Mathematics Grade 1

Integrated Resource Package 2007

British Columbia
The Best Place on Earth

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This Integrated Resource Package (IRP) provides basic information teachers will require in order to implement Mathematics K to 7. Once fully implemented, this document will supersede Mathematics K to 7 (1995).

> The information contained in this document is also available on the Internet at www.bced.gov.bc.ca/irp/irp.htm

The following paragraphs provide brief descriptions of the components of the IRP.

Introduction

The Introduction provides general information about Mathematics K to 7, including special features and requirements.

Included in this section are

- a rationale for teaching Mathematics K to 7 in BC schools
- goals for Mathematics K to 7
- descriptions of the curriculum organizers – groupings for prescribed learning outcomes that share a common focus
- a suggested timeframe for each grade
- a graphic overview of the curriculum content from K to 7
- additional information that sets the context for teaching Mathematics K to 7

Considerations for Program delivery

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners.

Prescribed Learning Outcomes

This section contains the *prescribed learning outcomes.* Prescribed learning outcomes are the legally required content standards for the provincial education system. They define the required attitudes, skills, and knowledge for each subject. The learning outcomes are statements of what students are expected to know and be able to do by the end of the grade.

The prescribed learning outcomes for the Mathematics K to 7 IRP are based on the Learning Outcomes contained within the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for K to 9 Mathematics available at www.wncp.ca.

Student Achievement

This section of the IRP contains information about classroom assessment and measuring student achievement, including sets of specific achievement indicators for each prescribed learning outcome. Achievement indicators are statements that describe what students should be able to do in order to demonstrate that they fully meet the expectations set out by the prescribed learning outcomes. Achievement indicators are not mandatory; they are provided to assist teachers in assessing how well their students achieve the prescribed learning outcomes.

The achievement indicators for the Mathematics K to 7 IRP are based on the achievement indicators contained within the WNCP Common Curriculum Framework for K to 9 Mathematics.

> The WNCP CCF for K to 9 Mathematics is available online at www.wncp.ca

Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

Classroom Assessment Model

This section contains a series of classroom units that address the learning outcomes. The units have been developed by BC teachers, and are provided to support classroom assessment. These units are suggestions only – teachers may use or modify the units to assist them as they plan for the implementation of this curriculum.

Each unit includes the prescribed learning outcomes and suggested achievement indicators, a suggested timeframe, a sequence of suggested assessment activities, and sample assessment instruments.

Learning Resources

This section contains general information on learning resources, providing a link to titles, descriptions, and ordering information for the recommended learning resources in the Mathematics K to 7 Grade Collections. [*Note: Grade Collections for Mathematics K to 7 will be updated as new resources matching the IRP are authorized.*]

Glossary

The glossary section provides a link to an online glossary that contains definitions for selected terms used in this Integrated Resource Package

INTRODUCTION

This Integrated Resource Package (IRP) sets out the provincially prescribed curriculum for Mathematics K to 7. The development of this IRP has been guided by the principles of learning:

- Learning requires the active participation of the student.
- People learn in a variety of ways and at different rates.
- Learning is both an individual and a group process.

In addition to these three principles, this document recognizes that British Columbia's schools include young people of varied backgrounds, interests, abilities, and needs. Wherever appropriate for this curriculum, ways to meet these needs and to ensure equity and access for all learners have been integrated as much as possible into the learning outcomes and achievement indicators.

The Mathematics K to 7 IRP is based on the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for Kindergarten to Grade 9 Mathematics (May 2006). A complete list of references used to inform the revisions of the WNCP CCF for K to 9 Mathematics as well as this IRP can be found at the end of this section of the IRP.

Mathematics K to 7, in draft form, was available for public review and response from September to November, 2006. Input from educators, students, parents, and other educational partners informed the development of this document.

Rationale

The aim of Mathematics K to 7 is to provide students with the opportunity to further their knowledge, skills, and attitudes related to mathematics.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Numeracy can be defined as the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world. Numeracy is more than knowing about numbers and number operations. (British Columbia Association of Mathematics Teachers 1998)

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of a variety of manipulatives and pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics. Information gathered from these discussions can be used for formative assessment to guide instruction.

As facilitators of learning educators are encouraged to highlight mathematics concepts as they occur within the K to 7 school environment and within home environments. Mathematics concepts are present within every school's subjects and drawing students' attention to these concepts as they occur can help to provide the "teachable moment."

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary. Positive learning experiences build self-confidence and develop attitudes that value learning mathematics.

Aboriginal perspective

Aboriginal students in British Columbia come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings including urban, rural, and isolated communities. Teachers need to understand the diversity of cultures and experiences of students.

Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized. Depending on their learning styles, students may look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students. *The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks 1993).*

Affective Domain

Bloom's taxonomy of learning behaviours identified three domains of educational activities, affective (growth in feelings or emotional areas – attitude), cognitive (mental skills – knowledge), and psychomotor (manual or physical skills – skills). The affective domain involves the way in which we perceive and respond to things emotionally, such as feelings, values, appreciation, enthusiasms, motivations, and attitudes.

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Research has shown that students who are more engaged with school and with mathematics are far more likely to be successful in school and in learning mathematics. (Nardi & Steward 2003). Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

Substantial progress has been made in research in the last decade that has examined the importance and use of the affective domain as part of the learning process. In addition there has been a parallel increase in specific research involving the affective domain and its' relationship to the learning of mathematics which has provided powerful evidence of the importance of this area to the learning of mathematics (McLeod 1988, 1992 & 1994; Hannula 2002 & 2006; Malmivuori 2001 & 2006). Teachers, students, and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Students who are feeling more comfortable with a subject, demonstrate more confidence and have the opportunity for greater academic achievement (Denton & McKinney 2004; Hannula 2006; Smith et al. 1998). Educators can include opportunities for active and co-operative learning in their mathematics lessons which has been shown in research to promote greater conceptual understanding, more positive attitudes and subsequently improved academic achievement from students (Denton & McKinney 2004). By allowing the sharing and discussion of answers and strategies used in mathematics, educators are providing rich opportunities for students mathematical development. Educators can foster greater conceptual understanding in students by having students practice certain topics and concepts in mathematics in a meaningful and engaging manner.

It is important for educators, students, and parents to recognize the relationship between the affective and cognitive domains and attempt to nurture those aspects of the affective domain that contribute to positive attitudes and success in learning.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that are integral to the nature of mathematics, including change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty. These components are woven throughout this curriculum.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, p. 184).

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program 2000, p. 146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms.

Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics.

These skills contribute to students' interaction with and understanding of their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Goals for Mathematics K to 7

Mathematics K to 7 represents the first formal steps that students make towards becoming life-long learners of mathematics.

Goals for Mathematics K to 7

The Mathematics K-7 curriculum is meant to start students toward achieving the main goals of mathematics education:

- using mathematics confidently to solve problems
- using mathematics to better understand the world around us
- communicating and reasoning mathematically
- appreciating and valuing mathematics
- making connections between mathematics and its applications
- committing themselves to lifelong learning
- becoming mathematically literate and using mathematics to participate in, and contribute to, society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- be able to use mathematics to make and justify decisions about the world around us
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

Curriculum Organizers

A curriculum organizer consists of a set of prescribed learning outcomes that share a common focus. The prescribed learning outcomes for Mathematics K to 7 progress in age-appropriate ways, and are grouped under the following curriculum organizers and suborganizers:

Curriculum Organizers and Suborganizers
Mathematics K-7
Number Patterns and Relations • Patterns • Variables and Equations Shape and Space • Measurement • 3-D Objects and 2-D Shapes • Transformations Statistics and Probability • Data Analysis • Chance and Uncertainty

These curriculum organizers reflect the main areas of mathematics that students are expected to address. The ordering of organizers, suborganizers, and outcomes in the Mathematics K to 7 curriculum does not imply an order of instruction. The order in which various topics are addressed is left to the professional judgment of teachers. Mathematics teachers are encouraged to integrate topics throughout the curriculum and within other subject areas to emphasize the connections between mathematics concepts.

Number

Students develop their concept of the number system and relationships between numbers. Concrete, pictorial and symbolic representations are used to help students develop their number sense. Computational fluency, the ability to connect understanding of the concepts with accurate, efficient and flexible computation strategies for multiple purposes, is stressed throughout the number organizer with an emphasis on the development of personal strategies, mental mathematics and estimation strategies.

The Number organizer does not contain any suborganizers.

Patterns and Relations

Students develop their ability to recognize, extend, create, and use numerical and non- numerical patterns to better understand the world around them as well as the world of mathematics. This organizer provides opportunities for students to look for relationships in the environment and to describe the relationships. These relationships should be examined in multiple sensory forms.

The Patterns and Relations organizer includes the following suborganizers:
- Patterns
- Variables and Equations

Shape and Space

Students develop their understanding of objects and shapes in the environment around them. This includes recognition of attributes that can be measured, measurement of these attributes, description of these attributes, the identification and use of referents, and positional change of 3-D objects and 2-D shapes on the environment and on the Cartesian plane.

The Shape and Space organizer includes the following suborganizers:
- Measurement
- 3-D Objects and 2-D Shapes
- Transformations

Statistics and Probability

Students collect, interpret and present data sets in relevant contexts to make decisions. The development of the concepts involving probability is also presented as a means to make decisions. The Shape and Space organizer includes the following suborganizers:
- Data Analysis
- Chance and Uncertainty

KEY CONCEPTS: OVERVIEW OF MATHEMATICS K TO 7 TOPICS

	Kindergarten	Grade 1	Grade 2	Grade 3
NUMBER	• number sequence to 10 • familiar number arrangements up to 5 objects • one-to-one correspondence • numbers in-depth to 10	• skip counting starting at 0 to 100 • arrangements up to 10 objects • numbers in-depth to 20 • addition & subtraction to 20 • mental math strategies to 18	• skip counting at starting points other than 0 to 100 • numbers in-depth to 100 • even, odd & ordinal numbers • addition & subtraction to 100 • mental math strategies to 18	• skip counting at starting points other than 0 to 1000 • numbers in-depth to 1000 • addition & subtraction to 1000 • mental math strategies for 2-digit numerals • multiplication up to 5 × 5 • representation of fractions
PATTERNS & RELATIONS *Patterns*	• repeating patterns of two or three elements	• repeating patterns of two to four elements • representation of pattern	• repeating patterns of three to five elements • increasing patterns	• increasing patterns • decreasing patterns
PATTERNS & RELATIONS *Variables & Equations*		• equalities & inequalities • symbol for equality	• equality & inequality • symbols for equality & inequality	• one-step addition and subtraction equations
SHAPE & SPACE *Measurement*	• direct comparison for length, mass & volume	• process of measurement using comparison	• days, weeks, months, & years • non-standard units of measure for length, height distance around, mass (weight)	• non-standard & standard units of time • measurements of length (cm, m) & mass (g, kg) • perimeter of regular & irregular shapes
SHAPE & SPACE *3-D Objects & 2-D Shapes*	• single attribute of 3-D objects	• one attribute of 3-D objects & 2-D shapes • composite 2-D shapes & 3-D objects • 2-D shapes in the environment	• two attributes of 3-D objects & 2-D shapes • cubes, spheres, cones, cylinders, pyramids • triangles, squares, rectangles, circles • 2-D shapes in the environment	• faces, edges & vertices of 3-D objects • triangles, quadrilaterals, pentagons, hexagons, octagons
SHAPE & SPACE *Transformations*				
STATISTICS & PROBABILITY *Data Analysis*			• data about self and others • concrete graphs and pictographs	• first-hand data • bar graphs
STATISTICS & PROBABILITY *Chance & Uncertainty*				

Grade 4	Grade 5	Grade 6	Grade 7
• numbers in-depth to 10 000 • addition & subtraction to 10 000 • multiplication & division of numbers • fractions less than or equal to one • decimals to hundredths	• numbers in-depth to 1 000 000 • estimation strategies for calculations & problem solving • mental mathematics strategies for multiplication facts to 81 & corresponding division facts • mental mathematics for multiplication • multiplication for 2-digit by 2-digit & division for 3-digit by 1-digit • decimal & fraction comparison • addition & subtraction of decimals	• numbers in-depth greater than 1 000 000 & smaller than one thousandth • factors & multiples • improper fractions & mixed numbers • ratio & whole number percent • integers • multiplication & division of decimals • order of operations excluding exponents	• divisibility rules • addition, subtraction, multiplication, & division of numbers • percents from 1% to 100% • decimal & fraction relationships for repeating & terminating decimals • addition & subtraction of positive fractions & mixed numbers • addition & subtraction of integers
• patterns in tables & charts	• prediction using a pattern rule	• patterns & relationships in graphs & tables including tables of value	• table of values & graphs of linear relations
• symbols to represent unknowns • one-step equations	• single-variable, one-step equations with whole number coefficients & solutions	• letter variable representation of number relationships • preservation of equality	• preservation of equality • expressions & equations • one-step linear equations
• digital clocks, analog clocks, & calendar dates • area of regular & irregular 2-D shapes	• perimeter & area of rectangles • length, volume, & capacity	• perimeter & area of rectangles • length, volume, & capacity	• properties of circles • area of triangles, parallelograms, & circles
• rectangular & triangular prisms	• parallel, intersecting, perpendicular, vertical & horizontal edges & faces • rectangles, squares, trapezoids, parallelograms & rhombuses	• types of triangles • regular & irregular polygons	• geometric constructions
• line symmetry	• 2-D shape single transformation	• combinations of transformations • single transformation in the first quadrant of the Cartesian plane	• four quadrants of the Cartesian plane • transformations in the four quadrants of the Cartesian plane
• many-to-one correspondence including bar graphs & pictographs	• first-hand & second-hand data • double bar graphs	• line graphs • methods of data collection • graph data	• central tendency, outliers & range • circle graphs
	• likelihood of a single outcome	• experimental & theoretical probability	• ratios, fractions, & percents to express probabilities • two independent events • tree diagrams for two independent events

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The following seven mathematical processes should be integrated within Mathematics K to 7.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students need to be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine 1991, p. 5).

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

Even more important than performing computational procedures or using calculators is the greater facility that students need – more than ever before – with estimation and mental mathematics (NCTM May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein 2001).

Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you...?" or "How could you...?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K to 3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong 1993, p. 10). The use of visualization in the study of mathematics provides students with the opportunity to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to decide when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Suggested Timeframe

Provincial curricula are developed in accordance with the amount of instructional time recommended by the Ministry of Education for each subject area. For Mathematics K to 7, the Ministry of Education recommends a time allotment of 20% (approximately 95 hours in Kindergarten and 185 hours in Grades 1 to 7) of the total instructional time for each school year. In the primary years, teachers determine the time allotments for each required area of study and may choose to combine various curricula to enable students to integrate ideas and see the application of mathematics concepts across curricula.

The Mathematics K to 7 IRP for grades 1 to 7 is based on approximately 170 hours of instructional time to allow flexibility to address local needs. For Kindergarten, this estimate is approximately 75 hours. Based on these recommendations, teachers should be spending about 2 to 2.5 hours each week on Mathematics in Kindergarten and 4.5 to 5 hours of instructional time each week on Mathematics grades 1 to 7.

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The following references have been used to inform the revisions of the BC Mathematics K to 7 IRP as well as the WNCP CCF for K-9 Mathematics upon which the Prescribed Learning Outcomes and Achievement Indicators are based.

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Considerations for Program Delivery

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners. Included in this section is information about

- alternative delivery policy
- inclusion, equity, and accessibility for all learners
- working with the Aboriginal community
- information and communications technology
- copyright and responsibility
- fostering the development of positive attitudes
- instructional focus
- applying mathematics

Alternative Delivery Policy

The Alternative Delivery policy does not apply to the Mathematics K to 7 curriculum.

The Alternative Delivery policy outlines how students, and their parents or guardians, in consultation with their local school authority, may choose means other than instruction by a teacher within the regular classroom setting for addressing prescribed learning outcomes contained in the Health curriculum organizer of the following curriculum documents:

- Health and Career Education K to 7, and Personal Planning K to 7 Personal Development curriculum organizer (until September 2008)
- Health and Career Education 8 and 9
- Planning 10

The policy recognizes the family as the primary educator in the development of children's attitudes, standards, and values, but the policy still requires that all prescribed learning outcomes be addressed and assessed in the agreed-upon alternative manner of delivery.

It is important to note the significance of the term "alternative delivery" as it relates to the Alternative Delivery policy. The policy does not permit schools to omit addressing or assessing any of the prescribed learning outcomes within the health and career education curriculum. Neither does it allow students to be excused from meeting any learning outcomes related to health. It is expected that students who arrange for alternative delivery will address the health-related learning outcomes and will be able to demonstrate their understanding of these learning outcomes.

For more information about policy relating to alternative delivery, refer to www.bced.gov.bc.ca/policy/

Inclusion, Equity, and Accessibility for All Learners

British Columbia's schools include young people of varied backgrounds, interests, and abilities. The Kindergarten to Grade 12 school system focuses on meeting the needs of all students. When selecting specific topics, activities, and resources to support the implementation of Mathematics K to 7, teachers are encouraged to ensure that these choices support inclusion, equity, and accessibility for all students. In particular, teachers should ensure that classroom instruction, assessment, and resources reflect sensitivity to diversity and incorporate positive role portrayals, relevant issues, and themes such as inclusion, respect, and acceptance.

Government policy supports the principles of integration and inclusion of students who have English as a second language and of students with special needs. Most of the prescribed learning outcomes and suggested achievement indicators in this IRP can be met by all students, including those with special needs and/or ESL needs. Some strategies may require adaptations to ensure that those with special and/or ESL needs can successfully achieve the learning outcomes. Where necessary, modifications can be made to the prescribed learning outcomes for students with Individual Education Plans.

For more information about resources and support for students with special needs, refer to www.bced.gov.bc.ca/specialed/

For more information about resources and support for ESL students, refer to www.bced.gov.bc.ca/esl/

Working with the Aboriginal Community

The Ministry of Education is dedicated to ensuring that the cultures and contributions of Aboriginal peoples in BC are reflected in all provincial curricula. To address these topics in the classroom in a way that is accurate and that respectfully reflects Aboriginal concepts of teaching and learning, teachers are strongly encouraged to seek the advice and support of local Aboriginal communities. Aboriginal communities are diverse in terms of language, culture, and available resources, and each community will have its own unique protocol to gain support for integration of local knowledge and expertise. To begin discussion of possible instructional and assessment activities, teachers should first contact Aboriginal education co-ordinators, teachers, support workers, and counsellors in their district who will be able to facilitate the identification of local resources and contacts such as Elders, chiefs, tribal or band councils, Aboriginal cultural centres, Aboriginal Friendship Centres, and Métis or Inuit organizations.

In addition, teachers may wish to consult the various Ministry of Education publications available, including the "Planning Your Program" section of the resource, *Shared Learnings* (2006). This resource was developed to help all teachers provide students with knowledge of, and opportunities to share experiences with, Aboriginal peoples in BC.

> For more information about these documents, consult the Aboriginal Education web site: www.bced.gov.bc.ca/abed/welcome.htm

Information and Communications Technology

The study of information and communications technology is increasingly important in our society. Students need to be able to acquire and analyze information, to reason and communicate, to make informed decisions, and to understand and use information and communications technology for a variety of purposes. Development of these skills is important for students in their education, their future careers, and their everyday lives.

Literacy in the area of information and communications technology can be defined as the ability to obtain and share knowledge through investigation, study, instruction, or transmission of information by means of media technology. Becoming literate in this area involves finding, gathering, assessing, and communicating information using electronic means, as well as developing the knowledge and skills to use and solve problems effectively with the technology. Literacy also involves a critical examination and understanding of the ethical and social issues related to the use of information and communications technology. Mathematics K to 7 provides opportunities for students to develop literacy in relation to information and communications technology sources, and to reflect critically on the role of these technologies in society.

Copyright and Responsibility

Copyright is the legal protection of literary, dramatic, artistic, and musical works; sound recordings; performances; and communications signals. Copyright provides creators with the legal right to be paid for their work and the right to say how their work is to be used. There are some exceptions in the law (i.e., specific things permitted) for schools but these are very limited, such as copying for private study or research. The copyright law determines how resources can be used in the classroom and by students at home

In order to respect copyright it is necessary to understand the law. It is unlawful to do the following, unless permission has been given by a copyright owner:

- photocopy copyrighted material to avoid purchasing the original resource for any reason
- photocopy or perform copyrighted material beyond a very small part – in some cases the copyright law considers it "fair" to copy whole works, such as an article in a journal or a photograph, for purposes of research and private study, criticism, and review
- show recorded television or radio programs to students in the classroom unless these are cleared for copyright for educational use (there are exceptions such as for news and news commentary taped within one year of broadcast that by law have record-keeping requirements – see the web site at the end of this section for more details)
- photocopy print music, workbooks, instructional materials, instruction manuals, teacher guides, and commercially available tests and examinations

- show video recordings at schools that are not cleared for public performance
- perform music or do performances of copyrighted material for entertainment (i.e., for purposes other than a specific educational objective)
- copy work from the Internet without an express message that the work can be copied

Permission from or on behalf of the copyright owner must be given in writing. Permission may also be given to copy or use all or some portion of copyrighted work through a licence or agreement. Many creators, publishers, and producers have formed groups or "collectives" to negotiate royalty payments and copying conditions for educational institutions. It is important to know what licences are in place and how these affect the activities schools are involved in. Some licences may also require royalty payments that are determined by the quantity of photocopying or the length of performances. In these cases, it is important to assess the educational value and merits of copying or performing certain works to protect the school's financial exposure (i.e., only copy or use that portion that is absolutely necessary to meet an educational objective).

It is important for education professionals, parents, and students to respect the value of original thinking and the importance of not plagiarizing the work of others. The works of others should not be used without their permission.

For more information about copyright, refer to www.cmec.ca/copyright/indexe.stm

Fostering the Development of Positive Attitudes in Mathematics

A positive attitude toward mathematics is often a result of a learning environment in the classroom that encourages students' own mathematical thinking and contributions to classroom activities and discussions. Teachers should provide a variety of instructional approaches used in the classroom in order to reach a variety of learning styles and dispositions. These include experiences that encourage students to

- enjoy and value mathematics
- develop mathematical habits of mind
- explore
- take risks
- exhibit curiosity
- make and correct errors
- persevere
- experience mathematics in non-threatening, engaging ways
- understand and appreciate the role of mathematics in human affairs

These learning opportunities enable students to gain confidence in their abilities to solve complex problems.

The assessment of attitudes is indirect, and based on inferences drawn from students' behaviour. We can see what students do and hear what they say, and from these observations make inferences and draw conclusions about their attitudes.

It is important for teachers to consider their role in developing a positive attitude in mathematics. Teachers and parents are role models from which students begin to develop their disposition toward mathematics. Teachers need to model these attitudes in order to help students develop them (Burns 2000). In this manner teachers need to "present themselves as problem solvers, as active learners who are seekers, willing to plunge into new situations, not always knowing the answer or what the outcome will be" (p. 29).

Instructional Focus

The Mathematics K to 7 courses are arranged into a number of organizers with mathematical processes integrated throughout. Students learn in different ways and at different rates. As in other subject areas, it is essential when teaching mathematics, that concepts are introduced to students in a variety of ways. Students should hear explanations, watch demonstrations, draw to represent their thinking, engage in experiences with concrete materials and be encouraged to visualize and discuss their understanding of concepts. Most students need a range of concrete or representational experiences with mathematics concepts before they develop symbolic or abstract understanding. The development of conceptual understanding should be emphasized throughout the curriculum as a means to develop students to become mathematical problem solvers.

Teaching through Problem Solving

Problem solving should be an integral part of all mathematics classrooms. Teachers are encouraged to weave problem solving throughout all curriculum organizers in the K to 7 mathematics curriculum on a regular basis. Problem solving provides a way of helping students learn mathematics.

Hiebert et al. (1996) encourage teachers to make mathematics problematic. A problem can be defined as any task or activity for which the students have not memorized a method or rule, nor is there an assumption by the students that there is only one correct way to solve the problem (Hiebert et al. 1997). Van de Walle (2006) notes that "a problem for learning mathematics also has these features:

- The problem must begin where the students are.
- The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.
- The problem must require justifications and explanations for answers and methods. (p. 11)

Why teach through problem solving?

- The math makes more sense. When using real world math problems, students are able to make the connections between what math is and how they can apply it.
- Problems are more motivating when they are challenging. Although some students are anxious when they are not directed by the teacher, most enjoy a challenge they can be successful in solving.
- Problem solving builds confidence. It maximizes the potential for understanding as each child makes his own sense out of the problem and allows for individual strategies.
- Problem solving builds perseverance. Because an answer is not instantaneous, many children think they are unable to do the math. Through the experience of problem solving they learn to apply themselves for longer periods of time and not give up.
- Problems can provide practice with concepts and skills. Good problems enable students to learn and apply the concepts in a meaningful way and an opportunity to practice the skills.
- Problem solving provides students with insight into the world of mathematics. Mathematicians struggle to find solutions to many problems and often need to go down more than one path to arrive at a solution. This is a creative process that is difficult to understand if one has never had to struggle.
- Problem solving provides the teacher with insight into a student's mathematical thinking. As students choose strategies and solve problems, the teacher has evidence of their thinking and can inform instruction based on this.
- Students need to practice problem solving. If we are expecting students to confront new situations involving mathematics, they need practice to become independent problem solvers (Small 2005).

Polya (1957) characterized a general method which can be used to solve problems, and to describe how problem-solving should be taught and learned. He advocated for the following steps in solving a mathematical problem:

- Understand the problem – What is unknown? What is known? Is enough information provided to determine the solution? Can a figure or model be used to represent the situation?
- Make a plan – Is there a similar problem that has been solved before? Can the problem be restated so it makes more sense?
- Carry out the plan – Have all of the steps been completed correctly?
- Look back – Do the results look correct? Is there another way to solve the problem that would verify the results?

While a number of variations of the problem solving model proposed by Polya (Van de Walle 2006; Small 2006; Burns 2000) they all have similar characteristics. The incorporation of a wide variety of strategies to solve problems is essential to developing students' ability to be flexible problem solvers.

The Mathematics K to 7 (1995) IRP provides a number of useful strategies that students can use to increase their flexibility in solving problems. These include:

- look for a pattern
- construct a table
- make an organized list
- act it out
- draw a picture
- use objects
- guess and check
- work backward
- write an equation
- solve a simpler (or similar) problem
- make a model (BC Ministry of Education 1995)

During problem-solving experiences, students are encouraged to solve problems using ways that make sense to them. As students share different ways of solving problems they can learn strategies from each other. Teachers are encouraged to facilitate this process to in an open and non-threatening environment. I this manner, students can develop a repertoire of strategies from which to draw upon when mathematical problems are presented to them.

Problem solving requires a shift in student attitudes and how teachers model these attitudes in the classroom. In order to be successful, students must develop, and teachers model, the following characteristics:

- interest in finding solutions to problems
- confidence to try various strategies
- willingness to take risks
- ability to accept frustration when not knowing
- understanding the difference between not knowing the answer and not having found it yet (Burns 2000)

Problems are not just simple computations embedded in a story nor are they contrived, that is, they do not exist outside the math classroom. Students will be engaged if the problems relate to their lives; their culture, interests, families, current events. They are tasks that are rich and open-ended so there is more than one way of arriving at a solution, or multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill or understanding. The students should not know the answer immediately. Problem solving takes time and effort on the part of the student and the teacher. Teaching thought problem solving is one of the ways that teachers can bring increased depth to the Mathematics K to 7 curriculum.

Instruction should provide an emphasis on mental mathematics and estimation to check the reasonableness of paper and pencil exercises, and the solutions to problems which are determined through the use of technology, including calculators and computers. (It is assumed that all students have regular access to appropriate technology such calculators, or computers with graphing software and standard spreadsheet programs.) Concepts should be introduced using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.

Applying Mathematics

For students to view mathematics as relevant and useful, they must see how it can be applied in a variety of contexts. Mathematics helps students understand and interpret their world and solve problems that occur in their daily lives both within and outside of the school context.

Teachers are encouraged to incorporate, and make explicit, mathematics concepts which naturally occur across the subject areas. Possible situations where cross curricular integration may occur in K to 7 include the following:

Fine Arts

- pattern, line, and form
- fractions in rhythm and metre
- spatial awareness in dance, drama, and visual arts
- geometric shapes in visual arts, drama, and dance
- symmetry and unison
- transformations
- perspective and proportion in visual arts
- measuring and proportional reasoning for mixing and applying materials in visual arts

Health and Career Education

- creating schedules
- interpreting statistical data
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables
- using mathematics to develop a logical argument to support a position on a topic or issue

Language Arts

- reading literature with a mathematics theme
- creating a picture book or writing a story with mathematical content
- listening to stories to decode mathematical contexts
- examine the plot of a story from a mathematical perspective
- create graphic organizers provide an explanation, proof, or justification for an argument
- role-play or oral presentations of problems and solutions
- creating word walls, personal dictionaries, or glossaries of mathematics terms
- examine the roots of mathematical terms

Physical Education

- examining the benefits of various physical activity (e.g. burning calories)
- examining patterns in physical movement
- measuring distances
- estimate distances and other quantise using referents
- reading and recording dates and time

Science

- discussing the magnitude of numbers
- classifying and sorting objects
- examining patterns to make a hypothesis
- measuring quantities
- use of referents for measurement
- units and conversions between units
- reading and writing quantities in multiple formats (e.g., numerals, words)
- collecting, organizing and interpreting data charts, graphs, diagrams, and tables
- creating a logical argument to support a hypothesis
- mental mathematics for calculations

Social Studies

- discussing the magnitude of numbers and building referents for numbers
- using concepts of area, perimeter, and distances when mapping
- graphing using the Cartesian plane
- using circle concepts to explain latitude and longitude, time zones, great circle routes
- interpreting statistical data
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables
- reading and recording dates and time
- examining the history of mathematics in context of world events
- using mathematics to develop a logical argument to support a position on a topic or issue

Students can also be encouraged to identify and examine the mathematics around them. In this way, students will come to see that mathematics is present outside of the classroom. There are many aspects of students' daily lives where they may encounter mathematic such as

- making purchases
- reading bus schedules
- reading sports statistics
- interpreting newspaper and media sources
- following a recipe
- estimating time to complete tasks
- estimating quantities
- creating patterns when doodling

Making these connections explicit for students helps to solidify the importance of mathematics.

PRESCRIBED LEARNING OUTCOMES

Prescribed learning outcomes are content standards for the provincial education system; they are the prescribed curriculum. Clearly stated and expressed in measurable and observable terms, learning outcomes set out the required attitudes, skills, and knowledge – what students are expected to know and be able to do – by the end of the subject and grade.

Schools have the responsibility to ensure that all prescribed learning outcomes in this curriculum are met; however, schools have flexibility in determining how delivery of the curriculum can best take place.

It is expected that student achievement will vary in relation to the learning outcomes. Evaluation, reporting, and student placement with respect to these outcomes are dependent on the professional judgment and experience of teachers, guided by provincial policy.

Prescribed learning outcomes for Mathematics K to 7 are presented by grade and by curriculum organizer and suborganizer, and are coded alphanumerically for ease of reference; however, this arrangement is not intended to imply a required instructional sequence.

Wording of Prescribed Learning Outcomes

All learning outcomes complete the stem, "It is expected that students will …."

When used in a prescribed learning outcome, the word "including" indicates that any ensuing item **must be addressed**. Lists of items introduced by the word "including" represent a set of minimum requirements associated with the general requirement set out by the outcome. The lists are not necessarily exhaustive, however, and teachers may choose to address additional items that also fall under the general requirement set out by the outcome.

Domains of Learning

Prescribed learning outcomes in BC curricula identify required learning in relation to one or more of the three domains of learning: cognitive, psychomotor, and affective. The following definitions of the three domains are based on Bloom's taxonomy.

The **cognitive domain** deals with the recall or recognition of knowledge and the development of intellectual abilities. The cognitive domain can be further specified as including three cognitive levels: knowledge, understanding and application, and higher mental processes. These levels are determined by the verb used in the learning outcome, and illustrate how student learning develops over time.

- Knowledge includes those behaviours that emphasize the recognition or recall of ideas, material, or phenomena.
- Understanding and application represents a comprehension of the literal message contained in a communication, and the ability to apply an appropriate theory, principle, idea, or method to a new situation.
- Higher mental processes include analysis, synthesis, and evaluation. The higher mental processes level subsumes both the knowledge and the understanding and application levels.

The **affective domain** concerns attitudes, beliefs, and the spectrum of values and value systems.

The **psychomotor domain** includes those aspects of learning associated with movement and skill demonstration, and integrates the cognitive and affective consequences with physical performances.

Domains of learning and cognitive levels also form the basis of the Assessment Overview Tables provided for each grade in the Classroom Assessment Model. In addition, domains of learning and, particularly, cognitive levels, inform the design and development of the Grades 4 and 7 Foundation Skills Assessment (FSA).

Prescribed Learning Outcomes

Grade 1

GRADE 1

It is expected that students will:

NUMBER

A1 say the number sequence, 0 to 100, by
- 1s forward and backward between any two given numbers
- 2s to 20, forward starting at 0
- 5s and 10s to 100, forward starting at 0 [C, CN, V, ME]

A2 recognize, at a glance, and name familiar arrangements of 1 to 10 objects or dots [C, CN, ME, V]

A3 demonstrate an understanding of counting by
- indicating that the last number said identifies "how many"
- showing that any set has only one count
- using the counting on strategy
- using parts or equal groups to count sets [C, CN, ME, R, V]

A4 represent and describe numbers to 20 concretely, pictorially, and symbolically [C, CN, V]

A5 compare sets containing up to 20 elements to solve problems using
- referents
- one-to-one correspondence [C, CN, ME, PS, R, V]

A6 estimate quantities to 20 by using referents [C, ME, PS, R, V]

A7 demonstrate, concretely and pictorially, how a given number can be represented by a variety of equal groups with and without singles [C, R, V]

A8 identify the number, up to 20, that is one more, two more, one less, and two less than a given number. [C, CN, ME, R, V]

A9 demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially, and symbolically by
- using familiar and mathematical language to describe additive and subtractive actions from their experience
- creating and solving problems in context that involve addition and subtraction
- modelling addition and subtraction using a variety of concrete and visual representations, and recording the process symbolically [C, CN, ME, PS, R, V]

A10 describe and use mental mathematics strategies (memorization not intended), such as
- counting on and counting back
- making 10
- doubles
- using addition to subtract

to determine the basic addition facts to 18 and related subtraction facts [C, CN, ME, PS, R, V]

PATTERNS AND RELATIONS

Patterns

B1 demonstrate an understanding of repeating patterns (two to four elements) by
- describing
- reproducing
- extending
- creating

patterns using manipulatives, diagrams, sounds, and actions [C, PS, R, V]

B2 translate repeating patterns from one representation to another [C, R, V]

GRADE 1

Variables and Equations

B3 describe equality as a balance and inequality as an imbalance, concretely, and pictorially (0 to 20) [C, CN, R, V]

B4 record equalities using the equal symbol [C, CN, PS, V]

SHAPE AND SPACE

Measurement

C1 demonstrate an understanding of measurement as a process of comparing by
- identifying attributes that can be compared
- ordering objects
- making statements of comparison
- filling, covering, or matching [C, CN, PS, R, V]

3-D Objects and 2-D Shapes

C2 sort 3-D objects and 2-D shapes using one attribute, and explain the sorting rule [C, CN, R, V]

C3 replicate composite 2-D shapes and 3-D objects [CN, PS, V]

C4 compare 2-D shapes to parts of 3-D objects in the environment [C, CN, V]

Transformations

not applicable at this grade level

STATISTICS AND PROBABILITY

Data Analysis

not applicable at this grade level

Chance and Uncertainty

not applicable at this grade level

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

Student Achievement

This section of the IRP contains information about classroom assessment and student achievement, including specific achievement indicators that may be used to assess student performance in relation to each prescribed learning outcome. Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

Classroom Assessment and Evaluation

Assessment is the systematic gathering of information about what students know, are able to do, and are working toward. Assessment evidence can be collected using a wide variety of methods, such as

- observation
- student self-assessments and peer assessments
- quizzes and tests (written, oral, practical)
- samples of student work
- projects and presentations
- oral and written reports
- journals and learning logs
- performance reviews
- portfolio assessments

Assessment of student achievement is based on the information collected through assessment activities. Teachers use their insight, knowledge about learning, and experience with students, along with the specific criteria they establish, to make judgments about student performance in relation to prescribed learning outcomes.

Three major types of assessment can be used in conjunction with each other to support student achievement.

- Assessment for learning is assessment for purposes of greater learning achievement.
- Assessment as learning is assessment as a process of developing and supporting students' active participation in their own learning.
- Assessment of learning is assessment for purposes of providing evidence of achievement for reporting.

Assessment for Learning

Classroom assessment for learning provides ways to engage and encourage students to become involved in their own day-to-day assessment – to acquire the skills of thoughtful self-assessment and to promote their own achievement.

This type of assessment serves to answer the following questions:

- What do students need to learn to be successful?
- What does the evidence of this learning look like?

Assessment for learning is criterion-referenced, in which a student's achievement is compared to established criteria rather than to the performance of other students. Criteria are based on prescribed learning outcomes, as well as on suggested achievement indicators or other learning expectations.

Students benefit most when assessment feedback is provided on a regular, ongoing basis. When assessment is seen as an opportunity to promote learning rather than as a final judgment, it shows students their strengths and suggests how they can develop further. Students can use this information to redirect their efforts, make plans, communicate with others (e.g., peers, teachers, parents) about their growth, and set future learning goals.

Assessment for learning also provides an opportunity for teachers to review what their students are learning and what areas need further attention. This information can be used to inform teaching and create a direct link between assessment and instruction. Using assessment as a way of obtaining feedback on instruction supports student achievement by informing teacher planning and classroom practice.

Assessment as Learning

Assessment as learning actively involves students in their own learning processes. With support and guidance from their teacher, students take responsibility for their own learning, constructing meaning for themselves. Through a process of continuous self-assessment, students develop the ability to take stock of what they have already learned, determine what they have not yet learned, and decide how they can best improve their own achievement.

Although assessment as learning is student-driven, teachers can play a key role in facilitating how this assessment takes place. By providing regular opportunities for reflection and self-assessment, teachers can help students develop, practise, and become comfortable with critical analysis of their own learning.

Assessment of Learning

Assessment of learning can be addressed through summative assessment, including large-scale assessments and teacher assessments. These summative assessments can occur at the end of the year or at periodic stages in the instructional process.

Large-scale assessments, such as Foundation Skills Assessment (FSA) and Graduation Program exams, gather information on student performance throughout the province and provide information

for the development and revision of curriculum. These assessments are used to make judgments about students' achievement in relation to provincial and national standards.

Assessment of learning is also used to inform formal reporting of student achievement.

For Ministry of Education reporting policy, refer to www.bced.gov.bc.ca/policy/policies/student_reporting.htm

Assessment *for* Learning	Assessment *as* Learning	Assessment *of* Learning
Formative assessment *ongoing in the classroom* • teacher assessment, student self-assessment, and/or student peer assessment • criterion-referenced criteria based on prescribed learning outcomes identified in the provincial curriculum, reflecting performance in relation to a specific learning task • involves both teacher and student in a process of continual reflection and review about progress • teachers adjust their plans and engage in corrective teaching in response to formative assessment	Formative assessment *ongoing in the classroom* • self-assessment • provides students with information on their own achievement and prompts them to consider how they can continue to improve their learning • student-determined criteria based on previous learning and personal learning goals • students use assessment information to make adaptations to their learning process and to develop new understandings	Summative assessment *occurs at end of year or at key stages* • teacher assessment • may be either criterion-referenced (based on prescribed learning outcomes) or norm-referenced (comparing student achievement to that of others) • information on student performance can be shared with parents/guardians, school and district staff, and other education professionals (e.g., for the purposes of curriculum development) • used to make judgments about students' performance in relation to provincial standards

For more information about assessment for, as, and of learning, refer to the following resource developed by the Western and Northern Canadian Protocol (WNCP): *Rethinking Assessment with Purpose in Mind.*

This resource is available online at www.wncp.ca

In addition, the BC Performance Standards describe levels of achievement in key areas of learning (reading, writing, numeracy, social responsibility, and information and communications technology integration) relevant to all subject areas. Teachers may wish to use the Performance Standards as resources to support ongoing formative assessment in mathematics.

BC Performance Standards are available at www.bced.gov.bc.ca/perf_stands/

Criterion-Referenced Assessment and Evaluation

In criterion-referenced evaluation, a student's performance is compared to established criteria rather than to the performance of other students. Evaluation in relation to prescribed curriculum requires that criteria be established based on the learning outcomes.

Criteria are the basis for evaluating student progress. They identify, in specific terms, the critical aspects of a performance or a product that indicate how well the student is meeting the prescribed learning outcomes. For example, weighted criteria, rating scales, or scoring guides (reference sets) are ways that student performance can be evaluated using criteria.

Wherever possible, students should be involved in setting the assessment criteria. This helps students develop an understanding of what high-quality work or performance looks like.

Criterion-referenced assessment and evaluation may involve these steps:

Step 1	Identify the prescribed learning outcomes and suggested achievement indicators (as articulated in this IRP) that will be used as the basis for assessment.
Step 2	Establish criteria. When appropriate, involve students in establishing criteria.
Step 3	Plan learning activities that will help students gain the attitudes, skills, or knowledge outlined in the criteria.
Step 4	Prior to the learning activity, inform students of the criteria against which their work will be evaluated.
Step 5	Provide examples of the desired levels of performance.
Step 6	Conduct the learning activities.
Step 7	Use appropriate assessment instruments (e.g., rating scale, checklist, scoring guide) and methods (e.g., observation, collection, self-assessment) based on the particular assignment and student.
Step 8	Review the assessment data and evaluate each student's level of performance or quality of work in relation to criteria.
Step 9	Where appropriate, provide feedback and/or a letter grade to indicate how well the criteria are met.
Step 10	Communicate the results of the assessment and evaluation to students and parents/guardians.

Key Elements

Key elements provide an overview of content in each curriculum organizer. They can be used to determine the expected depth and breadth of the prescribed learning outcomes.

Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for developmental learning.

Achievement Indicators

To support the assessment of provincially prescribed curricula, this IRP includes sets of achievement indicators in relation to each learning outcome.

Achievement indicators, taken together as a set, define the specific level of attitudes demonstrated, skills applied, or knowledge acquired by the student in relation to a corresponding prescribed learning outcome. They describe what evidence to look for to determine whether or not the student has fully met the intent of the learning outcome. Since each achievement indicator defines only one aspect of the corresponding learning outcome, the entire set of achievement indicators should be considered when determining whether students have fully met the learning outcome.

In some cases, achievement indicators may also include suggestions as to the type of task that would provide evidence of having met the learning outcome (e.g., a constructed response such as a list, comparison, or analysis; a product created and presented such as a report, poster, letter, or model; a particular skill demonstrated such as map making or critical thinking).

Achievement indicators support the principles of assessment for learning, assessment as learning, and assessment of learning. They provide teachers and parents with tools that can be used to reflect on what students are learning, as well as provide students with a means of self-assessment and ways of defining how they can improve their own achievement.

Achievement indicators are not mandatory; they are suggestions only, provided to assist in the assessment of how well students achieve the prescribed learning outcomes.

The following pages contain the suggested achievement indicators corresponding to each prescribed learning outcome for the Mathematics K to 7 curriculum. The achievement indicators are arranged by curriculum organizer for each grade; however, this order is not intended to imply a required sequence of instruction and assessment.

Student Achievement

Grade 1

KEY ELEMENTS: GRADE 1

MATHEMATICAL PROCESS (INTEGRATED)

The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the grade: communication, connections, mental mathematics and estimation, problem solving, reasoning, technology, and visualization.

NUMBER – develop number sense

- number sequence forward and backward to 100
- skip counting
- representation of number
- referents and one-to one-correspondence for sets up to 20 elements
- addition to 20 and basic addition and subtraction facts

PATTERNS AND RELATIONS – use patterns to describe the world and solve problems

Patterns

- repeating patterns of two to four elements
- representation of pattern
- equalities and inequalities

SHAPE AND SPACE – use direct and indirect measurement to solve problems

Measurement

- process of measurement using comparison

3-D Objects and 2-D Shapes

- one attribute of 3-D objects and 2-D shapes
- composite 2-D shapes and 3-D objects
- 2-D shapes in the environment

NUMBER

General Outcome: Develop number sense.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
A1 say the number sequence, 0 to 100, by - 1s forward and backward between any two given numbers - 2s to 20, forward starting at 0 - 5s and 10s to 100, forward starting at 0 [C, CN, V, ME]	❑ recite forward by 1s the number sequence between two given numbers (0 to 100) ❑ recite backward by 1s the number sequence between two given numbers ❑ record a given numeral (0 to 100) symbolically when it is presented orally ❑ read a given numeral (0 to 100) when it is presented symbolically ❑ skip count by 2s to 20 starting at 0 ❑ skip count by 5s to 100 starting at 0 ❑ skip count forward by 10s to 100 starting at 0 ❑ identify and correct errors and omissions in a given number sequence
A2 recognize, at a glance, and name familiar arrangements of 1 to 10 objects or dots [C, CN, ME, V]	❑ look briefly at a given familiar arrangement of objects or dots and identify the number represented without counting ❑ look briefly at a given familiar arrangement and identify how many objects there are without counting ❑ identify the number represented by a given arrangement of objects or dots on a ten frame
A3 demonstrate an understanding of counting by - indicating that the last number said identifies "how many" - showing that any set has only one count - using the counting on strategy - using parts or equal groups to count sets [C, CN, ME, R, V]	❑ answer the question, "How many are in the set?" using the last number counted in a given set ❑ identify and correct counting errors in a given counting sequence ❑ show that the count of the number of objects in a given set does not change regardless of the order in which the objects are counted ❑ count the number of objects in a given set, rearrange the objects, predict the new count, and recount to verify the prediction ❑ determine the total number of objects in a given set, starting from a known quantity and counting on ❑ count quantity using groups of 2s, 5s, or 10s and counting on

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

Prescribed Learning Outcomes	Suggested Achievement Indicators
A4 represent and describe numbers to 20 concretely, pictorially, and symbolically [C, CN, V]	❑ represent a given number up to 20 using a variety of manipulatives, including ten frames and base ten materials ❑ read given number words to 20 ❑ partition any given quantity up to 20 into 2 parts and identify the number of objects in each part ❑ model a given number using two different objects (e.g., 10 desks represents the same number as 10 pencils) ❑ place given numerals on a number line with benchmarks 0, 5, 10, and 20
A5 compare sets containing up to 20 elements to solve problems using - referents - one-to-one correspondence [C, CN, ME, PS, R, V]	❑ build a set equal to a given set that contains up to 20 elements ❑ build a set that has more, fewer, or as many elements as a given set ❑ build several sets of different objects that have the same given number of elements in the set ❑ compare two given sets using one-to-one correspondence and describe them using comparative words, such as more, fewer, or as many ❑ compare a set to a given referent using comparative language ❑ solve a given story problem (pictures and words) that involves the comparison of two quantities
A6 estimate quantities to 20 by using referents [C, ME, PS, R, V]	❑ estimate a given quantity by comparing it to a given referent (known quantity) ❑ select an estimate for a given quantity by choosing between at least two possible choices and explain the choice
A7 demonstrate, concretely and pictorially, how a given number can be represented by a variety of equal groups with and without singles [C, R, V]	❑ represent a given number in a variety of equal groups with and without singles (e.g., 17 can be represented by 8 groups of 2 and one single, 5 groups of 3 and two singles, 4 groups of 4 and one single, and 3 groups of 5 and two singles ❑ recognize that for a given number of counters, no matter how they are grouped, the total number of counters does not change ❑ group a set of given counters into equal groups in more than one way
A8 identify the number, up to 20, that is one more, two more, one less, and two less than a given number. [C, CN, ME, R, V]	❑ name the number that is one more, two more, one less, or two less than a given number, up to 20 ❑ represent a number on a ten frame that is one more, two more, one less or two less than a given number

Prescribed Learning Outcomes	Suggested Achievement Indicators
A9 demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially, and symbolically by - using familiar and mathematical language to describe additive and subtractive actions from their experience - creating and solving problems in context that involve addition and subtraction - modelling addition and subtraction using a variety of concrete and visual representations, and recording the process symbolically [C, CN, ME, PS, R, V]	❑ act out a given story problem presented orally or through shared reading ❑ indicate if the scenario in a given story problem represents additive or subtractive action ❑ represent the numbers and actions presented in a given story problem by using manipulatives, and record them using sketches and/or number sentences ❑ create a story problem for addition that connects to student experience and simulate the action with counters ❑ create a story problem for subtraction that connects to student experience and simulate the action with counters ❑ create a word problem for a given number sentence ❑ represent a given story problem pictorially or symbolically to show the additive or subtractive action and solve the problem
A10 describe and use mental mathematics strategies (memorization not intended), such as - counting on and counting back - making 10 - doubles - using addition to subtract to determine the basic addition facts to 18 and related subtraction facts [C, CN, ME, PS, R, V]	*(It is not intended that students recall the basic facts but become familiar with strategies to mentally determine sums and differences.)* ❑ use and describe a personal strategy for determining a given sum ❑ use and describe a personal strategy for determining a given difference ❑ write the related subtraction fact for a given addition fact ❑ write the related addition fact for a given subtraction fact

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

PATTERNS AND RELATIONS (PATTERNS)

General Outcome: Use patterns to describe the world and solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
B1 demonstrate an understanding of repeating patterns (two to four elements) by - describing - reproducing - extending - creating patterns using manipulatives, diagrams, sounds, and actions [C, PS, R, V]	❑ describe a given repeating pattern containing two to four elements in its core ❑ identify errors in a given repeating pattern ❑ identify the missing element(s) in a given repeating pattern ❑ create and describe a repeating pattern using a variety of manipulatives, musical instruments, and actions ❑ reproduce and extend a given repeating pattern using manipulatives, diagrams, sounds, and actions ❑ identify and describe a repeating pattern in the environment (e.g., classroom, outdoors) using everyday language ❑ identify repeating events (e.g., days of the week, birthdays, seasons)
B2 translate repeating patterns from one representation to another [C, R, V]	❑ represent a given repeating pattern using another mode (e.g., actions to sound, colour to shape, ABC ABC to blue yellow green blue yellow green ❑ describe a given repeating pattern using a letter code (e.g., ABC ABC…)

PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

General Outcome: Represent algebraic expressions in multiple ways.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
B3 describe equality as a balance and inequality as an imbalance, concretely, and pictorially (0 to 20) [C, CN, R, V]	❑ construct two equal sets using the same objects (same shape and mass) and demonstrate their equality of number using a balance scale ❑ construct two unequal sets using the same objects (same shape and mass) and demonstrate their inequality of number using a balance scale ❑ determine if two given concrete sets are equal or unequal and explain the process used
B4 record equalities using the equal symbol [C, CN, PS, V]	❑ represent a given equality using manipulatives or pictures ❑ represent a given pictorial or concrete equality in symbolic form ❑ provide examples of equalities where the given sum or difference is on either the left or right side of the equal symbol (=) ❑ record different representations of the same quantity (0 to 20) as equalities

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SHAPE AND SPACE (MEASUREMENT)

General Outcome: Use direct or indirect measurement to solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
C1 demonstrate an understanding of measurement as a process of comparing by - identifying attributes that can be compared - ordering objects - making statements of comparison - filling, covering, or matching [C, CN, PS, R, V]	❑ identify common attributes, such as length (height), mass (weight), volume (capacity), and area, that could be used to compare a given set of two objects ❑ compare two given objects and identify the attributes used to compare ❑ determine which of two or more given objects is longest/shortest by matching and explain the reasoning ❑ determine which of two or more given objects is heaviest/lightest by comparing and explain the reasoning ❑ determine which of two or more given objects holds the most/least by filling and explain the reasoning ❑ determine which of two or more given objects has the greatest/least area by covering and explain the reasoning

SHAPE AND SPACE (3-D OBJECTS AND 2-D SHAPES)

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
C2 sort 3-D objects and 2-D shapes using one attribute, and explain the sorting rule [C, CN, R, V]	❑ sort a given set of familiar 3-D objects or 2-D shapes using a given sorting rule ❑ sort a given set of familiar 3-D objects using a single attribute determined by the student and explain the sorting rule ❑ sort a given set of 2-D shapes using a single attribute determined by the student and explain the sorting rule ❑ determine the difference between two given pre-sorted sets of familiar 3-D objects or 2-D shapes and explain a possible sorting rule used to sort them
C3 replicate composite 2-D shapes and 3-D objects [CN, PS, V]	❑ select 2-D shapes from a given set of 2-D shapes to reproduce a given composite 2-D shape ❑ select 3-D objects from a given set of 3-D objects to reproduce a given composite 3-D object ❑ predict and select the 2-D shapes used to produce a composite 2-D shape, and verify by deconstructing the composite shape ❑ predict and select the 3-D objects used to produce a composite 3-D object, and verify by deconstructing the composite object
C4 compare 2-D shapes to parts of 3-D objects in the environment [C, CN, V]	❑ identify 3-D objects in the environment that have parts similar to a given 2-D shape

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

Classroom Assessment Model

The Classroom Assessment Model outlines a series of assessment units for Mathematics K to 7.

These units have been structured by grade level and theme. Collectively the units address all of the prescribed learning outcomes for each grade, and provide one suggested means of organizing, ordering, and delivering the required content. This organization is not intended to prescribe a linear means of delivery. Teachers are encouraged to reorder the learning outcomes and to modify, organize, and expand on the units to meet the needs of their students, to respond to local requirements, and to incorporate relevant recommended learning resources as applicable. (See the Learning Resources section later in this IRP for information about the recommended learning resources for Mathematics K to 7). In addition, teachers are encouraged to consider ways to adapt assessment strategies from one grade to another.

Considerations for Instruction and Assessment in Mathematics K to 7

It is highly recommended that parents and guardians be kept informed about all aspects of Mathematics K to 7. Suggested strategies for involving parents and guardians are found in the Introduction to this IRP.

Teachers are responsible for setting a positive classroom climate in which students feel comfortable learning about and discussing topics in Mathematics K to 7. Guidelines that may help educators establish a positive climate that is open to free inquiry and respectful of various points of view can be found in the section on Establishing a Positive Classroom Climate in the Introduction to this IRP.

Teachers may also wish to consider the following:

- Involve students in establishing guidelines for group discussion and presentations. Guidelines might include using appropriate listening and speaking skills, respecting students who are reluctant to share personal information in group settings, and agreeing to maintain confidentiality if sharing of personal information occurs.
- Promote critical thinking and open-mindedness, and refrain from taking sides on one point of view.
- Develop and discuss procedures associated with recording and using personal information that may be collected as part of students' work for the purposes of instruction and/or assessment (e.g., why the information is being collected, what the information will be used for, where the information will be kept; who can access it – students, administrators, parents; how safely it will be kept).
- Ensure students are aware that if they disclose personal information that indicates they are at risk for harm, then that information cannot be kept confidential. For more information, see the section on Confidentiality in the Introduction to this IRP.

Classroom Assessment and Evaluation

Teachers should consider using a variety of assessment instruments and techniques to assess students' abilities to meet the prescribed learning outcomes. Tools and techniques for assessment in Mathematics K to 7 can include

- teacher assessment tools such as observation checklists, rating scales, and scoring guides
- self-assessment tools such as checklists, rating scales, and scoring guides
- peer assessment tools such as checklists, rating scales, and scoring guides
- journals or learning logs
- video (to record and critique student demonstration or performance)
- written tests, oral tests (true/false, multiple choice, short answer)
- questionnaires, worksheets
- portfolios
- student-teacher conferences

Assessment in Mathematics K to 7 can also occur while students are engaged in, and based on the product of, activities such as

- class and group discussions
- interviews and questioning
- sharing strategies
- object manipulation
- models and constructions
- charts, graphs, diagrams
- games
- experiments
- artwork, songs/stories, dramas
- centres/stations
- demonstrations and presentations
- performance tasks
- projects

For more information about student assessment, refer to the section on Student Achievement, as well as to the Assessment Overview Tables in each grade of the Classroom Assessment Model.

Information and Communications Technology

The Mathematics K to 7 curriculum requires students to be able to use and analyse the most current information to make informed decisions on a range of topics. This information is often found on the Internet as well as in other information and communications technology resources. When organizing for instruction and assessment, teachers should consider how students will best be able to access the relevant technology, and ensure that students are aware of school district policies on safe and responsible Internet and computer use.

Contents of the Model

Assessment Overview Tables

The Assessment Overview Tables provide teachers with suggestions and guidelines for assessment of each grade of the curriculum. These tables identify the domains of learning and cognitive levels of the learning outcomes, along with a listing of suggested assessment activities and a suggested weight for grading for each curriculum organizer.

Overview

Each grade includes an overview of the assessment units:

- Learning at Previous Grades, indicating any relevant learning based on prescribed learning outcomes from earlier grades of the same subject area. It is assumed that students will have already acquired this learning; if they have not, additional introductory instruction may need to take place before undertaking the suggested assessment outlined in the unit. Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for reinforcement and developmental learning.
- Curriculum Correlation – a table that shows which curriculum organizers and suborganizers are addressed by each unit in this grade of the Classroom Assessment Model.

Prescribed Learning Outcomes

Each unit begins with a listing of the prescribed learning outcomes that are addressed by that unit. Collectively, the units address all the learning outcomes for that grade; some outcomes may appear in more than one unit. The units may not address all of the achievement indicators for each of the outcomes.

Suggested Assessment Activities

Assessment activities have been included for each set of prescribed learning outcomes and corresponding achievement indicators. Each assessment activity consists of two parts:

- Planning for Assessment – outlining the background information to explain the classroom context, opportunities for students to gain and practise learning, and suggestions for preparing the students for assessment
- Assessment Strategies – describing the assessment task, the method of gathering assessment information, and the assessment criteria as defined by the learning outcomes and achievement indicators.

A wide variety of activities have been included to address a variety of learning and teaching styles. The assessment activities describe a variety of tools and methods for gathering evidence of student performance. These assessment activities are also referenced in the Assessment Overview Tables, found at the beginning of each grade in the Model.

These strategies are suggestions only, designed to provide guidance for teachers in planning instruction and assessment to meet the prescribed learning outcomes.

Assessment Instruments

Sample assessment instruments have been included at the end of each grade where applicable, and are provided to help teachers determine the extent to which students are meeting the prescribed learning outcomes. These instruments contain criteria specifically keyed to one or more of the suggested assessment activities contained in the units. Ongoing formative assessment will be required throughout the year to guide instruction and provide evidence that students have met the breadth and depth of the prescribed learning outcomes.

Classroom Assessment Model

Grade 1

Grade 1: Assessment Overview Table

The purpose of this table is to provide teachers with suggestions and guidelines for formative and summative classroom-based assessment and grading of Grade 1 Mathematics.

Curriculum Organizers	Suggested Assessment Activities		Suggested Weight for Grading	Number of Outcomes	Number of Outcomes by Domain*		
					K	U&A	HMP
Number	• class discussions • interviews • sharing strategies • questioning • observation • self assessment • journals • anecdotal comments	• object manipulation • games • portfolios • models • artwork • songs/stories • student conferences • peer assessment	65-75%	10	3	3	4
Patterns and Relations	• dramas • observation • object manipulation • peer assessment • anecdotal comments • photo evidence • interviews	• class discussion • student work • artwork • models • self assessment • portfolios	10-20%	4	0	3	1
Shape and Space	• class discussion • models/ constructions • self assessment • peer assessment • anecdotal comments	• centres/stations • artwork • portfolios • photo evidence • interviews	10-20%	4	1	3	0
		Totals	100%	18	4	9	5

* The following abbreviations are used to represent the three cognitive levels within the cognitive domain: K = Knowledge; U&A = Understanding and Application; HMP = Higher Mental Processes.

Grade 1

Overview

Learning at Previous Grades

- number sequence forward and backward to 10
- familiar number arrangements
- one-to-one correspondence
- repeating patterns of 2 or 3 elements
- direct comparison for length, mass and volume
- single attribute of a 3-D objects

Curriculum Correlation

The following table shows which curriculum organizers and suborganizers are addressed by each unit in this grade of the Classroom Assessment Model. Note that some curriculum organizers/suborganizers are addressed in more than one unit. Grey shading on the table indicates that the organizer or suborganizer in question is not addressed at this grade level.

	Early Numeracy	Number of the Day	Comparing Quantities	Math Story Time	Everyday Estimating	Number Balancing	Patterns in Your World	Sort By Length	Copy Me
Number	X	X	X	X	X	X			
Patterns and Relations *Patterns*	X					X	X		
Variables and Equations									
Space and Shape *Measurement*								X	
3-D Objects and 2-D Shapes									X
Transformations									
Statistics and Probability *Data Analysis*									
Chance and Uncertainty									

Early Numeracy

Prescribed Learning Outcomes

It is expected that students will:

A1 say the number sequence, 0 to 100, by
- 1s forward and backward between any two given numbers
- 2s to 20, forward starting at 0
- 5s and 10s to 100, forward starting at 0 [C, CN, V, ME]

A2 recognize, at a glance, and name familiar arrangements of 1 to 10 objects or dots [C, CN, ME, V]

A3 demonstrate an understanding of counting by
- indicating that the last number said identifies "how many"
- showing that any set has only one count
- using the counting on strategy
- using parts or equal groups to count sets [C, CN, ME, R, V]

A4 represent and describe numbers to 20 concretely, pictorially, and symbolically [C, CN, V]

A5 compare sets containing up to 20 elements to solve problems using
- referents
- one-to-one correspondence [C, CN, ME, PS, R, V]

A6 estimate quantities to 20 by using referents [C, ME, PS, R, V]

A7 demonstrate, concretely and pictorially, how a given number can be represented by a variety of equal groups with and without singles [C, R, V]

A8 identify the number, up to 20, that is one more, two more, one less, and two less than a given number. [C, CN, ME, R, V]

B1 demonstrate an understanding of repeating patterns (two to four elements) by
- describing
- reproducing
- extending
- creating

patterns using manipulatives, diagrams, sounds, and actions [C, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES

- Periodically, students may need more detailed, individual, or small-group assessments as indicators of performance level and areas of weakness. The BC Early Numeracy Project (K-1) "…was designed to be used at the end of Kindergarten or early grade one, with a focus on identifying children at risk in mathematics." (Assessing Early Numeracy(RB 0152): BC Early Numeracy Project (K-1), 2003, p.3)

  Use assessment resources developed as part of the BC Early Numeracy Project (K-1) to set appropriate tasks and assess students' level of understanding and/or areas of weakness with respect to chosen concepts: *Assessing Early Numeracy* (RB 0152), *Supporting Early Numeracy* (RB 0153), *Whole Group Follow-Up* (RB 0154)

Number of the Day

Prescribed Learning Outcomes

It is expected that students will:

A1 say the number sequence, 0 to 100, by
- 1s forward and backward between any two given numbers
- 2s to 20, forward starting at 0
- 5s and 10s to 100, forward starting at 0 [C, CN, V, ME]

A2 recognize, at a glance, and name familiar arrangements of 1 to 10 objects or dots [C, CN, ME, V]

A3 demonstrate an understanding of counting by
- indicating that the last number said identifies "how many"
- showing that any set has only one count
- using the counting on strategy
- using parts or equal groups to count sets [C, CN, ME, R, V]

A4 represent and describe numbers to 20 concretely, pictorially, and symbolically [C, CN, V]

A6 estimate quantities to 20 by using referents [C, ME, PS, R, V]

A8 identify the number, up to 20, that is one more, two more, one less, and two less than a given number. [C, CN, ME, R, V]

A9 demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially, and symbolically by
- using familiar and mathematical language to describe additive and subtractive actions from their experience
- creating and solving problems in context that involve addition and subtraction
- modelling addition and subtraction using a variety of concrete and visual representations, and recording the process symbolically [C, CN, ME, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Use calendar activities to provide opportunity for practice of number sequences and number patterns. Choose a number of the day (e.g., from calendar, attendance, days in school, weather tally). Have individual students - count up/down from that number in a variety of ways (e.g., by 1s, 2s, 5s). - identify the numbers before and after the given number (later, small groups of students can play What's My number? where clues are given (e.g., My number is 2 more/less than 5.) - count onward to a special event on the calendar and use counters to represent the number of days until that event - add tally marks (e.g., ~~////~~, forming groups of five) to represent each day's weather over a period of time (e.g., weeks); eventually, use the tallies to have students practice counting by 5s and group tallies into 10 groups with a circle.	• Students should be observed for evidence that they can - follow the counting sequence - recognize that the last number said identifies "how many" - count forwards by 1s from any number - count backwards by 1s from any number - identify the numbers 1 or 2 more and less from the number (1-20) - count by 2s, 5s, or 10s Have students keep a math portfolio as a way of organizing evidence of their learning. The portfolio can be as simple as a scrapbook, file folder, or accordion file. This portfolio can include work samples, photos, anecdotal notes/ evidence, self/peer assessments, checklists, etc. Anecdotal records could be kept and added to their portfolio.

<table>
<tr><th>Planning for Assessment</th><th>Assessment Strategies</th></tr>
<tr><td>
• After choosing the Number of the Day, students can be challenged to make a collage of the number by cutting photos out of a magazine. Use this opportunity to highlight the importance of numbers in a variety of cultures. For example, the number four has significance in Aboriginal cultures when examining the seasons, directions, elements (air, fire, wind and water).
• In their Journal/work page the class can be given the task of printing the Number of the Day and then finding 10 different ways of making that number using simple addition and subtraction facts. These numeric sentences could then be read.
</td><td>
• Using the work samples, look for evidence that the students are
- accurately representing the numbers concretely, pictorially, and symbolically
- using familiar mathematical language for addition actions
- using familiar mathematical language for subtraction actions
- able to justify their solutions using concrete objects or pictures
Work samples can be added to students' math portfolios.
</td></tr>
<tr><td>
• The 100th day of school is an opportunity to motivate the students to use mathematics in a meaningful way. Have students
- make a number line to count the days in school; numbers can be coloured, underlined, circled, or bolded indicating counting by 5s, and 10s; students can be a part of printing/coding the numbers; the class counts and claps as they reach marked numbers depending on the counting pattern
- collect 100 things from home; at school on the 100th day they sort the items into groups of 10 on a Sorting Mat similar to the following (with decades printed):
<table>
<tr><td>50</td><td>100</td></tr>
<tr><td>40</td><td>90</td></tr>
<tr><td>30</td><td>80</td></tr>
<tr><td>20</td><td>70</td></tr>
<tr><td>10</td><td>60</td></tr>
</table>
- work in pairs to print one decade of numbers on individual cards; the class can glue their numbers in the appropriate spot to make a complete 100's chart.
- working in pairs, one partner picks 2 numbers one line apart on a 100 chart; the other needs to say the numbers between the 2
</td><td>
• Students should be observed for evidence that they can
- follow the counting sequence
- count forwards by 1s from any number
- count backwards by 1s from any number
- count by 5s
- count by 10s
- read the numerals 0-100
- write the numerals 0-100
Notes can be added to students' math portfolios.
</td></tr>
</table>

Comparing Quantities

Prescribed Learning Outcomes

It is expected that students will:

A5 compare sets containing up to 20 elements to solve problems using
- referents
- one-to-one correspondence [C, CN, ME, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Using an interview with individual or small group of students, present a dot cards with given quantities and manipulatives. Ask the student(s) a set of questions designed to assess their level of understanding of creating equal sets, sets with more or less, and solving a problem involving the comparison of 2 quantities.	• Questions could include the following: - Make a group of counters that has the same number as mine. How do you know it's the same? - Present 2 different dot cards. Which group has more? Less? How do you know? - Make a group with 2 more than mine. How do you know? - Make a group with 2 fewer than mine. How do you know?
• Present a story problem using 2 different dot cards (e.g., The first dock has this many canoes, and the second dock has this many. Which dock has more/fewer canoes?) You may find that using little pictures of canoes on the cards instead of dots can help reduce confusion for students. Challenge the students to create their own more/less/same problem stories including a visual representation of the numbers included in the story (e.g., How many hands in my family? Which bear has more honey pots?).	• Students should be able to - identify the card with more dots by either using one-to-one correspondence or counting - present a clear problem - make an accurate visual representation - be able to explain their solution Work samples can be added to students' math portfolios.

Math Story Time

Prescribed Learning Outcomes

It is expected that students will:

A10 describe and use mental mathematics strategies (memorization not intended), such as
- counting on and counting back
- making 10
- doubles
- using addition to subtract

to determine the basic addition facts to 18 and related subtraction facts [C, CN, ME, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• When reading and listening to math literature, ask students to represent the mathematics presented in the story. For example, represent 5 monkeys jumping on the bed with 1 falling off, concretely and pictorially. Follow up by posing additional problems related to the story to build on other strategies for addition or subtraction (e.g., Doubles: How many eyes are on the bed?). Students can also represent these new numbers in that problem concretely or pictorially. Ask students to suggest a fast way to find the answer.	• Students should be observed for evidence that they can solve problems using - making a visual of the problem - building up and down - counting on and back - knowing/using doubles - using anchors of 5 and 10 - connecting addition and subtraction. Individual conferences may be needed in order to determine students' abilities to use the above strategies or where errors are occurring.
• Model the process of creating their own story problems using a similar format, which they can pose to the class (e.g., If a boy had 2 wagons and 1 bicycle, how many wheels in total?). Ask students to consider how they figured it out, and whether there is another way. Then have students create their own story using the same pattern as the presented story. The students should create 2 questions related to their story to present to a partner to solve. The partner then tries to solve the problem in 2 different ways and explains how they did it. Partner share information about - how easy the problem was to understand and solve - the method used to solve the problem - how they came up with the idea for the problem.	• Ask students to think about their own learning by asking them whether it was easier to solve a problem or create a problem. Journal responses can be placed in students' math portfolios.

Everyday Estimating

Prescribed Learning Outcomes

It is expected that students will:

A6 estimate quantities to 20 by using referents [C, ME, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Using everyday classroom activities present a referent of 5 or 10 items to students and then ask them to estimate whether a set is enough for given purpose. For example: "This is 5 crayons. How many do you think are in this can? Do you think this is enough for the group?" To model and encourage the use of comparative language, use examples such as: "Take 2 steps. Now estimate how many you think it would be to cross the court. Would it be more or fewer than 15?"	• During a variety of class activities, look for - the mathematical language students use - their ability to use a referent - ability to make reasonable estimates - use of comparative language (e.g., more or less, closer to ____, about ____) One way to record student responses and understanding is to use sticky notes on a clipboard or folder, where each sticky is particular to an individual student. Make anecdotal comments about student learning and the particular language they use. Once a sticky is full it is placed in students' math portfolios.

Number Balance

Prescribed Learning Outcomes

It is expected that students will:

A2 recognize, at a glance, and name familiar arrangements of 1 to 10 objects or dots [C, CN, ME, V]

A4 represent and describe numbers to 20 concretely, pictorially, and symbolically [C, CN, V]

A7 demonstrate, concretely and pictorially, how a given number can be represented by a variety of equal groups with and without singles [C, R, V]

A9 demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially, and symbolically by
- using familiar and mathematical language to describe additive and subtractive actions from their experience
- creating and solving problems in context that involve addition and subtraction
- modelling addition and subtraction using a variety of concrete and visual representations, and recording the process symbolically [C, CN, ME, PS, R, V]

B3 describe equality as a balance and inequality as an imbalance, concretely, and pictorially (0 to 20) [C, CN, R, V]

B4 record equalities using the equal symbol [C, CN, PS, V]

PLANNING FOR ASSESSMENT

- Present students with 2 different colour stickers and sturdy paper cookie shapes. Ask the students to decorate cookie shapes with a specific number of stickers. Continue decorating cookie shapes that represent different numbers. These decorated cookies can be used in several activities. After each number is completed, analyse the different ways the number is configured (familiar arrangements) and represented by the 2 types of items (5 represented as 3 squares and 2 circles).

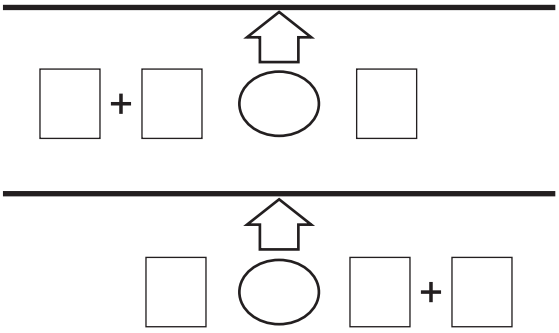
  After completing this activity for several numbers, the cookies can be used as flash cards for games or group practice recognizing at a glance familiar arrangements of numbers. They could also be used in a matching game to find cookies with the same number regardless of the configuration.

  Students print addition and subtraction sentences to match the arrangements of decorations on the cookies using an equal symbol appropriately.

ASSESSMENT STRATEGIES

- Preliminary assessment of students' understanding of the concept of conservation of number can be observed and assessed in many different classroom circumstances such as the following:
  - We are working in groups of 4 today, show me on your fingers how many books do you need for your group.
  - In the gym, organize the class in groups of 5. Have one student from each group go and get a beanbag for every child in their group.

  As students engage in activities that illustrate different ways to configure the same number, observe for evidence that they can
  - represent numbers pictorially and concretely accurately
  - recognize equal values
  - recognize quickly familiar arrangements of numbers.
  - use familiar and mathematical language to describe additive and subtractive actions
  - record equalities using the equal symbol

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• After modelling and practice with objects and a 2-pan balance, provide each student with a 2-pan balance with equal weight objects. Provide also a set of diagrams such as the following for students to complete:  Working in small groups, ask students to manipulate the weights to find one or more configurations that would match each diagram. Students would record their representation by drawing pictures on the line and putting the numerals in the boxes, including the appropriate symbol in the oval (=), making a complete number sentence.	• Circulate and have students explain their work, focussing on - representing the quantities correctly - equality and balance - inequality and imbalance - solving simple addition problems - appropriate use of the equal symbol Take notes on each student's level of understanding. Students can add their sheet to their math portfolios. Student conferences may be necessary to ask more probative questions to diagnosis areas of difficulty.

Patterns in Your World

Prescribed Learning Outcomes

It is expected that students will:

B1 demonstrate an understanding of repeating patterns (two to four elements) by
- describing
- reproducing
- extending
- creating

patterns using manipulatives, diagrams, sounds, and actions [C, PS, R, V]

B2 translate repeating patterns from one representation to another [C, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Show a variety of patterns and ask to describe why (in what way) each is a pattern. Then have students - use concrete classroom materials to create a repeating pattern at their work stations - work in pairs, taking turns to describe each other's pattern using a letter code and then show further understanding by extending the pattern on both ends - draw a pictorial reproduction of that same pattern using a different representation (e.g., colours to letters); partners then exchange papers and extend each others' patterns by at least 4 elements, identifying their work with their name to hand in; students could also discuss with their partners whether they figured out the pattern and extended it correctly.	• While students are working, look for - complexity of patterns (ABBABB vs. ABABAB) - extending the pattern on both ends - number of elements used (ABCABC vs. ABABAB) - ability to describe their pattern Early on in the student's understanding focus will be on 2-element patterns (ABABAB). As their sense of pattern grows they will begin creating complex patterns using more elements. The pictorial reproduction and some photographic evidence of completed patterns can be placed in students' math portfolios. Have students complete a pictorial self-assessment checklist to record their abilities to - find a pattern - tell about the pattern they find - change the pattern - extend the pattern Provide opportunities for students to share their self-assessments with partners. Conduct interviews to ensure the checklist is completed properly and accurately. The self assessment can be a part of students' math portfolios.
• Interview students to check their understanding and justify their self-assessment.	• Possible interview questions may include - Can you identify the missing element (cover 1 or 2 elements)? How do you know? - I have extended your pattern. Have I done a good job? Tell me why. - You said you changed how you showed your pattern. How did you show this change?

Sort by Length

Prescribed Learning Outcomes

It is expected that students will:

C1 demonstrate an understanding of measurement as a process of comparing by
- identifying attributes that can be compared
- ordering objects
- making statements of comparison
- filling, covering, or matching [C, CN, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Provide opportunities for students to explore and practise measurement using direct comparison, by looking and touching concrete materials to compare their length, weight, and area. Set up a sort by length activity consisting of a can of straws, scissors, tape, and blank paper. Students take 3 straws and cut 2 of them to get a total of 5 segments. They take the segments and order them from shortest to longest by matching. Students then tape the ordered straw segments onto a blank page. Replicate this idea but replace sorting objects by length with mass (heaviest/lightest), volume (holding most/least), or area (being covered by most/least tiles).	• Watch for evidence that the students are able to - use common attributes of measurement (length, mass, volume, etc.) when measuring - use comparative language (longer, heavier, holds more) - can order objects by attribute (e.g., from largest to smallest and smallest to largest) - directly compare objects to verify the comparison and justify the solution - explain their reasoning when making statements involving comparative measurements During an interview ask students why they ordered the way they did, and how they decided where to put their straws? Students can add the page to their math portfolios. Not all activities would need a collectable work sample for a portfolio; instead, students could give a verbal explanation after completing the task. Anecdotal records could be kept and added to their portfolio or file.

Copy Me

Prescribed Learning Outcomes

It is expected that students will:

C2 sort 3-D objects and 2-D shapes using one attribute, and explain the sorting rule [C, CN, R, V]
C3 replicate composite 2-D shapes and 3-D objects [CN, PS, V]
C4 compare 2-D shapes to parts of 3-D objects in the environment [C, CN, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Set out a variety of 2-D shapes on a surface with a ring for the purpose of categorizing the shapes. Without telling the students your sorting rule, choose 2 or 3 shapes that have a same property to sort into the ring. Then ask students continue to sort appropriate shapes to match the rule. Then have students try and verbalize the sorting rule you used.	• As students participate in the sorting activity, check the extent to which they are able to - sort, following an established sorting rule or principle - verbalize what they are doing (i.e., identify individual attributes that are similar or different) and explain their reasoning
• After sufficient practice, have the students work in small groups with one child creating a 'secret rule' for sorting their shapes. The rest of the group tries to sort and predict the rule. Give each child a turn to be the 'secret rule maker.' This same game can be repeated using 3-D objects. Repeat the process using 2-D shapes and 3-D objects together. Students need to explain the sorting rule with specific mention to why some 2-D shapes and 3-D objects are together in the sort.	• Circulate to observe and make notes whether students are able to - create a valid "secret" sorting rule - find another student's sorting rule and follow it - explain the sorting rule Ask students to show the parts that are the same/different. Student responses might resemble the following: - There is a curve on the cone just like all the other shapes. - All these shapes and objects have pointy parts.
• Show the students a composite of 2-D shapes. Provide students with a set of shapes (paper shapes, pattern blocks, tangram shapes) and ask them to make one just like the one shown. Next, students are divided into pairs and given a limited number of shapes. One student chooses which shapes to use and constructs a composite 2-D shape. The other student will then try to duplicate the design. This activity can be repeated using 3-D objects.	• Circulate and makes notes regarding the accuracy of the duplication and level of understanding. Watch for students who may have difficulty - moving from 2-D shapes to 3-D objects - finding appropriate shapes to include - duplicating a partner's work The partner then tries to copy the shape. Partners share information about - how easy it was to make the shape - any parts that were more difficult - whether they found 2-D or 3-D more challenging Photo evidence can be used and added to students' math portfolios.
• Provide students with a set of paper 2-D shapes and have them circulate finding parts of 3-D objects in the classroom or another environment. Students should be able to explain why they made their matches.	• The Copy Me rubric (see sample supplied at the end of this grade) provides sample criteria for assessing students' level of understanding of 2-D shapes and 3-D objects.

Copy Me

4	• Evidence is clearly communicated, students can explain their understanding without clues. • Student is able to sort, copy, explain, find and compare 2-D shapes and 3-D objects independently and shows creativity or original thinking.
3	• Evidence is clearly communicated, and students can explain their understanding with minimal clues. • Student is able to sort, copy, explain, find and compare 2-D shapes and 3-D objects independently with minimal clues.
2	• Evidence is not clearly communicated, and understanding is limited or not present. • Student is willing to attempt and complete the tasks of sorting, copying, explaining, finding and comparing 2-D shapes and 3-D objects but needs significant help to complete many tasks.
1	• Evidence is not clearly communicated, and understanding is limited or not present. • There may be attempts to sort, copy, explain, find and compare 2-D shapes and 3-D objects but has little success without one-on-one help.

LEARNING RESOURCES

This section contains general information on learning resources, and provides a link to the titles, descriptions, and ordering information for the recommended learning resources in the Mathematics K to 7 Grade Collections.

What Are Recommended Learning Resources?

Recommended learning resources are resources that have undergone a provincial evaluation process using teacher evaluators and have Minister's Order granting them provincial recommended status. These resources may include print, video, software and CD-ROMs, games and manipulatives, and other multimedia formats. They are generally materials suitable for student use, but may also include information aimed primarily at teachers.

Information about the recommended resources is organized in the format of a Grade Collection. A Grade Collection can be regarded as a "starter set" of basic resources to deliver the curriculum. In many cases, the Grade Collection provides a choice of more than one resource to support curriculum organizers, enabling teachers to select resources that best suit different teaching and learning styles. Teachers may also wish to supplement Grade Collection resources with locally approved materials.

How Can Teachers Choose Learning Resources to Meet Their Classroom Needs?

Teachers must use either

- provincially recommended resources
  OR
- resources that have been evaluated through a local, board-approved process

Prior to selecting and purchasing new learning resources, an inventory of resources that are already available should be established through consultation with the school and district resource centres. The ministry also works with school districts to negotiate cost-effective access to various learning resources.

What Are the Criteria Used to Evaluate Learning Resources?

The Ministry of Education facilitates the evaluation of learning resources that support BC curricula, and that will be used by teachers and/or students for instructional and assessment purposes. Evaluation criteria focus on content, instructional design, technical considerations, and social considerations.

> Additional information concerning the review and selection of learning resources is available from the ministry publication, Evaluating, Selecting and Managing Learning Resources: A Guide (Revised 2002) www.bced.gov.bc.ca/irp/resdocs/esm_guide.pdf

What Funding is Available for Purchasing Learning Resources?

As part of the selection process, teachers should be aware of school and district funding policies and procedures to determine how much money is available for their needs. Funding for various purposes, including the purchase of learning resources, is provided to school districts. Learning resource selection should be viewed as an ongoing process that requires a determination of needs, as well as long-term planning to co-ordinate individual goals and local priorities.

What Kinds of Resources Are Found in a Grade Collection?

The Grade Collection charts list the recommended learning resources by media format, showing links to the curriculum organizers. Each chart is followed by an annotated bibliography. Teachers should check with suppliers for complete and up-to-date ordering information. Most suppliers maintain web sites that are easy to access.

Mathematics K to 7 Grade Collections

The Grade Collections for Mathematics K to 7 include newly recommended learning resources as well as relevant resources previously recommended for prior versions of the Mathematics K to 7 curriculum. The ministry updates the Grade Collections on a regular basis as new resources are developed and evaluated.

> Please check the following ministry web site for the most current list of recommended learning resources in the Grade Collections for each IRP: www.bced.gov.bc.ca/irp_resources/lr/resource/gradcoll.htm

GLOSSARY

The British Columbia Ministry of Education recognizes the limitation of a glossary available only in print format. An online glossary has been developed by Alberta Education to support the implementation of their revised Kindergarten to Grade 9 Program of Studies. This glossary is based on the WNCP CCF for K-9 Mathematics and therefore also supports the British Columbia Mathematics K to 7 IRP.

This online glossary provides additional supports for teachers indlucing definitions, diagrams, pictures, and interactive applets that cannot be provided through a conventional print glossary. As a result, the Ministry of Education encourages educational stakeholders to access the glossary through a link which is provided on the British Columbia Ministry of Education website.

To access the glossary, follow the links for curriculum support material from the mathematics IRP main page at www.bced.gov.bc.ca/irp/irp_math.htm