Math Circles Library

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In November 2007 a group of 19 US mathematicians went to Russia on a 5-day trip to study their math circles. One of the many revelations from that trip has been finding their extensive library of books specifically aimed at participants and leaders of math circles. (Incidentally, a vast majority of the texts is available on line – if you read Russian you would enjoy checking http://www.mccme.ru/free-books/)

It was that library that inspired creation of a new series of books published by AMS together with the Mathematical Science Research Institute (MSRI):

MSRI Mathematical Circles Library

THE PRECURSOR TO THE MSRI-MCL:

this book was published in 1996 – before the advent of the MSRI-MCL or even creation of most US math circles. It remains a valuable and cherished resource for many circles.

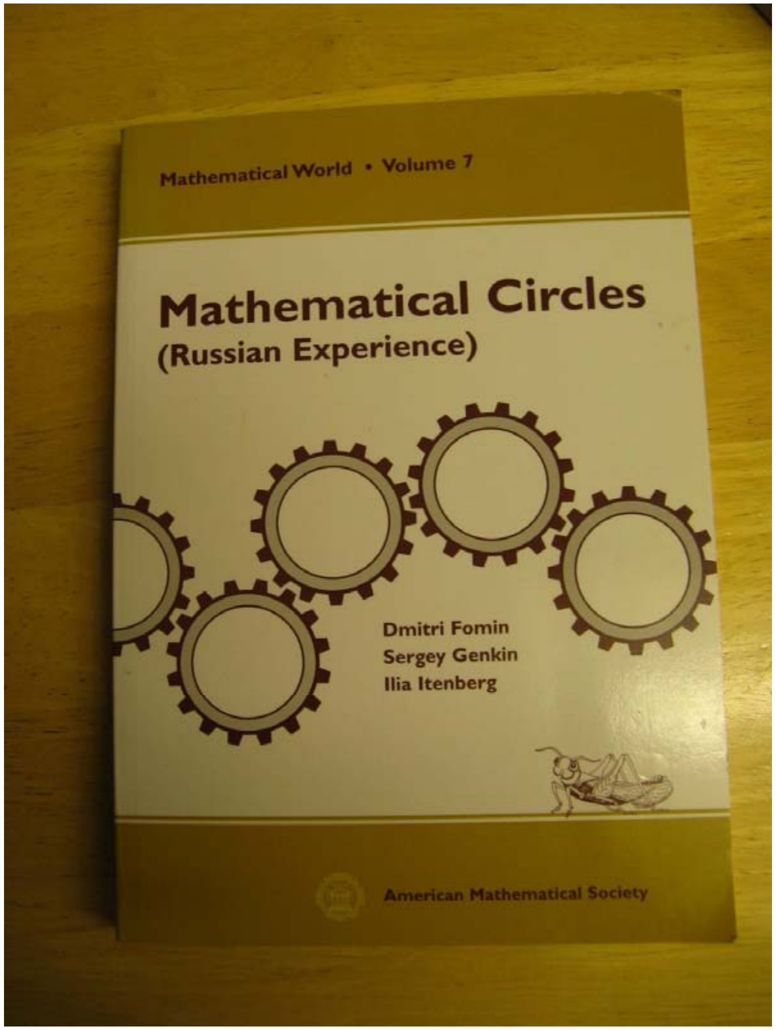


Table of Contents of *Mathematical Circles (Russian Experience)*:

the book is organized in two parts; each part provides materials suitable for a year-long math circle.

- *Year one*

- *Year two*

  Induction

  Divisibility: Congruences and Diophantine Eqs

  Combinatorics – 2

  Invariants

  Graphs – 2

  Geometry

  Number Bases

  Inequalities

  Problems for the second Year

As stated in the preface, this book is written for (listed in order chosen by the authors):

- School teachers
- University professors participating in mathematical education programs
- Various enthusiasts running mathematical circles
- People who just wanted to read something both mathematical and recreational
- And certainly, students can also use this book independently

The list on the previous slide is significant – it reflects one of the most important features of math circles –

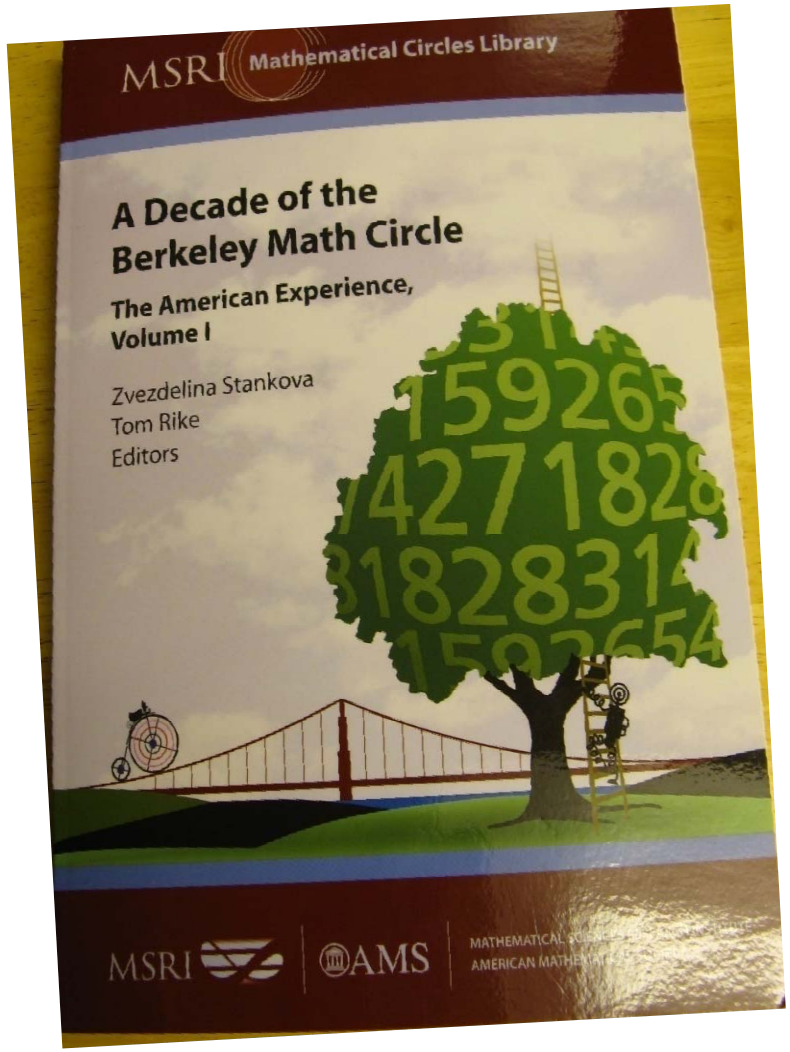
VIRTICAL INTEGRATION :

Math Circle bring together precollege, undergraduate and graduate students, school teachers, college professors, research mathematicians, and all other people who like mathematics and enjoy deep thinking.

The first two books of the MSRI- MCL:

- Circle in a Box, *Sam Vandervelde*
- A Decade of the Berkeley Math Circle. The American Experience, Volume 1, *Edited by Zvezdelina Stankova and Tom Rike* (a.k.a. the BMC-1 book)

BMC-1 BOOK:


MSRI Mathematical Circles Library
A Decade of the Berkeley Math Circle
The American Experience, Volume I
Zvezdelina Stankova
Tom Rike
Editors
MSRI
AMS

BMC-1 book Table of Contents:

According to the Editors, BMC-1 book is written for (in this order):

- The middle and high school student
- The middle and high school teacher who wishes to start a math circle
- The parents of a middle/high school student

Forthcoming Books:

1. Children and Mathematics, *Alexander Zvonkin, Paul Zeitz, translation editor*
2. Berkeley Math Circle, v.2
3. Moscow Math Olympiads, 1993-2005, in two volumes
4. Moscow Math Circle Curriculum in Day-by-Day Sets of Problems, *Sergei Dorichenko*
5. Introduction to Functional Equations and Inequalities, *Costas Efthimiou*
6. Geometry, A Guide for Teachers and Teachers of Teachers, *Paul Sally and Judith Sally*

While both *Mathematical Circles (Russian Experience)* and *BMC books* are organized by topics, one of the forthcoming books, Sergei Dorichenko's *Moscow Math Circle Curriculum in Day-by-Day Sets of Problems* has a distinctly different structure.

As suggested by the title, it consists (mostly) of transcriptions of a year-long math circle meetings for 7-grade Moscow students.

At the end of each meeting, students are given a list of problems; they are expected to work on the problems at home on their own. At the next meeting, they present their solutions either for the entire class, or more typically, talking individually with an instructor.

Example of a Chapter in Dorichenko's Book: Meeting 8

1. Which number is bigger,

   1+2+4+8+16+32+64+128+256+512+1024 or 2048? By how much?

2. Each day at noon, a mail steamer leaves Savannah for Belfast, while a steamer of the same line leaves Belfast for Savannah. Each steamer spends exactly 7 (24-hour) days at sea, and travels along the same route. How many steamers of the same company will the Belfast-Savannah steamer meet while underway?

3. The length of one side of a triangle is 3.8 inches, the length of another 0.6 inches. Find the length of the third side, if it is known that it is a whole number of inches.

4. Simplify the fraction:

$$\frac{1\cdot 2\cdot 3+2\cdot 4\cdot 6+4\cdot 8\cdot 12+7\cdot 14\cdot 21}{1\cdot 3\cdot 5+2\cdot 6\cdot 10+4\cdot 12\cdot 20+7\cdot 21\cdot 35}$$

5. Using a pencil, ruler, and sheet of graph paper, how can you draw a square with area (a) double and (b) 5 times larger than the area of one square?

6. Which is greater: the sum of the lengths of the sides of a quadrilateral or the sum of the lengths of its diagonals?

7. The menu in a school cafeteria constantly has 10 different options. To change up his meal options, George decides to buy a different meal for every lunch (he can eat anywhere between 0 and 10 different meals for lunch). (a) How many days can he eat for without repeating a meal? (b) How many total meals will he eat in that time?

8. Will it be possible to write more than 50 different two-digit numbers on a blackboard so that there won't be two numbers on the board that total 100?

ADDITIONAL PROBLEMS

9. Prove that the region of the right pentagonal star between two horizontal lines in the diagram below is exactly half of the total area.

10. Find the sum 6+66+666+6666+66666+-…+6666..[100 digits].

11. Is it possible to find a number of the form 11..1100…00, that is divisible by 2003?

12. A straight bar 2 meters in length is cut into 5 pieces, each no less than 17 cm long. Prove that amongst these pieces there are three that can be put together to form a triangle.

Problems for each meeting are chosen with great care: at each set, there are several topics and/or techniques at play.

Typically, one (or more) new topic/technique is introduced while there are a few problems dealing with topics that have been introduced earlier. In the latter case, problems might call for slight variation of the previously learned technique or for a different viewpoint on an old topic.

Problem structure of Meeting 8:

Problems 1, 4, 10 all require clever rewriting of a given expression to simplify computation

(note: none of them calls for the use of any formulas such as sum of a geometric series, etc).

For example, #1 can be done as follows.

If we add 1 to the sum it can be rewritten as

1+1+2+4+8+16+32+…+1024=

2+2+4+8+16+32+…+1024=

4+4+8+16+32+…+1024=8+8+16+…+1024=

…=1024+1024=2048.

Hence the sum is 1 less than 2048.

Problems 3, 6,12 - triangle inequality (and #12 introduces Fibonacci numbers).

Problems 5, 9 – area without any formulas (just cutting regions and counting parts).

Problems 8, 11 – pigeonhole principle (#8 makes use of remainders).

Problem 2 – introduces some new ideas (such as using 1-dimensional (or 2-dimensional) graph).

Problem 7 – reinforces earlier notions and (counting) techniques.

MSRI-MCL is always on the lookout for a good project!

If you would like to suggest a book for publication in the MSRI-MCL series, in any language, whether authored by you or someone else, please write the series managing editor,

Silvio Levy levy@msri.org,

with a brief description of the work.