

MATHEMATICS GRADE 4

Integrated Resource Package 2007



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This Integrated Resource Package (IRP) provides basic information teachers will require in order to implement Mathematics K to 7. Once fully implemented, this document will supersede Mathematics K to 7 (1995).

The information contained in this document is also available on the Internet at www.bced.gov.bc.ca/irp/irp.htm

The following paragraphs provide brief descriptions of the components of the IRP.

INTRODUCTION

The Introduction provides general information about Mathematics K to 7, including special features and requirements.

Included in this section are

- a rationale for teaching Mathematics K to 7 in BC schools
- goals for Mathematics K to 7
- descriptions of the curriculum organizers groupings for prescribed learning outcomes that share a common focus
- a suggested timeframe for each grade
- a graphic overview of the curriculum content from K to 7
- additional information that sets the context for teaching Mathematics K to 7

CONSIDERATIONS FOR PROGRAM DELIVERY

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners.

PRESCRIBED LEARNING OUTCOMES

This section contains the *prescribed learning outcomes*. Prescribed learning outcomes are the legally required content standards for the provincial education system. They define the required attitudes, skills, and knowledge for each subject. The learning outcomes are statements of what students are expected to know and be able to do by the end of the grade. The prescribed learning outcomes for the Mathematics K to 7 IRP are based on the Learning Outcomes contained within the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for K to 9 Mathematics available at www.wncp.ca.

STUDENT ACHIEVEMENT

This section of the IRP contains information about classroom assessment and measuring student achievement, including sets of specific achievement indicators for each prescribed learning outcome. Achievement indicators are statements that describe what students should be able to do in order to demonstrate that they fully meet the expectations set out by the prescribed learning outcomes. Achievement indicators are not mandatory; they are provided to assist teachers in assessing how well their students achieve the prescribed learning outcomes.

The achievement indicators for the Mathematics K to 7 IRP are based on the achievement indicators contained within the WNCP Common Curriculum Framework for K to 9 Mathematics.

The WNCP CCF for K to 9 Mathematics is available online at www.wncp.ca

Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

CLASSROOM ASSESSMENT MODEL

This section contains a series of classroom units that address the learning outcomes. The units have been developed by BC teachers, and are provided to support classroom assessment. These units are suggestions only – teachers may use or modify the units to assist them as they plan for the implementation of this curriculum.

Each unit includes the prescribed learning outcomes and suggested achievement indicators, a suggested timeframe, a sequence of suggested assessment activities, and sample assessment instruments.

LEARNING RESOURCES

This section contains general information on learning resources, providing a link to titles, descriptions, and ordering information for the recommended learning resources in the Mathematics K to 7 Grade Collections. [*Note: Grade Collections for Mathematics K to 7 will be updated as new resources matching the IRP are authorized.*]

GLOSSARY

The glossary section provides a link to an online glossary that contains definitions for selected terms used in this Integrated Resource Package



INTRODUCTION

his Integrated Resource Package (IRP) sets out the provincially prescribed curriculum for Mathematics K to 7. The development of this IRP has been guided by the principles of learning:

- Learning requires the active participation of the student.
- People learn in a variety of ways and at different rates.
- Learning is both an individual and a group process.

In addition to these three principles, this document recognizes that British Columbia's schools include young people of varied backgrounds, interests, abilities, and needs. Wherever appropriate for this curriculum, ways to meet these needs and to ensure equity and access for all learners have been integrated as much as possible into the learning outcomes and achievement indicators.

The Mathematics K to 7 IRP is based on the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for Kindergarten to Grade 9 Mathematics (May 2006). A complete list of references used to inform the revisions of the WNCP CCF for K to 9 Mathematics as well as this IRP can be found at the end of this section of the IRP.

Mathematics K to 7, in draft form, was available for public review and response from September to November, 2006. Input from educators, students, parents, and other educational partners informed the development of this document.

RATIONALE

The aim of Mathematics K to 7 is to provide students with the opportunity to further their knowledge, skills, and attitudes related to mathematics.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences. Numeracy can be defined as the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world. Numeracy is more than knowing about numbers and number operations. (British Columbia Association of Mathematics Teachers 1998)

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of a variety of manipulatives and pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics. Information gathered from these discussions can be used for formative assessment to guide instruction.

As facilitators of learning educators are encouraged to highlight mathematics concepts as they occur within the K to 7 school environment and within home environments. Mathematics concepts are present within every school's subjects and drawing students' attention to these concepts as they occur can help to provide the "teachable moment."

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary. Positive learning experiences build self-confidence and develop attitudes that value learning mathematics.

ABORIGINAL PERSPECTIVE

Aboriginal students in British Columbia come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings including urban, rural, and isolated communities. Teachers need to understand the diversity of cultures and experiences of students.

Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized. Depending on their learning styles, students may look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students. *The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks 1993).*

AFFECTIVE DOMAIN

Bloom's taxonomy of learning behaviours identified three domains of educational activities, affective (growth in feelings or emotional areas – attitude), cognitive (mental skills – knowledge), and psychomotor (manual or physical skills – skills). The affective domain involves the way in which we perceive and respond to things emotionally, such as feelings, values, appreciation, enthusiasms, motivations, and attitudes.

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Research has shown that students who are more engaged with school and with mathematics are far more likely to be successful in school and in learning mathematics. (Nardi & Steward 2003). Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

Substantial progress has been made in research in the last decade that has examined the importance and use of the affective domain as part of the learning process. In addition there has been a parallel increase in specific research involving the affective domain and its' relationship to the learning of mathematics which has provided powerful evidence of the importance of this area to the learning of mathematics (McLeod 1988, 1992 & 1994; Hannula 2002 & 2006; Malmivuori 2001 & 2006). Teachers, students, and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Students who are feeling more comfortable with a subject, demonstrate more confidence and have the opportunity for greater academic achievement (Denton & McKinney 2004; Hannula 2006; Smith et al. 1998). Educators can include opportunities for active and co-operative learning in their mathematics lessons which has been shown in research to promote greater conceptual understanding, more positive attitudes and subsequently improved academic achievement from students (Denton & McKinney 2004). By allowing the sharing and discussion of answers and strategies used in mathematics, educators are providing rich opportunities for students mathematical development. Educators can foster greater conceptual understanding in students by having students practice certain topics and concepts in mathematics in a meaningful and engaging manner.

It is important for educators, students, and parents to recognize the relationship between the affective and cognitive domains and attempt to nurture those aspects of the affective domain that contribute to positive attitudes and success in learning.

NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that are integral to the nature of mathematics, including change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty. These components are woven throughout this curriculum.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, ... can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain (Steen 1990, p. 184).

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program 2000, p. 146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms.

Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics.

These skills contribute to students' interaction with and understanding of their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

GOALS FOR MATHEMATICS K TO 7

Mathematics K to 7 represents the first formal steps that students make towards becoming life-long learners of mathematics.

GOALS FOR MATHEMATICS K TO 7

The Mathematics K-7 curriculum is meant to start students toward achieving the main goals of mathematics education:

- using mathematics confidently to solve problems
- using mathematics to better understand the world around us
- communicating and reasoning mathematically
- appreciating and valuing mathematics
- making connections between mathematics and its applications
- committing themselves to lifelong learning
- becoming mathematically literate and using mathematics to participate in, and contribute to, society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- be able to use mathematics to make and justify decisions about the world around us
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

CURRICULUM ORGANIZERS

A curriculum organizer consists of a set of prescribed learning outcomes that share a common focus. The prescribed learning outcomes for Mathematics K to 7 progress in age-appropriate ways, and are grouped under the following curriculum organizers and suborganizers:

Curriculum Organizers and Suborganizers

MATHEMATICS K-7

Number

PATTERNS AND RELATIONS

- Patterns
- Variables and Equations

Shape and Space

- Measurement
- 3-D Objects and 2-D Shapes
- Transformations

STATISTICS AND PROBABILITY

- Data Analysis
- Chance and Uncertainty

These curriculum organizers reflect the main areas of mathematics that students are expected to address. The ordering of organizers, suborganizers, and outcomes in the Mathematics K to 7 curriculum does not imply an order of instruction. The order in which various topics are addressed is left to the professional judgment of teachers. Mathematics teachers are encouraged to integrate topics throughout the curriculum and within other subject areas to emphasize the connections between mathematics concepts.

Number

Students develop their concept of the number system and relationships between numbers. Concrete, pictorial and symbolic representations are used to help students develop their number sense. Computational fluency, the ability to connect understanding of the concepts with accurate, efficient and flexible computation strategies for multiple purposes, is stressed throughout the number organizer with an emphasis on the development of personal strategies, mental mathematics and estimation strategies.

The Number organizer does not contain any suborganizers.

Patterns and Relations

Students develop their ability to recognize, extend, create, and use numerical and non- numerical patterns to better understand the world around them as well as the world of mathematics. This organizer provides opportunities for students to look for relationships in the environment and to describe the relationships. These relationships should be examined in multiple sensory forms.

The Patterns and Relations organizer includes the following suborganizers:

- Patterns
- Variables and Equations

Shape and Space

Students develop their understanding of objects and shapes in the environment around them. This includes recognition of attributes that can be measured, measurement of these attributes, description of these attributes, the identification and use of referents, and positional change of 3-D objects and 2-D shapes on the environment and on the Cartesian plane.

The Shape and Space organizer includes the following suborganizers:

- Measurement
- 3-D Objects and 2-D Shapes
- Transformations

Statistics and Probability

Students collect, interpret and present data sets in relevant contexts to make decisions. The development of the concepts involving probability is also presented as a means to make decisions. The Shape and Space organizer includes the following suborganizers:

- Data Analysis
- Chance and Uncertainty

Key Concepts: Overview of Mathematics K to 7 Topics

	Kindergarten	Grade 1	Grade 2	Grade 3
Number	 number sequence to 10 familiar number arrangements up to 5 objects one-to-one correspondence numbers in- depth to 10 	 skip counting starting at 0 to 100 arrangements up to 10 objects numbers in- depth to 20 addition & subtraction to 20 mental math strategies to 18 	 skip counting at starting points other than 0 to 100 numbers in-depth to 100 even, odd & ordinal numbers addition & subtraction to 100 mental math strategies to 18 	 skip counting at starting points other than 0 to 1000 numbers in-depth to 1000 addition & subtraction to 1000 mental math strategies for 2-digit numerals multiplication up to 5 × 5 representation of fractions
PATTERNS & RELATIONS Patterns	• repeating patterns of two or three elements	 repeating patterns of two to four elements representation of pattern 	repeating patterns of three to five elementsincreasing patterns	increasing patternsdecreasing patterns
PATTERNS & RELATIONS Variables & Equations		 equalities & inequalities symbol for equality 	 equality & inequality symbols for equality & inequality 	• one-step addition and subtraction equations
SHAPE & SPACE Measurement	 direct comparison for length, mass & volume 	• process of measurement using comparison	 days, weeks, months, & years non-standard units of measure for length, height distance around, mass (weight) 	 non-standard & standard units of time measurements of length (cm, m) & mass (g, kg) perimeter of regular & irregular shapes
Shape & Space 3-D Objects & 2-D Shapes	• single attribute of 3-D objects	 one attribute of 3-D objects & 2-D shapes composite 2-D shapes & 3-D objects 2-D shapes in the environment 	 two attributes of 3-D objects & 2-D shapes cubes, spheres, cones, cylinders, pyramids triangles, squares, rectangles, circles 2-D shapes in the environment 	 faces, edges & vertices of 3-D objects triangles, quadrilaterals, pentagons, hexagons, octagons
SHAPE & SPACE Transformations				
STATISTICS & PROBABILITY Data Analysis			 data about self and others concrete graphs and pictographs 	first-hand databar graphs
STATISTICS & PROBABILITY Chance & Uncertainty				

Grade 4	Grade 5	Grade 6	Grade 7
 numbers in-depth to 10 000 addition & subtraction to 10 000 multiplication & division of numbers fractions less than or equal to one decimals to hundredths 	 numbers in-depth to 1 000 000 estimation strategies for calculations & problem solving mental mathematics strategies for multiplication facts to 81 & corresponding division facts mental mathematics for multiplication multiplication for 2-digit by 2-digit & division for 3-digit by 1-digit decimal & fraction comparison addition & subtraction of decimals 	 numbers in-depth greater than 1 000 000 & smaller than one thousandth factors & multiples improper fractions & mixed numbers ratio & whole number percent integers multiplication & division of decimals order of operations excluding exponents 	 divisibility rules addition, subtraction, multiplication, & division of numbers percents from 1% to 100% decimal & fraction relationships for repeating & terminating decimals addition & subtraction of positive fractions & mixed numbers addition & subtraction of integers
• patterns in tables & charts	• prediction using a pattern rule	 patterns & relationships in graphs & tables including tables of value 	• table of values & graphs of linear relations
symbols to represent unknownsone-step equations	• single-variable, one-step equations with whole number coefficients & solutions	 letter variable representation of number relationships preservation of equality 	 preservation of equality expressions & equations one-step linear equations
 digital clocks, analog clocks, & calendar dates area of regular & irregular 2-D shapes 	 perimeter & area of rectangles length, volume, & capacity 	 perimeter & area of rectangles length, volume, & capacity 	 properties of circles area of triangles, parallelograms, & circles
• rectangular & triangular prisms	 parallel, intersecting, perpendicular, vertical & horizontal edges & faces rectangles, squares, trapezoids, parallelograms & rhombuses 	 types of triangles regular & irregular polygons 	geometric constructions
line symmetry	• 2-D shape single transformation	 combinations of transformations single transformation in the first quadrant of the Cartesian plane 	 four quadrants of the Cartesian plane transformations in the four quadrants of the Cartesian plane
 many-to-one correspondence including bar graphs & pictographs 	 first-hand & second-hand data double bar graphs 	 line graphs methods of data collection graph data 	 central tendency, outliers & range circle graphs
	likelihood of a single outcome	• experimental & theoretical probability	 ratios, fractions, & percents to express probabilities two independent events tree diagrams for two independent events

MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The following seven mathematical processes should be integrated within Mathematics K to 7.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students need to be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding... Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine 1991, p. 5).

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

Even more important than performing computational procedures or using calculators is the greater facility that students need – more than ever before – with estimation and mental mathematics (NCTM May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein 2001).

Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you...?" or "How could you...?" the problem-solving approach is being modelled. Students develop their own problemsolving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K to 3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong 1993, p. 10). The use of visualization in the study of mathematics provides students with the opportunity to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to decide when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt 1989). Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

SUGGESTED TIMEFRAME

Provincial curricula are developed in accordance with the amount of instructional time recommended by the Ministry of Education for each subject area. For Mathematics K to 7, the Ministry of Education recommends a time allotment of 20% (approximately 95 hours in Kindergarten and 185 hours in Grades 1 to 7) of the total instructional time for each school year. In the primary years, teachers determine the time allotments for each required area of study and may choose to combine various curricula to enable students to integrate ideas and see the application of mathematics concepts across curricula.

The Mathematics K to 7 IRP for grades 1 to 7 is based on approximately 170 hours of instructional time to allow flexibility to address local needs. For Kindergarten, this estimate is approximately 75 hours. Based on these recommendations, teachers should be spending about 2 to 2.5 hours each week on Mathematics in Kindergarten and 4.5 to 5 hours of instructional time each week on Mathematics grades 1 to 7.

References

The following references have been used to inform the revisions of the BC Mathematics K to 7 IRP as well as the WNCP CCF for K-9 Mathematics upon which the Prescribed Learning Outcomes and Achievement Indicators are based.

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Considerations for Program Delivery

his section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners. Included in this section is information about

- alternative delivery policy
- inclusion, equity, and accessibility for all learners
- working with the Aboriginal community
- information and communications technology
- copyright and responsibility
- fostering the development of positive attitudes
- instructional focus
- applying mathematics

ALTERNATIVE DELIVERY POLICY

The Alternative Delivery policy does not apply to the Mathematics K to 7 curriculum.

The Alternative Delivery policy outlines how students, and their parents or guardians, in consultation with their local school authority, may choose means other than instruction by a teacher within the regular classroom setting for addressing prescribed learning outcomes contained in the Health curriculum organizer of the following curriculum documents:

- Health and Career Education K to 7, and Personal Planning K to 7 Personal Development curriculum organizer (until September 2008)
- Health and Career Education 8 and 9
- Planning 10

The policy recognizes the family as the primary educator in the development of children's attitudes, standards, and values, but the policy still requires that all prescribed learning outcomes be addressed and assessed in the agreed-upon alternative manner of delivery.

It is important to note the significance of the term "alternative delivery" as it relates to the Alternative Delivery policy. The policy does not permit schools to omit addressing or assessing any of the prescribed learning outcomes within the health and career education curriculum. Neither does it allow students to be excused from meeting any learning outcomes related to health. It is expected that students who arrange for alternative delivery will address the health-related learning outcomes and will be able to demonstrate their understanding of these learning outcomes.

For more information about policy relating to alternative delivery, refer to www.bced.gov.bc.ca/policy/

INCLUSION, EQUITY, AND ACCESSIBILITY FOR ALL LEARNERS

British Columbia's schools include young people of varied backgrounds, interests, and abilities. The Kindergarten to Grade 12 school system focuses on meeting the needs of all students. When selecting specific topics, activities, and resources to support the implementation of Mathematics K to 7, teachers are encouraged to ensure that these choices support inclusion, equity, and accessibility for all students. In particular, teachers should ensure that classroom instruction, assessment, and resources reflect sensitivity to diversity and incorporate positive role portrayals, relevant issues, and themes such as inclusion, respect, and acceptance.

Government policy supports the principles of integration and inclusion of students who have English as a second language and of students with special needs. Most of the prescribed learning outcomes and suggested achievement indicators in this IRP can be met by all students, including those with special needs and/or ESL needs. Some strategies may require adaptations to ensure that those with special and/or ESL needs can successfully achieve the learning outcomes. Where necessary, modifications can be made to the prescribed learning outcomes for students with Individual Education Plans.

For more information about resources and support for students with special needs, refer to www.bced.gov.bc.ca/specialed/

For more information about resources and support for ESL students, refer to www.bced.gov.bc.ca/esl/

WORKING WITH THE ABORIGINAL COMMUNITY

The Ministry of Education is dedicated to ensuring that the cultures and contributions of Aboriginal peoples in BC are reflected in all provincial curricula. To address these topics in the classroom in a way that is accurate and that respectfully reflects Aboriginal concepts of teaching and learning, teachers are strongly encouraged to seek the advice and support of local Aboriginal communities. Aboriginal communities are diverse in terms of language, culture, and available resources, and each community will have its own unique protocol to gain support for integration of local knowledge and expertise. To begin discussion of possible instructional and assessment activities, teachers should first contact Aboriginal education co-ordinators, teachers, support workers, and counsellors in their district who will be able to facilitate the identification of local resources and contacts such as Elders, chiefs, tribal or band councils, Aboriginal cultural centres, Aboriginal Friendship Centres, and Métis or Inuit organizations.

In addition, teachers may wish to consult the various Ministry of Education publications available, including the "Planning Your Program" section of the resource, *Shared Learnings* (2006). This resource was developed to help all teachers provide students with knowledge of, and opportunities to share experiences with, Aboriginal peoples in BC.

For more information about these documents, consult the Aboriginal Education web site: www.bced.gov.bc.ca/abed/welcome.htm

INFORMATION AND COMMUNICATIONS TECHNOLOGY

The study of information and communications technology is increasingly important in our society. Students need to be able to acquire and analyze information, to reason and communicate, to make informed decisions, and to understand and use information and communications technology for a variety of purposes. Development of these skills is important for students in their education, their future careers, and their everyday lives. Literacy in the area of information and communications technology can be defined as the ability to obtain and share knowledge through investigation, study, instruction, or transmission of information by means of media technology. Becoming literate in this area involves finding, gathering, assessing, and communicating information using electronic means, as well as developing the knowledge and skills to use and solve problems effectively with the technology. Literacy also involves a critical examination and understanding of the ethical and social issues related to the use of information and communications technology. Mathematics K to 7 provides opportunities for students to develop literacy in relation to information and communications technology sources, and to reflect critically on the role of these technologies in society.

COPYRIGHT AND RESPONSIBILITY

Copyright is the legal protection of literary, dramatic, artistic, and musical works; sound recordings; performances; and communications signals. Copyright provides creators with the legal right to be paid for their work and the right to say how their work is to be used. There are some exceptions in the law (i.e., specific things permitted) for schools but these are very limited, such as copying for private study or research. The copyright law determines how resources can be used in the classroom and by students at home

In order to respect copyright it is necessary to understand the law. It is unlawful to do the following, unless permission has been given by a copyright owner:

- photocopy copyrighted material to avoid purchasing the original resource for any reason
- photocopy or perform copyrighted material beyond a very small part – in some cases the copyright law considers it "fair" to copy whole works, such as an article in a journal or a photograph, for purposes of research and private study, criticism, and review
- show recorded television or radio programs to students in the classroom unless these are cleared for copyright for educational use (there are exceptions such as for news and news commentary taped within one year of broadcast that by law have record-keeping requirements – see the web site at the end of this section for more details)
- photocopy print music, workbooks, instructional materials, instruction manuals, teacher guides, and commercially available tests and examinations

- show video recordings at schools that are not cleared for public performance
- perform music or do performances of copyrighted material for entertainment (i.e., for purposes other than a specific educational objective)
- copy work from the Internet without an express message that the work can be copied

Permission from or on behalf of the copyright owner must be given in writing. Permission may also be given to copy or use all or some portion of copyrighted work through a licence or agreement. Many creators, publishers, and producers have formed groups or "collectives" to negotiate royalty payments and copying conditions for educational institutions. It is important to know what licences are in place and how these affect the activities schools are involved in. Some licences may also require royalty payments that are determined by the quantity of photocopying or the length of performances. In these cases, it is important to assess the educational value and merits of copying or performing certain works to protect the school's financial exposure (i.e., only copy or use that portion that is absolutely necessary to meet an educational objective).

It is important for education professionals, parents, and students to respect the value of original thinking and the importance of not plagiarizing the work of others. The works of others should not be used without their permission.

For more information about copyright, refer to www.cmec.ca/copyright/indexe.stm

FOSTERING THE DEVELOPMENT OF POSITIVE ATTITUDES IN MATHEMATICS

A positive attitude toward mathematics is often a result of a learning environment in the classroom that encourages students' own mathematical thinking and contributions to classroom activities and discussions. Teachers should provide a variety of instructional approaches used in the classroom in order to reach a variety of learning styles and dispositions. These include experiences that encourage students to

- enjoy and value mathematics
- develop mathematical habits of mind

- explore
- take risks
- exhibit curiosity
- make and correct errors
- persevere
- experience mathematics in non-threatening, engaging ways
- understand and appreciate the role of mathematics in human affairs

These learning opportunities enable students to gain confidence in their abilities to solve complex problems.

The assessment of attitudes is indirect, and based on inferences drawn from students' behaviour. We can see what students do and hear what they say, and from these observations make inferences and draw conclusions about their attitudes.

It is important for teachers to consider their role in developing a positive attitude in mathematics. Teachers and parents are role models from which students begin to develop their disposition toward mathematics. Teachers need to model these attitudes in order to help students develop them (Burns 2000). In this manner teachers need to "present themselves as problem solvers, as active learners who are seekers, willing to plunge into new situations, not always knowing the answer or what the outcome will be" (p. 29).

INSTRUCTIONAL FOCUS

The Mathematics K to 7 courses are arranged into a number of organizers with mathematical processes integrated throughout. Students learn in different ways and at different rates. As in other subject areas, it is essential when teaching mathematics, that concepts are introduced to students in a variety of ways. Students should hear explanations, watch demonstrations, draw to represent their thinking, engage in experiences with concrete materials and be encouraged to visualize and discuss their understanding of concepts. Most students need a range of concrete or representational experiences with mathematics concepts before they develop symbolic or abstract understanding. The development of conceptual understanding should be emphasized throughout the curriculum as a means to develop students to become mathematical problem solvers.

Teaching through Problem Solving

Problem solving should be an integral part of all mathematics classrooms. Teachers are encouraged to weave problem solving throughout all curriculum organizers in the K to 7 mathematics curriculum on a regular basis. Problem solving provides a way of helping students learn mathematics.

Hiebert et al. (1996) encourage teachers to make mathematics problematic. A problem can be defined as any task or activity for which the students have not memorized a method or rule, nor is there an assumption by the students that there is only one correct way to solve the problem (Hiebert et al. 1997). Van de Walle (2006) notes that "a problem for learning mathematics also has these features:

- The problem must begin where the students are.
- The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.
- The problem must require justifications and explanations for answers and methods. (p. 11)

Why teach through problem solving?

- The math makes more sense. When using real world math problems, students are able to make the connections between what math is and how they can apply it.
- Problems are more motivating when they are challenging. Although some students are anxious when they are not directed by the teacher, most enjoy a challenge they can be successful in solving.
- Problem solving builds confidence. It maximizes the potential for understanding as each child makes his own sense out of the problem and allows for individual strategies.
- Problem solving builds perseverance. Because an answer is not instantaneous, many children think they are unable to do the math. Through the experience of problem solving they learn to apply themselves for longer periods of time and not give up.
- Problems can provide practice with concepts and skills. Good problems enable students to learn and apply the concepts in a meaningful way and an opportunity to practice the skills.
- Problem solving provides students with insight into the world of mathematics. Mathematicians struggle to find solutions to many problems and often need to go down more than one path to arrive at a

solution. This is a creative process that is difficult to understand if one has never had to struggle.

- Problem solving provides the teacher with insight into a student's mathematical thinking. As students choose strategies and solve problems, the teacher has evidence of their thinking and can inform instruction based on this.
- Students need to practice problem solving. If we are expecting students to confront new situations involving mathematics, they need practice to become independent problem solvers (Small 2005).

Polya (1957) characterized a general method which can be used to solve problems, and to describe how problem-solving should be taught and learned. He advocated for the following steps in solving a mathematical problem:

- Understand the problem What is unknown? What is known? Is enough information provided to determine the solution? Can a figure or model be used to represent the situation?
- Make a plan Is there a similar problem that has been solved before? Can the problem be restated so it makes more sense?
- Carry out the plan Have all of the steps been completed correctly?
- Look back Do the results look correct? Is there another way to solve the problem that would verify the results?

While a number of variations of the problem solving model proposed by Polya (Van de Walle 2006; Small 2006; Burns 2000) they all have similar characteristics. The incorporation of a wide variety of strategies to solve problems is essential to developing students' ability to be flexible problem solvers.

The Mathematics K to 7 (1995) IRP provides a number of useful strategies that students can use to increase their flexibility in solving problems. These include:

- look for a pattern
- construct a table
- make an organized list
- act it out
- draw a picture
- use objects
- guess and check
- work backward
- write an equation
- solve a simpler (or similar) problem
- make a model (BC Ministry of Education 1995)

During problem-solving experiences, students are encouraged to solve problems using ways that make sense to them. As students share different ways of solving problems they can learn strategies from each other. Teachers are encouraged to facilitate this process to in an open and non-threatening environment. I this manner, students can develop a repertoire of strategies from which to draw upon when mathematical problems are presented to them.

Problem solving requires a shift in student attitudes and how teachers model these attitudes in the classroom. In order to be successful, students must develop, and teachers model, the following characteristics:

- interest in finding solutions to problems
- confidence to try various strategies
- willingness to take risks
- ability to accept frustration when not knowing
- understanding the difference between not knowing the answer and not having found it yet (Burns 2000)

Problems are not just simple computations embedded in a story nor are they contrived, that is, they do not exist outside the math classroom. Students will be engaged if the problems relate to their lives; their culture, interests, families, current events. They are tasks that are rich and open-ended so there is more than one way of arriving at a solution, or multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill or understanding. The students should not know the answer immediately. Problem solving takes time and effort on the part of the student and the teacher. Teaching thought problem solving is one of the ways that teachers can bring increased depth to the Mathematics K to 7 curriculum.

Instruction should provide an emphasis on mental mathematics and estimation to check the reasonableness of paper and pencil exercises, and the solutions to problems which are determined through the use of technology, including calculators and computers. (It is assumed that all students have regular access to appropriate technology such calculators, or computers with graphing software and standard spreadsheet programs.) Concepts should be introduced using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.

APPLYING MATHEMATICS

For students to view mathematics as relevant and useful, they must see how it can be applied in a variety of contexts. Mathematics helps students understand and interpret their world and solve problems that occur in their daily lives both within and outside of the school context.

Teachers are encouraged to incorporate, and make explicit, mathematics concepts which naturally occur across the subject areas. Possible situations where cross curricular integration may occur in K to 7 include the following:

Fine Arts

- pattern, line, and form
- fractions in rhythm and metre
- spatial awareness in dance, drama, and visual arts
- geometric shapes in visual arts, drama, and dance
- symmetry and unison
- transformations
- perspective and proportion in visual arts
- measuring and proportional reasoning for mixing and applying materials in visual arts

Health and Career Education

- creating schedules
- interpreting statistical data
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables
- using mathematics to develop a logical argument to support a position on a topic or issue

Language Arts

- reading literature with a mathematics theme
- creating a picture book or writing a story with mathematical content
- listening to stories to decode mathematical contexts
- examine the plot of a story from a mathematical perspective
- create graphic organizers provide an explanation, proof, or justification for an argument
- role-play or oral presentations of problems and solutions
- creating word walls, personal dictionaries, or glossaries of mathematics terms
- examine the roots of mathematical terms

Physical Education

- examining the benefits of various physical activity (e.g. burning calories)
- examining patterns in physical movement
- measuring distances
- estimate distances and other quantise using referents
- reading and recording dates and time

Science

- discussing the magnitude of numbers
- classifying and sorting objects
- examining patterns to make a hypothesis
- measuring quantities
- use of referents for measurement
- units and conversions between units
- reading and writing quantities in multiple formats (e.g., numerals, words)
- collecting, organizing and interpreting data charts, graphs, diagrams, and tables
- creating a logical argument to support a hypothesis
- mental mathematics for calculations

Social Studies

- discussing the magnitude of numbers and building referents for numbers
- using concepts of area, perimeter, and distances when mapping

- graphing using the Cartesian plane
- using circle concepts to explain latitude and longitude, time zones, great circle routes
- interpreting statistical data
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables
- reading and recording dates and time
- examining the history of mathematics in context of world events
- using mathematics to develop a logical argument to support a position on a topic or issue

Students can also be encouraged to identify and examine the mathematics around them. In this way, students will come to see that mathematics is present outside of the classroom. There are many aspects of students' daily lives where they may encounter mathematic such as

- making purchases
- reading bus schedules
- reading sports statistics
- interpreting newspaper and media sources
- following a recipe
- estimating time to complete tasks
- estimating quantities
- creating patterns when doodling

Making these connections explicit for students helps to solidify the importance of mathematics.



Prescribed Learning Outcomes

Prescribed learning outcomes are content standards for the provincial education system; they are the prescribed curriculum. Clearly stated and expressed in measurable and observable terms, learning outcomes set out the required attitudes, skills, and knowledge – what students are expected to know and be able to do – by the end of the subject and grade.

Schools have the responsibility to ensure that all prescribed learning outcomes in this curriculum are met; however, schools have flexibility in determining how delivery of the curriculum can best take place.

It is expected that student achievement will vary in relation to the learning outcomes. Evaluation, reporting, and student placement with respect to these outcomes are dependent on the professional judgment and experience of teachers, guided by provincial policy.

Prescribed learning outcomes for Mathematics K to 7 are presented by grade and by curriculum organizer and suborganizer, and are coded alphanumerically for ease of reference; however, this arrangement is not intended to imply a required instructional sequence.

Wording of Prescribed Learning Outcomes

All learning outcomes complete the stem, "It is expected that students will"

When used in a prescribed learning outcome, the word "including" indicates that any ensuing item **must be addressed**. Lists of items introduced by the word "including" represent a set of minimum requirements associated with the general requirement set out by the outcome. The lists are not necessarily exhaustive, however, and teachers may choose to address additional items that also fall under the general requirement set out by the outcome.

Domains of Learning

Prescribed learning outcomes in BC curricula identify required learning in relation to one or more of the three domains of learning: cognitive, psychomotor, and affective. The following definitions of the three domains are based on Bloom's taxonomy.

The **cognitive domain** deals with the recall or recognition of knowledge and the development of intellectual abilities. The cognitive domain can be further specified as including three cognitive levels: knowledge, understanding and application, and higher mental processes. These levels are determined by the verb used in the learning outcome, and illustrate how student learning develops over time.

- Knowledge includes those behaviours that emphasize the recognition or recall of ideas, material, or phenomena.
- Understanding and application represents a comprehension of the literal message contained in a communication, and the ability to apply an appropriate theory, principle, idea, or method to a new situation.
- Higher mental processes include analysis, synthesis, and evaluation. The higher mental processes level subsumes both the knowledge and the understanding and application levels.

The **affective domain** concerns attitudes, beliefs, and the spectrum of values and value systems.

The **psychomotor domain** includes those aspects of learning associated with movement and skill demonstration, and integrates the cognitive and affective consequences with physical performances.

Domains of learning and cognitive levels also form the basis of the Assessment Overview Tables provided for each grade in the Classroom Assessment Model. In addition, domains of learning and, particularly, cognitive levels, inform the design and development of the Grades 4 and 7 Foundation Skills Assessment (FSA).



PRESCRIBED LEARNING OUTCOMES Grade 4

It is expected that students will:

NUMBER

- A1 represent and describe whole numbers to 10 000, pictorially and symbolically [C, CN, V]
- A2 compare and order numbers to 10 000 [C, CN]
- A3 demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by
 - using personal strategies for adding and subtracting
 - estimating sums and differences
 - solving problems involving addition and subtraction [C, CN, ME, PS, R]
- A4 explain the properties of 0 and 1 for multiplication, and the property of 1 for division [C, CN, R]
- A5 describe and apply mental mathematics strategies, such as
 - skip counting from a known fact
 - using doubling or halving
 - using doubling or halving and adding or subtracting one more group
 - using patterns in the 9s facts
 - using repeated doubling

to determine basic multiplication facts to 9 × 9 and related division facts [C, CN, ME, PS, R]

- A6 demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by
 - using personal strategies for multiplication with and without concrete materials
 - using arrays to represent multiplication
 - connecting concrete representations to symbolic representations
 - estimating products [C, CN, ME, PS, R, V]
- A7 demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by
 - using personal strategies for dividing with and without concrete materials
 - estimating quotients
 - relating division to multiplication [C, CN, ME, PS, R, V]
- A8 demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to
 - name and record fractions for the parts of a whole or a set
 - compare and order fractions
 - model and explain that for different wholes, two identical fractions may not represent the same quantity
 - provide examples of where fractions are used [C, CN, PS, R, V]
- A9 describe and represent decimals (tenths and hundredths) concretely, pictorially, and symbolically [C, CN, R, V]
- A10 relate decimals to fractions (to hundredths) [CN, R, V]
- A11 demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by
 - using compatible numbers
 - estimating sums and differences
 - using mental math strategies
 - to solve problems [C, ME, PS, R, V]

PATTERNS AND RELATIONS

Patterns

- B1 identify and describe patterns found in tables and charts, including a multiplication chart [C, CN, PS, V]
- B2 reproduce a pattern shown in a table or chart using concrete materials [C, CN, V]
- B3 represent and describe patterns and relationships using charts and tables to solve problems [C, CN, PS, R, V]
- B4 identify and explain mathematical relationships using charts and diagrams to solve problems [CN, PS, R, V]

Variables and Equations

- B5 express a given problem as an equation in which a symbol is used to represent an unknown number [CN, PS, R]
- B6 solve one-step equations involving a symbol to represent an unknown number [C, CN, PS, R, V]

SHAPE AND SPACE

Measurement

- C1 read and record time using digital and analog clocks, including 24-hour clocks [C, CN, V]
- C2 read and record calendar dates in a variety of formats [C, V]
- C3 demonstrate an understanding of area of regular and irregular 2-D shapes by
 - recognizing that area is measured in square units
 - selecting and justifying referents for the units cm² or m²
 - estimating area by using referents for cm² or m²
 - determining and recording area (cm² or m²)
 - constructing different rectangles for a given area (cm² or m²) in order to demonstrate that many different rectangles may have the same area [C, CN, ME, PS, R, V]

3-D Objects and 2-D Shapes

C4 describe and construct rectangular and triangular prisms [C, CN, R, V]

Transformations

C5 demonstrate an understanding of line symmetry by

- identifying symmetrical 2-D shapes
- creating symmetrical 2-D shapes
- drawing one or more lines of symmetry in a 2-D shape [C, CN, V]

[C]	Communication	[ME] Mental	[PS]	Problem Solving	[T]	Technology
[CN]	Connections	Mathematics and Estimation	[R]	Reasoning	[V]	Visualization

STATISTICS AND PROBABILITY

Data Analysis

- D1 demonstrate an understanding of many-to-one correspondence [C, R, T, V]
- D2 construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions [C, PS, R, V]

CHANCE AND UNCERTAINTY

not applicable at this grade level



STUDENT ACHIEVEMENT

This section of the IRP contains information about classroom assessment and student achievement, including specific achievement indicators that may be used to assess student performance in relation to each prescribed learning outcome. Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

CLASSROOM ASSESSMENT AND EVALUATION

Assessment is the systematic gathering of information about what students know, are able to do, and are working toward. Assessment evidence can be collected using a wide variety of methods, such as

- observation
- student self-assessments and peer assessments
- quizzes and tests (written, oral, practical)
- samples of student work
- projects and presentations
- oral and written reports
- journals and learning logs
- performance reviews
- portfolio assessments

Assessment of student achievement is based on the information collected through assessment activities. Teachers use their insight, knowledge about learning, and experience with students, along with the specific criteria they establish, to make judgments about student performance in relation to prescribed learning outcomes.

Three major types of assessment can be used in conjunction with each other to support student achievement.

- Assessment for learning is assessment for purposes of greater learning achievement.
- Assessment as learning is assessment as a process of developing and supporting students' active participation in their own learning.
- Assessment of learning is assessment for purposes of providing evidence of achievement for reporting.

Assessment for Learning

Classroom assessment for learning provides ways to engage and encourage students to become involved in their own day-to-day assessment – to acquire the skills of thoughtful self-assessment and to promote their own achievement.

This type of assessment serves to answer the following questions:

- What do students need to learn to be successful?
- What does the evidence of this learning look like?

Assessment for learning is criterion-referenced, in which a student's achievement is compared to established criteria rather than to the performance of other students. Criteria are based on prescribed learning outcomes, as well as on suggested achievement indicators or other learning expectations.

Students benefit most when assessment feedback is provided on a regular, ongoing basis. When assessment is seen as an opportunity to promote learning rather than as a final judgment, it shows students their strengths and suggests how they can develop further. Students can use this information to redirect their efforts, make plans, communicate with others (e.g., peers, teachers, parents) about their growth, and set future learning goals.

Assessment for learning also provides an opportunity for teachers to review what their students are learning and what areas need further attention. This information can be used to inform teaching and create a direct link between assessment and instruction. Using assessment as a way of obtaining feedback on instruction supports student achievement by informing teacher planning and classroom practice.

Assessment as Learning

Assessment as learning actively involves students in their own learning processes. With support and guidance from their teacher, students take responsibility for their own learning, constructing meaning for themselves. Through a process of continuous self-assessment, students develop the ability to take stock of what they have already learned, determine what they have not yet learned, and decide how they can best improve their own achievement.

Although assessment as learning is student-driven, teachers can play a key role in facilitating how this assessment takes place. By providing regular opportunities for reflection and self-assessment, teachers can help students develop, practise, and become comfortable with critical analysis of their own learning.

Assessment of Learning

Assessment of learning can be addressed through summative assessment, including large-scale assessments and teacher assessments. These summative assessments can occur at the end of the year or at periodic stages in the instructional process.

Large-scale assessments, such as Foundation Skills Assessment (FSA) and Graduation Program exams, gather information on student performance throughout the province and provide information

STUDENT ACHIEVEMENT

for the development and revision of curriculum. These assessments are used to make judgments about students' achievement in relation to provincial and national standards.

Assessment of learning is also used to inform formal reporting of student achievement.

For Ministry of Education reporting policy, refer to www.bced.gov.bc.ca/policy/policies/ student_reporting.htm

Assessment for Learning	Assessment as Learning	Assessment of Learning
 Formative assessment ongoing in the classroom teacher assessment, student self-assessment, and/or student peer assessment criterion-referenced criteria based on prescribed learning outcomes identified in the provincial curriculum, reflecting performance in relation to a specific learning task involves both teacher and student in a process of continual reflection and review about progress teachers adjust their plans and engage in corrective teaching in response to formative assessment 	 Formative assessment ongoing in the classroom self-assessment provides students with information on their own achievement and prompts them to consider how they can continue to improve their learning student-determined criteria based on previous learning and personal learning goals students use assessment information to make adaptations to their learning process and to develop new understandings 	 Summative assessment occurs at end of year or at key stages teacher assessment may be either criterion- referenced (based on prescribed learning outcomes) or norm-referenced (comparing student achievement to that of others) information on student performance can be shared with parents/guardians, school and district staff, and other education professionals (e.g., for the purposes of curriculum development) used to make judgments about students' performance in relation to provincial standards

For more information about assessment for, as, and of learning, refer to the following resource developed by the Western and Northern Canadian Protocol (WNCP): *Rethinking Assessment with Purpose in Mind*.

This resource is available online at www.wncp.ca

In addition, the BC Performance Standards describe levels of achievement in key areas of learning (reading, writing, numeracy, social responsibility, and information and communications technology integration) relevant to all subject areas. Teachers may wish to use the Performance Standards as resources to support ongoing formative assessment in mathematics.

BC Performance Standards are available at www.bced.gov.bc.ca/perf_stands/

Criterion-Referenced Assessment and Evaluation

In criterion-referenced evaluation, a student's performance is compared to established criteria rather than to the performance of other students. Evaluation in relation to prescribed curriculum requires that criteria be established based on the learning outcomes.

Criteria are the basis for evaluating student progress. They identify, in specific terms, the critical aspects of a performance or a product that indicate how well the student is meeting the prescribed learning outcomes. For example, weighted criteria, rating scales, or scoring guides (reference sets) are ways that student performance can be evaluated using criteria.

Wherever possible, students should be involved in setting the assessment criteria. This helps students develop an understanding of what high-quality work or performance looks like.

Criterion	Criterion-referenced assessment and evaluation may involve these steps:			
Step 1	Identify the prescribed learning outcomes and suggested achievement indicators (as articulated in this IRP) that will be used as the basis for assessment.			
Step 2	Establish criteria. When appropriate, involve students in establishing criteria.			
Step 3	Plan learning activities that will help students gain the attitudes, skills, or knowledge outlined in the criteria.			
Step 4	Prior to the learning activity, inform students of the criteria against which their work will be evaluated.			
Step 5	Provide examples of the desired levels of performance.			
Step 6	Conduct the learning activities.			
Step 7	Use appropriate assessment instruments (e.g., rating scale, checklist, scoring guide) and methods (e.g., observation, collection, self-assessment) based on the particular assignment and student.			
Step 8	Review the assessment data and evaluate each student's level of performance or quality of work in relation to criteria.			
Step 9	Where appropriate, provide feedback and/or a letter grade to indicate how well the criteria are met.			
Step 10	Communicate the results of the assessment and evaluation to students and parents/guardians.			

Key Elements

Key elements provide an overview of content in each curriculum organizer. They can be used to determine the expected depth and breadth of the prescribed learning outcomes.

Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for developmental learning.

ACHIEVEMENT INDICATORS

To support the assessment of provincially prescribed curricula, this IRP includes sets of achievement indicators in relation to each learning outcome.

Achievement indicators, taken together as a set, define the specific level of attitudes demonstrated, skills applied, or knowledge acquired by the student in relation to a corresponding prescribed learning outcome. They describe what evidence to look for to determine whether or not the student has fully met the intent of the learning outcome. Since each achievement indicator defines only one aspect of the corresponding learning outcome, the entire set of achievement indicators should be considered when determining whether students have fully met the learning outcome. In some cases, achievement indicators may also include suggestions as to the type of task that would provide evidence of having met the learning outcome (e.g., a constructed response such as a list, comparison, or analysis; a product created and presented such as a report, poster, letter, or model; a particular skill demonstrated such as map making or critical thinking).

Achievement indicators support the principles of assessment for learning, assessment as learning, and assessment of learning. They provide teachers and parents with tools that can be used to reflect on what students are learning, as well as provide students with a means of self-assessment and ways of defining how they can improve their own achievement.

Achievement indicators are not mandatory; they are suggestions only, provided to assist in the assessment of how well students achieve the prescribed learning outcomes.

The following pages contain the suggested achievement indicators corresponding to each prescribed learning outcome for the Mathematics K to 7 curriculum. The achievement indicators are arranged by curriculum organizer for each grade; however, this order is not intended to imply a required sequence of instruction and assessment.



STUDENT ACHIEVEMENT Grade 4

KEY ELEMENTS: GRADE 4

MATHEMATICAL PROCESS (INTEGRATED)

The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the grade: communication, connections, mental mathematics and estimation, problem solving, reasoning, technology, and visualization.

NUMBER – develop number sense

- whole numbers to 10 000
- addition with answers to 10 000 and corresponding subtraction
- multiplication by 0 and 1 and division by 1
- mental mathematics strategies for multiplication facts to 9 × 9 and corresponding division facts
- multiplication of 2- or 3- digit by 1-digit
- division of 2-digit divisor by 1-digit dividend
- fractions less than or equal to one
- decimal representation to hundredths and relation to fractions
- addition and subtraction of decimals to hundredths

PATTERNS AND RELATIONS – use patterns to describe the world and solve problems

Patterns

• pattern relationships in tables and charts

Variables and Equations

- symbols to represent unknowns
- one-step equations

SHAPE AND SPACE – use direct and indirect measurement to solve problems

Measurement

- digital, analog and 24 hour clocks and calendar dates
- area of regular and irregular 2-D shapes

3-D Objects and 2-D Shapes

• rectangular and triangular prisms

Transformations

• line symmetry

STATISTICS AND PROBABILITY – collect, display and analyze data to solve problems

Data Analysis

• many-to-one correspondence including bar graphs and pictographs

NUMBER

General Outcome: Develop number sense.

Prescribed Learning Outcomes	Suggested Achievement Indicators
	The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.
It is expected that students will:	Students who have fully met the prescribed learning outcome are able to:
A1 represent and describe whole numbers to 10 000, pictorially and symbolically [C, CN, V]	 read a given four-digit numeral without using the word "and" (e.g., 5321 is five thousand three hundred twenty one, not five thousand three hundred and twenty one) write a given numeral using proper spacing without commas (e.g., 4567 or 4 567, 10 000) write a given numeral 0 – 10 000 in words represent a given numeral using a place value chart or diagrams describe the meaning of each digit in a given numeral express a given numeral in expanded notation (e.g., 321 = 300 + 20 + 1) write the numeral represented by a given expanded notation explain and show the meaning of each digit in a given 4-digit numeral with all digits the same, (e.g., for the numeral 2222, the first digit represents two thousands, the second digit two hundreds, the third digit two tens, and the fourth digit two ones)
A2 compare and order numbers to 10 000 [C, CN]	 order a given set of numbers in ascending or descending order and explain the order by making references to place value create and order three different 4-digit numerals identify the missing numbers in an ordered sequence or on a number line identify incorrectly placed numbers in an ordered sequence or on a number line
 A3 demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by using personal strategies for adding and subtracting estimating sums and differences solving problems involving addition and subtraction [C, CN, ME, PS, R] 	 explain how to keep track of digits that have the same place value when adding numbers, limited to 3- and 4-digit numerals explain how to keep track of digits that have the same place value when subtracting numbers, limited to 3- and 4-digit numerals describe a situation in which an estimate rather than an exact answer is sufficient estimate sums and differences using different strategies (e.g., front-end estimation and compensation) solve problems that involve addition and subtraction of more than 2 numbers

[C] (Communication	[ME] Mental Mathematics and	[PS]	Problem Solving	[T]	Technology
[CN] (Connections	Estimation	[R]	Reasoning	[V]	Visualization

Presc	ribed Learning Outcomes	Suggested Achievement Indicators
1 : pi	xplain the properties of 0 and for multiplication, and the roperty of 1 for division C, CN, R]	 explain the property for determining the answer when multiplying numbers by one explain the property for determining the answer when multiplying numbers by zero explain the property for determining the answer when dividing numbers by one
m - - - to m re	escribe and apply mental hathematics strategies, such as skip counting from a known fact using doubling or halving using doubling or halving and adding or subtracting one more group using patterns in the 9s facts using repeated doubling determine basic pultiplication facts to 9 × 9 and elated division facts C, CN, ME, PS, R]	 □ provide examples for applying mental mathematics strategies: - doubling (e.g., for 4 × 3, think 2 × 3 = 6, and 4 × 3 = 6 + 6 - doubling and adding one more group (e.g., for 3 × 7, think 2 × 7 = 14, and 14 + 7 = 21 - use ten facts when multiplying by 9 (e.g., for 9 × 6, think 10 × 6 = 60, and 60 - 6 = 54; for 7 × 9, think 7 × 10 = 70, and 70 - 7 = 63 - halving (e.g., if 4 × 6 is equal to 24, then 2 × 6 is equal to 12 - relating division to multiplication (e.g., for 64 ÷ 8, think 8 × □ = 64)
of 1- - -	emonstrate an understanding f multiplication (2- or 3-digit by digit) to solve problems by using personal strategies for multiplication with and without concrete materials using arrays to represent multiplication connecting concrete representations to symbolic representations estimating products C, CN, ME, PS, R, V]	 model a given multiplication problem using the distributive property (e.g., 8 × 365 = (8 × 300) + (8 × 60) + (8 × 5)) use concrete materials, such as base ten blocks or their pictorial representations, to represent multiplication and record the process symbolically create and solve a multiplication problem that is limited to 2- or 3-digits by 1-digit estimate a product using a personal strategy (e.g., 2 × 243 is close to or a little more than 2 × 200, or close to or a little less than 2 × 250) model and solve a given multiplication problem using an array and record the process solve a given multiplication problem and record the process
of uj pi - -	emonstrate an understanding f division (1-digit divisor and p to 2-digit dividend) to solve roblems by using personal strategies for dividing with and without concrete materials estimating quotients elating division to multiplication <i>C</i> , CN, ME, PS, R, V]	 (It is not intended that remainders be expressed as decimals or fractions.) solve a given division problem without a remainder using arrays or base ten materials solve a given division problem with a remainder using arrays or base ten materials solve a given division problem using a personal strategy and record the process create and solve a word problem involving a 1- or 2-digit dividend estimate a quotient using a personal strategy (e.g., 86 ÷ 4 is close to 80 ÷ 4 or close to 80 ÷ 5)

Prescribed Learning Outcomes	Suggested Achievement Indicators
 A8 demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to name and record fractions for the parts of a whole or a set compare and order fractions model and explain that for different wholes, two identical fractions may not represent the same quantity provide examples of where fractions are used [C, CN, PS, R, V] 	 represent a given fraction using concrete materials identify a fraction from its given concrete representation name and record the shaded and non-shaded parts of a given set name and record the shaded and non-shaded parts of a given set represent a given fraction pictorially by shading parts of a given set represent a given fraction pictorially by shading parts of a given whole explain how denominators can be used to compare two given unit fractions with numerator 1 order a given set of fractions that have the same numerator and explain the ordering identify which of the benchmarks 0, ½, or 1 is closer to a given fraction name fractions between two given benchmarks on a number line order a given set of fractions by placing them on a number line with given benchmarks provide examples of when two identical fractions may not represent the same quantity (e.g., half of a large apple is not equivalent to half of a small apple; half of ten cloudberries is not equivalent to half of sixteen cloudberries) provide an example of a fraction that represents part of a set and a fraction that represents part of a whole from everyday contexts
A9 describe and represent decimals (tenths and hundredths) concretely, pictorially, and symbolically [C, CN, R, V]	 write the decimal for a given concrete or pictorial representation of part of a set, part of a region, or part of a unit of measure represent a given decimal using concrete materials or a pictorial representation explain the meaning of each digit in a given decimal with all digits the same represent a given decimal using money values (dimes and pennies) record a given money value using decimals provide examples of everyday contexts in which tenths and hundredths are used model, using manipulatives or pictures, that a given tenth can be expressed as hundredths (e.g., 0.9 is equivalent to 0.90 or 9 dimes is equivalent to 90 pennies)

[C] Communication	Mental [ME] Mental Mathematics and	[PS]	Problem Solving	[T]	Technology
[CN] Connections	Estimation	[R]	Reasoning	[V]	Visualization

Prescribed Learning Outcomes	Suggested Achievement Indicators
A10 relate decimals to fractions (to hundredths) [CN, R, V]	 read decimals as fractions (e.g., 0.5 is zero and five tenths) express orally and in written form a given decimal in fractional form express orally and in written form a given fraction with a denominator of 10 or 100 as a decimal express a given pictorial or concrete representation as a fraction or decimal (e.g., 15 shaded squares on a hundred grid can be expressed as 0.15 or ¹⁵/₁₀₀) express orally and in written form the decimal equivalent for a given fraction (e.g., ⁵⁰/₁₀₀ can be expressed as 0.50)
 A11 demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by using compatible numbers estimating sums and differences using mental math strategies to solve problems [C, ME, PS, R, V] 	 read decimals as fractions (e.g., 0.5 is zero and five tenths) express orally and in written form a given decimal in fractional form express orally and in written form a given fraction with a denominator of 10 or 100 as a decimal express a given pictorial or concrete representation as a fraction or decimal (e.g., 15 shaded squares on a hundred grid can be expressed as 0.15 or) express orally and in written form the decimal equivalent for a given fraction (e.g., can be expressed as 0.50)

PATTERNS AND RELATIONS (PATTERNS)

General Outcome: Use patterns to describe the world and solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
	The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.
It is expected that students will:	Students who have fully met the prescribed learning outcome are able to:
B1 identify and describe patterns found in tables and charts, including a multiplication chart [C, CN, PS, V]	 identify and describe a variety of patterns in a multiplication chart determine the missing element(s) in a given table or chart identify error(s) in a given table or chart describe the pattern found in a given table or chart
B2 reproduce a pattern shown in a table or chart using concrete materials [C, CN, V]	 create a concrete representation of a given pattern displayed in a table or chart explain why the same relationship exists between the pattern in a table and its concrete representation
B3 represent and describe patterns and relationships using charts and tables to solve problems [C, CN, PS, R, V]	 extend patterns found in a table or chart to solve a given problem translate the information provided in a given problem into a table or chart identify and extend the patterns in a table or chart to solve a given problem
B4 identify and explain mathematical relationships using charts and diagrams to solve problems [CN, PS, R, V]	 complete a Carroll diagram by entering given data into correct squares to solve a given problem determine where new elements belong in a given Carroll diagram solve a given problem using a Carroll diagram identify a sorting rule for a given Venn diagram describe the relationship shown in a given Venn diagram when the circles intersect, when one circle is contained in the other, and when the circles are separate determine where new elements belong in a given Venn diagram solve a given problem by using a chart or diagram to identify mathematical relationships

[C]	Communication	[ME] Mental Mathematics and	[PS]	Problem Solving	[T]	Technology
[CN]	Connections	Estimation	[R]	Reasoning	[V]	Visualization

PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

General Outcome: Represent algebraic expressions in multiple ways.

Prescribed Learning Outcomes	Suggested Achievement Indicators
	The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.
It is expected that students will:	Students who have fully met the prescribed learning outcome are able to:
B5 express a given problem as an equation in which a symbol is used to represent an unknown number [CN, PS, R]	 explain the purpose of the symbol, such as a triangle or circle, in a given addition, subtraction, multiplication, or division equation with one unknown (e.g. 36 ÷ □ = 6) express a given pictorial or concrete representation of an equation in symbolic form identify the unknown in a story problem, represent the problem with an equation, and solve the problem concretely, pictorially, or symbolically create a problem in context for a given equation with one unknown
B6 solve one-step equations involving a symbol to represent an unknown number [C, CN, PS, R, V]	 solve a given one-step equation using manipulatives solve a given one-step equation using guess and test describe orally the meaning of a given one-step equation with one unknown solve a given equation when the unknown is on the left or right side of the equation represent and solve a given addition or subtraction problem involving a "part-part-whole" or comparison context using a symbol to represent the unknown represent and solve a given multiplication or division problem involving equal grouping or partitioning (equal sharing) using symbols to represent the unknown

SHAPE AND SPACE (MEASUREMENT)

General Outcome: Use direct or indirect measurement to solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
	The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.
It is expected that students will:	Students who have fully met the prescribed learning outcome are able to:
C1 read and record time using digital and analog clocks, including 24-hour clocks [C, CN, V]	 state the number of hours in a day express the time orally and numerically from a 12-hour analog clock express the time orally and numerically from a 24-hour analog clock express the time orally and numerically from a 12-hour digital clock describe time orally and numerically from a 24-hour digital clock describe time orally and numerically from a 24-hour digital clock describe time orally and numerically from a 24-hour digital clock describe time orally as "minutes to" or "minutes after" the hour explain the meaning of AM and PM, and provide an example of an activity that occurs during the AM and another that occurs during the PM
C2 read and record calendar dates in a variety of formats [C, V]	 write dates in a variety of formats (e.g., <i>yyyy/mm/dd</i>, <i>dd/mm/yyyy</i>, March 21, 2006, <i>dd/mm/yy</i>) relate dates written in the format <i>yyyy/mm/dd</i> to dates on a calendar identify possible interpretations of a given date (e.g., 06/03/04)
 C3 demonstrate an understanding of area of regular and irregular 2-D shapes by recognizing that area is measured in square units selecting and justifying referents for the units cm² or m² estimating area by using referents for cm² or m² determining and recording area (cm² or m²) constructing different rectangles for a given area (cm² or m²) in order to demonstrate that many different rectangles may have the same area [C, CN, ME, PS, R, V] 	 describe area as the measure of surface recorded in square units identify and explain why the square is the most efficient unit for measuring area provide a referent for a square centimetre and explain the choice provide a referent for a square metre and explain the choice determine which standard square unit is represented by a given referent estimate the area of a given 2-D shape using personal referents determine the area of a regular 2-D shape and explain the strategy determine the area of an irregular 2-D shape and explain the strategy construct a rectangle for a given area demonstrate that many rectangles are possible for a given area by drawing at least two different rectangles for the same given area

[C] Communication	[ME] Mental Mathematics and	[PS]	Problem Solving	[T]	Technology
[CN] Connections	Estimation	[R]	Reasoning	[V]	Visualization

SHAPE AND SPACE (3-D OBJECTS AND 2-D SHAPES)

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Prescribed Learning Outcomes	Suggested Achievement Indicators
	The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.
It is expected that students will:	Students who have fully met the prescribed learning outcome are able to:
C4 describe and construct rectangular and triangular prisms [C, CN, R, V]	 identify and name common attributes of rectangular prisms from given sets of rectangular prisms identify and name common attributes of triangular prisms from given sets of triangular prisms sort a given set of rectangular and triangular prisms using the shape of the base construct and describe a model of rectangular and triangular prisms using materials such as pattern blocks or modelling clay construct rectangular prisms from their nets construct triangular prisms from their nets identify examples of rectangular and triangular prisms found in the environment

SHAPE AND SPACE (TRANSFORMATIONS)

General Outcome: Describe and analyze position and motion of objects and shapes.

Prescribed Learning Outcomes	Suggested Achievement Indicators
	The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.
It is expected that students will:	Students who have fully met the prescribed learning outcome are able to:
 C5 demonstrate an understanding of line symmetry by identifying symmetrical 2-D shapes creating symmetrical 2-D shapes drawing one or more lines of symmetry in a 2-D shape [C, CN, V] 	 identify the characteristics of given symmetrical and non-symmetrical 2-D shapes sort a given set of 2-D shapes as symmetrical and non-symmetrical complete a symmetrical 2-D shape given half the shape and its line of symmetry identify lines of symmetry of a given set of 2-D shapes and explain why each shape is symmetrical determine whether or not a given 2-D shape is symmetrical by using a Mira or by folding and superimposing create a symmetrical shape with and without manipulatives provide examples of symmetrical shapes found in the environment and identify the line(s) of symmetry sort a given set of 2-D shapes as those that have no lines of symmetry, one line of symmetry, or more than one line of symmetry

[C]	Communication	[ME] Mental	[PS]	Problem Solving	[T]	Technology
[CN]	Connections	Mathematics and Estimation	[R]	Reasoning	[V]	Visualization

STATISTICS AND PROBABILITY (DATA ANALYSIS)

General Outcome: Collect, display and analyze data to solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
	The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.
It is expected that students will:	Students who have fully met the prescribed learning outcome are able to:
D1 demonstrate an understanding of many-to-one correspondence [C, R, T, V]	 compare graphs in which different intervals or correspondences are used and explain why the interval or correspondence was used compare graphs in which the same data has been displayed using one-to-one and many-to-one correspondences, and explain how they are the same and different explain why many-to-one correspondence is sometimes used rather than one-to-one correspondence find examples of graphs in which many-to-one correspondence is used in print and electronic media, such as newspapers, magazines and the Internet, and describe the correspondence used
D2 construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions [C, PS, R, V]	 identify an interval and correspondence for displaying a given set of data in a graph and justify the choice create and label (with categories, title, and legend) a pictograph to display a given set of data using many-to-one correspondence, and justify the choice of correspondence used create and label (with axes and title) a bar graph to display a given set of data using many-to-one correspondence, and justify the choice of area graph to display a given set of data using many-to-one correspondence, and justify the choice of interval used answer a given question using a given graph in which data is displayed using many-to-one correspondence



CLASSROOM ASSESSMENT MODEL

The Classroom Assessment Model outlines a series of assessment units for Mathematics K to 7.

These units have been structured by grade level and theme. Collectively the units address all of the prescribed learning outcomes for each grade, and provide one suggested means of organizing, ordering, and delivering the required content. This organization is not intended to prescribe a linear means of delivery. Teachers are encouraged to reorder the learning outcomes and to modify, organize, and expand on the units to meet the needs of their students, to respond to local requirements, and to incorporate relevant recommended learning resources as applicable. (See the Learning Resources section later in this IRP for information about the recommended learning resources for Mathematics K to 7). In addition, teachers are encouraged to consider ways to adapt assessment strategies from one grade to another.

Considerations for Instruction and Assessment in Mathematics K to 7

It is highly recommended that parents and guardians be kept informed about all aspects of Mathematics K to 7. Suggested strategies for involving parents and guardians are found in the Introduction to this IRP.

Teachers are responsible for setting a positive classroom climate in which students feel comfortable learning about and discussing topics in Mathematics K to 7. Guidelines that may help educators establish a positive climate that is open to free inquiry and respectful of various points of view can be found in the section on Establishing a Positive Classroom Climate in the Introduction to this IRP.

Teachers may also wish to consider the following:

- Involve students in establishing guidelines for group discussion and presentations. Guidelines might include using appropriate listening and speaking skills, respecting students who are reluctant to share personal information in group settings, and agreeing to maintain confidentiality if sharing of personal information occurs.
- Promote critical thinking and open-mindedness, and refrain from taking sides on one point of view.
- Develop and discuss procedures associated with recording and using personal information that may

be collected as part of students' work for the purposes of instruction and/or assessment (e.g., why the information is being collected, what the information will be used for, where the information will be kept; who can access it – students, administrators, parents; how safely it will be kept).

• Ensure students are aware that if they disclose personal information that indicates they are at risk for harm, then that information cannot be kept confidential. For more information, see the section on Confidentiality in the Introduction to this IRP.

Classroom Assessment and Evaluation

Teachers should consider using a variety of assessment instruments and techniques to assess students' abilities to meet the prescribed learning outcomes. Tools and techniques for assessment in Mathematics K to 7 can include

- teacher assessment tools such as observation checklists, rating scales, and scoring guides
- self-assessment tools such as checklists, rating scales, and scoring guides
- peer assessment tools such as checklists, rating scales, and scoring guides
- journals or learning logs
- video (to record and critique student demonstration or performance)
- written tests, oral tests (true/false, multiple choice, short answer)
- questionnaires, worksheets
- portfolios
- student-teacher conferences

Assessment in Mathematics K to 7 can also occur while students are engaged in, and based on the product of, activities such as

- class and group discussions
- interviews and questioning
- sharing strategies
- object manipulation
- models and constructions
- charts, graphs, diagrams
- games
 - experiments
 - artwork, songs/stories, dramas
 - centres/stations
 - demonstrations and presentations
 - performance tasks
 - projects

For more information about student assessment, refer to the section on Student Achievement, as well as to the Assessment Overview Tables in each grade of the Classroom Assessment Model.

Information and Communications Technology

The Mathematics K to 7 curriculum requires students to be able to use and analyse the most current information to make informed decisions on a range of topics. This information is often found on the Internet as well as in other information and communications technology resources. When organizing for instruction and assessment, teachers should consider how students will best be able to access the relevant technology, and ensure that students are aware of school district policies on safe and responsible Internet and computer use.

CONTENTS OF THE MODEL

Assessment Overview Tables

The Assessment Overview Tables provide teachers with suggestions and guidelines for assessment of each grade of the curriculum. These tables identify the domains of learning and cognitive levels of the learning outcomes, along with a listing of suggested assessment activities and a suggested weight for grading for each curriculum organizer.

Overview

Each grade includes an overview of the assessment units:

- Learning at Previous Grades, indicating any relevant learning based on prescribed learning outcomes from earlier grades of the same subject area. It is assumed that students will have already acquired this learning; if they have not, additional introductory instruction may need to take place before undertaking the suggested assessment outlined in the unit. Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for reinforcement and developmental learning.
- Curriculum Correlation a table that shows which curriculum organizers and suborganizers are addressed by each unit in this grade of the Classroom Assessment Model.

Prescribed Learning Outcomes

Each unit begins with a listing of the prescribed learning outcomes that are addressed by that unit. Collectively, the units address all the learning outcomes for that grade; some outcomes may appear in more than one unit. The units may not address all of the achievement indicators for each of the outcomes.

Suggested Assessment Activities

Assessment activities have been included for each set of prescribed learning outcomes and corresponding achievement indicators. Each assessment activity consists of two parts:

- Planning for Assessment outlining the background information to explain the classroom context, opportunities for students to gain and practise learning, and suggestions for preparing the students for assessment
- Assessment Strategies describing the assessment task, the method of gathering assessment information, and the assessment criteria as defined by the learning outcomes and achievement indicators.

A wide variety of activities have been included to address a variety of learning and teaching styles. The assessment activities describe a variety of tools and methods for gathering evidence of student performance. These assessment activities are also referenced in the Assessment Overview Tables, found at the beginning of each grade in the Model.

These strategies are suggestions only, designed to provide guidance for teachers in planning instruction and assessment to meet the prescribed learning outcomes.

Assessment Instruments

Sample assessment instruments have been included at the end of each grade where applicable, and are provided to help teachers determine the extent to which students are meeting the prescribed learning outcomes. These instruments contain criteria specifically keyed to one or more of the suggested assessment activities contained in the units. Ongoing formative assessment will be required throughout the year to guide instruction and provide evidence that students have met the breadth and depth of the prescribed learning outcomes.



CLASSROOM ASSESSMENT MODEL Grade 4

GRADE 4: ASSESSMENT OVERVIEW TABLE

The purpose of this table is to provide teachers with suggestions and guidelines for formative and summative classroom-based assessment and grading of Grade 4 Mathematics.

Curriculum Organizers	Suggested Asses	Suggested Assessment Activities	Suggested Weight for	Number of Outcomes	Numb	Number of Outcomes by Domain*	mes by
			Grading	Carconica	K	U&A	HMP
NUMBER	 checklists observations interviews, questioning portfolios games models 	 journals/logs demonstrations presentations projects self-assessment peer assessment 	45-55%	11	4	a	7
PATTERNS AND RELATIONS	 observations interviews portfolios journals/logs graphic organizers 	 games checklists models questioning self-assessment peer assessment 	10-20%	9	0	4	2
SHAPE AND SPACE	 interviews portfolios journals/logs self-assessments peer assessments games 	 checklists models demonstrations presentations graphic organizers discussions 	25-35%	u	n	1	1
STATISTICS AND PROBABILITY	 observations interviews discussions self-assessments checklists 	 journals/logs questioning presentations performance tasks projects 	5-10%	2	0	1	1
		Totals	100%	24	7	11	9

* The following abbreviations are used to represent the three cognitive levels within the cognitive domain: K = Knowledge; U & A = Understanding and Application; HMP = Higher Mental Processes.

OVERVIEW

Learning at Previous Grades

- whole numbers to 1000
- skip counting
- referents to 1000
- place value to 1000
- mental mathematics for adding and subtracting 2-digit numerals
- addition with answers to 1000 and corresponding subtraction
- mental math strategies for addition facts to 18 and corresponding subtraction facts
- multiplication to 5×5 and corresponding division
- fraction representation
- increasing patterns
- decreasing patterns
- one-step addition and subtraction equations involving symbols for the unknown
- non-standard and standard units of time
- measurements of length (cm, m) and mass (g, kg)
- perimeter of regular and irregular shapes
- faces, edges and vertices of 3-D objects
- triangles, quadrilaterals, pentagons, hexagons, octagons
- first hand data
- bar graphs

Curriculum Correlation

The following table shows which curriculum organizers and suborganizers are addressed by each unit in this grade of the Classroom Assessment Model. Note that some curriculum organizers/suborganizers are addressed in more than one unit. Grey shading on the table indicates that the organizer or suborganizer in question is not addressed at this grade level.

	Shapes Around Us	TV Program Infomercial	Schedule	Number Game	Equation Challenges	Racing to 100	Writing a Math Book	Concentration Game	Crossword Puzzle	Constructing Rectangles	Patterns	Data Analysis	Show What You Know	Fractions and Decimals	Can You Spot the Errors?
Number		Х		х		x	Х	Х	Х	Х			Х	Х	Х
Patterns and Relations <i>Patterns</i>	x										x				
Variables and Equations					х										
Space and Shape <i>Measurement</i>			x				х	х		х					
3-D Objects and 2-D Shapes	X														
Transformations	X						х								
Statistics and Probability Data Analysis												x			
Chance and Uncertainty															

Shapes Around Us

Prescribed Learning Outcomes

It is expected that students will:

- C4 describe and construct rectangular and triangular prisms [C, CN, R, V]
- C5 demonstrate an understanding of line symmetry by
 - identifying symmetrical 2-D shapes
 - creating symmetrical 2-D shapes
 - drawing one or more lines of symmetry in a 2-D shape [C, CN, V]
- B4 identify and explain mathematical relationships using charts and diagrams to solve problems [CN, PS, R, V]

PLANNING FOR ASSESSMENT	Assessment Strategies
Have students identify real-world examples of triangular and rectangular prisms. They then describe their common attributes and record this information on a graphic organizer such as a Frayer model:	 Look for evidence that the students are providing real world examples of triangular and rectangular prisms clearly describing common attributes of triangular and rectangular prisms and are using appropriate vocabulary such as faces, edges, vertices able to identify non-examples of rectangular and triangular prisms and give reasons why these are not prisms able to sort these prisms using the shapes of their bases able to explain why the entry for a particular part of the Frayer model is correct
Examples Non-examples Have them sort these on a Venn diagram using the shapes of the bases.	

PLANNING FOR ASSESSMENT	Assessment Strategies
 Have students construct models of rectangular and triangular prisms. Provide materials such as modelling clay, toothpicks, paper, or cardboard to create models of prisms. Nets for rectangular and triangular prisms could be given to students to create paper models of these prisms. Encourage students to use vocabulary such as edges, vertices, parallel faces, etc to describe their creations. 	 Interview, conversation, and discussion prompts might include the following: Tell me about your model. Describe it. How do you know this is an example of a rectangular (or triangular) prism? What features or attributes of a rectangular/ triangular prism does it have? Which objects in the real world look like these prisms? How do you know? Why do you think that most containers, packages are shaped like rectangular prisms? When assessing student models of triangular and rectangular prisms, look for evidence that models constructed include the attributes of rectangular and triangular prisms and are examples of rectangular or triangular prisms student persevered during the activity student engaged in conversation with peers about the activity and used mathematical vocabulary to describe his/her creations Include photographs of student models in their math portfolios. Suggested comments for students to attach to photographs might include the following: This is an example of I want you to notice that Some things I know about rectangular (or triangular prisms) are Some things I know about rectangular (or triangular prisms because I know that Some things that are different about rectangular prisms because I know that Some things that are different about rectangular prisms and triangular prisms are Here are some examples of things around me that are rectangular prisms. I think most packages and containers are shaped like rectangular prisms because

PLANNING FOR ASSESSMENT	Assessment Strategies
 Present students with examples of Canadian Aboriginal art. These may include paintings, tapestries or totem poles. On a graphic organizer such as a t-chart, have them sort the shapes into these categories: non-symmetrical, 1 line of symmetry, 2 or more lines of symmetry. Students could use Miras or folding strategies to determine whether or not shapes are symmetrical. At the bottom of the chart, have students explain in writing how they know a shape is symmetrical or not. Have students use the shapes that are common to Northwest coastal Aboriginal art to draw their own design of an animal. Students should use both symmetrical (with one or more lines of symmetry) and non-symmetrical shapes. (Students may draw these by hand or using a computer drawing program.) A unit on symmetry has also been created in <i>Shared Learnings</i> (pp. 134-136) which teachers may want to adapt to meet the needs of students at a Grade 4 level. 	 Circulate as students complete the task and verify through conversations and student demonstrations that students are indeed able to identify symmetrical shapes and their respective lines of symmetry. Prompts to guide conversations might include the following: Tell me how you know that shape is symmetrical or not? What is the difference between a symmetrical shape and a shape that is not symmetrical? Tell me how you decided that the shape is not symmetrical? What strategies did you use, Miras, folding? Show me the lines of symmetry on this shape. Are there other lines of symmetry? How do you know? Have students write in their learning logs. They may use these sentence prompts: Something I know about symmetrical shapes are that Here is a shape that is not symmetrical Iknow I did a good job because If I had to explain symmetry to someone else I would say
 With the whole class, have students play a sorting game with geometric solids or attribute blocks. Use a Carroll diagram as a gameboard. Post the gameboard for all to see. Give each student an object. On his or her turn, have the student place the object in the appropriate cell and explains his or her reasons for doing so. Play a similar game using the Venn diagram. As a variation of this game, have students play in pairs. Partner A uses a blank Carroll or Venn diagram and places several objects according to a rule that he or she has created. Partner B identifies the rule then places an object in the appropriate box. Partners switch roles. 	 While students are playing the game, look for evidence that students are able to sort shapes into Venn diagram or Carroll diagram identify a sorting rule for Venn or Carroll diagram determine where a new element belongs in a Carroll or Venn diagram explain the relationships among elements in the Venn/Carroll diagram

TV Program Infomercial

Prescribed Learning Outcomes It is expected that students will: A3 demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by - using personal strategies for adding and subtracting - estimating sums and differences - solving problems involving addition and subtraction [C, CN, ME, PS, R] A6 demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by - using personal strategies for multiplication with and without concrete materials - using arrays to represent multiplication - connecting concrete representations to symbolic representations - estimating products [C, CN, ME, PS, R, V] A7 demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by - using personal strategies for dividing with and without concrete materials - estimating quotients relating division to multiplication [C, CN, ME, PS, R, V] PLANNING FOR ASSESSMENT Assessment Strategies Have students prepare an infomercial for a • While students are presenting their infomercial, fictitious TV show, Math News, explaining their look for evidence that personal strategies for solving a given addition, - personal strategies were clearly described personal strategies were effective and solved subtraction, multiplication or division problem. Students may use concrete materials or pictures the problem accurately to demonstrate personal strategies. students included models, illustrations, symbolic representations in their descriptions of personal strategies Have students assess each others' work by completing a peer assessment sheet, using criteria established as a class such as the following: The presentation and explanations were clear. I understood what ____ was trying to say. - Here is what I think ____ said. This group used graphic presentations that were clear and had something important to show. 's strategy of ____ solved the problem correctly. - Everyone in the group worked well together. This group used appropriate mathematical vocabulary. - Something that ____ did really well was ___ - A question I would like to ask ____ about is

Schedule

Prescribed Learning Outcomes

- C1 read and record time using digital and analog clocks, including 24-hour clocks [C, CN, V] C2 read and record calendar dates in a variety of formats [C, V]

PLANNING FOR ASSESSMENT	Assessment Strategies
• Have students use a table or chart to record a schedule of their typical daily activities. Have students record the date in the format <i>yyyy/mm/dd</i> , draw the clock and state the time using both digital and 24-hour format. Activities should be labelled am or pm. Alternatively, students could construct a schedule of their ideal day; a timetable for the class; a practise schedule for sports team, etc.	 When assessing student schedules, consider whether the student is able to use or draw an analog clock and show the time accurately use digital or 24 format to represent a given time illustrate the meaning of am and pm and labelled daily activities appropriately. (e.g., breakfast is labelled using am) record the date using a variety of formats such as <i>yyyy/mm/dd</i> and <i>dd/mm/yy</i> identify possible interpretations of a given date (e.g., 06/03/04)

Number Game

Prescribed Learning Outcomes

- A1 represent and describe whole numbers to 10 000, pictorially and symbolically [C, CN, V]
- A2 compare and order numbers to 10 000 [C, CN]

Planning for Assessment	Assessment Strategies
 Have students play a card game in groups of four. Before starting the game have each player create a deck of 10 cards, labelling each card with a single digit 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9. Combine each player's deck to create a team deck of 40 cards. Have one student shuffle the deck and deal out four cards to each player. Each player then uses the digits to form the largest number possible. (e.g., If a player is dealt 3, 5, 1, 9, then the largest number he or she can create using those digits is 9531). Players compare their numbers. The player with largest number scores 4 points. The player with second largest number scores 3 points, and so on. The player with the smallest number scores 1 point. The game continues for several rounds with players adding up their scores from previous rounds. Students should record the numbers they have been creating. This information will be used at the end of the game when they reflect on their learning in their math journals. Students can decide how many rounds to play before a champion is declared. Tell students that you will be circulating and listening to see if students justify their numbers (I know this is the largest number I can create because) and challenge inaccuracies with good reasons. 	 Circulate and observe students playing the game. Look for evidence that students are able to understand that the order of the digits determines the size of the number (student should create the largest number possible using the digits he/she has been dealt) order all players' numbers from largest to smallest understand that the each digit represents a different quantity (e.g., in 2457, 2 represents 2 thousands, 4 = 4 hundreds) give reasons why their number is the largest possible one they can make given the digits they were dealt challenge other players if they have ordered the players' numbers incorrectly Students should reflect on the game by writing in their math journals, using prompts such as the following: Today I (describe how to play the game) Here are the numbers that I got The largest number I got today was (have them write out the number in at least 2 ways: numerical form, expanded notation, written form, or pictorially) I know that this is the largest number I got because Here is a drawing of that number using Dienes blocks. Circulate and listen as students play the game. Suggested prompts: Tell me how you know this is the biggest number you could create using the same digits?

Equation Challenges

Prescribed Learning Outcomes

- B5 express a given problem as an equation in which a symbol is used to represent an unknown number [CN, PS, R]
- B6 solve one-step equations involving a symbol to represent an unknown number [C, CN, PS, R, V]

PLANNING FOR ASSESSMENT • Given a story problem, have students write an equation to match the problem (e.g., there are 4 sandwiches on a tray; there were 13 at the start; some are missing). Students create an equation to match (e.g., $4 + n = 13$). This activity can be reversed. Given an equation	 Assessment Strategies Observe to what extent students were able to create an equation to match the story create a story to match the equation explain the meaning of the unknown variable solve the problem in one or more ways Have students write in their math journals using
(e.g., 5 <i>n</i> = 15), students create a story problem.	 the following the prompts: I know I am right because I Something I learned was Some strategies I used to solve the problems were I wonder Something challenging was When I don't know what to do, I
 Have students solve equations with one unknown variable such as the following (include all operations +, -, ÷, × with the unknown on both right and left sides of the equation): 16 + n = 20 23 - n = 18 5n = 25 n - 3 = 7 17 - 9 = n 12 ÷ n = 4 Given these as examples, have students create their own equations. Have students pair up and solve their partner's equations. In a variation of this task, Partner A creates a word problem from a given equation and hands the word problem only for his or her partner to solve. Partner B represents the word problem with an equation and then solves it concretely, pictorially or symbolically. 	 Verify that students are able to solve equations using the four operations with the unknown on the right or on the left side of the equation create a word problem in context for a given equation solve a given one step equation with one unknown using manipulatives, guess and test, and other strategies

PLANNING	G FOI	R As	SESSMENT	Assessment Strategies
• Give students a "function machine" such as the following:		achine" such as the	• Verify to what extent students are able to identify the mistake in the pattern and explain	
	1	4		how they correct the problem. Verify to what extent students are able to explain how they know they are right.
	2	8		
-	3	12		Have students include this entry from their journals into their math portfolios.
	4	16		
	5	20		
	6	22		
	7	28		
	8	32		
	9	36		
or journals) where	the r	nachi	fy (in their math logs ine breaks down. iting how they know	

Racing to 100

Prescribed Learning Outcomes

- A3 demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by
 - using personal strategies for adding and subtracting
 - estimating sums and differences
 - solving problems involving addition and subtraction [C, CN, ME, PS, R]
- A5 describe and apply mental mathematics strategies, such as
 - skip counting from a known fact
 - using doubling or halving
 - using doubling or halving and adding or subtracting one more group
 - using patterns in the 9s facts
 - using repeated doubling
 - to determine basic multiplication facts to 9 × 9 and related division facts [C, CN, ME, PS, R]
- A6 demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by
 - using personal strategies for multiplication with and without concrete materials
 - using arrays to represent multiplication
 - connecting concrete representations to symbolic representations
 - estimating products [C, CN, ME, PS, R, V]
- A7 demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by
 - using personal strategies for dividing with and without concrete materials
 - estimating quotients
 - relating division to multiplication [C, CN, ME, PS, R, V]

PLANNING FOR ASSESSMENT	Assessment Strategies
• Have students play a dice game using 2 dice. On his or her turn, player A may choose to add, subtract, multiply or divide the dice outcome. This is player As score for that round. On his or her next turn, Player A obtains a score for that round in the same way. He or she may then choose to add, subtract, multiply or divide the new score to his or her previous score. The first player to reach 100 wins.	 Circulate as students play game and look for evidence that students are adding, subtracting, multiplying or dividing accurately understand that adding and multiplying increase scores and that subtracting and dividing reduce scores are using mental math strategies to arrive at totals are able to estimate quotients as they are tallying their scores using mental math strategies are able to estimate products are able to estimate products are able to estimate products are using personal strategies to determine sums, differences, products and quotients are using the properties of 0 and 1 to determine products are using the property of 1 to determine quotients when dividing Circulate and interview students or invite them into a conversation to explain their thinking and justify their actions as they play the game. Suggested prompts for interviews, conversations and discussions. What would be your best move given those dice outcomes? Explain why you think that. Explain why you chose to add (subtract, multiply or divide) those dice outcomes. How will this affect your score for this round? How will this affect your total score? Explain why you chose to add (subtract, multiply or divide your scores. How will this affect your score for this round? How will this affect your total score? Explain why you chose to add (subtract, multiply or divide your scores. How will this affect your score for this round? How will this affect your total score? Explain why you chose to add (subtract, multiply or divide your scores. How will this affect your score for this round? How will it affect your score for this round? How will this affect your score? Explain why you

Writing a Math Book

Prescribed Learning Outcomes

It is expected that students will:

- A3 demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by
 - using personal strategies for adding and subtracting
 - estimating sums and differences
 - solving problems involving addition and subtraction [C, CN, ME, PS, R]
- A4 explain the properties of 0 and 1 for multiplication, and the property of 1 for division [C, CN, R]
- A6 demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by
 - using personal strategies for multiplication with and without concrete materials
 - using arrays to represent multiplication
 - connecting concrete representations to symbolic representations
 - estimating products [C, CN, ME, PS, R, V]

A7 demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by

- using personal strategies for dividing with and without concrete materials
- estimating quotients
- relating division to multiplication [C, CN, ME, PS, R, V]
- A8 demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to
 - name and record fractions for the parts of a whole or a set
 - compare and order fractions
 - model and explain that for different wholes, two identical fractions may not represent the same quantity
 - provide examples of where fractions are used [C, CN, PS, R, V]
- C1 read and record time using digital and analog clocks, including 24-hour clocks [C, CN, V]
- C5 demonstrate an understanding of line symmetry by
 - identifying symmetrical 2-D shapes
 - creating symmetrical 2-D shapes
 - drawing one or more lines of symmetry in a 2-D shape [C, CN, V]

PLANNING FOR ASSESSMENT	Assessment Strategies
• Have students create a picture book about fractions for younger students. Included in this book, will be illustrations of fractions from everyday contexts, illustrations of fractions depicting both fractional parts of a set and fractional parts of a whole.	 Students could include this picture book in their portfolios to demonstrate their learning, adding comments about what they learned from this activity. Work with students to create an assessment tool for the picture book. Ask students how they will you know they have done a good job. The following are criteria to consider including: fractions depicted include examples of fractional parts of a set as well as fractional parts of a set as well as fractional parts of a whole models of fractions match their symbolic representations contexts from real life are included as well as fractions illustrated on a model

PLANNING FOR ASSESSMENT	Assessment Strategies
 Have students create a picture book for the class library explaining any one of the following concepts: the properties of 0 and 1 for multiplication and the properties of 1 for division personal strategies for computation in addition, subtraction, multiplication and division telling time using analog, digital, and 24-hour clocks including examples of activities that occur in the am and in the pm a picture book showing symmetrical and a symmetrical designs that they have drawn as well as examples from real life contexts 	 When reviewing the books that students have created, look for evidence that they clearly explained the properties of 0 and 1 for multiplication and the property of 1 for division clearly described (verbally and pictorially) personal strategies for computation in addition, subtraction, multiplication and division clearly described how to tell time using analog, digital, and 24-hour clocks and provided examples of activities that occurred in the am and pm. were able to draw symmetrical and asymmetrical objects that are symmetrical and asymmetrical

Concentration Game

Prescribed Learning Outcomes

- A1 represent and describe whole numbers to 10 000, pictorially and symbolically [C, CN, V]
- A9 describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically [C, CN, R, V]
- A10 relate decimals to fractions (to hundredths) [CN, R, V]
- C1 read and record time using digital and analog clocks, including 24-hour clocks [C, CN, V]
- C2 read and record calendar dates in a variety of formats [C, V]

	Assessment Strategies
 PLANNING FOR ASSESSMENT Have students play a concentration game to match decimals to their pictorial representations (e.g., using a 10 × 10 grid to equal 1) or the decimal fractional form (e.g., .50, 5, ⁵/₁₀, ⁵⁰/₁₀₀) 	 ASSESSMENT STRATEGIES Circulate and look for evidence that students are matching decimals and decimal fractional forms correctly and are able to justify the matches by using models or illustrations to demonstrate equivalence of decimals and their corresponding decimal fraction form. able to explain the meaning of each digit in a given decimal able to provide examples of everyday contexts for the decimals and decimal fractions that they are pairing
• Have students play a concentration game to match a whole number to its expanded or written form (e.g., 9456 = 9 000 + 400 + 50 + 6 = nine thousand four hundred fifty six).	 Circulate and look for evidence that each student recognizes a given numeral in its expanded form can explain and show the meaning of each digit in a given numeral can read a given four digit numeral without using the word and (e.g., 5321 is five thousand three hundred twenty one, not five thousand three hundred and twenty one) can express a given numeral in written words recognizes a given numeral represented by its expanded form
• Have students play a concentration game to match calendar dates that are written in a variety of formats (e.g., <i>yyyy/mm/dd;</i> 08/03/07; <i>dd/mm/yy;</i> March 8, 2007)	 Circulate and look for evidence that students are able to write the date using a variety of formats given a date written in one format, are able to recognize the date written in different formats
• Have students play a concentration game to match times from an analog, digital and 24-hour clock.	 Observe students to note evidence of the extent to which they are able to identify the correct time when given a variety of formats recognize and correctly use am and pm provide an example of an activity that occurs in the am and in the pm

Crossword Puzzle

Prescribed Learning Outcomes

- A1 represent and describe whole numbers to 10 000, pictorially and symbolically [C, CN, V]
- A2 compare and order numbers to 10 000 [C, CN]
- A9 describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically [C, CN, R, V]
- A10 relate decimals to fractions (to hundredths) [CN, R, V]

PLANNING FOR ASSESSMENT	Assessment Strategies
 Have students design a crossword type puzzle and create clues as well as an answer key. Have students trade crosswords with partners and complete the partner's puzzle. Clues could, for example, have the numbers written out in words (e.g., two thousand two hundred twenty two) include numbers written out in expanded notation (e.g., 3000 + 500 + 60 = 8) include equality statements e.g., 25/100 is the same as what decimal? be pictorial representations (e.g., drawings of Dienes blocks showing 3 456) require the identification of missing numbers in a sequence (e.g., 7 542, 7 642, 7 742,, 7 942) 	 When assessing student work, verify to what extent the student was able to vary the clues given (clues were varied: some pictorial representations, some expanded notations clues, some written words, some sequencing, etc.) the student was able to provide an accurate answer key (e.g., How well did he/she answer his own questions) the student was able to solve a partner's puzzle As a class, create a peer assessment sheet to assess each others' puzzles. The assessment might include criteria such as the following: The clues were clear. I understood what was trying to say. 1 2 3 4 5 6 7 8 9 10 (score out of 10) Something that did really well was A questions I would like to ask is

Constructing Rectangles

Prescribed Learning Outcomes

- A6 demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by
 - using personal strategies for multiplication with and without concrete materials
 - using arrays to represent multiplication
 - connecting concrete representations to symbolic representations
 - estimating products [C, CN, ME, PS, R, V]
- C3 demonstrate an understanding of area of regular and irregular 2-D shapes by
 - recognizing that area is measured in square units
 - selecting and justifying referents for the units cm² or m²
 - estimating area by using referents for cm² or m²
 - determining and recording area (cm² or m²)
 - constructing different rectangles for a given area (cm² or m²) in order to demonstrate that many different rectangles may have the same area [C, CN, ME, PS, R, V]

PLANNING FOR ASSESSMENT	Assessment Strategies
• Given an area in square units, have students use manipulatives such as cm cubes (e.g., Dienes blocks) to construct different sized rectangles.	 Look for evidence that students are able to create many different rectangles for a given area. Some suggested questions to elicit student understanding: What are some things in the real world that have these dimensions and or this area? How do you know that you have come up with all the possible rectangles for this area? Tell me why we measure area in square units instead of units?
• Give students dimensions (length and width) of a rectangle in cm/m, and have them estimate the area of a rectangle (e.g., Estimate the area of a 13 \times 15 rectangle. Tell why you think this is a reasonable estimate. Give some examples of things in your environment that are this size.)	 When reviewing student responses look for evidence that estimates are reasonable students are able to justify their estimates the referents they have chosen are reasonable
 Present class with a pair of old jeans with a stain on it. Have students determine the area of the stain so that a patch can cover it. Have students record in their math journals the method used to determine area of the stain the area in square units how they know results are reasonable 	 When reviewing student responses to this problem, look for evidence that the method used to determine the area leads to a reasonable solution that a student's answer is reasonable that a student is able to choose appropriate tools and units to measure the stain of how clearly a student can justify his or her answer

Patterns

Prescribed Learning Outcomes

- B1 identify and describe patterns found in tables and charts, including a multiplication chart [C, CN, PS, V]
- B2 reproduce a pattern shown in a table or chart using concrete materials [C, CN, V]
- B3 represent and describe patterns and relationships using charts and tables to solve problems [C, CN, PS, R, V]

PLANNING FOR ASSESSMENT	Assessment Strategies
 Given this numerical pattern, have students extend the pattern and explain how they determined the pattern and its missing elements what real world situation could be described by this pattern. A B 1 2 2 4 3 4 Have students use manipulatives to illustrate this pattern. Ask them to describe how the concrete representation illustrates this pattern. 	 Verify that students are able to identify the pattern in the table extend the pattern determine the missing element describe the pattern by identifying a real world situations that could reflect this pattern create a concrete representation of this pattern using manipulatives
 Using a 1 – 144 grid (use a 12 × 12 grid with numbers 1 through 144), have students find all the multiples of 2 and colour them in. Have students describe the pattern (e.g., It looks like a checkerboard.). Repeat this for the multiples of 3, 4, 5, 6, 7, 8, and 9. Ask students to describe what changes they notice as the numbers increase. 	 When reviewing student work, notice to what extent students identify all (some or none) of the multiples of the given number are able to predict and extend the pattern of multiples describe pattern (clearly, partially, with difficulty) by relating it to similar designs in the real and world
 Given the 9 times table (e.g., 9 × 1 = 9, 9 × 2 = 18, 9 × 3 = 27 etc.), have students describe in writing all the patterns they can find. (e.g., the sum digits of the products equal 9) 	 Students may include this in their portfolios as a sample of their thinking. Encourage students to reflect on how their work demonstrates that they were good mathematicians (e.g., by looking for patterns, using mathematical vocabulary to describe my thinking, persevering even though the task was difficult, accepting a challenge, asking good questions; offering a conjecture) When reviewing student responses, look for evidence that the student is able to identify and describe patterns found in the 9 times table.

Data Analysis

Prescribed Learning Outcomes

- It is expected that students will:
- D1 demonstrate an understanding of many-to-one correspondence [C, R, T, V]
- D2 construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions [C, PS, R, V]

PLANNING FOR ASSESSMENT	Assessment Strategies
 Pose a question such as the following: Are you watching too much television? Have students estimate about how many hours of television (or video games/computer time) they have watched in a week, in a month. Have students construct 2 graphs for the same data: a bar graph and pictograph. The intervals in the bar graph could be drawn using one-to-one correspondence. The pictograph could be drawn using many-to-one correspondence (e.g., <i>n</i> = 5 hours). Have students use this data to draw conclusions about whether or not they are watching too much television. Have students explain which of the 2 graphs, the bar graph or the pictograph best represents their data. Students should give reasons for their choices. 	 Have students self-assess their graphs by writing in a math journal, using sentence stems such as the following: I know I constructed a good graph because

Show What You Know

Prescribed Learning Outcomes		
 It is expected that students will: A5 describe and apply mental mathematics strategies, such as skip counting from a known fact using doubling or halving using doubling or halving and adding or subtracting one more group using patterns in the 9s facts using repeated doubling to determine basic multiplication facts to 9 × 9 and related division facts [C, CN, ME, PS, R] A6 demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by using personal strategies for multiplication connecting concrete representations to symbolic representations estimating products [C, CN, ME, PS, R, V] A7 demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by using personal strategies for dividing with and without concrete materials estimating quotients relating division to multiplication [C, CN, ME, PS, R, V] 		
PLANNING FOR ASSESSMENT	Assessment Strategies	
 Conduct interviews with individuals to determine students' abilities to use mental math and/or personal strategies such as the following when solving multiplication and division equations: To assess the mental math strategy of doubling, ask students to show how they can use 2 × 3 = 6 to help find the answer to 4 × 3 (e.g., think 2 × 3 = 6, and 4 × 3 = 6 + 6). To assess the mental math strategy of doubling and adding one more group, ask students to show how you would solve 3 × 7 using this strategy (e.g., think 2 × 7 = 14, and 14 + 7 = 21). To assess skip counting from a known fact, ask students to show how they would solve 7 × 9 using this strategy (e.g., think 7 × 10 = 70, and 70 - 7 = 63). To assess relating division to multiplication, ask students to solve this problem 64 ÷ 8 by using a related multiplication fact (e.g., think 8 × □ = 64). 	 During the interview, make and record observations about how students apply mental mathematics strategies. Look for evidence of skip counting from a known fact using double or halving using doubling or halving and adding or subtracting one more group using the patterns in the 9s facts using repeated doubling student using personal strategies in combination with the mental math strategies described above student's confidence in solving the problem 	

PLANNING FOR ASSESSMENT	Assessment Strategies
 Conduct interviews with individual students to determine their understanding of multiplication. Ask students to solve problems such as Show 3 different ways you could solve this equation using concrete materials, personal strategies, distributive property (200 × 5 + 60 × 5), arrays or algorithms. Explain your thinking. Create a multiplication problem for this equation: 260 × 5 	 Look for evidence to what extent the student is able to show multiple ways to solve this problem explain his or her thinking use an array to solve this problem demonstrate personal strategies use concrete materials to solve the problem
 Conduct interviews with individual students to determine their understanding of division. Ask students to solve problems such as Show 3 different ways you could solve this equation using concrete materials, personal strategies or algorithms: 89 ÷ 5. Explain your thinking. Create a division problem for this equation. 	 Look for evidence to what extent the student is able to show multiple ways to solve this problem explain his or her thinking use an array to solve this problem demonstrate personal strategies use concrete materials to solve the problem

Fractions and Decimals

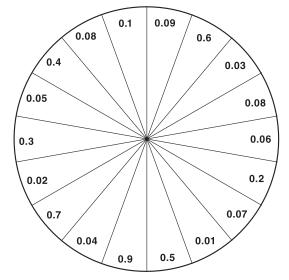
Prescribed Learning Outcomes

- A8 demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to
 - name and record fractions for the parts of a whole or a set
 - compare and order fractions
 - model and explain that for different wholes, two identical fractions may not represent the same quantity
 - provide examples of where fractions are used [C, CN, PS, R, V]
- A9 describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically [C, CN, R, V]
- A11 demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by
 - using compatible numbers
 - estimating sums and differences
 - using mental math strategies
 - to solve problems [C, ME, PS, R, V]

Assessment Strategies
 When reviewing student interpretations of the blueprints, look for evidence that students can name and record fractions for parts of a whole can name and record fractions for parts of a set explain that the denominator specifies how many pieces the whole is set is divided into explain that the numerator specifies how many parts of the whole or set we are interested in are able to solve the problem in more than one way (e.g., provides more than one example of each blueprint)
 Pose various comparison questions and look for evidence that students are making the appropriate choices (e.g., call on individual students to justify their choices). Conversation prompts might include the following Explain why you think that B is larger than A. Why would you say that the halves are not equal?

PLANNING FOR ASSESSMENT

• Have students in pairs play a game using Dienes blocks and a spinner such as the following



Students should use the Dienes blocks as follows:

- a flat $(10 \times 10 \text{ piece}) = 1.0 \text{ or the whole}$
- rods = .10
- cubes = .01

Each player starts off with a "flat" (1.0), which serves as a gameboard. The goal is to be the first player in the pair to cover the flat and reach 1.0 exactly. On their turn, students spin the spinner to determine the amount that they place on their gameboard (e.g., if the spinner lands on .03, the student adds 3 cubes onto the gameboard). Students should read the decimal on the spinner out loud to their partners (e.g., for .03 say three hundredths). If on his or her turn a player exceeds 1.0, he or she should subtract the amount and continue until the 10 × 10 gameboard is covered.

During the game, have students keep track of their progress on a recording sheet. For each turn, they write the decimal amount earned on that turn, add it to the previous amounts, and colour the corresponding quantity on a printed version of a 10×10 grid, using a different colour for each turn. Once again, if on a turn, adding the amount indicated by the spinner were to cause the student's total to exceed 1.0 (on the 10 $\times 10$ grid), the student will instead subtract this from her or his previous total and carry on.

ASSESSMENT **S**TRATEGIES

- Circulate while students play the game. Ask students to explain the meaning of each digit in a given decimal. Ask students to suggest everyday contexts in which tenths and hundredths are used. Assess students on the basis of their abilities to
 - write the decimal for a given pictorial representation of part of a region
 - use concrete materials or pictorial representations to illustrate a given decimal
 - explain the meaning of each digit in a given decimal (e.g., given .03, 0 stands for no tenths and 3 stands for 3 out of hundred or three hundredths)
 - add or subtract decimal amounts
 - use money values to represent a given decimal (e.g., If the pieces on your gameboard were worth money, how much would you have right now?)
 - provide examples of everyday contexts in which tenths and hundredths are used – real-life situations that could be true for the amount represented on the gameboard (e.g., .25 = 25 cents or 25 out of 100 candies from a package)
 - illustrate using manipulatives or pictures how a given tenth can be expressed as equivalent hundredths (e.g., 0.9 is equivalent to 0.9 or 9 dimes or 90 pennies)

PLANNING FOR ASSESSMENT	Assessment Strategies
 Play a game with the whole class in which students use tile or pattern blocks to illustrate fractional amounts. Place fractions on a number line labelled 0, ½ and 1. Place some in the correct spots, some in the wrong places (e.g., place % between 0 and ½). Have students illustrate the specified amount with their manipulatives. Then have them close their eyes and respond by showing thumbs up to indicate agreement with your placement or thumbs down to show that they disagree. Students can then play this game in pairs taking turns placing fractions on the number line and responding. 	 As students play the game, look for evidence that they have a sense of the size of a fraction, .i.e. that it is larger than ½, smaller than a half, closer to one, closer to zero understand that the denominator specifies how many pieces the whole is set is divided into understand that the numerator specifies how many parts of the whole or set Notice which students need continuing support with manipulatives in order to complete this task.

Can You Spot the Errors?

Prescribed Learning Outcomes

- A11 demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by
 - using compatible numbers
 - estimating sums and differences
 - using mental math strategies
 - to solve problems [C, ME, PS, R, V]

PLANNING FOR ASSESSMENT	Assessment Strategies
 Have students correct a fictitious student's worksheet, supplying written explanations about why a given answer is incorrect. The fictitious worksheet might contain entries such as the following: (a) 56 ×15/280 +56/336 (b) 1.560 1.23 18 17.01 (c) 250 ÷ 5 = 50 58.4 -1.45/56.95 (d) 4 517 063 = (4 × 1 000 000) + (5 × 100 000) + (1 × 1000) + (7 × 1000) + (6 × 10) + (3 × 1) (e) 60 000 + 7000 + 500 + 2 = 67 502 Have students make up their own version using the sample provided as a template. Students then exchange sheets with a partner and repeat the exercise.	 When reviewing students' work, look for evidence of their understanding of multiplication of 2-digit by 2-digit problems understanding of division (3 digit by 1 digit) with and without remainders ability to explain why keeping track of place value positions is important when adding and subtracting decimals ability to solve a problem that involves addition and subtraction of decimals, limited to thousandths ability to represent and describe whole numbers to 10 000 by expressing a given numeral in expanded notation ability to represent and describe whole numbers to 10 000 by describing the meaning of each digit in a given numeral ability to represent and describe whole numbers to 10 000 by writing a given numeral represented by expanded notation ability to use more than one strategy to determine errors



LEARNING RESOURCES

This section contains general information on learning resources, and provides a link to the titles, descriptions, and ordering information for the recommended learning resources in the Mathematics K to 7 Grade Collections.

What Are Recommended Learning Resources?

Recommended learning resources are resources that have undergone a provincial evaluation process using teacher evaluators and have Minister's Order granting them provincial recommended status. These resources may include print, video, software and CD-ROMs, games and manipulatives, and other multimedia formats. They are generally materials suitable for student use, but may also include information aimed primarily at teachers.

Information about the recommended resources is organized in the format of a Grade Collection. A Grade Collection can be regarded as a "starter set" of basic resources to deliver the curriculum. In many cases, the Grade Collection provides a choice of more than one resource to support curriculum organizers, enabling teachers to select resources that best suit different teaching and learning styles. Teachers may also wish to supplement Grade Collection resources with locally approved materials.

How Can Teachers Choose Learning Resources to Meet Their Classroom Needs?

Teachers must use either

- provincially recommended resources OR
- resources that have been evaluated through a local, board-approved process

Prior to selecting and purchasing new learning resources, an inventory of resources that are already available should be established through consultation with the school and district resource centres. The ministry also works with school districts to negotiate cost-effective access to various learning resources.

What Are the Criteria Used to Evaluate Learning Resources?

The Ministry of Education facilitates the evaluation

of learning resources that support BC curricula, and that will be used by teachers and/or students for instructional and assessment purposes. Evaluation criteria focus on content, instructional design, technical considerations, and social considerations.

Additional information concerning the review and selection of learning resources is available from the ministry publication, Evaluating, Selecting and Managing Learning Resources: A Guide (Revised 2002) www.bced.gov.bc.ca/irp/resdocs/esm_guide.pdf

What Funding is Available for Purchasing Learning Resources?

As part of the selection process, teachers should be aware of school and district funding policies and procedures to determine how much money is available for their needs. Funding for various purposes, including the purchase of learning resources, is provided to school districts. Learning resource selection should be viewed as an ongoing process that requires a determination of needs, as well as long-term planning to co-ordinate individual goals and local priorities.

What Kinds of Resources Are Found in a Grade Collection?

The Grade Collection charts list the recommended learning resources by media format, showing links to the curriculum organizers. Each chart is followed by an annotated bibliography. Teachers should check with suppliers for complete and up-to-date ordering information. Most suppliers maintain web sites that are easy to access.

$M \text{ATHEMATICS} \ K \ \text{to} \ 7 \ G \text{rade} \ Collections$

The Grade Collections for Mathematics K to 7 include newly recommended learning resources as well as relevant resources previously recommended for prior versions of the Mathematics K to 7 curriculum. The ministry updates the Grade Collections on a regular basis as new resources are developed and evaluated.

Please check the following ministry web site for the most current list of recommended learning resources in the Grade Collections for each IRP: www.bced.gov.bc.ca/irp_resources/lr/resource/gradcoll.htm



GLOSSARY

The British Columbia Ministry of Education recognizes the limitation of a glossary available only in print format. An online glossary has been developed by Alberta Education to support the implementation of their revised Kindergarten to Grade 9 Program of Studies. This glossary is based on the WNCP CCF for K-9 Mathematics and therefore also supports the British Columbia Mathematics K to 7 IRP.

This online glossary provides additional supports for teachers indlucing definitions, diagrams, pictures, and interactive applets that cannot be provided through a conventional print glossary. As a result, the Ministry of Education encourages educational stakeholders to access the glossary through a link which is provided on the British Columbia Ministry of Education website.

To access the glossary, follow the links for curriculum support material from the mathematics IRP main page at www.bced.gov.bc.ca/irp/irp_math.htm