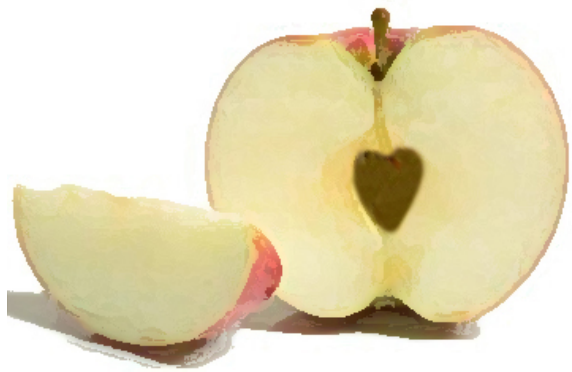

Getting to the Core

Grade 5 Unit of Study

Multiplication and Division of Fractions

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Multiplication & Division of Fractions

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Santa Ana Unified School District Common Core Unit Planner-Mathematics

Unit Title:	Multiplying and Dividing Fractions	
Grade Level:	5th Grade	**Time Frame**: 3 weeks
Big Idea (Enduring Understandings):	**The properties of multiplication and division of whole numbers apply also to the multiplication and division of fractions.**	
Essential Questions:	• How are fractions related to division? • How can the area of a rectangle with fractional sides be represented? • How can a visual model help to show multiplication of a fraction by a whole number? • How does multiplying by a fraction or by a mixed number affect the size of the product? • How can multiplication of fractions and mixed numbers be used in real life situations? • How can division of fractions be used in real life situations?	
21st Century Skills:	**Learning and Innovation:** ☒ Critical Thinking & Problem Solving ☒ Communication & Collaboration ☐ Creativity & Innovation **Information, Media and Technology:** ☒ Online Tools ☒ Software ☐ Hardware	
Essential Academic Language:	**Tier II:** Contrast However Although Nevertheless Moreover In addition Similarly	**Tier III:** Multiply Divide Simplest form Mixed number Denominator Numerator Unit fraction Improper fraction Mixed number Equivalent fraction Reciprocal

What pre-assessment will be given? Prerequisite Skills Test	**How will pre-assessment guide instruction?** Students missing two or more in any section will need intervention through the Preparing the Learner lessons.

Instructional Activities:

(What learning experiences will students engage in? How will you use these learning experiences to drive responsive teaching?)

Preparing the Learner Lesson A Preparing a Fraction Bar Toolkit	Preparing the Learner Lesson B Launching Mathematical Discourse	Preparing the Learner Lesson C Learning the Language of Contrast

CCS 5.3 Interpret Fractions as Division:
Fractions are defined as division of the numerator by the denominator.

CCS 5.4.a Multiply fractions by whole numbers and by other fractions:
Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
When multiplying a fraction times a whole, the parts of the fraction are partitioned among the whole number.

CCS 5.4.b Multiply fractions by whole numbers and by other fractions:
Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
The area of a rectangle with fractional lengths can be found by multiplying the length times the width, just as with whole numbers.

CCS 5.5.a,b Scaling:
Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
Multiplying by a fraction reduces the size of the product, while multiplying by a mixed number increases the size of the product.

Assessment or Performance Task

CCS 5. 6 Real world problems with multiplication of fractions and mixed numbers:
Visual and numeric models of multiplication of fractions and mixed numbers are used to solve problems in daily life.

CCS5 .7.a,b,c Real world problems with division of fractions and whole numbers:
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions:
Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.
Interpret division of a whole number by a unit fraction, and compute such quotients.
Division of fractions is used to solve problems in daily life.

Performance Task

Standards	Assessment of Standards	
Common Core Learning Standards Taught and Assessed	**What assessments will be utilized for this unit? (F = formative, S = summative)**	**What does the assessment tell us?**
Common Core Mathematics Content Standards: **Number and Operations–Fractions** **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **3.** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions, mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **4**. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15.(In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **5**. Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n\, b)$ to the effect of multiplying a/b by 1. **6**. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **7**. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship	F: Problem solving journal F: Visual representation of thinking F: Performance Task : Lesson 1-4 Review Tasks F: Lesson 7 Performance Task S: Performance Task: Culminating Task S: End of Unit Assessment S: Benchmark Tests **Other Evidence:** Teacher observations	Ongoing evidence of students' understanding of the concepts presented Diagnostic information for intervention or acceleration Student comprehension of unit concepts and the big idea: **The properties of multiplication and division of whole numbers apply also to the multiplication and division of fractions.**

between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$. b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?		
Bundled Language Standard(s): 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. 6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases, including those that signal contrast, addition, and other logical relationships (e.g., however, although, nevertheless, similarly, moreover, in addition).	**F:** Teacher evaluation of student use of appropriate mathematical academic language during partner, small group, and class discussions. **S**: Use of accurate mathematical terms and appropriate relationship language in culminating written word problem and its solution.	Do students use the appropriate academic language when speaking in class discussions and presentations and when writing in their daily math journals?
Bundled Speaking and Listening Standard(s): 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 5 topics and texts, building on others' ideas and expressing their own clearly. a. Come to discussions prepared having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion. b. Follow agreed-upon rules for discussions and carry out assigned roles. c. Pose and respond to specific questions by making comments that contribute to the discussion and elaborate on the remarks of others. d. Review the key ideas expressed and draw conclusions in light of information and knowledge gained from the discussions. 4. Report on a topic or text, or present an opinion, sequencing ideas logically and using appropriate facts and relevant, descriptive details to support main ideas or themes; speak clearly at an understandable pace.	Teacher Evaluation of student speaking and listening: F: Ask and answer questions in pairs and small groups during and after lessons. F: Work collaboratively to solve complex problems while treating each other with respect. F: Participation in presentations of solutions for group work. S: Design and write a recipe using fractional parts which must be multiplied or divided to change the quantity of the recipe.	When talking about mathematics in pairs and groups, do students follow protocol/rules/ routines for collaborative discussions? Can students plan and deliver an informative presentation with appropriately detailed sequencing? Do all students participate in the thinking, conversation, and final product? Do they follow rules and

		guidelines for collaboration?
Standards of Mathematical Practice:	(*Check all that apply*) ☒ 1. Make sense of problems and persevere in solving them. ☐ 2. Reason abstractly and quantitatively. ☐ 3. Construct viable arguments and critique the reasoning of others. ☒ 4. Model with mathematics. ☐ 5. Use appropriate tools strategically. ☐ 6. Attend to precision. ☒ 7. Look for and make use of structure. ☐ 8. Look for and express regularity in repeated reasoning.	**Opportunities for Observable Data** *(How will students demonstrate these Mathematical Practices?)* 1. Students analyze fractional parts and understand how they are related to multiplication and division. 4. Students will create visual models of operations with fractions. 8. Students will notice that Multiplication Properties apply to the multiplication of fractions and mixed numbers.
Resources/ Materials:	**Mathematical Tools:** tiles or counters, fraction bars, graph paper, colored water-based markers, colored pencils **Media/Technology:** ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division; NCTM Illuminations Website <http //illuminations.nctm.org> (Fractions games: *Drop Zone*, *Fraction Feud*, *Dig It, Equivalent Fractions, Fraction Game, Fraction Models)* **Supplementary Materials:**	
Interdisciplinary Connections:	**Cite several interdisciplinary or cross-content connections made in this unit of study (i.e. literature, science, social studies, art, etc.)** Art projects using tessellations of geometric figures showing fractional parts. Data analysis where statistics are related as fractions.	

Differentiated Instruction:	**Based on desired student outcomes, what instructional variation will be used to address the needs of English Learners by language proficiency level?** • Use of sentence frames (appropriate for language level) to facilitate academic language and conversations. Use of visual organizers to assist processing mathematical ideas • Explicitly teach key academic vocabulary. • Use of manipulatives to facilitate conceptual understanding • Flexible grouping to support language acquisition and target instruction • Use of collaboration to promote socio-cultural learning • Opportunities for verbal rehearsal of concepts	**Based on desired student outcomes, what instructional variation will be used to address the needs of students with special needs, including gifted and talented?** **Special Needs-** • Use of visual organizers in organizing and evaluating evidence. • Explicitly teach key academic vocabulary. • Monitor student responses for corrective teaching • Use of games • ST Math • Opportunities for verbal rehearsal of concepts **GATE-** • Use of pre-assessment results to accelerate/compact curriculum and instruction for students who demonstrate mastery (85% +).

5^{th} Grade Fractions Prerequisite Skills Test

Name ___________________________________

Write each fraction in simplest form.

1. 24/28 ____________ 2. 6/16 _______________ 3. 3 18/24 _______________

Solve. Write your answer in simplest form.

4. Jake had 10 apples. He ate some of the apples. He has 6 apples left. What fraction of the original apples does he have left?

   ____

5. Juan and his friend bought 77 pieces of pizza. They ate 55 pieces. What fraction of the pizza did they eat?

   ____

Write as a fraction in simplest form.

6. 20 ÷ 40 = ____________________________ 7. 15 ÷ 20 = _______________________________

8. Are 3/8 and 9/24 equivalent fractions? Why or why not?

____

9. What is 2 ½ written as an improper fraction? ___________________________________

10. What is 25/6 written as a mixed number? ___________________________________

Solve. Write your answer as a fraction in simplest form or a mixed number.

11. Mabel shared some apples with her friends. She handed out ½ apple to each of 15 friends. How many apples did she hand out?

____

Name ___________________________________

12. Jane wants to make popcorn balls. Each popcorn ball requires 1/3 of a bag of popcorn. If she has 2 2/3 bags of popcorn, how many popcorn balls can she make?

____

13. José has 84 model trucks. He wants to divide them into 4 sets, giving three sets to friends and keeping one set for himself. How many trucks will be in each set?

____

14. If a farmer has 54 mangos, how many boxes can he fill with 6 mangos in each box?

____

15. Jenna is giving her marble collection away to her friends. She wants to divide 28 marbles equally among four friends. She is planning to give 6 marbles to each friend. Has she figured out the right number to give to each person? Why or why not?

____

Solve.

16. What is 125/125 in simplest form? ___________________________________

17. What is 327/1 in simplest form? ___________________________________

18. If 45 X 75 is 3375, what is 3375 ÷ 45? ___________________________________

Divide the numbers given. Check your work to show your answer is correct.

19. 85 ÷ 5 = _________________________ 20. 1728 ÷ 4 = ___________________________

Prerequisite Skills Test

Answer Key

1. 6/7
2. 3/8
3. 3 ¾
4. 3/5
5. 5/7
6. ½
7. ¾
8. Yes, the second fraction is found by multiplying the first fraction by 3/3.
9. 17/8
10. 4 1/6
11. 7 ½
12. 8
13. 21
14. 9
15. No, 6 X 4 is 24. She should give 7 marbles to each friend.
16. 1
17. 327
18. 75
19. 17
20. 432

This test measures the following prerequisite skills:

Items 1–8—Write factors in simplest form

Items 9–12—Change mixed numbers to improper fractions

Items 13–16—Understands division as making equal sets or repeated subtraction

Items 17–20—Knows how to check division of whole numbers with multiplication. Understands n/n = 1 and n X 1 = n.

Any students that miss two or more items in any given area should be given appropriate intervention instruction.

<table>
<tr><td colspan="2">Grade Level/Course
5th Grade</td><td>Duration: 60 min.
Date:</td><td colspan="2">Unit: Multiplication and Division of Fractions
Preparing the Learner Lesson # A
Preparing a Fraction Bar Tool Kit</td></tr>
<tr><td colspan="2">Common Core Standards</td><td colspan="3">Number and Operations–Fractions
3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions, mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</td></tr>
<tr><td colspan="2">Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: Strips of colored construction paper, rulers, math journals
Media/Technology to be used to deepen learning: ST Math Fraction Concepts; Fraction Concepts L1; NCTM Illuminations Website <http //illuminations.nctm.org> (Fractions games: Drop Zone, Fraction Feud, Dig It, Equivalent Fractions, Fraction Game, Fraction Models)</td></tr>
<tr><td colspan="2">Objectives</td><td colspan="2">Content:
Students will understand that equivalent fractions may be divided into more or less equal parts, but still be the same amount.</td><td>Language:
Students will use language of equivalence in describing fractional parts (is the same as, is equivalent to, is the same amount).</td></tr>
<tr><td colspan="2">Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒Level 4: Extended Thinking</td></tr>
<tr><td colspan="2">Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td colspan="2">Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="3">Academic Vocabulary (Tier II & Tier III)</td><td rowspan="2">TEACHER PROVIDES SIMPLE EXPLANATION</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td colspan="2">Fraction
Bar
Equivalent</td><td>Represented by
Same as</td></tr>
<tr><td>STUDENTS FIGURE OUT THE MEANING</td><td colspan="2">Fold
Measure
Strips</td><td></td></tr>
<tr><td colspan="2">Pre-teaching Considerations</td><td colspan="3">Students should have measurement skills and basic unit fraction knowledge.</td></tr>
<tr><td colspan="5">Lesson Delivery</td></tr>
<tr><td colspan="2">Instructional Methods</td><td colspan="3">Check method(s) used in the lesson:
☒ Modeling ☒ Guided Practice ☒ Collaboration
☐ Independent Practice ☐ Guided Inquiry ☒ Reflection</td></tr>
</table>

Lesson Continuum

Lesson Opening

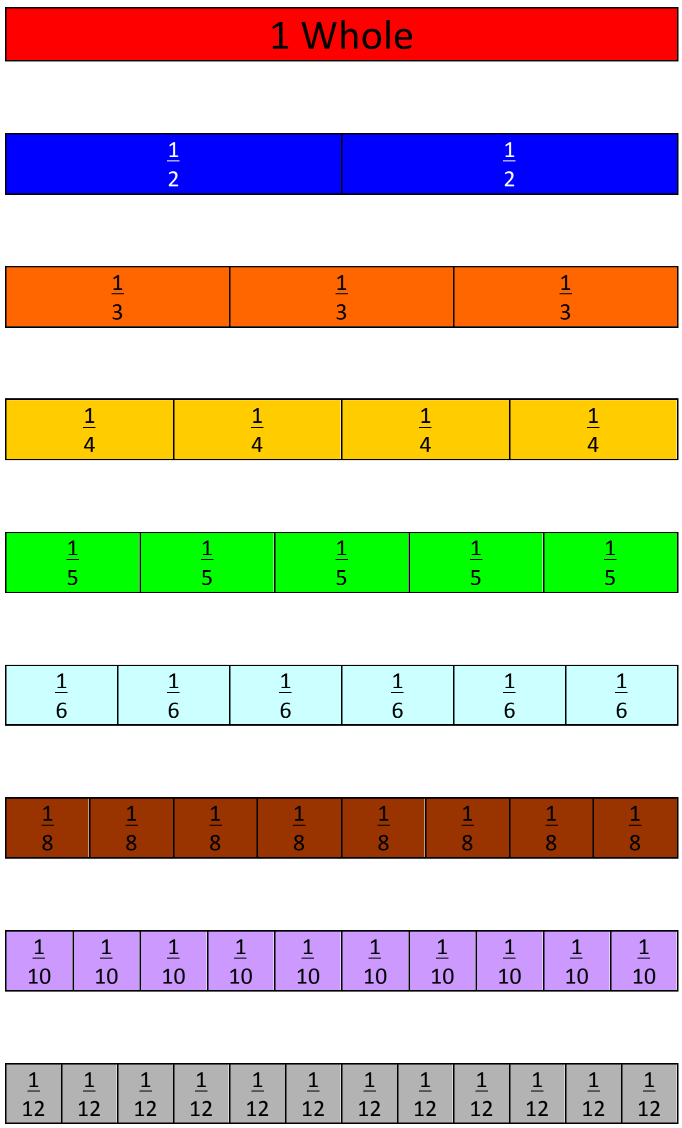
Prior Knowledge: Measurement skills, basic unit fraction knowledge.

Context and Motivation:

Teacher Preparation: Cut six-inch strips of the following colors of paper: red, blue, orange, yellow, green, turquoise, brown, purple, and gray. Make each strip one-inch in width. You will need one six-inch strip of each color for each student in your class. Provide an envelope or baggie for each student. Make one set of fraction bars yourself, so you will have it as a model. (Cut a few extra strips for those who make mistakes.)

"Today we are going to prepare a Fraction Bar set that we can use for the rest of this unit. I have cut strips of paper into 6-inch lengths, so that all the fraction bars will be the same size. Each of you will need one strip of paper of each color. We will all make each our fraction bars the same color, so we can have a common understanding and description for each fraction bar.
The colors for each bar will be: 1 whole—red, 1 half—blue, 1 third—orange, 1 fourth—yellow, 1 fifth—green, 1 sixth—turquoise, 1 eighth—brown, 1 tenth—purple, and 1 twelfth—gray."

Post the colors and sizes on a chart, so students can reference them as they work. (Colors, of course, may be adjusted, if these particular colors are not available at your site.)



<table>
<tr>
<td></td>
<td></td>
<td colspan="2">“Mark the red bar with 1 Whole, and set it aside. Now how can we make the blue bar into halves? How many equal parts is that? What would be the best way to make sure that both parts are exactly the same?” “We could measure it, or we could fold it very carefully. Why don’t some of you try it by measuring and some by folding, and we’ll compare the results?” Allow students to mark their halves, first with pencil, then with a pen or marker to show two equal pieces. “Mark each half with ½, and set the halves bar aside.”

Allow students time to fold or measure their strips. Some will be easier to make by measuring, some by folding. Allow students to experiment and compare their results using various methods. Halves, fourths, and eighths can be most easily folded. Thirds, sixths, and twelfths can be measured (2 inches, 1 inch, and ½ inch).

Do not cut the strips into pieces. Make it very clear that each strip must equal six inches in order to make it a valuable fraction tool. Comparisons can be made by overlaying bars, placing bars alongside, or folding strips to their fractional parts.

Fifths and tenths will cause the most difficulty. Try to determine the closest possible measurement you can for these two fraction bars. The actual measurement for a 1/5 bar would be 1 1/5 inch (which is very close to 1 3/16 inch). The measurement for a 1/10 bar would be 3/5 inch (which is very close to 10/16 of an inch).</td>
</tr>
<tr>
<td></td>
<td>Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding</td>
<td>When all the students have finished preparing their fraction bar sets, spend a few minutes acquainting the students with them. Lay out all the fraction bars in sequence by fractional size.

“Which numbers did we not use for our fraction bars? (7, 9, 11) why do you think that is? The fraction bars we made today are the most commonly used fractions. If we know how to use these, we can figure out how to compare fractions using those other numbers, when we come across them.”

“Find all the strips that can be folded to equal one half. How many can you find? What are the names of these strips?” (2/4, 4/8, 3/6, 6/12) Some children may realize that 1/6 and 1/3 equal ½ as well. They can also combine twelfths with the other strips to equal ½.

(Students should sketch and record findings in their math journals.)

“Find all the strips that equal 1/3. (2/6, 4/12) These are called equivalent fractions. Can you find any other equivalent fractions?” (1/4 and 2/8, 1/5 and 2/10, 1/6 and 2/12)

“Let’s pose a problem for you. If you share a candy bar that is cut into six equal pieces with just one friend, how many sixths will you each eat? Can you show me with your fraction bars?” (3/6) What is another name for this fraction?

“I will give a few problems, and I want you to use your fraction bars to show the answer to the problems. Use your fraction bars to make candy bars that are cut into different numbers of pieces.

“How many ways can you show to share 2 candy bars among 4 friends? How would you write the fractions to show this?” (1/2, 2/4, 3/6, 4/8, 6/12, 5/10)</td>
<td>Differentiated Instruction:

English Learners:
Modeling halves, thirds, fourths, etc.

Sentence frames:
_____ can be represented by __________.

_______ is equivalent to _______.

_________ is the same as __________.

Special Needs:
Pair up to complete the work.

Same sentence frames as EL Learners.

Use of hands-on materials</td>
</tr>
</table>

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	"How can 4 friends share 3 candy bars? Is there more than one way? (3/4, 6/8, 9/12) How can 3 friends share 4 candy bars? (1 ¼ , 1 2/8, 1 3/12) How can 8 friends share 3 candy bars? Is there more than one way? (With these fraction bars, only 3/8, or ¼ and 1/8) Students should record their representations in their math journals. Can you make up a problem using your fraction bars to represent candy bars? Ask students to write a problem, then select students to share their problems with the rest of the class. <u>Math Meeting:</u> Gather students together to share fractional equivalence problems they wrote using the fraction bars. Ask other students to solve the given problems and give reasons for their answers. Possible sentence frames: If _____ friends share _____ candy bars, each will get ______ candy bar, because _______________. ________ is equivalent to _______ because ______________.	**Accelerated Learners:** Make fraction bars for sevenths, ninths, elevenths. Use calculators to divide 6 inches into equal lengths and determine the length of each segment. Convert tenths to sixteenths.
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

Fraction Bars

1 Whole

$\frac{1}{2}$	$\frac{1}{2}$

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

Grade Level/Course 5th Grade	**Duration:** 60 min. **Date:**	**Unit: Multiplication and Division of Fractions** **Preparing the Learner Lesson # B** Launching Mathematical Discourse

Common Core Standards	**5th Grade Number and Operations—Fractions 5.NF 3** **3.** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions, mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **Speaking and Listening Standard:** 4. Report on a topic or text, or present an opinion, sequencing ideas logically and using appropriate facts and relevant, descriptive details to support main ideas or themes; speak clearly at an understandable pace.
Materials/ Resources/ Lesson Preparation	**Mathematical Tools:** Fraction bars **Media/Technology to be used to deepen learning:** ST Math Fraction Concepts; Fraction Concepts L1; NCTM Illuminations Website <http //illuminations.nctm.org> (Fractions games: *Drop Zone*, *Fraction Feud*, *Dig It, Equivalent Fractions, Fraction Game, Fraction Models)* **Supplementary Materials**: Problems about Equivalence
Objectives	**Content:** Students will solve problems about equivalence using fraction bars and other visuals. **Language:** Students will express their solution strategies using collaborative behaviors of taking turns, adding on to another's thinking, and disagreeing respectfully.
Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☒**Level 4: Extended Thinking**
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**

Academic Vocabulary (Tier II & Tier III)		KEY WORDS ESSENTIAL TO UNDERSTANDING	WORDS WORTH KNOWING
	TEACHER PROVIDES SIMPLE EXPLANATION	Equivalence Portions Visual representation	Adding on Disagreeing with
	STUDENTS FIGURE OUT THE MEANING	Sharing equally	

Pre-teaching Considerations	Students will have a set of Fraction Bars to assist their thinking in this lesson.

<table>
<tr><th colspan="4">Lesson Delivery</th></tr>
<tr><td colspan="2">Instructional Methods</td><td colspan="2">Check method(s) used in the lesson:
☒ Modeling ☐ Guided Practice ☒ Collaboration
☐ Independent Practice ☒ Guided Inquiry ☒ Reflection</td></tr>
<tr><td rowspan="2">Lesson Continuum</td><td>Lesson Opening</td><td colspan="2">Prior Knowledge: Students will have a set of Fraction Bars to assist their thinking in this lesson.
Context and Motivation:
Today we are going to practice having a productive classroom discussion through “math talk”. Math talk is like telling someone else how you made something and why. Who can tell me the steps to making a peanut butter sandwich? (Ask for volunteers. According to responses, act really dense, and do only what they say, with no thought of your own.) “Put the peanut butter on the bread.” (Place the jar of peanut butter on top of the loaf of bread. Children will laugh.) “Open the jar first.” Then what? (Open the jar, then place the open jar on top of the loaf of bread.) “Use the knife,” etc.

If I didn’t know anything about how to make a peanut butter sandwich, could I make one with these directions? Math talk has to be the same way. You have to tell exactly what you did to solve a problem, not leaving out any of the steps.
Today, we’ll practice telling each other what we did to solve a problem.</td></tr>
<tr><td>Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding</td><td>The purpose of this lesson is to launch quality discourse in the mathematics classroom.

<u>Preparation for the lesson:</u>
Run copies of the Problems about Equivalence for each group.

<u>Post and Discuss Group Norms:</u>
1) Listen respectfully.
2) Only one person can talk at a time.
3) Everyone must get a turn to speak.
4) Show a visual representation of your solution.

<u>Guided Practice--Fishbowl:</u>
Select one group of three or four students to demonstrate their thinking process, while everyone else watches. Place this group in the center, with everyone else seated in a circle around them.

Give them this problem to solve: “A group of 3 children are sharing 2 burritos. At another table, 6 children are sharing burritos. How many burritos should the second group receive so that each child gets the same portion as the first group?”

“Share your thinking one at a time. If you have something to add to another person’s thinking, say it with respect. You can say, I agree with what _______ is saying, but in addition, I would like to say that _________.

If you wish to disagree with someone you can say, Although _______ said ________, I am thinking about this differently. I think that ________.</td><td>Differentiated Instruction:

English Learners:
Use sentence frames.

Special Needs:
Work in small groups.
Use fraction bars
Use sentence frames.
Adjust numbers in problems used.

Accelerated Learners:
Adjust numbers used in equivalence problems.

Students can write their own equivalence problems to solve.</td></tr>
</table>

		Make sure everyone has a turn to speak. When you have solved the problem, make a visual representation of the solution to share with the rest of the class." Teacher charts discussion/visuals shared on chart paper or whiteboard. <u>Fishbowl Reflection:</u> Did the members of this group take turns speaking? Did everyone have a turn to talk? Were the others quiet while one person was speaking? How did they decide on the correct solution? Did they check their work? Did they show a visual representation in more than one way? Do you have any suggestions for this group? Who can tell this group one thing they did that made their discussion interesting? <u>Independent Practice:</u> Place students in groups of three or four, with a variety of levels in each group (high, medium, and low, if possible). Make sure that students in each group are seated close enough together to see clearly and to share materials. Review Instructions: 1) Listen respectfully. 2) Only one person can talk at a time. 3) Everyone must get a turn to speak. 4) Show a visual representation of your solution. Give the following word problems to each group to solve: A. 8 children want to share 6 pizzas so that everyone gets the same amount. How much pizza can each child have? B. Some girls were sharing bananas. Each girl got ¼ banana. How many bananas, and how many girls might be in the group? Show more than one solution. C. 24 football players wanted to share 6 pies. One football player started to cut each pie into 24 pieces and give each of the others one piece from each pie. Another football player complained that the pieces would be too small. He wanted to cut the pies into bigger pieces. How can they cut the pies into larger pieces, and still share the pies equally? D. 4 children are sharing 3 bottles of juice. At another table, 12 children are sharing juice. How many bottles of juice should they get, so that each child gets the same amount of juice? E. David used exactly 8 cups of flour to make 6 loaves of bread. How many loaves of bread can he make with 12 cups of flour? Allow students time to work on the problems. Circulate the room watching for examples of students working collaboratively, taking turns speaking, adding on to what another child has said, disagreeing respectfully, and other examples of collaborative conversation.	

		<u>Math Meeting:</u> Bring students together to discuss their solutions and how they worked together. Select student groups to share. Did the group clearly state the reasoning for their solution? Did they make an appropriate visual representation of their solution? Did they use their fraction bars to assist them? Did the members of the group take turns speaking? Did everyone have a turn? Did they add on to what another had said? Did they disagree respectfully? Possible sentence frames to post: If the children in the first group shared ________ for ___ children, then the children in the second group should share ____________ for _______ children. ________ for __________ is the same as _______ for _________. ______ and _______ are equivalent fractions. They are both the same amount. Did students clearly express their thinking? What positive comment can you make for this group?	
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

Problems about Equivalence

A. 8 children want to share 6 pizzas so that everyone gets the same amount. How much pizza can each child have?

B. Some girls were sharing bananas. Each girl got ¼ banana. How many bananas, and how many girls might be in the group? Show more than one solution.

C. 24 football players wanted to share 6 pies. One football player started to cut each pie into 24 pieces and give each of the others one piece from each pie. Another football player complained that the pieces would be too small. He wanted to cut the pies into bigger pieces. How can they cut the pies into larger pieces, and still share the pies equally?

D. 4 children are sharing 3 bottles of juice. At another table, 12 children are sharing juice. How many bottles of juice should they get, so that each child gets the same amount of juice?

E. David used exactly 8 cups of flour to make 6 loaves of bread. How many loaves of bread can he make with 12 cups of flour?

Grade Level/Course 5th Grade	**Duration:** 60 min. **Date:**	**Unit: Multiplication and Division of Fractions** **Preparing the Learner Lesson # C** Learning the Language of Contrast
Common Core Standards	**Number and Operations–Fractions** **3.** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions, mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **Bundled Language Standards:** 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. 6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases, including those that signal contrast, addition, and other logical relationships (e.g., however, although, nevertheless, similarly, moreover, in addition).	
Materials/ Resources/ Lesson Preparation	**Mathematical Tools:** Fraction bars **Media/Technology to be used to deepen learning:** ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division; NCTM Illuminations Website <http //illuminations.nctm.org> (Fractions games: *Drop Zone*, *Fraction Feud*, *Dig It, Equivalent Fractions, Fraction Game, Fraction Models)* **Supplementary Materials**: Comparison Problems	
Objectives	**Content:** Students will compare fractional amounts and determine which are equivalent and which are not.	**Language:** Students will use the language of contrast in discussing fraction equivalence (moreover, however, similarly, in addition to, whereas, although, nevertheless).
Depth of Knowledge Level	☒ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☒**Level 4: Extended Thinking**	
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	
Academic Vocabulary (Tier II & Tier / **TEACHER PROVIDES SIMPLE EXPLANATION**	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**
	Moreover However In addition to Although On the other hand Similarly Nevertheless Whereas	

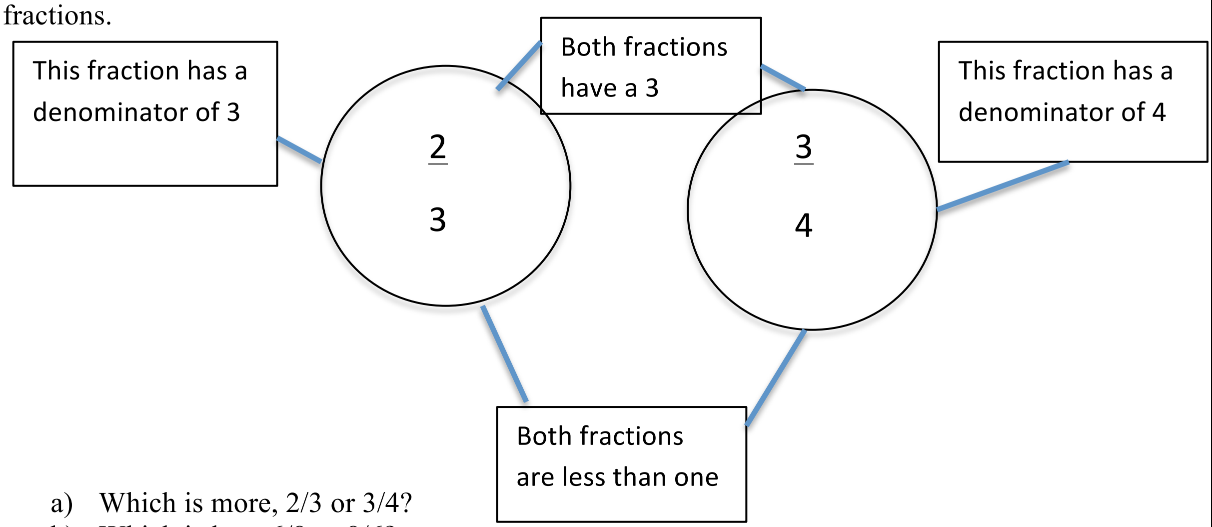
	STUDENTS FIGURE OUT THE MEANING		
Pre-teaching Considerations	Students have an understanding of unit fractions. They also have a set of Fraction Bars.		

Lesson Delivery

Instructional Methods	**Check method(s) used in the lesson:** ☒ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**

Lesson Continuum — **Lesson Opening**

Prior Knowledge: Students have an understanding of unit fractions. They also have a set of Fraction Bars.
Context and Motivation:
Today we are going to compare fractional amounts. Let's use a Double Bubble to compare two fractions.



a) Which is more, 2/3 or 3/4?
b) Which is less, 6/8 or 8/6?
c) Which is more, 8/10 or 4/5?

Introduce language of contrast: whereas, therefore, however, in addition to, similarly, nevertheless, on the other hand. Start with using *but* and *however*, then move to *therefore* and *although.* Include the higher-level vocabulary as the students are ready.

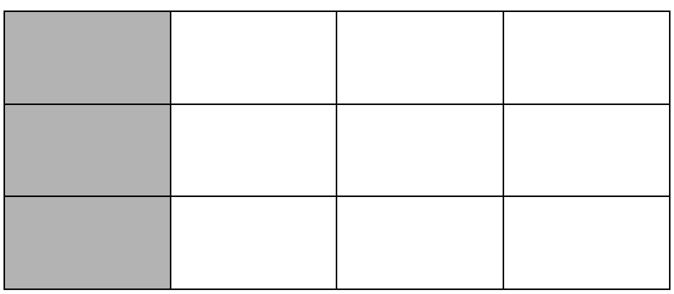
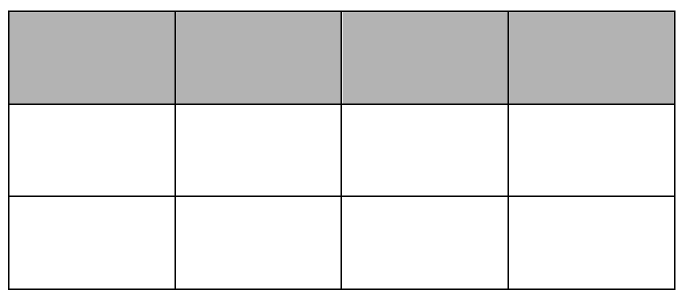
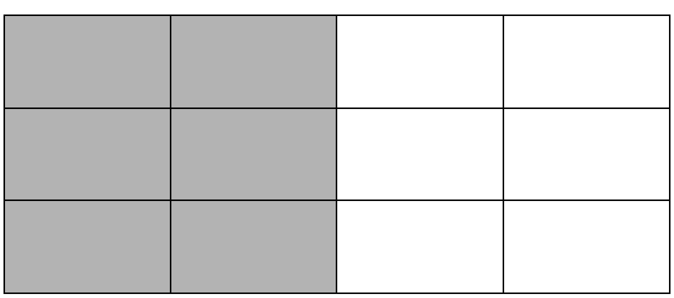
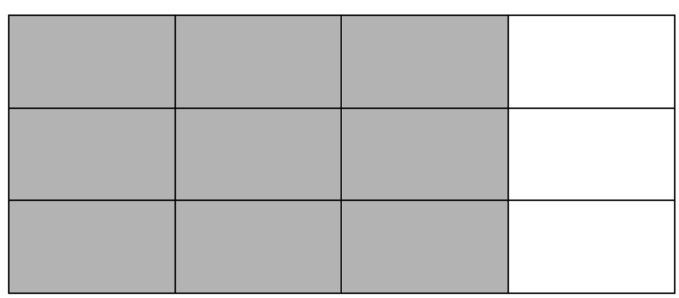
"1/3 is greater than ¼, but 2/3 is less than ¾."
"Although 1/3 is a larger fraction than ¼, 2/3 is less than ¾."
"Whereas ¾ is closer to one whole than 2/3, it is a greater amount."
"2/3 is less than one whole. Similarly, ¾ is less than one whole."
"Both fractions have a 3. However, the 3 is in the numerator for ¾ and in the denominator for 2/3."

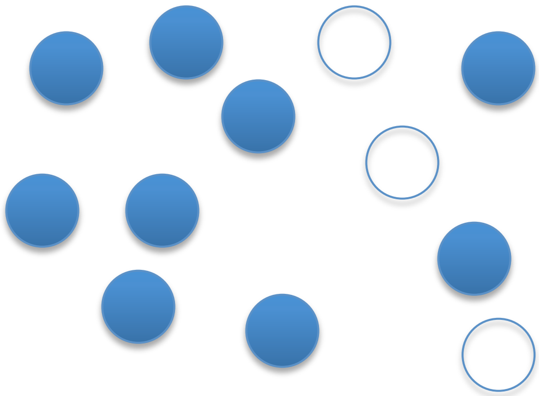
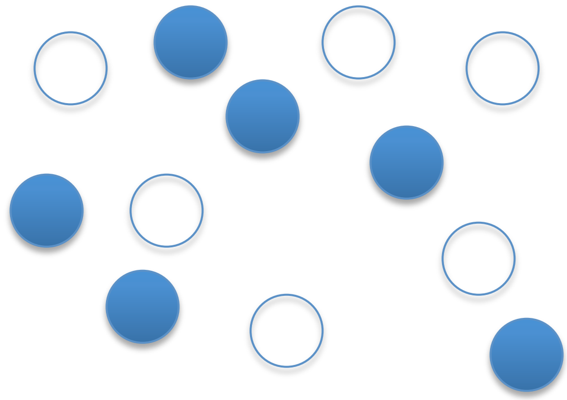
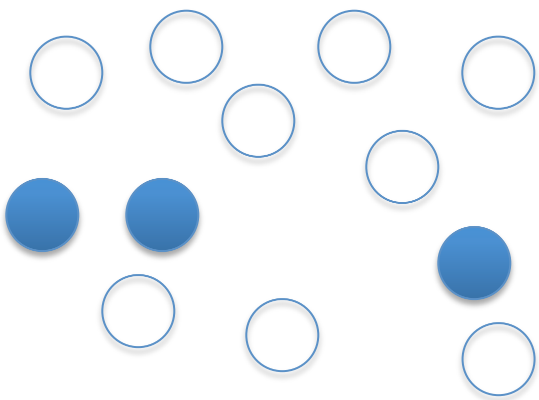
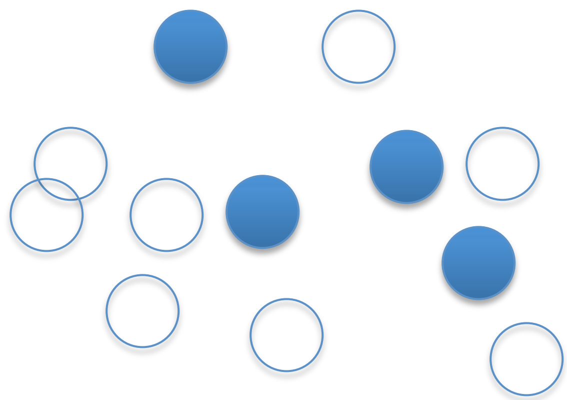
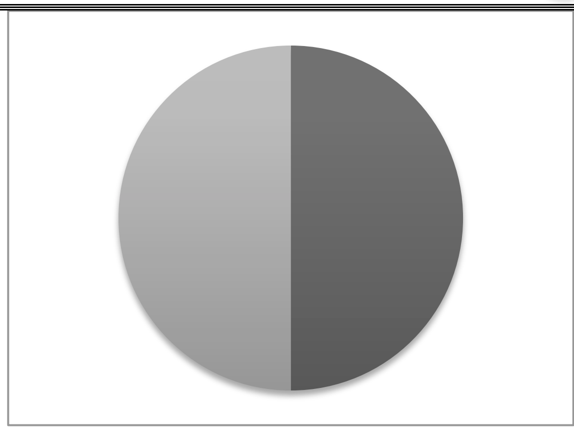
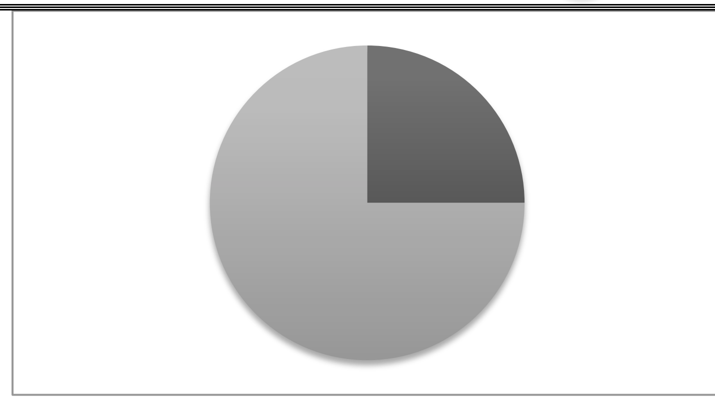
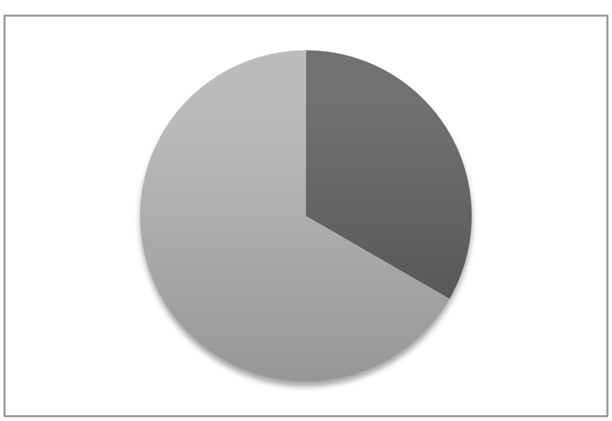
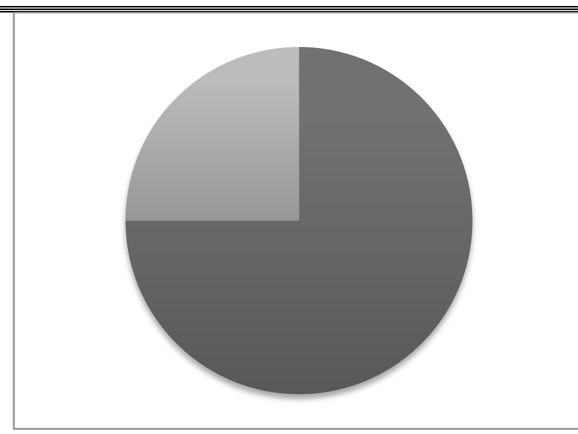
You and your partner will work together to determine which is the greater fraction, and explain your thinking. What are some ways to determine which is larger? We can lay the fraction bars next to each other. We can think about the size of each fractional piece. We can imagine each fraction cut up into many smaller pieces of equal size. See if you can think of another way to compare these fractions.

Allow students to work, and share their strategies for comparing the fractions.

		Now, let's practice saying sentences to compare these two amounts: "Although 1/3 is larger than ¼, ¾ is greater than 2/3." "¾ is closer to 1 whole than 2/3. Therefore, ¾ is larger than 2/3" "Whereas 6/8 is less than 1 whole, 8/6 is greater than one whole. Therefore, 6/8 is less than 8/6." "If a bar is cut into 5 equal parts, each part is 1/5. However, if each of those parts is cut in half, we will have ten equal parts, and each part will be called 1/10. 4/5 is the same as 8/10. Each piece is just cut in half."	
	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	<u>Collaborative Group Work:</u> Form groups of no more than four students. "We are going to play a game to practice working together as a team, and helping everyone to be successful." Distribute one set of Fraction Match Cards to each collaborative group. Students pass the cards out so that each student has four cards. (If there are only three members to a team, they can make up a "dummy hand" and all members can help that missing person build a complete set of cards.) <u>Rules of the game:</u> 1) Every member of the team has to end up with four cards that are related in a similar way. 2) No one is finished until all members of the team have a completed set. 3) No one may ask another member for a card. 4) Anyone may offer a card to another member of the team. 5) Keep your cards on the table, so they are visible to all the other members of your team. 6) Everyone must remain silent until the whole team has a complete set of related cards. "Did everyone end up with a complete set of related cards? Now discuss how you decided which cards belonged together. Did anyone have to give up a set to help someone else be successful? How did you realize that? How did it make you feel to give up your set to help someone else?" Post and use these sentence frames to help EL Learners to put their ideas into words. Model how to use the sentence frames with the Fraction Match cards. Tell students they could also use the sentence frames for journal responses. <u>Sentence frames:</u> "____ colored dots out of 12 is the same as ___ shaded boxes out of 12. Therefore, I put them in the same set." "This shaded circle shows ______ . Similarly, this shaded box shows ______.	**Differentiated Instruction:** **English Learners:** Use sentence frames. "____ colored dots out of 12 is the same as ___ shaded boxes out of 12. Therefore, I put them in the same set." "This shaded circle shows ______ . Similarly, this shaded box shows ______. **Special Needs:** Visuals help students of varying abilities to see similarities. **Accelerated Learners:** Practice with higher-level vocabulary
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

Fraction Match

One half	**One third**
$\frac{1}{4}$	$\frac{3}{4}$
	
	

<table>
<tr><td>Grade Level/Course
5th Grade</td><td>Duration: 60 min.
Date:</td><td colspan="3">Unit: Multiplication & Division of Fractions
Lesson # 1
Quotients of Whole Numbers</td></tr>
<tr><td colspan="2">Common Core Standards</td><td colspan="3">5th Grade Number and Operations—Fractions 5.NF 3
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions, mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</td></tr>
<tr><td colspan="2">Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: fractions bars, 1–inch wide strips of paper with lengths 6, 12, and 18 inches, pencils and paper
Media/Technology to be used to deepen learning: ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division</td></tr>
<tr><td colspan="2">Objectives</td><td colspan="2">Content:
Students will be able to divide whole numbers by whole numbers so that the resulting quotients are fractions.</td><td>Language:
Students will be able state conjectures and then make a generalization about the patterns observed in the equations.</td></tr>
<tr><td colspan="2">Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☐ Level 4: Extended Thinking</td></tr>
<tr><td colspan="2">Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
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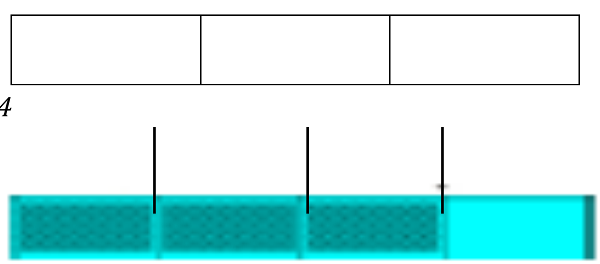
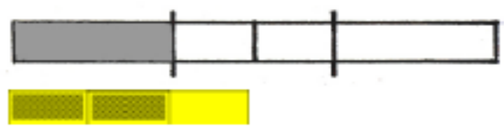
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td colspan="2">Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="3">Academic Vocabulary (Tier II & Tier III)</td><td rowspan="2">TEACHER PROVIDES SIMPLE EXPLANATION</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td colspan="2">Conjecture
Generalization
Equations
Quotients</td><td></td></tr>
<tr><td>STUDENTS FIGURE OUT THE MEANING</td><td colspan="2">Equal parts</td><td></td></tr>
</table>

<table>
<tr><td colspan="2">Pre-teaching Considerations</td><td colspan="2">Students should have knowledge of division, multiplication and division facts, ability to identify fractions.</td></tr>
<tr><td colspan="4">Lesson Delivery</td></tr>
<tr><td colspan="2">Instructional Methods</td><td colspan="2">Check method(s) used in the lesson:
☒ Modeling ☐ Guided Practice ☒ Collaboration
☐ Independent Practice ☐ Guided Inquiry ☒ Reflection</td></tr>
<tr><td rowspan="2">Lesson Continuum</td><td>Lesson Opening</td><td colspan="2">Prior Knowledge: Knowledge of division, multiplication and division facts, ability to identify fractions
Context, Motivation :
Essential Question: How are fractions related to division?
Tell students: Today you are going to prove how fractions are related to division.
In their table groups ask students to define fraction. Make sure that students understand that fractions are parts of a whole.</td></tr>
<tr><td>Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding</td><td><u>Modeling to Make Conjectures</u>
Each pair will need sets of bars and strips of paper the width of a Fraction Bar and lengths of 6 inches, 12 inches, and 18 inches.
Tell students:
Use your Fraction Bars to measure the length of each strip of paper to see how many whole bars it represents. (The strips represent 1 bar, 2 bars, and 3 bars.) Label each strip as a 1-bar, 2-bar or 3-bar.
Ask students to take their paper 3-bar and place the other paper bars aside for now.
How can a 3-bar be divided into 4 equal parts by paper-folding? (Fold the entire bar in half and then fold the result in half.)
Fold your 3-bar into 4 equal parts and then open the 3-bar and mark the crease lines to show the 4 parts. Shade one of these parts. What Fraction Bar has the same shaded amount as one of the 4 parts of the 3-bar? (The blue 3/4 bar.)
Students can show this by placing the 3/4 bar next to the shaded part of their 3-bar.
Write the division equation for 3 ÷ 4. (3 ÷ 4 = 3/4)
3-bar
3÷4=3/4
</td><td>Differentiated Instruction:
English Learners:
Use visuals, realia
Special Needs:
Use visuals, realia
Accelerated Learners:
Give students multiple opportunities to explore arrays such as in an online investigation.</td></tr>
</table>

Ask students to use their paper 2-bar for the next activity.
How can a 2-bar be paper-folded into 3 equal parts? (The two ends of the strip can be folded toward each other to overlap in 3 parts.)
Fold your 2-bar into 3 equal parts, open the parts and mark the creases, and shade one of the parts. What Fraction Bar has the same amount of shading as the shaded part of your 2-bar? (2/3 bar)
Write the division equation for 2 ÷ 3. (2 ÷ 3 = 2/3)

2-bar



$2 \div 3 = \frac{2}{3}$

Fold your paper 1-bar into either 2 or 3 or 4 equal parts and shade one of the parts. Describe the Fraction Bar with the same amount of shading. (Either green, yellow or blue bar with 1 part shaded.)
Write the division equation for either 1 ÷ 2 or 1 ÷ 3 or 1 ÷ 4. (1 ÷ 2 = 1/2;
1 ÷ 3 = 1/3; and 1 ÷ 4 = 1/4)
Generalizing: List the equations from Activity 1.
3 ÷ 4 = ¾   2 ÷ 3 = 2/3   1 ÷ 2 = ½   1 ÷ 3 = 1/3   1 ÷ 4 = ¼

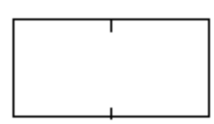
Look for patterns and write a conjecture for dividing one whole number by another. ***(When one whole number is divided by another, the resulting fraction has the first whole number as the numerator and the second whole number as the denominator.)***

Using the Conjecture to Solve Problems

Tell students to select from the following problems. Make sure students illustrate their problem solution.

If 8 pound of grass seed are divided equally into 5 piles, what is the weight of one of these piles? (8/5 or $1\frac{3}{5}$ pounds)
If a piece of wood with a length of 5 feet is cut into 6 equal pieces, what is the length of each piece? (5/6 of a foot)
If three chicken pies are shared equally among 5 people, what fraction of a pie will each person have? (3/5 of a pie)

Show students this sketch and present the following problem.



If Mr. Green used 32 feet of fence to build a rabbit pen and the length of the pen is twice the width, what is the width of the pen? Explain your reasoning.
(Some students may notice that the perimeter of the pen has 6 equal parts and compute 32 ÷ 6 = 32/6 = 5 2/6 feet.)

		Math Meeting: Ask several dyad teams to share their problem solutions. Have a discussion about their conjectures. Did they find their conjectures to be true? If they are for all situations we call them generalizations. **Reflection:** **Record today's generalization into your journal and draw an illustration or model of it.** Follow-up activity is on the next page. Students could complete the page for homework or for independent time (#1-3). (#4-5 advanced learners)	

Lesson Reflection	
Teacher Reflection Evidenced by Student Learning/ Outcomes	

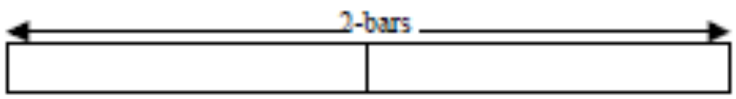
Name: _____________________________ Date: __________________

1 At a pizza party, 5 people will equally share 3 pizzas.

a. Draw lines (as best you can) to divide each pizza into 5 equal parts. What is the fraction for the total amount of pizza each person will receive? ____

b. Complete the following equation. 3 ÷ 5 = ____

2. Three people wish to share 2 banana bread cakes. Draw lines to divide each cake into 3 equal parts.



a. What is the fraction for the total amount of banana bread each person will receive? _____

b. This activity illustrates 2 divided by 3. Complete this equation: 2 ÷ 3 = ____

3. Nine people will equally share 50 pounds of potatoes.

a. What is the amount of potatoes each person will receive? _________

b. The amount each person will receive is between what two whole numbers? ____ and ____

Solve each of the following problems. Write a fraction if the answer is less than 1 or write a mixed number if the answer is greater than 1.

4. Taylor plans to use 2 cups of brown sugar in making 3 loaves of whole wheat bread. If this amount of brown sugar is divided equally into 3 parts, what fraction of a cup will there be for each loaf of bread?

5. Ashley's mother will make 4 pineapple fruitcakes for a bake sale to raise money for the school band. If 25 ounces of crushed pineapple are divided equally into 4 parts, how much pineapple will there be for each fruitcake?

Grade Level/Course 5th Grade	**Duration:** 60 min. **Date:**	**Unit: Multiplication & Division of Fractions** **Lesson # 2** Multiplying Whole Numbers and Fractions
Common Core Standards	5th Grade Number and Operations—Fractions 5NF.4.a Apply and extend previous understandings of multiplication and division to multiply and divide fractions a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show* $(2/3) \times 4 = 8/3$, *and create a story context for this equation. Do the same with* $(2/3) \times (4/5) = 8/15$.*(In general,* $(a/b) \times (c/d) = ac/bd$.*)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	
Materials/ Resources/ Lesson Preparation	**Mathematical Tools:** colored pencils, student math journals **Media/Technology to be used to deepen learning:** ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division; *http://www.visual**fractions**.com/; http://www.learner.org/courses/learningmath/number/session9/part_a/try.htm*	
Objectives	**Content:** Students will use what they know about multiplying whole numbers to begin developing an understanding of what occurs when multiplying fractions.	**Language:** Students will b state why a statement about multiplying whole numbers is true or false, and make new generalizations about multiplying with fractions.
Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☐**Level 4: Extended Thinking**	
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	
Academic Vocabulary (Tier II & Tier III) **TEACHER PROVIDES SIMPLE EXPLANATION**	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**
	Factor	

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<tr><td colspan="2">Instructional Methods</td><td colspan="2">Check method(s) used in the lesson:
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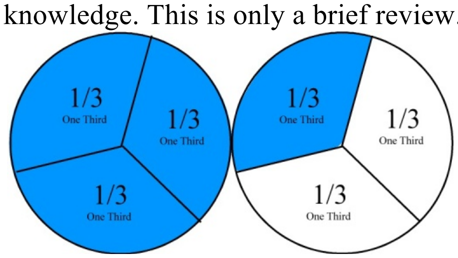
Lesson Continuum

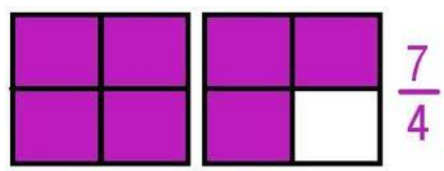
Lesson Opening

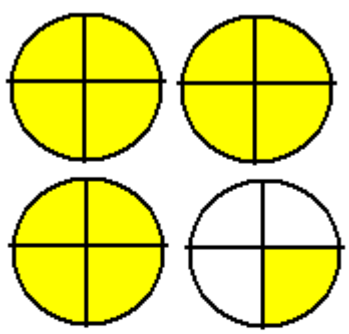
Prior Knowledge: Students should be able to change mixed numbers to improper fractions; write fractions in simplest form.

Context and Motivation:

Open the lesson with a brief review of mixed numbers. Show the following slides of improper fractions/ mixed number and ask students to **think** of how to write the number as a mixed number and improper fraction or either respectively. Then students should share their numbers with an **elbow neighbor**. Ask students to **share** their responses. Discuss any differences and how the an improper fractions can be changed to a mixed number and vise versa. Remember they should already have this knowledge. This is only a brief review.







Tell students that we used visual models to show how a mixed number is related to improper fraction. Now they **will show how a visual model can show multiplication of a fraction by a whole number. (essential question)** The lesson should guide students to the enduring understanding: When multiplying a fraction times a whole number, the parts of the fraction are partitioned among the whole number.

Concept Attainment

Begin the lesson by posting a chart of "true" statements about multiplying whole numbers (these statements are in student language collected when we were learning about multiplication. Since you do not have statements, we will use these):

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

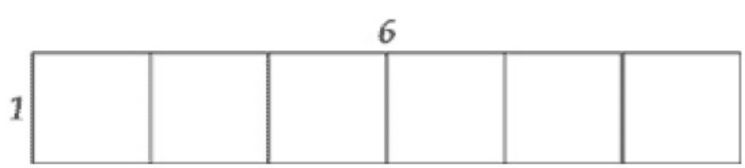
1. Multiplication is the same as repeated addition when you add the same number again and again.
2. Times means "groups of."
3. A multiplication problem can be shown as a rectangle or array.
4. You can reverse the order of the factors and the product stays the same.
5. You can break numbers apart to make multiplying easier.
6. When you multiply two numbers, the product is larger than the factors unless one of the factors is zero or one.

Point to the first statement:
1. Multiplication is the same as repeated addition when you add the same number again and again.
Ask students: Do you think this is true when we think about fractions? Write on the board: *6* x ½
Tell students: Talk with your neighbor about how you might make sense of this problem with repeated addition.
After a few minutes, call on a student. A student may respond, "I think you can do it by adding one-half over and over again. I did one-half plus one-half, like that, six times. I think the answer is three."
Write students' responses on the board, i.e. 6 x 1/2 = 1/2 + 1/2 + 1/2 + 1/2 + 1/2 + 1/2 = 3
Ask students: How did you get the answer of three? A student may respond: One-half plus one-half is one whole, and you can do that three times, and you get three.

$$6 \times 12 = \underbrace{1/2 + 1/2}_{1} + \underbrace{1/2 + 1/2}_{1} + \underbrace{1/2 + 1/2}_{1} = 3$$

Ask students: So, does this first statement work for multiplying with a fraction? Students should agree. Write true next to the statement. Students should write one of the agreed upon explanations inside the fold of the respective statement.
Then point to the second statement: *2. Times means "groups of."*
Ask students: Does it make sense to read 'six times one-half' as 'six groups of one-half'?
Most of the students should agree. If there is disagreement draw a picture. When everyone agrees, write true next to the statement. Students should draw the explanation inside the fold of the respective statement.

3. A multiplication problem can be shown as a rectangle or array.
Ask students: Can we draw a rectangle to show six times one-half? Students may not be sure. **Suppose the problem were six times one**. Write *6 x 1* on the board. The students should be familiar with using rectangles or arrays for whole number multiplication. Draw a rectangle saying: **One side of the rectangle is six units long and the other side of the rectangle is one unit long**. Label the sides *6* and *1* and then divide the rectangle into six small squares.



Differentiated Instruction:

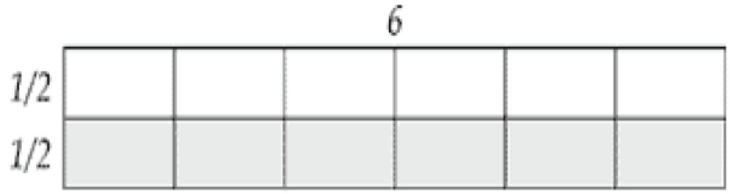
English Learners:
Using sentence frames
Teacher's use of visuals.
Give students tiles or rods to work through proving the statements.
Post the wording of the expressions:
4 groups of one-third
6 groups of one half.
Using a variety of guided questions:

Special Needs:

Working in pairs
Selecting appropriate numbers
Using sentence frames
Guide students through the drawing of visuals models.
Give students tiles or rods to work through proving the statements.

Accelerated Learners:
Students should draw two models of the reflection question.

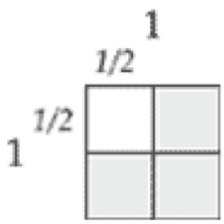
Say to students: **See if this rectangle helps you think about how I might draw a rectangle to show six times one-half. Ask: Which way should I cut the rectangle?** Wait for student responses (Think-Pair- Share). Split the rectangle then erase or cross out the 1 and replaced it with ½ written twice. Also, shade in the bottom half to indicate that it wasn't part of the problem.



The top half of the rectangle is six units by one-half unit and shows the problem six times one-half. The bottom shaded half shows the same problem again, but we don't need to consider both.

How many squares are there in the unshaded rectangle? Does this still give an answer of three?
Possible discussion: Two halves make a whole, and you do that three times, so the six halves make three whole squares. Three is still the answer.

Let's try one, I write on the board: ½ x ½
Show the students a way to think about representing the problem with a rectangle. **When I draw a rectangle for a multiplication problem with fractions, I find it easier first to draw a rectangle with whole number sides. So, for this problem, I think about a rectangle that is one by one**. Draw a square on the board, labeled each side with a *1,* and continue: **This rectangle is a square because both factors are the same. It shows that one times one is one. Now watch as I draw a rectangle inside this one with sides that each measure one-half**. **I divided the square, shaded in the part we didn't need to consider to show the ½ by ½ portion in the upper left corner, and labeled each side ½ .**



Tell students: **The part that isn't shaded has sides that are each one-half of a unit. How much of the one-by-one square isn't shaded? (**One-fourth) Some students may not get this. The next statement may help clarify the model.
Ask students: **But do you agree that the unshaded rectangle has sides that are each one-half?** (But they may not be sure that the answer of one-fourth is correct.)

Let's see if the other statements can help you see why. If the students thought of the problem as "one-half of one-half," they may agree with the answer of one fourth. We are developing this idea.

Point to the next statement:
4. You can reverse the order of the factors and the product stays the same.
Ask students to give a whole number example of this statement.
Let's think about this statement for the first problem we solved—six times one-half. What about if we think about the problem as one-half times six?
Write on the board: ½ x 6
If we think about the times sign as 'groups of,' then one-half times six should be 'one-half groups of six.' But that doesn't sound right. It does make sense, however, to say 'one *half of a group* of six,' or 'one-half of six,' and leave off the groups' part. Both sound better and they're still the same idea. What do you think 'one-half of six' could mean? (One-half of six is three, so one-half times six is three, and that's the same as six times one-half.)
Say: Let's think about one-half times one-half the same way. What is one-half of one-half?
Possible student responses: a fourth; a quarter; one-fourth.
So what do you think about reversing the order of the factors when the factors are fractions?
The students should agree that it would work, so write *OK* next to Statements 3 and 4. Students should draw the visuals inside the fold of the respective statements.
Tell students: **Let's look at the fifth statement.**
5. You can break numbers apart to make multiplying easier.
Talk with your neighbor about how you could apply this statement to the problem six times one-half. (take responses)
You could break the six into twos, and then you do two times one-half three times. Two times one-half is one. One plus one plus one is three. So it works." Write on the board:
6 x ½
6 = 2 + 2 + 2
6 x ½ = (2 x ½) + (2 x ½) + (2 x ½)
= 1 + 1 + 1 = 3
and**--- We could split the six into four and two. Half of four is two and half of two is one and two plus one is three.**
6 = 4 + 2
4 x ½ = 2
2 x ½ = 1
2 + 1 = 3
Therefore, the statement is true. Write true next to the statement. Students should write one of the agreed upon explanations inside the fold of the respective statement.

We have one statement left:
6. When you multiply two numbers, the product is larger than the factors unless one of the factors is zero or one.
Ask**: Does this statement hold true for six times one-half?** Give students time to think and share with peers.
It doesn't work because 3 is smaller than six, so it doesn't work.
Could we change the statement so that it does work?
Posed a problem that has a fraction as one of the factors for which the answer is greater than both of the factors. Think about this problem—six times three-halves. That's the same as six groups of three-halves.
Write on the board: $6 \times \frac{3}{2}$ $\frac{3}{2}+\frac{3}{2}+\frac{3}{2}+\frac{3}{2}+\frac{3}{2}+\frac{3}{2}$
Talk with your neighbor about what the answer would be to this problem.
Possible discussion: students may conclude: We knew that three-halves is the same as one and a half, and one and a half plus one and a half is three, and three plus three plus three is nine, so the answer is nine.

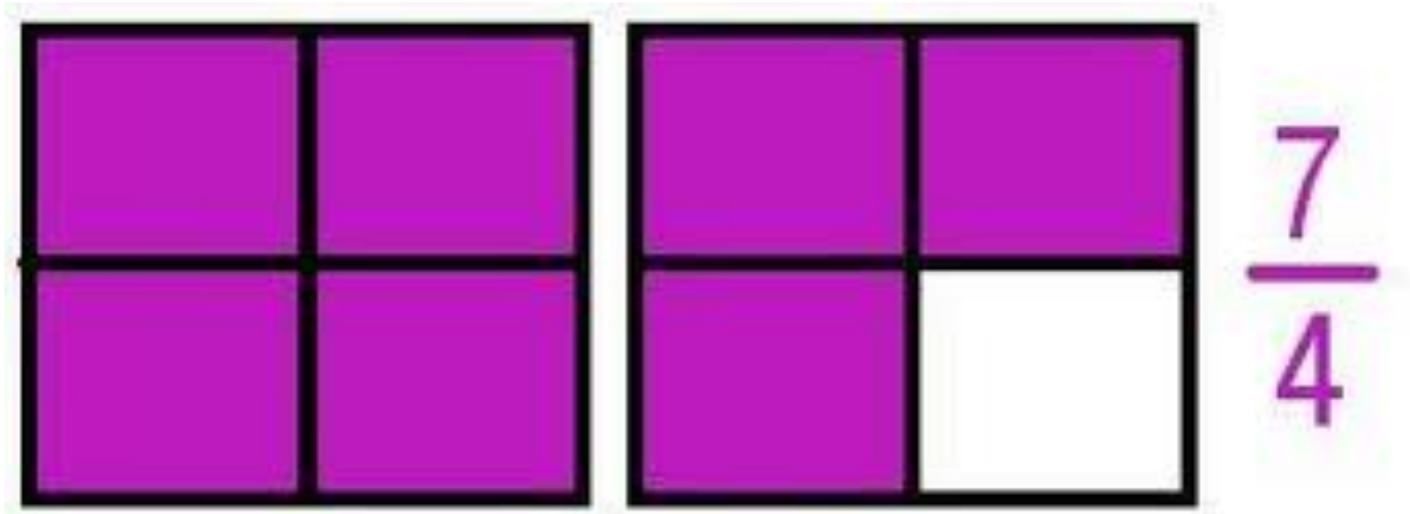
$$6 \times \frac{3}{2}$$
$$\frac{3}{2}+\frac{3}{2}+\frac{3}{2}+\frac{3}{2}+\frac{3}{2}+\frac{3}{2}$$
$$\frac{3}{2} = 1\frac{1}{2}$$
$$\underbrace{1\frac{1}{2}+1\frac{1}{2}}_{3}+\underbrace{1\frac{1}{2}+1\frac{1}{2}}_{3}+\underbrace{1\frac{1}{2}+1\frac{1}{2}}_{3}$$
9

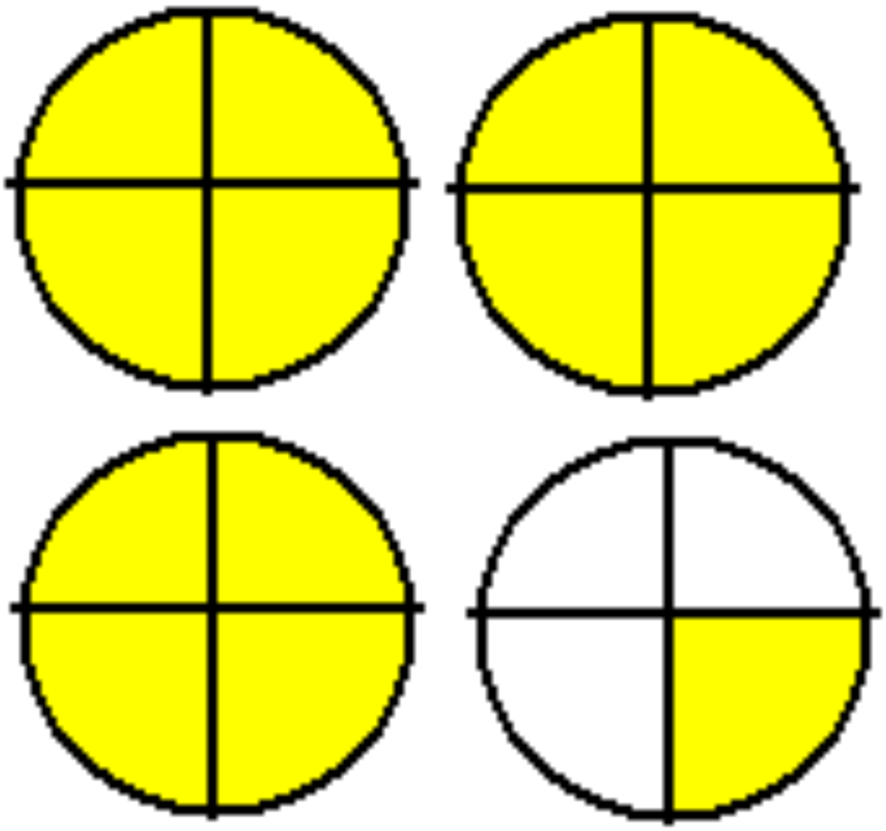
And nine is bigger than six or three-halves. So the statement doesn't work.
How could we fix it? The fraction has to be smaller than one. So any number that is zero or one or in between makes an answer that is smaller than the factors. At the end, write 'zero or one or a fraction that's smaller than one.'

6. When you multiply two numbers, the product is larger than the factors unless one of the factors is zero or one or a fraction smaller than one.
Write true next to the revised statement. Students should write one of the agreed upon explanations inside the fold of the respective statement.

Reflection
Ask students to draw a visual model to solve for $4 \times \frac{1}{3}$ (Students may use rectangles, number lines, circles..)
What do you understand about multiplying whole numbers by fractions?

Lesson Reflection	
Teacher Reflection Evidenced by Student Learning/ Outcomes	


$\frac{7}{4}$

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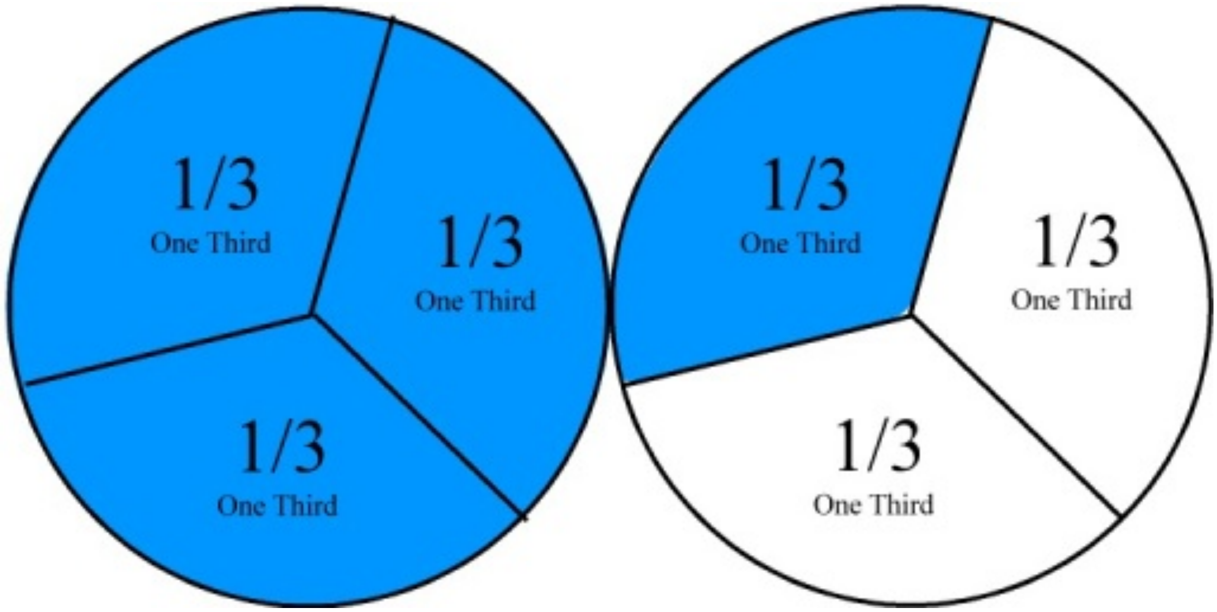
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Grade Level/Course **5th Grade**	**Duration:** 60 min. **Date:**	**Unit: Multiplication & Division of Fractions** **Lesson # 3** Multiplying Fractions with Whole Numbers
Common Core Standards	5th Grade Number and Operations—Fractions **5NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **a.** Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.) **5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	
Materials/ Resources/ Lesson Preparation	**Mathematical Tools:** colored pencils, graph paper, fraction bars **Media/Technology to be used to deepen learning:** ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division; *http://www.visual**fractions**.com/; http://www.learner.org/courses/learningmath/number/session9/part_a/try.htm*	
Objectives	**Content:** Students will be able to decompose fractions additively and relate repeated addition to multiplication (4 x 1/3 = 1/3+1/3+1/3+1/3), and relate partitioning and sharing contexts to fractions (division of numerator by the denominator).	**Language:** Students will be able to interpret and create visual models for multiplying fractions (number lines and fraction bars) and interpret and create story contexts for multiplying fractions.
Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☐ **Level 4: Extended Thinking**	
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	
Academic Vocabulary (Tier II & Tier III) **TEACHER PROVIDES SIMPLE EXPLANATION**	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**
	Factor product fraction	

	STUDENTS FIGURE OUT THE MEANING	fraction less than one number line	

Pre-teaching Considerations

Students will be able to:

- Understand that fractions are numbers that represent quantities less than a whole
- Understand the meaning of multiplication with whole numbers
- Familiarity with number lines and bar models
- Able to decompose fractions additively (3/4= 1/4+1/4+1/4)

Lesson Delivery

Instructional Methods

Check method(s) used in the lesson:

☒ **Modeling**	☐ **Guided Practice**	☒ **Collaboration**
☐ **Independent Practice**	☐ **Guided Inquiry**	☒ **Reflection**

Lesson Continuum

Lesson Opening

Prior Knowledge:

Context and Motivation:

During this part of the lesson, students will be introduced to the story context for the next two lessons. This part of the lesson should move quickly—8 to 10 minutes.

- About How Much? Begin by engaging students in an estimation routine. This launch will be used as an assessment and as an opportunity to model how estimating will be useful as students solve problems.
- Let students know that they will be solving problems about goals that students made for a recycling drive and servings at an ice cream party.
- Share sheet "Ice Cream". Reveal and discuss one problem at a time (also on Power point). Ask students to predict how much ice cream was served and to explain their thinking.
- Ask students how they are making their estimates. Take note of strategies and misconceptions.

<u>Guiding Questions:</u>

- What models or pictures were you visualizing?
- How did you decide if your estimate would be more or less than 1?
- Which estimates were not reasonable and why?

Note: During this phase of the lesson it is not necessary to identify that the story contexts were multiplication problems or to record any equations. This is an opportunity for students to recognize the importance of the strategies they use to estimate and how the strategies allow them to check the reasonableness of their answers

<u>Lesson Delivery:</u>

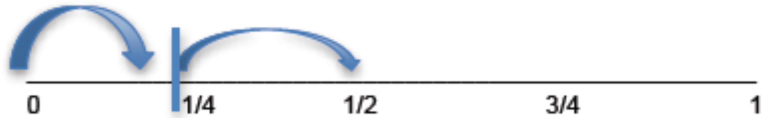
During this phase of the lesson, students will be introduced to a number line model for solving problems that involve thinking about groups of unit fractions and then groups of non-unit fractions. They will be developing the idea that (a) groups of size b/d is an accumulation of b/d +b/d ….

Introduce the story context: A fifth grade class was collecting recyclables for a drive and then celebrated their success with an ice cream party. We are going to be solving problems about how much ice cream was served at the party. We will be using number lines representations to solve the problems. Let's look at this first story context together: (Write the context on a chart or board, or use the power point)

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Amy helped serve mint chocolate chip ice cream. Her booth had paper bowls that hold ¼ cup servings. In the first five minutes, Amy served 3 students their bowls of ice cream. How much ice cream has she served?

Introduce a fraction number line. Draw a number line that has two tick marks for 0 and 1. Ask students how they could represent ¼ on the number line. How many ¼ would there be on this number line? Students may think of the idea that ¼ is ½ of ½, so they would mark ½. Then they would split the space between 0 and ½ in half again and mark that space as ¼.



Return to the story context. Ask students how they could represent the three scoops of size 1/5. Mark 3 hops or moves of size 1/5 on the number line. Ask students how they might be able to record the three hops represented on the number line in an equation or in words. Possible responses are : *¼ + ¼+ ¼ = ¾ or 3 groups of ¼ or 3 x ¼*

Students work with partners or in small groups to solve the four ice cream problems on handout "The Fifth Grade Ice Cream Party".
As students work, ask them:
How does your number line model show the amount of ice cream that was served?
How did you decide how to divide your number line?
Which landmark numbers were useful as you found parts of the bar?

<u>**Guiding Questions:**</u>

- **What is 4 groups of 1/5?**
- **What is 2 groups of 1/3?**
- **How does your drawing help someone see amount of ice cream that was served? Is there**
- **something you could add or do to your model to make it clearer?**

Note: Problem 4 shifts the focus to multiplying a whole number by a non-unit fraction. Observe how students use their strategies for the first three problems in solving this problem. As students are working, begin selecting strategies and ideas for students to share in the whole group discussion.
While students are working, the teacher will select and sequence student strategies and engage the class in the math meeting that compares the methods that students used to solve the problems. The discussion will begin to assist students in relating the number of groups (as hops on the number line) and the size of the groups to the final product.

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Math Meeting

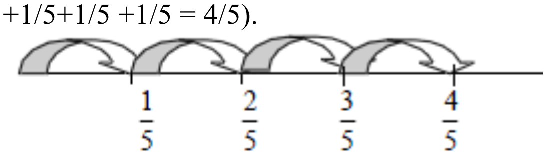
Bring students back together and discuss the ways in which students used the number lines to solve the problems.
Begin with problem 1.
One of the tables at the party has mint chocolate chip ice cream. The servings are 1/5 of cup. After five minutes, Ms. Cruz had scooped out 4 servings. How much ice cream has she served?

As students share their ideas, listen for opportunities to talk about how the story contexts are asking for students to consider several groups of the same sized fraction.

Guiding Questions:

- **Is the amount of ice cream more or less than 1 full cup?**
- **How do you know? Why is it less than ½ a cup?**
- **How does the number line model show you that there have been four servings scooped out?**

Involve students in thinking about a form of notation: the whole number x unit fraction. (4 x 1/5)
Ask students what addition problem is equivalent to 4 x 1/5. (1/5 +1/5+1/5 +1/5 = 4/5).



Ask students what they notice about the size of the product related to the size of the two factors. Begin by asking what they notice about the product of 4x5=20(product is larger than both factors)

Then move to: 4 x 1/5 = 4/5 (the product is smaller than one factor (4) and larger than the other factor (1/5). Ask them to consider the other two problems. Chart these and begin to articulate a general statement about what they are noticing and why.
When you multiply a whole number (not 0) by a fraction less than 1, your product is smaller than the whole number and larger than the fraction. Note: This can be part two of the lesson continued on the following day.
The last problem in the set asks students to consider a non unit fraction. Begin by asking the
students how this problem is the same or different than the first three. (It has a whole number multiplied by a non-unit fraction.)

Ask students how Problem 4 is related to Problem 3.
At the sundae table, Lauren was serving mini marshmallows. She used 1/3 cup for each sundae. How much of the marshmallows has she used after making 2 sundaes?

During clean up time, Mr. Diaz found 2 gallon containers that were 2/3 full. How much ice cream was left?

*Problem 3 and 4 both have 2 groups of fractions that are thirds
*Problem 3 has a unit fraction 1/3. Problem 4 has 2/3.
*Problem 4 has a larger product than Problem 3

Differentiated Instruction:

English Learners:
Provide sentence frames.
Visuals as the number line.

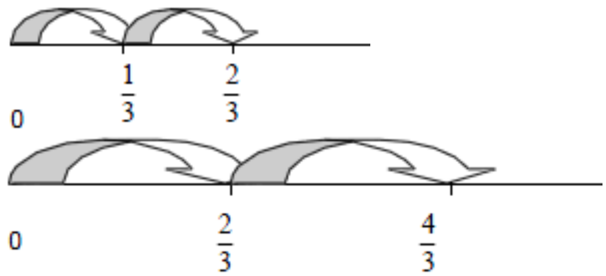
Guide students to answer the guiding questions by starting the answer with a answer stem, e.g. 2 groups of 1/3 is _______.

Special Needs:
Pair student with another student who will be able to provide support during the lesson.
Provide sentence frames.
Visuals as the number line.

If students are struggling they can use fraction manipulatives, such as fraction strips or fraction tiles. Sometimes those manipulatives, which are region models, are easier than number line models.

Accelerated Learners:
For students in need of enrichment, use an odd number of servings, such as 3 or 5 so that the fractional pieces are more complex.

Ask students to assist you in drawing a number line that models problem 3 and then a number line that models problem 4.



Pose these questions:

- **Which number line has larger hops? How much larger are the hops?**
- **How many 1/3 hops are there in 2/3? If you hop 4 hops of 1/3 where would you land on the**
- **number line? If you hop 2 hops of 2/3 where do you land?**

As the students share responses to the questions, use the models of the number lines to make explicit the idea that although both problems involve thirds and 2 groups that problem 4 has hops that are double the size so the product is double the size. (Note: this work draws upon multiplication ideas of doubling with whole numbers and use of the distributive property.)

The following idea will continue to be explored in subsequent lessons but students should begin to consider how:
2 x 2/3 = 2 x (1/3 +1/3) = 2 x (2 x 1/3)
Discuss that this means 2 groups of 1/3 and 1/3 or four groups of 1/3. Ask students to point out where there are two groups of 1/3 and 1/3 on the number line model for problem 4.
Pose questions such as: ***How are 2 groups of 1/3 and 1/3 (2/3) similar to 4 groups of 1/3?***

Reflection

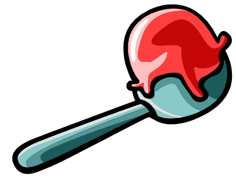
Ask students to consider the following: **How can we use the number line model to justify that 2 x 2/3 = 4 x 1/3?**
Close by having students share any more ideas. Record these so that they are visible and so they can be revisited in the next lesson. Students will be asked to rewrite problems 1 and 2 with non unit fractions and to compare the products of each of the problems. Students will be challenged to work on problems where the result is known and they must decide the number of servings that were served.

At the sundae table, Lauren was serving mini marshmallows. She used 1/3 cup for each sundae. One bag only had 2/3 of a cup. How many sundaes can she top?

Mr. Diaz has 4/6 of a container of chocolate ice cream. He wants to serve 4 mini servings. What size should each serving be?

Lesson Reflection	
Teacher Reflection Evidenced by Student Learning/ Outcomes	

Name: ____________



Ice Cream

How much ice cream was served? Choose the closest estimate.

We served 4 boxes that had 12 ice cream cones each.

4 6 40 400

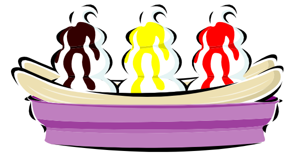
We served ½ a box that had 12 ice cream cones.

2 6 12 24

We had ½ a container of ice cream and ½ of what was in the container was scooped out. How much was scooped out?

1 container ½ of the container ¼ of the container

Name_________________ Date__________



The Fifth Grade Ice Cream Party

Use fraction number lines to find out how much ice cream was served at the fifth grade party.

1. One of the tables at the party has mint chocolate chip ice cream. The servings are 1/5 of cup. After five minutes, Ms. Cruz had scooped out 4 servings. How much ice cream has she served?

2. Hot fudge was a popular topping! At the end of the party, there were 3 containers left with ½ cup each of hot fudge. How much hot fudge was left?

3. At the sundae table, Lauren was serving mini marshmallows. She used 1/3 cup for each sundae. How much of the marshmallows has she used after making 2 sundaes?

<table>
<tr><td>Grade Level/Course
5th Grade</td><td>Duration: 60 min.
Date:</td><td>Unit: Multiplication & Division of Fractions
Lesson # 4
Multiplying Fractions with Fractions</td></tr>
<tr><td>Common Core Standards</td><td colspan="2">5th Grade Number and Operations—Fractions
5NF.4a.b. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
a. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="2">Textbook: Houghton Mifflin 10.2
Mathematical Tools: graph paper, pencils Tiling the Art Room handout, tiles, white paper
Media/Technology to be used to deepen learning: ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division</td></tr>
<tr><td>Objectives</td><td>Content:
Students will be able to relate ideas about multiplication of whole numbers on arrays to multiplication of fractions less than one, and understand that unit squares represent a measurement of surface area</td><td>Language:
Students will be able to interpret a story context involving multiplication of fractions and represent a story context using an area model.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="2">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="2">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="2">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
</table>

Academic Vocabulary (Tier II & Tier III)		KEY WORDS ESSENTIAL TO UNDERSTANDING	WORDS WORTH KNOWING
	TEACHER PROVIDES SIMPLE EXPLANATION	array, area, unit square, length, product, factor, fraction, fraction less than one	
	STUDENTS FIGURE OUT THE MEANING		

Pre-teaching Considerations	Students will draw upon previous work with arrays with sides of whole number lengths. They will also make use of the ideas that a fractional amount originates from a whole and that when operating with two fractions it is understood that those two fractions originate from the same whole. Students will also use their knowledge and observations from the previous lesson as they continue investigating the size of the products.

Lesson Delivery

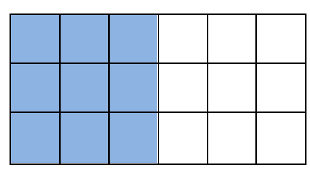
Instructional Methods	**Check method(s) used in the lesson:** ☒ **Modeling** ☒ **Guided Practice** ☒ **Collaboration** ☒ **Independent Practice** ☐ **Guided Inquiry** ☒ **Reflection**

Prior Knowledge: Students will use their knowledge and observations from the previous lesson as they continue investigating the size of the products. Students will draw upon previous work with arrays with sides of whole number lengths.

Context and Motivation:

Students will be engaged with a story context about a new multipurpose room whose walls will be tiled in a various ways. They will be introduced to a situation in which one of the dimensions of a tiling is a fractional amount.

Tell students: **You will be thinking about ways that a group of fifth grade students planned the tiling of a new art room at their school.** Show students an arrangement (array) of tiles. Ask the students to arrange the tiles in different ways so that there are always equal rows and columns (3x 6, 9x2 1x 18). Discuss how the rows and columns are part of the whole area (the 18 square units). Show the following array and ask students what ½ of the area would be.



Discuss how the dimensions of this array are 3 by 6 and the whole array is 18 square units but that we can shade in half of the tiles/ squares or 9 square units.

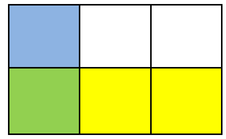
Let students know that they will be working with arrays in the next part of the lesson but that the dimensions of the arrays will be fractional amounts.

Lesson Continuum

Lesson Opening

Give each child a piece of white paper. Ask them to fold the paper horizontally into three equal parts (thirds). Tell them to choose a color and color one of the thirds.

Then ask students to fold the paper vertically in half and color one of the halves in a different color.



Ask: ***What part of the paper has both colors? (1 out of the six parts or 1/6) Why did we end up with six parts? What is the denominator of the horizontal dimension? 3 What is the denominator of the vertical dimension? 2 Why does only one of the parts have both colors? What are the dimensions of the two colored part? 1/3 by ½ and 1/3 x ½ = 1/6***

Today we are going to answer this question: How can the area of a rectangle with fractional sides be represented?

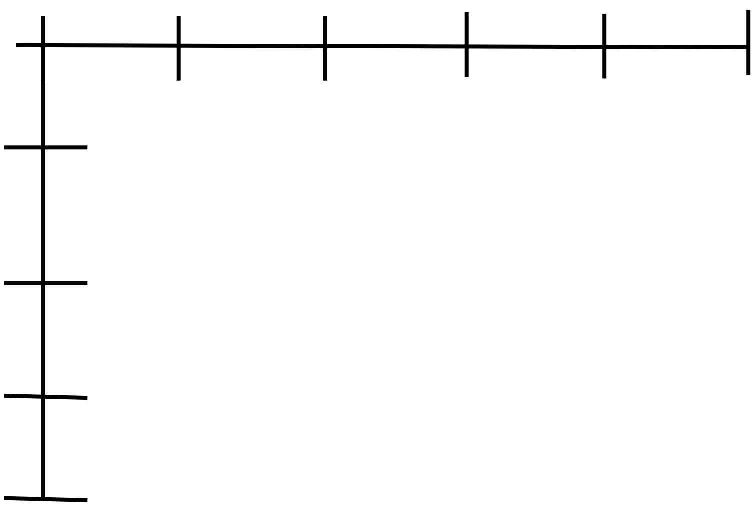
Lesson Delivery

Write on the board: 3 ½ x 5. First ask the students to estimate the answer and then ask for ideas about how to arrive at an exact answer. (17 ½) As students explain their ideas, record on the board or have students come to the board to show their thinking. If students do not have a strategy the following activities will give them some ideas.

After solution strategies have been shared, demonstrate for the class how to find the answer by drawing a 3 ½ by 5 rectangle. Tell them that **you will start by rounding three and a half to four so that both sides of the rectangle will be a whole number.** The follow the steps:

1. Draw the horizontal line, mark off 5 units, and erase any extra.

1. Draw a second side 4 units long.



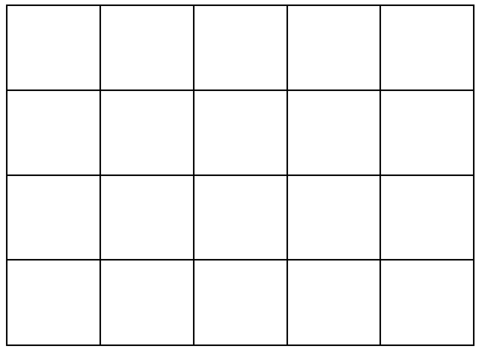
Differentiated Instruction:

English Learners:
Provide arrays that have one of the factors partitioned and labeled on the array and ask them to partition and label the second factor.

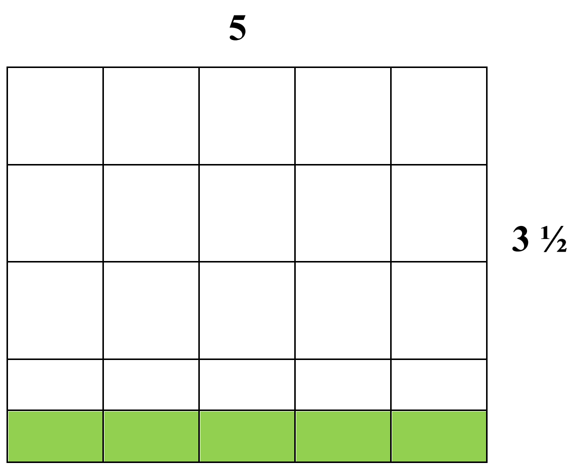
Special Needs:
For students who are struggling give them a context and have them describe to you the situation, what numbers are important, and what the operations in the equation may be. After helping students write an equation for various contexts, see if they can then create a story context or picture for a given equation.

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

3. Complete the 4 X 5 rectangle and divide it into small squares. Check that the students know that there are twenty squares and, therefore the answer to 3 ½ x 5 must be less than 20.



4. Draw a horizontal line to create a 3 ½ by 5 rectangle. Label the sides and shade the bottom portion to show that it is not part of the problem. Outline the 3 ½ by 5 rectangle with a darker line.



5. Have students help you count the number of squares in the unshaded section. First count the whole squares, then count the partial squares.

6. Point out that each of the five columns in the rectangle is 3 ½ units, which shows how you could solve the problem by adding three and a half five times. Also show the partial products 3 x 5 (the fifteen whole squares) and ½ x 5 (the five partial squares).

Accelerated Learners:
Challenge accelerated learners to represent their thinking in two to three ways of their choice.
The 3x6 dimensions are friendly since the area is 18. For students who are in need of an extension give them dimensions that will yield a total area that is an odd number, such as 3x7 or 3x9.

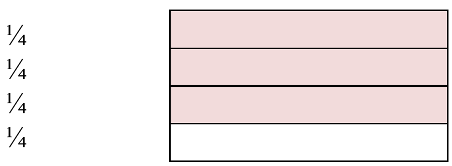
Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Mural Activity:

During this part of the lesson, students will consider how to partition a rectangle (the mural) into fractional parts and then find a part of those parts. They should apply how to use a visual model to solve a problem in context. They will represent parts of parts on with an area model. The idea of keeping track of all the parts in the whole is made explicit both in the diagrams and in the discussion of the values of the parts. Pose this situation (write on board / chart):

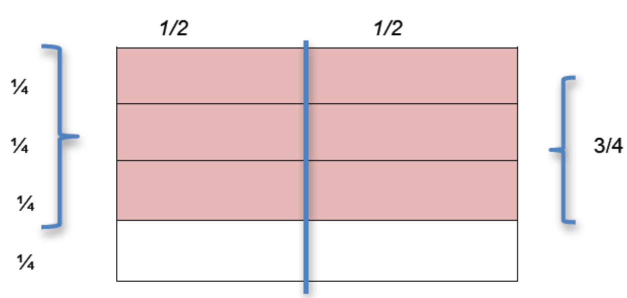
Our Elementary School asked the fifth grade students to help design some tile murals for the new art room. One of the murals was a 6 by 3 design like the one we made with tiles. Another mural is going have ¾ of the design as red tiles and ½ of those will have flowers on them.

Draw a rectangle ask students how we could show that ¾ of a mural would have red tiles. Partition a rectangle into four parts; the dimension of each part is ¼ by 1. Shade in 3 of the 4 parts (3/4) red



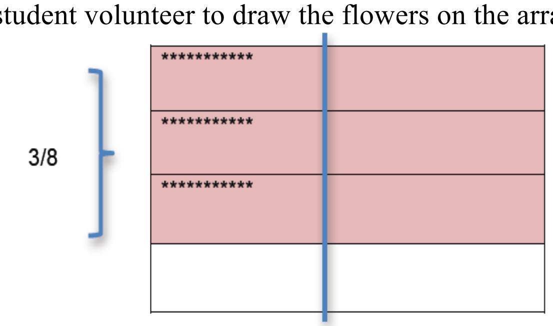
Ask students to share ideas for how they could represent that ½ of those red tiles would have flowers. Divide each of the four sections in half; divide the whole mural in half.

Ask: **How many parts has the whole mural been divided into? How do you know? What part of the mural are red tiles? (6/8 or ¾) Where do you see 6 out of 8 parts that are red?**



Show the above representation and ask students which part of the mural would have flowers. (½ of the red parts)

Ask for a student volunteer to draw the flowers on the array.



	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	Ask students: **What part of the whole mural has flowers? (3/8 of the mural has flowers.) What does the 8 represent? What does the 3 represent? Which part of the mural were you finding? (We found ¾ of the mural and then we looked for ½ of that part) What multiplication equation does this array represent? (½ x ¾).** Students will now work in small groups or with partners to solve additional problems about the murals in the multipurpose room. As students work pose questions such as the ones you posed in this part of the lesson. While students are working, the teacher will select and sequence student strategies for solving the problems. The discussion will focus on what it means to find a part of a part and how the ARRAY representations both show the whole, the part and the part of that part. Students will begin to notice that the result of multiplying a fraction by a fraction (both less than one) results in a fraction that is less than both fractions. **<u>Math Meeting</u>** Begin the whole group discussion with Problem 2. Select students so that they may share their strategies for making the array representations and how the representation connects to the story context. *The students decided to create a tile arrangement with geometric shapes. 1/5 of the tiles will be triangles. ½ of the triangle tiles will be painted blue. What part of this mural will be blue triangles?* Continue the whole group discussion with a comparison of the representation for Problem 2 and Problem 3. *The students decided to create a tile arrangement with geometric shapes. 2/5 of the tiles will be triangles. ½ of the triangle tiles will be painted blue. What part of this mural will be blue triangles?* Ask questions: **How are the representations different? How are they similar? How do the two equations for the problems compare?** *(2/5 x ½= 2/10 and 1/5 x ½ = 1/10)* **Why is the product for the situation in Problem 2 twice as much as the product for Problem 1?** **<u>Reflection</u>** Ask students to answer the question in their journals: How can the area of a rectangle with fractional sides be represented? **<u>Assessment (Formal or Informal)</u>** While students are working, pose questions and observe them to check for their understanding. Suggested things to observe or ask about: • Can students correctly translate a story context into a picture and an equation? • Can students clearly and accurately explain why they chose certain operations for fractions? • Can students correctly translate an equation into a story problem or equation?	

Lesson Reflection	
Teacher Reflection Evidenced by Student Learning/ Outcomes	

Name ___________________ Date _____________

Tiling the Multipurpose Room

Use an array model to show how the fifth grade students completed the tile murals for their new multipurpose room. As you work, use what you know about arrays with whole numbers.

1. One of the murals in the multipurpose room will fit over the sink. This mural will have a pattern of light blue and black tiles. The black tiles will cover 2/3 of the design. The students will paint yellow suns on ¼ of those black tiles. What part of the whole mural will be black with yellow suns?

2. The students decided to create a tile arrangement with geometric shapes. 1/5 of the tiles will be triangles. ½ of the triangle tiles will be painted blue. What part of this mural will be blue triangles?

3. The art teacher asked the students to design a mural with their handprints. The students will cover 2/5 of the mural with handprints. ½ of those handprint tiles will be painted red. What part of this mural will be red handprints?

Name ___________________ Date ________________

More Tiling of the Multipurpose Room

1. A large mural made up of handprint tiles will go on the left wall. This mural will measure 2 ½ feet by 4 feet. How large will the mural be?

2. The door of the multipurpose room measures 6 feet by 4 ½ feet. The art teacher is considering asking a group of fifth graders to paint the door with designs. How large an area will they be painting?

3. A small area above a window is available for a tiling design. The space measures 5 inches by ½ inch. How large is the area above the window?

<table>
<tr><td>Grade Level/Course
5th Grade</td><td>Duration: 60 min.
Date:</td><td colspan="2">Unit: Multiplication & Division of Fractions
Lesson # 5
Multiplying Fractions by Fractions</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">5th Grade Number and Operations—Fractions
5NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
a. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: counters, fraction bars, water-based markers
Media/Technology to be used to deepen learning: ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division
Supplementary Materials: PowerPoint</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will be able to multiply fractions by whole numbers and other fractions and conjecture about it product.</td><td>Language:
Students will be able to create a story context for multiplying with fractions.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☐ Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="3">Academic Vocabulary (Tier II & Tier III)</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td colspan="2">TEACHER PROVIDES SIMPLE EXPLANATION: Numerator, denominator, canceling, commutative property for multiplication, mixed number, improper fraction</td><td></td></tr>
<tr><td colspan="2">STUDENTS FIGURE OUT THE MEANING</td><td></td></tr>
<tr><td>Pre-teaching Considerations</td><td colspan="3">Students should be able to change a mixed number to improper fraction and explain that a fraction represents division.</td></tr>
</table>

<table>
<tr><th colspan="3">Lesson Delivery</th></tr>
<tr><td colspan="2">Instructional Methods</td><td>Check method(s) used in the lesson:
☐ Modeling ☒ Guided Practice ☒ Collaboration
☐ Independent Practice ☒ Guided Inquiry ☒ Reflection</td></tr>
<tr><td>Lesson Continuum</td><td>Lesson Opening</td><td>Prior Knowledge: Students should be able to change a mixed number to improper fraction and explain that a fraction represents division.
Context and Motivation:
This lesson will focus on definitions and mathematical conventions.
Read the essential question with the students: “How does multiplying by a fraction or by a mixed number affect the size of the product?”
Tell students that they will answer the essential question and make some conjectures about what happens to the product when multiplying with different fractions. Tell students to notice the numerator and the denominator in relationship to the product.
Each pair of students should have counters or tiles—about 20 to 30.
Show an array of 12 counters or tiles.
How can these markers be used to find 2/3 of 12? (Divide the markers into 3 equal groups and take 2 of the groups.)
What is 2/3 of 12 markers? (8)
What is 1/3 of 12 markers? (4)
Point out that these results can be written as multiplication equations.
$2/3 \times 12 = 8$ and $1/3 \times 12 = 4$
Students should work with a partner. Ask students the following questions:
Count out 15 blue markers. What is 2/5 of these markers? Ask students for their responses. If no one shares the following explanation share it and write the multiplication equation. (Divide 15 markers into 5 equal groups and the total of two of the groups is 6 markers. $2/5 \times 15 = 6$)
What is 4/5 of 15 markers? Write the multiplication equation. ($4/5 \times 15 = 12$)
Review some examples of multiplying a whole number times a fraction if necessary..
What is the rule you discovered for multiplying a whole number times a fraction? Multiply the whole number times the numerator and keep the denominator. Write this on a chart after the first generalization made during the lesson 1 for future reference.
Will this rule also work for multiplying a fraction times a whole number? Try it for computing $2/3 \times 12 = 8$. (Yes, multiply 2 times 12 and divide by 3 to get 8.)

Summarizing for relationships and generalizations: State a rule that holds for both multiplying a whole number times a fraction and multiplying a fraction times a whole number. (Multiply the numerator and the whole number and keep the denominator.) Discuss the commutative property for multiplication that holds for fractions, as well as whole numbers. For example, $2/3 \times 12 = 12 \times 2/3$, so only one rule is needed. Rewrite this rule under the other two rules. Tell students that they will look for more relationships between numerators and denominators when multiplying by whole number and fractions to make their computations more efficient.</td></tr>
</table>

Activities/Tasks/ Strategies/Technology/
Questioning/Engagement/Writing/Checking for Understanding

Show students 20 tiles.
What is ¾ of 20 counters? (15)
Explain how you arrived at this answer. (One way is to use the rule stated above. Another is to divide the 20 markers into 4 equal groups of 5 markers each and take 3 of the groups.)
The process of dividing the whole number by the denominator and then multiplying by the numerator, suggests a m **<u>canceling</u>.**

$$\frac{3}{\cancel{4}_{1}} \times \cancel{20}^{5} = 15$$

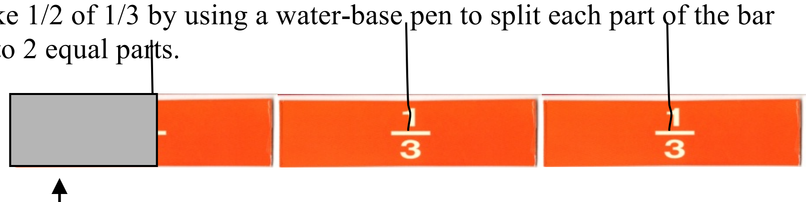
Ask students to use canceling to compute the following products.
3/5 × 50 1/4 × 40 2/3 × 27 5/6 × 24

Discuss the convenience of canceling. Show an example of canceling a common factor from the denominator and the whole number when the denominator does not evenly divide into the whole number and an example using compatible numbers and canceling.

$$\frac{3}{\cancel{4}_{2}} \times \cancel{10}^{5} = \frac{15}{2} = 7\frac{1}{2} \qquad \frac{3}{4} \times 19 \approx \frac{3}{\cancel{4}_{1}} \times \cancel{20}^{5} = 15$$

Tell them they have a**t least three ways to visually model how to work with fractions: rectangles and arrays, fraction bars, and number lines**. Although the teacher will now use fraction bars, any other model will be acceptable. Using a variety of models will help make students flexible thinkers and problem solvers. Also some students can visualize fractions better with one model better than another.

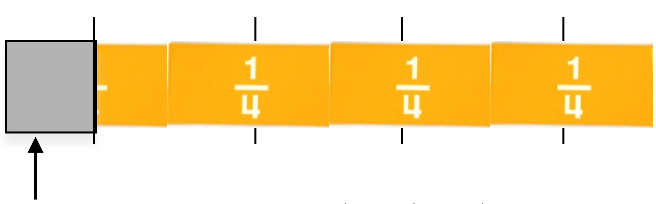
Show students and have them find a 1/3 bar and demonstrate how to take 1/2 of 1/3 by using a water-base pen to split each part of the bar into 2 equal parts.



1/2 × 1/3 = 1/6

Tell students: **One of these new split shaded parts is 1/6 of a bar because a whole bar now contains 6 equal parts. The resulting product is 1/2 × 1/3 = 1/6.**

Select any bar with just one part shaded. For example**: Explain how the bar can be used to find ½ of the fraction for the bar. Write the multiplication equation.** Lines from water-base can be washed off the bars.



1/2 × 1/4 = 1/8

Differentiated Instruction:

English Learners:
Houghton Mifflin:
Universal Access: English Learners
T.E., p. 216B
Visuals, realia

Special Needs:
Houghton Mifflin:
Reteach 10.3,
Strategic Intervention book
NS 24, 26, 27, 28
Visuals, realia

Accelerated Learners:
Houghton Mifflin:
Enrichment 10.3

Find the bar for 1/2 and explain how to use this bar to find 1/3 of 1/2. Write the resulting multiplication equation. (Split each part into 3 equal parts. One of the new shaded parts equals 1/6 because there are now 6 equal parts in a whole bar. 1/3 × 1/2 = 1/6)

Discuss **the fact that 1/2 of 1/3 is equal to 1/3 of 1/2 and this is an example of the commutative property. In this case 1/2 × 1/3 = 1/3 × 1/2 = 1/6.**

Use your 1/2 bar and show how to take 2/3 of the shaded amount. Remember, taking 2/3 of something means to divide it into 3 equal parts and take 2 of the new parts.

2/3 × 1/2 = 2/6

<u>**Math Meeting**</u>

Distribute copies of "Multiplying Fractions from Fraction Bars." If a copy of this activity sheet is projected, students can illustrate drawing lines on the bars to take parts of the shaded amounts and explain their reasoning. This should not take long. Observe students and note students who still do not understand the relationship between the models and equations.

Summarizing to see relationships and generalizations. List the above multiplication equations in one spot to help students compare and look for relationships.
(1) 1/2 × 1/3 = 1/6 (2) 1/2 × 1/4 = 1/8 (3) 1/3 × 1/2 = 1/6 (4) 2/3 × 1/2 = 2/6

Tell students: **Study these multiplication equations and state a generalization for multiplying a fraction times a fraction.** (When students come to this conclusion--*Multiply the numerator times the numerator and the denominator times the denominator*—add it to the chart of generalizations.)

Tell students: **Look for other relationships and patterns in these four equations.** (1) In each product we are multiplying by a fraction less than 1; (2) In each product, the answer is less than the number being multiplied. For example, in 1/2 × 1/3 = 1/6, we see that 1/6 is less than 1/3.

Discuss the **fact that multiplying by fractions less than 1 results in taking part of something, so the product is always less than the number being multiplied**.

Connecting: Ask students to apply "canceling" to multiplying fractions with fractions.
Ask students to compute the product 2/3 × 7/12 and write the answer in lowest terms.
(2/3 × 7/12 = 14/36 = 7/18)
Compute the same product using canceling.

$$\frac{\overset{1}{\cancel{2}}}{3} \times \frac{7}{\underset{6}{\cancel{12}}} = \frac{7}{18}$$

Discuss the convenience of canceling.

2. Write this product: 5/6 × 9/10. Sometimes it is possible to cancel more than once. Compute this product by canceling.

$$\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}} = \frac{3}{4}$$

Reflection

In your journals write a math story to go with any one of the problems we worked on together.

Assessment (Formal or Informal)

Student journals
Student activity sheet

Lesson Reflection	
Teacher Reflection Evidenced by Student Learning/ Outcomes	

Name: ____________________ Date:_ ____

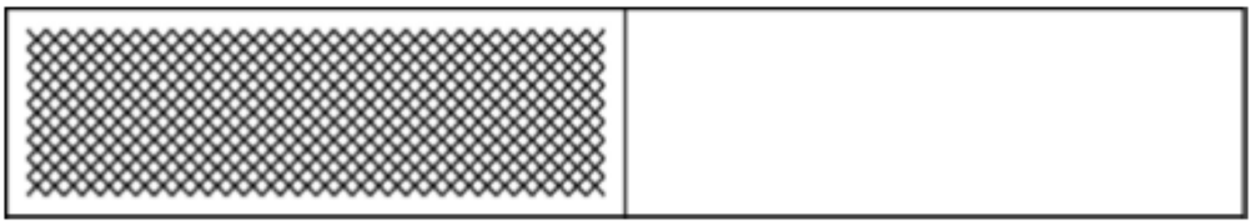
Activity Sheet "Multiplying Fractions from Fraction Bars"

Draw lines on the shaded part of the bar to determine the fraction of the shaded amount. Then complete the equation.

a.

$\frac{1}{2}$ of $\frac{1}{6}$ = $\frac{1}{2} \times \frac{1}{6}$ =

b.



$\frac{1}{4}$ of $\frac{1}{2}$ = $\frac{1}{4} \times \frac{1}{2}$ =

c.

$\frac{2}{3}$ of $\frac{1}{4}$ = $\frac{2}{3} \times \frac{1}{4}$ =

d.

$\frac{3}{4}$ of $\frac{1}{3}$ = $\frac{3}{4} \times \frac{1}{3}$ =

Lesson 1-5 Review

Name: _______________

1. Complete each product. You may find it helpful to use the given figures.

a. 

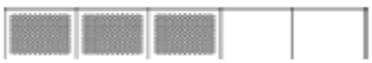
$\frac{1}{2} \times 3 =$ ____

b. 

$\frac{1}{3} \times \frac{1}{2} =$ ____

c. 

$\frac{3}{4} \times 2 =$ ____

d. 

$\frac{1}{2} \times \frac{3}{5} =$ ____

e. 

$\frac{1}{2} \times 8 =$ ____

f. 

$\frac{1}{5} \times \frac{1}{3} =$ ____

2. Compute each product.

a. $\frac{1}{3} \times 25 =$

b. $\frac{1}{2} \times \frac{3}{4} =$

c. $\frac{1}{4} \times 9 =$

d. $\frac{3}{4} \times \frac{2}{3} =$

e. $\frac{2}{3} \times 17 =$

f. $\frac{1}{3} \times \frac{1}{5} =$

3. Taylor used $\frac{2}{3}$ of 12 stamps to send cards to family members. How many stamps were left?

____

4. One-half of a fence was damages by a storm on Tuesday, and $\frac{1}{3}$ of the damaged part was repaired on Wednesday. What fraction of the whole fence was repaired on Wednesday?

____

5. The Highway Department decided that $\frac{2}{3}$ of a 16-mile stretch of road needed a new surface. What length of the road needed a new surface?

____

<table>
<tr><td colspan="2">Grade Level/Course
5th Grade</td><td>Duration: 60 min.
Date:</td><td colspan="2">Unit: Multiplication & Division of Fractions
Lesson # 6
Comparing Size of Products</td></tr>
<tr><td colspan="2">Common Core Standards</td><td colspan="3">5th Grade Number and Operations—Fractions
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
a. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)
5. Interpret multiplication as scaling (resizing), by:
a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</td></tr>
<tr><td colspan="2">Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: graph paper, tiles, fraction bars, colored pencils, journals, die
Media/Technology to be used to deepen learning: ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division</td></tr>
<tr><td colspan="2">Objectives</td><td colspan="2">Content:
Students will be able to solve problems using the multiplication of fractions and compare sizes of products by comparing the sizes of factors.</td><td>Language:
Students will take notes about the variety of strategies and models that could be used to solve problems using the multiplication of fractions.</td></tr>
<tr><td colspan="2">Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒Level 4: Extended Thinking</td></tr>
<tr><td colspan="2">Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td colspan="2">Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="2">Academic Vocabulary (Tier II & Tier III)</td><td rowspan="2">TEACHER PROVIDES SIMPLE EXPLANATION</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td colspan="2">Review of previous vocabulary</td><td></td></tr>
</table>

<table>
<tr><td></td><td>STUDENTS FIGURE OUT THE MEANING</td><td></td><td></td></tr>
<tr><td colspan="2">Pre-teaching Considerations</td><td colspan="2">Students should have knowledge of multiplication of fractions.</td></tr>
<tr><td colspan="4">Lesson Delivery</td></tr>
<tr><td colspan="2">Instructional Methods</td><td colspan="2">Check method(s) used in the lesson:
☒ Modeling ☒ Guided Practice ☒ Collaboration
☐ Independent Practice ☒ Guided Inquiry ☒ Reflection</td></tr>
<tr><td>Lesson Continuum</td><td>Lesson Opening</td><td colspan="2">Prior Knowledge: Students should have knowledge of multiplication of fractions.
Context, and Motivation:
Students will journal the models for solving multiplying with fractions and whole numbers and then discuss the strategy of comparing sizes of products and approximating products.
Tell students: Today we will journal the models for solving with the multiplication of fractions. However, before we begin journaling you want to show them how to play a game.
Students play the game in pairs. The object of the game is to get the smallest possible answer.
First students need to make a game sheet to play three rounds.
Student A   Student B
$\square \quad \frac{\square}{\square} \times \frac{\square}{\square} =$ $\frac{\square}{\square} \times \frac{\square}{\square} = \quad \square$
$\square \quad \frac{\square}{\square} \times \frac{\square}{\square} =$ $\frac{\square}{\square} \times \frac{\square}{\square} = \quad \square$
$\square \quad \frac{\square}{\square} \times \frac{\square}{\square} =$ $\frac{\square}{\square} \times \frac{\square}{\square} = \quad \square$

<u>Game Instructions:</u>
Take turns rolling the die and writing the number in one of your spaces. Once a number is written, it cannot be changed. The boxes to the side are reject boxes that give one chance to write a number that you do not want to use in the problem.

After writing a number pass the die to your partner. Do this until you both have recorded two fractions for round 1. Multiply your fractions. Check each other’s answers. The winner of the round is the player with the smaller product. Play for three rounds.

Tell students they could play this on menu days and after they finish their tasks.</td></tr>
</table>

		Note-taking Foldable Students will take an inventory of the various ways to conceptualize the multiplication of fractions. Guide students through each story problem and its related conceptualization. Colored pencils are helpful in note-taking as is creating compartments for each strategy through folding and snipping. Note-taking through folding compartments makes information easily accessible to students. Use the teacher sample to talk students through creating the note-taking journal page. Once the note-taking page is folded and cut, write the title of the page on the top front: Models of Multiplication with Fractions. Begin with modeling with tiles. The problems have been provided for students. Some students may need the actual manipulatives in front of them. Make sure they are available.	

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

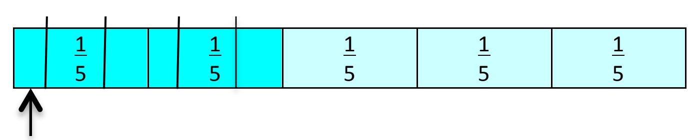
Tiles:
A cook used 2/3 of 2 squares of chocolate to make chocolate cheesecake. How much of these squares was used for the cheesecake?
Ask students**: What does it mean to take 2/3 of something? (Divide it into 3 equal parts and take 2.)**
Ask for suggestions as to how to take 2/3 of 2 squares. **(Take 2/3 of each square.)**

Distributive Property:
Now the cook wanted to use 2/3 of 19 chocolate squares. How much chocolate would be needed? Since it is not practical to take 2/3 of each of the 19 squares, what is 2/3 of 18?

1/3 of 18 is 6, and 2/3 of 18 is 12
Then we can take 2/3 of the one remaining square. Write a multiplication equation for 2/3 of 19. (2/3 × 19 = 12 2/3)
We have computed 2/3 x 19 by computing 2/3 of 18 and 2/3 of 1. This is an example of the distributive property: 2/3 × (18 + 1) = 2/3 × 18 + 2/3 × 1
Knowing that 2/3 × 2 = 4/3, state a rule for computing this product. "Multiply numerator times the whole number and keep the denominator." (Ask students to repeat this with you.)

Fraction Bars:
In science class two-fifths of an ounce of calcium sulfate is available in the lab, and 2/3 of this amount is needed for an experiment. How much of the calcium sulfate is needed for the experiment?

Ask students: **What does it mean to take 2/3 of something?**
Possible student response: Divide the amount into 3 equal parts and take 2 of them.
Ask for suggestions as to how to take 2/3 of 2/5. **Take 2/3 of each of the two parts**.
Split two parts of the fifths bar into 3 equal parts to obtain a bar of 6/15. Then take 2/3 of the shaded part.



1/15

2/3 of each 1/5 is 2/15 So, 2/3 × 2/5 = 2/15 + 2/15 = 4/15

Rectangles/Arrays:
(Students use the gird lines on the note paper)
If a tablecloth has dimensions of 2 yards by 3 ½ yards, what is its area in square yards?

Tell students: **Outline a rectangle on your grid paper with dimensions of 2 by 4 and label the lengths of two its sides. Remember one side is 3 ½ so we will need to halve 2 squares.**

Differentiated Instruction:

English Learners:
Using sentence frames
Using visuals
Working in pairs
Houghton Mifflin:
Universal Access: English Learners
T.E., p. 216B
Visuals, realia

Special Needs:
Working in pairs
Modifying numbers given
Using sentence frames
Houghton Mifflin:
Reteach 10.3,
Strategic Intervention book
NS 24, 26, 27, 28
Visuals, realia

Accelerated Learners:
Houghton Mifflin:
Enrichment 10.3

What is the area of the tablecloth in square yards? (7 square yards) **Can this area be found by multiplying the lengths of the two sides?** (Yes) **Compute their product**. (2 × 3½ = 2 × 7/2 = 14/2 = 7 square yards)

Under the arrays fold:
A 1 ¾ foot by 1 ½ foot rectangular sheet of metal is cut from a 2 foot by 2 foot sheet. What is the area of the sheet metal?
Tell students: **Use four of the unit squares on the grid sheet to sketch a rectangle with dimensions of 1 ¾ by 1 ½.**
Subdivide the four unit squares as shown and label each part with a number for its area. For the unit square with 4 parts, each part is ¼ square foot, and for the unit square with 8 parts, each part is 1/8 square foot.

The area of the 1 ¾ by 1 ½ rectangle is the sum of the areas for the eight parts. What is this sum? (2 5/8, so the area of the metal sheet is 2 5/8 square feet).

Can the area also be found by computing the product of the lengths of the sides of the rectangle? (Yes)
Compute the product 1 ¾ × 1 ½ by using improper fractions. Write the answer as a mixed number.
(1 ¾ × 1 ½ = 7/4 × 3/2 = 21/8 = 2 5/8)

Number Line:
David the baker had ½ pound of butter and used 1/3 of it in a batch of cookies. How much of the butter did he use?

Tell students**: Draw a number line 0 to 1. Divide the number line into halves.**
How much of the ½ pound of butter did David use? (1/3)
So we need 3 parts. Divide both halves into 3 parts. How many total parts are there? (6)
Draw jumps with a light color on the bottom of the number line. How many parts of the three did he use? (1)
Draw 1 jump on the top of the number line. So we draw 1/3 of ½. Compute 1/3 x ½ = 1/6

Circles:
Tell students: **Model 1 ¾ x 2.**
Draw 2 circles and divide them into fourths. Shade 1 whole circle and ¾ of the second circle. This makes 1 ¾. How many times? (2)
So we have to draw another set. How many fourths do we have? (14)
What is the improper fraction? (14/4)
How many wholes? (2) **How many fourths are left?** (2/4 or ½)
Ask students to write a story for the problem. (Allow students to work with a partner).

Math Meeting

Have a short math meeting and ask students to share the problems to go with the problem they just modeled with circles.

In the second half of class spend time analyzing the products by comparing the sizes of factors. Use the activity sheet to have this discussion.

1. Multiplying by fractions less than 1

Distribute copies of the activity sheet "Comparing Sizes of Products" so students can use the figures in #1a, b, and c to model the following information.

Ricardo, Jasmin, and Jordan each have 12 stamps. They each use the following amounts of their stamps: Ricardo uses 1/2; Jasmin uses 1/3; and Jordan uses 1/4.

Write a multiplication equation to represent the number of stamps used by each person.

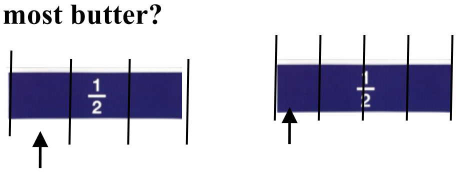
(½ × 12 = 6; 1/3 × 12 = 4, and ¼ × 12 = 3)

As the size of the fractions become smaller, what happens to the size of the products? (The products become smaller.)

2. **Multiplying by fractions less than 1**

Use the bars in #2 on the activity sheet to model the following information.

On Day 1, David had ½ pound of butter and used 1/3 of it in a batch of cookies. On Day 2, he had another ½ pound of butter and used ¼ of it in making a batch of waffles. On which day did he use the most butter?



Day 1 Day 2

a. **This information can be illustrated by using two ½ bars. Write a multiplication equation for the amount of butter used on each day**. (Day 1: 1/3 × ½ = 1/6; and Day 2: ¼ × ½ = 1/8)

b. **On which day was the greater amount of butter used?** (Day 1)

c. **If we continued multiplying by smaller and smaller fractions, such as 1/5 × ½, 1/6 × ½, etc., what happens to the size of the products?** (The products become smaller.)

d. **In general if any given number, whole number or fraction, is multiplied by a fraction less than one, what can be said about the size of the product?** (It is smaller than the given number.)

3. Multiplying by fractions greater than 1

Use the figures in **#3a**, **b** and **c** on the activity sheet to model the following information.

a. Beatriz has 12 stamps and Pepe has one and one-half times the number of Beatriz's stamps. Draw the number of stamps that Pepe has on your activity sheet. How many stamps does Pepe have? (18)

How can 1 ½ x 12 be computed? (Using the meaning of 1½, we can take one group of 12 and then half of the group of 12 to get 18 stamps. Or, we can replace the mixed number 1 ½ by the fraction 3/2 and compute 3/2 × 12 = 36/2 = 18.)

		Kennedy has one and one-third times the number of Beatriz's stamps. Draw the number of stamps that Kennedy has on your activity sheet. How can we determine 1 and 1/3 of 12 stamps? (Take the whole collection of stamps and then add 1/3 of 12 stamps.) **How can 1 1/3 × 12 be computed?** (Replace the mixed number 1 1/3 by the fraction 4/3 and compute 4/3 × 12 = 48/3 = 16.) **Nelli has one and one-fourth times the number of Beatriz's stamps. Draw the number of stamps that Nelli has on your activity sheet. How can we determine 1 and ¼ of 12 stamps?** (Replace the mixed number 1 1/4 by the fraction 5/4 and compute 5/4 × 12 = 60/4 = 15.) **In general, if any number is multiplied by a fraction greater than 1, what can you say about the size of the product?** (It is larger than the given number.) **<u>Reflection</u>** Note in your journal or behind your activity sheet: What I got from this lesson: What I still need to get: **<u>Assessment (Formal or Informal)</u>** Teacher observations Activity sheet responses	

Lesson Reflection	
Teacher Reflection Evidenced by Student Learning/ Outcomes	

Name: ________________________ Date:_____

Activity Sheet "Comparing the Sizes of Products"

Ricardo, Jasmin, and Jordan each have 12 stamps. They each use the following amounts of their stamps:

1. Circle the number of stamps for each fraction and complete the equation.

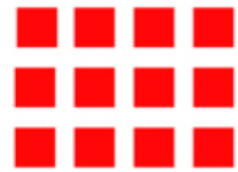
a. Ricardo's stamps

½ × 12 =

b. Jasmin's stamps

1/3 × 12 =

c. Jordan's stamps

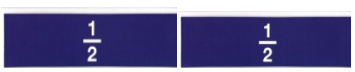


¼ × 12 =

2. Draw lines on the bars to show 1/3 of ½ and ¼ of ½ and complete the equations.

On Day 1, David had ½ pound of butter and used 1/3 of it in a batch of cookies. On Day 2, he had another ½ pound of butter and used ¼ of it in making a batch of waffles. On which day did he use the most butter?

a. Day 1



1/3 × ½ =

b. Day 2

¼ × ½ =

3. **Beatriz has 12 stamps. Sketch the stamps in the boxes for Pepe, Kennedy, and Nelli and complete the equations.**

a. Beatriz's stamps

Pepe's stamps (1 ½ times the number of Beatriz's stamps)

1 ½ × 12 =

---

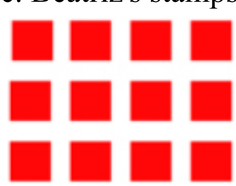
b. Beatriz's stamps

Kennedy's stamps (1 1/3 times the number of Beatriz's stamps)

1 1/3 × 12 =

---

c. Beatriz's stamps



Nelli's stamps (1 1/4 times the number of Beatriz's stamps)

1 ¼ x 12 =

A cook used 2/3 of 2 squares of chocolate to make chocolate cheesecake. How much of these squares was used for the cheesecake?

Now the cook wanted to use 2/3 of 19 chocolate squares. How much chocolate would be needed? Since it is not practical to take 2/3 of each the 19 squares, what is 2/3 of 18?

David the baker had ½ pound of butter and used 1/3 of it in a batch of cookies. How much of the butter did he use?

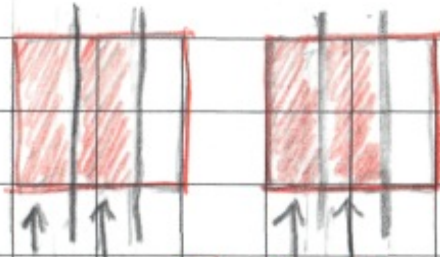
In science class two-fifths of an ounce of calcium sulfate is available in the lab, and 2/3 of this amount is needed for an experiment. How much of the calcium sulfate is needed for the experiment?

If a table cloth has dimensions of 2 yards by 3 ½ yards, what is its area in square yards?

A 1 ¾ foot by 1 ½ foot rectangular sheet of metal is cut from a 2 foot by 2 foot sheet. What is the area of the sheet metal?


Models of Multiplication with Fractions
Circle
Tiles
Number Line
0
Distributive Property
Fraction Bars
1/2
1/2
Arrays
Rectangles

A cook used 2/3 of 2 squares of chocolate to make chocolate cheesecake. How much of these squares was used for the cheesecake?



$\frac{2}{3} \times 2 = \frac{4}{3}$

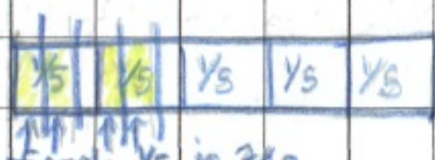
Now the cook wanted to use 2/3 of 19 chocolate squares. How much chocolate would be needed? Since it is not practical to take 2/3 of each the 19 squares, what is 2/3 of 18?

- $\frac{1}{3}$ of 18 is 6, so $\frac{2}{3}$ of 18 is 12
- Now we have 1 remaining square. $\frac{2}{3} \times 1 = \frac{2}{3}$

$\therefore$ he needs $12\frac{2}{3}$ chocolate

$\frac{2}{3} \times (18+1) = (\frac{2}{3} \times 18) + (\frac{2}{3} \times 1)$

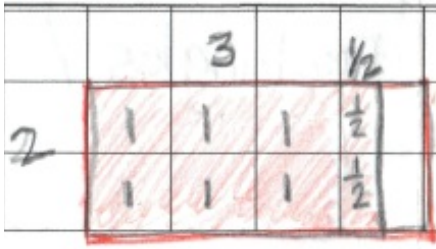
In science class two-fifths of an ounce of calcium sulfate is available in the lab, and 2/3 of this amount is needed for an experiment. How much of the calcium sulfate is needed for the experiment?



$\frac{2}{3}$ of each $\frac{1}{5}$ is $\frac{2}{15}$

so $\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$

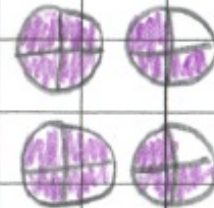
If a table cloth has dimensions of 2 yards by 3 ½ yards, what is its area in square yards?



$2 \times 3\frac{1}{2} = 7$

$2 \times \frac{7}{2} = \frac{14}{2} = 7$

Write a story for the circle model.



$1\frac{3}{4} \times 2 = 2\frac{2}{4} \rightarrow 2\frac{1}{2}$

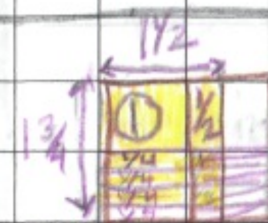
$\frac{7}{4} \times \frac{2}{1} = \frac{14}{4} \rightarrow 2\frac{2}{4}$

$2\frac{1}{2}$

David the baker had ½ pound of butter and used 1/3 of it in a batc of cookies. How much the butter did he use?

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

A 1 ¾ foot by 1 ½ foot rectangular sheet of metal is cut from a 2 foot by 2 foot sheet. What is the area of the sheet metal?



$1\frac{3}{4} \times 1\frac{1}{2}$

$\frac{7}{4} \times \frac{3}{2} = \frac{21}{8} = 2\frac{5}{8}$

$\frac{1}{4} + \frac{1}{4} + \frac{1}{2} = 1$

$\frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}$

$1 + 1 + \frac{5}{8} = 2\frac{5}{8}$

<table>
<tr><td colspan="2">Grade Level/Course
5th Grade</td><td>Duration: 60 min.
Date:</td><td colspan="2">Unit: Multiplication & Division of Fractions
Lesson # 7
Multiplying Fractions with Whole Numbers and Fractions</td></tr>
<tr><td colspan="2">Common Core Standards</td><td colspan="3">5th Grade Number and Operations—Fractions
5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</td></tr>
<tr><td colspan="2">Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: tiles counters, fraction bars, paper, pencil, journals
Media/Technology to be used to deepen learning: ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division</td></tr>
<tr><td colspan="2">Objectives</td><td colspan="2">Content:
Students will be able to solve problems involving multiplication of fractions and mixed numbers.</td><td>Language:
Students will be able to explain the strategies and procedures they used to solve problems with fractions.</td></tr>
<tr><td colspan="2">Depth of Knowledge Level</td><td colspan="3">☒ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒Level 4: Extended Thinking</td></tr>
<tr><td colspan="2">Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
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☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
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<tr><td colspan="2">Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
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<tr><td colspan="2">Whole numbers, mixed number, improper fractions, simplest form</td><td></td></tr>
<tr><td>STUDENTS FIGURE OUT THE MEANING</td><td colspan="2"></td><td></td></tr>
<tr><td colspan="2">Pre-teaching Considerations</td><td colspan="3">Students show understand multiplication of fractions with whole, improper numbers, mixed numbers and fractions.</td></tr>
</table>

<table>
<tr><th colspan="3">Lesson Delivery</th></tr>
<tr><td colspan="2">Instructional Methods</td><td>Check method(s) used in the lesson:
☐ Modeling ☐ Guided Practice ☒ Collaboration
☐ Independent Practice ☒ Guided Inquiry ☒ Reflection</td></tr>
<tr><td rowspan="2">Lesson Continuum</td><td>Lesson Opening</td><td>Prior Knowledge: Students show understand multiplication of fractions with whole, improper numbers, mixed numbers and fractions.
Context and Motivation:
Today’s big idea (or enduring understanding) is “Multiplication of fractions and mixed numbers is used to solve problems in daily life.” So you will use your knowledge to solve problems.

Tell students that today they will solve a variety of problems. They should use strategies and models that we have been studying. You should see models and algorithms and students should be able to explain how they checked their solution.
Review all vocabulary, especially simplest form. Remind students as they find solutions, they should also reduce the fraction to its simplest form if necessary.</td></tr>
<tr><td></td><td><table><tr><td><u>Lesson</u>
It is not necessary that all students solve every problem. Students will be at various level of understanding. As the class goes through the problem set assign problems to groups of students according to their levels of proficiency. After each problem set have a math meeting and ask students from each level share their solutions.

Set 1: Whole numbers times fractions and mixed numbers
Advanced: Marie poured 12 pitchers of water into her fish tank, and each pitcher held 2 1/3 quarts of water. How much water did she put into her fish tank? (28 quarts)

Proficient: One lap around the track at the King Elementary School is 1/12 of a mile. If Kristen runs 18 laps, how far has she run? (1 ½ miles)

Basic: How many miles of tunnel can engineers drill in 6 months, if they can drill ¾ of a mile each month? (4 ½ miles)

Guide students that need support. Ask students questions about their thinking:
Guiding Questions:
How do you know?
What does (this) ____________ represent?
How did you know where…?
How did you know which…?
What strategy are you using?
What math words can you use or learn?
What the steps involved?
(Students should use a visual model and an algorithm to solve the problems.)</td><td>Differentiated Instruction:

English Learners:
Using sentence frames
Working in pairs or small groups
Support students at intermediate and below with the guiding questions and prompts.

Special Needs:
Working in pairs or small groups
Using sentence frames
The problems are leveled by proficiency levels.

Accelerated Learners:
The problems are leveled by proficiency levels.</td></tr></table></td></tr>
</table>

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	**<u>Math Meeting</u>** Bring students together. Have at least three students from each level share their solutions. **TO HELP STUDENTS RETELL** (and tell/list/recite/name/find/describe/explain/illustrate/summarize) **Guiding Questions:** • How did you solve the problem? • What did you do? • What strategy did you use? • What math words did you use or learn? • What were the steps involved? • What did you learn today? • What do(es) __________________ mean to you? **Prompts to use:** • I solved the problem by … • The math words I used were … • The steps I followed were … • My strategy was successful because … • Explain to a young child or someone that wasn't involved … • Draw a picture to show how you solved the problem. **Set 2 Fractions times whole numbers** Advanced: On an 18 day vacation, Ruby practiced her guitar and on some days and her harmonica on all of the other days. If she practiced her guitar on 2/3 of the days, on how many days did she spend practicing her harmonica? (6 days) Proficient: It is 40 miles from Los Angles to Irvine. If the Garcia family drove 3/5 of the distance to Irvine before getting a flat tire, how far were they from Irvine? (16 miles) Basic: It costs $150 to stay at the scout camp for one week. If Elena earned 2/3 of this amount, how much money did she earn for the cost of the camp? ($100) Guide students that need support. Ask students questions about their thinking. **<u>Math Meeting</u>** Bring students together. Have at least three students from each level share their solutions. Use guiding questions and prompts when necessary. **Set 3 Mixed numbers times mixed numbers** Advanced: If a farmer can plow 5 ¼ acres of land in one day, how many acres of land can she plow in 2 2/3 days? (14 acres) Proficient: What is the area of a rectangular greeting card, if its dimensions are 7 ½ inches by 4 2/5 inches? (33 square inches) Basic: If a spaceship orbits a planet in 1 2/5 days, how many days will it take to orbit the planet 5 times? (7 days) Guide students that need support. Ask students questions about their thinking.	

		Math Meeting Bring students together. Have at least three students from each level share their solutions. Use guiding questions and prompts when necessary **Set 4 Variety of types of multiplication problems** Proficient/Advanced: Tony ordered 4 Classic Fruit Gift Baskets online and each weighed 5 ¾ pounds. What was the total shipping weight? (23 pounds) Proficient/Advanced: Mistie's mother paid $180 for a cell phone, but Mistie purchased one for 2/3 of the cost of her mother's. What was the cost of Mistie's cell phone? ($120) Proficient/Advanced: A school's enrollment of 300 students decreased by ¼ because of a new district organization. What was the school's new enrollment? (225) Basic/Proficient: A town purchased 48 acres of land for its new school complex. How many acres of the land were for athletic fields if they occupied 5/6 of the land? (40 acres) Basic/Proficient: If 1/6 of the people in a city of 30,000 people have diabetes, how many people in that city have this disease? (5000) Guide students that need support. Ask students questions about their thinking. **Math Meeting** Bring students together. Have at least three students from each level share their solutions. Use guiding questions and prompts when necessary. **Set 4 Approximating products of mixed numbers** Ask students to create a multiplication problem involving this information. A person weighs 240 pounds and must loose either 1/3 or 1/4 or 1/5 of their weight. Each large cake requires 1 1/8 cups of sugar and several cakes will be needed. Approximate the products by first rounding the mixed numbers to whole numbers. On January 15, it snowed 2 7/8 inches every hour for 5 1/5 hours. The record for that date was 19 inches. Was this a new record for that date? (No) An experiment calls for 8 1/8 ounces of sulfate. If 45 ounces of sulfate are available, is that enough for 5 experiments? (Yes)	

		Math Meeting Bring students together. Have at least three students from each level share their problems, and approximations. Use guiding questions and prompts when necessary **Reflection** What was the most challenging part of problem solving? And why? How does knowing fraction models, equivalent fractions, and simplest form help you to solve problems? **Assessment (Formal or Informal)** Students' solutions strategies.	
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

Multiplication of Fractions and Mixed Numbers

Name: ______________________________________

Set 1: Whole numbers times fractions and mixed numbers

A. Marie poured 12 pitchers of water into her fish tank, and each pitcher held 2 1/3 quarts of water. How much water did she put into her fish tank?

B. Proficient: One lap around the track at the King Elementary School is 1/12 of a mile. If Kristen runs 18 laps, how far has she run?

C. How many miles of tunnel can engineers drill in 6 months, if they can drill ¾ of a mile each month?

Set 2 Fractions times whole numbers

A. On an 18 day vacation, Ruby practiced her guitar and on some days and her harmonica on all of the other days. If she practiced her guitar on 2/3 of the days, on how many days did she spend practicing her harmonica?

B. It is 40 miles from Los Angles to Irvine. If the Garcia family drove 3/5 of the distance to Irvine before getting a flat tire, how far were they from Irvine?

C. It costs $150 to stay at the scout camp for one week. If Elena earned 2/3 of this amount, how much money did she earn for the cost of the camp?

Set 3 Mixed numbers times mixed numbers

A. If a farmer can plow 5 ¼ acres of land in one day, how many acres of land can she plow in 2 2/3 days?

B. What is the area of a rectangular greeting card, if its dimensions are 7 ½ inches by 4 2/5 inches?

C. If a spaceship orbits a planet in 1 2/5 days, how many days will it take to orbit the planet 5 times?

Set 4 Approximating products of mixed numbers

Create a multiplication problem involving this information.

A person weighs 240 pounds and must loose either 1/3 or 1/4 or 1/5 of their weight.

Each large cake requires 1 1/8 cups of sugar and several cakes will be needed.

Approximate the products by first rounding the mixed numbers to whole numbers.

On January 15, it snowed 2 7/8 inches every hour for 5 1/5 hours. The record for that date was 19 inches. Was this a new record for that date?

An experiment calls for 8 1/8 ounces of sulfate. If 45 ounces of sulfate are available, is that enough for 5 experiments?

Standard 5 NF. 6 **Homework Practice**

Name: ______________________________ **Date:** ________________________

1. Shoes are made in 5 different widths. Each width differs by $\frac{1}{12}$ of an inch from the next width. Mark's shoe is 4 widths greater than his sister's. What fraction of an inch greater is the width of Mark's shoe?

2. A leveling screw on a washing machine has 12 threads to the inch. Therefore, each full turn of the screw extends the leg $\frac{1}{12}$ of an inch. How much will the leg be extended for 3 complete turns of the leveling screw?

3. The earth travels around the sun every $365\frac{1}{4}$ days. There are 24 hours in one day. How many hours is $\frac{1}{4}$ of a day?

4. The moon travels around the earth every $27\frac{1}{3}$ days. How many hours is $\frac{1}{3}$ of a day?

5. On Wednesday it rained $\frac{6}{10}$ of an inch. On Thursday it rained only $\frac{1}{3}$ as much as it rained on Wednesday. What fraction of an inch did it rain on Thursday?

6. For each turn of a steel stock in a lathe, $\frac{1}{32}$ of an inch is cut off. What thickness of steel is cut off in 8 turns?

7. The glacier on Mount Blanc in Switzerland moves $\frac{1}{25}$ of a mile each year. How far does it move in $3\frac{3}{4}$ years?

8. In 1897 48 million pounds of blue shad were caught in the ocean between Maine and Florida. The yearly catch is now $\frac{1}{6}$ of the 1897 catch. How many pounds of blue shad are now caught yearly?

Fifth Grade Performance Tasks
Multiplication of Fractions

Name: ________________________

1. In the statement below the word _____ means multiply.
   What is 1/5 of 5/6 ?

    - Multiplication allows you to find a fraction of a fraction.

2. Solve the following problem. Show your solution two ways (numerically and with a model).

James has 3/4 of a pizza. He eats 1/3 of what is left. What fraction of the whole pizza did James just eat?

Visual Model:

Algorithm or equation:

Reasoning in writing:

3. $\frac{7}{9} \div \frac{9}{8}$

4. $\frac{1}{6} \times 4$

5. $13 \times 2/13$

6. $\frac{5}{12} \times 2$

7. What is $\frac{5}{7} of\ 11/12$?

Solve the following problem. Show your solution two ways (numerically and with a model)

8. Mr. Martinez is driving from San Diego to Santa Ana. When he leaves he has 7/8 of a tank of gas. During the drive he uses 3/5 of this gas. What fraction of the whole tank does Mr. Martinez use on his drive?

Visual Model:

Algorithm or equation:

Reasoning in writing:

Fifth Grade Performance Task
Multiplication of Fractions

Student Name: ________________________

	Exceeds (6 points)	**Proficient (4 pts)**	**Below Expectations (3 pts)**
Manipulatives or Visual Model / Concepts	Understands visual concept of the fraction and applies it to the problem.	Sees the fraction in the visual, but cannot apply it to the problem.	Cannot see the visual of the fraction.
Arithemetic / Procedures	Follows mathematical procedure to solve the problem without help.	Follows mathematical procedure with some assistance.	Requires assistance on every step when working the problem.
Mathematical Reasoning/	Student explanation is coherent and logical. Shows understanding of mathematical concept and process. Uses mathematical language correctly. Student expresses insight.	Student explanation is coherent and logical. Shows understanding of mathematical concept and process. Uses some mathematical language correctly.	Requires assistance in performing the task. Cannot explain why procedures are used.

<table>
<tr><td colspan="2">Grade Level/Course
5th Grade</td><td>Duration: 60 min.
Date:</td><td colspan="2">Unit: Multiplication & Division of Fractions
Lesson # 8
Division of Fractions</td></tr>
<tr><td colspan="2">Common Core Standards</td><td colspan="3">5th Grade Number and Operations—Fractions 5NF.7.a
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1
b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.</td></tr>
<tr><td colspan="2">Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: fraction bars, grid paper
Media/Technology to be used to deepen learning: ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division
Supplemental Materials: J. Gregg; D. Gregg, (2007). Mathematics Teaching, pp. 490-496</td></tr>
<tr><td colspan="2">Objectives</td><td colspan="2">Content:
Students will be able to divide whole numbers by unit fractions and relate their division equation to multiplication.</td><td>Language:
Students will be able to illustrate or create a visual, write an equation, and explain the process orally and in writing.</td></tr>
<tr><td colspan="2">Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☐Level 4: Extended Thinking</td></tr>
<tr><td colspan="2">Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td colspan="2">Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="3">Academic Vocabulary (Tier II & Tier III)</td><td rowspan="2">TEACHER PROVIDES SIMPLE EXPLANATION</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td colspan="2">Whole numbers
Unit fractions
Portions
Fractional part</td><td></td></tr>
<tr><td>STUDENTS FIGURE OUT THE MEANING</td><td colspan="2"></td><td></td></tr>
<tr><td colspan="2">Pre-teaching Considerations</td><td colspan="3">Students should be familiar with multiple fraction models, including but not limited to, fraction bars/strips, color tiles, number lines, and circle fractions, dividend ÷ divisor = quotient.</td></tr>
</table>

<table>
<tr><th colspan="4">Lesson Delivery</th></tr>
<tr><td colspan="2">Instructional Methods</td><td colspan="2">Check method(s) used in the lesson:
☐ Modeling ☒ Guided Practice ☒ Collaboration
☒ Independent Practice ☒ Guided Inquiry ☒ Reflection</td></tr>
<tr><td rowspan="2">Lesson Continuum</td><td>Lesson Opening</td><td colspan="2">Prior Knowledge: Students should be familiar with multiple fraction models, including but not limited to, fraction bars/strips, color tiles, number lines, and circle fractions, dividend ÷ divisor = quotient.
Context and Motivation:
Students will be able to divide whole numbers by unit fractions and relate their division equation to multiplication.
Define unit fraction for students by doing a quick sort: is/is not on a tree map: 1/3, 1/7, 1/9, 2/3, 3/7, 5/9. Then ask students what the difference between the two groups is? Guide students to the conclusion that a fraction in which the numerator is 1.
Continue by telling the students the learning objective and you will start with analyzing unit fraction word problems models. Their job is to interpret the models. Then they will solve problems with unit fractions, and finally they will analyze how to check their solutions by relating the multiplication equation to their division equation. They will begin by learning to interpret fractional situations that involve fractions and then make sense of the algorithmic procedures for dividing fractions.
Introduce the idea of serving sizes using couple of labels from packages, noting that the serving size is not always a whole number. For example, a serving size can be 1 ½ cookies. Tell students for the first part of the lesson they just started working for a bakery. They cater desserts by portions or serving sizes. They also tie the desserts into packages using ribbon bows for larger portions and ties for smaller portions. So, they will need to learn about serving sizes by fractions.</td></tr>
<tr><td></td><td>Students will explore interpreting problems using different models. Assign each group a different model. Students “solve” the problem models by passing the chart around the table so that each team member can participate. You will need charts for 3 models (see examples). Make two of each (depending on the number and size of the coop groups). This activity will help students visualize and interpret fractions using different models. Students use the models or drawings to help them solve the problem.

Whole numbers, n divided in groups of unit fraction (a fraction in which the numerator is 1).
What is the _____ (portion/fractional part, size)?
What is the number of groups/pieces?

<u>Math Meeting:</u>
The speaker in each group should share their groups’ solutions. The first three should be easier for students to express; the last three students need to express the division of a fraction into a whole although you have not named division of fractions yet.</td><td></td></tr>
</table>

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	After the discussion present students similar problems, but to move toward a computational algorithm. You will not provide them drawings. Show each problem one at a time. Ask students: How can we view this problem as a division problem? How can you write a division number sentence for each of these problems? Ask students to discuss their thinking with a partner before having a whole group discussion. **<u>Guiding Questions:</u>** For problems three and four clarify the language and interpretation: **How many 7s are in 30? How many 1/2s are in 30? What is the problem question asking? "How many portions/servings/pieces can be made or created?** **1.** A serving is 6 cookies. How many servings can I make from 30 cookies? **2.** A serving is 7 cookies. How many servings can I make from 30 cookies? Students have to figure out what to do with the leftovers. **3.** A serving is ½ cookies. How many servings can I make from 30 cookies? **4.** A serving is ¼ cookies. How many servings can I make from 30 cookies? Numbers 3 and 4: 30 ÷ ¼ -- students may notice • they are multiplying the denominator in the divisor by the whole number • They are figuring out how many portions are in the dividend Have students solve the following problems. They should apply what they have noticed so far about dividing fractions. For each problem they should solve with a drawing and write an equation. They should also express their answer in a complete sentence: Beatrice can frost ___ small cakes. Each small cake takes 1/5 of a cup of frosting. If Beatrice made 8 cups of frosting, how many small cakes can she frost? One of the bakery's clients asked that Benny and Jerry's ice cream be served for a party along with the cupcakes. You bought 6 pints of ice cream. If you serve each guest 1/3 of a pint of ice cream, how many guests can you serve? Finally, place mats need to be made for the tables. You bought 4 yards of material have the placemats made. Each placemat requires 1/6 yards of material. How many placemats will be able to be made from the material?	**Differentiated Instruction:** **English Learners:** It is important that students' visuals are learned through the creation of visual models. **Special Needs:** The lesson has been scaffolded to assist students struggling at the conceptual level. However, some students may need to be talked through the problem solving process. Houghton Mifflin: *Reteach* 11.1 **Accelerated Learners:** Give students different number sets. The fractions could easily be changed to mixed numbers. Houghton Mifflin: *Enrichment* 11.1

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Math Meeting

Ask at least three students to share their solutions.

Guiding Questions:

- What questions arose as you worked?
- What were you thinking when you made decisions or selected strategies to solve the problem? How have you shown your thinking (e.g., picture, model, number, sentence)?
- Which way (e.g., picture, model, number, sentence) best shows what you know?
- How have you used math words to describe your experience?

I decided to use a …

A graph (table, T-chart, picture) shows this the best because …

What is the problem question asking? e.g., "How many portions/servings/pieces can be made or created?"

Have the following whole group discussion about *checking their work and relating division of unit fractions to multiplication*. (Students should be note taking.)Write the students' equations.

$8 \div 1/5 = 40$ because $1/5 \times 40 = 8$

$6 \div 1/3 = 18$ because $1/3 \times 18 = 6$

$4 \div 1/6 = 24$ because $1/6 \times 24 = 4$

Ask students to state the pattern and use it to work the following sequence of open number sentences.

$5 \div \frac{1}{4} = a$ because $\frac{1}{4} \times a = 5$ $a =$

$7 \div 1/3 = a$ because ____________

$2 \div 1/7 = a$ because ____________

Reflection

What have you/we discovered about ___________ while solving this problem?

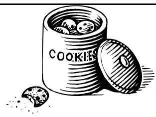
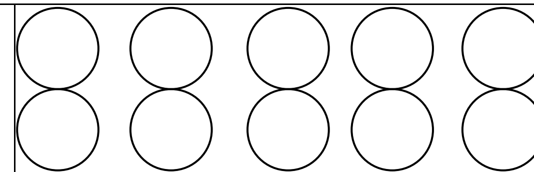
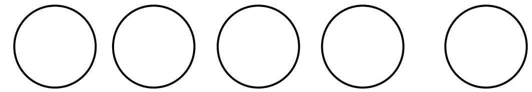
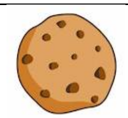
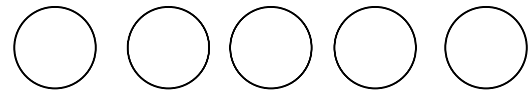
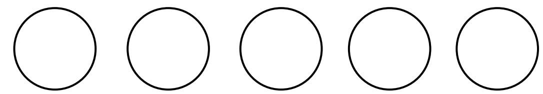
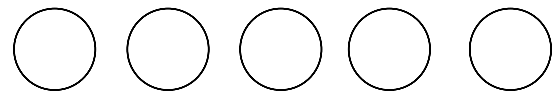
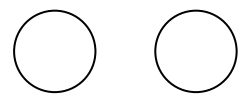
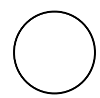
What have you/we learned today?

Assessment (Formal or Informal)

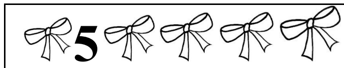
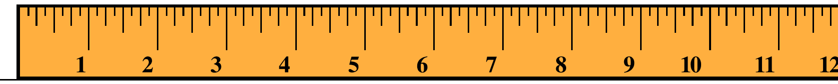
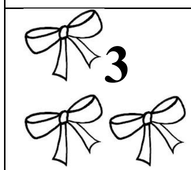
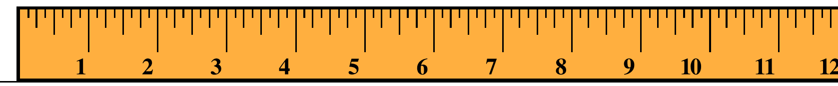
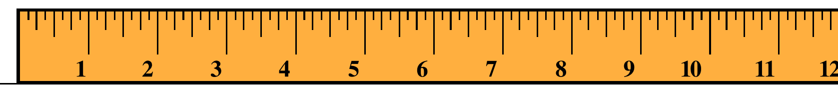
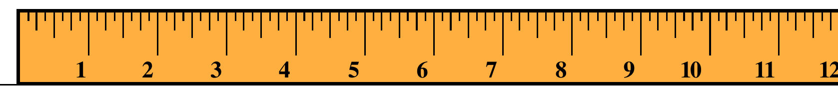
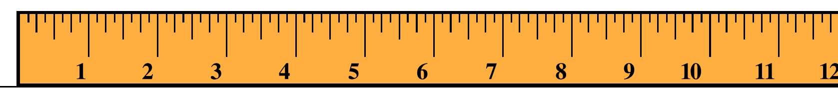
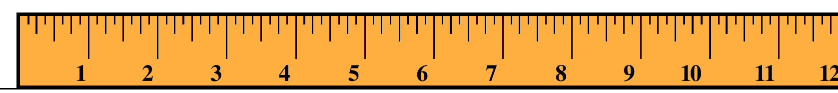
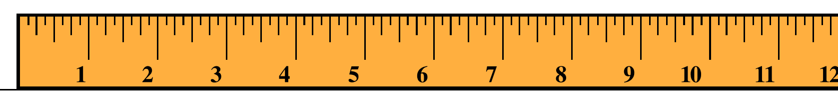
Students' oral presentations

Students' journals and problem solutions

Lesson Reflection	
Teacher Reflection Evidenced by Student Learning/ Outcomes	

 **5**	A serving is 5 cookies. How many servings can I make from 10 cookies?	
 **3**	A serving is 3 cookies. How many servings can I make from 5 cookies?	
 **1**	A serving is 1 cookie. How many servings can I make from 5 cookies?	
 **½**	A serving is ½ cookie. How many servings can I make from 5 cookies?	
 **¼**	A serving is ¼ cookie. How many servings can I make from 5 cookies?	
 **½**	A serving is ½ cookie. How many servings can I make from 2 cookies?	
 **½**	A serving is ½ cookie. How many servings can I make from 1 cookies?	

 5	A serving is 5 brownies. How many servings can I make from 10 brownies?	
 3	A serving is 3 brownies. How many servings can I make from 5 brownies?	
 1	A serving is 1 brownie. How many servings can I make from 5 brownies?	
 ½	A serving is ½ brownie. How many servings can I make from 5 brownies?	
 ¼	A serving is ¼ brownie. How many servings can I make from 5 brownies?	
 ½	A serving is ½ brownie. How many servings can I make from 2 brownies?	
 ½	A serving is ½ brownie. How many servings can I make from 1 brownies?	

	A ribbon tie is 5 inches long. How many ribbon ties can I make from 10 inches of ribbon?	
	A ribbon tie is 3 inches long. How many ribbon ties can I make from 5 inches of ribbon?	
 1	A ribbon tie is 1 inch long. How many ribbon ties can I make from 5 inches of ribbon?	
 ½	A ribbon tie is ½ inch long. How many ribbon ties can I make from 5 inches of ribbon?	
 ¼	A ribbon tie is ¼ inch long. How many ribbon ties can I make from 5 inches of ribbon?	
 ½	A ribbon tie is ½ inch long. How many ribbon ties can I make from 2 inches of ribbon?	
 ½	A ribbon tie is ½ inch long. How many ribbon ties can I make from 5 inches of ribbon?	

<table>
<tr><td>Grade Level/Course
5th Grade</td><td>Duration: 60 min.
Date:</td><td colspan="2">Unit: Multiplication & Division of Fractions
Lesson # 9
Dividing Unit Fractions by Whole Numbers</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">5th Grade Number and Operations—Fractions
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: have accessible fraction bars, gird paper
Media/Technology to be used to deepen learning: ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will be able to divide unit fractions by whole numbers and relate their division equation to multiplication.</td><td>Language:
Students will be able to illustrate or create a visual, write an equation, and explain the process orally and in writing.</td></tr>
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<tr><td rowspan="2">Academic Vocabulary (Tier II & Tier III)
TEACHER PROVIDES SIMPLE EXPLANATION</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td colspan="2">Portion
Container
Fractional part
Divisor
Unit fraction</td><td></td></tr>
</table>

<table>
<tr><td></td><td>STUDENTS FIGURE OUT THE MEANING</td><td>Whole number</td><td></td></tr>
<tr><td colspan="2">Pre-teaching Considerations</td><td colspan="2">Students should be familiar with multiple fraction models, including but not limited to, fraction bars/strips, color tiles, number lines, and circle fractions, dividend ÷ divisor = quotient.</td></tr>
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<tr><td colspan="2">Instructional Methods</td><td colspan="2">Check method(s) used in the lesson:
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☐ Independent Practice ☒ Guided Inquiry ☒ Reflection</td></tr>
<tr><td rowspan="2">Lesson Continuum</td><td>Lesson Opening</td><td colspan="2">Prior Knowledge: Students should be familiar with multiple fraction models, including but not limited to, fraction bars/strips, color tiles, number lines, and circle fractions, dividend ÷ divisor = quotient.
Context and Motivation:
Students will be able to divide unit fractions by whole numbers and relate their division equation to multiplication.

Tell students the learning objective and that you will start with analyzing unit fraction word problems models that are similar but different from the prior lesson.. Their job is to interpret the models. Then they will solve problems with unit fractions, and finally they will analyze how to check their solutions by relating the multiplication equation to their division equation. They will begin by learning to interpret fractional situations that involve fractions and then make sense of the algorithmic procedures for dividing fractions. They should try and the notice the difference from the previous lesson. (The previous lesson's work should have been placed in their journals).

Tell students for the first part of the lesson they working for a bakery. However, they cater desserts by portions or serving sizes. However, they also need to deal with the leftovers after the party is over. How will they divide the leftovers evenly among their clients? So, they will need to learn about dividing a portion into portions. (You might want to chart this situational context for students to read with you).</td></tr>
<tr><td></td><td>Students will explore interpreting the problems using different models. Assign each group a different model. Students “solve” the problem models by passing the chart around the table so that each team member can participate. You will need charts for 3 models (see examples). Make two of each (depending on the number and size of the coop groups). This activity will help students visualize and interpret fractions using different models. Students use the models or drawings to help them solve the problem.

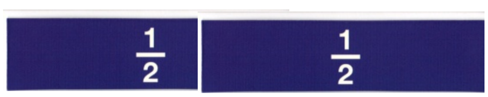
<u>Math Meeting:</u>
The speaker in each group should share their groups’ solutions. (Make sure students place their notes and papers in their math notebooks or journals.) Ask students what they notice about this set of problems.</td><td></td></tr>
</table>

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

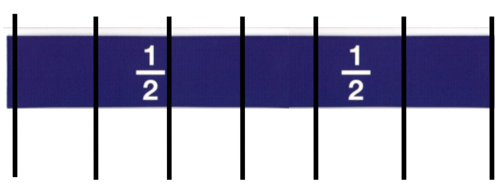
What patterns do they notice? *They should notice that they are not dividing the whole but dividing a unit fraction of the whole. They should also notice that the denominator in the quotient is the multiple of the dividend and the whole number. The whole number is the divisor that tells them how many portions in which they should divide the whole therefore their fractional quotient will be smaller though the denominator is larger. This concept often escapes students. These key insights should be charted for future discussions. If they do not notice these patterns direct their attention to them.*

After the discussion present students the following similar problems, but to move toward a computational algorithm. You will not provide them drawings. The problems are not in a catering contexts. Students will be exposed to different contexts to interpret. Focus on the first problem to scaffold students' thinking. It may help to tell students that we are now using the fair-sharing interpretation of division since we are distributing (sharing) a certain amount of cake among some number of containers and want to know how much cake will be in one container.

1. Three families planned to camp in Yosemite National Forest. They reserved 1/2 acre of camp ground lots. If the families share this land equally, what fraction of an acre will each family have for setting up camp?
Guiding questions:
How can the information in the problem be represented by a visual fraction?



If the shared part of the model representing 1/2 is divided into 3 equal parts, what is the fraction for one of these parts?



Write a division equation to express 1/2 divided into 3 equal parts.
(1/2 ÷ 3 = 1/6)
What might this look like in an algorithm according to what we discovered from the problems we did together?
½÷ 3 → ½ ÷ 3/1 →
3 – ½= 2 ½
2 ½ – ½= 2
2 – ½= 1 ½
1 ½– ½= 1
1- ½ = ½
½ - ½ = 0 → so we subtracted ½ 6 times or 1/6.
OR
½÷ 3 → ½ ÷ 3/1 →

Differentiated Instruction:

English Learners:
It is important that students' visuals are learned through the creation of visual models.

Special Needs:
The lesson has been scaffolded to assist students struggling at the conceptual level. However, some students may need to be talked through the problem solving process.

Accelerated Learners:
Give students different number sets. The fractions could easily be changed to mixed numbers.

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

By this point, some students may figure that they multiply 2 x 3 or the inverse of the divisor.
Have students solve the next two problems. They should apply what they have noticed so far about dividing fractions. For each problem they should **solve with a drawing and write an equation. They should also express their answer in a complete sentence: Each family will have 1/6 of the acre to set up camp.**

Two of the campers went cycling. One has 1/5 gallon of water. If this water is shared equally between the two people, what fraction of a gallon will each person have?

Seven of the campers volunteered to pick up waste along the side of a 1/2-mile stretch of highway coming into the camping village. If each camper cleans one of 7 equal parts of this 1/2-mile highway, what fraction of a mile will each camper be assigned to cleanup?

<u>Math Meeting</u>
Ask at least two students to share their solutions.
Guiding Questions:

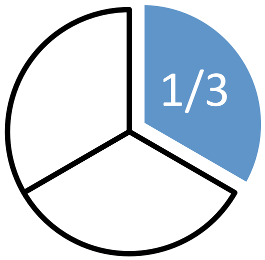
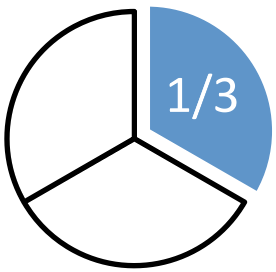
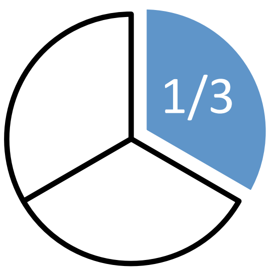
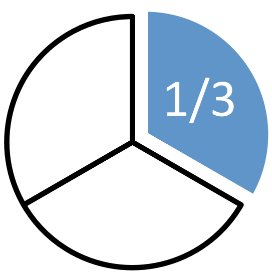
- What questions arose as you worked?
- What were you thinking when you made decisions or selected strategies to solve the problem? How have you shown your thinking (e.g., picture, model, number, sentence)?
- Which way (e.g., picture, model, number, sentence) best shows what you know?
- How have you used math words to describe your experience?

I decided to use a …
A graph (table, T-chart, picture) shows this the best because …
What is the problem question asking? e.g., "What is the portion/fraction of each group?"
Have the following whole group discussion about *checking their work and relating division of unit fractions to multiplication*. (Students should be note taking.)Write the students' equations.
$\frac{1}{2} \div 3 = 1/6$ because $3 \times 1/6 = 3/6$ or $\frac{1}{2}$
$1/5 \div 2 = 1/10$ because $2 \times 1/10 = 2/10$ or $1/5$
$\frac{1}{2} \div 7 = 1/14$ because $7 \times 1/14 = 7/14$ or $\frac{1}{2}$
Ask students to state the pattern and use it to work the following sequence of open number sentences.
$1/3 \div 4 = a$ because $4 \times a = 1/3$ $a = 1/12$
$1/9 \div 2 = a$ because ____________
$1/6 \div 3 = a$ because ____________

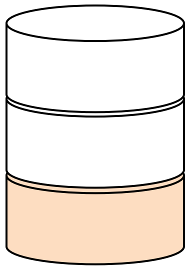
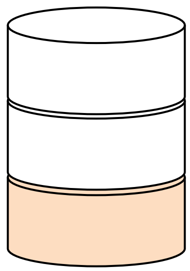
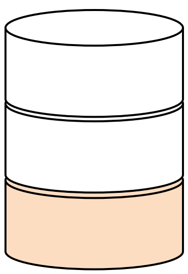
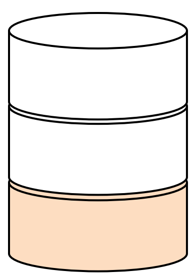
<u>Reflection</u>
What have you/we discovered about unit fractions by whole numbers while solving this problem?
What have you/we learned today?

<u>Assessment (Formal or Informal)</u>
Students' orally presentations
Students' journals and problem solutions

Lesson Reflection	
Teacher Reflection Evidenced by Student Learning/ Outcomes	

	You have 1/3 of a whole cake. You want to divide it equally into 3 containers. How much cake will be in each container?
	You have 1/3 of a whole cake. You want to divide it equally into 4 containers. How much cake will be in each container?
	You have 1/3 of a whole cake. You want to divide it equally into 8 containers. How much cake will be in each container?
	You have 1/3 of a whole cake. You want to divide it equally into 2 containers. How much cake will be in each container?

 	You have 1/3 of a whole brownie pan. You want to divide it equally into 3 containers. How much brownie will be in each container?
 	You have 1/3 of a whole brownie pan. You want to divide it equally into 4 containers. How much brownie will be in each container?
 	You have 1/3 of a whole brownie pan. You want to divide it equally into 8 containers. How much brownie will be in each container?
 	You have 1/3 of a whole brownie pan. You want to divide it equally into 2 containers. How much brownie will be in each container?

	You have 1/3 of a whole iced tea server. You want to divide it equally into 3 servings. How much tea will be poured into each container?
	You have 1/3 of a whole iced tea server. You want to divide it equally into 4 servings. How much tea will be poured into each container?
	You have 1/3 of a whole iced tea server. You want to divide it equally into 8 servings. How much tea will be poured into each container?
	You have 1/3 of a whole iced tea server. You want to divide it equally into 2 servings. How much tea will be poured into each container?

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<tr><td>Grade Level/Course
5th Grade</td><td colspan="2">Duration: 60 min.
Date:</td><td colspan="2">Unit: Multiplication & Division of Fractions
Lesson # 10
Dividing Unit Fractions by Whole Numbers and Whole Numbers by Unit Fractions</td></tr>
<tr><td>Common Core Standards</td><td colspan="4">5th Grade Number and Operations—Fractions 5NF.7.c
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1
c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="4">Mathematical Tools: have accessible fraction bars, gird paper
Media/Technology to be used to deepen learning: ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division</td></tr>
<tr><td>Objectives</td><td colspan="3">Content:
Students will be able to divide unit fractions by whole numbers and whole numbers by unit fractions.</td><td>Language:
Students will be able to illustrate or create a visual, write an equation, and explain the process orally and in writing.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="4">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☐Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="4">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="4">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="2">Academic Vocabulary (Tier II & Tier III)</td><td rowspan="2">TEACHER PROVIDES SIMPLE EXPLANATION</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td colspan="2">Dividend
Divisor
Quotient
Split into</td><td></td></tr>
</table>

<table>
<tr><td></td><td>STUDENTS FIGURE OUT THE MEANING</td><td></td><td></td></tr>
<tr><td colspan="2">Pre-teaching Considerations</td><td colspan="2"></td></tr>
<tr><td colspan="4">Lesson Delivery</td></tr>
<tr><td colspan="2">Instructional Methods</td><td colspan="2">Check method(s) used in the lesson:
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☒ Independent Practice ☐ Guided Inquiry ☒ Reflection</td></tr>
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Context and Motivation:
Review: How many groups/containers are involved? (related to the divisor; could be a fraction)
What is the number portion/fractional part, size? (related to the quotient)
What is the _____ (portion/fractional part, size)? (related to the divisor)
What is the number of groups/pieces? (related to the quotient)

Tell students that we have been learning how to divide fractions in real situations. Ask students for more ideas of how division of fractions is used in daily life. Give students some think-pair-share time. Chart way situations students share out. Tell them they will use their ideas in tomorrow's lesson.

Today they will apply what they have learned to a variety of division of fractions problems. They will receive several problems to solve. They should try and solve all four. All students may not be able to complete all the tasks in time. Differentiate the tasks' expectations. Students who are struggling should at least complete 2 of the problems completely. More advanced students should complete all four.</td></tr>
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<tr>
<td></td>
<td>Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding</td>
<td>

<u>Lesson Delivery:</u>
Tell students that they should solve each problem two ways: with a visual model and numerically. They should also explain their reasoning in writing. Review the task rubric with students before you distribute the problems. While students are working observe students' uses of visual models and if they decompose fractions or use a traditional equation or algorithm. Note which student work will make for a good example for students to learn through and ask those students if they would be willing to share.

Problem 1: A relay race that is 1/3 mile will be run by 4 fifth graders. How far will each person run if their distances are equal?

Problem 2: Ten bananas were used for making pies for a bake sale. If 2½ bananas were used for each pie, how many pies were made?

Problem 3: Josie is making tomato sauce for pizza. Her recipe calls for 2/4 cup of tomato paste. The recipe makes enough for 6 pizzas. How much tomato paste is on each pizza?

Problem 4: You need $25 to buy a new scooter and you receive 1/4 dollar each week for washing the floor. How many weeks will it take to earn enough money to buy the scooter?

<u>English Learners</u>
First, I drew _________.....Then I drew …….in order to _______ into _____. Next, I ----.
Word bank: divide, spilt into, each person will…, dividend, divisor, quotient, the number of pies __,
The number of weeks, the amount of tomato paste, for/on each ___.

<u>Math Meeting</u>:
Ask at least one student to share his/her problem solutions. After each student, ask students if anyone had solved the problem differently or with a different strategy or model. Note students' numerical representations.

<u>Reflection</u>
Journal question:
Which problem was more difficult to solve or represent? Why?
Which problem was more fun to solve? Why?
Did the rubric help you think about the quality of your work?

<u>Assessment (Formal or Informal)</u>
The problem set can be used as part of your formal summative assessment.
</td>
<td>
Differentiated Instruction:

English Learners:
Provide these learners with sentence frames for the written portion of the exercise.
A vocabulary bank will also help these of students.

Special Needs:
The vocabulary bank would be helpful for struggling students. Teacher may also orally rehearse with these students what they might write. The teacher may expect these students to get through problems 1 and 2.

Accelerated Learners:
Give these students the option of a challenge problem:
In summer you can earn $2 ½ a day cutting grass. How many days will it take to earn $60?
</td>
</tr>
<tr>
<td colspan="4">Lesson Reflection</td>
</tr>
<tr>
<td colspan="2">Teacher Reflection Evidenced by Student Learning/ Outcomes</td>
<td colspan="2"></td>
</tr>
</table>

DIVIDING FRACTIONS

Name: ______________________

Solve the problem using a visual model and numerically. Then explain your reasoning.

Problem 1: A relay race that is 1/3 mile will be run by 4 fifth graders. How far will each person run if their distances are equal?

Visual Model:

Problem 2: Ten bananas were used for making pies for a bake sale. If 2½ bananas were used for each pie, how many pies were made?

Visual Model:

Name: ______________________

Problem 3: Josie is making tomato sauce for pizza. Her recipe calls for 2/4 cup of tomato paste. The recipe makes enough for 6 pizzas. How much tomato paste is on each pizza?
Visual Model:

____
____
____
____
____
____
____

Problem 4: You need $25 to buy a new scooter and you receive 1/4 dollar each week for washing the floor. How many weeks will it take to earn enough money to buy the scooter?
Visual Model:

____
____
____
____
____
____
____

Math Task Rubric

Name: ____________________________

Problem 1: ______ Problem 2: ______ Problem 3: _______ Problem 4: ________

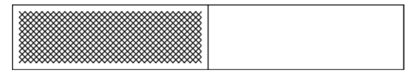
	Needs Improvement	Approaches Proficiency	Demonstrates Proficiency	Exemplary Distinction
Mathematics Skills	Little or no success with the mathematics skill. No workable solution is provided.	Part of the task is correct however gaps in skill and/or understanding are apparent.	Demonstrates solid execution of mathematical skill presenting a solution, which is correct and complete.	Work demonstrates rigorous mathematical skills and mastery that exceeds expectations.
Conceptual Understanding	Very little understanding of the mathematical concepts involved and/or misunderstood the task.	Some understanding of the relevant concepts is demonstrated.	Demonstrates knowledge of the mathematical concepts involved.	Work shows precise and thorough use of the mathematical concepts critical to successful completion of the task. Special insights or other exceptional qualities are included.
Mathematical Practice	Shows little or no progress toward demonstrating the mathematical practice.	Includes incomplete responses that demonstrate mathematics progress toward the mathematical practice.	Work demonstrates solid mathematical thinking and the ability to successfully use the mathematical practice.	Shows in-depth understanding of essential mathematical practice and eloquence or insight in the explanations of the practice.
Communication	Writing is confusing or absent.	There is some confusion in the writing and/or charts, diagrams. Mathematics is not clearly explained.	Addresses all processes and components of the task. Explanations are reasonable and clear to the audience.	Writes a comprehensive, compelling, and thoughtful solution. Diagrams are illuminating. Every component of the product is obvious to the audience.

NCSM Great Tasks for Mathematics

More Practice for Dividing Fractions 5.NF.7

SET 1

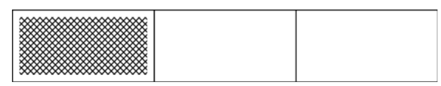
1. If 1/2 of a storage locker is available and will be shared equally by 3 students, then each student will have what fractional part of the storage locker?



available amount of storage

2. Courtney has 2 cups of orange juice and a batch of orange muffins takes 1/4 cup. How many batches of orange muffins can be made?

3. If 1/3 gallon of paint is available to paint 2 chairs, and each chair takes the same amount of paint, what fraction of a gallon of paint will be used for each chair?



available amount of paint

4. If 4 ounces of potassium are ordered for a crystal growing experiment, and each experiment requires 1/2 ounce, how many experiments can be carried out?

SET 2

1. Sounds travels 1/5 of a mile in 1 second. How many seconds will it take to travel 2 miles?

2. Each batch of popcorn takes 1/4 of a pound of butter. How many batches can be made from 3 pounds of butter?

3. If a glacier moves 1/8 of a mile in one year, how far will it move in 20 years?

4. Kelsey has 4 pounds of cheese and wants slices that weigh 1/10 of a pound. How many slices can be obtained?

Grade Level/Course 5th Grade	**Duration:** 60 min. **Date:**	**Unit: Multiplication & Division of Fractions** **Lesson # 11** Dividing Unit Fractions and Whole Numbers
Common Core Standards	5th Grade Number and Operations—Fractions 5NF.7.a, b **7**. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1 a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) =	
Materials/ Resources/ Lesson Preparation	**Mathematical Tools:** fraction bars, grid paper **Media/Technology to be used to deepen learning:** ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division	
Objectives	**Content:** Students will be able to divide unit fractions by whole numbers and whole numbers by unit fractions.	**Language:** Students will be able to illustrate or create a visual, write story context for an expression and explain their solution orally.
Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☒ **Level 4: Extended Thinking**	
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	

Academic Vocabulary (Tier II & Tier III)		KEY WORDS ESSENTIAL TO UNDERSTANDING	WORDS WORTH KNOWING
	TEACHER PROVIDES SIMPLE EXPLANATION		
	STUDENTS FIGURE OUT THE MEANING		

Pre-teaching Considerations	Students should be familiar with multiple fraction models, including but not limited to, fraction bars/strips, color tiles, number lines, and circle fractions, dividend ÷ divisor = quotient

Lesson Delivery

Instructional Methods	**Check method(s) used in the lesson:** ☐ **Modeling** ☒ **Guided Practice** ☒ **Collaboration** ☒ **Independent Practice** ☐ **Guided Inquiry** ☒ **Reflection**

Lesson Continuum

Lesson Opening

Prior Knowledge: Students should be familiar with multiple fraction models, including but not limited to, fraction bars/strips, color tiles, number lines, and circle fractions, dividend ÷ divisor = quotient.

Context and Motivation:

Review: How many groups/containers are involved? (related to the divisor; could be a fraction)
What is the number portion/fractional part, size? (related to the quotient)
What is the ______ (portion/fractional part, size)? (related to the divisor)
What is the number of groups/pieces? (related to the quotient)
Chart of possible language to be used for this lesson—see context/motivation

Tell students: **Yesterday we charted some ideas for how we use fractions in our daily lives. Today's question is: How can division of fractions be used in real life situations? Today you will create your own division with fractions math story problems. You can use those ideas to write your own story.**
You should write the story and then solve the problem with a visual model and numerically. We will look at a story problem structure to give you a pattern to follow. I will provide the expression and you will build your story around the expression.

Review the rubric with students before you do one together.
Let's do one together. The expression is 12 ÷ 1/4.
Now refer to the tree map chart that was prepared before class without the writing and build the following story. Write in the story context as in the example.

Create your own story context

Who (is in the problem)?	What (is the problem about)?	Which (amounts, numbers, portions, shares, fractions will be used)?	What is the unknown or what do you need to solve for?
Bella	Graphic novels	12 graphic novels, 1/4 read	How many has she read?

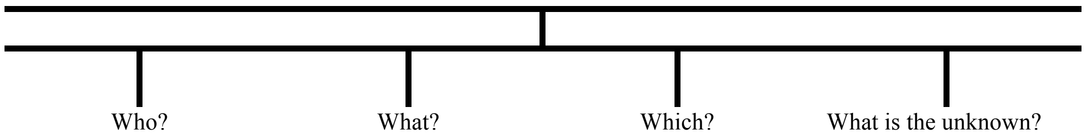
Then write a complete story: **Bella loves to read graphic novels. She got 12 graphic novels for her birthday. She has read 1/4 of the books. How many of the graphic novels has she read so far?**

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding		Differentiated Instruction:
		Tell students: **Use some of our ideas for how we use fractions in our daily lives.** **Write one on your own. When you and a work partner have each finished your own story problems, read them to each other to make sure they make sense. If you are unsure ask me.** Guide students through the process of writing the first problem with the tree map. This time they choose their own who, what, which and, the unknown, and then write their story. Problem 1: 1/6 ÷ 4 Problem 2: 6 ÷ 1/7 Problem 3: ½ ÷ 7 Problem 4: 8 ÷ ¼ **<u>English Learners</u>** Word bank: divide, spilt into, each person will…, dividend, divisor, quotient, the number of __ __, The number or weeks, the amount of _______, for/on each ___. **<u>Math Meeting</u>** Ask at least one student to share his/her problem solutions of each problem. Note students' numerical representations and models. Compare them with other students' who had similar structures. It is also a good time to check for accurate computation. **<u>Reflection</u>** What was difficult about writing story problems? What was easy about writing story problems? **<u>Assessment (Formal or Informal)</u>** Student's completed stories. Use the rubric to measure student learning for at least one of the problems. Notice which of the students needed teacher support or peer support. They may need more practice creating stories.	**English Learners:** Students work with a partner. If necessary, provide these learners with sentence frames for the written portion of the exercise. A vocabulary bank and the tree map will also help this group of students. If necessary have these students draw a tree map for each story. **Special Needs:** Students work with a partner. Students may need similar support as English Learners. This group of students may not be able to complete all four exercises. Make sure they complete at least two. **Accelerated Learners:** Challenge students to use mixed numbers.
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

DIVIDING FRACTIONS

Name: ____________________

Write a story for the expression and solve your problem using a visual model and numerically. Then explain you reasoning.



Problem 1: 1/6 ÷ 4

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Visual Model:

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Problem 2: 6 ÷ 1/7

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____

Visual Model:

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5NF.7.a, b

DIVIDING FRACTIONS

Name: ____________________

Write a story for the expression and solve your problem using a visual model and numerically. Then explain your reasoning.

Problem 3: $\frac{1}{2} \div 7$

____

____

Visual Model:

____
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Problem 4: $8 \div \frac{1}{4}$

____

____

Visual Model:

____
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____
____
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____

5NF.7.a, b

Name:_______________

Writing Math Word Problems				
	Advanced 3 pts	**Proficient 2 pts**	**Basic 1 pts**	**Strategic 0 pts**
Content	Advanced Appropriate content is used for each word problem. Student clearly understands the mathematical concepts.	Proficient Appropriate content is used for each word problem. Student shows some understanding of the mathematical concepts.	Basic Appropriate content may be used. Student shows little understanding of the mathematical concepts.	Strategic Appropriate content is not observed. Student does not demonstrate an understanding of the mathematical concepts.
Organization	Advanced The word problem is written in clear and coherent language. The word problem includes a correct answer key that is neat and legible.	Proficient The word problem is written in clear and coherent language. The word problem includes an answer key.	Basic The word problem is not written in clear and coherent language. The word problem may or may not include an answer key.	Strategic The word problem is not written in clear and coherent language, or may not be observed. The word problem does not include an answer key.
Mechanics	Advanced Mathematical language, capitalization and punctuation are present with no mistakes.	Proficient Mathematical language, capitalization and punctuation are present with no more than two mistakes.	Basic Mathematical language, capitalization and punctuation may be used, but more than two mistakes.	Strategic Mathematical language, capitalization and punctuation are not observed.
Visual Model	Advanced Visual model clearly represents the topic of the problem.	Proficient Visual model somewhat represents the topic of the problem.	Basic Visual model attempts to represent the problem.	Strategic Visual model is not observed.

Grade Level/Course 5th Grade	**Duration:** 60 min. **Date:**	**Unit: Multiplication & Division of Fractions** **Lesson # 12** Culminating Task and Final Assessment
Common Core Standards	5th Grade Number and Operations—Fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions, mixed numbers, e.g., by using visual fraction models or equations to represent the problem. 4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15.(In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. 5. Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n\ b)$ to the effect of multiplying a/b by 1. 6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. 7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1 a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?	
Materials/ Resources/ Lesson Preparation	**Mathematical Tools:** fraction bars, grid paper **Media/Technology to be used to deepen learning:** ST Math Fraction Concepts; Fraction Concepts L1; Fractions Multiplication, Fraction Division	

Objectives		**Content:** Students will be able to apply their understanding of operations on fractions to adjust a recipe.	**Language:** Students will be able to illustrate or create a visual, write an equation, and explain the process orally and in writing.
Depth of Knowledge Level		☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☒**Level 4: Extended Thinking**	
Standards for Mathematical Practice		☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics		☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	
Academic Vocabulary (Tier II & Tier III)		**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**
	TEACHER PROVIDES SIMPLE EXPLANATION	Recipe Adjust	Servings cream Ingredients Teaspoonfuls
	STUDENTS FIGURE OUT THE MEANING	Half Twice	Stir blend
Pre-teaching Considerations		Students should have had multiple experiences with fractions addition and subtraction, multiplication and division of fractions by whole numbers and whole numbers by fractions similar to problems in previous tasks.	
Lesson Delivery			
Instructional Methods		**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**	
Lesson Continuum	**Lesson Opening**	**Prior Knowledge:** Students should have had multiple experiences with fractions addition and subtraction, multiplication and division of fractions by whole numbers and whole numbers by fractions similar to problems in previous tasks. **Context and Motivation:** Review: how to read a recipe. Half of a _____(recipe) Makes ___ servings (cookies) Twice as many Three times as many	

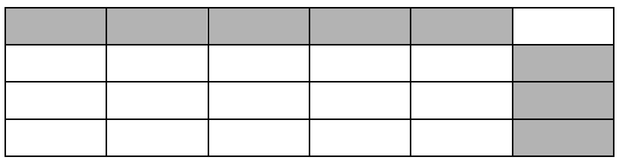
		Students could work in pairs or small groups. Introduce the problem and be sure everyone is clear with the context. Facilitate a preliminary discussion with the class to make sure students understand all vocabulary as well as the context of the problem before students get to work. After allowing students to share their initial thoughts, ask them to work in pairs or individually to investigate the following: How would you rewrite the recipe for 120 cookies? How would you rewrite the recipe for half as many cookies? Challenge: Is it possible to adjust the recipe for 60 servings?	
	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	**The Recipe Task:** Explain how you would adjust the recipe to serve a family of 6 so that each family member gets one cookie. Explain how you would adjust the recipe to serve a class of 30 so that each student receives 1 cookie or as close as possible. Notice some ways students may be confused: Students who when working on halving the recipe, divide by ½ rather than by 2. Use some guiding questions. **Guiding Questions:** How can you tell that your answer is correct? Does dividing by 2 (or ½) help solve this problem? How do you know? Did you develop a strategy to find your answers? Did you identify any patterns or rules? Explain. **Math Meeting:** Choose a few students to share their recipe adjustments. Did anyone use estimation? **Reflection** How did modeling help you make sense of the problem? Did you use equivalent fractions? How? Did you make any connections between the multiplication and division of fractions? **Formal Assessment** Students will take the End of Unit Fraction Test	**Differentiated Instruction:** **English Learners:** Visuals and graphics Math manipulatives are available. Task is completed with a partner. **Special Needs:** Students may be required to complete only one part of the task. **Accelerated Learners:** Challenge: Is it possible to adjust the recipe for 60 servings?
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

Fifth Grade End of Unit Fraction Test

Name ______________________________________

Work each problem in the space provided. Circle the correct answer for each problem

1. Use the area model below to answer the question. Which expression is shown?



a. ½ X ¾ =
b. 2/3 X 5/8 =
c. ¾ X 4/5 =
d. ¾ X 5/6 =

2. 3 X 2/3 =

a. 2/9
b. 1/3
c. 1 1/3
d. 2

3. 2/5 X 10 =

a. 4
b. 2 2/5
c. 2
d. 1/25

4. 4/7 X 3/8 =

a. 3/14
b. 7/15
c. 3/5
d. 4/5

5. 6/7 X 2/3 =

a. 4/15
b. 4/7
c. 4/5
d. 1 1/5

6. 1 4/5 X 1 1/6

a. 2 5/11
b. 2 1/10
c. 1 5/11
d. 1 2/15

Fifth Grade End of Unit Fraction Test, page 2

Name ____

7. 1 1/8 X 2 2/3 a. 3 1/8 b. 3 c. 2 1/8 d. 2 1/12	8. The Franklins had ¾ gallon of milk . They used ½ of the milk they had for breakfast. How much milk was used for breakfast? a. 1/3 gallon b. 3/8 gallon c. 3/7 gallon d. 2/3 gallon
9. Kenesha has read 4/5 of a book. She read 2/3 of that amount while at school. How much of the book has she read at school? a. 1/5 b. 1/3 c. 8/15 d. ¾	10. Hana had a rope that was 2/3 yard long. She used ½ of it. How much rope did she use? a. 3/5 yard b. 3/7 yard c. 2/5 yard d. 1/3 yard
11. While walking, Ella averages 3 ½ miles per hour. At that speed, how many miles could she go in 1 2/7 hours? a. 1 4/9 miles b. 3 1/3 miles c. 4 ½ miles d. 5 miles	12. How many fourths are in 6? a. 24 b. 4 c. 2 ½ d. 1 ½

Fifth Grade End of Unit Fraction Test, page 3

Name _______________________________________

13. How many halves are in 3? a. 6 b. 5 c. 4 d. 2	14. 3/5 ÷ 6 = a. 1/10 b. 1 4/5 c. 3 3/5 d. 10
15. 7/8 ÷ 3 = a. 7/24 b. 8/21 c. 3 7/8 d. 4	16. Cora is making casseroles. She needs 2/3 cup of corn for each casserole. How many casseroles can she make if she has 10 cups of corn? a. 1/15 b. 4 c. 6 2/3 d. 15
17. Kay has 4 meters of ribbon. She wants to make bows that use 4/5 meter of ribbon each. How many bows can she make? a. 1/5 b. 1 3/5 c. 1 ¾ d. 5	18. 9/10 ÷ ¾ = a. 27/40 b. 1 1/6 c. 1 1/5 d. 1 3/10

Fifth Grade End of Unit Fraction Test, page 4

Name ____________________________________

19. 5/6 ÷5/11 = a. 1/66 b. 25/66 c. 6/11 d. 1 5/6	20. 5/7 ÷ ½ a. ¾ b. 13/17 c. 1 3/7 d. 1 ¾
21. Which of the following is equal to ½ ÷ 7/8? a. 2/1 X 8/7 b. 7/8 X ½ c. ½ X 7/8 d. ½ X 8/7	22. 2 2/3 ÷ 8/9 = a. 1/3 b. 17/24 c. 2 10/28 d. 3
23. Janet just mulched her yard and had 2 ¼ bags of mulch left. She divided it evenly and gave 3/8 of a bag to each of the people on her block. How many people live on Janet's block? a. 3 b. 6 c. 8 d. 27	24. Aretha has 3 ½ bags of nuts for her party. She has invited 14 people to her party. How many nuts can she give to each person at her party? a. ½ bag b. 3/8 bag c. ¼ bag d. 1/8 bag

Fifth Grade End of Unit Fraction Test

Answer Key

1. d ¾ X 5/6
2. d 2
3. a 4
4. a 3/14
5. b 4/7
6. b 2 1/10
7. b 3
8. b 3/8 gallon
9. c 8/15
10. d 1/3
11. c 4 ½
12. a 24
13. a 6
14. a 1/10
15. a 7/24
16. d 15
17. d 5
18. c 1 1/5
19. d 1 5/6
20. c 1 3/7
21. d ½ X 8/7
22. d 3
23. b 6
24. c ¼ bag

Name: ___________________________

Making Sugar Cookies

(Makes 12)

Ingredients:

2/3 cup flour
¼ teaspoon baking soda
1/8 teaspoon baking powder
¼ cup butter, softened
¾ cup white sugar
1 small egg
¼ teaspoon vanilla extract

Directions:

1. In a small bowl, stir together flour, baking soda, and baking powder. Set aside.
2. In a large bowl, cream together the butter and sugar until smooth. Beat in egg and vanilla.
3. Gradually blend in the dry ingredients.
4. Roll rounded teaspoonfuls of dough into balls, and place onto ungreased cookie sheets. Bake 8 to 10 minutes in the preheated oven, or until golden.
5. Let stand on cookie sheet two minutes before removing to cool on wire racks.

Recipe adapted from http://allrecipes.com/recipe/easy-sugar-cookies/

The Recipe Task:

1. Explain how you would adjust the recipe to serve a family of 6 so that each family member gets one cookie.
2. Explain how you would adjust the recipe to serve a class of 30 so that each student receives 1 cookie or as close as possible

Complete your tasks on grid paper. Then, rewrite the recipes as a real recipe on a recipe card.

Recipe: ______________________________

From the Kitchen of: ________________________

www.hooverwebdesign.com

Recipe

Dish: ________________ Serves: ______________

Math Menu

5th Grade Common Core Mathematics – Multiplying and Dividing Fractions

Math Menu Centers are provided so that teachers can take time to meet with students who may need extra attention.

Starting Menu Activities

If you set up your choices at stations, list the materials students will find at each station. Students can keep track of their choices on their own choice lists. You can add or replace any of the choice activities with other related ones. (http://www.math-play.com/Fractions-Jeopardy/fractions-jeopardy.html)

Make copies of game directions available or simply post each sheet. Students may refer to the directions when in doubt about the rules of the game.

During Choice Time, circulate among the groups and observe students as they are involved with an activity, or use the time to meet with small groups of students who are having difficulty with a particular activity.
Some things you might look for are the following:

- How are students making decisions about choosing an activity and organizing their time and materials?
- Are there too many or not enough activities going on at once?
- Are students keeping track of the choices they have completed?

Introduce the following stations after Lessons 1-4.

Houghton Mifflin Math Centers Chapter 10 pp. 208 C

- Working in Circles
- Measurement Matters
- Mixed Fun

Multiplying with Rectangles (adapted from M. Burns)

Introduce After Lesson 8

The Multiplying Game (adapted from M. Burns)

Houghton Mifflin Math Centers Chapter 11 pp. 226 C

- Fraction Fix Up
- Fruitful Fractions
- Mixed Fractions

MATH MENU ACTIVITIES

Name: ___________________________

<u>Menu Activities after Lesson 4</u>

- ☐ **Choice 1: Working in Circles**
- ☐ **Choice 2: Measurement Matters**
- ☐ **Choice 3: Mixed Matters**
- ☐ **Choice 4: Multiplying with Rectangles**

<u>Menu Activities after Lesson 8</u>

- ☐ **Choice 5: The Multiplying Game**
- ☐ **Choice 6: Fraction Fix Up**
- ☐ **Choice 7: Fruitful Fractions**
- ☐ **Choice 8: Mixed Fractions**
- ☐ _______________________________
- ☐ _______________________________

Multiplying with Rectangles

① $\frac{1}{2} \times \frac{1}{3} =$

② $\frac{1}{2} \times \frac{2}{3} =$

③ $\frac{1}{2} \times \frac{5}{8} =$

④ $\frac{1}{3} \times \frac{1}{3} =$

⑤ $\frac{1}{3} \times \frac{2}{3} =$

⑥ $\frac{1}{3} \times \frac{5}{8} =$

⑦ $\frac{3}{4} \times \frac{1}{3} =$

⑧ $\frac{3}{4} \times \frac{2}{3} =$

⑨ $\frac{3}{4} \times \frac{5}{8} =$

⑩ $\frac{2}{3} \times \frac{1}{3} =$

⑪ $\frac{2}{3} \times \frac{2}{3} =$

⑫ $\frac{2}{3} \times \frac{5}{8} =$

The Multiplying Game

You need:
- a partner
- a die

Rules

1. You need a game board with three rounds like this.

$$\frac{\square}{\square} \times \frac{\square}{\square} = \qquad \frac{\square}{\square} \times \frac{\square}{\square} =$$

$\square$ $\qquad\qquad\qquad$ $\square$

2. Players take turns rolling the die and writing the number in one of their spaces for that round. Once a number is written, it cannot be changed. The boxes to the side are reject boxes that give one chance to write a number that you don't want to use in the problem.

3. After writing a number, pass the die to the other player.

4. Play until both players have recorded two fractions. (Your reject box may be empty if you used your first four numbers for the fractions.)

5. Multiply your two fractions. Check each other's answers.

6. The winner of the round is the player with the smaller product. Explain how you know which answer is smaller.

7. Play three rounds.

Unit Resources

Burns, M. (2003). *Lessons for Multiplying and Dividing Fractions*. Math Solutions, Scholastic.

Gregg, J.; Gregg, D. (2007). *Mathematics Teaching*, pp. 490-496

Houghton Mifflin (2009). California Math, 5th Grade

Accountability and Curriculum Reform Effort in Response to a *Framework for Change* (2010)

NCSM Great Tasks for Mathematics

Council of Great City Schools