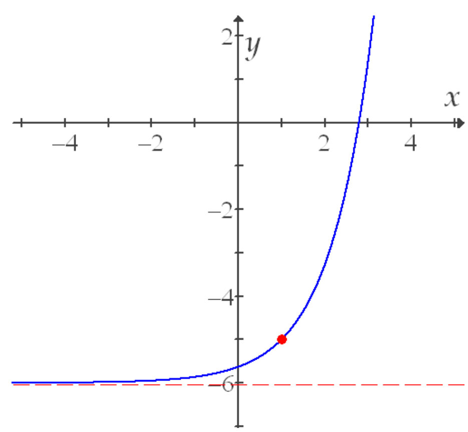
(d) y-intercept: 1

63.



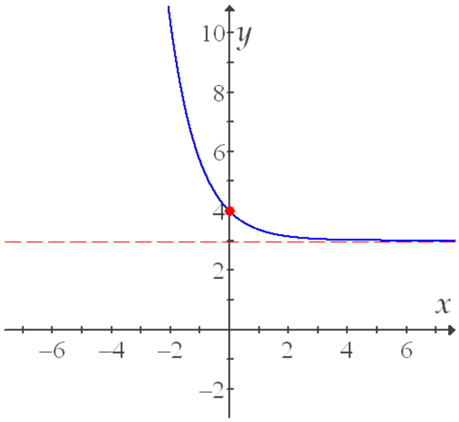
Domain: $(-\infty, \infty)$; Range: $(-\infty, 0)$

65.



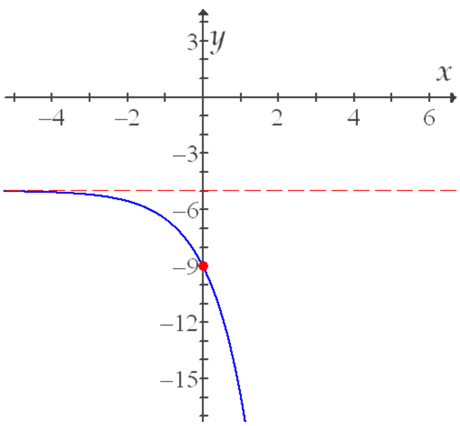
Domain: $(-\infty, \infty)$; Range: $(-6, \infty)$

67.



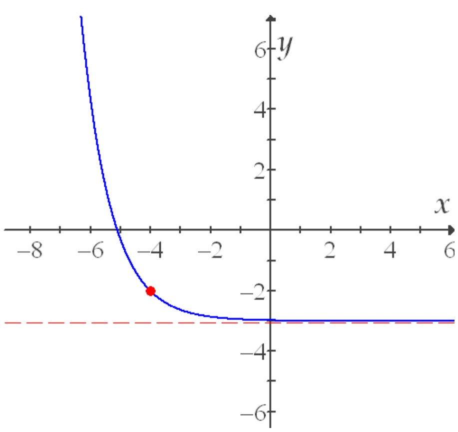
Domain: $(-\infty, \infty)$; Range: $(3, \infty)$

69.



Domain: $(-\infty, \infty)$; Range: $(-\infty, -5)$

71.



Domain: $(-\infty, \infty)$; Range: $(-3, \infty)$

Odd-Numbered Answers to Exercise Set 3.2: Logarithmic Functions

1. **(a)** $3^4 = 81$ **(b)** $2^6 = 64$

3. **(a)** $5^x = 7$ **(b)** $e^8 = x$

5. **(a)** $\log_6(1) = 0$ **(b)** $\log_{10}\left(\frac{1}{10{,}000}\right) = -4$

7. **(a)** $\log_7(7) = 1$ **(b)** $\ln(x) = 4$

9. **(a)** $\ln(7) = x$ **(b)** $\ln(y) = x - 2$

11. **(a)** 3 **(b)** 0

13. **(a)** 1 **(b)** 2

15. **(a)** $\dfrac{1}{2}$ **(b)** -1

17. **(a)** $\dfrac{1}{3}$ **(b)** -2

19. **(a)** -1 **(b)** $\dfrac{1}{4}$

21. **(a)** $-\dfrac{1}{2}$ **(b)** $\dfrac{5}{3}$

23. **(a)** 1 **(b)** 4

25. **(a)** -6 **(b)** $\dfrac{1}{3}$

27. **(a)** -4 **(b)** x

29. **(a)** 5 **(b)** $\sqrt{3}$

31. **(a)** 6 **(b)** x^4

33. **(a)** $x = 5$ **(b)** $x = 3$

35. **(a)** $x = 49$ **(b)** $x = 9$

37. **(a)** $x = \dfrac{3}{2}$ **(b)** $x = 2$

39. **(a)** $x = -97;\ x = 103$ **(b)** $x = 4$

41. **(a)** $x = e^4$ **(b)** $x = e^2 - 5$

43. **(a)** $x = \log(20) \approx 1.301$

(b) $x = \ln(20) \approx 2.996$

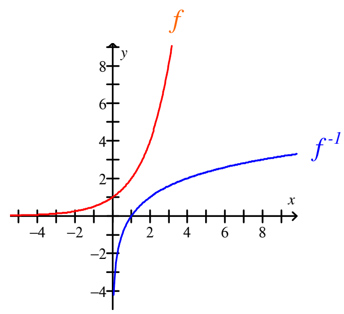
45. **(a)** $x = \log\left(\dfrac{2}{5}\right) \approx -0.398$

(b) $x = \ln\left(\dfrac{2}{5}\right) = -0.916$

47. $x = \dfrac{2 + \log(6)}{3} \approx 0.926$

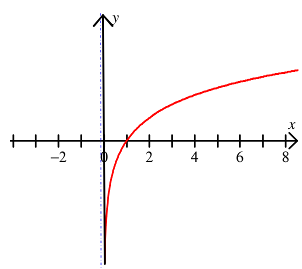
49. $x = \pm\dfrac{\sqrt{\ln(40)}}{2} \approx \pm 0.960$

51. **(a) and (b): See graph below**



(c) The inverse of $y = 2^x$ is $\underline{y = \log_2(x)}$.

53. **(a)** **Graph:**

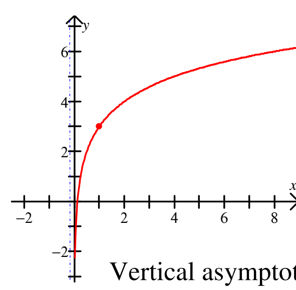


Note: The scale on the *y*-axis varies, depending on the value of *b*.

(b) The *x*-intercept is 1. There are no *y*-intercepts.

(c) There is a vertical asymptote at $x = 0$.

55. **(a)**

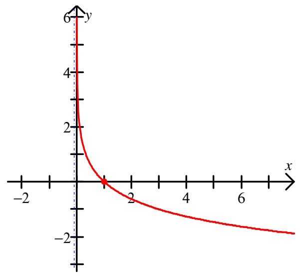


Vertical asymptote at $x = 0$.
Graph passes through $(1, 3)$

(b) Domain: $(0, \infty)$; Range: $(-\infty, \infty)$

(c) Increasing

57. (a)

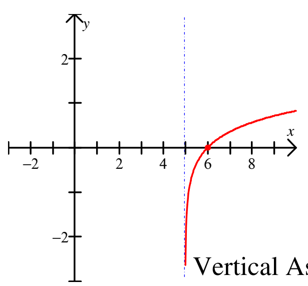


Vertical Asymptote at $x = 0$
Graph passes through $(1, 0)$

(b) Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

(c) Decreasing

59. (a)

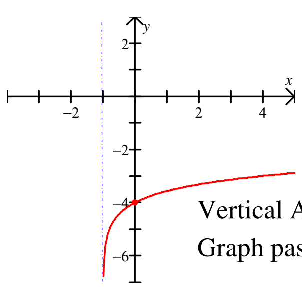


Vertical Asymptote at $x = 5$
Graph passes through $(6, 0)$

(b) Domain: $(5, \infty)$
Range: $(-\infty, \infty)$

(c) Increasing

61. (a)

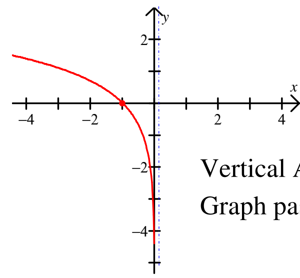


Vertical Asymptote at $x = -1$
Graph passes through $(0, -4)$

(b) Domain: $(-1, \infty)$
Range: $(-\infty, \infty)$

(c) Increasing

63. (a)

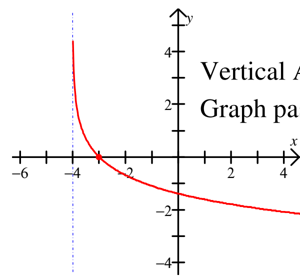


Vertical Asymptote at $x = 0$
Graph passes through $(-1, 0)$

(b) Domain: $(-\infty, 0)$
Range: $(-\infty, \infty)$

(c) Decreasing

65. (a)

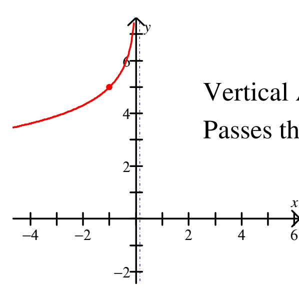


Vertical Asymptote at $x = -4$
Graph passes through $(-3, 0)$

(b) Domain: $(-4, \infty)$
Range: $(-\infty, \infty)$

(c) Decreasing

67. (a)



Vertical Asymptote at $x = 0$
Passes through $(-1, 5)$

(b) Domain: $(-\infty, 0)$
Range: $(-\infty, \infty)$

(c) Increasing

69. $(0, \infty)$

71. $(0, \infty)$

73. $(-2, \infty)$

75. $(-\infty, 0)$

77. $\left(-\infty, \frac{5}{2}\right)$

79. $(-\infty, \infty)$

81. $(-\infty, -2) \cup (3, \infty)$

Odd-Numbered Answers to Exercise Set 3.3: Laws of Logarithms

1. True

3. False

5. False

7. False

9. True

11. False

13. False

15. True

17. True

19. False

21. $\log_5(9)+\log_5(C)+\log_5(D)$

23. $\log_7(K)+\log_7(P)-\log_7(L)$

25. $2\ln(B)+3\ln(P)+\frac{1}{6}\ln(K)$

27. $\log(9)+2\log(k)+3\log(m)-4\log(p)-\log(w)$

29. $\frac{1}{2}\ln(x)+\ln(y+3)$

31. $\frac{1}{4}\log_3(7)+\frac{1}{4}\log_3(x)$

33. $4\ln(x)+\ln(x-5)-\frac{1}{2}\ln(x+3)$

35. $\frac{1}{2}\log(x^2-7)-\frac{1}{2}\log(x^2+3)-\log(x-4)$

37. **(a)** $\log_5\left(\frac{AB}{C}\right)$

(b) $1=\log_5(5)$

(c) $\log_5\left(\frac{5AB}{C}\right)$

(d) $\log_5\left(\frac{AB}{5C}\right)$

39. $\ln\left(\frac{YW}{Z}\right)$

41. $\log\left(\frac{K^4P^3}{\sqrt{Q}}\right)$

43. $\log_7\left[\frac{(x-2)^3}{\sqrt[5]{x^2-3}\,(x+5)^4}\right]$

45. $\log_7(40)$

47. $\ln(x^2+5x+6)$

49. $\ln\left[\frac{\sqrt[6]{x^5(x+3)^2}}{(x^2+5)^4}\right]$

51. 3

53. $2\frac{1}{4}$, or $\frac{9}{4}$

55. 3

57. 3

59. $\frac{3}{2}$

61. $\frac{3}{2}$

63. 25

65. 81

67. 34

69. 320

71. 30

73. $y=x^3$

75. $y=\frac{7}{x^5}$

77. $y=\frac{x^4(x+2)^3}{7}$

79. **(a)** $\dfrac{\ln(2)}{\ln(5)} \approx 0.431$

(b) $\dfrac{\ln(17.2)}{\ln(6)} \approx 1.588$

81. **(a)** $\dfrac{\log(4)}{\log(9)} \approx 0.631$

(b) $\dfrac{\log(8.9)}{\log(4)} \approx 1.577$

83. 3

85. 24

87. **(a)** $\ln(2)+\ln(3)$

(b) $2\log(2)+\log(7)$

89. **(a)** $2\log(2)+\log(3)+\log(5)$

(b) $3\ln(3)+\ln(2)+\ln(5)$

91. **(a)** $A+C$

(b) $-B$

93. **(a)** $A-C$

(b) $2C$

95. **(a)** $2B+C$

(b) $3B-2A-C$

97. **(a)** $\dfrac{2}{3}A+\dfrac{1}{3}B$

(b) $2B-2C-2A$

99. **(a)** $2A+2C+1$

(b) $\dfrac{C}{B}$

Odd-Numbered Answers to Exercise Set 3.4: Exponential and Logarithmic Equations and Inequalities

1. **(a)** $x=\dfrac{\ln(12)}{\ln(7)}$

(b) $x=\dfrac{2\ln(2)+\ln(3)}{\ln(7)}$

(c) $x\approx 1.277$

3. **(a)** $x=\ln(22)$

(b) $x=\ln(2)+\ln(11)$

(c) $x\approx 3.091$

5. **(a)** $x=\dfrac{\ln(14)}{\ln(6)}$

(b) $x=\dfrac{\ln(2)+\ln(7)}{\ln(2)+\ln(3)}$

(c) $x\approx 1.473$

7. **(a) – (c)** No solution

9. **(a)** $x=-\dfrac{\ln(7)}{3\ln(19)}$

(b) $x=-\dfrac{\ln(7)}{3\ln(19)}$

(c) $x\approx -0.220$

11. **(a)** $x=\ln\left(\dfrac{13}{6}\right)$

(b) $x=\ln(13)-\ln(2)-\ln(3)$

(c) $x\approx 0.773$

13. **(a)** $x=\dfrac{\ln(36)-4}{5}$

(b) $x=\dfrac{2\ln(2)+2\ln(3)-4}{5}$

(c) $x\approx -0.083$

15. **(a)** $x=\dfrac{\ln(10)-4\ln(7)}{\ln(7)}$

(b) $x=\dfrac{\ln(2)+\ln(5)-4\ln(7)}{\ln(7)}$

(c) $x\approx -2.817$

17. **(a)** $x=\dfrac{3\ln(20)}{\ln(32)}$

(b) $x=\dfrac{6\ln(2)+3\ln(5)}{5\ln(2)}$

(c) $x\approx 2.593$

19. **(a)** $x=\dfrac{-\ln(6)}{4\ln(5)-7\ln(6)}=\dfrac{\ln(6)}{7\ln(6)-4\ln(5)}$

(b) $x=\dfrac{-\ln(2)-\ln(3)}{4\ln(5)-7\ln(2)-7\ln(3)}$

$=\dfrac{\ln(2)+\ln(3)}{7\ln(2)+7\ln(3)-4\ln(5)}$

(c) $x\approx 0.294$

21. **(a)** $x=\dfrac{\ln(2)+5\ln(8)}{3\ln(2)-2\ln(8)}$

(b) $x=\dfrac{16\ln(2)}{-3\ln(2)}=-\dfrac{16}{3}$ $\left(-\dfrac{16}{3}\text{ is the final answer}\right)$

(c) $x\approx -5.333$

23. **(a)** $x=-\ln\left(\dfrac{16}{7}\right)=\ln\left(\dfrac{16}{7}\right)^{-1}=\ln\left(\dfrac{7}{16}\right)$

(b) $x=\ln(7)-4\ln(2)$

(c) $x\approx -0.827$

25. **(a)** $x=\ln(8)$

(b) $x=3\ln(2)$

(c) $x\approx 2.079$

27. **(a) – (c)** $x=1$

Odd-Numbered Answers to Exercise Set 3.4: Exponential and Logarithmic Equations and Inequalities

29. **(a)** $x=\dfrac{\ln(3)}{\ln(7)}$

(b) $x=\dfrac{\ln(3)}{\ln(7)}$

(c) $x\approx 0.565$

31. **(a) – (c)** $x=-3,\ x=3$

33. **(a) – (c)** $x=-1,\ x=5$

35. **(a) – (c)** $x>\dfrac{7}{3}$

Note: The answer is not all real numbers. Recall the domain of $\log_2(3x-7)$. The inner portion of a logarithmic function must be positive, i.e. $3x-7>0$.

37. $x=e^8$

39. $x=e^{-7}=\dfrac{1}{e^7}$

41. $x=1{,}000$

43. $x=24$

45. $x=\dfrac{101}{4}$

47. $x=\dfrac{e^9-8}{3}$

49. $x=6$ (Note: $x=-1$ is not a solution; if it is plugged into the original equation, the resulting equation contains logarithms of negative numbers, which are undefined.)

51. $x=4$ (Note: $x=-2$ is not a solution; if it is plugged into the original equation, the resulting equation contains logarithms of negative numbers, which are undefined.)

53. $x=-1$ (Note: $x=-4$ is not a solution; if it is plugged into the original equation, the resulting equation contains logarithms of negative numbers, which are undefined.)

55. $x=\dfrac{28}{3}$

57. $x=3$

59. No solution. (Note: $x=-\dfrac{36}{7}$ is not a solution; if it is plugged into the original equation, the resulting equation contains logarithms of negative numbers, which are undefined.)

61. $x=\dfrac{27}{2}$

63. $x=6$ (Note: $x=-4$ is not a solution; if it is plugged into the original equation, the resulting equation contains the logarithms of negative numbers, which are undefined.)

65. $x=e^{e^{-5}}=e^{\left(\frac{1}{e^5}\right)}$

67. $x=5$

69. $x>0$ (Note: The answer is not all real numbers. Recall the domain of $\ln(x)$. The inner portion of a logarithmic function must be positive, i.e. $x>0$.)

71. No solution. (Note: $x=0$ is not a solution; if it is plugged into the original equation, the resulting equation contains the logarithm of zero, which is undefined.)

73. $x=5$ (Note: $x=-5$ is not a solution; if it is plugged into the original equation, the resulting equation contains the logarithm of a negative number, which is undefined.)

75. **(a)** The following answers are equivalent; either one is acceptable:

$$x>\frac{\ln(15)-3\ln(8)}{\ln(8)};\quad x>\frac{\ln(15)}{\ln(8)}-3$$

(b) The following answers are equivalent; either one is acceptable:

$$x>\frac{\ln(5)+\ln(3)-9\ln(2)}{3\ln(2)}$$

$$x>\frac{\ln(5)+\ln(3)}{3\ln(2)}-3$$

Continued on the next page…

Odd-Numbered Answers to Exercise Set 3.4: Exponential and Logarithmic Equations and Inequalities

Interval Notation:

$$\left(\frac{\ln(5)+\ln(3)-9\ln(2)}{3\ln(2)},\ \infty\right)$$

Also acceptable: $\left(\frac{\ln(5)+\ln(3)}{3\ln(2)}-3,\ \infty\right)$

(c) $x > -1.698$; Interval Notation: $(-1.698, \infty)$

77. **(a)** The solution is all real numbers. We cannot take the logarithm of both sides of the inequality $7^x > 0$, since $\ln(0)$ is undefined. Regardless of the value of *x*, the function $f(x) = 7^x$ is always positive. Therefore, the solution is all real numbers.

(b) All real numbers; $(-\infty, \infty)$

(c) All real numbers; $(-\infty, \infty)$

79. **(a)** $x \geq 2 - \ln(9)$

(b) $x \geq 2 - 2\ln(3)$

Interval Notation: $[2 - 2\ln(3), \infty)$

(c) $x \geq -0.197$; Interval Notation: $[-0.197, \infty)$

81. **(a)** $x > \frac{\ln(7)}{\ln(0.2)}$

Note: $\ln(0.2)$ is negative; dividing by $\ln(0.2)$ has reversed the inequality symbol.

(b) $x > -\frac{\ln(7)}{\ln(5)}$

Interval Notation: $\left(-\frac{\ln(7)}{\ln(5)}, \infty\right)$

(c) $x > -1.209$; Interval Notation: $(-1.209, \infty)$

83. **(a)** $x < \frac{\ln\left(\frac{125}{8}\right)}{\ln(0.4)}$

Note: $\ln(0.4)$ is negative; dividing by $\ln(0.4)$ has reversed the inequality symbol.

(b) Solution: $x < -3$

The final simplification steps for this problem are shown below:

$$x < \frac{3\ln(5) - 3\ln(2)}{\ln(2) - \ln(5)}$$

$$x < \frac{3[\ln(5) - \ln(2)]}{\ln(2) - \ln(5)} \qquad \textit{Note}: \frac{\ln(5)-\ln(2)}{\ln(2)-\ln(5)} = -1$$

Therefore, $x < -3$.

Interval Notation: $(-\infty, -3)$

(c) $x < -3$; Interval Notation: $(-\infty, -3)$

85. **(a)** No solution

Note: The solution process yields the equation $e^x \leq -\frac{46}{5}$. There is no solution, since e^x is always positive.

(b) No solution

(c) No solution

87. **(a)** Solution: $x \geq 1$

Explanation:

First, find the domain of $\log_2(x)$. The domain of $\log_2(x)$ is $x > 0$.

Next, isolate *x* in the given inequality:

$$\log_2(x) \geq 0$$
$$2^{\log_2(x)} \geq 2^0$$
$$x \geq 1$$

The solution must satisfy both $x > 0$ and $x \geq 1$. The intersection of these two sets is $x \geq 1$, which is the solution to the inequality.

Continued on the next page…

(b) $x \geq 1$**;** Interval Notation: $[1, \infty)$

(c) $x \geq 1$**;** Interval Notation: $[1, \infty)$

89. **(a)** Solution: $2 \leq x < \frac{7}{3}$

Explanation:

First find the domain of $\ln(7-3x)$:

$$7-3x > 0$$
$$-3x > -7$$
$$x < \frac{7}{3}$$

Next, isolate *x* in the given inequality:

$$\ln(7-3x) \leq 0$$
$$e^{\ln(7-3x)} \leq e^{0}$$
$$7-3x \leq 1$$
$$-3x \leq -6$$
$$x \geq 2$$

The solution must satisfy both $x < \frac{7}{3}$ and $x \geq 2$.

The intersection of these two sets is $2 \leq x < \frac{7}{3}$,

which is the solution to the inequality.

(b) $2 \leq x < \frac{7}{3}$; Interval Notation: $\left[2, \frac{7}{3}\right)$

(c) $2 \leq x < \frac{7}{3}$; Interval Notation: $\left[2, \frac{7}{3}\right)$

Odd-Numbered Answers to Exercise Set 3.5: Applications of Exponential Functions

1. $A(t)$ = the total amount of money in the investment after t years

P = the principal, i.e. the original amount of money invested

r = the interest rate, expressed as a decimal (rather than a percent)

n = the number of times that the interest is compounded each year

t = the number of years for which the money is invested

3. **(a)** P
(b) $A(t)$

5. **(a)** \$5,901.45
(b) \$5,969.37
(c) \$6,004.79
(d) \$6,028.98
(e) \$6,041.26

7. \$10,133.46

9. 6.14%

11. Option (a) is a better investment (7.5%, compounded quarterly).

13. **(a)** 250 rabbits
(b) 2% per day
(c) 271 rabbits
(d) 380 rabbits
(e) 34.66 days

15. **(a)** $N(t) = 18{,}000e^{0.75t}$
(b) 7,261,718 bacteria
(c) 2.27 hours

17. 3,113 people

19. **(a)** 317 buffalo
(b) 387 buffalo
(c) 57.44 years

21. **(a)** 70 grams
(b) 27.62 grams
(c) 39.46%
(d) 11.18 days

23. **(a)** \$28,319.85
(b) 13.95 years

25. **(a)** \$17,926.76
(b) 5.60 years

27. **(a)** $m(t) = 75e^{-0.0139t}$
(b) 24.67 grams
(c) 144.96 years

Odd-Numbered Answers to Exercise Set 4.1: Special Right Triangles and Trigonometric Ratios

1. angles

3. 180

5. largest, smallest

7. **(a)** $x = 5\sqrt{2}$

(b)
$$\begin{aligned} 5^2 + 5^2 &= x^2 \\ 25 + 25 &= x^2 \\ 50 &= x^2 \\ x &= \sqrt{50} = \sqrt{25}\sqrt{2} \\ x &= 5\sqrt{2} \end{aligned}$$

9. **(a)** $x = \dfrac{4\sqrt{2}}{\sqrt{2}}$, so $x = 4$

(b)
$$\begin{aligned} x^2 + x^2 &= \left(4\sqrt{2}\right)^2 \\ 2x^2 &= 16(2) = 32 \\ x^2 &= 16 \\ x &= 4 \end{aligned}$$

11. **(a)** $x = \dfrac{8}{\sqrt{2}} = \dfrac{8}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{8\sqrt{2}}{2}$, so $x = 4\sqrt{2}$

(b)
$$\begin{aligned} x^2 + x^2 &= 8^2 \\ 2x^2 &= 64 \\ x^2 &= 32 \\ x &= \sqrt{32} = \sqrt{16}\sqrt{2} \\ x &= 4\sqrt{2} \end{aligned}$$

13. **(a)** $x = \dfrac{9}{\sqrt{2}} = \dfrac{9}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$, so $x = \dfrac{9\sqrt{2}}{2}$

(b)
$$\begin{aligned} x^2 + x^2 &= 9^2 \\ 2x^2 &= 81 \\ x^2 &= \frac{81}{2} \\ x &= \sqrt{\frac{81}{2}} = \frac{9}{\sqrt{2}} = \frac{9}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ x &= \frac{9\sqrt{2}}{2} \end{aligned}$$

15. **(a)** $x = \left(8\sqrt{2}\right)\sqrt{2}$, so $x = 16$

(b)
$$\begin{aligned} \left(8\sqrt{2}\right)^2 + \left(8\sqrt{2}\right)^2 &= x^2 \\ 64(2) + 64(2) &= x^2 \\ 128 + 128 &= x^2 \\ 256 &= x^2 \\ x &= 16 \end{aligned}$$

17. **(a)** $x = \dfrac{2\sqrt{3}}{\sqrt{2}} = \dfrac{2\sqrt{3}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2\sqrt{6}}{2}$, so $x = \sqrt{6}$

(b)
$$\begin{aligned} x^2 + x^2 &= \left(2\sqrt{3}\right)^2 \\ 2x^2 &= 4(3) = 12 \\ x^2 &= 6 \\ x &= \sqrt{6} \end{aligned}$$

19. 60°

21. **(a)** $a = b = 5$

(b) The altitude divides the equilateral triangle into two congruent 30°-60°-90° triangles. Therefore, $a = b$. Since $a + b = 10$, then $a + a = 10$, so $a = 5$. Hence $a = b = 5$.

(c)
$$\begin{aligned} 5^2 + c^2 &= 10^2 \\ 25 + c^2 &= 100 \\ c^2 &= 75 \\ c &= \sqrt{75} = \sqrt{25}\sqrt{3} \\ c &= 5\sqrt{3} \end{aligned}$$

23. $x = 14,\ y = 7\sqrt{3}$

25. $x = 12,\ y = 6$

27. $x = \dfrac{5}{2},\ y = \dfrac{5\sqrt{3}}{2}$

29. $x = 4\sqrt{3},\ y = 2\sqrt{3}$

31. $x = 15,\ y = 10\sqrt{3}$

Odd-Numbered Answers to Exercise Set 4.1: Special Right Triangles and Trigonometric Ratios

33. **(a)**
$$\begin{aligned}(BC)^2+15^2&=17^2\\(BC)^2+225&=289\\(BC)^2&=64\\BC&=8\end{aligned}$$

(b) $\sin(A)=\dfrac{8}{17}$ $\qquad$ $\sin(B)=\dfrac{15}{17}$

$\cos(A)=\dfrac{15}{17}$ $\qquad$ $\cos(B)=\dfrac{8}{17}$

$\tan(A)=\dfrac{8}{15}$ $\qquad$ $\tan(B)=\dfrac{15}{8}$

35. $\cos(\theta)=\dfrac{2\sqrt{6}}{7},\ \tan(\theta)=\dfrac{5\sqrt{6}}{12}$

37. cosecant

39. cotangent

41. cosine

43. **(a)**
$$\begin{aligned}x^2+5^2&=6^2\\x^2+25&=36\\x^2&=11\\x&=\sqrt{11}\end{aligned}$$

(b) $\sin(\alpha)=\dfrac{5}{6}$ $\qquad$ $\csc(\alpha)=\dfrac{6}{5}$

$\cos(\alpha)=\dfrac{\sqrt{11}}{6}$ $\qquad$ $\sec(\alpha)=\dfrac{6\sqrt{11}}{11}$

$\tan(\alpha)=\dfrac{5\sqrt{11}}{11}$ $\qquad$ $\cot(\alpha)=\dfrac{\sqrt{11}}{5}$

(c) $\sin(\beta)=\dfrac{\sqrt{11}}{6}$ $\qquad$ $\csc(\beta)=\dfrac{6\sqrt{11}}{11}$

$\cos(\beta)=\dfrac{5}{6}$ $\qquad$ $\sec(\beta)=\dfrac{6}{5}$

$\tan(\beta)=\dfrac{\sqrt{11}}{5}$ $\qquad$ $\cot(\beta)=\dfrac{5\sqrt{11}}{11}$

45. **(a)**
$$\begin{aligned}4^2+8^2&=x^2\\16+64&=x^2\\x^2&=80\\x&=\sqrt{80}=\sqrt{16}\sqrt{5}\\x&=4\sqrt{5}\end{aligned}$$

(b) $\sin(\alpha)=\dfrac{2\sqrt{5}}{5}$ $\qquad$ $\csc(\alpha)=\dfrac{\sqrt{5}}{2}$

$\cos(\alpha)=\dfrac{\sqrt{5}}{5}$ $\qquad$ $\sec(\alpha)=\sqrt{5}$

$\tan(\alpha)=2$ $\qquad$ $\cot(\alpha)=\dfrac{1}{2}$

(c) $\sin(\beta)=\dfrac{\sqrt{5}}{5}$ $\qquad$ $\csc(\beta)=\sqrt{5}$

$\cos(\beta)=\dfrac{2\sqrt{5}}{5}$ $\qquad$ $\sec(\beta)=\dfrac{\sqrt{5}}{2}$

$\tan(\beta)=\dfrac{1}{2}$ $\qquad$ $\cot(\beta)=2$

47. $\sin(\theta)=\dfrac{3}{7}$ $\qquad$ $\csc(\theta)=\dfrac{7}{3}$

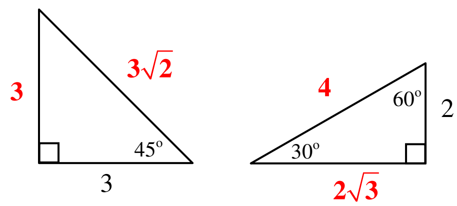
$\cos(\theta)=\dfrac{2\sqrt{10}}{7}$ $\qquad$ $\sec(\theta)=\dfrac{7\sqrt{10}}{20}$

$\tan(\theta)=\dfrac{3\sqrt{10}}{20}$ $\qquad$ $\cot(\theta)=\dfrac{2\sqrt{10}}{3}$

Continued on the next page…

Odd-Numbered Answers to Exercise Set 4.1: Special Right Triangles and Trigonometric Ratios

49. **(a)**



(b) $\sin(45^\circ) = \dfrac{\sqrt{2}}{2}$ $\qquad \csc(45^\circ) = \sqrt{2}$

$\cos(45^\circ) = \dfrac{\sqrt{2}}{2}$ $\qquad \sec(45^\circ) = \sqrt{2}$

$\tan(45^\circ) = 1$ $\qquad \cot(45^\circ) = 1$

(c) $\sin(30^\circ) = \dfrac{1}{2}$ $\qquad \csc(30^\circ) = 2$

$\cos(30^\circ) = \dfrac{\sqrt{3}}{2}$ $\qquad \sec(30^\circ) = \dfrac{2\sqrt{3}}{3}$

$\tan(30^\circ) = \dfrac{\sqrt{3}}{3}$ $\qquad \cot(30^\circ) = \sqrt{3}$

(d) $\sin(60^\circ) = \dfrac{\sqrt{3}}{2}$ $\qquad \csc(60^\circ) = \dfrac{2\sqrt{3}}{3}$

$\cos(60^\circ) = \dfrac{1}{2}$ $\qquad \sec(60^\circ) = 2$

$\tan(60^\circ) = \sqrt{3}$ $\qquad \cot(60^\circ) = \dfrac{\sqrt{3}}{3}$

51. Answers vary, but some important observations are below.

First, notice that when comparing Exercises 49 and 50:

The answers to part (b) are identical, for each respective trigonometric function. So, for example, $\sin(45^\circ) = \dfrac{\sqrt{2}}{2}$ in both examples. Similarly, the answers to part (c) are identical. So, for example, $\tan(30^\circ) = \dfrac{\sqrt{3}}{3}$ regardless of the triangle being used to compute the ratio. Along the same lines, the answers to part (d) in both exercises are identical. Notice, for example, that $\cos(60^\circ) = \dfrac{1}{2}$ in both exercises. In fact, the value of $\cos(60^\circ)$ is always $\dfrac{1}{2}$, and the trigonometric ratio of any given number is constant.

In addition to the observations above, the following formulas, known as the cofunction identities, are illustrated in parts (b) – (d), and will be covered in more detail in Chapter 6.

$\sin(\theta) = \cos(90^\circ - \theta)$,

e.g. $\sin(30^\circ) = \cos(60^\circ)$

$\sin(60^\circ) = \cos(30^\circ)$

$\sin(45^\circ) = \cos(45^\circ)$

$\sec(\theta) = \csc(90^\circ - \theta)$,

e.g. $\sec(30^\circ) = \csc(60^\circ)$

$\sec(60^\circ) = \csc(30^\circ)$

$\sec(45^\circ) = \csc(45^\circ)$

$\tan(\theta) = \cot(90^\circ - \theta)$,

e.g. $\tan(30^\circ) = \cot(60^\circ)$

$\tan(60^\circ) = \cot(30^\circ)$

$\tan(45^\circ) = \cot(45^\circ)$

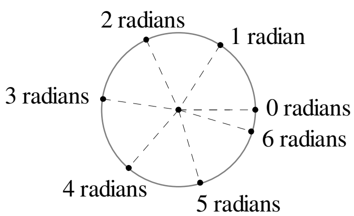
Odd-Numbered Answers to Exercise Set 4.2: Radians, Arc Length, and the Area of a Sector

1. $\theta = 4$

3. $\theta = 240$

5. $\theta = \dfrac{1}{2}$

7. **(a)** There are slightly more than 6 radians in one revolution, as shown in the figure below.



(b) There are 2π radians in a complete revolution.

Justification:
The arc length in this case is the circumference C of the circle, so $s = C = 2\pi r$. Therefore,

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi .$$

Comparison to part (a):
$2\pi \approx 6.28$, and it can be seen from part (a) that there are slightly more than 6 radians in one revolution.

9. **(a)** $\dfrac{\pi}{6} \approx 0.52$ **(b)** $\dfrac{\pi}{2} \approx 1.57$ **(c)** $\dfrac{3\pi}{4} \approx 2.36$

11. **(a)** $\dfrac{2\pi}{3} \approx 2.09$ **(b)** $\dfrac{5\pi}{4} \approx 3.93$ **(c)** $\dfrac{11\pi}{6} \approx 5.76$

13. **(a)** $\dfrac{19\pi}{180} \approx 0.33$ **(b)** $\dfrac{2\pi}{9} \approx 0.70$ **(c)** $\dfrac{2\pi}{5} \approx 1.26$

15. **(a)** 45° **(b)** 240° **(c)** 150°

17. **(a)** 180° **(b)** 165° **(c)** 305°

19. **(a)** 143.24° **(b)** 28.99°

21. $\dfrac{31\pi}{60}$

23. $s = 4\pi$ cm

25. $s = 5\pi$ ft

27. $s = \dfrac{21\pi}{2}$ yd

29. $s = 2\pi$ ft

31. $\theta = \dfrac{\pi}{3}$; $s = 4\pi$ cm (Notice that $s = 4\pi$ cm in the solution for Exercise 23.)

33. $\theta = \dfrac{5\pi}{4}$; $s = 5\pi$ ft (Notice that $s = 5\pi$ ft in the solution for Exercise 25.)

35. $s = \dfrac{15\pi}{2}$ in

37. $s = 10\pi$ cm

39. $r = \dfrac{24}{5}$ m $= 4.8$ m

41. $r = \dfrac{28}{3\pi}$ ft ≈ 2.97 ft

43. $r = \dfrac{12}{5}$ in $= 2.4$ in

45. $r = \dfrac{10}{\pi}$ m ≈ 3.18 m

47. $P = \left(\dfrac{4\pi}{3} + 16\right)$ cm ≈ 20.19 cm

49. $A = 24\pi \text{ cm}^2$

51. $A = 10\pi \text{ ft}^2$

53. $A = \dfrac{189\pi}{4} \text{ yd}^2$

55. $A = 2\pi \text{ ft}^2$

57. $\theta = 3$; $A = 24\pi \text{ cm}^2$ (Notice that $A = 24\pi \text{ cm}^2$ in the solution for Exercise 49.)

Odd-Numbered Answers to Exercise Set 4.2: Radians, Arc Length, and the Area of a Sector

59. $\theta = \dfrac{5\pi}{4}$; $A = 10\pi \text{ ft}^2$ (Notice that $A = 10\pi \text{ ft}^2$ in the solution for Exercise 51.)

61. $r = 7 \text{ cm}$

63. $r = \dfrac{4}{5} \text{ in}$

65. $\theta = \dfrac{7\pi}{4}$

67. **(a)** Two methods of solution are shown below.

Method 1: (shown in the text)
The angular speed ω can be found using the formula $\omega = \dfrac{\theta}{t}$, where θ is in radians. Since the rate of turn is $900^\circ/\text{sec} = \dfrac{900^\circ}{1\text{ sec}}$, we note that $\theta = 900^\circ$ and $t = 1\text{ sec}$.

First, convert 900° to radians:

$$\theta = 900^\circ \times \frac{\pi}{180^\circ} = \overset{5}{\cancel{900^\circ}} \times \frac{\pi}{\cancel{180^\circ}} = 5\pi$$

Then $\omega = \dfrac{\theta}{t} = \dfrac{5\pi \text{ radians}}{1\text{ sec}} = 5\pi$ radians/sec

Method 2:
Take the given rate of turn, $900^\circ/\text{sec}$, equivalent to $\dfrac{900^\circ}{1\text{ sec}}$, and use unit conversion ratios to change this rate of turn from degrees/sec to radians/sec, as shown below:

$$\frac{900^\circ}{1\text{ sec}} \times \frac{\pi \text{ radians}}{180^\circ} = \frac{\overset{5}{\cancel{900^\circ}}}{1\text{ sec}} \times \frac{\pi \text{ radians}}{\cancel{180^\circ}}$$

$$= 5\pi \text{ radians/sec}$$

(b) Three methods of solution are shown below:

Method 1: (shown in the text)
The linear speed v can be found using the formula $v = \dfrac{d}{t}$, where d is the distance traveled by the point on the CD in time t.

Since the rate of turn is $900^\circ/\text{sec} = \dfrac{900^\circ}{1\text{ sec}}$, we note that $\theta = 900^\circ$ and $t = 1\text{ sec}$.

To find d, use the formula $d = r\theta$ (equivalent to the formula for arc length, $s = r\theta$). We must first convert $\theta = 900^\circ$ to radians:

$$\theta = 900^\circ \times \frac{\pi}{180^\circ} = \overset{5}{\cancel{900^\circ}} \times \frac{\pi}{\cancel{180^\circ}} = 5\pi$$

Then $d = r\theta = (6\text{cm})(5\pi) = 30\pi \text{ cm}$.

Finally, use the formula for linear speed:
$v = \dfrac{d}{t} = \dfrac{30\pi \text{ cm}}{1\text{ sec}} = 30\pi$ cm/sec

Method 2:
Another formula for the linear speed v is $v = r\omega$, where ω is the angular speed. (This can be derived from previous formulas, since $v = \dfrac{d}{t} = \dfrac{r\theta}{t} = r\left(\dfrac{\theta}{t}\right) = r\omega$.)

It is given that $r = 6\text{ cm}$, and it is known from part (a) that $w = 5\pi$ radians/sec, so:
$v = rw = (6\text{ cm})(5\pi \text{ radians/sec}) = 30\pi \text{ cm/sec}$.
(Remember that units are not written for radians, with the exception of angular speed.)

Method 3:
Take the given rate of turn, $900^\circ/\text{sec}$, equivalent to $\dfrac{900^\circ}{1\text{ sec}}$, and use unit conversion ratios to change this rate of turn from degrees/sec to cm/sec, as shown below.

Continued on the next page…

Odd-Numbered Answers to Exercise Set 4.2: Radians, Arc Length, and the Area of a Sector

(Note that in one revolution, the circumference of the circle with a 6 cm radius is $C = 2\pi r = 2\pi(6) = 12\pi$ cm, hence the unit conversion ratio $\frac{12\pi \text{ cm}}{1 \text{ revolution}}$).

$$\frac{900^\circ}{1 \text{ sec}} \times \frac{\pi \text{ radians}}{180^\circ} \times \frac{1 \text{ revolution}}{2\pi \text{ radians}} \times \frac{12\pi \text{ cm}}{1 \text{ revolution}}$$

$$= \frac{\overset{5}{\cancel{900^\circ}}}{1 \text{ sec}} \times \frac{\pi \cancel{\text{ radians}}}{\cancel{180^\circ}} \times \frac{1 \cancel{\text{ revolution}}}{\cancel{2\pi} \cancel{\text{ radians}}} \times \frac{\overset{6}{\cancel{12}}\pi \text{ cm}}{1 \cancel{\text{ revolution}}}$$

$$= 30\pi \text{ cm/sec}$$

(c) This problem can be solved in one of three ways:

<u>Method 1:</u>
The linear speed v can be found using the formula $v = \frac{d}{t}$, where d is the distance traveled by the point on the CD in time t.

Since the rate of turn is $900^\circ/\text{sec} = \frac{900^\circ}{1 \text{ sec}}$, we note that $\theta = 900^\circ$ and $t = 1$ sec.

To find d, use the formula $d = r\theta$ (equivalent to the formula for arc length, $s = r\theta$). We must first convert $\theta = 900^\circ$ to radians:

$$\theta = 900^\circ \times \frac{\pi}{180^\circ} = \overset{5}{\cancel{900^\circ}} \times \frac{\pi}{\cancel{180^\circ}} = 5\pi$$

The CD has a radius of 6 cm, so the desired radius of the point halfway between the center of the CD and its outer edge is $r = \frac{1}{2}(6) = 3$ cm.

Then $d = r\theta = (3\text{cm})(5\pi) = 15\pi$ cm.

Finally, use the formula for linear speed:
$$v = \frac{d}{t} = \frac{15\pi \text{ cm}}{1 \text{ sec}} = 15\pi \text{ cm/sec}$$

<u>Method 2:</u>
Another formula for the linear speed v is $v = r\omega$, where ω is the angular speed. (This can be derived from previous formulas, since $v = \frac{d}{t} = \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right) = r\omega$.)

The CD has a radius of 6 cm, so the desired radius of the point halfway between the center of the CD and its outer edge is $r = \frac{1}{2}(6) = 3$ cm.

It is known from part (a) that $w = 5\pi$ radians/sec, so:
$v = rw = (3 \text{ cm})(5\pi \text{ radians/sec}) = 15\pi$ cm/sec.
(Remember that units are not written for radians, with the exception of angular speed.)

<u>Method 3:</u>
Take the given rate of turn, $900^\circ/\text{sec}$, equivalent to $\frac{900^\circ}{1 \text{ sec}}$, and use unit conversion ratios to change this rate of turn from degrees/sec to cm/sec, as shown below.

(Note that in one revolution, the circumference of the circle with a 3 cm radius is $C = 2\pi r = 2\pi(3) = 6\pi$ cm, hence the unit conversion ratio $\frac{6\pi \text{ cm}}{1 \text{ revolution}}$).

$$\frac{900^\circ}{1 \text{ sec}} \times \frac{\pi \text{ radians}}{180^\circ} \times \frac{1 \text{ revolution}}{2\pi \text{ radians}} \times \frac{6\pi \text{ cm}}{1 \text{ revolution}}$$

$$= \frac{\overset{5}{\cancel{900^\circ}}}{1 \text{ sec}} \times \frac{\pi \cancel{\text{ radians}}}{\cancel{180^\circ}} \times \frac{1 \cancel{\text{ revolution}}}{\cancel{2\pi} \cancel{\text{ radians}}} \times \frac{\overset{3}{\cancel{6}}\pi \text{ cm}}{1 \cancel{\text{ revolution}}}$$

$$= 15\pi \text{ cm/sec}$$

69. Since the diameter is 26 inches, the radius, $r = 13$ in. Take the given rate of turn, 20 miles/hr, equivalent to $\frac{20 \text{ miles}}{1 \text{ hr}}$, and use unit conversion ratios to change this rate of turn from miles/hr to rev/min, as shown below.

Continued on the next page…

(Note that in one revolution, the circumference of the circle with a 13 in radius is $C = 2\pi r = 2\pi(13) = 26\pi$ in, hence the unit conversion ratio $\frac{1 \text{ revolution}}{26\pi \text{ in}}$).

$$\frac{20 \text{ miles}}{1 \text{ hr}} \times \frac{5280 \text{ ft}}{1 \text{ mile}} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ revolution}}{26\pi \text{ in}}$$

$$= \frac{20 \cancel{\text{miles}}}{1 \cancel{\text{hr}}} \times \frac{5280 \cancel{\text{ft}}}{1 \cancel{\text{mile}}} \times \frac{12 \cancel{\text{in}}}{1 \cancel{\text{ft}}} \times \frac{1 \cancel{\text{hr}}}{60 \text{ min}} \times \frac{1 \text{ revolution}}{26\pi \cancel{\text{in}}}$$

≈ 258.57 rev/min, or 258.57 rpm

71. Numbers 67 and 68 show various methods for solving these types of problems. One method is shown below for each question.

(a) Take the given rate of turn, 4 revolutions per second, equivalent to $\frac{4 \text{ revolutions}}{1 \text{ sec}}$, and use unit conversion ratios to change this rate of turn from revolutions/sec to radians/sec, as shown below.

$$\frac{4 \text{ revolutions}}{1 \text{ second}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$

$$= \frac{4 \cancel{\text{revolutions}}}{1 \text{ second}} \times \frac{2\pi \text{ radians}}{1 \cancel{\text{revolution}}}$$

$= 8\pi$ radians/sec

(b) $v = r\omega = (10 \text{ in})(8\pi \text{ radians/sec})$, so $v = 80\pi$ in/sec

(c) Use the linear speed from part (b), $v = 80\pi$ in/sec, and then use unit conversion ratios to change this from in/sec to miles/hr, as shown below:

$$\frac{80\pi \text{ in}}{1 \text{ sec}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ mile}}{5280 \text{ ft}} \times \frac{60 \text{ sec}}{1} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1}{60 \text{ min}}$$

$$= \frac{80\pi \cancel{\text{in}}}{1 \cancel{\text{sec}}} \times \frac{1 \cancel{\text{ft}}}{12 \cancel{\text{in}}} \times \frac{1 \text{ mile}}{5280 \cancel{\text{ft}}} \times \frac{60 \cancel{\text{sec}}}{1 \cancel{\text{min}}} \times \frac{60 \cancel{\text{min}}}{1 \text{ hr}}$$

$= 14.28$ miles per hour (mph)

73. Hour Hand:
$r = 4$ in

Rate of turn: $\frac{1 \text{ revolution}}{12 \text{ hours}}$

(Note that in one revolution, the circumference of the circle with a 4 in radius is $C = 2\pi r = 2\pi(4) = 8\pi$ in, hence the unit conversion ratio $\frac{1 \text{ revolution}}{8\pi \text{ in}}$).

$$20 \text{ min} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ revolution}}{12 \text{ hr}} \times \frac{8\pi \text{ in}}{1 \text{ revolution}}$$

$$= 20 \cancel{\text{min}} \times \frac{1 \cancel{\text{hr}}}{60 \cancel{\text{min}}} \times \frac{1 \cancel{\text{revolution}}}{12 \cancel{\text{hr}}} \times \frac{8\pi \text{ in}}{1 \cancel{\text{revolution}}}$$

$= \frac{2\pi}{9}$ in

Minute Hand:
$r = 5$ in

Rate of turn: $\frac{1 \text{ revolution}}{60 \text{ min}}$

(Note that in one revolution, the circumference of the circle with a 5 in radius is $C = 2\pi r = 2\pi(5) = 10\pi$ in, hence the unit conversion ratio $\frac{1 \text{ revolution}}{10\pi \text{ in}}$).

$$20 \text{ min} \times \frac{1 \text{ revolution}}{60 \text{ min}} \times \frac{10\pi \text{ in}}{1 \text{ revolution}}$$

$$= 20 \cancel{\text{min}} \times \frac{1 \cancel{\text{revolution}}}{60 \cancel{\text{min}}} \times \frac{10\pi \text{ in}}{1 \cancel{\text{revolution}}}$$

$= \frac{10\pi}{3}$ in

Second Hand:
$r = 6$ in

Rate of turn: $\frac{1 \text{ revolution}}{1 \text{ min}}$

Continued on the next page…

(Note that in one revolution, the circumference of the circle with a 6 in radius is $C = 2\pi r = 2\pi(6) = 12\pi$ in , hence the unit conversion ratio $\dfrac{1 \text{ revolution}}{12\pi \text{ in}}$).

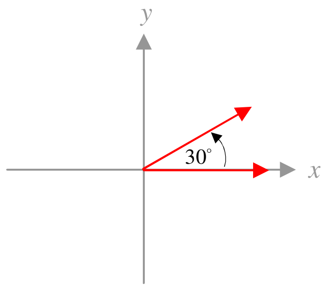
$$20 \text{ min} \times \frac{1 \text{ revolution}}{1 \text{ min}} \times \frac{12\pi \text{ in}}{1 \text{ revolution}}$$

$$= 20 \cancel{\text{min}} \times \frac{1 \cancel{\text{revolution}}}{1 \cancel{\text{min}}} \times \frac{12\pi \text{ in}}{1 \cancel{\text{revolution}}}$$

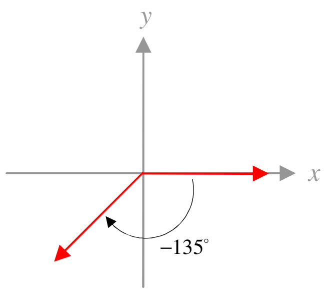
$$= 240\pi \text{ in}$$

Odd-Numbered Answers to Exercise Set 4.3: Unit Circle Trigonometry

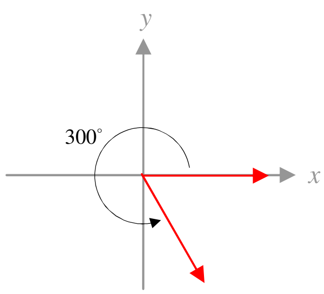
1. **(a)**



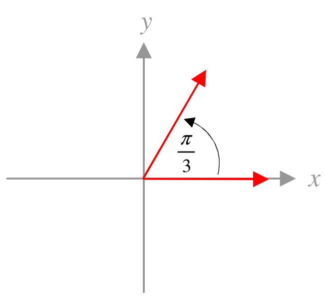
(b)



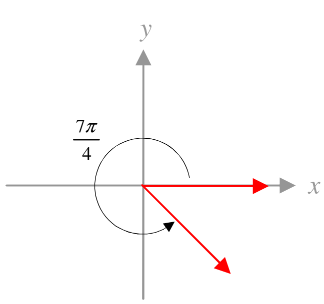
(c)



3. **(a)**

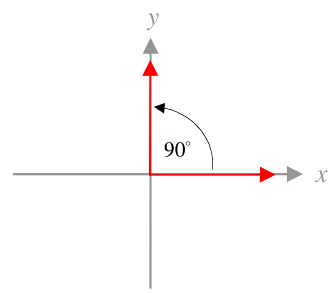


(b)

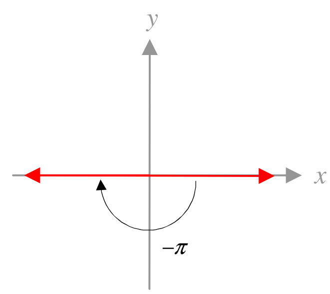


(c)

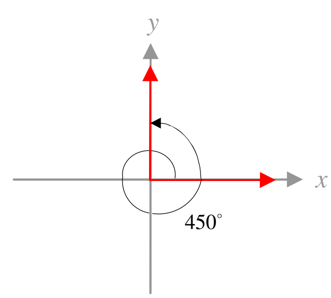
5. **(a)**



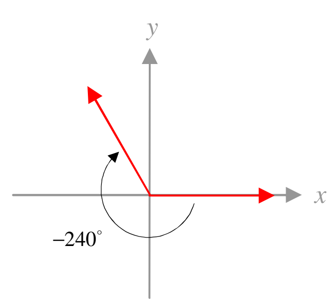
(b)



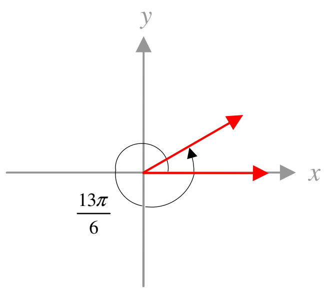
(c)



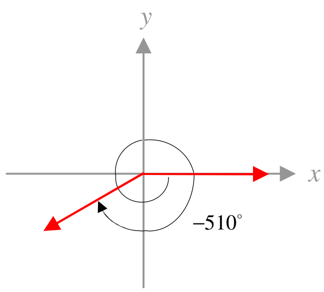
7. **(a)**



(b)



(c)



9. **(a)** $-310^\circ,\ 410^\circ,\ 770^\circ$

(b) $-560^\circ,\ 160^\circ,\ 520^\circ$

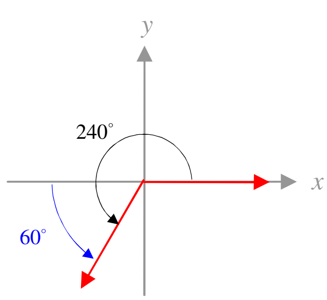
11. **(a)** $-\frac{8\pi}{5},\ \frac{12\pi}{5},\ \frac{22\pi}{5}$

(b) $-\frac{3\pi}{2},\ \frac{\pi}{2},\ \frac{5\pi}{2}$

13. $0^\circ,\ 90^\circ,\ 180^\circ,\ 270^\circ$

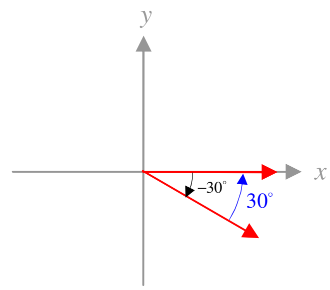
15. $\frac{7\pi}{4},\ \frac{9\pi}{4},\ \frac{11\pi}{4},\ \frac{13\pi}{4}$

17. **(a)** The reference angle is 60°, shown in blue below.

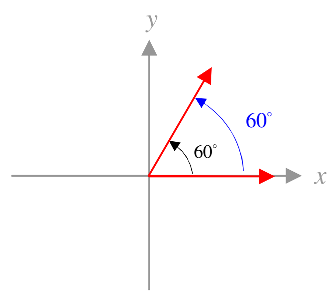


Continued in the next column…

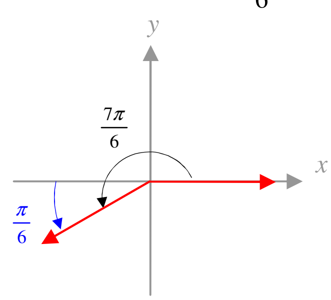
(b) The reference angle is 30°, shown in blue below.



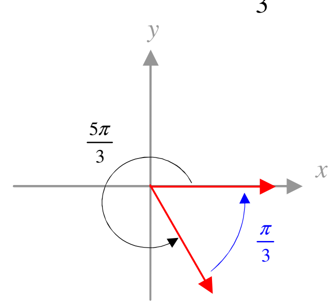
(c) The reference angle is 60°, shown in blue below.



19. **(a)** The reference angle is $\frac{\pi}{6}$, shown in blue below.



(b) The reference angle is $\frac{\pi}{3}$, shown in blue below.



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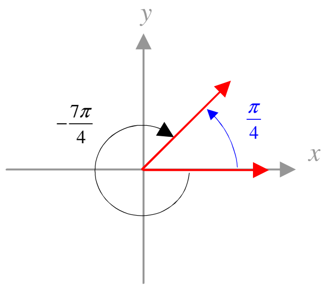
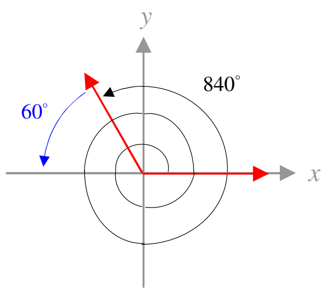
Odd-Numbered Answers to Exercise Set 4.3: Unit Circle Trigonometry

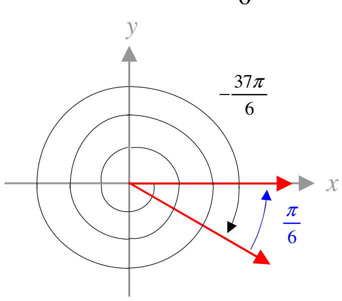
(c) The reference angle is $\dfrac{\pi}{4}$, shown in blue below.



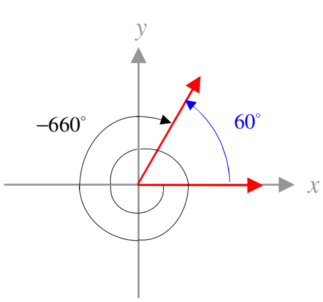
21. (a) The reference angle is 60°, shown in blue below.



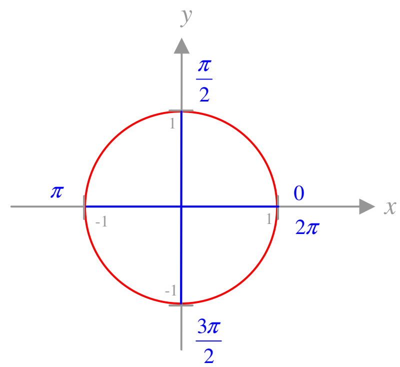
(b) The reference angle is $\dfrac{\pi}{6}$, shown in blue below.



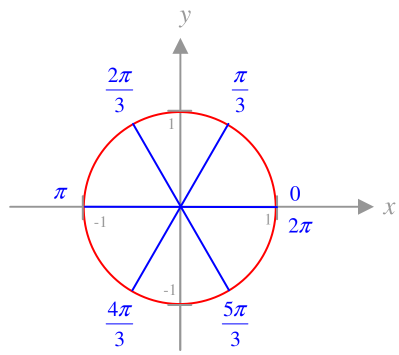
(c) The reference angle is 60°, shown in blue below.



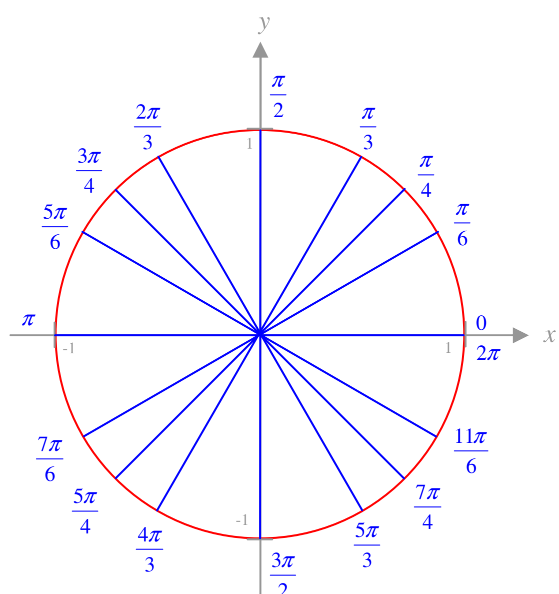
23.



25.



27.



29. II

31. III

33. IV

35. =

Odd-Numbered Answers to Exercise Set 4.3: Unit Circle Trigonometry

37. <

39. >

41. $\sin(\theta) = \frac{2}{3}$ $\csc(\theta) = \frac{3}{2}$

$\cos(\theta) = \frac{\sqrt{5}}{3}$ $\sec(\theta) = \frac{3\sqrt{5}}{5}$

$\tan(\theta) = \frac{2\sqrt{5}}{5}$ $\cot(\theta) = \frac{\sqrt{5}}{2}$

43. $\sin(\theta) = -\frac{3}{5}$ $\csc(\theta) = -\frac{5}{3}$

$\cos(\theta) = \frac{4}{5}$ $\sec(\theta) = \frac{5}{4}$

$\tan(\theta) = -\frac{3}{4}$ $\cot(\theta) = -\frac{4}{3}$

45. $\sin(\theta) = \frac{2\sqrt{6}}{5}$ $\csc(\theta) = \frac{5\sqrt{6}}{12}$

$\cos(\theta) = -\frac{1}{5}$ $\sec(\theta) = -5$

$\tan(\theta) = -2\sqrt{6}$ $\cot(\theta) = -\frac{\sqrt{6}}{12}$

47. The terminal side of the angle intersects the unit circle at the point $(0, 1)$.

$\sin(90^\circ) = 1$ $\csc(90^\circ) = 1$

$\cos(90^\circ) = 0$ $\sec(90^\circ)$ is undefined

$\tan(90^\circ)$ is undefined $\cot(90^\circ) = 0$

49. The terminal side of the angle intersects the unit circle at the point $(1, 0)$.

$\sin(-2\pi) = 0$ $\csc(-2\pi)$ is undefined

$\cos(-2\pi) = 1$ $\sec(-2\pi) = 1$

$\tan(-2\pi) = 0$ $\cot(-2\pi)$ is undefined

51. The terminal side of the angle intersects the unit circle at the point $(0, -1)$.

$\sin\left(-\frac{5\pi}{2}\right) = -1$ $\csc\left(-\frac{5\pi}{2}\right) = -1$

$\cos\left(-\frac{5\pi}{2}\right) = 0$ $\sec\left(-\frac{5\pi}{2}\right)$ is undefined

$\tan\left(-\frac{5\pi}{2}\right)$ is undefined $\cot\left(-\frac{5\pi}{2}\right) = 0$

53. **(a)** $\cos(300^\circ) = \cos(60^\circ)$

(b) $\tan(135^\circ) = -\tan(45^\circ)$

55. **(a)** $\sin(140^\circ) = \sin(40^\circ)$

(b) $\sec\left(-\frac{2\pi}{3}\right) = -\sec\left(\frac{\pi}{3}\right)$

57. **(a)** $\csc\left(\frac{25\pi}{6}\right) = \csc\left(\frac{\pi}{6}\right)$

(b) $\cot(-460^\circ) = \cot(80^\circ)$

59. The terminal side of the angle intersects the unit circle at the point $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\sin(45^\circ) = \frac{\sqrt{2}}{2}$ $\csc(45^\circ) = \sqrt{2}$

$\cos(45^\circ) = \frac{\sqrt{2}}{2}$ $\sec(45^\circ) = \sqrt{2}$

$\tan(45^\circ) = 1$ $\cot(45^\circ) = 1$

61. The terminal side of the angle intersects the unit circle at the point $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$\sin(210^\circ) = -\frac{1}{2}$ $\csc(210^\circ) = -2$

$\cos(210^\circ) = -\frac{\sqrt{3}}{2}$ $\sec(210^\circ) = -\frac{2\sqrt{3}}{3}$

$\tan(210^\circ) = \frac{\sqrt{3}}{3}$ $\cot(210^\circ) = \sqrt{3}$

63. The terminal side of the angle intersects the unit circle at the point $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ $\qquad$ $\csc\left(\frac{5\pi}{3}\right) = -\frac{2\sqrt{3}}{3}$

$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$ $\qquad$ $\sec\left(\frac{5\pi}{3}\right) = 2$

$\tan\left(\frac{5\pi}{3}\right) = -\sqrt{3}$ $\qquad$ $\cot\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{3}$

65. (a)

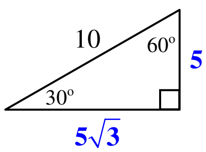


Diagram 1

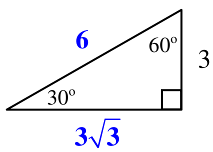


Diagram 2

(b) $\sin(30^\circ) = \frac{5}{10} = \frac{1}{2}$ $\qquad$ $\sin(60^\circ) = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$

$\cos(30^\circ) = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$ $\qquad$ $\cos(60^\circ) = \frac{5}{10} = \frac{1}{2}$

$\tan(30^\circ) = \frac{5}{5\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\qquad$ $\tan(60^\circ) = \frac{5\sqrt{3}}{5} = \sqrt{3}$

(c) $\sin(30^\circ) = \frac{3}{6} = \frac{1}{2}$ $\qquad$ $\sin(60^\circ) = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

$\cos(30^\circ) = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$ $\qquad$ $\cos(60^\circ) = \frac{3}{6} = \frac{1}{2}$

$\tan(30^\circ) = \frac{3}{3\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\qquad$ $\tan(60^\circ) = \frac{3\sqrt{3}}{3} = \sqrt{3}$

(d) $\sin(30^\circ) = \frac{1}{2}$ $\qquad$ $\sin(60^\circ) = \frac{\sqrt{3}}{2}$

$\cos(30^\circ) = \frac{\sqrt{3}}{2}$ $\qquad$ $\cos(60^\circ) = \frac{1}{2}$

$\tan(30^\circ) = \frac{\sqrt{3}}{3}$ $\qquad$ $\tan(60^\circ) = \sqrt{3}$

(e) The methods in parts (b) – (d) all yield the same results.

67. **(a)** $-\frac{\sqrt{3}}{2}$ **(b)** -1

69. **(a)** $\frac{\sqrt{2}}{2}$ **(b)** $\frac{2\sqrt{3}}{3}$

71. **(a)** $\frac{\sqrt{3}}{3}$ **(b)** $-\frac{\sqrt{2}}{2}$

73. **(a)** -2 **(b)** -1

75. **(a)** 0 **(b)** $-\frac{\sqrt{2}}{2}$

77. **(a)** Undefined **(b)** $-\frac{1}{2}$

79. **(a)** $\frac{2\sqrt{3}}{3}$ **(b)** -1

81. **(a)** $-\frac{\sqrt{3}}{3}$ **(b)** $\sqrt{2}$

83. **(a)** 0.6018 **(b)** −0.7813

85. **(a)** −5.2408 **(b)** 2.6051

87. **(a)** 0.8090 **(b)** 1.0257

89. **(a)** −4.6373 **(b)** −1.0101

Odd-Numbered Answers to Exercise Set 4.4: Trigonometric Expressions and Identities

1. $\cos^2(\theta)+\sin^2(\theta)=1$

$$\frac{\cos^2(\theta)}{\cos^2(\theta)}+\frac{\sin^2(\theta)}{\cos^2(\theta)}=\frac{1}{\cos^2(\theta)}$$

$$1+\tan^2(\theta)=\sec^2(\theta)$$

3. $\sin(\theta)=-\frac{12}{13};\quad \tan(\theta)=-\frac{12}{5}$

5. $\sin(\theta)=-\frac{1}{9}\ (given)$ $\quad \csc(\theta)=-9$

$\cos(\theta)=-\frac{4\sqrt{5}}{9}$ $\quad \sec(\theta)=-\frac{9\sqrt{5}}{20}$

$\tan(\theta)=\frac{\sqrt{5}}{20}$ $\quad \cot(\theta)=4\sqrt{5}$

7. $\cot(\theta)=-\frac{\sqrt{65}}{4};\quad \sin(\theta)=-\frac{4}{9};$

9. $\csc(\theta)=\frac{7}{6};\quad \sin(\theta)=\frac{6}{7}$

11. $\sin(\theta)=-\frac{3}{5}$ $\quad \csc(\theta)=-\frac{5}{3}$

$\cos(\theta)=-\frac{4}{5}$ $\quad \sec(\theta)=-\frac{5}{4}$

$\tan(\theta)=\frac{3}{4}\ (given)$ $\quad \cot(\theta)=\frac{4}{3}$

13. $\sin^2(\theta)+2\sin(\theta)-15$

15. $\csc^2(x)-6\csc(x)+9$

17. $\frac{3\sin(\theta)-7\cos(\theta)}{\cos(\theta)\sin(\theta)}$

19. $-3\tan(\theta)\sin(\theta)$

21. $[5\sin(\theta)+7\cos(\theta)][5\sin(\theta)-7\cos(\theta)]$

23. $[\sec(\theta)-3][\sec(\theta)-4]$

25. $[5\cot(\theta)+1][2\cot(\theta)-3]$

27. $\sin(\theta)$

29. $\sec^2(x)$

31. $\sec(x)$

33. $\cos^2(x)$

35. $\tan^2(\theta)$

37. $\cos^2(\theta)$

39. 1

41. $\sec(x)$

43. 1

45. $\sin(x)$

47. $\tan(x)$

49. $2\tan(x)$

51. -1

Notes for 53-69: The symbol ∴ means, "therefore." Q.E.D is an abbreviation for the Latin term, "Quod Erat Demonstrandum"—Latin for "which was to be demonstrated"—and is frequently written when a proof is complete.

53. $\sec(x)-\sin(x)\tan(x)\overset{?}{=}\cos(x)$

Left-Hand Side	**Right-Hand Side**
$\sec(x)-\sin(x)\tan(x)$	$\cos(x)$
$=\frac{1}{\cos(x)}-\sin(x)\frac{\sin(x)}{\cos(x)}$	
$=\frac{1-\sin^2(x)}{\cos(x)}$	
$=\frac{\cos^2(x)}{\cos(x)}$	
$=\cos(x)$	

$\therefore\ \sec(x)-\sin(x)\tan(x)=\cos(x)$ Q.E.D.

55. $$\frac{\sec(\theta)\csc(\theta)}{\tan(\theta)+\cot(\theta)} \stackrel{?}{=} 1$$

Left-Hand Side	**Right-Hand Side**
$\frac{\sec(\theta)\csc(\theta)}{\tan(\theta)+\cot(\theta)}$	1
$=\frac{\frac{1}{\cos(\theta)}\cdot\frac{1}{\sin(\theta)}}{\frac{\sin(\theta)}{\cos(\theta)}+\frac{\cos(\theta)}{\sin(\theta)}}$	
$=\frac{\frac{1}{\cos(\theta)\sin(\theta)}}{\frac{\sin^2(\theta)+\cos^2(\theta)}{\cos(\theta)\sin(\theta)}}$	
$=\frac{\frac{1}{\cos(\theta)\sin(\theta)}}{\frac{1}{\cos(\theta)\sin(\theta)}}$	
$=\frac{1}{\cos(\theta)\sin(\theta)}\cdot\frac{\cos(\theta)\sin(\theta)}{1}$	
$=1$	

$$\therefore\ \frac{\sec(\theta)\csc(\theta)}{\tan(\theta)+\cot(\theta)} = 1 \quad \text{Q.E.D.}$$

57. $$\tan(x)-\sin(x)\cos(x) \stackrel{?}{=} \tan(x)\sin^2(x)$$

Left-Hand Side	**Right-Hand Side**
$\tan(x)-\sin(x)\cos(x)$	$\tan(x)\sin^2(x)$
$=\frac{\sin(x)}{\cos(x)}-\sin(x)\cos(x)$	
$=\frac{\sin(x)-\sin(x)\cos^2(x)}{\cos(x)}$	
$=\frac{\sin(x)\left[1-\cos^2(x)\right]}{\cos(x)}$	
$=\frac{\sin(x)\sin^2(x)}{\cos(x)}$	
$=\tan(x)\sin^2(x)$	

$$\therefore\ \tan(x)-\sin(x)\cos(x) = \tan(x)\sin^2(x)$$
Q.E.D.

59. $$\cot^2(x)-\cos^2(x) \stackrel{?}{=} \cot^2(x)\cos^2(x)$$

Left-Hand Side	**Right-Hand Side**
$\cot^2(x)-\cos^2(x)$	$\cot^2(x)\cos^2(x)$
$=\frac{\cos^2(x)}{\sin^2(x)}-\cos^2(x)$	
$=\frac{\cos^2(x)-\cos^2(x)\sin^2(x)}{\sin^2(x)}$	
$=\frac{\cos^2(x)\left[1-\sin^2(x)\right]}{\sin^2(x)}$	
$=\frac{\cos^2(x)\cos^2(x)}{\sin^2(x)}$	
$=\cot^2(x)\cos^2(x)$	

$$\therefore\ \cot^2(x)-\cos^2(x)=\cot^2(x)\cos^2(x) \quad \text{Q.E.D.}$$

61. $$\frac{\cot(x)-\tan(x)}{\sin(x)\cos(x)} \stackrel{?}{=} \csc^2(x)-\sec^2(x)$$

Left-Hand Side	**Right-Hand Side**
$\frac{\cot(x)-\tan(x)}{\sin(x)\cos(x)}$	$\csc^2(x)-\sec^2(x)$
$=\frac{\frac{\cos(x)}{\sin(x)}-\frac{\sin(x)}{\cos(x)}}{\sin(x)\cos(x)}$	
$=\frac{\frac{\cos^2(x)-\sin^2(x)}{\sin(x)\cos(x)}}{\sin(x)\cos(x)}$	
$=\frac{\cos^2(x)-\sin^2(x)}{\sin(x)\cos(x)}\cdot\frac{1}{\sin(x)\cos(x)}$	
$=\frac{\cos^2(x)-\sin^2(x)}{\sin^2(x)\cos^2(x)}$	
$=\frac{\cos^2(x)}{\sin^2(x)\cos^2(x)}-\frac{\sin^2(x)}{\sin^2(x)\cos^2(x)}$	
$=\frac{1}{\sin^2(x)}-\frac{1}{\cos^2(x)}$	
$=\csc^2(x)-\sec^2(x)$	

$$\therefore\ \frac{\cot(x)-\tan(x)}{\sin(x)\cos(x)}=\csc^2(x)-\sec^2(x)$$
Q.E.D.

Odd-Numbered Answers to Exercise Set 4.4: Trigonometric Expressions and Identities

63. $$2\cos^2(x)+5\sin^2(x) \stackrel{?}{=} 2+3\sin^2(x)$$

Left-Hand Side	**Right-Hand Side**
$2\cos^2(x)+5\sin^2(x)$	$2+3\sin^2(x)$
$=2\cos^2(x)+2\sin^2(x)+3\sin^2(x)$	
$=2\left[\cos^2(x)+\sin^2(x)\right]+3\sin^2(x)$	
$=2(1)+3\sin^2(x)$	
$=2+3\sin^2(x)$	

$$\therefore\ 2\cos^2(x)+5\sin^2(x) = 2+3\sin^2(x)$$

Q.E.D.

65. $$\tan^4(\theta)+\tan^2(\theta) \stackrel{?}{=} \sec^4(\theta)-\sec^2(\theta)$$

Left-Hand Side	**Right-Hand Side**
$\tan^4(\theta)+\tan^2(\theta)$	$\sec^4(\theta)-\sec^2(\theta)$
$=\tan^2(\theta)\left[\tan^2(\theta)+1\right]$	
$=\tan^2(\theta)\sec^2(\theta)$	
$=\left[\sec^2(\theta)-1\right]\sec^2(\theta)$	
$=\sec^4(\theta)-\sec^2(\theta)$	

$$\therefore\ \tan^4(\theta)+\tan^2(\theta) = \sec^4(\theta)-\sec^2(\theta)$$

Q.E.D.

67. Two methods of proof are shown. (Either method is sufficient.)

Method 1: Work with the left-hand side and show that it is equal to the right-hand side.

$$\frac{\sin(x)}{1+\cos(x)} \stackrel{?}{=} \csc(x)-\cot(x)$$

Left-Hand Side	**Right-Hand Side**
$\dfrac{\sin(x)}{1+\cos(x)}$	$\csc(x)-\cot(x)$
$=\dfrac{\sin(x)}{1+\cos(x)}\cdot\dfrac{1-\cos(x)}{1-\cos(x)}$	
$=\dfrac{\sin(x)\left[1-\cos(x)\right]}{1-\cos^2(x)}$	
$=\dfrac{\sin(x)\left[1-\cos(x)\right]}{\sin^2(x)}$	
$=\dfrac{1-\cos(x)}{\sin(x)}$	
$=\dfrac{1}{\sin(x)}-\dfrac{\cos(x)}{\sin(x)}$	
$=\csc(x)-\cot(x)$	

$$\therefore\ \frac{\sin(x)}{1+\cos(x)}=\csc(x)-\cot(x)$$ Q.E.D.

Continued on the next page…

Method 2: Work with the right-hand side and show that it is equal to the left-hand side.

$$\frac{\sin(x)}{1+\cos(x)} \stackrel{?}{=} \csc(x)-\cot(x)$$

Left-Hand Side	**Right-Hand Side**
$\frac{\sin(x)}{1+\cos(x)}$	$\csc(x)-\cot(x)$
	$=\frac{1}{\sin(x)}-\frac{\cos(x)}{\sin(x)}$
	$=\frac{1-\cos(x)}{\sin(x)}$
	$=\frac{[1-\cos(x)]}{\sin(x)}\cdot\frac{[1+\cos(x)]}{[1+\cos(x)]}$
	$=\frac{1-\cos^2(x)}{\sin(x)[1+\cos(x)]}$
	$=\frac{\sin^2(x)}{\sin(x)[1+\cos(x)]}$
	$=\frac{\sin(x)}{1+\cos(x)}$

$$\therefore\ \frac{\sin(x)}{1+\cos(x)}=\csc(x)-\cot(x) \quad \text{Q.E.D.}$$

69. Two methods of proof are shown. (Either method is sufficient.)

Method 1: Work with the left-hand side and show that it is equal to the right-hand side.

$$\frac{\cos(x)}{1+\sin(x)} \stackrel{?}{=} \frac{1-\sin(x)}{\cos(x)}$$

Left-Hand Side	**Right-Hand Side**
$\frac{\cos(x)}{1+\sin(x)}$	$\frac{1-\sin(x)}{\cos(x)}$
$=\frac{\cos(x)}{1+\sin(x)}\cdot\frac{1-\sin(x)}{1-\sin(x)}$	
$=\frac{\cos(x)[1-\sin(x)]}{1-\sin^2(x)}$	
$=\frac{\cos(x)[1-\sin(x)]}{\cos^2(x)}$	
$=\frac{1-\sin(x)}{\cos(x)}$	

$$\therefore\ \frac{\cos(x)}{1+\sin(x)} = \frac{1-\sin(x)}{\cos(x)} \quad \text{Q.E.D.}$$

Continued on the next page…

Method 2: Work with the right-hand side and show that it is equal to the left-hand side.

$$\frac{\cos(x)}{1+\sin(x)} \stackrel{?}{=} \frac{1-\sin(x)}{\cos(x)}$$

Left-Hand Side

$$\frac{\cos(x)}{1+\sin(x)}$$

Right-Hand Side

$$\frac{1-\sin(x)}{\cos(x)}$$

$$=\frac{[1-\sin(x)]}{\cos(x)}\cdot\frac{[1+\sin(x)]}{[1+\sin(x)]}$$

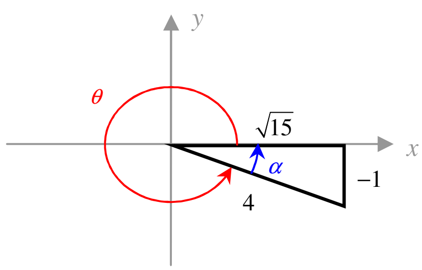
$$=\frac{1-\sin^2(x)}{\cos(x)[1+\sin(x)]}$$

$$=\frac{\cos^2(x)}{\cos(x)[1+\sin(x)]}$$

$$=\frac{\cos(x)}{1+\sin(x)}$$

$$\therefore \quad \frac{\cos(x)}{1+\sin(x)} = \frac{1-\sin(x)}{\cos(x)} \qquad \text{Q.E.D.}$$

71.



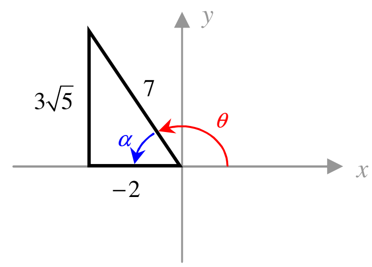
Note: The triangle may not be drawn to scale.

$\sin(\theta) = -\frac{1}{4}$ (*given*) $\qquad \csc(\theta) = -4$

$\cos(\theta) = \frac{\sqrt{15}}{4} \qquad \sec(\theta) = \frac{4\sqrt{15}}{15}$

$\tan(\theta) = -\frac{\sqrt{15}}{15} \qquad \cot(\theta) = -\sqrt{15}$

73.



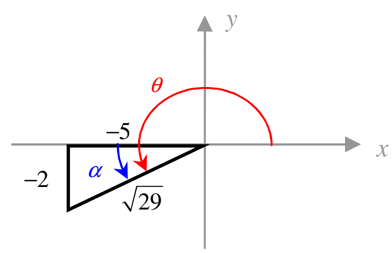
Note: The triangle may not be drawn to scale.

$\sin(\theta) = \frac{3\sqrt{5}}{7} \qquad \csc(\theta) = \frac{7\sqrt{5}}{15}$

$\cos(\theta) = -\frac{2}{7} \qquad \sec(\theta) = -\frac{7}{2}$ (*given*)

$\tan(\theta) = -\frac{3\sqrt{5}}{2} \qquad \cot(\theta) = -\frac{2\sqrt{5}}{15}$

75.



Note: The triangle may not be drawn to scale.

$\sin(\theta) = -\frac{2\sqrt{29}}{29} \qquad \csc(\theta) = -\frac{\sqrt{29}}{2}$

$\cos(\theta) = -\frac{5\sqrt{29}}{29} \qquad \sec(\theta) = -\frac{\sqrt{29}}{5}$

$\tan(\theta) = \frac{2}{5}$ (*given*) $\qquad \cot(\theta) = \frac{5}{2}$

77. $\sin(\theta) = -\frac{5}{13} \qquad \csc(\theta) = -\frac{13}{5}$

$\cos(\theta) = \frac{12}{13} \qquad \sec(\theta) = \frac{13}{12}$

$\tan(\theta) = \frac{-5}{12} \qquad \cot(\theta) = -\frac{12}{5}$

Odd-Numbered Answers to Exercise Set 4.4: Trigonometric Expressions and Identities

79. $\sin(\theta) = -\dfrac{\sqrt{10}}{10}$ $\qquad$ $\csc(\theta) = -\sqrt{10}$

$\cos(\theta) = -\dfrac{3\sqrt{10}}{10}$ $\qquad$ $\sec(\theta) = -\dfrac{\sqrt{10}}{3}$

$\tan(\theta) = \dfrac{1}{3}$ $\qquad$ $\cot(\theta) = 3$

Odd-Numbered Answers to Exercise Set 5.1: Trigonometric Functions of Real Numbers

1. $\sin(t) = -\frac{2}{7}$ $(given)$ $\quad \csc(t) = -\frac{7}{2}$

$\cos(t) = \frac{3\sqrt{5}}{7}$ $\quad \sec(t) = \frac{7\sqrt{5}}{15}$

$\tan(t) = -\frac{2\sqrt{5}}{15}$ $\quad \cot(t) = -\frac{3\sqrt{5}}{2}$

3. $\sin(t) = \frac{\sqrt{15}}{4}$ $\quad \csc(t) = \frac{4\sqrt{15}}{15}$

$\cos(t) = -\frac{1}{4}$ $\quad \sec(t) = -4$ $(given)$

$\tan(t) = -\sqrt{15}$ $\quad \cot(t) = -\frac{\sqrt{15}}{15}$

5. $\sin(t) = -\frac{3}{5}$ $\quad \csc(t) = -\frac{5}{3}$

$\cos(t) = -\frac{4}{5}$ $\quad \sec(t) = -\frac{5}{4}$

$\tan(t) = \frac{3}{4}$ $\quad \cot(t) = \frac{4}{3}$ $(given)$

7. $\sin(t) = -\frac{2\sqrt{10}}{7}$ $\quad \csc(t) = -\frac{7\sqrt{10}}{20}$

$\cos(t) = \frac{3}{7}$ $\quad \sec(t) = \frac{7}{3}$

$\tan(t) = -\frac{2\sqrt{10}}{3}$ $(given)$ $\quad \cot(t) = -\frac{3\sqrt{10}}{20}$

9. **(a)** $\frac{\sqrt{2}}{2}$ **(b)** -2

11. **(a)** $-\frac{\sqrt{3}}{2}$ **(b)** Undefined

13. **(a)** -2 **(b)** $\sqrt{3}$

15. **(a)** $\frac{\sqrt{2}}{2}$ **(b)** $-\frac{\sqrt{3}}{3}$

17. **(a)** 0 **(b)** 2

19. **(a)** Undefined **(b)** $\frac{1}{2}$

21. **(a)** $\frac{\sqrt{2}}{2}$ **(b)** $\frac{3}{4}$

23. **(a)** 1 **(b)** $-\frac{\sqrt{3}}{4}$

25. -1

27. $-\csc(t)$

29. $-\sec(t)$

31. $\sec(t)$

33. $-\csc(t)$

35. 1

37. $\tan(t)$

Continued on the next page…

Odd-Numbered Answers to Exercise Set 5.1: Trigonometric Functions of Real Numbers

Notes for 39-45: The symbol ∴ means, "therefore." Q.E.D is an abbreviation for the Latin term, "Quod Erat Demonstrandum"—Latin for "which was to be demonstrated"—and is frequently written when a proof is complete.

39. Two methods of proof are shown. (Either method is sufficient.)

Method 1: Work with the left-hand side and show that it is equal to the right-hand side.

$$\frac{\cos(-t)}{1-\sin(-t)} \overset{?}{=} \sec(-t)+\tan(-t)$$

Left-Hand Side	Right-Hand Side
$\frac{\cos(-t)}{1-\sin(-t)}$	$\sec(-t)+\tan(-t)$
$=\frac{\cos(t)}{1+\sin(t)}$	
$=\frac{\cos(t)}{1+\sin(t)}\cdot\frac{1-\sin(t)}{1-\sin(t)}$	
$=\frac{\cos(t)\left[1-\sin(t)\right]}{1-\sin^2(t)}$	
$=\frac{\cos(t)\left[1-\sin(t)\right]}{\cos^2(t)}$	
$=\frac{1-\sin(t)}{\cos(t)}$	
$=\frac{1}{\cos(t)}-\frac{\sin(t)}{\cos(t)}$	
$=\sec(t)-\tan(t)$	
$=\sec(-t)+\tan(-t)$	

$$\therefore \frac{\cos(-t)}{1-\sin(-t)} = \sec(-t)+\tan(-t)$$

Q.E.D.

Method 2: Work with the right-hand side and show that it is equal to the left-hand side.

$$\frac{\cos(-t)}{1-\sin(-t)} \overset{?}{=} \sec(-t)+\tan(-t)$$

Left-Hand Side	Right-Hand Side
$\frac{\cos(-t)}{1-\sin(-t)}$	$\sec(-t)+\tan(-t)$
	$=\sec(t)-\tan(t)$
	$=\frac{1}{\cos(t)}-\frac{\sin(t)}{\cos(t)}$
	$=\frac{1-\sin(t)}{\cos(t)}$
	$=\frac{1-\sin(t)}{\cos(t)}\cdot\frac{1+\sin(t)}{1+\sin(t)}$
	$=\frac{1-\sin^2(t)}{\cos(t)\left[1+\sin(t)\right]}$
	$=\frac{\cos^2(t)}{\cos(t)\left[1+\sin(t)\right]}$
	$=\frac{\cos(t)}{1+\sin(t)}$
	$=\frac{\cos(-t)}{1-\sin(-t)}$

$$\therefore \frac{\cos(-t)}{1-\sin(-t)} = \sec(-t)+\tan(-t)$$

Q.E.D.

Continued on the next page…

Odd-Numbered Answers to Exercise Set 5.1: Trigonometric Functions of Real Numbers

41. Two methods of proof are shown. (Either method is sufficient.)

Method 1: Work with the left-hand side and show that it is equal to the right-hand side.

$$\frac{\sin(t)}{\cos(-t)-1} \stackrel{?}{=} \frac{1+\cos(-t)}{\sin(-t)}$$

Left-Hand Side	**Right-Hand Side**
$\dfrac{\sin(t)}{\cos(-t)-1}$	$\dfrac{1+\cos(-t)}{\sin(-t)}$
$=\dfrac{\sin(t)}{\cos(t)-1}$	
$=\dfrac{\sin(t)}{[\cos(t)-1]}\cdot\dfrac{[\cos(t)+1]}{[\cos(t)+1]}$	
$=\dfrac{\sin(t)[\cos(t)+1]}{\cos^2(t)-1}$	
$=\dfrac{-\sin(t)[\cos(t)+1]}{1-\cos^2(t)}$	
$=\dfrac{-\sin(t)[\cos(t)+1]}{\sin^2(t)}$	
$=\dfrac{\cos(t)+1}{-\sin(t)}$	
$=\dfrac{1+\cos(-t)}{\sin(-t)}$	

$$\therefore\quad \frac{\sin(t)}{\cos(-t)-1} = \frac{1+\cos(-t)}{\sin(-t)} \qquad \text{Q.E.D.}$$

Method 2: Work with the right-hand side and show that it is equal to the left-hand side.

$$\frac{\sin(t)}{\cos(-t)-1} \stackrel{?}{=} \frac{1+\cos(-t)}{\sin(-t)}$$

Left-Hand Side	**Right-Hand Side**
$\dfrac{\sin(t)}{\cos(-t)-1}$	$\dfrac{1+\cos(-t)}{\sin(-t)}$
	$=\dfrac{1+\cos(t)}{-\sin(t)}$
	$=\dfrac{[1+\cos(t)]}{-\sin(t)}\cdot\dfrac{[1-\cos(t)]}{[1-\cos(t)]}$
	$=\dfrac{1-\cos^2(t)}{-\sin(t)[1-\cos(t)]}$
	$=\dfrac{\sin^2(t)}{-\sin(t)[1-\cos(t)]}$
	$=\dfrac{-\sin(t)}{1-\cos(t)}$
	$=\dfrac{\sin(t)}{\cos(t)-1}$
	$=\dfrac{\sin(t)}{\cos(-t)-1}$

$$\therefore\quad \frac{\sin(t)}{\cos(-t)-1} = \frac{1+\cos(-t)}{\sin(-t)} \qquad \text{Q.E.D.}$$

Continued on the next page…

Odd-Numbered Answers to Exercise Set 5.1: Trigonometric Functions of Real Numbers

43. $$\frac{1+\sec(-t)}{\csc(-t)} \stackrel{?}{=} \sin(-t)+\tan(-t)$$

Left-Hand Side	Right-Hand Side
$\frac{1+\sec(-t)}{\csc(-t)}$	$\sin(-t)+\tan(-t)$
$=\frac{1+\sec(t)}{-\csc(t)}$	
$=\frac{1+\frac{1}{\cos(t)}}{-\frac{1}{\sin(t)}}$	
$=\frac{\frac{\cos(t)+1}{\cos(t)}}{-\frac{1}{\sin(t)}}$	
$=\frac{\cos(t)+1}{\cos(t)}\cdot\frac{[-\sin(t)]}{1}$	
$=\frac{-\sin(t)\cos(t)-\sin(t)}{\cos(t)}$	
$=\frac{-\sin(t)\cos(t)}{\cos(t)}-\frac{\sin(t)}{\cos(t)}$	
$=-\sin(t)-\tan(t)$	
$=-\sin(-t)+\tan(-t)$	

$$\therefore \frac{1+\sec(-t)}{\csc(-t)} = \sin(-t)+\tan(-t)$$

Q.E.D.

45. $$\frac{\sin(t-4\pi)}{1+\cos(t+2\pi)}+\frac{1+\cos(t-2\pi)}{\sin(t+6\pi)} \stackrel{?}{=} 2\csc(t+4\pi)$$

Left-Hand Side	Right-Hand Side
$\frac{\sin(t-4\pi)}{1+\cos(t+2\pi)}+\frac{1+\cos(t-2\pi)}{\sin(t+6\pi)}$	$2\csc(t+4\pi)$
$=\frac{\sin(t)}{1+\cos(t)}+\frac{1+\cos(t)}{\sin(t)}$	
$=\frac{\sin^2(t)+[1+\cos(t)]^2}{\sin(t)[1+\cos(t)]}$	
$=\frac{\sin^2(t)+1+2\cos(t)+\cos^2(t)}{\sin(t)[1+\cos(t)]}$	
$=\frac{1+[\sin^2(t)+\cos^2(t)]+2\cos(t)}{\sin(t)[1+\cos(t)]}$	
$=\frac{1+1+2\cos(t)}{\sin(t)[1+\cos(t)]}$	
$=\frac{2+2\cos(t)}{\sin(t)[1+\cos(t)]}$	
$=\frac{2[1+\cos(t)]}{\sin(t)[1+\cos(t)]}$	
$=\frac{2}{\sin(t)}$	
$=2\csc(t)$	
$=2\csc(t+4\pi)$	

$$\therefore \frac{\sin(t-4\pi)}{1+\cos(t+2\pi)}+\frac{1+\cos(t-2\pi)}{\sin(t+6\pi)}=2\csc(t+4\pi)$$

Q.E.D.

Odd-Numbered Answers to Exercise Set 5.2: Graphs of the Sine and Cosine Functions

1. **(a)** Period: 6
(b) Amplitude: 2

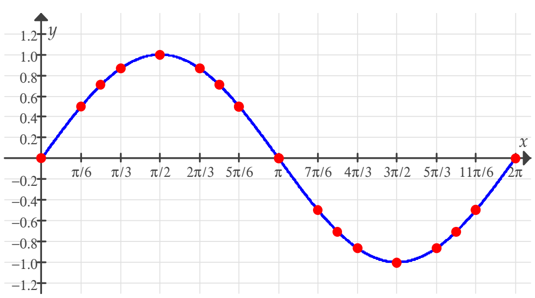
3. **(a)** Period: 2
(b) Amplitude: 5

5. **(a)** Period: π
(b) Amplitude: 4

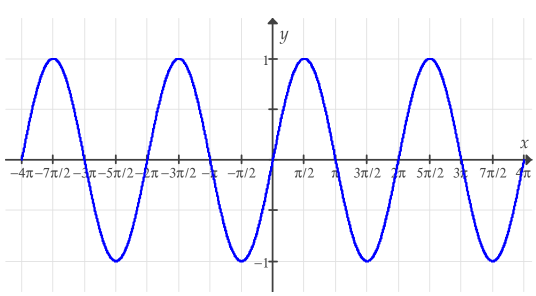
7. **(a)**

x	$y=\sin(x)$	x	$y=\sin(x)$
0	$\mathbf{0}$	π	$\mathbf{0}$
$\frac{\pi}{6}$	$\frac{1}{2}=0.5$	$\frac{7\pi}{6}$	$-\frac{1}{2}=-0.5$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}\approx 0.71$	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}\approx -0.71$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}\approx 0.87$	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}\approx -0.87$
$\frac{\pi}{2}$	1	$\frac{3\pi}{2}$	-1
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}\approx 0.87$	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}\approx -0.87$
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}\approx 0.71$	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}\approx -0.71$
$\frac{5\pi}{6}$	$\frac{1}{2}=0.5$	$\frac{11\pi}{6}$	$-\frac{1}{2}=-0.5$
		2π	0

(b)



(c)



(d) Domain: $(-\infty,\infty)$
Range: $[-1,1]$

(e) Amplitude: 1
Period: 2π

(f) Answers vary. Following are some intervals on which $f(x)=\sin(x)$ is increasing. (Any one interval is sufficient.)

$$\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \quad \left(\frac{3\pi}{2},\frac{5\pi}{2}\right) \quad \left(\frac{7\pi}{2},\frac{9\pi}{2}\right)$$

$$\left(-\frac{5\pi}{2},-\frac{3\pi}{2}\right) \quad \left(-\frac{9\pi}{2},-\frac{7\pi}{2}\right) \quad \text{etc...}$$

Note: Some textbooks indicate closed intervals rather than open intervals for the solutions above, e.g. $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, *etc.*

9. $A\left(-\frac{3\pi}{2},0\right)$; $B(\pi,-1)$; $C(2\pi,1)$; $D\left(\frac{5\pi}{2},0\right)$

11. **(a)** 1
(b) 0
(c) 1
(d) 0

13. **(a)** $x=\frac{\pi}{2}$

(b) $x=0,\ x=\pi$

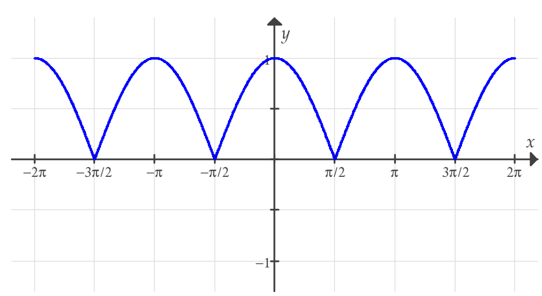
(c) $x=\frac{3\pi}{2}$

15. **(a)** $x=-\pi$

(b) $x=-2\pi$

(c) $x=-\frac{\pi}{2},\quad x=-\frac{3\pi}{2}$

17.



Odd-Numbered Answers to Exercise Set 5.2: Graphs of the Sine and Cosine Functions

19. D

21. A

23. K

25. B

27. H

29. C

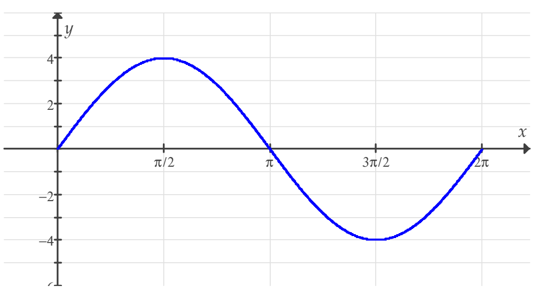
31. F

33. C

35. A

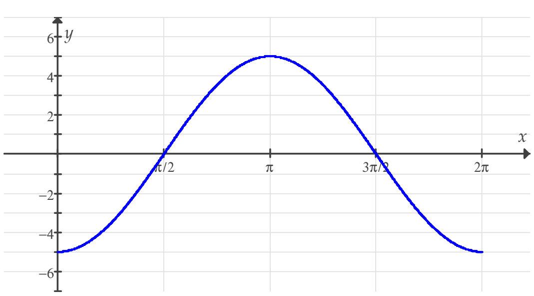
37.
(a) Period: 2π
(b) Amplitude: 4
(c) Phase Shift: 0
(d) Vertical Shift: 0

(e)



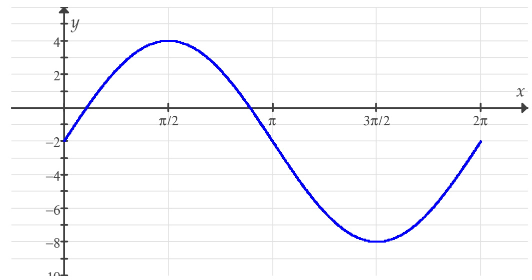
39.
(a) Period: 2π
(b) Amplitude: 5
(c) Phase Shift: 0
(d) Vertical Shift: 0

(e)



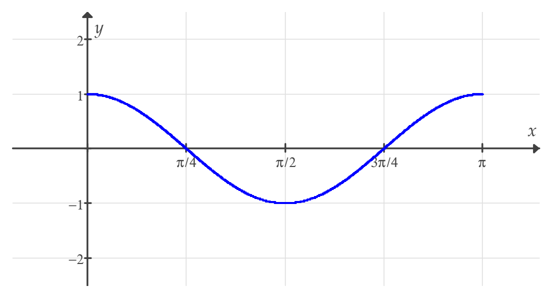
41.
(a) Period: 2π
(b) Amplitude: 6
(c) Phase Shift: 0
(d) Vertical Shift: -2

(e)



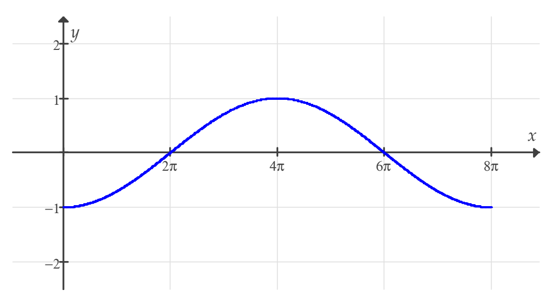
43.
(a) Period: π
(b) Amplitude: 1
(c) Phase Shift: 0
(d) Vertical Shift: 0

(e)



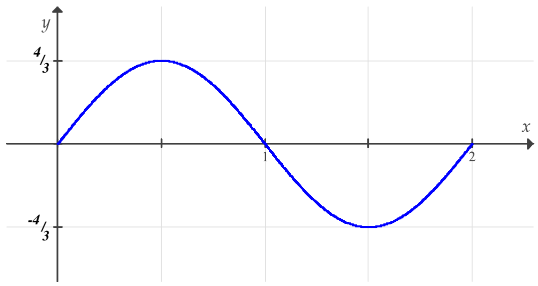
45.
(a) Period: 8π
(b) Amplitude: 1
(c) Phase Shift: 0
(d) Vertical Shift: 0

(e)

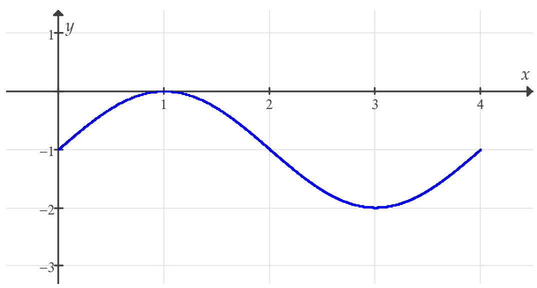


Odd-Numbered Answers to Exercise Set 5.2: Graphs of the Sine and Cosine Functions

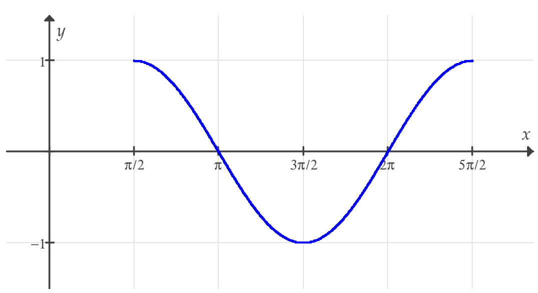
47. **(a)** Period: 2
(b) Amplitude: $\frac{4}{3}$
(c) Phase Shift: 0
(d) Vertical Shift: 0
(e)



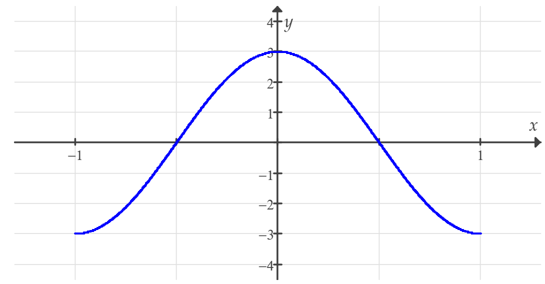
49. **(a)** Period: 4
(b) Amplitude: 1
(c) Phase Shift: 0
(d) Vertical Shift: −1
(e)



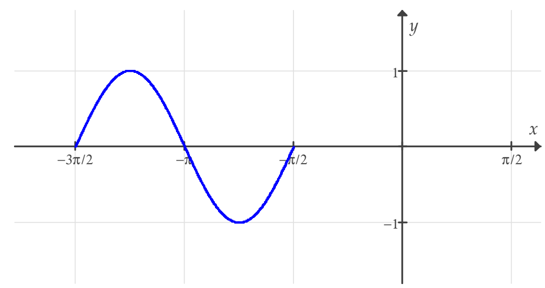
51. **(a)** Period: 2π
(b) Amplitude: 1
(c) Phase Shift: $\frac{\pi}{2}$
(d) Vertical Shift: 0
(e)



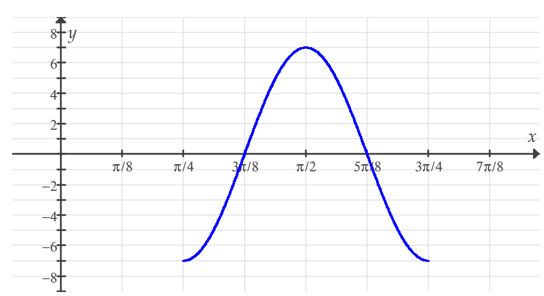
53. **(a)** Period: 2
(b) Amplitude: 3
(c) Phase Shift: −1
(d) Vertical Shift: 0
(e)



55. **(a)** Period: π
(b) Amplitude: 1
(c) Phase Shift: $-\frac{3\pi}{2}$
(d) Vertical Shift: 0
(e)

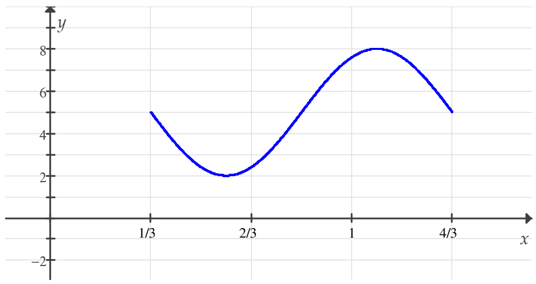


57. **(a)** Period: $\frac{\pi}{2}$
(b) Amplitude: 7
(c) Phase Shift: $\frac{\pi}{4}$
(d) Vertical Shift: 0
(e)

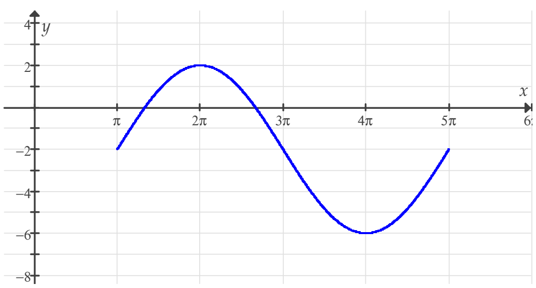


Odd-Numbered Answers to Exercise Set 5.2: Graphs of the Sine and Cosine Functions

59. **(a)** Period: 1
(b) Amplitude: 3
(c) Phase Shift: $\frac{1}{3}$
(d) Vertical Shift: 5
(e)



61. **(a)** Period: 4π
(b) Amplitude: 4
(c) Phase Shift: π
(d) Vertical Shift: -2
(e)

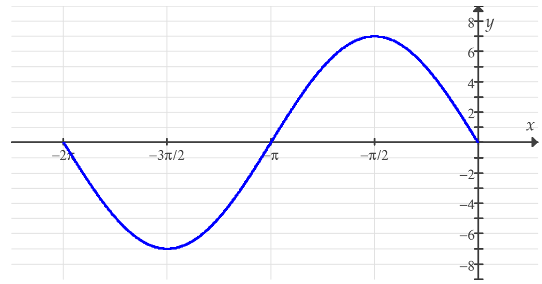


Notes for 63-67: For the equations $f(x) = A\sin(Bx - C) + D$ and $f(x) = A\cos(Bx - C) + D$:

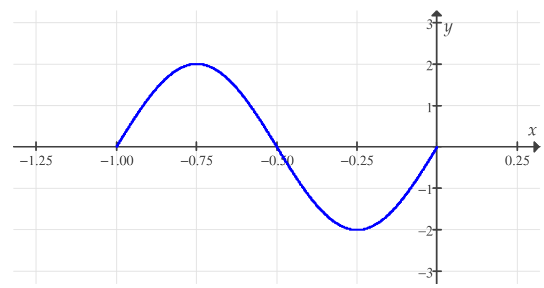
* **The period of the function is $\frac{2\pi}{|B|}$ when $B < 0$.**

** **The text only defines phase shift for $B > 0$. Therefore, the phase shift portion of the exercises has been omitted.**

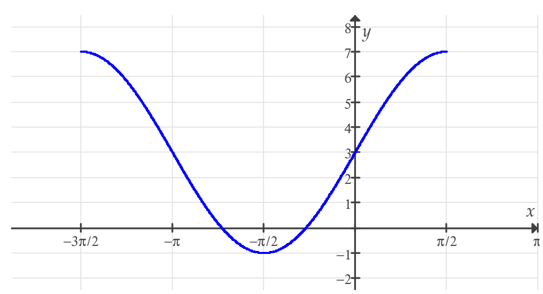
63. **(a)** Period: 2π (*See note in left column)
(b) Amplitude: 7
(c) Phase Shift: Omit (**See note in left column)
(d) Vertical Shift: 0
(e)



65. **(a)** Period: 1 (*See note in left column)
(b) Amplitude: 2
(c) Phase Shift: Omit (**See note in left column)
(d) Vertical Shift: 0
(e)



67. **(a)** Period: 2π (*See note in left column)
(b) Amplitude: 4
(c) Phase Shift: Omit (**See note in left column)
(d) Vertical Shift: 3
(e)



Odd-Numbered Answers to Exercise Set 5.2: Graphs of the Sine and Cosine Functions

69. (a) Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = 4\sin(2x)$$

$$f(x) = -4\sin\left[2\left(x-\frac{\pi}{2}\right)\right] = -4\sin(2x-\pi)$$

$$f(x) = -4\sin\left[2\left(x+\frac{\pi}{2}\right)\right] = -4\sin(2x+\pi)$$

$$f(x) = 4\sin\left[2(x-\pi)\right] = 4\sin(2x-2\pi)$$

(b) Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = 4\cos\left[2\left(x-\frac{\pi}{4}\right)\right] = 4\cos\left(2x-\frac{\pi}{2}\right)$$

$$f(x) = 4\cos\left[2\left(x+\frac{3\pi}{4}\right)\right] = 4\cos\left(2x+\frac{3\pi}{2}\right)$$

$$f(x) = -4\cos\left[2\left(x+\frac{\pi}{4}\right)\right] = -4\cos\left(2x+\frac{\pi}{2}\right)$$

$$f(x) = -4\cos\left[2\left(x-\frac{3\pi}{4}\right)\right] = -4\cos\left(2x-\frac{3\pi}{2}\right)$$

71. (a) Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = 3\sin\left[\frac{\pi}{2}(x-1)\right] = 3\sin\left(\frac{\pi}{2}x-\frac{\pi}{2}\right)$$

$$f(x) = 3\sin\left[\frac{\pi}{2}(x+3)\right] = 3\sin\left(\frac{\pi}{2}x+\frac{3\pi}{2}\right)$$

$$f(x) = 3\sin\left[\frac{\pi}{2}(x-5)\right] = 3\sin\left(\frac{\pi}{2}x-\frac{5\pi}{2}\right)$$

$$f(x) = -3\sin\left[\frac{\pi}{2}(x+1)\right] = -3\sin\left(\frac{\pi}{2}x+\frac{\pi}{2}\right)$$

$$f(x) = -3\sin\left[\frac{\pi}{2}(x-3)\right] = -3\sin\left(\frac{\pi}{2}x-\frac{3\pi}{2}\right)$$

$$f(x) = -3\sin\left[\frac{\pi}{2}(x+5)\right] = -3\sin\left(\frac{\pi}{2}x+\frac{5\pi}{2}\right)$$

Continued in the next column…

(b) Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = -3\cos\left(\frac{\pi}{2}x\right)$$

$$f(x) = 3\cos\left[\frac{\pi}{2}(x-2)\right] = 3\cos\left(\frac{\pi}{2}x-\pi\right)$$

$$f(x) = -3\cos\left[\frac{\pi}{2}(x-4)\right] = -3\cos\left[\frac{\pi}{2}x-2\pi\right]$$

$$f(x) = 3\cos\left[\frac{\pi}{2}(x+2)\right] = 3\cos\left(\frac{\pi}{2}x+\pi\right)$$

$$f(x) = -3\cos\left[\frac{\pi}{2}(x+4)\right] = -3\cos\left[\frac{\pi}{2}x+2\pi\right]$$

73. (a) Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = -5\sin(4x)-2$$

$$f(x) = -5\sin\left[4\left(x-\frac{\pi}{2}\right)\right]-2 = -5\sin(4x-2\pi)-2$$

$$f(x) = 5\sin\left[4\left(x-\frac{\pi}{4}\right)\right]-2 = 5\sin(4x-\pi)-2$$

$$f(x) = 5\sin\left[4\left(x+\frac{\pi}{4}\right)\right]-2 = 5\sin(4x+\pi)-2$$

(b) Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = 5\cos\left[4\left(x+\frac{\pi}{8}\right)\right]-2 = 5\cos\left(4x+\frac{\pi}{2}\right)-2$$

$$f(x) = 5\cos\left[4\left(x-\frac{3\pi}{8}\right)\right]-2 = 5\cos\left(4x-\frac{3\pi}{2}\right)-2$$

$$f(x) = -5\cos\left[4\left(x-\frac{\pi}{8}\right)\right]-2 = -5\cos\left(4x-\frac{\pi}{2}\right)-2$$

$$f(x) = -5\cos\left[4\left(x+\frac{3\pi}{8}\right)\right]-2 = -5\cos\left(4x+\frac{3\pi}{2}\right)-2$$

$$f(x) = -5\cos\left[4\left(x-\frac{5\pi}{8}\right)\right]-2 = -5\cos\left(4x-\frac{5\pi}{2}\right)-2$$

Continued on the next page…

Odd-Numbered Answers to Exercise Set 5.2: Graphs of the Sine and Cosine Functions

75. **(a)** Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = -2\sin\left[\frac{\pi}{4}(x+1)\right]+3 = -2\sin\left[\frac{\pi}{4}x+\frac{\pi}{4}\right]+3$$

$$f(x) = 2\sin\left[\frac{\pi}{4}(x-3)\right]+3 = 2\sin\left[\frac{\pi}{4}x-\frac{3\pi}{4}\right]+3$$

$$f(x) = 2\sin\left[\frac{\pi}{4}(x+5)\right]+3 = 2\sin\left[\frac{\pi}{4}x+\frac{5\pi}{4}\right]+3$$

(b) Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = 2\cos\left[\frac{\pi}{4}(x+3)\right]+3 = 2\cos\left[\frac{\pi}{4}x+\frac{3\pi}{4}\right]+3$$

$$f(x) = 2\cos\left[\frac{\pi}{4}(x-5)\right]+3 = 2\cos\left[\frac{\pi}{4}x-\frac{5\pi}{4}\right]+3$$

$$f(x) = -2\cos\left[\frac{\pi}{4}(x-1)\right]+3 = -2\cos\left[\frac{\pi}{4}x-\frac{\pi}{4}\right]+3$$

77. **(a)** $D(t) = 2\cos\left[\frac{2\pi}{13}(t-2)\right]+4 = 2\cos\left(\frac{2\pi t}{13}-\frac{4\pi}{13}\right)+4$

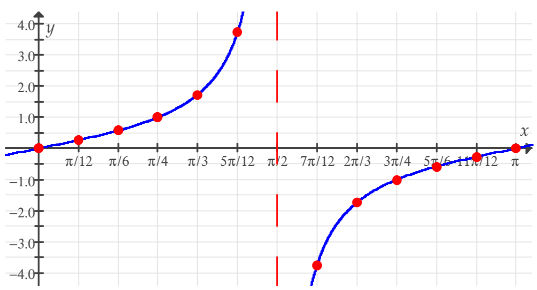
(b) 3.52 meters

Odd-Numbered Answers to Exercise Set 5.3:
Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

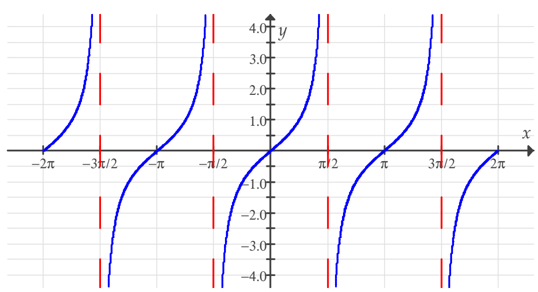
1. (a)

x	$y=\tan(x)$	x	$y=\tan(x)$
0	**0**	$\frac{\pi}{2}$	**Undefined**
$\frac{\pi}{12}$	**0.27**	$\frac{7\pi}{12}$	**–3.73**
$\frac{\pi}{6}$	**0.58**	$\frac{2\pi}{3}$	**–1.73**
$\frac{\pi}{4}$	**1**	$\frac{3\pi}{4}$	**–1**
$\frac{\pi}{3}$	**1.73**	$\frac{5\pi}{6}$	**–0.58**
$\frac{5\pi}{12}$	**3.73**	$\frac{11\pi}{12}$	**–0.27**
		π	**0**

(b)



(c)



(d) Domain: $\left\{x \,\middle|\, x \neq \pm\frac{\pi}{2},\ \pm\frac{3\pi}{2},\ \pm\frac{5\pi}{2},\ldots\right\}$

Range: $(-\infty,\infty)$

(e) Period: π

Continued in the next column…

(f) Answers vary. Following are some intervals on which $f(x)=\tan(x)$ is increasing. (Any one interval is sufficient.)

$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ $\left(\frac{3\pi}{2},\frac{5\pi}{2}\right)$

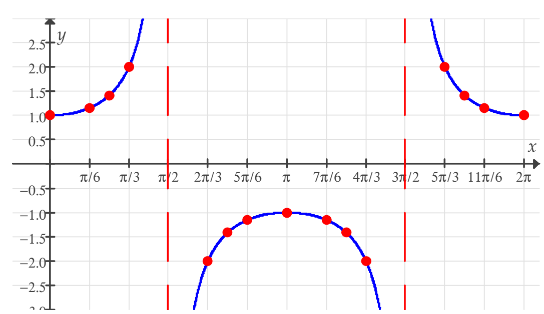
$\left(-\frac{3\pi}{2},-\frac{\pi}{2}\right)$ $\left(-\frac{5\pi}{2},-\frac{3\pi}{2}\right)$ etc…

There are no intervals on which $f(x)=\tan(x)$ is decreasing.

3. (a)

x	$y=\sec(x)$	x	$y=\sec(x)$
0	**1**	π	**–1**
$\frac{\pi}{6}$	**1.15**	$\frac{7\pi}{6}$	**–1.15**
$\frac{\pi}{4}$	**1.41**	$\frac{5\pi}{4}$	**–1.41**
$\frac{\pi}{3}$	**2**	$\frac{4\pi}{3}$	**–2**
$\frac{\pi}{2}$	**Undefined**	$\frac{3\pi}{2}$	**Undefined**
$\frac{2\pi}{3}$	**–2**	$\frac{5\pi}{3}$	**2**
$\frac{3\pi}{4}$	**–1.41**	$\frac{7\pi}{4}$	**1.41**
$\frac{5\pi}{6}$	**–1.15**	$\frac{11\pi}{6}$	**1.15**
		2π	**1**

(b)



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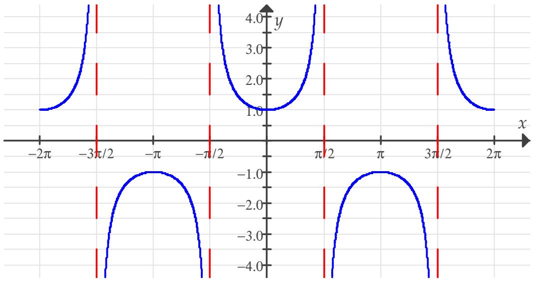
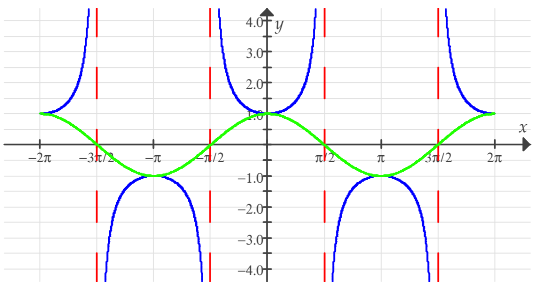
Odd-Numbered Answers to Exercise Set 5.3:
Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

(c)



(d) The graph of $h(x) = \cos(x)$ is shown in green below, superimposed on the graph of $f(x) = \sec(x)$, shown in blue. (Asymptotes of $f(x) = \sec(x)$ are shown in red.)



Notice that the graphs of $f(x) = \sec(x)$ and $h(x) = \cos(x)$ coincide at the points on the graphs where $y = \pm 1$. The asymptotes of $f(x) = \sec(x)$ occur at the same x-values as the x-intercepts of $h(x) = \cos(x)$.

When the graph of $f(x) = \sec(x)$ is desired, one can begin with the graph of $h(x) = \cos(x)$, and then use the key points mentioned above to create the graph of $f(x) = \sec(x)$.

(e) Domain: $\left\{x \,\middle|\, x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots\right\}$

Range: $(-\infty, 1] \cup [1, \infty)$

(f) Period: 2π

Continued in the next column…

(g) Answers vary. Following are some intervals on which $f(x) = \sec(x)$ is increasing. (Any two of these intervals are sufficient.)

$\left(0, \frac{\pi}{2}\right)$ $\left(\frac{\pi}{2}, \pi\right)$ $\left(2\pi, \frac{5\pi}{2}\right)$

$\left(-\frac{3\pi}{2}, -\pi\right)$ $\left(-2\pi, -\frac{3\pi}{2}\right)$ etc…

5. C

7. A

9. G

11. F

13. D

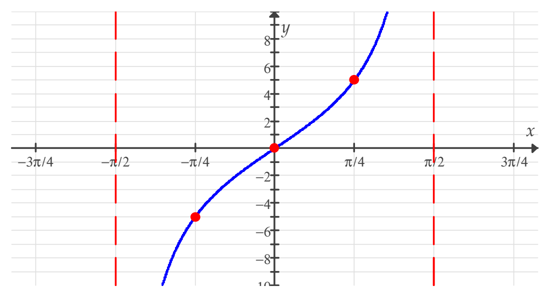
15. B

17. C

19. F

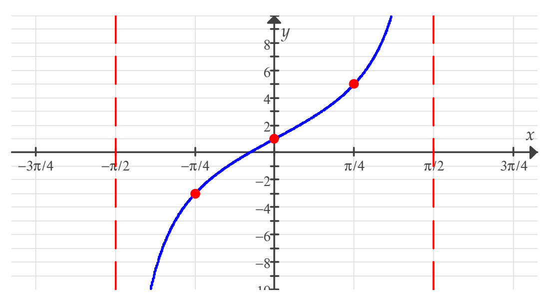
21. **(a)** Period: π

(b)



23. **(a)** Period: π

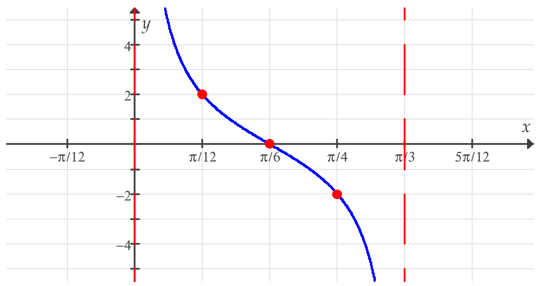
(b)



Odd-Numbered Answers to Exercise Set 5.3:
Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

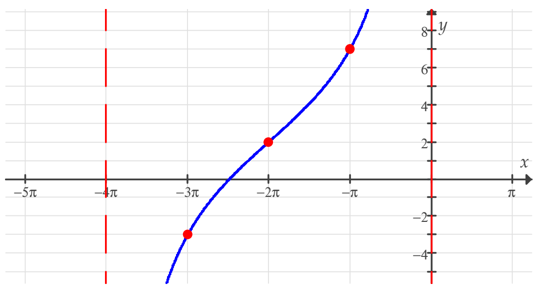
25. **(a)** Period: $\frac{\pi}{3}$

(b)



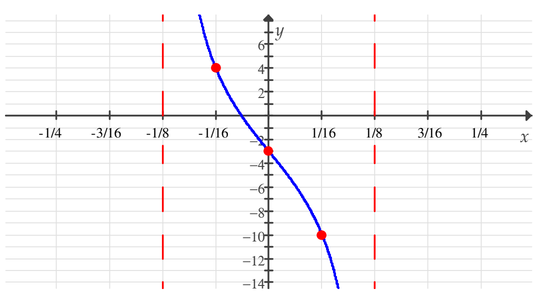
27. **(a)** Period: 4π

(b)



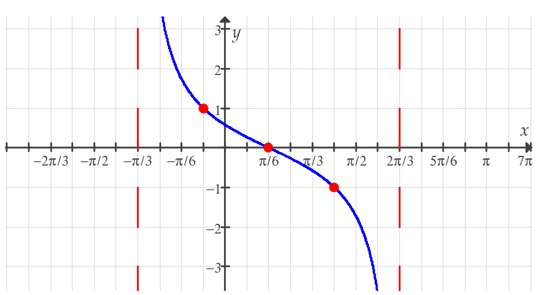
29. **(a)** Period: $\frac{1}{4}$

(b)



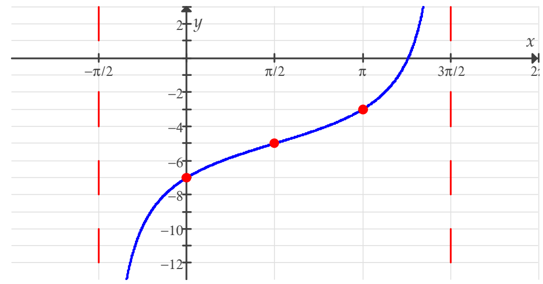
31. **(a)** Period: π

(b)



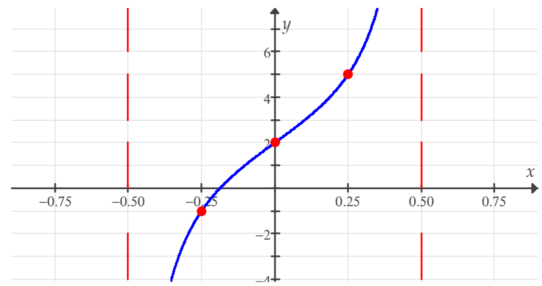
33. **(a)** Period: 2π

(b)

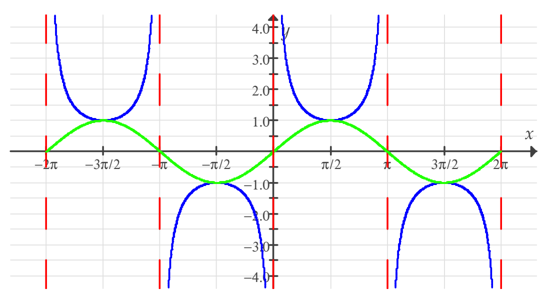


35. **(a)** Period: 1

(b)



37. **(a)** The graph of $f(x) = \sin(x)$ is shown in green below, superimposed on the graph of $g(x) = \csc(x)$, shown in blue. (Asymptotes of $g(x) = \csc(x)$ are shown in red.)



(b) The x-intercepts of $f(x) = \sin(x)$ occur at $x = -2\pi,\ -\pi,\ 0,\ \pi,\ 2\pi$.

(c) The asymptotes of $g(x) = \csc(x)$ are $x = -2\pi,\ x = -\pi,\ x = 0,\ x = \pi,\ x = 2\pi$. The asymptotes of $g(x) = \csc(x)$ occur at the same x-values as the x-intercepts of $f(x) = \sin(x)$.

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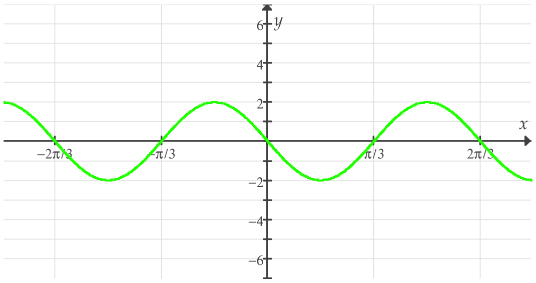
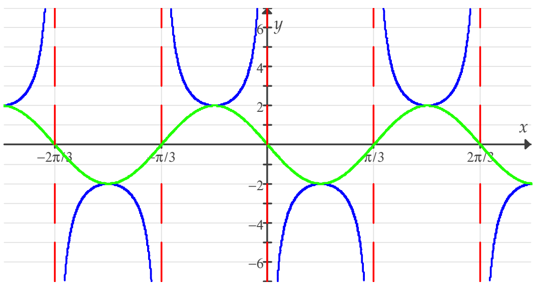
Odd-Numbered Answers to Exercise Set 5.3: Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

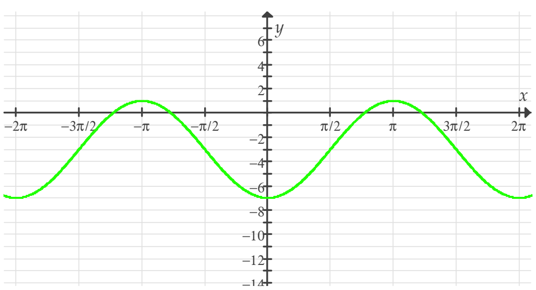
(d)



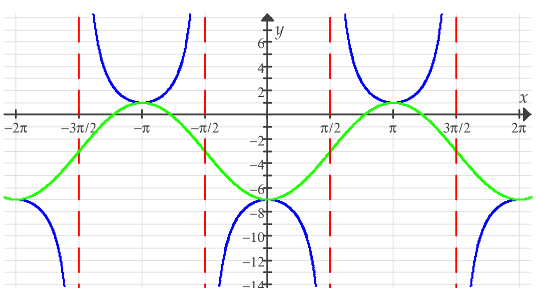
(e)



(f)

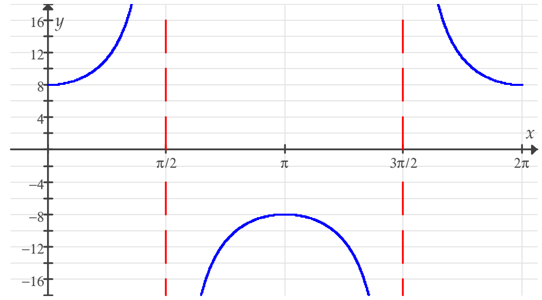


(g)



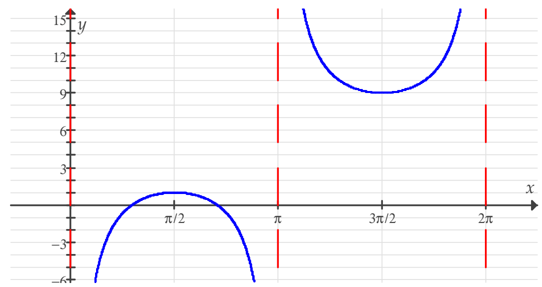
39. **(a)** Period: 2π

(b)



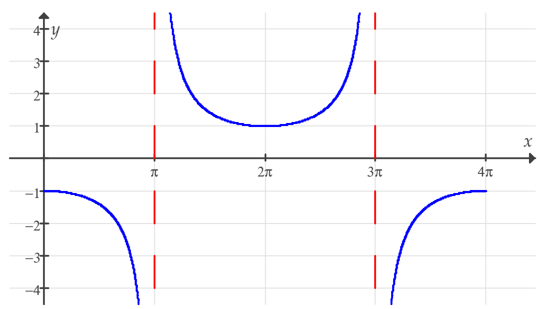
41. **(a)** Period: 2π

(b)



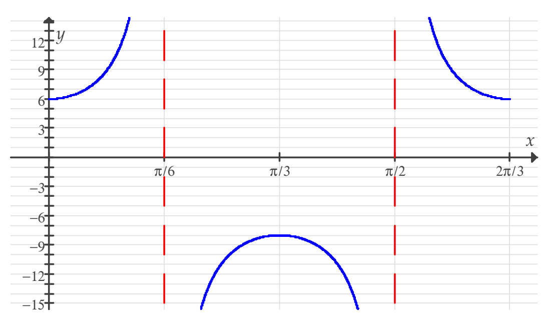
43. **(a)** Period: 4π

(b)



45. **(a)** Period: $\frac{2\pi}{3}$

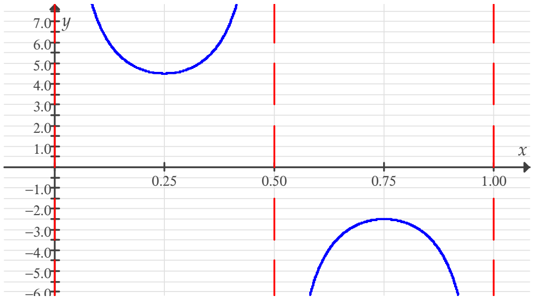
(b)



Odd-Numbered Answers to Exercise Set 5.3:
Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

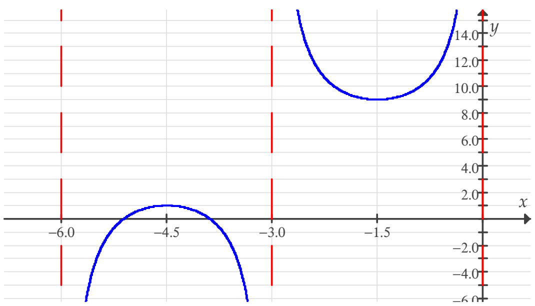
47. (a) Period: 1

(b)



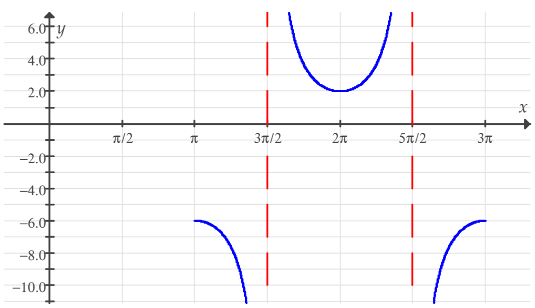
49. (a) Period: 6

(b)



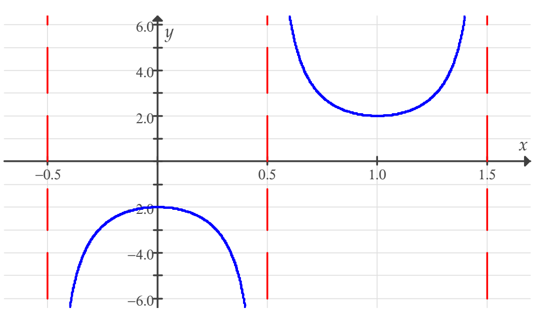
51. (a) Period: 2π

(b)



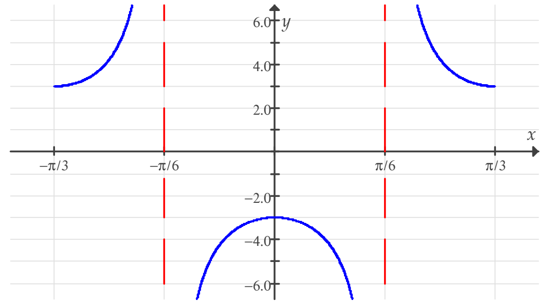
53. (a) Period: 2

(b)



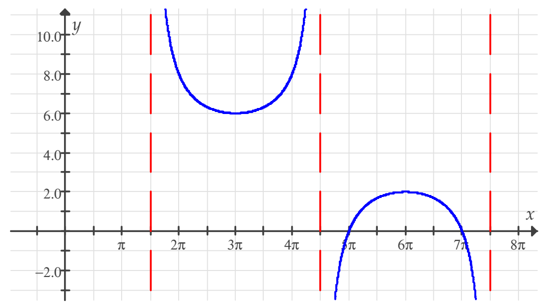
55. (a) Period: $\frac{2\pi}{3}$

(b)



57. (a) Period: 6π

(b)



59. (a) Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = 5\tan(x) + 3$$
$$f(x) = 5\tan(x - \pi) + 3$$
$$f(x) = 5\tan(x + \pi) + 3$$

(b) Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = -5\cot\left(x - \frac{\pi}{2}\right) + 3$$
$$f(x) = -5\cot\left(x + \frac{\pi}{2}\right) + 3$$
$$f(x) = -5\cot\left(x + \frac{3\pi}{2}\right) + 3$$

Continued on the next page…

Odd-Numbered Answers to Exercise Set 5.3: Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

61. **(a)** Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = -3\tan\left[2\left(x+\frac{\pi}{8}\right)\right]-3 = -3\tan\left(2x+\frac{\pi}{4}\right)-3$$

$$f(x) = -3\tan\left[2\left(x-\frac{3\pi}{8}\right)\right]-3 = -3\tan\left(2x-\frac{3\pi}{4}\right)-3$$

$$f(x) = -3\tan\left[2\left(x+\frac{5\pi}{8}\right)\right]-3 = -3\tan\left(2x+\frac{5\pi}{4}\right)-3$$

(b) Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = 3\cot\left[2\left(x-\frac{\pi}{8}\right)\right]-3 = 3\cot\left(2x-\frac{\pi}{4}\right)-3$$

$$f(x) = 3\cot\left[2\left(x+\frac{3\pi}{8}\right)\right]-3 = 3\cot\left(2x+\frac{3\pi}{4}\right)-3$$

$$f(x) = 3\cot\left[2\left(x+\frac{7\pi}{8}\right)\right]-3 = 3\cot\left(2x+\frac{7\pi}{4}\right)-3$$

63. **(a)** Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = 3\sec\left(x-\frac{\pi}{2}\right)-1$$

$$f(x) = -3\sec\left(x+\frac{\pi}{2}\right)-1$$

$$f(x) = 3\sec\left(x+\frac{3\pi}{2}\right)-1$$

$$f(x) = -3\sec\left(x-\frac{3\pi}{2}\right)-1$$

(b) Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = 3\csc(x)-1$$

$$f(x) = -3\csc(x-\pi)-1$$

$$f(x) = -3\csc(x+\pi)-1$$

$$f(x) = 3\csc(x+2\pi)-1$$

65. **(a)** Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = 4\csc\left[\frac{\pi}{4}(x+1)\right]+3 = 4\csc\left(\frac{\pi}{4}x+\frac{\pi}{4}\right)+3$$

$$f(x) = -4\csc\left[\frac{\pi}{4}(x+5)\right]+3 = -4\csc\left(\frac{\pi}{4}x+\frac{5\pi}{4}\right)+3$$

(b) Answers vary. Some solutions are shown below. (Intermediate steps are shown in gray.)

$$f(x) = 4\sec\left[\frac{\pi}{4}(x-1)\right]+3 = 4\sec\left(\frac{\pi}{4}x-\frac{\pi}{4}\right)+3$$

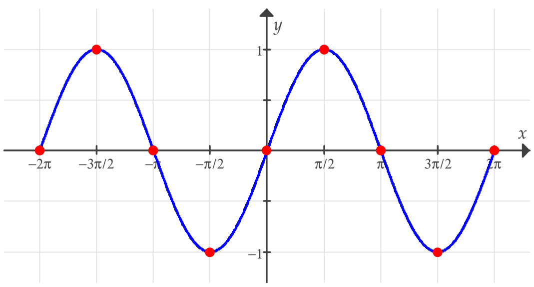
$$f(x) = -4\sec\left[\frac{\pi}{4}(x+3)\right]+3 = -4\sec\left(\frac{\pi}{4}x+\frac{3\pi}{4}\right)+3$$

Odd-Numbered Answers to Exercise Set 5.4: Inverse Trigonometric Functions

1. **(a)**

x	$y=\cos(x)$	x	$y=\cos(x)$
0	**1**		
$\frac{\pi}{2}$	**0**	$-\frac{\pi}{2}$	**0**
π	**–1**	$-\pi$	**–1**
$\frac{3\pi}{2}$	**0**	$-\frac{3\pi}{2}$	**0**
2π	**1**	-2π	**1**

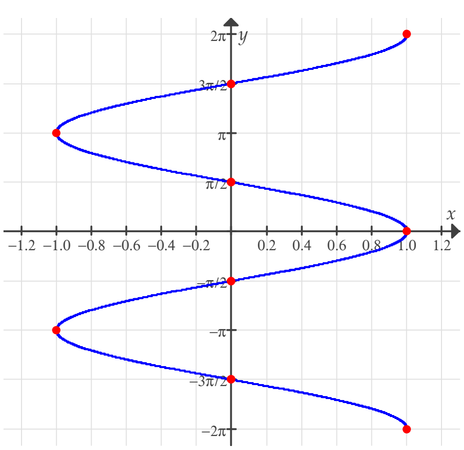
(b)



(c)

$x=\cos(y)$	y	$x=\cos(y)$	y
1	0		
0	$\frac{\pi}{2}$	**0**	$-\frac{\pi}{2}$
–1	π	**–1**	$-\pi$
0	$\frac{3\pi}{2}$	**0**	$-\frac{3\pi}{2}$
1	2π	**1**	-2π

(d)



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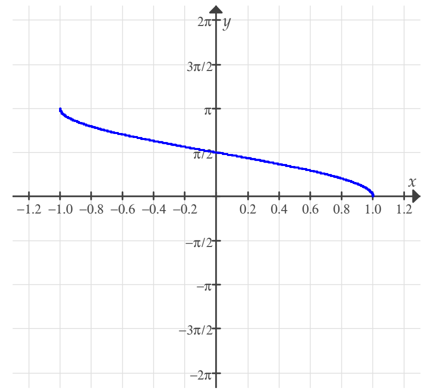
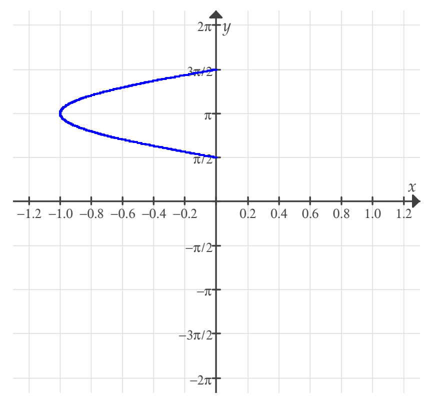
(e) The inverse relation in part (d) is not a function. The graph does not pass the vertical line test.

(f) **i.**



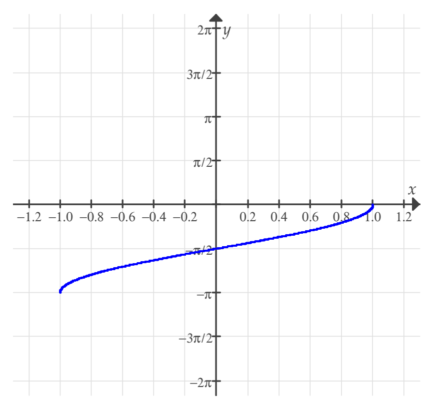
The above graph represents a function.

ii.



The above graph does not represent a function.

iii.



The above graph represents a function.

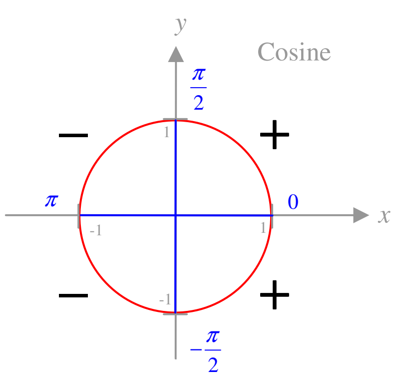
Continued on the next page…

iv.

The above graph does not represent a function.

(g) The graph of $f(x)=\cos^{-1}x$ can be seen in subsection *i* of part (f), with a range of $[0,\pi]$. Of the graphs in part (f), only two are functions – one with a range of $[0,\pi]$ and the other with a range of $[-\pi,0]$. The range of $[0,\pi]$ encompasses angles with positive measure rather than negative measure. In terms of the unit circle, $[0,\pi]$ encompasses first quadrant angles (along with second quadrant angles as well), while the range of $[-\pi,0]$ encompasses third and fourth quadrant angles.

(h) i.



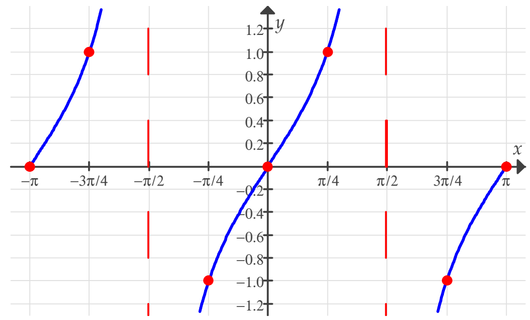
ii. Quadrants I and II; Quadrants III and IV

iii. Quadrants I and II; the range of $f(x)=\cos^{-1}(x)$ is $[0,\pi]$.

3. (a)

x	$y=\tan(x)$	x	$y=\tan(x)$
0	**0**		
$\frac{\pi}{4}$	**1**	$-\frac{\pi}{4}$	**–1**
$\frac{\pi}{2}$	**Undefined**	$-\frac{\pi}{2}$	**Undefined**
$\frac{3\pi}{4}$	**–1**	$-\frac{3\pi}{4}$	**1**
π	**0**	$-\pi$	**0**

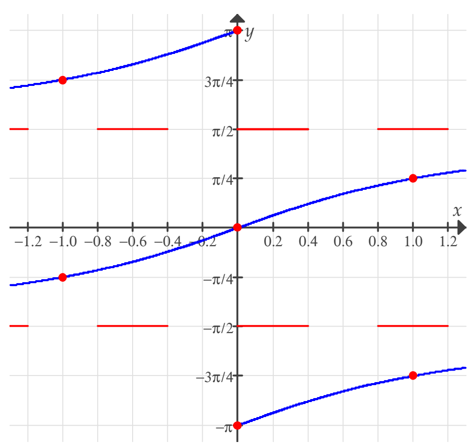
(b)



(c)

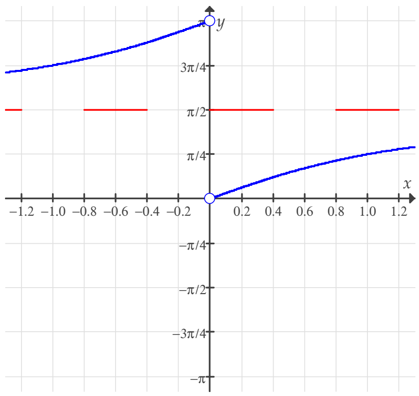
$x=\tan(y)$	y	$x=\tan(y)$	y
0	0		
1	$\frac{\pi}{4}$	**–1**	$-\frac{\pi}{4}$
Undefined	$\frac{\pi}{2}$	**Undefined**	$-\frac{\pi}{2}$
–1	$\frac{3\pi}{4}$	**1**	$-\frac{3\pi}{4}$
0	π	**0**	$-\pi$

(d)



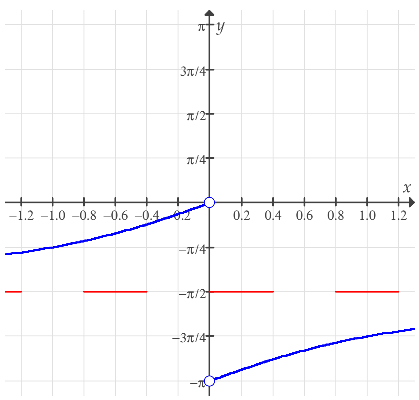
(e) The inverse relation in part (d) is not a function. The graph does not pass the vertical line test.

(f) **i.**



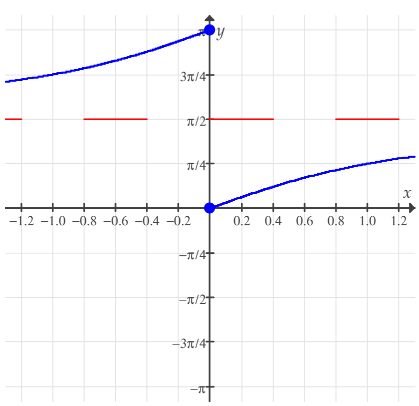
The above graph represents a function.

ii.



The above graph represents a function.

iii.



The above graph does not represent a function.

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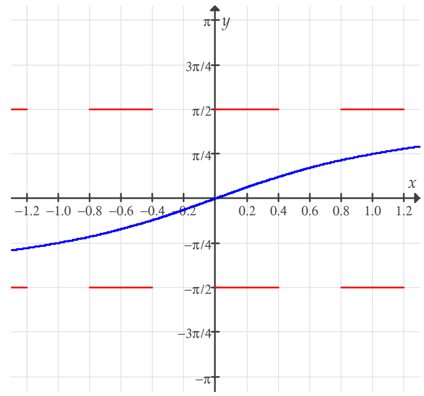
iv.



The above graph represents a function.

(g) The graph of $f(x)=\tan^{-1}x$ can be seen in subsection *iv* of part (f), with a range of $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Of the graphs in part (f), the following represent functions:

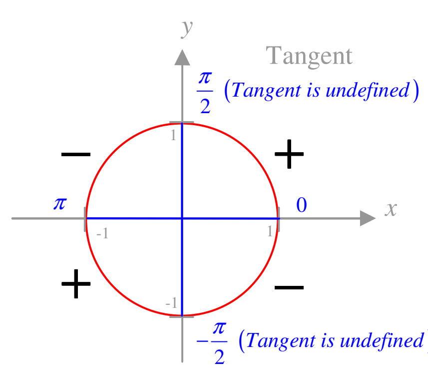
i. $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right)$

ii. $\left(-\pi,-\frac{\pi}{2}\right)\cup\left(-\frac{\pi}{2},0\right)$

iv. $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Unlike the other two functions in part (f), $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ can be represented by a single interval. In terms of the unit circle, the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ also encompasses <u>first quadrant angles</u> (along with second quadrant angles as well).

(h) **i.**



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ii. Quadrants I and II; Quadrants II and III; Quadrants III and IV; Quadrants I and IV

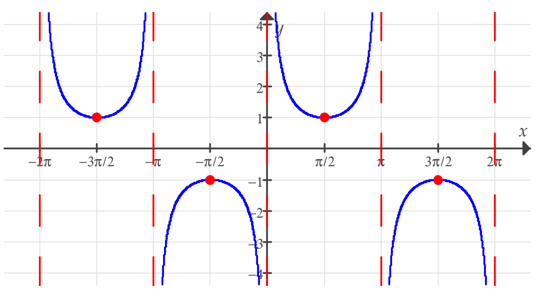
iii. Quadrants I and II; Quadrants I and IV.

Quadrants I and II would produce a range of $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$. Quadrants I and IV would produce a range of $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. The range of $f(x) = \tan^{-1}(x)$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

5. **(a)**

x	$y = \csc(x)$	x	$y = \csc(x)$
0	**Undefined**		
$\frac{\pi}{2}$	**1**	$-\frac{\pi}{2}$	**–1**
π	**Undefined**	$-\pi$	**Undefined**
$\frac{3\pi}{2}$	**–1**	$-\frac{3\pi}{2}$	**1**
2π	**Undefined**	-2π	**Undefined**

(b)

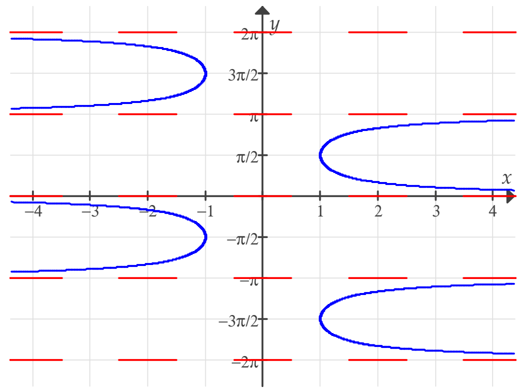


(c)

$x = \csc(y)$	y	$x = \csc(y)$	y
Undefined	0		
1	$\frac{\pi}{2}$	**–1**	$-\frac{\pi}{2}$
Undefined	π	**Undefined**	$-\pi$
–1	$\frac{3\pi}{2}$	**1**	$-\frac{3\pi}{2}$
Undefined	2π	**Undefined**	-2π

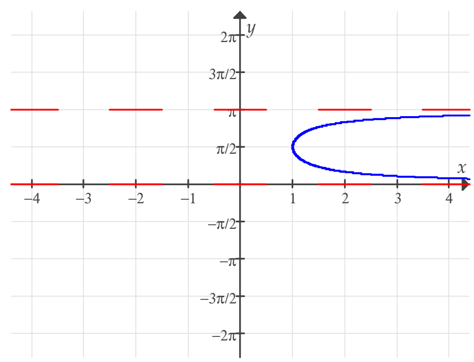
Continued in the next column…

(d)



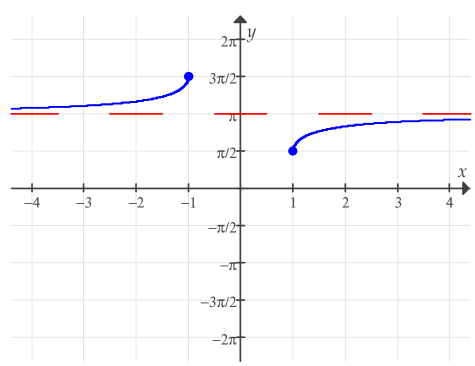
(e) The inverse relation in part (d) is not a function. The graph does not pass the vertical line test.

(f) **i.**



The above graph does not represent a function.

ii.



The above graph represents a function.

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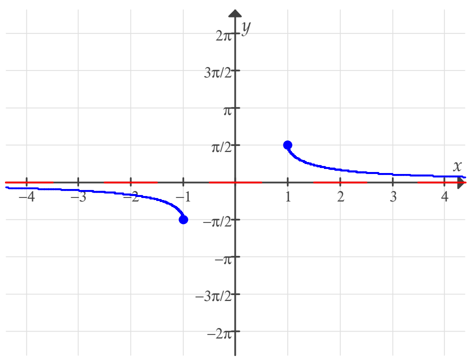
iii.

The above graph does not represent a function.

iv.



The above graph represents a function.

(g) The graph of $f(x)=\csc^{-1} x$ can be seen in subsection *iv* of part (f), with a range of $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$. Of the graphs in part (f), the following represent functions:

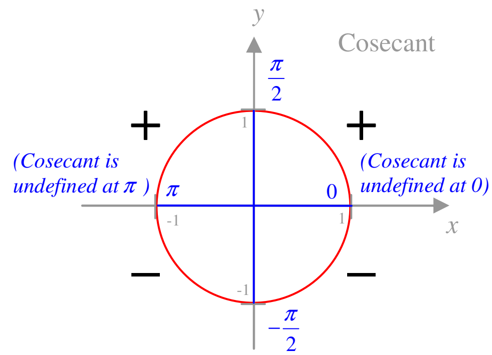
ii. $\left[\frac{\pi}{2}, \pi\right) \cup \left(\pi, \frac{3\pi}{2}\right]$

iv. $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

In terms of the unit circle, the interval $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ encompasses <u>first quadrant angles</u> (along with second quadrant angles as well).

Recall that $\csc(x)=\dfrac{1}{\sin(x)}$. The range of $g(x)=\sin^{-1}(x)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and the range of $f(x)=\csc^{-1} x$ is $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$. These two ranges are identical with the exception of $y=0$, where $f(x)=\csc^{-1} x$ is undefined.

(h) **i.**



ii. Quadrants I and IV; Quadrants II and III

iii. Quadrants I and IV; the range of $f(x)=\csc^{-1} x$ is $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$.

7. **(a)** $\cos^{-1}\left(-\frac{1}{2}\right)=$ the number (in the interval $[0, \pi]$) whose cosine is $-\frac{1}{2}$.

Therefore, $\cos^{-1}\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$.

Continued in the next column…

(b) $\tan^{-1}(1)=$ the number (in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$) whose tangent is 1.

Therefore, $\tan^{-1}(1)=\frac{\pi}{4}$.

9. **(a)** $\sin^{-1}(-1)=$ the number (in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$) whose sine is -1.

Therefore, $\sin^{-1}(-1)=-\frac{\pi}{2}$.

(b) $\sec^{-1}\left(\sqrt{2}\right)=$ the number (in the interval $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$) whose secant is $\sqrt{2}$.

Therefore, $\sec^{-1}\left(\sqrt{2}\right)=\frac{\pi}{4}$.

Odd-Numbered Answers to Exercise Set 5.4: Inverse Trigonometric Functions

11. $\sin\left[\sin^{-1}(0.2)\right]=$ the sine of the number (in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$) whose sine is 0.2.

Therefore, $\sin\left[\sin^{-1}(0.2)\right]=0.2$.

13. $\tan\left[\arctan(4)\right]=$ the tangent of the number (in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$) whose tangent is 4.

Therefore, $\tan\left[\arctan(4)\right]=4$.

15. **(a)** $\frac{\pi}{6}$ **(b)** 0

17. **(a)** Undefined **(b)** 0

19. **(a)** $-\frac{\pi}{4}$ **(b)** $\frac{\pi}{3}$

21. **(a)** $\frac{2\pi}{3}$ **(b)** $\frac{\pi}{2}$

23. **(a)** $\frac{2\pi}{3}$ **(b)** $-\frac{\pi}{2}$

25. **(a)** Undefined **(b)** $\frac{5\pi}{6}$

27. **(a)** False **(b)** True

29. **(a)** False **(b)** False

31. **(a)** 1.192 **(b)** -0.927

33. **(a)** -0.608 **(b)** -2.462

35. **(a)** 1.326 **(b)** -0.119

37. **(a)** 1.212 **(b)** 2.246

39. **(a)** 1.768 $\left(\pi-\tan^{-1}(5)\right)$ or

1.769 $(\pi-1.373)$

(b) 3.042

41. **(a)** 0.430 **(b)** 1.875

43. **(a)** Undefined **(b)** -0.162

45. **(a)** -0.84 **(b)** Undefined

47. **(a)** 3.5 **(b)** -7

49. **(a)** $\frac{4}{5}$ **(b)** $\frac{8}{15}$

51. **(a)** $\frac{\sqrt{53}}{7}$ **(b)** $2\sqrt{6}$

53. **(a)** $\frac{5}{13}$ **(b)** $-\sqrt{35}$

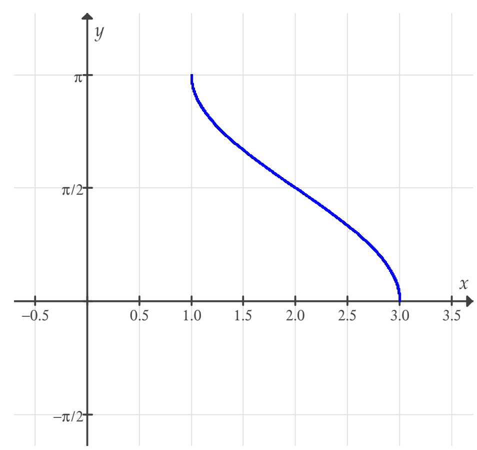
55. **(a)** $-\frac{\sqrt{2}}{2}$ **(b)** $\frac{\sqrt{2}}{2}$

57. **(a)** -2 **(b)** $-\sqrt{3}$

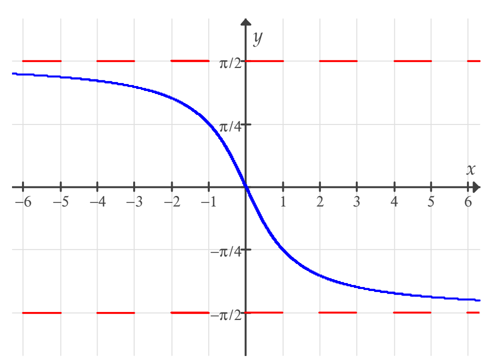
59. **(a)** $\frac{2\pi}{3}$ **(b)** $\frac{\pi}{4}$

61. **(a)** $\frac{5\pi}{6}$ **(b)** $-\frac{\pi}{4}$

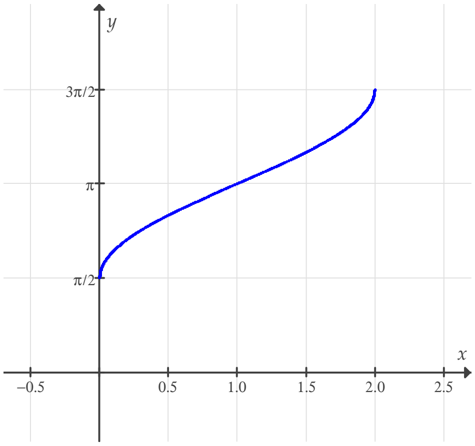
63.



65.



67.



Odd-Numbered Answers to Exercise Set 6.1: Sum and Difference Formulas

1. $\sin(x)$

3. $\sin(x)$

5. $\sqrt{3}\cos(\theta)$

7. $\sqrt{2}\cos(x)$

9. $-2\sin(\theta)$

11. $\frac{5}{3}$

13. $-\frac{1}{5}$

15. $-\frac{11}{29}$

17. $-\frac{13}{11}$

19. $\frac{\sqrt{2}}{2}$

21. $\frac{1}{2}$

23. $\cos\left(\frac{19\pi}{90}\right)$

25. $-\frac{1}{2}$

27. $\sin(A)$

29. $\tan(34^\circ)$

31. $\sqrt{3}$

33. $\tan(a)$

35. **(a)** $\frac{\pi}{6}=\frac{2\pi}{12}$ **(b)** $\frac{\pi}{4}=\frac{3\pi}{12}$ **(c)** $\frac{\pi}{3}=\frac{4\pi}{12}$

(d) $\frac{2\pi}{3}=\frac{8\pi}{12}$ **(e)** $\frac{3\pi}{4}=\frac{9\pi}{12}$ **(f)** $\frac{5\pi}{6}=\frac{10\pi}{12}$

37. **(a)** $\frac{5\pi}{12}=\frac{8\pi}{12}-\frac{3\pi}{12}=\frac{2\pi}{3}-\frac{\pi}{4}$ *OR*

$\frac{5\pi}{12}=\frac{9\pi}{12}-\frac{4\pi}{12}=\frac{3\pi}{4}-\frac{\pi}{3}$

(b) $\frac{7\pi}{12}=\frac{9\pi}{12}-\frac{2\pi}{12}=\frac{3\pi}{4}-\frac{\pi}{6}$ *OR*

$\frac{7\pi}{12}=\frac{10\pi}{12}-\frac{3\pi}{12}=\frac{5\pi}{6}-\frac{\pi}{4}$

(c) $-\frac{7\pi}{12}=\frac{2\pi}{12}-\frac{9\pi}{12}=\frac{\pi}{6}-\frac{3\pi}{4}$ *OR*

$-\frac{7\pi}{12}=\frac{3\pi}{12}-\frac{10\pi}{12}=\frac{\pi}{4}-\frac{5\pi}{6}$

(d) $-\frac{5\pi}{12}=\frac{3\pi}{12}-\frac{8\pi}{12}=\frac{\pi}{4}-\frac{2\pi}{3}$ *OR*

$-\frac{5\pi}{12}=\frac{4\pi}{12}-\frac{9\pi}{12}=\frac{\pi}{3}-\frac{3\pi}{4}$

39. **(a)** $\frac{19\pi}{12}=\frac{15\pi}{12}+\frac{4\pi}{12}=\frac{5\pi}{4}+\frac{\pi}{3}$ *OR*

$\frac{19\pi}{12}=\frac{9\pi}{12}+\frac{10\pi}{12}=\frac{3\pi}{4}+\frac{5\pi}{6}$

(b) $\frac{25\pi}{12}=\frac{15\pi}{12}+\frac{10\pi}{12}=\frac{5\pi}{4}+\frac{5\pi}{6}$ *OR*

$\frac{25\pi}{12}=\frac{21\pi}{12}+\frac{4\pi}{12}=\frac{7\pi}{4}+\frac{\pi}{3}$

(c) $\frac{35\pi}{12}=\frac{27\pi}{12}+\frac{8\pi}{12}=\frac{9\pi}{4}+\frac{2\pi}{3}$ *OR*

$\frac{35\pi}{12}=\frac{33\pi}{12}+\frac{2\pi}{12}=\frac{11\pi}{4}+\frac{\pi}{6}$

(d) $\frac{43\pi}{12}=\frac{33\pi}{12}+\frac{10\pi}{12}=\frac{11\pi}{4}+\frac{5\pi}{6}$

Odd-Numbered Answers to Exercise Set 6.1: Sum and Difference Formulas

41. (a) $-\dfrac{7\pi}{12}=-\dfrac{3\pi}{12}-\dfrac{4\pi}{12}=-\dfrac{\pi}{4}-\dfrac{\pi}{3}$

$=-\dfrac{4\pi}{12}-\dfrac{3\pi}{12}=-\dfrac{\pi}{3}-\dfrac{\pi}{4}$

(b) $-\dfrac{13\pi}{12}=-\dfrac{9\pi}{12}-\dfrac{4\pi}{12}=-\dfrac{3\pi}{4}-\dfrac{\pi}{3}$

$=-\dfrac{4\pi}{12}-\dfrac{9\pi}{12}=-\dfrac{\pi}{3}-\dfrac{3\pi}{4}$

OR

$-\dfrac{13\pi}{12}=-\dfrac{10\pi}{12}-\dfrac{3\pi}{12}=-\dfrac{5\pi}{6}-\dfrac{\pi}{4}$

$=-\dfrac{3\pi}{12}-\dfrac{10\pi}{12}=-\dfrac{\pi}{4}-\dfrac{5\pi}{6}$

(c) $-\dfrac{29\pi}{12}=-\dfrac{2\pi}{12}-\dfrac{27\pi}{12}=-\dfrac{\pi}{6}-\dfrac{9\pi}{4}$

$=-\dfrac{27\pi}{12}-\dfrac{2\pi}{12}=-\dfrac{9\pi}{4}-\dfrac{\pi}{6}$

OR

$-\dfrac{29\pi}{12}=-\dfrac{8\pi}{12}-\dfrac{21\pi}{12}=-\dfrac{2\pi}{3}-\dfrac{7\pi}{4}$

$=-\dfrac{21\pi}{12}-\dfrac{8\pi}{12}=-\dfrac{7\pi}{4}-\dfrac{2\pi}{3}$

(d) $-\dfrac{41\pi}{12}=-\dfrac{33\pi}{12}-\dfrac{8\pi}{12}=-\dfrac{11\pi}{4}-\dfrac{2\pi}{3}$

$=-\dfrac{8\pi}{12}-\dfrac{33\pi}{12}=-\dfrac{2\pi}{3}-\dfrac{11\pi}{4}$

43. $\sin(-\theta) \overset{?}{=} -\sin(\theta)$

Left-Hand Side	**Right-Hand Side**
$\sin(-\theta)$	$-\sin(\theta)$
$=\sin(0-\theta)$	
$=\sin(0)\cos(\theta)-\cos(0)\sin(\theta)$	
$=0\cdot\cos(\theta)-1\cdot\sin(\theta)$	
$=-\sin(\theta)$	

$\therefore\ \sin(-\theta) = -\sin(\theta)$ Q.E.D.

45. $\tan(-\theta) \overset{?}{=} -\tan(\theta)$

Left-Hand Side	**Right-Hand Side**
$\tan(-\theta)$	$-\tan(\theta)$
$=\tan(0-\theta)$	
$=\dfrac{\tan(0)-\tan(\theta)}{1+\tan(0)\tan(\theta)}$	
$=\dfrac{0-\tan(\theta)}{1+0\cdot\tan(\theta)}$	
$=-\tan(\theta)$	

$\therefore\ \tan(-\theta) = -\tan(\theta)$ Q.E.D.

47. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

49. $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

51. $-2-\sqrt{3}$

53. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

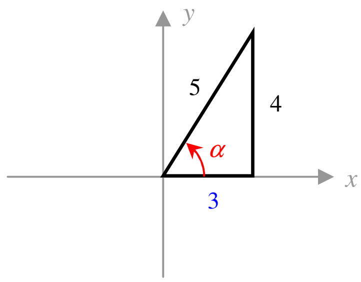
55. $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

57. $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

59. $-2+\sqrt{3}$

61. $2-\sqrt{3}$

63. The following diagrams, though not required, may be helpful in setting up the problems:

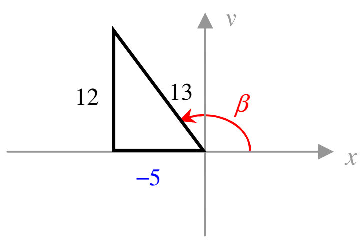


$\sin(\alpha)=\frac{4}{5}$

$\cos(\alpha)=\frac{3}{5}$

$\tan(\alpha)=\frac{4}{3}$

Note: The triangle may not be drawn to scale.



$\sin(\beta)=\frac{12}{13}$

$\cos(\beta)=-\frac{5}{13}$

$\tan(\beta)=-\frac{12}{5}$

Note: The triangle may not be drawn to scale.

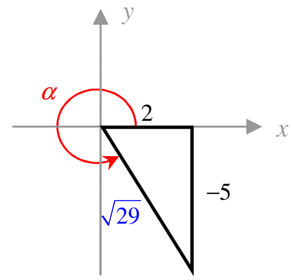
Answers:

(a) $-\frac{56}{65}$

(b) $-\frac{63}{65}$

(c) $-\frac{56}{33}$

65. The following diagrams, though not required, may be helpful in setting up the problems:

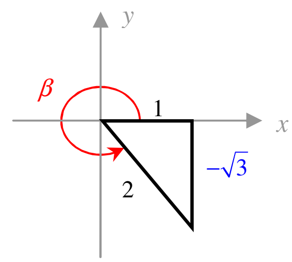


$\sin(\alpha)=-\frac{5}{\sqrt{29}}=-\frac{5\sqrt{29}}{29}$

$\cos(\alpha)=\frac{2}{\sqrt{29}}=\frac{2\sqrt{29}}{29}$

$\tan(\alpha)=-\frac{5}{2}$

Note: The triangle may not be drawn to scale.



$\sin(\beta)=-\frac{\sqrt{3}}{2}$

$\cos(\beta)=\frac{1}{2}$

$\tan(\beta)=-\sqrt{3}$

Note: The triangle may not be drawn to scale.

Answers:

(a) $\frac{-5\sqrt{29}-2\sqrt{87}}{58}$

(b) $\frac{2\sqrt{29}-5\sqrt{87}}{58}$

(c) $\frac{40-29\sqrt{3}}{71}$

67. $\frac{\sqrt{2}}{2}$

69. $\frac{33}{56}$

71. $\cos(A-B)$

73. $\sin^2(A)-\sin^2(B)$ *OR*
$\cos^2(B)-\cos^2(A)$

75. $$\sin(x-y)+\sin(x+y) \stackrel{?}{=} 2\sin(x)\cos(y)$$

Left-Hand Side	**Right-Hand Side**
$\sin(x-y)+\sin(x+y)$	$2\sin(x)\cos(y)$
$=\sin(x)\cos(y)-\cos(x)\sin(y)+$	
$\sin(x)\cos(y)+\cos(x)\sin(y)$	
$=2\sin(x)\cos(y)$	

$$\therefore\ \sin(x-y)+\sin(x+y) = 2\sin(x)\cos(y)$$

Q.E.D.

77. $$\frac{\cos(x-y)+\cos(x+y)}{\sin(x)\sin(y)} \stackrel{?}{=} 2\cot(x)\cot(y)$$

Left-Hand Side	**Right-Hand Side**
$\dfrac{\cos(x-y)+\cos(x+y)}{\sin(x)\sin(y)}$	$2\cot(x)\cot(y)$
$=\dfrac{\cos(x)\cos(y)+\sin(x)\sin(y)}{\sin(x)\sin(y)}+$	
$\dfrac{\cos(x)\cos(y)-\sin(x)\sin(y)}{\sin(x)\sin(y)}$	
$=\dfrac{2\cos(x)\cos(y)}{\sin(x)\sin(y)}$	
$=\dfrac{2\cos(x)}{\sin(x)}\cdot\dfrac{\cos(y)}{\sin(y)}$	
$=2\cot(x)\cot(y)$	

$$\therefore\ \frac{\cos(x-y)+\cos(x+y)}{\sin(x)\sin(y)} = 2\cot(x)\cot(y)$$

79. $$\frac{\sin(x-y)}{\sin(x+y)} \stackrel{?}{=} \frac{\tan(x)-\tan(y)}{\tan(x)+\tan(y)}$$

Left-Hand Side	**Right-Hand Side**
$\dfrac{\sin(x-y)}{\sin(x+y)}$	$\dfrac{\tan(x)-\tan(y)}{\tan(x)+\tan(y)}$
	$=\dfrac{\frac{\sin(x)}{\cos(x)}-\frac{\sin(y)}{\cos(y)}}{\frac{\sin(x)}{\cos(x)}+\frac{\sin(y)}{\cos(y)}}$
	$=\dfrac{\frac{\sin(x)\cos(y)-\cos(x)\sin(y)}{\cos(x)\cos(y)}}{\frac{\sin(x)\cos(y)+\cos(x)\sin(y)}{\cos(x)\cos(y)}}$
	$=\dfrac{\frac{\sin(x-y)}{\cos(x)\cos(y)}}{\frac{\sin(x+y)}{\cos(x)\cos(y)}}$
	$=\dfrac{\sin(x-y)}{\cos(x)\cos(y)}\ \dfrac{\cos(x)\cos(y)}{\sin(x+y)}$
	$=\dfrac{\sin(x-y)}{\sin(x+y)}$

$$\therefore\ \frac{\sin(x-y)}{\sin(x+y)} = \frac{\tan(x)-\tan(y)}{\tan(x)+\tan(y)}$$

Q.E.D.

81. **(a)** $\sin(A)=\dfrac{a}{c}$

(b) $\cos(B)=\dfrac{a}{c}$

(c) $\sin(A)=\cos(B)$

(d) $A=90^\circ-B$

$B=90^\circ-A$

(e) $\sin(A)=\cos\left(90^\circ-A\right)$

$\cos(A)=\sin\left(90^\circ-A\right)$

83. **(a)** $\sec(A) = \frac{c}{b}$

(b) $\csc(B) = \frac{c}{b}$

(c) $\sec(A) = \csc(B)$

(d) $A = 90^\circ - B$

$B = 90^\circ - A$

(e) $\sec(A) = \csc(90^\circ - A)$

$\csc(A) = \sec(90^\circ - A)$

85. $x = 15^\circ$

87. $x = \frac{\pi}{10}$

89. $x = 18^\circ$

91. 1

93. $\sin(x)$

95. $\cot(\theta)$

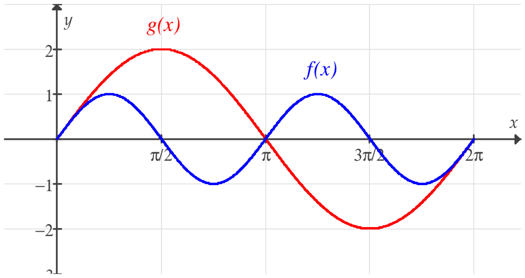
97. -1

99. -1

Odd-Numbered Answers to Exercise Set 6.2: Double-Angle and Half-Angle Formulas

1. **(a)** $\frac{\sqrt{3}}{2}$ **(b)** 1 **(c)** No

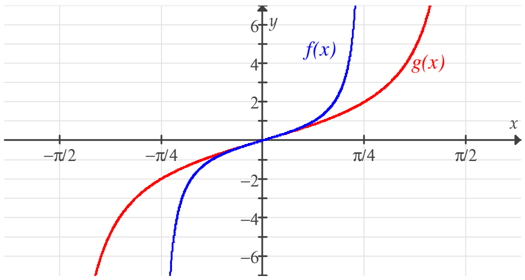
(d)



(e) No

3. **(a)** $\sqrt{3}$ **(b)** $\frac{2\sqrt{3}}{3}$ **(c)** No

(d)



Notes:
Only one 'branch' of each graph is shown. In the portion of each graph shown,

$f(x)$ has asymptotes at $x = \pm\frac{\pi}{4}$, and

$g(x)$ has asymptotes at $x = \pm\frac{\pi}{2}$.

(e) No

5.
$$\begin{aligned}\cos(2\theta) &= \cos(\theta+\theta)\\ &= \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta)\\ &= \cos^2(\theta) - \sin^2(\theta)\end{aligned}$$

7. **(a)** $\cos^2(\theta) = 1 - \sin^2(\theta)$

(b)
$$\begin{aligned}\cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta)\\ &= \left[1-\sin^2(\theta)\right] - \sin^2(\theta)\\ &= 1 - 2\sin^2(\theta)\end{aligned}$$

9. **(a)** $-\frac{120}{169}$ **(b)** $\frac{119}{169}$ **(c)** $-\frac{120}{119}$

11. **(a)** $-\frac{4\sqrt{21}}{25}$ **(b)** $\frac{17}{25}$ **(c)** $\frac{-4\sqrt{21}}{17}$

13. $\frac{1}{2}$

15. $\cos(68^\circ)$

17. $\frac{\sqrt{3}}{3}$

19. $\sqrt{2}$

21. $-\frac{\sqrt{2}}{2}$

23. $\frac{\sqrt{3}}{2}$

25. $-\tan(82^\circ)$

27. $\cos(\beta)$

29. **(a)** Quadrant II; Negative
(b) Quadrant I; Positive

31. **(a)** Quadrant IV; Positive
(b) Quadrant III; Negative

33. **(a)** $\sin\left(\frac{s}{2}\right) = \pm\sqrt{\frac{1-\cos(s)}{2}}$

(b) $\cos\left(\frac{s}{2}\right) = \pm\sqrt{\frac{1+\cos(s)}{2}}$

(c)
$$\begin{aligned}\tan\left(\frac{s}{2}\right) &= \frac{\pm\sqrt{\frac{1-\cos(s)}{2}}}{\pm\sqrt{\frac{1+\cos(s)}{2}}} = \pm\sqrt{\frac{\frac{1-\cos(s)}{2}}{\frac{1+\cos(s)}{2}}}\\ &= \pm\sqrt{\frac{1-\cos(s)}{2}\cdot\frac{2}{1+\cos(s)}}\\ &= \pm\sqrt{\frac{1-\cos(s)}{1+\cos(s)}}\end{aligned}$$

Odd-Numbered Answers to Exercise Set 6.2: Double-Angle and Half-Angle Formulas

35. (a) $\tan\left(\frac{s}{2}\right)=\pm\sqrt{\frac{1-\cos(s)}{1+\cos(s)}}$

$=\pm\frac{\sqrt{1-\cos(s)}}{\sqrt{1+\cos(s)}}\cdot\frac{\sqrt{1-\cos(s)}}{1-\cos(s)}=\pm\frac{1-\cos(s)}{\sqrt{1-\cos^2(s)}}$

$=\pm\frac{1-\cos(s)}{\sqrt{\sin^2(s)}}=\pm\frac{1-\cos(s)}{\sin(s)}$

(b) $\tan\left(\frac{s}{2}\right)=\frac{1-\cos(s)}{\sin(s)}$

(c) $\tan\left(\frac{s}{2}\right)=\pm\sqrt{\frac{1-\cos(s)}{1+\cos(s)}}$

$\tan\left(\frac{s}{2}\right)=\frac{\sin(s)}{1+\cos(s)}$

$\tan\left(\frac{s}{2}\right)=\frac{1-\cos(s)}{\sin(s)}$

Of the formulas above, $\tan\left(\frac{s}{2}\right)=\frac{1-\cos(s)}{\sin(s)}$ is generally easiest to use, since the denominator contains only one term. This is particularly true when the denominator needs to be rationalized. In the case where the denominator does not contain a radical sign, $\tan\left(\frac{s}{2}\right)=\frac{\sin(s)}{1+\cos(s)}$ is equally easy to use.

Of the formulas above, $\tan\left(\frac{s}{2}\right)=\pm\sqrt{\frac{1-\cos(s)}{1+\cos(s)}}$ is generally the most difficult to use. One must always choose whether or not the answer is positive or negative, as well as rationalizing the denominator. (One advantage of this formula is that it does not require the user to find $\sin(s)$ before computing the answer.)

37. (a) $\cos(75^\circ)=\cos(45^\circ+30^\circ)$

$=\cos(45^\circ)\cos(30^\circ)-\sin(45^\circ)\sin(30^\circ)$

$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}$

$=\frac{\sqrt{6}-\sqrt{2}}{2}$

Other numbers may be chosen that have a sum or difference of 75°, such as $\cos(105^\circ-30^\circ)$, but the end result is the same.

(b) $\cos(75^\circ)=+\sqrt{\frac{1+\cos(150^\circ)}{2}}=\sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}}$

$=\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}=\sqrt{\frac{2-\sqrt{3}}{4}}$

$=\frac{\sqrt{2-\sqrt{3}}}{2}$

(c) $\frac{\sqrt{6}-\sqrt{2}}{4}\approx 0.26$; $\frac{\sqrt{2-\sqrt{3}}}{2}\approx 0.26$.
They are the same.

39. (a) $\frac{\sqrt{2+\sqrt{2}}}{2}$

(b) $-\frac{\sqrt{2-\sqrt{2}}}{2}$

(c) $\sqrt{2}-1$

(d) $\sqrt{2}-1$

41. (a) $-\frac{\sqrt{2-\sqrt{3}}}{2}$

(b) $-\frac{\sqrt{2+\sqrt{3}}}{2}$

(c) $2-\sqrt{3}$

Odd-Numbered Answers to Exercise Set 6.2: Double-Angle and Half-Angle Formulas

43. **(a)** $-\dfrac{\sqrt{2+\sqrt{2}}}{2}$

(b) $-\dfrac{\sqrt{2-\sqrt{2}}}{2}$

(c) $\sqrt{2}+1$

45. **(a)** $-\dfrac{\sqrt{2+\sqrt{3}}}{2}$

(b) $\dfrac{\sqrt{2-\sqrt{3}}}{2}$

(c) $-2-\sqrt{3}$

47. **(a)** IV

(b) $\dfrac{3\pi}{4}<\dfrac{\theta}{2}<\pi$

(c) II

(d) Positive

(e) Negative

(f) $\dfrac{\sqrt{2}}{3}$

(g) $-\dfrac{\sqrt{7}}{3}$

(h) $-\dfrac{\sqrt{14}}{7}$

49. **(a)** $\dfrac{\sqrt{14}}{4}$

(b) $\dfrac{\sqrt{2}}{4}$

(c) $\sqrt{7}$

51. $\dfrac{1-\cos(2x)}{\sin(2x)} \stackrel{?}{=} \tan(x)$

Left-Hand Side	**Right-Hand Side**
$\dfrac{1-\cos(2x)}{\sin(2x)}$	$\tan(x)$
$=\dfrac{1-\left[1-2\sin^2(x)\right]}{2\sin(x)\cos(x)}$	
$=\dfrac{1-1+2\sin^2(x)}{2\sin(x)\cos(x)}$	
$=\dfrac{2\sin^2(x)}{2\sin(x)\cos(x)}$	
$=\dfrac{\sin(x)}{\cos(x)}$	
$=\tan(x)$	

$\therefore\ \dfrac{1-\cos(2x)}{\sin(2x)} = \tan(x)$ Q.E.D.

53. $\left[1-\sqrt{2}\sin(x)\right]\left[1+\sqrt{2}\sin(x)\right] \stackrel{?}{=} \cos(2x)$

Left-Hand Side	**Right-Hand Side**
$\left[1-\sqrt{2}\sin(x)\right]\left[1+\sqrt{2}\sin(x)\right]$	$\cos(2x)$
$=1-2\sin^2(x)$	
$=\cos(2x)$	

$\therefore\ \left[1-\sqrt{2}\sin(x)\right]\left[1+\sqrt{2}\sin(x)\right]=\cos(2x)$

Q.E.D.

Continued on the next page…

55. $$\csc(x) - \cot(x) \stackrel{?}{=} \tan\left(\frac{x}{2}\right)$$

Left-Hand Side	**Right-Hand Side**
$\csc(x) - \cot(x)$	$\tan\left(\frac{x}{2}\right)$
$= \frac{1}{\sin(x)} - \frac{\cos(x)}{\sin(x)}$	
$= \frac{1-\cos(x)}{\sin(x)}^{*}$	
$= \frac{[1-\cos(x)]}{\sin(x)} \cdot \frac{[1+\cos(x)]}{[1+\cos(x)]}$	
$= \frac{1-\cos^2(x)}{\sin(x)[1+\cos(x)]}$	
$= \frac{\sin^2(x)}{\sin(x)[1+\cos(x)]}$	
$= \frac{\sin(x)}{1+\cos(x)}$	
$= \tan\left(\frac{x}{2}\right)$	

$$\csc(x) - \cot(x) = \tan\left(\frac{x}{2}\right) \qquad \text{Q.E.D.}$$

*Note about the proof above: Exercise 35 establishes the identity $\tan\left(\frac{x}{2}\right) = \frac{1-\cos(x)}{\sin(x)}$. If this identity is used, several steps can be omitted from the proof above. In the proof above, the identity from the text, $\tan\left(\frac{x}{2}\right) = \frac{\sin(x)}{1+\cos(x)}$ was used.

Odd-Numbered Answers to Exercise Set 6.3: Solving Trigonometric Equations

1. No

3. Yes

5. **(a)** $x = 120^\circ$, $x = 240^\circ$

(b) $x = \frac{2\pi}{3}$, $x = \frac{4\pi}{3}$

(c) $x = \frac{2\pi}{3} + 2k\pi$ and $x = \frac{4\pi}{3} + 2k\pi$, where k is any integer.

7. **(a)** $x = 135^\circ$, $x = 315^\circ$

(b) $x = \frac{3\pi}{4}$, $x = \frac{7\pi}{4}$

(c) $x = \frac{3\pi}{4} + k\pi$, where k is any integer.

9. **(a)** $x = 90^\circ$

(b) $x = \frac{\pi}{2}$

(c) $x = \frac{\pi}{2} + 2k\pi$, where k is any integer.

11. **(a)** $x = 150^\circ$, $x = 210^\circ$

(b) $x = \frac{5\pi}{6}$, $x = \frac{7\pi}{6}$

(c) $x = \frac{5\pi}{6} + 2k\pi$ and $x = \frac{7\pi}{6} + 2k\pi$, where k is any integer.

13. **(a)** $x = 45^\circ$, $x = 315^\circ$

(b) $x = \frac{\pi}{4}$, $x = \frac{7\pi}{4}$

(c) $x = \frac{\pi}{4} + 2k\pi$ and $x = \frac{7\pi}{4} + 2k\pi$, where k is any integer.

15. **(a)** $x = 210^\circ$, $x = 330^\circ$

(b) $x = \frac{7\pi}{6}$, $x = \frac{11\pi}{6}$

(c) $x = \frac{7\pi}{6} + 2k\pi$ and $x = \frac{11\pi}{6} + 2k\pi$, where k is any integer.

17. **(a)** $x = 0^\circ$

(b) $x = 0$

(c) $x = 2k\pi$, where k is any integer.

19. **(a)** 53.1°

(b) $x \approx 233.1^\circ$, $x \approx 306.9^\circ$

21. **(a)** 27.4°

(b) $x \approx 207.4^\circ$, $x \approx 332.6^\circ$

23. **(a)** $x \approx 115.4^\circ$, $x \approx 244.6^\circ$

(b) $x \approx 45.6^\circ$, $x \approx 134.4^\circ$

25. **(a)** $x \approx 161.0^\circ$, $x \approx 341.0^\circ$

(b) No solution

27. **(a)** $x = \frac{\pi}{6}$, $x = \frac{11\pi}{6}$

(b) $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, $x = \frac{\pi}{6}$, $x = \frac{11\pi}{6}$

(c) No

(d) Part (b) is correct. Part (a) is incorrect because dividing by $\cos(x)$ causes loss of the solutions of $\cos(x) = 0$.

29. $x = 0^\circ$, $x = 180^\circ$, $x = 45^\circ$, $x = 135^\circ$

31. $x = 0^\circ$, $x = 180^\circ$, $x = 150^\circ$, $x = 330^\circ$

33. $x = 90^\circ$, $x = 270^\circ$, $x = 30^\circ$, $x = 150^\circ$, $x = 210^\circ$, $x = 330^\circ$

35. $x = 90^\circ$, $x = 270^\circ$, $x = 120^\circ$, $x = 240^\circ$, $x = 180^\circ$

37. **(a)** $0 \le 2x < 4\pi$

(b) Let $u = 2x$;
$u = \frac{\pi}{4}$, $u = \frac{3\pi}{4}$, $u = \frac{9\pi}{4}$, $u = \frac{11\pi}{4}$

(c) $x = \frac{\pi}{8}$, $x = \frac{3\pi}{8}$, $x = \frac{9\pi}{8}$, $x = \frac{11\pi}{8}$

Odd-Numbered Answers to Exercise Set 6.3: Solving Trigonometric Equations

39. **(a)** $-\frac{\pi}{2} \le x - \frac{\pi}{2} < \frac{3\pi}{2}$

(b) Let $u = x - \frac{\pi}{2}$; $u = -\frac{\pi}{3}$, $u = \frac{4\pi}{3}$

(c) $x = \frac{\pi}{6}$, $x = \frac{11\pi}{6}$

41. **(a)** $\pi \le 3x + \pi < 7\pi$

(b) Let $u = 3x + \pi$;

$u = \pi$, $u = 2\pi$, $u = 3\pi$, $u = 4\pi$, $u = 5\pi$, $u = 6\pi$

(c) $x = 0$, $x = \frac{\pi}{3}$, $x = \frac{2\pi}{3}$, $x = \pi$, $x = \frac{4\pi}{3}$, $x = \frac{5\pi}{3}$

43. $x = \frac{\pi}{8}$, $x = \frac{7\pi}{8}$, $x = \frac{9\pi}{8}$, $x = \frac{15\pi}{8}$

45. $x = 112.5^\circ$, $x = 157.5^\circ$, $x = 292.5^\circ$, $x = 337.5^\circ$

47. No solution

49. $x = 80^\circ$, $x = 100^\circ$, $x = 200^\circ$, $x = 220^\circ$, $x = 320^\circ$, $x = 340^\circ$

51. $x = \frac{5\pi}{24}$, $x = \frac{7\pi}{24}$, $x = \frac{17\pi}{24}$, $x = \frac{19\pi}{24}$, $x = \frac{29\pi}{24}$,

$x = \frac{31\pi}{24}$, $x = \frac{41\pi}{24}$, $x = \frac{43\pi}{24}$

53. $x = 135^\circ$, $x = 315^\circ$

55. $x = 0$, $x = \frac{5\pi}{3}$

57. $x = 0^\circ$, $x = 180^\circ$

59. $x = 330^\circ$

61. $x = 7.5^\circ$, $x = 37.5^\circ$, $x = 187.5^\circ$, $x = 217.5^\circ$

63. $x = 0^\circ$, $x = 180^\circ$, $x \approx 23.6^\circ$, $x \approx 156.4^\circ$

65. $x \approx 108.4^\circ$, $x \approx 288.4^\circ$, $x \approx 26.6^\circ$, $x \approx 206.6^\circ$

67. $x \approx 75.5^\circ$, $x \approx 104.4^\circ$, $x \approx 255.5^\circ$, $x \approx 284.5^\circ$

69. $x = \frac{\pi}{3}$, $x = \frac{5\pi}{3}$, $x = \pi$

71. $x = \frac{7\pi}{6}$, $x = \frac{11\pi}{6}$, $x = \frac{\pi}{2}$

73. $x = \frac{\pi}{3}$, $x = \frac{2\pi}{3}$, $x = \frac{4\pi}{3}$, $x = \frac{5\pi}{3}$

75. $x = \frac{\pi}{3}$, $x = \frac{5\pi}{3}$

77. $x = \frac{\pi}{4}$, $x = \frac{5\pi}{4}$

Odd-Numbered Answers to Exercise Set 7.1: Solving Right Triangles

1. **(a)** $\cos\left(30^\circ\right)=\dfrac{x}{10}$; $\sin\left(30^\circ\right)=\dfrac{y}{10}$

(b) $x=5\sqrt{3}$; $y=10$

(c) In a 30°-60°-90° triangle, the length of the hypotenuse (in this case, 10), is two times the length of the shorter leg (in this case, *y*). Therefore, $10=2y$, so $y=5$. The length of the longer leg (in this case, *x*) is $\sqrt{3}$ times the length of the shorter leg (in this case, 5). Therefore, $x=5\sqrt{3}$.

3. **(a)** $\tan\left(60^\circ\right)=\dfrac{7\sqrt{3}}{x}$; $\sin\left(60^\circ\right)=\dfrac{7\sqrt{3}}{y}$

(b) $x=7$; $y=14$

(c) In a 30°-60°-90° triangle, the length of the longer leg (in this case, $7\sqrt{3}$) is $\sqrt{3}$ times the length of the shorter leg (in this case, *x*). Therefore, $7\sqrt{3}=x\sqrt{3}$, so $x=7$. The length of the hypotenuse (in this case, *y*), is two times the length of the shorter leg (in this case, 7). Therefore, $y=2\left(7\right)=14$.

5. **(a)** $\tan\left(45^\circ\right)=\dfrac{5}{x}$; $\sin\left(45^\circ\right)=\dfrac{5}{y}$

(b) $x=5$; $y=5\sqrt{2}$

(c) In a 45°-45°-90° triangle, the legs are congruent, so $x=5$. The length of the hypotenuse (in this case, *y*) is $\sqrt{2}$ times the length of either leg (in this case, 5). Therefore, $y=5\sqrt{2}$.

7. **(a)** $\cos\left(45^\circ\right)=\dfrac{x}{7\sqrt{2}}$; $\sin\left(45^\circ\right)=\dfrac{y}{7\sqrt{2}}$

(b) $x=7$; $y=7$

(c) In a 45°-45°-90° triangle, the legs are congruent, so $x=y$. The length of the hypotenuse (in this case, $7\sqrt{2}$) is $\sqrt{2}$ times the length of either leg (in this case, we will use *y*). Therefore, $7\sqrt{2}=y\sqrt{2}$, so $y=7$. Since the legs are congruent, $x=y=7$.

9. **(a)** 0.7431 **(b)** 0.2924

11. **(a)** 0.3640 **(b)** 0.7818

13. $x\approx 48.43^\circ$

15. $x\approx 75.96^\circ$

17. $x\approx 19.75^\circ$

19. $x\approx 18.12^\circ$

21. $AC\approx 9.18$ in

23. $\angle H\approx 49.88^\circ$

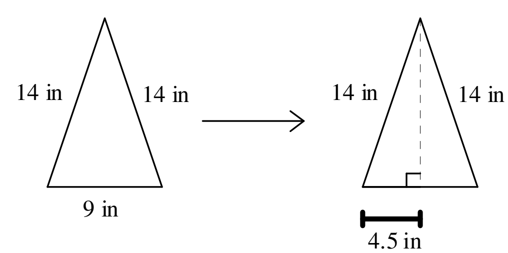
25. $\angle G\approx 37.87^\circ$

27. $\angle M=61^\circ$ (given)
$\angle D=90^\circ$ (given)
$\angle J=29^\circ$
$JD=3$ (given)
$MD\approx 1.66$
$JM\approx 3.43$

29. $\angle H=90^\circ$ (given)
$\angle L\approx 22.02^\circ$
$\angle P\approx 67.98^\circ$
$PH=3$ (given)
$PL=8$ (given)
$HL=\sqrt{55}\approx 7.42$

Note: The diagrams in 31-45 may not be drawn to scale.

31. **(a)**

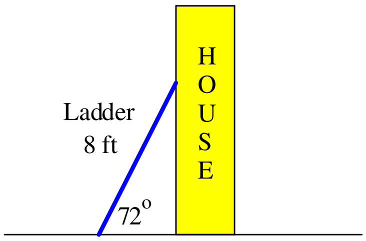


The second diagram above may be useful, since right triangle trigonometry can be used to find the missing angles.

(b) The measures of the angles are:
71.3°, 71.3°, and 31.4° (or 31.5°, depending on rounding and the method of solution).

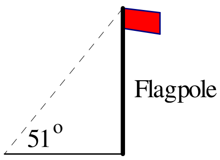
Odd-Numbered Answers to Exercise Set 7.1: Solving Right Triangles

33. (a)



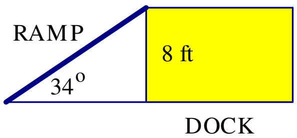
(b) 7.6 ft

35. (a)



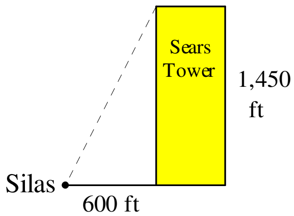
(b) 49.4 ft

37. (a)



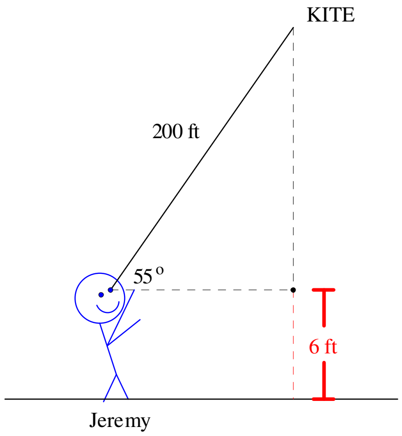
(b) 14.3 ft

39. (a)



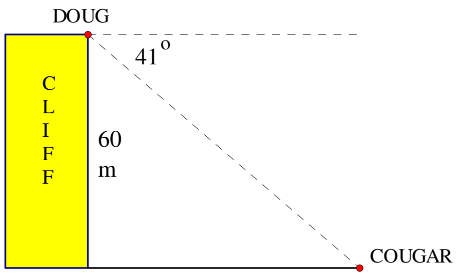
(b) 67.5°

41. (a)



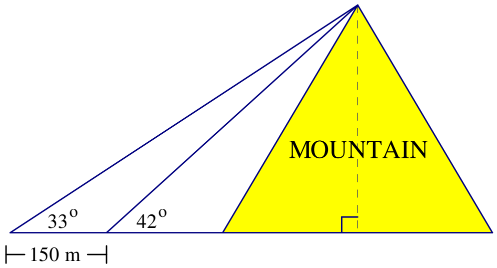
(b) 169.8 ft

43. (a)



(b) 69.0 m

45. (a)



(b) 388.1 m

Odd-Numbered Answers to Exercise Set 7.2: Area of a Triangle

1. $A = 40\ \text{cm}^2$

3. $A = \dfrac{3\sqrt{55}}{2}\ \text{ft}^2$

5. $A = \dfrac{25\sqrt{3}}{2}\ \text{m}^2$

7. $A \approx 62.36\ \text{in}^2$

9. **(a)** $\sin(C) = \dfrac{h}{b}$

(b) $h = b\sin(C)$

(c) $K = \frac{1}{2}ab\sin(C)$

11. $A = 15\sqrt{3}\ \text{in}^2$

13. $A = 18\sqrt{2}\ \text{m}^2$

15. $A \approx 22.53\ \text{m}^2$

17. $A \approx 11.48\ \text{m}^2$

19. $A = \dfrac{49\sqrt{2}}{4}\ \text{in}^2$

21. $A = 30\sqrt{3}\ \text{cm}^2$

23. $A \approx 36.88\ \text{m}^2$

25. **(a)** $x = 30^\circ,\quad x = 150^\circ$

(b) $\angle S = 30^\circ$ or $\angle S = 150^\circ$

27. $\angle R \approx 47.73^\circ$ or $\angle R \approx 132.27^\circ$

29. $A = 288\sqrt{2}\ \text{in}^2$

31. $A = 72\sqrt{3}\ \text{in}^2$

33. $4\sqrt{3}\ \text{ft}^2$

35. $492.43\ \text{in}^2$

37. $A \approx 11.89\ \text{cm}^2$

39. $A = \left(\dfrac{8\pi}{3} - 4\sqrt{3}\right)\ \text{in}^2$

Odd-Numbered Answers to Exercise Set 7.3: The Law of Sines and the Law of Cosines

1. $b \approx 8.76$ m

3. $c = 3\sqrt{6}$ in

5. $\angle N \approx 27.66^\circ$

7. $s \approx 8.34$ cm

9. $\angle C \approx 51.75^\circ$

11. No such triangle exists.

13. $t = 6\sqrt{2}$ cm

15. $\angle D \approx 43.56^\circ$ or $\angle D \approx 136.44^\circ$

17. $\angle C \approx 41.62^\circ$

19. No such triangle exists.

21. $\angle A \approx 12.56^\circ$

23. $\angle D = 20^\circ$
$\angle E = 125^\circ$ (given)
$\angle F = 35^\circ$ (given)
$d \approx 5.62$ ft
$e \approx 13.45$ ft
$f = 9.42$ ft (given)

25.

$\angle W \approx 82.13^\circ$	$\angle W \approx 41.87^\circ$
$\angle H \approx 69.87^\circ$	$\angle H \approx 110.13^\circ$
$\angle Y = 28^\circ$ (given)	$\angle Y = 28^\circ$ (given)
$w \approx 8.44$ in	$w \approx 5.69$ in
$h = 8$ in (given)	$h = 8$ in (given)
$y = 4$ in (given)	$y = 4$ in (given)

27. No such triangle exists.

29. $\angle P \approx 27.42^\circ$
$\angle E = 130^\circ$ (given)
$\angle Z \approx 22.58^\circ$
$p = 12$ mm (given)
$e \approx 19.96$ mm
$z = 10$ mm (given)

31. $\angle P \approx 27.36^\circ$
$\angle I \approx 95.22^\circ$
$\angle G \approx 57.42^\circ$
$p = 6$ cm (given)
$i = 13$ cm (given)
$g = 11$ cm (given)

33. No such triangle exists.

35. $653.3\ \text{cm}^2$

37. $277.5\ \text{in}^2$

39. **(a)** 36.87°, 53.13°
(b) 36.87°, 53.13°

41. $\angle E \approx 14.06^\circ$, $\angle Y \approx 40.94^\circ$

43. $AD \approx 5.64$ mm

Odd-Numbered Answers to Exercise Set 8.1: Parabolas

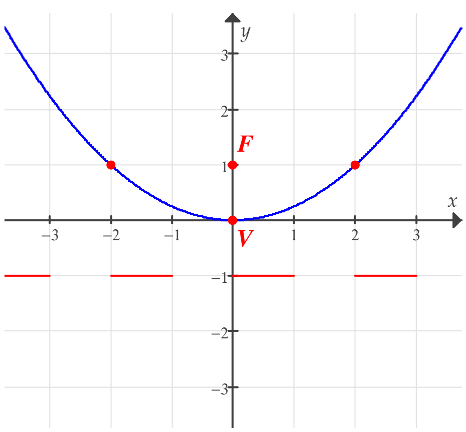
1. $(y-7)^2 = 2(x+3)$

3. $(x-4)^2 = -9\left(y - \frac{26}{9}\right)$

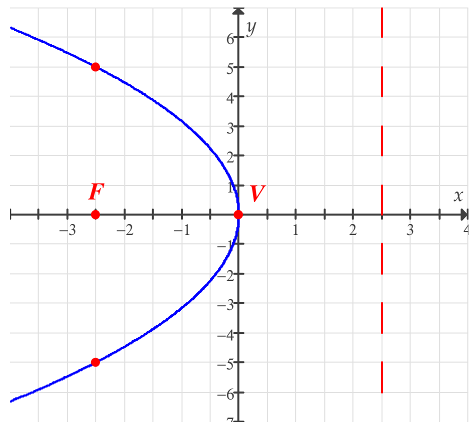
5. $(y-4)^2 = \frac{1}{3}(x-2)$

7. $\left(x - \frac{1}{2}\right)^2 = \frac{1}{3}\left(y + \frac{23}{4}\right)$

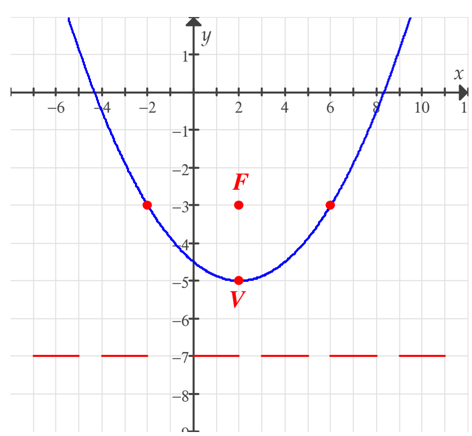
9. **(a)** Equation: $x^2 = 4y$
(b) Axis: $x = 0$
(c) Vertex: $(0, 0)$
(d) Directrix: $y = -1$
(e) Focus: $(0, 1)$
(f) Focal Width: 4
(g) Endpoints of focal chord: $(-2, 1)$, $(2, 1)$
(h) Graph:



11. **(a)** Equation: $y^2 = -10x$
(b) Axis: $y = 0$
(c) Vertex: $(0, 0)$
(d) Directrix: $x = 2\frac{1}{2}$
(e) Focus: $\left(-2\frac{1}{2}, 0\right)$
(f) Focal Width: 10
(g) Endpoints of focal chord: $\left(-2\frac{1}{2}, -5\right)$, $\left(-2\frac{1}{2}, 5\right)$
(h) Graph:



13. **(a)** Equation: $(x-2)^2 = 8(y+5)$
(b) Axis: $x = 2$
(c) Vertex: $(2, -5)$
(d) Directrix: $y = -7$
(e) Focus: $(2, -3)$
(f) Focal Width: 8
(g) Endpoints of focal chord: $(-2, -3)$, $(6, -3)$
(h) Graph:



15. **(a)** Equation: $y^2 = 4(x-3)$

(b) Axis: $y = 0$

(c) Vertex: $(3, 0)$

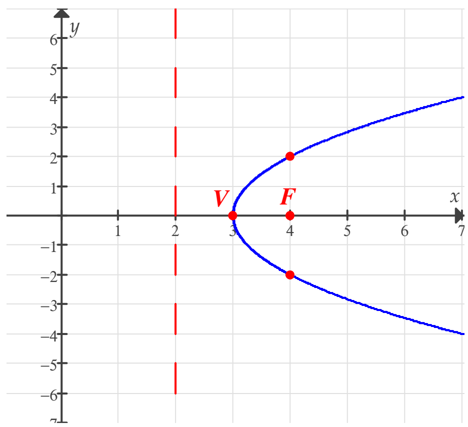
(d) Directrix: $x = 2$

(e) Focus: $(4, 0)$

(f) Focal Width: 4

(g) Endpoints of focal chord: $(4, -2)$, $(4, 2)$

(h) Graph:



17. **(a)** Equation: $(x+5)^2 = -2(y-4)$

(b) Axis: $x = -5$

(c) Vertex: $(-5, 4)$

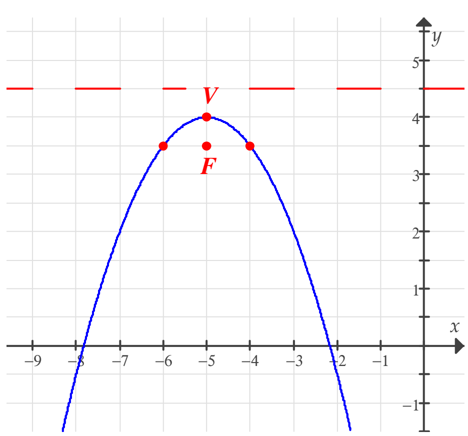
(d) Directrix: $y = 4\frac{1}{2}$

(e) Focus: $(-5, 3\frac{1}{2})$

(f) Focal Width: 2

(g) Endpoints of focal chord: $(-6, 3\frac{1}{2})$, $(-4, 3\frac{1}{2})$

(h) Graph:



19. **(a)** Equation: $(y-6)^2 = -1(x-2)$

(b) Axis: $y = 6$

(c) Vertex: $(2, 6)$

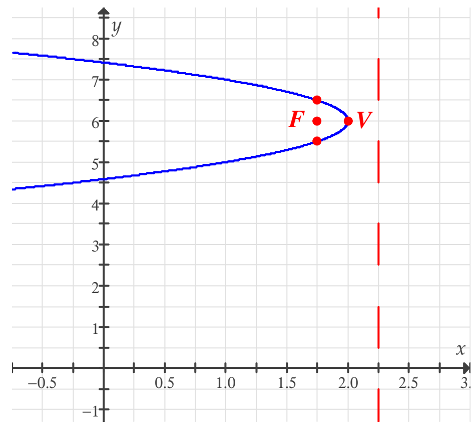
(d) Directrix: $x = 2\frac{1}{4}$

(e) Focus: $(1\frac{3}{4}, 6)$

(f) Focal Width: 1

(g) Endpoints of focal chord: $(1\frac{3}{4}, 5\frac{1}{2})$, $(1\frac{3}{4}, 6\frac{1}{2})$

(h) Graph:



21. **(a)** Equation: $(x+6)^2 = 6(y+2)$

(b) Axis: $x = -6$

(c) Vertex: $(-6, -2)$

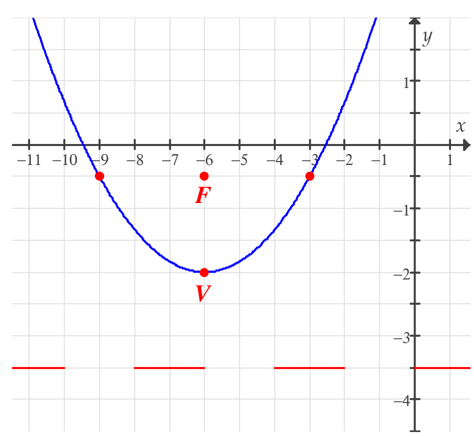
(d) Directrix: $y = -3\frac{1}{2}$

(e) Focus: $(-6, -\frac{1}{2})$

(f) Focal Width: 6

(g) Endpoints of focal chord: $(-9, -\frac{1}{2})$, $(-3, -\frac{1}{2})$

(h) Graph:



23. **(a)** Equation: $(y-2)^2 = 8(x+5)$

(b) Axis: $y = 2$

(c) Vertex: $(-5, 2)$

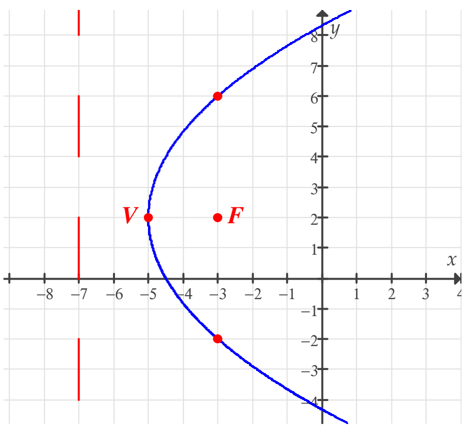
(d) Directrix: $x = -7$

(e) Focus: $(-3, 2)$

(f) Focal Width: 8

(g) Endpoints of focal chord: $(-3, -2)$, $(-3, 6)$

(h) Graph:



25. **(a)** Equation: $(x+5)^2 = -16y$

(b) Axis: $x = -5$

(c) Vertex: $(-5, 0)$

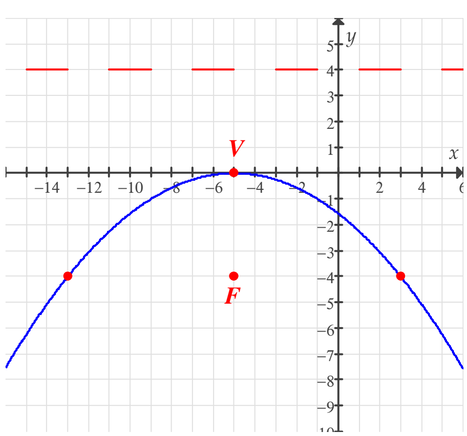
(d) Directrix: $y = 4$

(e) Focus: $(-5, -4)$

(f) Focal Width: 16

(g) Endpoints of focal chord: $(-13, -4)$, $(3, -4)$

(h) Graph:



27. **(a)** Equation: $(y-2)^2 = -2(x-4)$

(b) Axis: $y = 2$

(c) Vertex: $(4, 2)$

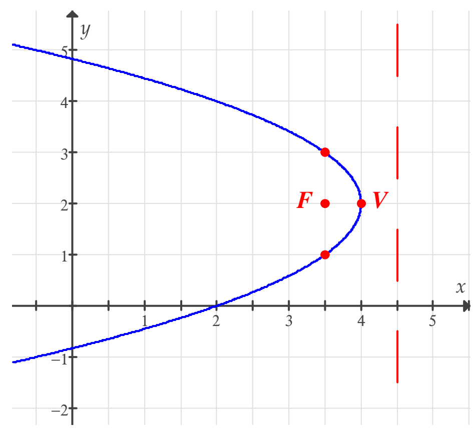
(d) Directrix: $x = 4\frac{1}{2}$

(e) Focus: $(3\frac{1}{2}, 2)$

(f) Focal Width: 2

(g) Endpoints of focal chord: $(3\frac{1}{2}, 1)$, $(3\frac{1}{2}, 3)$

(h) Graph:



29. **(a)** Equation: $(y+5)^2 = \frac{8}{3}(x+1)$

(b) Axis: $y = -5$

(c) Vertex: $(-1, -5)$

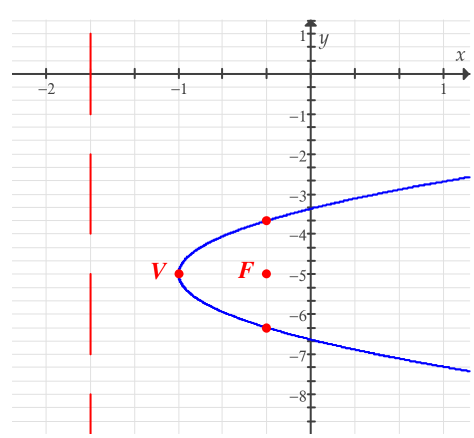
(d) Directrix: $x = -1\frac{2}{3}$

(e) Focus: $(-\frac{1}{3}, -5)$

(f) Focal Width: $\frac{8}{3}$

(g) Endpoints of focal chord:
$(-\frac{1}{3}, -6\frac{1}{3})$, $(-\frac{1}{3}, -3\frac{2}{3})$

(h) Graph:



Odd-Numbered Answers to Exercise Set 8.1: Parabolas

31. **(a)** Equation: $(x-2)^2 = -\frac{7}{2}(y-6)$

(b) Axis: $x = 2$

(c) Vertex: $(2, 6)$

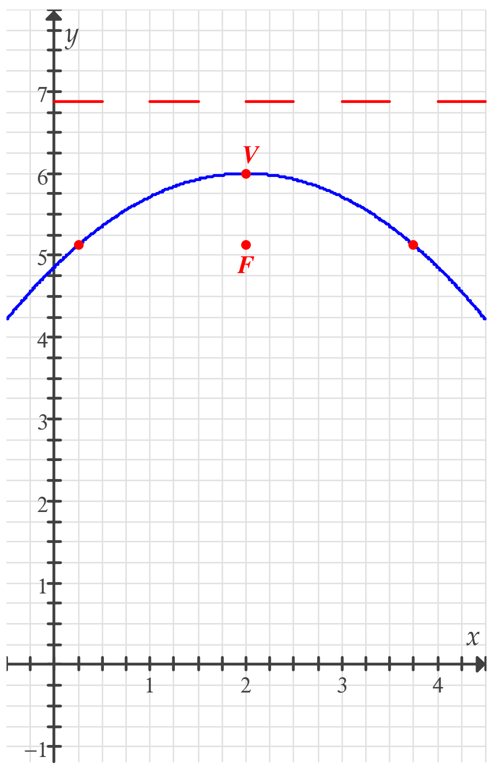
(d) Directrix: $y = 6\frac{7}{8}$

(e) Focus: $(2, 5\frac{1}{8})$

(f) Focal Width: $\frac{7}{2}$

(g) Endpoints of focal chord:
$(\frac{1}{4}, 5\frac{1}{8}),\ (3\frac{3}{4}, 5\frac{1}{8})$

(h) Graph:



33. $(y-5)^2 = 24(x+2)$

35. $(x-2)^2 = 16y$

37. $(x+2)^2 = 12(y+6)$

39. $(y+1)^2 = -7(x-5\frac{3}{4})$

41. $(x+4)^2 = -28(y-5)$

43. $(x-5)^2 = 6(y-6)$

45. $(y-2)^2 = x+3$

47. $y^2 = -10(x-2\frac{1}{2})$

49. **(a)** $y = 11x-5$
(b) $y = x$

51. **(a)** $y = x-3$
(b) $y = 7x-\frac{3}{2}$

53. $y = 3x-7$

55. $y = 4x+26$

57. $(2, 7)$ and $(7, 32)$

59. $(-1, 1)$

61. $(-3, -32)$ and $(3, 4)$

63. $(2, 1)$ and $(1, 4)$

Odd-Numbered Answers to Exercise Set 8.2: Ellipses

1. $\dfrac{x^2}{4}+\dfrac{y^2}{25}=1$

3. $\dfrac{(x-2)^2}{4}+\dfrac{(y-4)^2}{9}=1$

5. $\dfrac{(x-5)^2}{2}+\dfrac{(y-3)^2}{3}=1$

7. $\dfrac{\left(x-\frac{1}{2}\right)^2}{8}+\dfrac{\left(y+\frac{3}{2}\right)^2}{16}=1$

9. **(a)** $e=\dfrac{c}{a}$

(b) elongated

(c) circle

11. **(a)** Equation: $\dfrac{x^2}{9}+\dfrac{y^2}{49}=1$

(b) Center: $(0,0)$

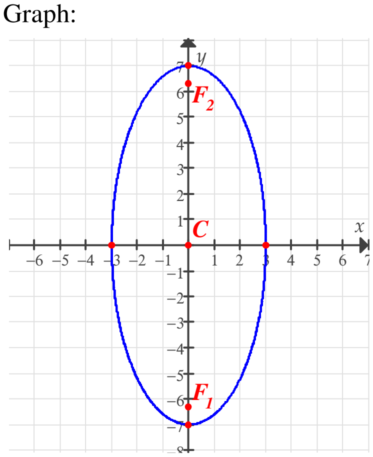
(c) Vertices of Major Axis: $(0,-7),\ (0,7)$
Length of Major Axis: 14

(d) Vertices of Minor Axis: $(-3,0),\ (3,0)$
Length of Minor Axis: 6

(e) Foci: $\left(0,-2\sqrt{10}\right),\ \left(0,2\sqrt{10}\right)$ *(exact)*
$(0,-6.3),\ (0,6.3)$ *(decimal)*

(f) Eccentricity: $e=\dfrac{2\sqrt{10}}{7}\approx 0.9$

(g) Graph:



13. **(a)** Equation: $\dfrac{(x-2)^2}{16}+\dfrac{y^2}{4}=1$

(b) Center: $(2,0)$

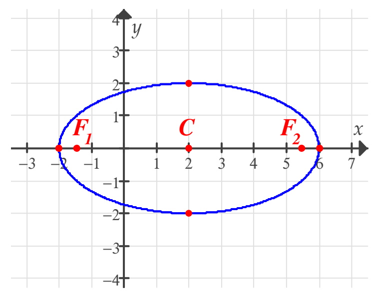
(c) Vertices of Major Axis: $(-2,0),\ (6,0)$
Length of Major Axis: 8

(d) Vertices of Minor Axis: $(2,-2),\ (2,2)$
Length of Minor Axis: 4

(e) Foci: $\left(2-2\sqrt{3},0\right),\ \left(2+2\sqrt{3},0\right)$ *(exact)*
$(-1.5,0),\ (5.5,0)$ *(decimal)*

(f) Eccentricity: $e=\dfrac{\sqrt{3}}{2}\approx 0.9$

(g) Graph:



15. **(a)** Equation: $\dfrac{(x-2)^2}{25}+\dfrac{(y+3)^2}{16}=1$

(b) Center: $(2,-3)$

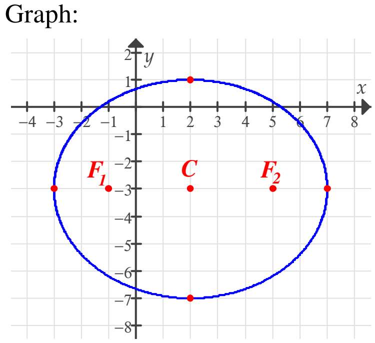
(c) Vertices of Major Axis: $(-3,-3),\ (7,-3)$
Length of Major Axis: 10

(d) Vertices of Minor Axis: $(2,-7),\ (2,1)$
Length of Minor Axis: 8

(e) Foci: $(-1,-3),\ (5,-3)$

(f) Eccentricity: $e=\dfrac{3}{5}$

(g) Graph:



Odd-Numbered Answers to Exercise Set 8.2: Ellipses

17. **(a)** Equation: $\dfrac{(x+4)^2}{9}+\dfrac{(y-3)^2}{1}=1$

(b) Center: $(-4,3)$

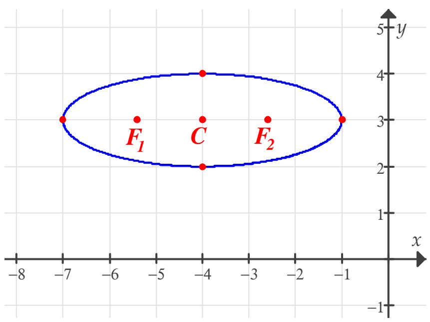
(c) Vertices of Major Axis: $(-7,3),\ (-1,3)$
Length of Major Axis: 6

(d) Vertices of Minor Axis: $(-4,2),\ (-4,4)$
Length of Minor Axis: 2

(e) Foci: $\left(-4-\sqrt{2},3\right),\ \left(-4+\sqrt{2},3\right)$ *(exact)*
$(-5.4,3),\ (-2.6,3)$ *(decimal)*

(f) Eccentricity: $e=\dfrac{\sqrt{2}}{3}\approx 0.5$

(g) Graph:



19. **(a)** Equation: $\dfrac{(x-2)^2}{11}+\dfrac{(y+4)^2}{36}=1$

(b) Center: $(2,-4)$

(c) Vertices of Major Axis: $(2,-10),\ (2,2)$
Length of Major Axis: 12

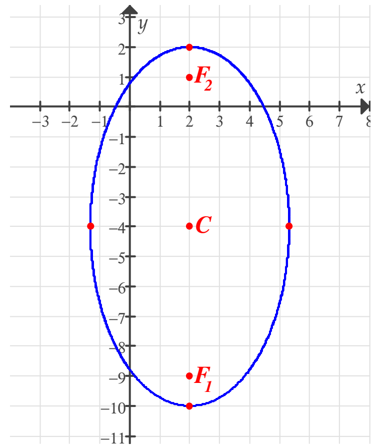
(d) Vertices of Minor Axis:
$\left(2-\sqrt{11},-4\right),\ \left(2+\sqrt{11},-4\right)$ *(exact)*
$(-1.3,-4),\ (5.3,-4)$ *(decimal)*
Length of Minor Axis: $2\sqrt{11}\approx 6.6$

(e) Foci: $(2,-9),\ (2,1)$

(f) Eccentricity: $e=\dfrac{5}{6}$

Continued in the next column…

(g) Graph:



21. **(a)** Equation: $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$

(b) Center: $(0,0)$

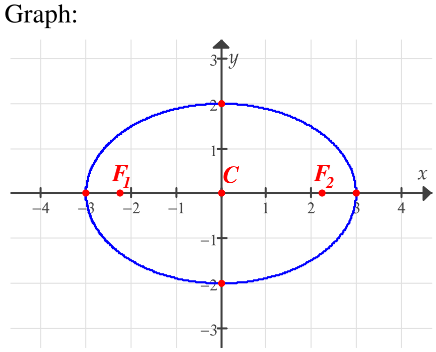
(c) Vertices of Major Axis: $(-3,0),\ (3,0)$
Length of Major Axis: 6

(d) Vertices of Minor Axis: $(0,-2),\ (0,2)$
Length of Minor Axis: 4

(e) Foci: $\left(-\sqrt{5},0\right),\ \left(\sqrt{5},0\right)$ *(exact)*
$(-2.2,0),\ (2.2,0)$ *(decimal)*

(f) Eccentricity: $e=\dfrac{\sqrt{5}}{3}\approx 0.7$

(g) Graph:



23. **(a)** Equation: $\dfrac{(x-1)^2}{16}+\dfrac{(y+2)^2}{25}=1$

(b) Center: $(1,-2)$

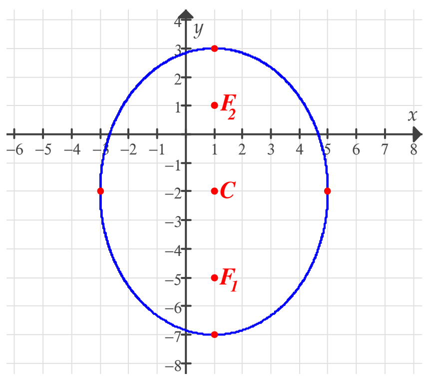
(c) Vertices of Major Axis: $(1,-7),\ (1,3)$
Length of Major Axis: 10

(d) Vertices of Minor Axis: $(-3,-2),\ (5,-2)$
Length of Minor Axis: 8

(e) Foci: $(1,-5),\ (1,1)$

(f) Eccentricity: $e=\dfrac{3}{5}$

(g) Graph:



25. **(a)** Equation: $\dfrac{(x-1)^2}{4}+\dfrac{(y+5)^2}{16}=1$

(b) Center: $(1,5)$

(c) Vertices of Major Axis: $(1,1),\ (1,9)$
Length of Major Axis: 8

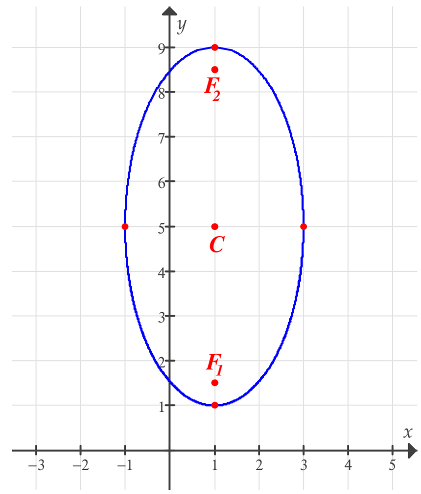
(d) Vertices of Minor Axis: $(-1,5),\ (3,5)$
Length of Minor Axis: 4

(e) Foci: $\left(1,5-2\sqrt{3}\right),\ \left(1,5+2\sqrt{3}\right)$ *(exact)*
$(1,8.5),\ (1,1.5)$ *(decimal)*

(f) Eccentricity: $e=\dfrac{\sqrt{3}}{2}\approx 0.9$

Continued in the next column…

(g) Graph:



27. **(a)** Equation: $\dfrac{(x+2)^2}{7}+\dfrac{(y-3)^2}{16}=1$

(b) Center: $(-2,3)$

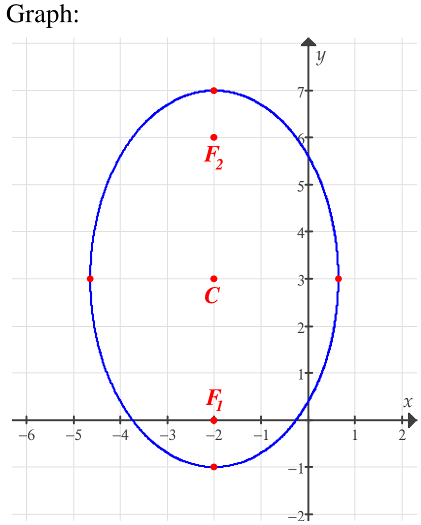
(c) Vertices of Major Axis: $(-2,-1),\ (-2,7)$
Length of Major Axis: 8

(d) Vertices of Minor Axis:
$\left(-2-\sqrt{7},3\right),\ \left(-2+\sqrt{7},3\right)$ *(exact)*
$(-4.6,3),\ (0.6,3)$ *(decimal)*
Length of Minor Axis: $2\sqrt{7}\approx 5.3$

(e) Foci: $(-2,0),\ (-2,6)$

(f) Eccentricity: $e=\dfrac{3}{4}$

(g) Graph:



29. $\dfrac{x^2}{64}+\dfrac{y^2}{25}=1$

31. $\dfrac{(x+4)^2}{9}+\dfrac{(y-7)^2}{25}=1$

33. $\dfrac{(x+3)^2}{9}+\dfrac{(y+5)^2}{4}=1$

35. $\dfrac{(x-2)^2}{56}+\dfrac{y^2}{81}=1$

37. $\dfrac{(x+3)^2}{36}+\dfrac{(y-1)^2}{11}=1$

39. $\dfrac{(x-3)^2}{25}+\dfrac{(y-2)^2}{9}=1$

41. $\dfrac{(x+5)^2}{28}+\dfrac{(y-2)^2}{64}=1$

43. $\dfrac{x^2}{32}+\dfrac{(y-6)^2}{36}=1$

45. $\dfrac{(x-4)^2}{25}+\dfrac{(y-3)^2}{21}=1$

47. $\dfrac{x^2}{16}+\dfrac{y^2}{7}=1$

49. $x^2+y^2=81$

51. $(x-7)^2+(y+2)^2=100$

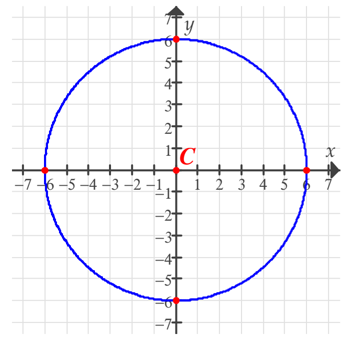
53. $(x+3)^2+(y+4)^2=18$

55. $(x-2)^2+(y+5)^2=26$

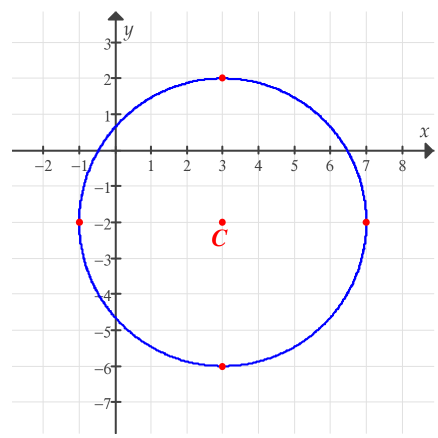
57. $(x+3)^2+(y+3)^2=10$

59. $(x+3)^2+(y-5)^2=25$

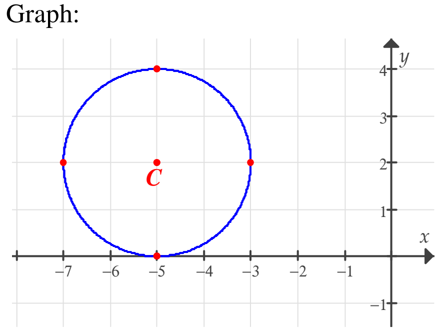
61. **(a)** Equation: $x^2+y^2=36$
(b) Center: $(0, 0)$
(c) Radius: 6
(d) Graph:



63. **(a)** Equation: $(x-3)^2+(y+2)^2=16$
(b) Center: $(3, -2)$
(c) Radius: 4
(d) Graph:



65. **(a)** Equation: $(x+5)^2+(y-2)^2=4$
(b) Center: $(-5, 2)$
(c) Radius: 2
(d) Graph:

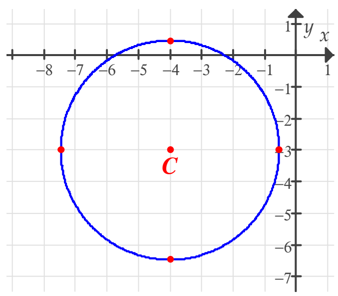


67. **(a)** Equation: $(x+4)^2+(y+3)^2=12$

(b) Center: $(-4,-3)$

(c) Radius: $2\sqrt{3}\approx 3.5$

(d) Graph:

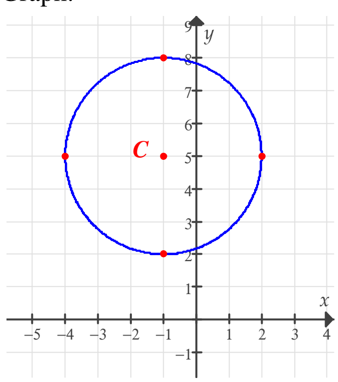


69. **(a)** Equation: $(x+1)^2+(y-5)^2=9$

(b) Center: $(-1,5)$

(c) Radius: 3

(d) Graph:

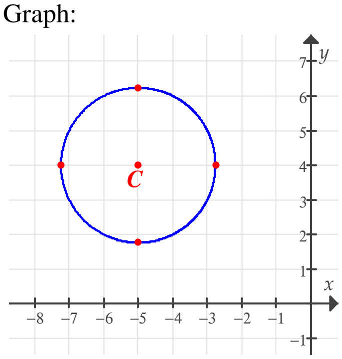


71. **(a)** Equation: $(x+5)^2+(y-4)^2=5$

(b) Center: $(-5,4)$

(c) Radius: $\sqrt{5}\approx 2.2$

(d) Graph:

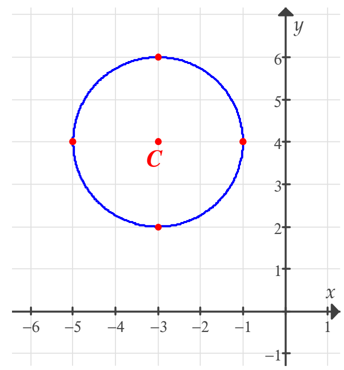


73. **(a)** Equation: $(x+3)^2+(y-4)^2=4$

(b) Center: $(-3,4)$

(c) Radius: 2

(d) Graph:



75. $(x-3)^2+(y-2)^2=32$

77. $(x-5)^2+(y+1)^2=13$

Odd-Numbered Answers to Exercise Set 8.3: Hyperbolas

1. Parabola

3. Hyperbola

5. Circle

7. Ellipse

9. $\dfrac{y^2}{8} - \dfrac{x^2}{1} = 1$

11. $\dfrac{(x-3)^2}{5} - \dfrac{(y-1)^2}{5} = 1$

13. $\dfrac{(x+1)^2}{5} - \dfrac{(y-2)^2}{7} = 1$

15. $2a$

17. **(a)** $y - y_1 = m(x - x_1)$

(b) $y - k = m(x - h)$

(c) Rise: $\pm a$
Run: $\pm b$

(d) Slopes: $\dfrac{a}{b},\ -\dfrac{a}{b};$ *i.e.* $\pm\dfrac{a}{b}$

(e) $y - k = \dfrac{a}{b}(x-h),\ y - k = -\dfrac{a}{b}(x-h);$

i.e. $y - k = \pm\dfrac{a}{b}(x-h)$

19. In the standard form for the equation of a hyperbola, a^2 represents the denominator of the first term.

21. **(a)** Equation: $\dfrac{y^2}{9} - \dfrac{x^2}{49} = 1$

(b) Center: $(0, 0)$

(c) Vertices: $(0, -3),\ (0, 3)$
Length of transverse axis: 6

(d) Endpoints of conjugate axis: $(-7, 0),\ (7, 0)$
Length of conjugate axis: 14

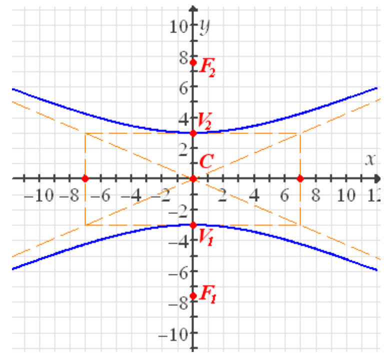
(e) Foci: $\left(0, -\sqrt{58}\right),\ \left(0, \sqrt{58}\right)$ *(exact)*
$(0, -7.6),\ (0, 7.6)$ *(decimal)*

Continued in the next column…

(f) Asymptotes: $y = \pm\dfrac{3}{7}x$

(g) Eccentricity: $e = \dfrac{\sqrt{58}}{3} \approx 2.5$

(h) Graph:



23. **(a)** Equation: $\dfrac{x^2}{25} - \dfrac{y^2}{9} = 1$

(b) Center: $(0, 0)$

(c) Vertices: $(-5, 0),\ (5, 0)$
Length of transverse axis: 10

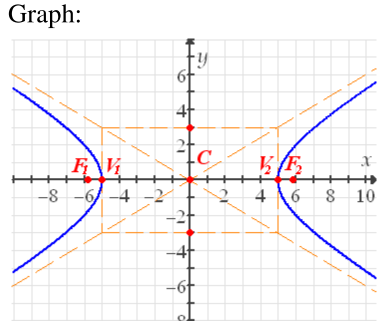
(d) Endpoints of conjugate axis: $(0, -3),\ (0, 3)$
Length of conjugate axis: 6

(e) Foci: $\left(-\sqrt{34}, 0\right),\ \left(\sqrt{34}, 0\right)$ *(exact)*
$(-5.8, 0),\ (5.8, 0)$ *(decimal)*

(f) Asymptotes: $y = \pm\dfrac{3}{5}x$

(g) Eccentricity: $e = \dfrac{\sqrt{34}}{5} \approx 1.2$

(h) Graph:



25. **(a)** Equation: $\dfrac{(x+1)^2}{16}-\dfrac{(y-5)^2}{9}=1$

(b) Center: $(-1,5)$

(c) Vertices: $(-5,5),\ (3,5)$
Length of transverse axis: 8

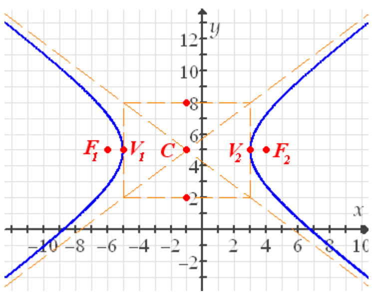
(d) Endpoints of conjugate axis: $(-1,2),\ (-1,8)$
Length of conjugate axis: 6

(e) Foci: $(-6,5),\ (4,5)$

(f) Asymptotes: $y-5=\pm\dfrac{3}{4}(x+1)$

(g) Eccentricity: $e=\dfrac{5}{4}$

(h) Graph:



27. **(a)** Equation: $\dfrac{(y-3)^2}{25}-\dfrac{(x-1)^2}{36}=1$

(b) Center: $(1,3)$

(c) Vertices: $(1,-2),\ (1,8)$
Length of transverse axis: 10

(d) Endpoints of conjugate axis: $(-5,3),\ (7,3)$
Length of conjugate axis: 12

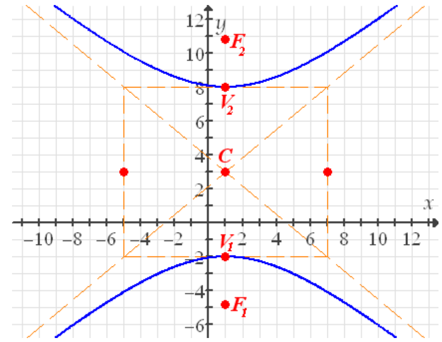
(e) Foci: $\left(1,\ 3\pm\sqrt{61}\right)$ *(exact)*
$(1,-4.8),\ (1,10.8)$ *(decimal)*

(f) Asymptotes: $y-3=\pm\dfrac{5}{6}(x-1)$

(g) Eccentricity: $e=\dfrac{\sqrt{61}}{5}\approx 1.6$

Continued in the next column…

(h) Graph:



29. **(a)** Equation: $\dfrac{(x+4)^2}{25}-\dfrac{(y+3)^2}{1}=1$

(b) Center: $(-4,-3)$

(c) Vertices: $(-9,-3),\ (1,-3)$
Length of transverse axis: 10

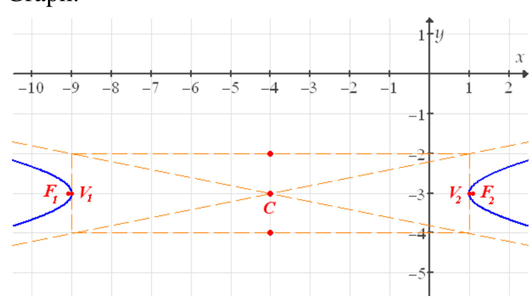
(d) Endpoints of conjugate axis: $(-4,-4),\ (-4,-2)$
Length of conjugate axis: 2

(e) Foci: $\left(-4-\sqrt{26},-3\right),\left(-4+\sqrt{26},-3\right)$ *(exact)*
$(-9.1,-3),\ (1.1,-3)$ *(decimal)*

(f) Asymptotes: $y+3=\pm\dfrac{1}{5}(x+4)$

(g) Eccentricity: $e=\dfrac{\sqrt{26}}{5}\approx 1.0$

(h) Graph:



Note: The foci are very close to the vertices in the above diagram.

31. **(a)** Equation: $\dfrac{(x+1)^2}{9}-\dfrac{(y-1)^2}{64}=1$

(b) Center: $(-1,1)$

(c) Vertices: $(-4,1)$, $(2,1)$

Length of transverse axis: 6

(d) Endpoints of conjugate axis: $(-1,-7)$, $(-1,9)$

Length of conjugate axis: 16

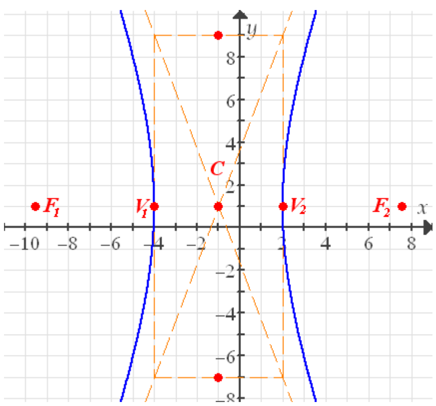
(e) Foci: $\left(-1-\sqrt{73},1\right)$, $\left(-1+\sqrt{73},1\right)$ *(exact)*

$(-9.5,1)$, $(7.5,1)$ *(decimal)*

(f) Asymptotes: $y-1=\pm\dfrac{8}{3}(x+1)$

(g) Eccentricity: $e=\dfrac{\sqrt{73}}{3}\approx 2.8$

(h) Graph:



33. **(a)** Equation: $\dfrac{(y-5)^2}{4}-\dfrac{(x+3)^2}{5}=1$

(b) Center: $(-3,5)$

(c) Vertices: $(-3,3)$, $(-3,7)$

Length of transverse axis: 4

(d) Endpoints of conjugate axis:

$\left(-3-\sqrt{5},5\right)$, $\left(-3+\sqrt{5},5\right)$ *(exact)*

$(-5.2,5)$, $(-0.7,5)$ *(decimal)*

Length of conjugate axis: $2\sqrt{5}\approx 4.5$

(e) Foci: $\left(-3,5-\sqrt{41}\right)$, $\left(-3,5+\sqrt{41}\right)$ *(exact)*

$(-3,-1.4)$, $(-3,11.4)$ *(decimal)*

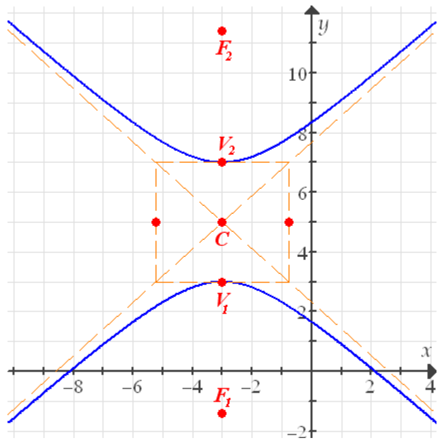
Continued in the next column…

(f) Asymptotes: $y-5=\pm\dfrac{2\sqrt{5}}{5}(x+3)$ *(exact)*

$y-5\approx\pm 0.9(x+3)$ *(decimal)*

(g) Eccentricity: $e=\dfrac{\sqrt{41}}{2}\approx 3.2$

(h) Graph:



Numbers 35-57 can be found on the next page…

Odd-Numbered Answers to Exercise Set 8.3: Hyperbolas

35. **(a)** Equation: $\dfrac{(x-9)^2}{108}-\dfrac{y^2}{36}=1$

(b) Center: $(9,0)$

(c) Vertices: $\left(9-6\sqrt{3},0\right),\ \left(9+6\sqrt{3},0\right)$ *(exact)*

$(-1.4,0),\ (19.4,0)$

Length of transverse axis: $12\sqrt{3}\approx 20.8$

(d) Endpoints of conjugate axis: $(9,-6),\ (9,6)$

Length of conjugate axis: 12

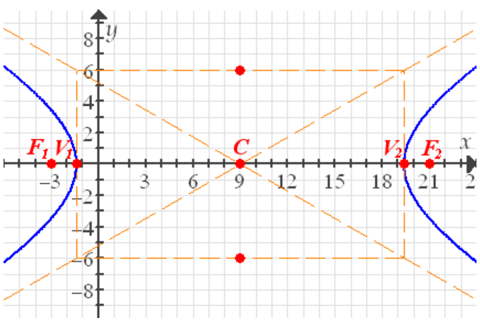
(e) Foci: $(-3,0),\ (21,0)$

(f) Asymptotes: $y=\pm\dfrac{\sqrt{3}}{3}(x-9)$ *(exact)*

$y\approx 0.6(x-9)$ *(decimal)*

(g) Eccentricity: $e=\dfrac{2\sqrt{3}}{3}\approx 1.1$

(h) Graph:



37. $\dfrac{x^2}{64}-\dfrac{y^2}{25}=1$

39. $\dfrac{(y+5)^2}{4}-\dfrac{(x+2)^2}{100}=1$

41. $\dfrac{(y-1)^2}{25}-\dfrac{(x+6)^2}{16}=1$

43. $\dfrac{y^2}{9}-\dfrac{x^2}{72}=1$

45. $\dfrac{(x-4)^2}{11}-\dfrac{(y-3)^2}{25}=1$

47. $\dfrac{(y-1)^2}{64}-\dfrac{(x-4)^2}{16}=1$

49. $\dfrac{(y-3)^2}{9}-\dfrac{(x-5)^2}{16}=1$

51. $\dfrac{(x+1)^2}{16}-\dfrac{(y-2)^2}{49}=1$

53. $\dfrac{(x-3)^2}{16}-\dfrac{(y-4)^2}{128}=1$

55. $\dfrac{(y+3)^2}{36}-\dfrac{(x-4)^2}{45}=1$

57. $\dfrac{(y-4)^2}{9}-\dfrac{(x-3)^2}{7}=1$

Odd-Numbered Answers to Exercise Set A.1: Factoring Polynomials

1. **(a)** 13
(b) No

3. **(a)** 100
(b) Yes

5. **(a)** 36
(b) Yes

7. **(a)** 81
(b) Yes

9. **(a)** −36
(b) No

11. $(x+5)(x-1)$

13. $(x-3)(x-2)$

15. $x^2-7x-12$

17. $(x+10)(x+2)$

19. $(x+3)(x-8)$

21. $(x+8)^2$

23. $(x-7)(x-8)$

25. $(x+4)(x-15)$

27. $(x+3)(x+14)$

29. $(x+7)(x-7)$

31. x^2-3

33. $9x^2+25$

35. $(2x+1)(x-3)$

37. $(2x+1)(4x-3)$

39. $(3x+4)(3x-1)$

41. $(4x+5)(x-2)$

43. $(4x-3)(3x-2)$

45. $x(x+9)$

47. $-5x(x-4)$

49. $2(x+3)(x-3)$

51. $-5x^2(x+2)(x-2)$

53. $2(x+4)(x+1)$

55. $-10(x-7)(x+6)$

57. $x(x+11)(x-2)$

59. $-x(x+2)^2$

61. $x^2\left(x^2+6x+6\right)$

63. $x^3(3x+10)(3x-10)$

65. $5(2x+1)(5x+3)$

67. $(x+2)(x+5)(x-5)$

69. $(x-5)\left(x^2+4\right)$

71. $(x+9)(2x+1)(2x-1)$

Odd-Numbered Answers to Exercise Set A.2
Dividing Polynomials

1. Quotient: $x-4$;
Remainder: 3

3. Quotient: $x+6$;
Remainder: -8

5. Quotient: x^2-5x-4;
Remainder: 0

7. Quotient: $3x^2+4x+5$;
Remainder: -7

9. Quotient: $2x+7$;
Remainder: $5x+14$

11. Quotient: $\frac{1}{2}x^3+8x^2-1$;
Remainder: -8

13. Quotient: $3x^2-x-15$;
Remainder: $5x+60$

15. Quotient: $x+2$;
Remainder: 24

17. Quotient: $3x^2-2x+4$;
Remainder: 8

19. Quotient: x^3-x^2+4x-4;
Remainder: 0

21. Quotient: $3x^3+4x^2-7x-17$;
Remainder: -75

23. Quotient: x^2-2x+4;
Remainder: 0

25. Quotient: $4x^2+2x-6$;
Remainder: 2

27. **(a)** Using substitution, $P(2)=4$
(b) The remainder is 4, so $P(2)=4$.

29. **(a)** $P(-1)=-7$
(b) The remainder is -7, so $P(-1)=-7$.

31. $P(5)=-97$

33. $P\left(-\frac{3}{4}\right)=-10$

35. $x^2-11x+24=(x-8)(x-3)$

37. $x^2-7x-18=(x+2)(x-9)$

39. $4x^2-25x-21=(x-7)(4x+3)$

41. $2x^2+7x+5=(x+1)(2x+5)$