Arithmetic Sequences

Supplemental Material Not Found in You Text

Math 34: Spring 2015

Do NOT print these slides!!

There are printer friendly files on the website.

January 28, 2015

Arithmetic Sequences

Arithmetic Sequences

- Recall an **Arithmetic Sequence** is a sequence where the *difference* between any two consecutive numbers in the sequence is constant.

  In other words: $a_{k+1} - a_k = d$ *where* d *is a constant.*

Arithmetic Sequences

- Recall an **Arithmetic Sequence** is a sequence where the *difference* between any two consecutive numbers in the sequence is constant.

  In other words: $a_{k+1} - a_k = d$ *where* d *is a constant.*

- Which of the following are Arithmetic Sequences?
  1. $1, 4, 7, 10, 13, \ldots$
  2. $2, 4, 8, 16, 32, \ldots$
  3. $-3, 7, 17, 27, \ldots$

Arithmetic Sequences

- Recall an **Arithmetic Sequence** is a sequence where the *difference* between any two consecutive numbers in the sequence is constant.

  In other words: $a_{k+1} - a_k = d$ *where* d *is a constant.*

- Which of the following are Arithmetic Sequences?
  1. $1, 4, 7, 10, 13, \ldots$
     IS arithmetic, with constant difference $d = 3$
  2. $2, 4, 8, 16, 32, \ldots$
  3. $-3, 7, 17, 27, \ldots$

Arithmetic Sequences

- Recall an **Arithmetic Sequence** is a sequence where the *difference* between any two consecutive numbers in the sequence is constant.

  In other words: $a_{k+1} - a_k = d$ *where* d *is a constant.*

- Which of the following are Arithmetic Sequences?
  1. $1, 4, 7, 10, 13, \ldots$
     IS arithmetic, with constant difference $d = 3$
  2. $2, 4, 8, 16, 32, \ldots$
     is NOT arithmetic
  3. $-3, 7, 17, 27, \ldots$

Arithmetic Sequences

- Recall an **Arithmetic Sequence** is a sequence where the *difference* between any two consecutive numbers in the sequence is constant.

  In other words: $a_{k+1} - a_k = d$ *where* d *is a constant.*

- Which of the following are Arithmetic Sequences?
  1. $1, 4, 7, 10, 13, \ldots$
     IS arithmetic, with constant difference $d = 3$
  2. $2, 4, 8, 16, 32, \ldots$
     is NOT arithmetic
  3. $-3, 7, 17, 27, \ldots$
     IS arithmetic, with constant difference $d = 10$

Real World Examples

1. You start a new job and you're told you salary is $\$29,000$ for the first year, and that you'll get a \$1700 raise each year. What will your salary be in the third year? What will your salary be in 10 years? How long does it take for your salary to (at least) double?

2. A new company has a loss of $\$2,500$ in its first month, but they expect their monthly profit to increase by \$400 each month. What is their profit in the 12^{th} month? What is their total profit/loss of the year?

Real World Examples

1. You start a new job and you're told you salary is $\$29,000$ for the first year, and that you'll get a \$1700 raise each year. What will your salary be in the third year? What will your salary be in 10 years? How long does it take for your salary to (at least) double?

2. A new company has a loss of $\$2,500$ in its first month, but they expect their monthly profit to increase by \$400 each month. What is their profit in the 12^{th} month? What is their total profit/loss of the year?

   Both these scenarios can be modeled by Arithmetic Sequences, and we will develop tools to help us answer these questions.

General Way to Write an Arithmetic Sequence

- Consider the Arithmetic Sequence below. Notice the first term is 5 and the common difference is 2:

$$5, 7, 9, 11, 13, \ldots$$

General Way to Write an Arithmetic Sequence

- Consider the Arithmetic Sequence below. Notice the first term is 5 and the common difference is 2:

$$5, 7, 9, 11, 13, \ldots$$

Look at the pattern that the common difference of 2 creates.

General Way to Write an Arithmetic Sequence

- Consider the Arithmetic Sequence below. Notice the first term is 5 and the common difference is 2:

$$5, 7, 9, 11, 13, \ldots$$

Look at the pattern that the common difference of 2 creates.

$$5, \qquad 7, \qquad 9, \qquad 11, \qquad 13, \qquad \ldots$$

General Way to Write an Arithmetic Sequence

- Consider the Arithmetic Sequence below. Notice the first term is 5 and the common difference is 2:

$$5, 7, 9, 11, 13, \ldots$$

Look at the pattern that the common difference of 2 creates.

$$\underbrace{5}_{a_1}, \quad \underbrace{5+(1)2}_{a_2}, \quad \underbrace{5+(2)2}_{a_3}, \quad \underbrace{5+(3)2}_{a_4}, \quad \underbrace{5+(4)2}_{a_5}, \quad \ldots$$

General Way to Write an Arithmetic Sequence

- Consider the Arithmetic Sequence below. Notice the first term is 5 and the common difference is 2:

$$5, 7, 9, 11, 13, \ldots$$

Look at the pattern that the common difference of 2 creates.

$$\underbrace{5}_{a_1}, \quad \underbrace{5+(1)2}_{a_2}, \quad \underbrace{5+(2)2}_{a_3}, \quad \underbrace{5+(3)2}_{a_4}, \quad \underbrace{5+(4)2}_{a_5}, \quad \ldots$$

$$5, \quad 7, \quad 9, \quad 11, \quad 13, \quad \ldots$$

We notice the pattern for this sequence $a_n = 5 + (n-1)2$

General Way to Write an Arithmetic Sequence

- Consider the Arithmetic Sequence below. Notice the first term is 5 and the common difference is 2:

$$5, 7, 9, 11, 13, \ldots$$

Look at the pattern that the common difference of 2 creates.

$$\underbrace{5}_{a_1}, \quad \underbrace{5+(1)2}_{a_2}, \quad \underbrace{5+(2)2}_{a_3}, \quad \underbrace{5+(3)2}_{a_4}, \quad \underbrace{5+(4)2}_{a_5}, \quad \ldots$$

$$5, \quad 7, \quad 9, \quad 11, \quad 13, \quad \ldots$$

We notice the pattern for this sequence $a_n = 5 + (n-1)2$

We also see that $a_n = a_{n-1} + 2$ (each term is 2 more than the previous term)

General Way to Write an Arithmetic Sequence

- **Way to Write a Formula for an Arithmetic Sequence:** Given that $a_1, a_2, a_3, \ldots$ is an arithmetic sequence with common difference d,

  We can rewrite the sequence as

  $$a_n = a_1 + (n-1)d$$

  where the index starts at $n = 1$.

  Here a_1 is the first term of the sequence (a constant) and d is the common difference (also a constant).

Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

1. Find the fifth term in the sequence.

2. Find the 20^{th} term in the sequence.

3. Find a formula for the n^{th} term in the sequence.

Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

To understand everything about this sequences we need to know:

1. Find the fifth term in the sequence.

2. Find the 20^{th} term in the sequence.

3. Find a formula for the n^{th} term in the sequence.

Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

To understand everything about this sequences we need to know:

It's Arithmetic

With common difference $d = 6$ And first term $a_1 = -10$

1. Find the fifth term in the sequence.

2. Find the 20^{th} term in the sequence.

3. Find a formula for the n^{th} term in the sequence.

Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

To understand everything about this sequences we need to know:

It's Arithmetic

With common difference $d = 6$ And first term $a_1 = -10$

1. Find the fifth term in the sequence.

   Since the first 4 terms are given, and the common difference is $d = 6$, we can see the 5^{th} term 6 more than 4^{th} term.

2. Find the 20^{th} term in the sequence.

3. Find a formula for the n^{th} term in the sequence.

Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

To understand everything about this sequences we need to know:

It's Arithmetic

With common difference $d = 6$ And first term $a_1 = -10$

1. Find the fifth term in the sequence.

   Since the first 4 terms are given, and the common difference is $d = 6$, we can see the 5^{th} term 6 more than 4^{th} term.

   i.e. $a_5 = a_4 + 6 = 8 + 6 = 14$

2. Find the 20^{th} term in the sequence.

3. Find a formula for the n^{th} term in the sequence.

Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

To understand everything about this sequences we need to know:

It's Arithmetic

With common difference $d = 6$ And first term $a_1 = -10$

1. Find the fifth term in the sequence.

   Since the first 4 terms are given, and the common difference is $d = 6$, we can see the 5^{th} term 6 more than 4^{th} term.

   i.e. $a_5 = a_4 + 6 = 8 + 6 = 14$

2. Find the 20^{th} term in the sequence.

   Use the formula: $a_n = a_1 + (n-1)d$

3. Find a formula for the n^{th} term in the sequence.

Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

To understand everything about this sequences we need to know:

It's Arithmetic

With common difference $d = 6$ And first term $a_1 = -10$

1. Find the fifth term in the sequence.

   Since the first 4 terms are given, and the common difference is $d = 6$, we can see the 5^{th} term 6 more than 4^{th} term.

   i.e. $a_5 = a_4 + 6 = 8 + 6 = 14$

2. Find the 20^{th} term in the sequence.

   Use the formula: $a_n = a_1 + (n-1)d$

   $a_n = -10 + (n-1)6$ with starting term $n = 1$

3. Find a formula for the n^{th} term in the sequence.

Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

To understand everything about this sequences we need to know:

It's Arithmetic

With common difference $d = 6$ And first term $a_1 = -10$

1. Find the fifth term in the sequence.

   Since the first 4 terms are given, and the common difference is $d = 6$, *we can see the* 5^{th} *term 6 more than* 4^{th} *term.*

   i.e. $a_5 = a_4 + 6 = 8 + 6 = 14$

2. Find the 20^{th} term in the sequence.

   Use the formula: $a_n = a_1 + (n-1)d$

   $a_n = -10 + (n-1)6$ with starting term $n = 1$

   This mean the 20^{th} term is: $a_{20} = -10 + (20-1)6 = 104$

3. Find a formula for the n^{th} term in the sequence.

Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

To understand everything about this sequences we need to know:

It's Arithmetic

With common difference $d = 6$ And first term $a_1 = -10$

1. Find the fifth term in the sequence.

   Since the first 4 terms are given, and the common difference is $d = 6$, we can see the 5^{th} term 6 more than 4^{th} term.

   i.e. $a_5 = a_4 + 6 = 8 + 6 = 14$

2. Find the 20^{th} term in the sequence.

   Use the formula: $a_n = a_1 + (n-1)d$

   $a_n = -10 + (n-1)6$ with starting term $n = 1$

   This mean the 20^{th} term is: $a_{20} = -10 + (20-1)6 = 104$

3. Find a formula for the n^{th} term in the sequence.

   Done above because shortcuts are awesome

Examples, Real World Arithmetic Sequences (Number 1)

Joan invests $\$3,000$ *in an account that pays* 2% *simple interest. Determine how much money is in her account after each of the first 5 years.*

Examples, Real World Arithmetic Sequences (Number 1)

Joan invests $\$3,000$ *in an account that pays* 2% *simple interest. Determine how much money is in her account after each of the first 5 years.*

- Using $I = PRT$ formula for simple interest.

Examples, Real World Arithmetic Sequences (Number 1)

Joan invests $\$3,000$ *in an account that pays* 2% *simple interest. Determine how much money is in her account after each of the first 5 years.*

- Using $I = PRT$ formula for simple interest.
  $P = \$3,000$
  $R = 0.02$
  $T =$ (depends which year we're talking about)

Year	**Interest (I = PRT)**	**Total In Account**
1	$\$3000 \cdot 0.02 \cdot 1 = \$60$	$\$3000 + \$60 = \$3060$

Examples, Real World Arithmetic Sequences (Number 1)

Joan invests $\$3,000$ *in an account that pays* 2% *simple interest. Determine how much money is in her account after each of the first 5 years.*

- Using $I = PRT$ formula for simple interest.
  $P = \$3,000$
  $R = 0.02$
  $T =$ (depends which year we're talking about)

Year	**Interest (I = PRT)**	**Total In Account**
1	$\$3000 \cdot 0.02 \cdot 1 = \$60$	$\$3000 + \$60 = \$3060$
2	$\$3000 \cdot 0.02 \cdot 2 = \$120$	$\$3000 + \$120 = \$3120$

Examples, Real World Arithmetic Sequences (Number 1)

Joan invests $\$3,000$ *in an account that pays* 2% *simple interest. Determine how much money is in her account after each of the first 5 years.*

- Using $I = PRT$ formula for simple interest.
  $P = \$3,000$
  $R = 0.02$
  $T =$ (depends which year we're talking about)

Year	**Interest (I = PRT)**	**Total In Account**
1	$\$3000 \cdot 0.02 \cdot 1 = \$60$	$\$3000 + \$60 = \$3060$
2	$\$3000 \cdot 0.02 \cdot 2 = \$120$	$\$3000 + \$120 = \$3120$
3	$\$3000 \cdot 0.02 \cdot 3 = \$180$	$\$3000 + \$180 = \$3180$
4	$\$3000 \cdot 0.02 \cdot 4 = \$240$	$\$3000 + \$240 = \$3240$
5	$\$3000 \cdot 0.02 \cdot 5 = \$300$	$\$3000 + \$300 = \$3300$

Examples, Real World Arithmetic Sequences (Number 1) Cont.

- Another way to think about it:

Examples, Real World Arithmetic Sequences (Number 1) Cont.

- Another way to think about it:
  The simple interest from each year is $\$3000 \cdot 0.02 \cdot 1 = \$60$, *so each year Joan has* \$60 *more than the previous year.*

Examples, Real World Arithmetic Sequences (Number 1) Cont.

- Another way to think about it:
  The simple interest from each year is $\$3000 \cdot 0.02 \cdot 1 = \$60$, *so each year Joan has* \$60 *more than the previous year.*

- This looks like an arithmetic sequence.
  With starting value $a_1 = \$3060$ and common difference $d = \$60$.

Examples, Real World Arithmetic Sequences (Number 1) Cont.

- Another way to think about it:
  The simple interest from each year is $\$3000 \cdot 0.02 \cdot 1 = \$60$, *so each year Joan has* \$60 *more than the previous year.*

- This looks like an arithmetic sequence.
  With starting value $a_1 = \$3060$ and common difference $d = \$60$.

$$\$3060, \$3120, \$3180, \$3240, \$3300, \ldots$$

Examples, Real World Arithmetic Sequences (Number 1) Cont.

- Another way to think about it:
  The simple interest from each year is $\$3000 \cdot 0.02 \cdot 1 = \$60$, *so each year Joan has* \$60 *more than the previous year.*

- This looks like an arithmetic sequence.
  With starting value $a_1 = \$3060$ and common difference $d = \$60$.

$$\$3060, \$3120, \$3180, \$3240, \$3300, \ldots$$

- So the amount of money in the account at (the end of) year n is: $\mathbf{a_n = \$3000 + (n - 1)\$60}$

Examples, Real World Arithmetic Sequences (Number 2)

You start a new job and you're told you salary is $\$29,000$ *for the first year, and that you'll get a* $\$1700$ *raise each year. What will your salary be in the third year? What will your salary be in 10 years? How long does it take for your salary to (at least) double?*

Examples, Real World Arithmetic Sequences (Number 2)

You start a new job and you're told you salary is $\$29,000$ *for the first year, and that you'll get a* $\$1700$ *raise each year. What will your salary be in the third year? What will your salary be in 10 years? How long does it take for your salary to (at least) double?*

- Fill in the table indicating your salary in the first several years:

Year	Salary in indicated year
1	
2	
3	
4	

Examples, Real World Arithmetic Sequences (Number 2)

You start a new job and you're told you salary is $\$29,000$ *for the first year, and that you'll get a* $\$1700$ *raise each year. What will your salary be in the third year? What will your salary be in 10 years? How long does it take for your salary to (at least) double?*

- Fill in the table indicating your salary in the first several years:

Year	Salary in indicated year
1	$\$29,000$
2	
3	
4	

Examples, Real World Arithmetic Sequences (Number 2)

You start a new job and you're told you salary is $\$29,000$ *for the first year, and that you'll get a* $\$1700$ *raise each year. What will your salary be in the third year? What will your salary be in 10 years? How long does it take for your salary to (at least) double?*

- Fill in the table indicating your salary in the first several years:

Year	Salary in indicated year
1	$\$29,000$
2	$\$30,700$
3	$\$32,400$
4	$\$34,100$

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- Notice that the list of your salaries year by year look like an Arithmetic Sequence.

  Identify the common difference, and the first term:

- Write a formula for a_n (your salary in year n).

- What will your salary be in the third year?

- What will your salary be in 10 years?

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- Notice that the list of your salaries year by year look like an Arithmetic Sequence.

  Identify the common difference, and the first term:

  $a_1 = \$29,000$ $d = \$1700$

  (This is the important bit. You make the table to help you with this.)
- Write a formula for a_n (your salary in year n).
- What will your salary be in the third year?
- What will your salary be in 10 years?

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- Notice that the list of your salaries year by year look like an Arithmetic Sequence.

  Identify the common difference, and the first term:

  $a_1 = \$29,000$ $d = \$1700$

  (This is the important bit. You make the table to help you with this.)

- Write a formula for a_n (your salary in year n).

  $a_n = \$29,000 + (n-1)\$1700$

  Where n is measured in years, and a_n is your salary in year n (measured in dollars)

- What will your salary be in the third year?

- What will your salary be in 10 years?

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- Notice that the list of your salaries year by year look like an Arithmetic Sequence.

  Identify the common difference, and the first term:

  $a_1 = \$29,000$ $d = \$1700$

  (This is the important bit. You make the table to help you with this.)
- Write a formula for a_n (your salary in year n).

  $a_n = \$29,000 + (n-1)\$1700$

  Where n is measured in years, and a_n is your salary in year n (measured in dollars)
- What will your salary be in the third year?

  $a_3 = \$29,000 + (3-1)\$1700 = \$32,400$
- What will your salary be in 10 years?

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- Notice that the list of your salaries year by year look like an Arithmetic Sequence.

  Identify the common difference, and the first term:

  $a_1 = \$29,000$ $d = \$1700$

  (This is the important bit. You make the table to help you with this.)
- Write a formula for a_n (your salary in year n).

  $a_n = \$29,000 + (n-1)\$1700$

  Where n is measured in years, and a_n is your salary in year n (measured in dollars)
- What will your salary be in the third year?

  $a_3 = \$29,000 + (3-1)\$1700 = \$32,400$
- What will your salary be in 10 years?

  $a_{10} = \$29,000 + (10-1)\$1700 = \$44,300$

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  Double your (starting) salary is $\$58,000$

  This is a value for a_n

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  Double your (starting) salary is $\$58,000$

  This is a value for a_n

$$\$58,000 = \$29,000 + (n-1)\$1700$$

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  Double your (starting) salary is $\$58,000$

  This is a value for a_n

$$\begin{aligned} \$58,000 &= \$29,000 + (n-1)\$1700 \\ \$58,000 - \$29,000 &= (n-1)\$1700 \end{aligned}$$

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  Double your (starting) salary is $\$58,000$

  This is a value for a_n

$$\begin{aligned} \$58,000 &= \$29,000 + (n-1)\$1700 \\ \$58,000 - \$29,000 &= (n-1)\$1700 \\ \$29,000 &= n \cdot \$1700 - \$1700 \end{aligned}$$

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  Double your (starting) salary is $\$58,000$

  This is a value for a_n

$$\begin{aligned} \$58,000 &= \$29,000 + (n-1)\$1700 \\ \$58,000 - \$29,000 &= (n-1)\$1700 \\ \$29,000 &= n \cdot \$1700 - \$1700 \\ \$29,000 + \$1700 &= n \cdot \$1700 \end{aligned}$$

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  Double your (starting) salary is $\$58,000$

  This is a value for a_n

$$\begin{aligned} \$58,000 &= \$29,000 + (n-1)\$1700 \\ \$58,000 - \$29,000 &= (n-1)\$1700 \\ \$29,000 &= n \cdot \$1700 - \$1700 \\ \$29,000 + \$1700 &= n \cdot \$1700 \\ \$30,700 &= n \cdot \$1700 \end{aligned}$$

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  Double your (starting) salary is $\$58,000$

  This is a value for a_n

$$\begin{aligned}
\$58,000 &= \$29,000 + (n-1)\$1700 \\
\$58,000 - \$29,000 &= (n-1)\$1700 \\
\$29,000 &= n \cdot \$1700 - \$1700 \\
\$29,000 + \$1700 &= n \cdot \$1700 \\
\$30,700 &= n \cdot \$1700 \\
\frac{\$30,700}{\$1700} &= n
\end{aligned}$$

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  Double your (starting) salary is $\$58,000$

  This is a value for a_n

$$\begin{aligned}
\$58,000 &= \$29,000 + (n-1)\$1700 \\
\$58,000 - \$29,000 &= (n-1)\$1700 \\
\$29,000 &= n \cdot \$1700 - \$1700 \\
\$29,000 + \$1700 &= n \cdot \$1700 \\
\$30,700 &= n \cdot \$1700 \\
\frac{\$30,700}{\$1700} &= n
\end{aligned}$$

  So $n \approx 18.0588$, We must round up to 19 years.

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  Double your (starting) salary is $\$58,000$

  This is a value for a_n

$$\begin{aligned}
\$58,000 &= \$29,000 + (n-1)\$1700 \\
\$58,000 - \$29,000 &= (n-1)\$1700 \\
\$29,000 &= n \cdot \$1700 - \$1700 \\
\$29,000 + \$1700 &= n \cdot \$1700 \\
\$30,700 &= n \cdot \$1700 \\
\frac{\$30,700}{\$1700} &= n
\end{aligned}$$

  So $n \approx 18.0588$, We must round up to 19 years.

  Double Check:

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  Double your (starting) salary is $\$58,000$

  This is a value for a_n

$$\begin{aligned}
\$58,000 &= \$29,000 + (n-1)\$1700 \\
\$58,000 - \$29,000 &= (n-1)\$1700 \\
\$29,000 &= n \cdot \$1700 - \$1700 \\
\$29,000 + \$1700 &= n \cdot \$1700 \\
\$30,700 &= n \cdot \$1700 \\
\frac{\$30,700}{\$1700} &= n
\end{aligned}$$

So $n \approx 18.0588$, We must round up to 19 years.

Double Check:

$a_{18} = \$29,000 + (18-1)\$1700 = \$57,900$ (less than double)

$a_{19} = \$29,000 + (19-1)\$1700 = \$59,600$ (more than double)

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  Double your (starting) salary is $\$58,000$

  This is a value for a_n

$$\begin{aligned}
\$58,000 &= \$29,000 + (n-1)\$1700 \\
\$58,000 - \$29,000 &= (n-1)\$1700 \\
\$29,000 &= n \cdot \$1700 - \$1700 \\
\$29,000 + \$1700 &= n \cdot \$1700 \\
\$30,700 &= n \cdot \$1700 \\
\frac{\$30,700}{\$1700} &= n
\end{aligned}$$

  So $n \approx 18.0588$, We must round up to 19 years.

  Double Check:

  $a_{18} = \$29,000 + (18-1)\$1700 = \$57,900$ (less than double)

  $a_{19} = \$29,000 + (19-1)\$1700 = \$59,600$ (more than double)

  So it takes 19 year for your salary to double.

Partial Sums of an Arithmetic Sequence

- Remember S_n is the sum of the first n terms of a sequence.

Partial Sums of an Arithmetic Sequence

- Remember S_n is the sum of the first n terms of a sequence.
- Let's work our a formula for the n^{th} partial sum of an Arithmetic Sequence

Partial Sums of an Arithmetic Sequence

- Here's one way to write our Arithmetic sequence:

$$a_1, (a_1 + d), (a_1 + 2d), (a_1 + 3d), \ldots$$

Partial Sums of an Arithmetic Sequence

- Here's one way to write our Arithmetic sequence:

$$a_1, (a_1 + d), (a_1 + 2d), (a_1 + 3d), \ldots$$

- So the n^{th} Partial Sum of the Arithmetic Series can be written as:

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-2)d) + \underbrace{(a_1 + (n-1)d)}_{a_n}$$

Partial Sums of an Arithmetic Sequence

- Here's one way to write our Arithmetic sequence:

$$a_1, (a_1 + d), (a_1 + 2d), (a_1 + 3d), \ldots$$

- So the n^{th} Partial Sum of the Arithmetic Series can be written as:

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-2)d) + \underbrace{(a_1 + (n-1)d)}_{a_n}$$

- Another way to name the terms:

$$\underbrace{a_1}_{a_n-(n-1)d} + \underbrace{(a_1 + d)}_{a_n-(n-2)d} + \underbrace{(a_1 + 2d)}_{a_n-(n-3)d} + \cdots + \underbrace{(a_1 + (n-2)d)}_{a_n-d} + \underbrace{(a_1 + (n-1)d)}_{a_n}$$

Partial Sums of an Arithmetic Sequence

- Here's one way to write our Arithmetic sequence:

$$a_1, (a_1 + d), (a_1 + 2d), (a_1 + 3d), \ldots$$

- So the n^{th} Partial Sum of the Arithmetic Series can be written as:

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-2)d) + \underbrace{(a_1 + (n-1)d)}_{a_n}$$

- Another way to name the terms:

$$\underbrace{a_1}_{a_n-(n-1)d} + \underbrace{(a_1 + d)}_{a_n-(n-2)d} + \underbrace{(a_1 + 2d)}_{a_n-(n-3)d} + \cdots + \underbrace{(a_1 + (n-2)d)}_{a_n-d} + \underbrace{(a_1 + (n-1)d)}_{a_n}$$

- This gives us another way to write S_n

$$S_n = a_n + (a_n - d) + (a_n - 2d) + \cdots + (a_n - (n-2)d) + (a_n - (n-1)d)$$

Partial Sums of an Arithmetic Sequence

- Here's one way to write our Arithmetic sequence:

$$a_1, (a_1 + d), (a_1 + 2d), (a_1 + 3d), \ldots$$

- So the n^{th} Partial Sum of the Arithmetic Series can be written as:

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-2)d) + \underbrace{(a_1 + (n-1)d)}_{a_n}$$

- Another way to name the terms:

$$\underbrace{a_1}_{a_n - (n-1)d} + \underbrace{(a_1 + d)}_{a_n - (n-2)d} + \underbrace{(a_1 + 2d)}_{a_n - (n-3)d} + \cdots + \underbrace{(a_1 + (n-2)d)}_{a_n - d} + \underbrace{(a_1 + (n-1)d)}_{a_n}$$

- This gives us another way to write S_n

$$S_n = a_n + (a_n - d) + (a_n - 2d) + \cdots + (a_n - (n-2)d) + (a_n - (n-1)d)$$

- Add the two ways to write S_n together.....

Partial Sums of an Arithmetic Sequence (Cont.)

- Add the two ways to write S_n together.....

$$\begin{array}{rcl} S_n & = & a_1 + (a_1 + d) + (a_1 + 2d) \quad + \cdots + (a_1 + (n-2)d) + (a_1 + (n-1)d) \\ +S_n & = & a_n + (a_n - d) + (a_n - 2d) \quad + \cdots + (a_n - (n-2)d) + (a_n - (n-1)d) \\ \hline 2S_n & = & (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) \quad + \cdots + (a_1 + a_n) \quad + (a_1 + a_n) \end{array}$$

Partial Sums of an Arithmetic Sequence (Cont.)

- Add the two ways to write S_n together.....

$$\begin{array}{rcll} S_n & = & a_1 + (a_1 + d) + (a_1 + 2d) & + \cdots + (a_1 + (n-2)d) + (a_1 + (n-1)d) \\ +S_n & = & a_n + (a_n - d) + (a_n - 2d) & + \cdots + (a_n - (n-2)d) + (a_n - (n-1)d) \\ \hline 2S_n & = & (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) & + \cdots + (a_1 + a_n) \qquad + (a_1 + a_n) \end{array}$$

We count the $(a_1 + a_n)$ terms on the right...

Partial Sums of an Arithmetic Sequence (Cont.)

- Add the two ways to write S_n together.....

$$\begin{array}{rcl} S_n & = & a_1 + (a_1 + d) + (a_1 + 2d) \quad + \cdots + (a_1 + (n-2)d) + (a_1 + (n-1)d) \\ +S_n & = & a_n + (a_n - d) + (a_n - 2d) \quad + \cdots + (a_n - (n-2)d) + (a_n - (n-1)d) \\ \hline 2S_n & = & (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) \quad + \cdots + (a_1 + a_n) \quad + (a_1 + a_n) \end{array}$$

We count the $(a_1 + a_n)$ terms on the right...
We see that $2S_n = n(a_1 + a_n)$ and...

Partial Sums of an Arithmetic Sequence (Cont.)

- Add the two ways to write S_n together.....

$$\begin{array}{rcll} S_n & = & a_1 + (a_1 + d) + (a_1 + 2d) & + \cdots + (a_1 + (n-2)d) + (a_1 + (n-1)d) \\ +S_n & = & a_n + (a_n - d) + (a_n - 2d) & + \cdots + (a_n - (n-2)d) + (a_n - (n-1)d) \\ \hline 2S_n & = & (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) & + \cdots + (a_1 + a_n) \qquad + (a_1 + a_n) \end{array}$$

We count the $(a_1 + a_n)$ terms on the right...
We see that $2S_n = n(a_1 + a_n)$ and...

$$S_n = \frac{n(a_1 + a_n)}{2} = \frac{n}{2}(a_1 + a_n)$$

Formula the n^{th} Partial Sum of an Arithmetic Sequence

- **The n^{th} partial sum of an Arithmetic Sequence** $a_1, a_2, a_3, \ldots$ is given by

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Where a_1 is the first term of the Arithmetic Sequence and a_n is the n^{th} term of the Arithmetic Series.

Using the Formula the n^{th} Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence $7, 10, 13, 16, 19, 22, \ldots$
  1. Find the 4^{th} Partial Sum of the Sequence.
  2. Find the 20^{th} term of the Sequence
  3. Find the 20^{th} Partial Sum of the Sequence

Using the Formula the n^{th} Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence $7, 10, 13, 16, 19, 22, \ldots$
  1. Find the 4^{th} Partial Sum of the Sequence.
     $S_4 = \frac{4}{2}(a_1 + a_4) = \frac{4}{2}(7 + 16) = 2(23) = 46$
  2. Find the 20^{th} term of the Sequence
  3. Find the 20^{th} Partial Sum of the Sequence

Using the Formula the n^{th} Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence $7, 10, 13, 16, 19, 22, \ldots$
  1. Find the 4^{th} Partial Sum of the Sequence.
     $S_4 = \frac{4}{2}(a_1 + a_4) = \frac{4}{2}(7 + 16) = 2(23) = 46$
     (We can double check that $7 + 10 + 13 + 16 = 46$)
  2. Find the 20^{th} term of the Sequence
  3. Find the 20^{th} Partial Sum of the Sequence

Using the Formula the n^{th} Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence $7, 10, 13, 16, 19, 22, \ldots$
  1. Find the 4^{th} Partial Sum of the Sequence.
     $S_4 = \frac{4}{2}(a_1 + a_4) = \frac{4}{2}(7 + 16) = 2(23) = 46$
     (We can double check that $7 + 10 + 13 + 16 = 46$)
  2. Find the 20^{th} term of the Sequence
     Our Arithmetic Sequence has $a_1 = 7$ and $d = 3$ so $a_n = 7 + (n - 1)3$, so ...
  3. Find the 20^{th} Partial Sum of the Sequence

Using the Formula the n^{th} Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence $7, 10, 13, 16, 19, 22, \ldots$
  1. Find the 4^{th} Partial Sum of the Sequence.
     $S_4 = \frac{4}{2}(a_1 + a_4) = \frac{4}{2}(7 + 16) = 2(23) = 46$
     (We can double check that $7 + 10 + 13 + 16 = 46$)
  2. Find the 20^{th} term of the Sequence
     Our Arithmetic Sequence has $a_1 = 7$ and $d = 3$ so
     $a_n = 7 + (n - 1)3$, so ...
     $a_{20} = 7 + (20 - 1)3 = 7 + 19 \cdot 3 = 64$
  3. Find the 20^{th} Partial Sum of the Sequence

Using the Formula the n^{th} Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence $7, 10, 13, 16, 19, 22, \ldots$
  1. Find the 4^{th} Partial Sum of the Sequence.
     $S_4 = \frac{4}{2}(a_1 + a_4) = \frac{4}{2}(7 + 16) = 2(23) = 46$
     (We can double check that $7 + 10 + 13 + 16 = 46$)
  2. Find the 20^{th} term of the Sequence
     Our Arithmetic Sequence has $a_1 = 7$ and $d = 3$ so
     $a_n = 7 + (n - 1)3$, so ...
     $a_{20} = 7 + (20 - 1)3 = 7 + 19 \cdot 3 = 64$
  3. Find the 20^{th} Partial Sum of the Sequence
     $S_{20} = \frac{20}{2}(7 + 64) = 10(71) = 710$

Using the Formula the n^{th} Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence $7, 10, 13, 16, 19, 22, \ldots$
  1. Find the 4^{th} Partial Sum of the Sequence.
     $S_4 = \frac{4}{2}(a_1 + a_4) = \frac{4}{2}(7 + 16) = 2(23) = 46$
     (We can double check that $7 + 10 + 13 + 16 = 46$)
  2. Find the 20^{th} term of the Sequence
     Our Arithmetic Sequence has $a_1 = 7$ and $d = 3$ so
     $a_n = 7 + (n - 1)3$, so ...
     $a_{20} = 7 + (20 - 1)3 = 7 + 19 \cdot 3 = 64$
  3. Find the 20^{th} Partial Sum of the Sequence
     $S_{20} = \frac{20}{2}(7 + 64) = 10(71) = 710$

Which is much faster than

Using the Formula the n^{th} Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence $7, 10, 13, 16, 19, 22, \ldots$

1. Find the 4^{th} Partial Sum of the Sequence.
   $S_4 = \frac{4}{2}(a_1 + a_4) = \frac{4}{2}(7 + 16) = 2(23) = 46$
   (We can double check that $7 + 10 + 13 + 16 = 46$)

2. Find the 20^{th} term of the Sequence
   Our Arithmetic Sequence has $a_1 = 7$ and $d = 3$ so
   $a_n = 7 + (n - 1)3$, so ...
   $a_{20} = 7 + (20 - 1)3 = 7 + 19 \cdot 3 = 64$

3. Find the 20^{th} Partial Sum of the Sequence
   $S_{20} = \frac{20}{2}(7 + 64) = 10(71) = 710$

Which is much faster than

$7 + 10 + 13 + 16 + 19 + 22 + 25 + 28 + 31 + 34 + 37 + 40 + 43 + 46 + 49 + 52 + 55 + 58 + 61 + 64 = 710$

Examples, Real World Arithmetic Sequences (Number 3)

A new company has a loss of $\$2,500$ *in its first month, but they expect their monthly profit to increase by* $\$400$ *each month. What is their profit in the* 12^{th} *month? What is their total profit/loss of the year?*

Examples, Real World Arithmetic Sequences (Number 3)

A new company has a loss of $\$2,500$ *in its first month, but they expect their monthly profit to increase by* \$400 *each month. What is their profit in the* 12^{th} *month? What is their total profit/loss of the year?*

Fill in the Table:

Month	Profit/Lost for Month in indicated month
1	
2	
3	

Examples, Real World Arithmetic Sequences (Number 3)

A new company has a loss of $\$2,500$ *in its first month, but they expect their monthly profit to increase by* \$400 *each month. What is their profit in the* 12^{th} *month? What is their total profit/loss of the year?*

Fill in the Table:

Month	Profit/Lost for Month in indicated month
1	$-\$2,500$

Examples, Real World Arithmetic Sequences (Number 3)

A new company has a loss of $\$2,500$ *in its first month, but they expect their monthly profit to increase by* \$400 *each month. What is their profit in the* 12^{th} *month? What is their total profit/loss of the year?*

Fill in the Table:

Month	Profit/Lost for Month in indicated month
1	$-\$2,500$
2	$-\$2,100$
3	$-\$1,700$

Examples, Real World Arithmetic Sequences (Number 3)

A new company has a loss of $\$2,500$ *in its first month, but they expect their monthly profit to increase by* \$400 *each month. What is their profit in the* 12^{th} *month? What is their total profit/loss of the year?*

Fill in the Table:

Month	Profit/Lost for Month in indicated month
1	$-\$2,500$
2	$-\$2,100$
3	$-\$1,700$

This is an Arithmetic Sequence with $a_1 = -\$2500$ and $d = 400$

So a_n represents the monthly profit/loss in month n and

$$a_n = -\$2500 + (n-1)\$400$$

Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12^{th} month:

- The total profit/loss for the year:

Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12^{th} month:
  is represented by a_{12}

- The total profit/loss for the year:

Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12^{th} month:
  is represented by a_{12}
  $a_{12} = -\$2500 + (12-1)\$400 = \$1900$

- The total profit/loss for the year:

Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12^{th} month:
  is represented by a_{12}
  $a_{12} = -\$2500 + (12-1)\$400 = \$1900$

- The total profit/loss for the year:

(profit/loss for Jan) + (profit/loss for Feb) + $\cdots$ + (profit/loss for Dec)

Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12^{th} month:
  is represented by a_{12}
  $a_{12} = -\$2500 + (12-1)\$400 = \$1900$

- The total profit/loss for the year:

(profit/loss for Jan) + (profit/loss for Feb) + $\cdots$ + (profit/loss for Dec)

which can be represented in symbols as $a_1 + a_2 + \cdots + a_{12} = S_{12}$

Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12^{th} month:
  is represented by a_{12}
  $a_{12} = -\$2500 + (12-1)\$400 = \$1900$

- The total profit/loss for the year:

(profit/loss for Jan) + (profit/loss for Feb) + $\cdots$ + (profit/loss for Dec)

which can be represented in symbols as $a_1 + a_2 + \cdots + a_{12} = S_{12}$

Total Profits for the Year are S_{12}

Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12^{th} month:
  is represented by a_{12}
  $a_{12} = -\$2500 + (12-1)\$400 = \$1900$

- The total profit/loss for the year:

(profit/loss for Jan) + (profit/loss for Feb) + $\cdots$ + (profit/loss for Dec)

which can be represented in symbols as $a_1 + a_2 + \cdots + a_{12} = S_{12}$

Total Profits for the Year are S_{12}
$S_{12} = \frac{12}{2}(a_1 + a_{12}) = \frac{12}{2}(-2500 + 1900) = 6(-600) = -\$3600$

Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12^{th} month:
  is represented by a_{12}
  $a_{12} = -\$2500 + (12-1)\$400 = \$1900$

- The total profit/loss for the year:

(profit/loss for Jan) + (profit/loss for Feb) + $\cdots$ + (profit/loss for Dec)

which can be represented in symbols as $a_1 + a_2 + \cdots + a_{12} = S_{12}$

Total Profits for the Year are S_{12}
$S_{12} = \frac{12}{2}(a_1 + a_{12}) = \frac{12}{2}(-2500 + 1900) = 6(-600) = -\$3600$

They lost a total of \$3600 for the year.

Examples, Real World Arithmetic Sequences (Number 2...again)

You start a new job and you're told you salary is $\$29,000$ *for the first year, and that you'll get a* $\$1700$ *raise each year. How much money will you make total your first 10 years on the job.*

Examples, Real World Arithmetic Sequences (Number 2...again)

You start a new job and you're told you salary is $\$29,000$ *for the first year, and that you'll get a* $\$1700$ *raise each year. How much money will you make total your first 10 years on the job.*

We saw earlier this is an Arithmetic sequence with
$a_1 = \$29,000,\ d = \$1700$

$a_n = 29000 + (n-1)1700$
$a_{10} = 29000 + (n-1)1700 = 44,300$

Examples, Real World Arithmetic Sequences (Number 2...again)

You start a new job and you're told you salary is $\$29,000$ *for the first year, and that you'll get a* $\$1700$ *raise each year. How much money will you make total your first 10 years on the job.*

We saw earlier this is an Arithmetic sequence with
$a_1 = \$29,000,\ d = \$1700$

$a_n = 29000 + (n-1)1700$
$a_{10} = 29000 + (n-1)1700 = 44,300$

Total you make in the first 10 years is S_{10}

Examples, Real World Arithmetic Sequences (Number 2...again)

You start a new job and you're told you salary is $\$29,000$ *for the first year, and that you'll get a* $\$1700$ *raise each year. How much money will you make total your first 10 years on the job.*

We saw earlier this is an Arithmetic sequence with
$a_1 = \$29,000,\ d = \$1700$

$a_n = 29000 + (n-1)1700$
$a_{10} = 29000 + (n-1)1700 = 44,300$

Total you make in the first 10 years is S_{10}
$S_{10} = \frac{n}{2}(a_1 + a_{10})$

$S_{10} = \frac{10}{2}(29000 + 44300) = \$366,500$

Homework

It is NOT in your book.

It IS at the end of the printout on the *course website*.