Arithmetic Sequences

Supplemental Material Not Found in You Text

Math 34: Fall 2016

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September 9, 2016

Arithmetic Sequences

- Recall an **Arithmetic Sequence** is a sequence where the *difference* between any two consecutive numbers in the sequence is constant.

  In other words: $a_{k+1} - a_k = d$ *where* d *is a constant.*

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  In other words: $a_{k+1} - a_k = d$ *where* d *is a constant.*

- Which of the following are Arithmetic Sequences?
  1. $1, 4, 7, 10, 13, \ldots$
  2. $2, 4, 8, 16, 32, \ldots$
  3. $-3, 7, 17, 27, \ldots$

Real World Examples

1. You start a new job and you're told you salary is $\$29,000$ for the first year, and that you'll get a $\$1700$ raise each year. What will your salary be in the third year? What will your salary be in 10 years?

2. A new company has a loss of $\$2,500$ in its first month, but they expect their monthly profit to increase by $\$400$ each month. What is their profit in the 12^{th} month? What is their total profit/loss of the year?

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Both these scenarios can be modeled by Arithmetic Sequences, and we will develop tools to help us answer these questions.

General Way to Write an Arithmetic Sequence

- Consider the Arithmetic Sequence below. Notice the first term is 5 and the common difference is 2:

$$5, 7, 9, 11, 13, \ldots$$

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$$\begin{array}{cccccc} 5, & 7, & 9, & 11, & 13, & \ldots \\ \underbrace{5}_{a_1}, & \underbrace{5+(1)2}_{a_2}, & \underbrace{5+(2)2}_{a_3}, & \underbrace{5+(3)2}_{a_4}, & \underbrace{5+(4)2}_{a_5}, & \ldots \end{array}$$

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We notice the pattern for this sequence $a_n = 5 + (n-1)2$

We also see that $a_n = a_{n-1} + 2$
(each term is 2 more than the previous term)

General Way to Write an Arithmetic Sequence

- **Way to Write a Formula for an Arithmetic Sequence:** Given that $a_1, a_2, a_3, \ldots$ is an arithmetic sequence with common difference d,

  We can rewrite the sequence as

  $$a_n = a_1 + (n-1)d$$

  where the index starts at $n = 1$.

  Here a_1 is the first term of the sequence (a constant) and d is the common difference (also a constant).

Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

1. Find the fifth term in the sequence.
2. Find the 20^{th} term in the sequence.
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   i.e. $a_5 = a_4 + 6 = 8 + 6 = 14$
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3. Find a formula for the n^{th} term in the sequence.

   Done above because shortcuts are awesome

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Joan invests $\$3,000$ *in an account that pays* 2% *simple interest. Determine how much money is in her account after each of the first 5 years.*

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  $P = \$3,000$
  $R = 0.02$
  $T =$ (depends which year we're talking about)

Year	**Interest (I = PRT)**	**Total In Account**
1	$\$3000 \cdot 0.02 \cdot 1 = \$60$	$\$3000 + \$60 \ = \$3060$

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3	$\$3000 \cdot 0.02 \cdot 3 = \$180$	$\$3000 + \$180 = \$3180$
4	$\$3000 \cdot 0.02 \cdot 4 = \$240$	$\$3000 + \$240 = \$3240$
5	$\$3000 \cdot 0.02 \cdot 5 = \$300$	$\$3000 + \$300 = \$3300$

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  The simple interest from each year is $\$3000 \cdot 0.02 \cdot 1 = \$60$, *so each year Joan has* $\$60$ *more than the previous year.*

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$$\$3060, \$3120, \$3180, \$3240, \$3300, \ldots$$

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$$\$3060, \$3120, \$3180, \$3240, \$3300, \ldots$$

- So the amount of money in the account at (the end of) year n is: $\mathbf{a_n = \$3060 + (n-1)\$60}$

Examples, Real World Arithmetic Sequences (Number 2)

You start a new job and you're told you salary is $\$29,000$ *for the first year, and that you'll get a* $\$1700$ *raise each year. What will your salary be in the third year? What will your salary be in 10 years? How long does it take for your salary to (at least) double?*

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2	\$30, 700
3	\$32, 400
4	\$34, 100

Examples, Real World Arithmetic Sequences (Number 2) Cont.

- Notice that the list of your salaries year by year look like an Arithmetic Sequence.
  Identify the common difference, and the first term:
- Write a formula for a_n (your salary in year n).
- What will your salary be in the third year?
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  Where n is measured in years, and a_n is your salary in year n (measured in dollars)

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  $a_3 = \$29,000 + (3-1)\$1700 = \$32,400$

- What will your salary be in 10 years?

  $a_{10} = \$29,000 + (10-1)\$1700 = \$44,300$

Partial Sums of an Arithmetic Sequence

- Remember S_n is the sum of the first n terms of a sequence.

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- Let's work our a formula for the n^{th} partial sum of an Arithmetic Sequence

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- So the n^{th} Partial Sum of the Arithmetic Series can be written as:

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-2)d) + \underbrace{(a_1 + (n-1)d)}_{a_n}$$

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- Another way to name the terms:

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$$S_n = a_n + (a_n - d) + (a_n - 2d) + \cdots + (a_n - (n-2)d) + (a_n - (n-1)d)$$

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Partial Sums of an Arithmetic Sequence (Cont.)

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Partial Sums of an Arithmetic Sequence (Cont.)

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+S_n & = & a_n + (a_n-d) + (a_n-2d) & + \cdots + (a_n-(n-2)d) + (a_n-(n-1)d) \\
\hline
2S_n & = & (a_1+a_n) + (a_1+a_n) + (a_1+a_n) & + \cdots + (a_1+a_n) \quad + (a_1+a_n)
\end{array}$$

We count the $(a_1 + a_n)$ terms on the right...

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We count the $(a_1 + a_n)$ terms on the right...
We see that $2S_n = n(a_1 + a_n)$ and...

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We see that $2S_n = n(a_1 + a_n)$ and...

$$S_n = \frac{n(a_1 + a_n)}{2} = \frac{n}{2}(a_1 + a_n)$$

Formula the n^{th} Partial Sum of an Arithmetic Sequence

- **The n^{th} partial sum of an Arithmetic Sequence** $a_1, a_2, a_3, \ldots$ is given by

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Where a_1 is the first term of the Arithmetic Sequence and a_n is the n^{th} term of the Arithmetic Series.

Using the Formula the n^{th} Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence $7, 10, 13, 16, 19, 22, \ldots$
  1. Find the 4^{th} Partial Sum of the Sequence.
  2. Find the 20^{th} term of the Sequence
  3. Find the 20^{th} Partial Sum of the Sequence

Using the Formula the n^{th} Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence $7, 10, 13, 16, 19, 22, \ldots$
  1. Find the 4^{th} Partial Sum of the Sequence.
     $S_4 = \frac{4}{2}(a_1 + a_4) = \frac{4}{2}(7 + 16) = 2(23) = 46$
  2. Find the 20^{th} term of the Sequence
  3. Find the 20^{th} Partial Sum of the Sequence

Using the Formula the n^{th} Partial Sum of an Arithmetic Sequence

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Using the Formula the n^{th} Partial Sum of an Arithmetic Sequence

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$7 + 10 + 13 + 16 + 19 + 22 + 25 + 28 + 31 + 34 + 37 + 40 + 43 + 46 + 49 + 52 + 55 + 58 + 61 + 64 = 710$

Examples, Real World Arithmetic Sequences (Number 3)

A new company has a loss of $\$2,500$ *in its first month, but they expect their monthly profit to increase by* $\$400$ *each month. What is their profit in the* 12^{th} *month? What is their total profit/loss of the year?*

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This is an Arithmetic Sequence with $a_1 = -\$2500$ and $d = 400$

So a_n represents the monthly profit/loss in month n and

$$a_n = -\$2500 + (n-1)\$400$$

Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12^{th} month:

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$-2500 - 2100 - 1700 - 1300 - 900 - 500 - 100 + 300 + 700 + 1100 + 1500 + 1900 = -3600$

Examples, Real World Arithmetic Sequences (Number 2...again)

You start a new job and you're told you salary is $\$29,000$ *for the first year, and that you'll get a* $\$1700$ *raise each year. How much money will you make total your first 10 years on the job.*

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Total you make in the first 10 years is S_{10}
$S_{10} = \frac{n}{2}(a_1 + a_{10})$

$S_{10} = \frac{10}{2}(29000 + 44300) = \$366,500$

Homework

It is NOT in your book.

It IS at the end of the printout on the *course website*.