Math 1710 Class 32

t - Inference incl. 2-Sample and MP

Dr. Back

Nov. 11, 2009

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In some cases we'll skip some details and come back to them over the next few classes.

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A coffee vending machine dispenses coffee into a paper cup. You're supposed to get 10 ounces of coffee., but the amount varies slightly form cup to cup. Here are the amounts measured in a random sample of 20 cups.

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9.9	9.6	9.8	9.8
10.0	9.5	9.7	10.1
9.9	9.6	10.2	9.8
10.0	9.9	9.5	9.9

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Is there evidence that the machine is shortchanging customers?

Coffee Machine

Is there evidence that the machine is shortchanging customers?

A natural HT situation.

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We'll summarize the data by its mean $\bar{x} = 9.845$ and its standard deviation $s = .1986$.

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Notation: Let μ denote the mean amount of coffee in a dispensed cup.

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Notation: Let μ denote the mean amount of coffee in a dispensed cup.

Hypotheses:

- H_0: $\mu = 10$ *(or $\mu \geq 10$)*
- H_a: $\mu < 10$

Coffee Machine

Is there evidence that the machine is shortchanging customers?

Recall by the CLT that the sampling distribution of $\bar{x}$ is

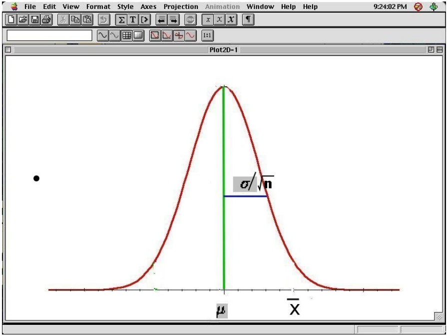
$$N(\mu, \frac{\sigma}{\sqrt{n}})$$

when n is large.

Coffee Machine

Is there evidence that the machine is shortchanging customers?

$$N(\mu, \frac{\sigma}{\sqrt{n}})$$



Coffee Machine

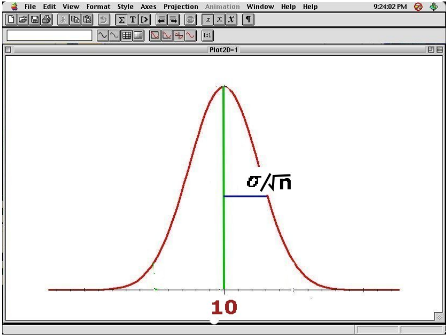
Is there evidence that the machine is shortchanging customers?

As usual with HT's, we are interested in whether the observed statistic of $\bar{x} = 9.845$ is reasonably consistent with the sampling distribution assuming H_0 is true.

Coffee Machine

Is there evidence that the machine is shortchanging customers?

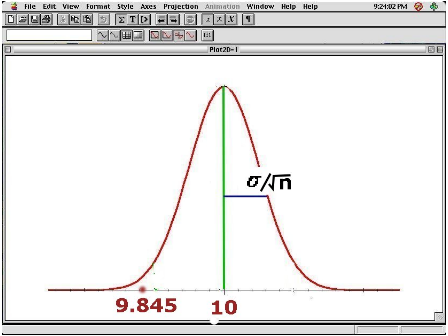
$$N(10, \frac{\sigma}{\sqrt{n}})$$



Coffee Machine

Is there evidence that the machine is shortchanging customers?

$$N(10, \tfrac{\sigma}{\sqrt{n}}) \text{ with } \bar{x}$$



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We know $n = 20$, but the major catch is not knowing σ.
What is the obvious approximation?

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Answer: Use $s = .1986$ instead of σ.

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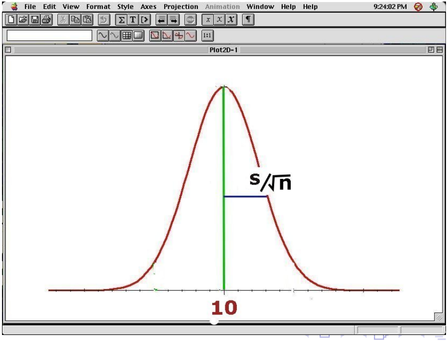
$$N(10, \frac{s}{\sqrt{n}})$$



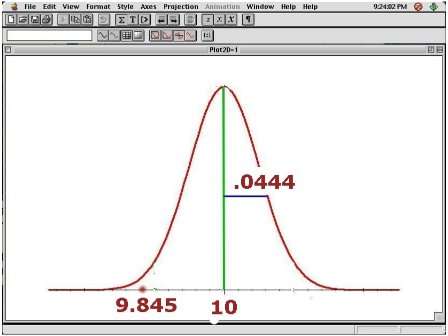
Coffee Machine

Is there evidence that the machine is shortchanging customers?

Since the standard error is

$$SE(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{.1986}{\sqrt{20}} = .0444$$

$$N(10, .0444) \text{ with } \bar{x} = 9.845.$$



Coffee Machine

Is there evidence that the machine is shortchanging customers?

If $s = \sigma$, we'd look at a Z-statistic

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{9.845 - 10}{\frac{.1986}{\sqrt{20}}}$$

where we've written H_0 more abstractly as $\mu = \mu_0$, μ_0 being the hypothesized value, 10 in this case.

Coffee Machine

Is there evidence that the machine is shortchanging customers?

Because s will not exactly match σ, we actually get a bit of extra error here. This is compensated for by viewing

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{9.845 - 10}{\frac{.1986}{\sqrt{20}}} = \frac{-.155}{.0444} = -3.49.$$

as a *t-Statistic*.

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Since the error in approximating σ by s varies with the sample size, there is a different t-distribution for each sample size.

These are labeled by the "degrees of freedom" which for a 1-sample t-test is:

$$df = n - 1.$$

Coffee Machine

Is there evidence that the machine is shortchanging customers?

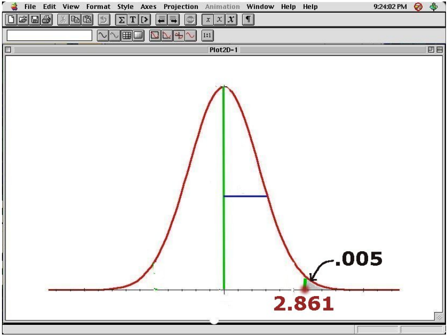
One tail probability		0.10	0.05	0.025	0.01	0.005
Table T Values of t_α	df					
	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831

These are all critical values t^*.
For example $P(t > 1.328) = .10$ for the t distribution with 19 df.

Coffee Machine

Is there evidence that the machine is shortchanging customers?

Our t-statistic of -3.49 is more extreme than any on the df=19 row of the table.
The picture



shows what the critical value $t^* = 2.861$ for a tail prob. of .005

Coffee Machine

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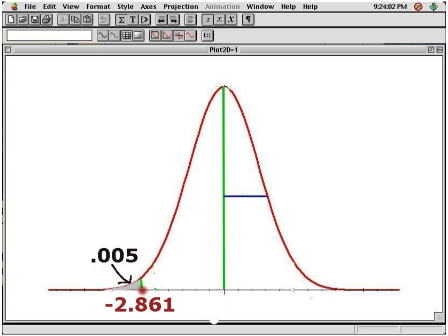
So by symmetry



$P(T < -2.861) = .005$ as well.

Coffee Machine

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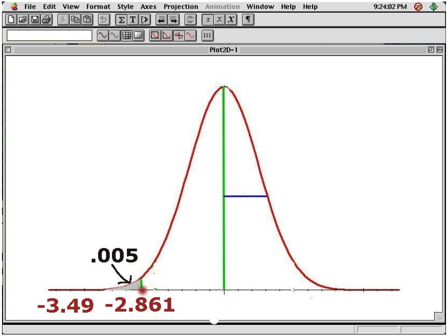
So our tail probability and p-value are both less than .005 and we reject the null. The machine does appear to be shortchanging.



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Sps. our t-statistic had been 2.00 with the same 1-sided hypotheses

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Answer: A tail probability and P-value of between .025 and .05.

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Sps. instead our t-statistic had been 2.00 with 2-sided hypotheses

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- H_a: $\mu \neq 10$

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Answer: Our tail probability is still between .025 and .05 but our P-value is now between .05 and .10.

t-CI's and HT's

A t-hypothesis test with

$$H_0 : \ \mu = m\mu_0$$

is resolved using a t-statistic of

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}.$$

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H_a can be any of the three

1. $\mu \neq \mu_0$
2. $\mu > \mu_0$
3. $\mu < \mu_0$

t-CI's and HT's

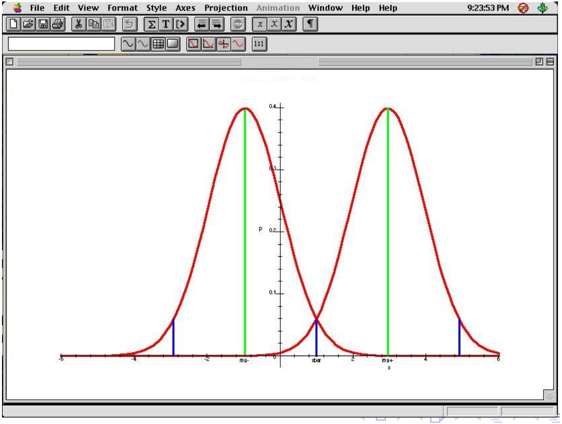
A t confidence interval for μ is

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

for the same reason as the corresponding formula in the proportion case:

t-CI’s and HT’s

The two sampling distributions at the border of the CI.
The blue lines delimit central regions on the sampling distributions.
The CI is between the green lines on the horizontal axis.



In the coffee machine example, this gives a 95% CI for μ of

$$9.845 \pm 2.093(.0444) = 9.845 \pm .093 = (9.752, 9.938)$$

. You did expect it to not include 10, didn't you?

t-Distributions

Let $X_0, X_1, \ldots X_d$ be $d+1$ independent standard normal random variables.

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Think of this as keeping track of the t-statistic

$$\frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{\frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}}{\frac{s}{\sigma}}$$

with the top X_0 keeping track of the numerator and the denominator $\sqrt{X_1^2 + X_2^2 + \ldots + X_d^2}$ keeping track of the ratio of s to σ.

t-Distributions

Just as the normal distribution has the density formula

$$f(x) = \frac{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}$$

t-Distributions

Methods of calculus show that the density formula for the t-distribution with d degrees of freedom is

$$f(x) = \frac{\Gamma(\frac{d+1}{2})}{\Gamma(\frac{d}{2})} \frac{1}{\sqrt{d\pi}} \frac{1}{(1+\frac{x^2}{d})^{\frac{d+1}{2}}}$$

where d is the number of degrees of freedom and

$$\Gamma(n) = \int_0^\infty t^{n-1} e^{-t}\, dt$$

is the *gamma* function, a generalized factorial.

t-Distributions

It will be important when we think about 2-sample tests to realize that the degrees of freedom is just a parameter in the above formula.

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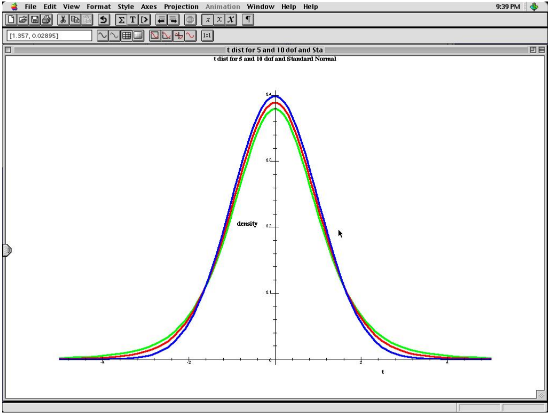
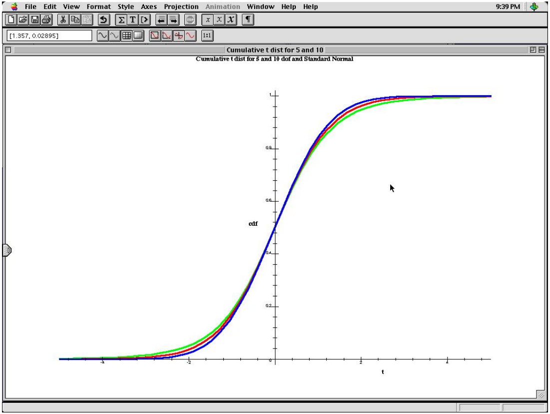
t-Distributions

A little more probability in the tails

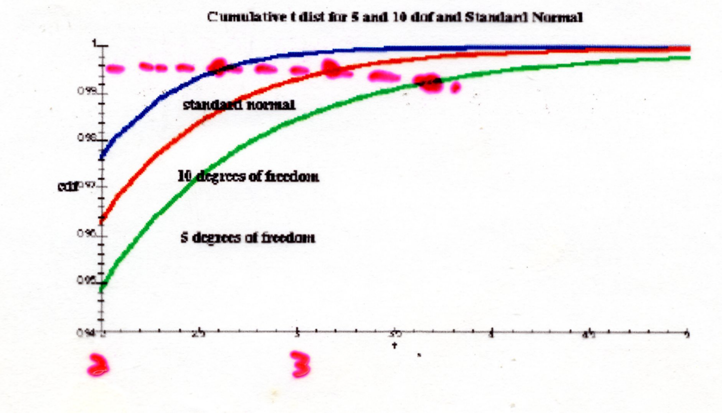


Cumulative Dist. Fcn (Graphical Form of Table Z)



t-Distributions

This picture shows why t^* can be very different from z^*.



t-Distributions

t distribution critical value

two tail	0.20	0.10	0.05	0.02	0.01
one tail	0.10	0.05	0.025	0.01	0.005
df					
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7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
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25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
32	1.309	1.694	2.037	2.449	2.738
35	1.306	1.690	2.030	2.438	2.725
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
60	1.296	1.671	2.000	2.390	2.660
75	1.293	1.665	1.992	2.377	2.643
100	1.290	1.660	1.984	2.364	2.626
120	1.289	1.658	1.980	2.358	2.617
140	1.288	1.656	1.977	2.353	2.611
180	1.286	1.653	1.973	2.347	2.603
250	1.285	1.651	1.969	2.341	2.596
400	1.284	1.649	1.966	2.336	2.588
1000	1.282	1.646	1.962	2.330	2.581
∞	1.282	1.645	1.960	2.326	2.576
levels / conf. levels	80%	90%	95%	98%	99%

Shaft Diameters

Next time.

A Sample Size Problem

Next time.

Haiku Poetry: Intrinsic or Extrinsic Motivation Different in Effectiveness?

haiku: A Japanese poem composed of three unrhymed lines of five, seven, and five syllables. Haiku often reflect on some aspect of nature.

Haiku Poetry: Intrinsic or Extrinsic Motivation Different in Effectiveness?

How can you study this?

Haiku Poetry: Intrinsic or Extrinsic Motivation Different in Effectiveness?

How can you study this?

Randomized Experiment on Creativity in poetry:

Grades on creativity recorded. (Higher is better.)

Both groups filled out questionnaires before writing poetry.

- Intrinsic group first did questionnaire emphasizing intrinsic motivations.
- Extrinsic group first did questionnaire emphasizing extrinsic motivations.

Haiku Poetry: Intrinsic or Extrinsic Motivation Different in Effectiveness?

How can you study this?

People then wrote Haiku poems which were graded by experienced poets.

Haiku Poetry: Intrinsic or Extrinsic Motivation Different in Effectiveness?

How can you study this?

The Results

Group	n	$\bar{x}$	s
Intrinsic	24	19.88	4.44
Extrinsic	23	15.73	5.25

Haiku Poetry: Intrinsic or Extrinsic Motivation Different in Effectiveness?

How can you study this?

Notation: Let μ_i and μ_e denote the respective mean haiku scores for the intrinsically and the extrinsically motivated groups.

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- H_0: $\mu_i = \mu_e$
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Under H_0, the sampling distribution of $\bar{x}_i - \bar{x}_e$ has mean 0 and approx. std deviation

$$SE(\bar{x}_i - \bar{x}_e) = \sqrt{\frac{s_i^2}{n_i} + \frac{s_e^2}{n_e}}$$

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$$SE(\bar{x}_i - \bar{x}_e) = \sqrt{\frac{4.44^2}{24} + \frac{5.25^2}{23}} = 1.42.$$

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$$SE(\bar{x}_i - \bar{x}_e) = \sqrt{\frac{4.44^2}{24} + \frac{5.25^2}{23}} = 1.42.$$

Our t-statistic is

$$t = \frac{19.88 - 15.73}{1.42} = 2.92.$$

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For homework and hand computation on exams, we suggest you use df the smaller of 24-1 and 23-1 to interpret the t-statistic. *To be explained later.*

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With $df = 22$, 2.92 is more extreme than the biggest t^* in table T (which is 2.819) so our tail probability is less than .005 and our P-value is less than .01.

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With $df = 22$, 2.92 is more extreme than the biggest t^* in table T (which is 2.819) so our tail probability is less than .005 and our P-value is less than .01.

We reject H_0. Intrinsic vs. extrinsic motivation does seem to make a difference.

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- H_0: $\mu_i = \mu_e$
- H_a: $\mu_i \neq \mu_e$

We reject H_0. Intrinsic vs. extrinsic motivation does seem to make a difference.

A 95% CI for $\mu_i \neq \mu_e$ would be

$$19.88 - 15.73 \pm 2.074 \cdot 1.42 = 4.15 \pm 2.95 = (1.20, 7.10).$$

Haiku Poetry: Intrinsic or Extrinsic Motivation Different in Effectiveness?

How can you study this?

Hypotheses:

- H_0: $\mu_i = \mu_e$
- H_a: $\mu_i \neq \mu_e$

Scope of the Inference:

- Causal relationship seems to be established.
- Doesn't say much about a population since not an SRS.

Satterthwaite

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

Satterthwaite

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But the sampling distribution of $\bar{x} - \bar{y}$ can be well approximated by a t-distribution with the following degrees of freedom:

Satterthwaite

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x - 1}\left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y - 1}\left(\frac{s_y^2}{n_y}\right)^2}$$

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Let $min(n_x - 1, n_y - 1)$ denote the smallest of $n_x - 1$ and $n_y - 1$. This df is always between $min(n_x - 1, n_y - 1)$ and $n_x + n_y - 2$.

Satterthwaite

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Special Case $s_x = s_y = s,\ n_x = n_y = n.$

$$df = \frac{\left(\frac{2s^2}{n}\right)^2}{\frac{1}{n-1}\frac{2s^4}{n^2}} = 2(n-1).$$

Satterthwaite

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x - 1}\left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y - 1}\left(\frac{s_y^2}{n_y}\right)^2}$$

Special Case $\frac{s_x^2}{n_x} << \frac{s_y^2}{n_y}$

$$df = \frac{\left(\frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_y - 1}\left(\frac{s_y^2}{n_y}\right)^2} = n_y - 1.$$

Satterthwaite

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x - 1}\left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y - 1}\left(\frac{s_y^2}{n_y}\right)^2}$$

Calculators and computers will use this df formula. *For hand computation on HW and exams, we suggest you never use this formula. Instead use the conservative min*$(n_x - 1, n_y - 1)$.

Satterthwaite

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Calculators and computers will use this df formula. *For hand computation on HW and exams, we suggest you never use this formula. Instead use the conservative min*$(n_x - 1, n_y - 1)$. Your t^* will be a little too big so your CI will be a little too wide and your HT a little harder to show statistical significance. But this is not so bad.

Satterthwaite

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$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x - 1}\left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y - 1}\left(\frac{s_y^2}{n_y}\right)^2}$$

In your published papers and lab reports, use a computer which will likely automatically use this formula.

Satterthwaite

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$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x - 1}\left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y - 1}\left(\frac{s_y^2}{n_y}\right)^2}$$

Satterthwaite derived this formula using the fact that the exact mean and variance of the sampling distribution of $\bar{x} - \bar{y}$ is easy to calculate. He asked, "What df in the t-distribution would give the right μ and σ." The answer is the above.

Satterthwaite

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$$f(x) = \frac{\Gamma\left(\frac{d+1}{2}\right)}{\Gamma\left(\frac{d}{2}\right)} \frac{1}{\sqrt{d\pi}} \frac{1}{\left(1 + \frac{x^2}{d}\right)^{\frac{d+1}{2}}}$$

Data Desk View of Creativity

Summary of **INTRINSIC:SCORE**
No Selector

Percentile 25

Count	24
Mean	19.8833
Median	20.4
StdDev	4.43951
Range	17.7
IntQRange	5.05
Lower ith %tile	17.35
Upper ith %tile	22.4

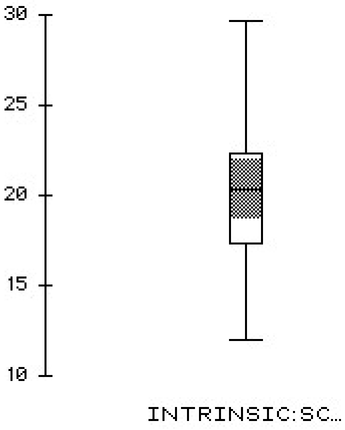
Data Desk View of Creativity

Summary of **EXTRINSIC:SCORE**
No Selector

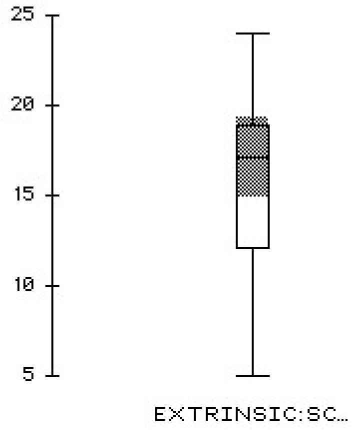
Percentile 25

Count	23
Mean	15.7391
Median	17.2
StdDev	5.2526
Range	19
IntQRange	7
Lower ith %tile	12.075
Upper ith %tile	19.075

Data Desk View of Creativity



Data Desk View of Creativity



Data Desk View of Creativity

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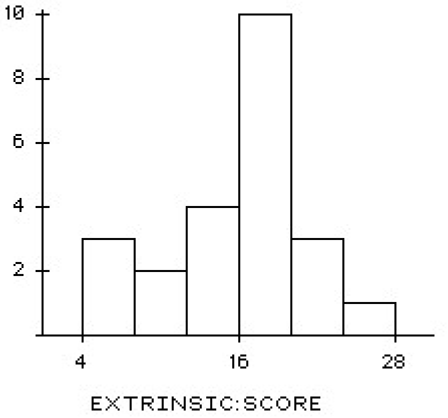
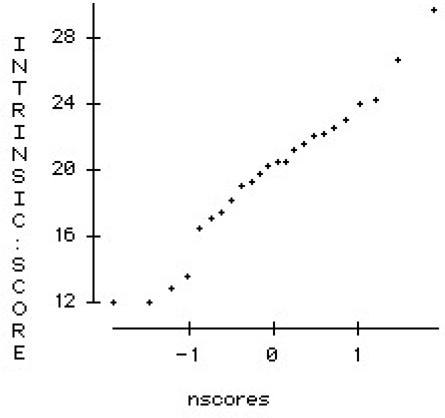
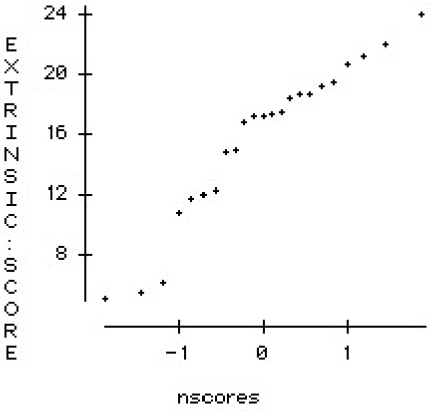


Data Desk View of Creativity



Data Desk View of Creativity



Data Desk View of Creativity

t-Interval for Individual μ's

No Selector

Individual Confidence 95.00%

Bounds: Lower Bound < μ < Upper Bound

With 95.00% Confidence, 18.008691 < μ(INTRINSIC:SCORE) < 21.757975

Data Desk View of Creativity

t-Interval for Individual μ's

No Selector

Individual Confidence 95.00%

Bounds: Lower Bound < μ < Upper Bound

With 95.00% Confidence, 13.467738 < μ(EXTRINSIC:SCORE) < 18.010523

Data Desk View of Creativity

2-Sample t-Interval for μ1−μ2

No Selector

Individual Confidence 95.00%

Bounds: Lower Bound < μ1−μ2 < Upper Bound

With 95.00% Confidence, −7.0108029 < μ(EXTRINSIC:SCORE)−μ(INTRINSIC:SCORE) < −1.2776029

Data Desk View of Creativity


```
2-Sample t-Test of μ1-μ2
No Selector
Individual Alpha Level 0.05
Ho: μ1-μ2  =  0 Ha:  μ1-μ2  ≠ 0

EXTRINSIC:SCORE  -  INTRINSIC:SCORE:
Test Ho: μ(EXTRINSIC:SCORE)-μ(INTRINSIC:SCORE) = 0 vs Ha: μ(EXTRINSIC:SCORE)-μ(INTRINSIC:SCORE) ≠ 0
Difference Between Means = -4.1442029 t-Statistic = -2.915 w/43 df
Reject Ho at Alpha = 0.05
p = 0.0056
```

Pulse Rate Example

Does regular exercise reduce resting pulse rates? 10 volunteers do 20 minutes of exercise 3 times a week for 6 weeks. Their resting pulse rates before and after were measured. (Beats/min.)

Pulse Rate Example

Subject	Before	After
Allen	73	73
Brandon	83	79
Carlos	85	81
David	87	86
Edwin	91	87
Franco	99	91
Graeme	87	84
Hans	85	83
Ivan	83	84
Jorge	79	76

Pulse Rate Example

Subject	Before	After
Allen	73	73
Brandon	83	79
Carlos	85	81
David	87	86
Edwin	91	87
Franco	99	91
Graeme	87	84
Hans	85	83
Ivan	83	84
Jorge	79	76

Notice that 8 out of 10 subjects had reduced RPR's. There is a non-parametric test called the sign test that could be used to conclude that RPR's tend to decrease as hypothesized. Can you see how binomial distribution ideas could be used for this?

Pulse Rate Example

Notation: Let μ_b be the mean resting pulse rate before the exercise program and μ_a the mean resting pulse rate after.

Pulse Rate Example

Notation: Let μ_b be the mean resting pulse rate before the exercise program and μ_a the mean resting pulse rate after.

Hypotheses:

- H_0: $\mu_b = \mu_a$ (or $\mu_b - \mu_a \leq 0$ or ...)
- H_a: $\mu_b > \mu_a$

Pulse Rate Example

Since individuals have important characteristics besides their exercise programs which determine their RPR's, statistical tests based on

$$Var(\bar{x}_b - \bar{x}_a) = Var(\bar{x}_b) + Var(\bar{x}_a)$$

would not be appropriate here.

Pulse Rate Example

In MP, we look at the differences and apply 1-sample ideas to the differences.

Pulse Rate Example

Differences d_i

Subject	Before - After
Allen	0
Brandon	4
Carlos	4
David	1
Edwin	4
Franco	8
Graeme	3
Hans	2
Ivan	-1
Jorge	3

Pulse Rate Example

The mean $\bar{d} = 2.8$ with a standard deviation s of 2.8.

Note you need to directly calculate s for the differences; it is not determined by s_a and s_b.

Pulse Rate Example

The mean $\bar{d} = 2.8$ with a standard deviation s of 2.8.

$$t = \frac{2.8}{\frac{2.53}{\sqrt{10}}} = -3.50$$

with df=9. Our P-value is $< .005$ and we reject H_0. Exercise does seem to reduce RPR's.

Matched Pairs vs. Indep. 2-Sample

Group1	**Group 2**	
x_1	y_1	
x_2	y_2	
$\dots$	$\dots$	
x_n	$\dots$	
	y_m	
$\bar{x}$	$\bar{y}$	**mean**
s_x	s_y	**std. dev**

Matched Pairs vs. Indep. 2-Sample

Group1	**Group 2**	
x_1	y_1	
x_2	y_2	
$\dots$	$\dots$	
x_n	$\dots$	
	y_m	
$\bar{x}$	$\bar{y}$	**mean**
s_x	s_y	**std. dev**

Independent 2-Sample

Each x_i and each y_j are (except to the extent that they come from one of the groups) independent of each other.

The sample sizes m and n can be different.

Matched Pairs vs. Indep. 2-Sample

Group1	**Group 2**	
x_1	y_1	
x_2	y_2	
$\dots$	$\dots$	
x_n	$\dots$	
	y_m	
$\bar{x}$	$\bar{y}$	**mean**
s_x	s_y	**std. dev**

Matched Pairs

The i'th observation x_i in group 1 is more closely related (in a relevant way) to the i'th observation y_i in group 2 than a general x_i is to a general y_j.

The sample sizes m and n must be the same.