Test 1
90 min
15 questions
4 FR questions

Need to have
1. Cougar card
2. something to write with
3. laptop OR TI-84 (bat.)

Math 2311
Test 1 Review

Know all definitions!

1. State whether each situation is categorical or quantitative. If quantitative, state whether it's discrete or continuous.

a. The amount a person grew (in height) in a year.
categorical, quantitative discrete or quantitative continuous

b. The number of classes a student missed.
categorical, quantitative discrete or quantitative continuous

c. Letter Grades.
categorical, quantitative discrete or quantitative continuous

2. Six people were asked how many movies they saw last month. The results were:
2 6 1 3 4 2

a. Find the mean and median.
Commands:

```
x = c(2,6,1,3,4.2)
mean(x) = 3
median(x) = 2.5
```

b. Find the variance and the standard deviation.
Commands:

```
var(x) = 3.2
sd(x) = 1.79
```

c. Find the five-number summary, IQR and range.
Commands:

```
fivenum(x)   1   2   2.5   4   6
             MIN Q1  Q2    Q3  MAX
                     median
```

IQR = Q3 − Q1 = 4 − 2 = 2

Range = MAX − MIN = 6 − 1 = 5

Outlier Boundaries: (Q1 − 1.5 * IQR, Q3 + 1.5 * IQR)

$$P(n,k) = \frac{n!}{(n-k)!}$$

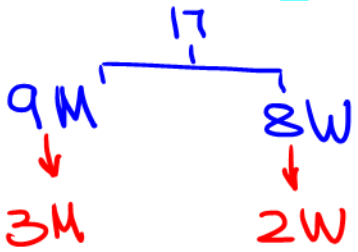
3. An organization needs to make up a social committee. If the organization has 25 members, in how many ways can a 10 person social committee be made? C(25,10)

Command: choose (25,10) Answer: 3,268,760

4. An organization has 30 members. In how many ways can the positions of president, vice-president, secretary, treasurer, and historian be filled if not one person can fill more than one position? P(30, 5) } 30 * 29 * 28 * 27 * 26

Command: factorial 30 / factorial (30-5) Answer: 17,100,720

5. Suppose that from a group of 9 men and 8 women, a committee of 5 people is to be chosen.
a. In how many ways can the committee be chosen so that there are exactly 3 men? Then find the probability of this event.



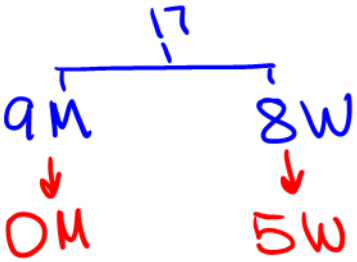
Commands: Answers:

C(9,3) * C(8,2)

choose (9,3) * choose (8,2) 2352

2352 / choose (17,5) .3801

b. In how many ways can the committee be chosen so that there are no men? Then find the probability of this event.

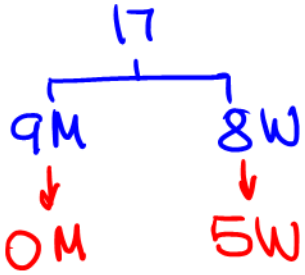


Commands: Answers:

choose (8,5) 56

56 / choose (17,5) .009

c. What is the probability that the committee contains at least one man? 1,2,3,4,5 OR Use complement 0Men



Command: Answer:

1 - .009 .991

6. Find $A \cap (B^c \cup C)$ using

$U = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{3,4,6,8\}$

$B = \{1,2,5,9\}$

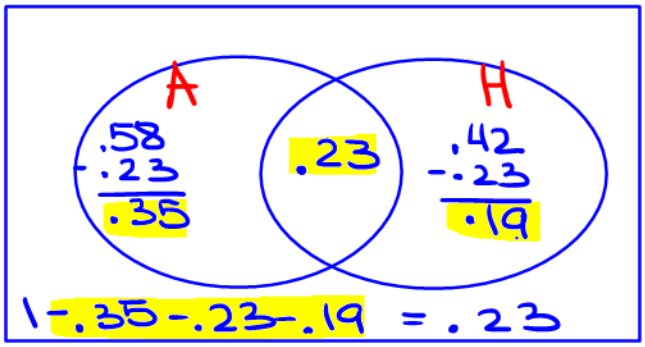
$C = \{2,4,6,8,10\}$

$B^c = \{3,4,6,7,8,10\}$

$B^c \cup C = \{2,3,4,6,7,8,10\}$

$A \cap (B^c \cup C) = \{3,4,6,8\}$

7. Suppose that 58% of all customers of a large insurance agency have automobile policies with the agency, 42% have homeowner's policies, and 23% have both. What proportion of the groups will:



a. none of the polices?

.23 or 23%

b. have auto or homeowners' or both?

.35 + .23 + .19 = .77 77%

c. have only homeowners'? 1 - .23 = .77

.19 or 19%

8. Suppose $P(A) = 0.72$, $P(B) = 0.46$ and $P(A \cup B) = 0.86$.

a. Find $P(A \cap B)$. Are A and B mutually exclusive (disjoint)? NO $P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.86 = .72 + .46 - P(A \cap B)$

$P(A \cap B) = .72 + .46 - .86 = .32 \neq 0$

b. Find $P(A|B)$.

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(A|B) = \frac{.32}{.46} = .6957$

c. Are A and B independent?

Check if: $P(A \cap B) = P(A) \cdot P(B)$ ⇐ Def

$.32 \stackrel{?}{\neq} .72 * .46$ NO

FR

9. Gabelli Partners is planning a major investment. The amount of profit X is uncertain but a probabilistic estimate gives the following distribution (in millions of dollars):

Profit	1	1.5	2	4	10
Probability	0.1	0.2	0.4	??	0.1

$P(X=4) = 1-(.1+.2+.4+.1)$
$= .2$

a. Find $P(1.5 \le X < 4)$

$= P(X=1.5) + P(X=2)$
$= .2 + .4 = .6$

b. Find the mean profit and the variance of the profit.

$\mu_X = E[X] = x_1p_1 + x_2p_2 + \cdots + x_np_n$

$\mu = 1(.1) + 1.5(.2) + 2(.4) + 4(.2) + 10(.1)$
$= 3$ $ millions

$\sigma_X^2 = Var[X] = E[X^2] - (E[X])^2$

$\sigma^2 = 1^2(.1) + 1.5^2(.2) + 2^2(.4) + 4^2(.2) + 10^2(.1) - (3)^2 = 6.35$

$\sigma = \sqrt{6.35}$
sqrt(6.35)

c. Gabelli Partners owes its source of capital a fee of $200,000 plus 10% of the profits X. So the firm actually retains Y = .9X – 0.2 from the investment. Find the mean and standard deviation of Y.

$a = .9$ & $b = -.2$

$E[Y] = aE[X] + b$

$E(Y) = .9(3) - .2$
$= 2.5$

$\sigma_Y = \sqrt{a^2 Var[X]} = \sqrt{.9^2 * 6.35} = 2.2679$

FR

10. A headache remedy is said to be 80% effective in curing headaches caused by simple nervous tension. An investigator tests this remedy on 6 randomly selected patients suffering from nervous tension.

a. What kind of distribution does X have? Binomial or Geometric

Know:
- Fixed number of trials
- Trials are independent
- Success or Failure
- Prob. of success is the same in each trial

$n = 6$

$p = .8$

b. Calculate the mean and standard deviation of X.

$\mu = np = 6(.8) = \boxed{4.8}$

$\sigma = \sqrt{np(1-p)} = \sqrt{6(.8)(1-.8)} = \sqrt{6(.8)(.2}$

$= \boxed{.9798}$

c. Determine the probability that three subjects experience headache relief with this remedy.

$P(X = 3)$ $\quad P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

Command: k, n, p

Answer:

dbinom(3, 6, .8)

.0819

$P(X=3) = C(6,3)(.8)^3(1-.8)^3$

d. Determine the probability that at most one subject experiences headache relief with this remedy.

$P(X \leq 1)$

0 or 1

Command:

Answer:

pbinom(1, 6, .8)

.0016

$C(6,0)(.8)^0(.2)^6 + C(6,1)(.8)^1(.2)^5$

1 1 6

11. A basketball player completes 64% of her free-throws. We want to observe this player during one game to see how many free-throw attempts she makes before completing one.

a. What type of distribution is this? Binomial or Geometric

Know the def!

b. What is the probability that the player misses 3 free-throws before she has makes one?

$P(X = 4)$

Command: dgeom(4-1,.64)

Answer: 0.0299

c. How many free-throw attempts can the player expect to throw before she gets a basket?

$\mu = \frac{1}{p} = \frac{1}{.64} = 1.5625$

2

d. Determine the probability that it takes more than 5 attempts before she makes a basket.

$P(X > 5) = 1 - P(X \leq 5)$

Command: 1-pgeom(5-1,.64)

Answer: .006

12. A manufacturer produces a large number of toasters. From past experience, the manufacturer knows that approximately 1% are defective. In a quality control procedure, we randomly select 50 toasters for testing. We want to determine the probability that no more than one of these toasters is defective.

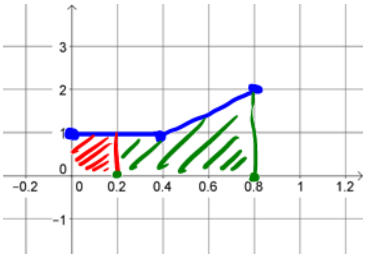
Binomial or Geometric

13. Draw a card from a standard deck of 52 playing cards, observe the card, and replace the card within the deck. Count the number of times you draw a card in this manner until you observe a jack.

Binomial or Geometric

14. Think about a density curve that consists of two line segments. The first goes from the point (0, 1) to the point (0.4, 1). The second goes from (0.4, 1) to (0.8, 2) in the *xy* plane. Let *X* be the continuous random variable.

Sketch the density curve.



What percent of observations:

a. fall below 0.2?

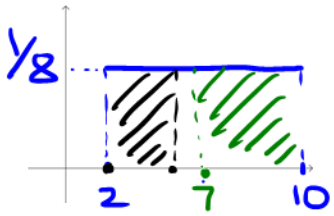
$P(X<0.2) = (.2)(1) = .2 \rightarrow$ **20%**

b. lie between 0.2 and 0.8?

$P(.2<X<.8) = 1-.2 = .8 \rightarrow$ **80%**

15. Consider a uniform density curve (has the same height all the way across) defined for $2 \le X \le 10$, where *X* is the continuous random variable.

Sketch the uniform density curve.



$\text{Area} = 1 = lw$

$1 = 8w$

$w = \frac{1}{8}$

a. What is the probability that *X* falls above 7?

$P(X>7) = 3\left(\frac{1}{8}\right) = .375$

b. What percent of the observations of *X* lie between 2 and 5?

$P(2<X<5) = 3\left(\frac{1}{8}\right) = .375 \rightarrow$ **37.5%**

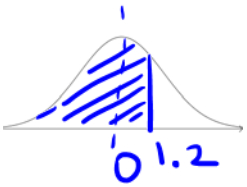
FR

16. The random variable Z is the standard normal random variable.

a. What is the mean and standard deviation for Z?

$\mu = 0$ $\sigma = 1$

b. Find $P(Z < 1.2)$ and draw the picture.



Command: normalcdf(1.2)

Answer: .8849

c. Find $P(Z > -1.39)$ and draw the picture.

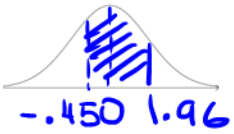
$1 - P(Z < -1.39)$

$1 - .0823 = .9177$

Command: 1-normalcdf(-1.39)

Answer: .9177

d. Find $P(-0.45 < Z < 1.96)$.



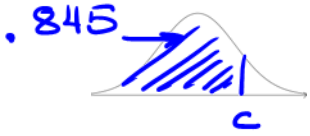
$P(Z < 1.96) - P(Z < -0.45)$

$= .9750 - .3264 = .6486$

Command: normalcdf(1.96) - normalcdf(-0.45)

Answer: .6486

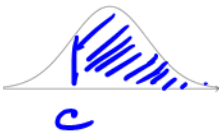
e. Find c such that $P(Z < c) = 0.845$ ← find in the table



Command: invNorm(.845)

Answer: 1.02

f. Find c such that $P(Z > c) = 0.845$



Command: invNorm(1-.845)

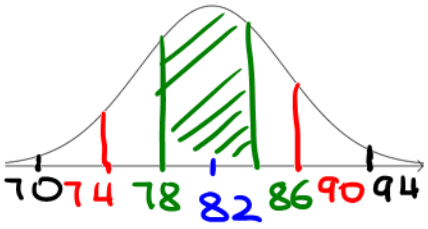
Answer: -1.02

$P(Z < c) = .155$

Convert to standard normal

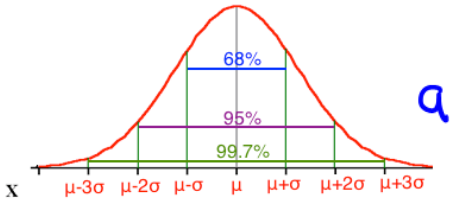
17. Let X be a normal random variable with $\mu = 82$ and $\sigma = 4$.

a. Sketch the distribution



b. According to the Empirical Rule, the middle 68% of the data falls between what values?

Recall:



[78, 86]

95% [74, 90]

99.7% [70, 94]

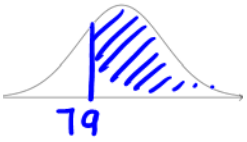
c. Find $P(X < 83)$ $= P\left(Z < \frac{83-82}{4}\right) = P(Z < .25)$

Z-Table

$Z = \frac{X-M}{\sigma}$

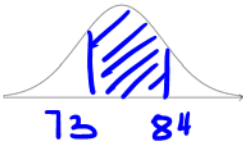
Command: normcdf(83, 82, 4) Answer: .5987

d. Find $P(X > 79)$



Command: 1-normcdf(79, 82, 4) Answer: .7734

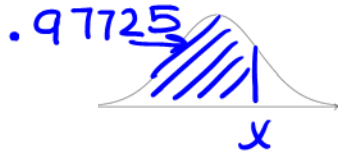
e. Find $P(73 < X < 84)$



Command: Answer:

normcdf(84, 82, 4) - normcdf(73, 82, 4) .6792

f. Find x such that $P(X < x) = .97725$? Given P, need to find x. Use: qnorm



Command: qnorm(.97725,82,4) Answer: 90

18. Suppose a sample of 100 subjects was taken and their scores on an exam recorded. If the population mean for the exam is 67 and population variance is 36,

var = 36 $\sigma = \sqrt{var} = \sqrt{36} = 6$

a. what is the mean and standard error of the sampling distribution, $\bar{X}$?

Commands: Answers:

$\mu_{\bar{x}} = \mu = 67$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{100}}$ 6/sqrt(100) .6

b. find the probability that the sample mean is less than 70.

$P(\bar{x} < 70)$

Command: Answer:

normalcdf(70,67,.6) .9999

c. find the probability that the sample mean exceeds 68.

$P(\bar{x} > 68)$

Command: Answer:

1-normalcdf(68,67,.6) .0478

19 In a large population, 67% of the households have cable tv. A simple random sample of 256 households is to be contacted and the sample proportion computed.

a. What is the mean and standard deviation of the sampling distribution of the sample proportions?

Commands: Answers:

$\mu_{\hat{p}} = p = .67$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$ sqrt(.67*(1-.67)/256) = .0294

b. What is the probability that the sampling distribution of sample proportions is less than 73%?

$P(\hat{p} < .73)$

Command: Answer:

phorm(.73, .67, .0294) .9794

Formulas to be provided.

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

$$s = \sqrt{s^2}$$

$${}_nP_n = n(n-1)(n-2)...3 \cdot 2 \cdot 1 = n!$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$$P = \frac{n!}{r!s!t!}$$

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(A \cup B)^c = A^c \cap B^c$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E^c) = 1 - P(E)$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

$$\mu_X = E[X] = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

$$\sigma_X^2 = Var[X] = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_n - \mu_X)^2 p_n$$

$$= \sum (x_i - \mu_X)^2 p_i$$

$$\sigma_X^2 = Var[X] = E[X^2] - (E[X])^2$$

$$E[W] = E[aX + b] = aE[X] + b$$

$$\sigma_W^2 = Var[W] = Var[aX + b] = a^2 Var[X]$$

$$E[X+Y]=E[X]+E[Y]$$
$$\sigma^2_{X+Y}=Var[X+Y]=Var[X]+Var[Y]$$
$$E[X-Y]=E[X-Y]=E[X]-E[Y]$$
$$\sigma^2_{X-Y}=Var[X-Y]=Var[X]+Var[Y]$$
$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$$
$$P(X\geq k)=1-P(X\leq(k-1))$$
$$\mu=E\left[X\right]=np$$
$$\sigma^2=np(1-p)$$
$$P(X=n)=(1-p)^{n-1}p$$
$$P(X>n)=(1-p)^n$$
$$E[X]=\mu=\frac{1}{p}$$
$$\sigma^2=\frac{1-p}{p^2}$$