Math Guided Self-Placement

Please answer the following questions and bring them to your counseling appointment.

1. How do you feel about math?

a) I like math and have been very successful in all my math classes.
b) I do not like math because I am not good at it, but I try my best to pass all my math classes with at least a C.
c) I do not like math, but I have been passing all my math classes with a C or better.
d) I hate math because I have been struggling with this subject since elementary school.

2. How do you study math?

a) I usually read my textbook or lecture notes and then do all assigned homework problems.
b) I usually start doing all assigned homework problems without reading my textbook or lecture notes.
c) It is very hard for me to do math homework, but I sometimes do it.
d) I never do my math homework.

3. If you have trouble doing a math assignment, how do you seek help?

a) I go to my instructor's office hours.
b) I go to the tutorial center to get help from a tutor.
c) I ask my classmates for help.
d) I never seek help.

4. How do you study for a math test?

a) I read my textbook, review my lecture notes, review related homework problems, and solve all problems on a study guide, if any.
b) I solve many math problems about concepts that are hard for me.
c) I go to a tutoring center and ask tutors to give me problems for practice.
d) I never study for math tests.

5. What would you do if you fail a math test?

a) I would go over my mistakes and correct all problems I did incorrectly.
b) I would go to my instructor or tutor and ask him or her to clarify my misunderstandings.
c) I would look at the test of one of my classmates who solved all problems correctly.
d) I would not look over the test and not ask others to explain what I did incorrectly.

6. Do you feel comfortable to go to your math instructors?

a) I feel comfortable to go to my instructors to ask math and non-math questions.
b) I feel comfortable to go to my instructor to discuss issues not related to math but feel embarrassed to ask math questions.
c) I do not feel comfortable to go to my math instructor but will do it if I am failing the course.
d) I do not feel comfortable to go to my math instructor and will not do it even if I am failing the course.

7. Do you go to every class meeting of your math class?

a) I go to every class meeting of my math class regardless of whether I am successful in the course or not.
b) I go to every math class meeting only if I am passing the course.
c) I make up excuses why I do not need to go to my math class.
d) I do not go to all my math classes meetings but try to be in class on test days.

8. Do you complete all your math assignments and submit them on time?

a) I do all my math assignments and submit them on time.
b) I do most of my math assignments and submit them on time.
c) I do most of my math assignments, but it is hard for me to submit them on time because I am a procrastinator.
d) I never complete my math assignments.

Are you Ready for Math 120?

Below are several problems and solutions to them. For each solution, identify whether:

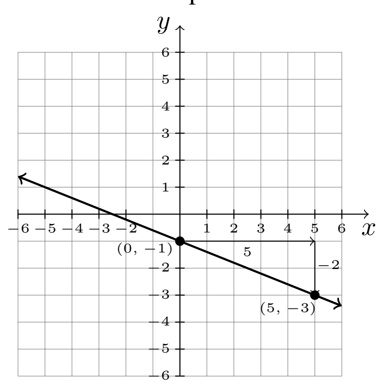
(a) You can solve such problems.
(b) You cannot solve such problems now but you have solved them before.
(c) You cannot solve such problems now and are sure you have never solved them before.

Example Number	Concept	Your Reflection Select one: (a) (b) (c)
1	Graphing a linear equation	
2	Finding an equation of a line	
3	Solving a system by substitution	
4	Simplifying a power expression	
5	Solving a rational equation	

Example 1 Graphing a Linear Function

Graph $f(x) = -\frac{2}{5}x - 1$.

Solution
The y-intercept is $(0, -1)$, so we plot that point. The slope is $-\frac{2}{5} = \frac{-2}{5} = \frac{\text{rise}}{\text{run}}$. So, starting from the point $(0, -1)$, we look 5 units to the right and 2 units down, and plot the point $(5, -3)$. Then we draw a straight line that contains the two points.



Example 2 Finding an Equation of a Line

Find an equation of the line that contains the points $(-2, 4)$ and $(3, 1)$.

Solution
First, we find the slope of the line:

$$\frac{1-4}{3-(-2)} = \frac{-3}{5} = -\frac{3}{5}$$

So, the equation is of the form $y = -\frac{3}{5}x + b$. To find b, we substitute the coordinates of the point $(-2, 4)$

into the equation $y = -\frac{3}{5}x + b$:

$$\begin{aligned}
4 &= -\frac{3}{5}(-2) + b \\
4 &= \frac{6}{5} + b \\
4 \cdot 5 &= \frac{6}{5} \cdot 5 + b \cdot 5 \\
20 &= 6 + 5b \\
20 - 6 &= 6 + 5b - 6 \\
14 &= 5b \\
\frac{14}{5} &= \frac{5b}{5} \\
\frac{14}{5} &= b
\end{aligned}$$

So, the equation is $y = -\frac{3}{5}x + \frac{14}{5}$.

Example 3 Solving a System by Substitution

Solve the system

$$\begin{aligned}
3x - 2y &= 7 \\
y &= 4x - 6
\end{aligned}$$

Solution
We substitute $4x - 6$ for y in the equation $3x - 2y = 7$ and solve for x:

$$\begin{aligned}
3x - 2(4x - 6) &= 7 \\
3x - 8x + 12 &= 7 \\
-5x + 12 &= 7 \\
-5x + 12 - 12 &= 7 - 12 \\
-5x &= -5 \\
\frac{-5x}{-5} &= \frac{-5}{-5} \\
x &= 1
\end{aligned}$$

Then we substitue 1 for x in the equation $y = 4x - 6$ and solve for y:

$$y = 4(1) - 6 = -2$$

So, the solution is $(1, -2)$.

Example 4 Simplifying a Power Expression

Simplify $\dfrac{(2y^2)^3}{4y^9}$.

Solution

$$\begin{aligned}\frac{(2y^2)^3}{4y^9} &= \frac{2^3(y^2)^3}{4y^9} \\ &= \frac{8y^6}{4y^9} \\ &= 2y^{6-9} \\ &= 2y^{-3} \\ &= \frac{2}{y^3}\end{aligned}$$

Example 5 Solving a Rational Equation

Solve $\dfrac{x}{x-5} + \dfrac{2}{x-6} = \dfrac{2}{x^2-11x+30}$.

Solution

$$\begin{aligned}\frac{x}{x-5} + \frac{2}{x-6} &= \frac{2}{x^2-11x+30} \\ \frac{x}{x-5} + \frac{2}{x-6} &= \frac{2}{(x-6)(x-5)} \\ (x-6)(x-5)\cdot\frac{x}{x-5} + (x-6)(x-5)\cdot\frac{2}{x-6} &= (x-6)(x-5)\cdot\frac{2}{(x-6)(x-5)} \\ (x-6)\cdot x + (x-5)\cdot 2 &= 2 \\ x^2 - 6x + 2x - 10 &= 2 \\ x^2 - 4x - 12 &= 0 \\ (x-6)(x+2) &= 0 \\ x - 6 = 0 \quad \text{or} \quad x + 2 &= 0 \\ x = 6 \quad \text{or} \quad x &= -2\end{aligned}$$

Because 6 is an excluded value, it is not a solution. The only solution is -2.

Are You Ready for Math 130?

Below are several problems and solutions to them. For each solution, identify whether:

(a) You can solve such problems.
(b) You cannot solve such problems now but you have solved them before.
(c) You cannot solve such problems now and are sure you have never solved them before.

Example Number	Concept	Your Reflection Select one: (a) (b) (c)
1	Simplifying a complex rational expression	
2	Solving a square root equation	
3	Applying the Pythagorean Theorem	
4	Graphing a quadratic function	
5	Finding the domain and the range of a function	
6	Sketching the graph of an inverse function	

Example 1 Simplifying a Complex Rational Expression

Simplify $\dfrac{\dfrac{1}{x^2}-\dfrac{1}{y}}{\dfrac{1}{x}+\dfrac{1}{y^2}}$.

Solution

$$\begin{aligned}
\frac{\frac{1}{x^2}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y^2}} &= \frac{\frac{1}{x^2}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y^2}}\cdot\frac{x^2y^2}{x^2y^2} \\
&= \frac{\frac{1}{x^2}\cdot\frac{x^2y^2}{1}-\frac{1}{y}\cdot\frac{x^2y^2}{1}}{\frac{1}{x}\cdot\frac{x^2y^2}{1}+\frac{1}{y^2}\cdot\frac{x^2y^2}{1}} \\
&= \frac{y^2-x^2y}{xy^2+x^2} \\
&= \frac{y(y-x^2)}{x(y^2+x)}
\end{aligned}$$

Example 2 Solving a Square Root Equation

Solve $x=\sqrt{x-1}+3$.

Solution

$$
\begin{aligned}
x &= \sqrt{x-1}+3 \\
x-3 &= \sqrt{x-1} \\
(x-3)^2 &= \left(\sqrt{x-1}\right)^2 \\
x^2-6x+9 &= x-1 \\
x^2-7x+10 &= 0 \\
(x-2)(x-5) &= 0 \\
x-2=0 \quad &\text{or} \quad x-5=0 \\
x=2 \quad &\text{or} \quad x=5
\end{aligned}
$$

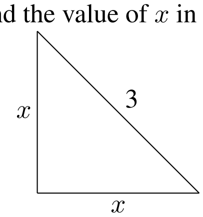
We check that 2 and 5 satisfy the original equation.

Check $x=2$	Check $x=5$
$x=\sqrt{x-1}+3$	$x=\sqrt{x-1}+3$
$2=\sqrt{2-1}+3$	$5=\sqrt{5-1}+3$
$2=4$	$5=5$
false	true

The only solution is 5.

Example 3 Applying the Pythagorean Theorem

Find the value of x in the following figure.



Solution
We apply the Pythagorean Theorem to get

$$
\begin{aligned}
x^2+x^2 &= 3^2 \\
2x^2 &= 9 \\
x^2 &= \frac{9}{2} \\
x &= \pm\sqrt{\frac{9}{2}} \\
x &= \pm\frac{\sqrt{9}}{\sqrt{2}} \\
x &= \pm\frac{3}{\sqrt{2}} \\
x &= \pm\frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \\
x &= \pm\frac{3\sqrt{2}}{2}
\end{aligned}
$$

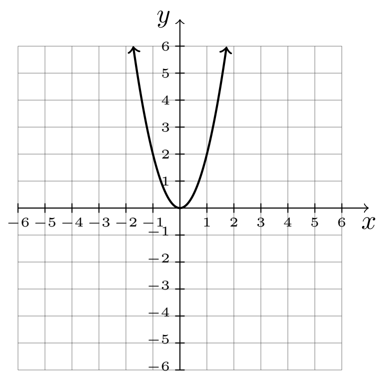
The length of a leg of triangle is positive, so the value of x is $\dfrac{3\sqrt{2}}{2}$.

Example 4 Graphing a Quadratic Function

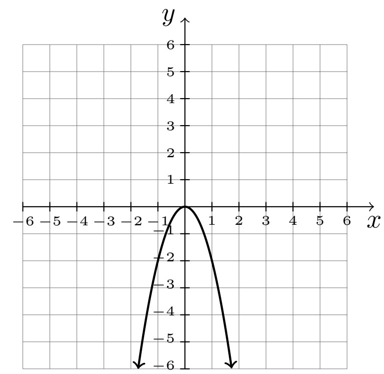
Graph the function $f(x) = -2(x-3)^2 + 5$.

Solution

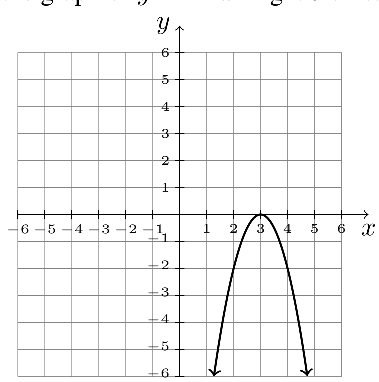
We start by graphing $y = 2x^2$.



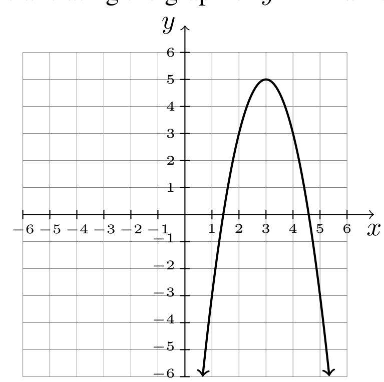
Then we graph $y = -2x^2$ by reflecting the graph of $y = x^2$ across the x-axis.



Next, we graph $y = -2(x-3)^2$ by translating the graph of $y = -2x^2$ right 3 units.

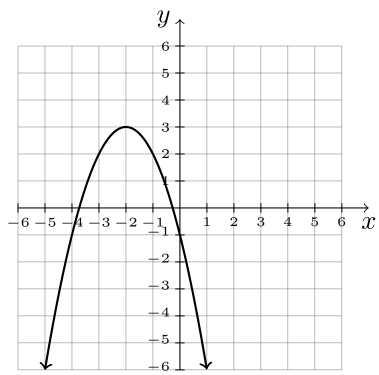


Finally, we graph $y = -2(x-3)^2 + 5$ by translating the graph of $y = -2x^2$ up 5 units.



Example 5 Finding the Domain and the Range of a Function

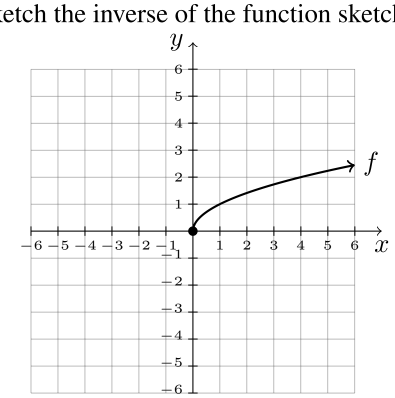
Find the domain and the range of the function sketched below.



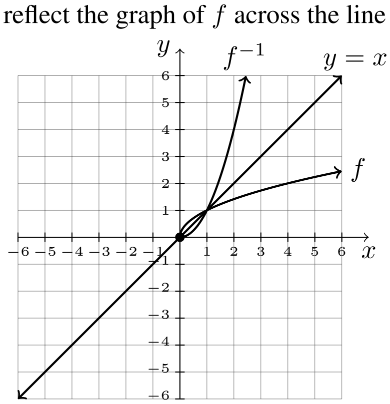
Solution
The domain is all real numbers. The range is $[3, -\infty)$.

Example 6 Sketching the Graph of an Inverse Function

Sketch the inverse of the function sketched below.



Solution
We reflect the graph of f across the line $y = x$ to get the graph of f^{-1}.



Are you ready for Math 200?

Below are several problems and solutions to them. For each solution, identify whether:

(a) You can solve such problems.
(b) You cannot solve such problems now but you have solved them before.
(c) You cannot solve such problems now and are sure you have never solved them before.

Example Number	Concept	Your reflection Select one: (a) (b) (c)
1	Finding a proportion and representing it as a fraction, decimal, and percent	
2	Graphing a linear equation and interpreting its slope and y-intercept	
3	Solving a linear equation	
4	Solving and inequality and representing its solution with interval notation	

Example 1 Finding a proportion and representing it as a fraction, decimal, and percent

In Fall 2018, 89 out of 128 pre-statistics students passed the course. Find the proportion of students who passed the course. Represent your answer as a fraction, decimal rounded to the nearest hundredths, and percent.

Solution

The proportion of the students who passed the course is $\frac{89}{128}$.

$\frac{89}{128} = 0.6953125 \approx 0.70$ or 70%

Therefore, about 70% of pre-statistics students passed the course in Fall 2018.

Example 2 Graphing a linear equation and interpreting its slope and y-intercept

A relationship between the age of a car and its price is represented by the equation

$y = -3.5x + 36.2$ where x is the age of a car in years and y is its price in thousands of dollars.

Graph the equation and interpret the slope and y-intercept.

Solution

The age and price of a car cannot be negative. We graph the portion of a line in the first quadrant, using x- and y-intercepts. When x = 0, y = 36.2.Therefore, the y-intercept has the coordinates (0, 36.2). When y = 0,

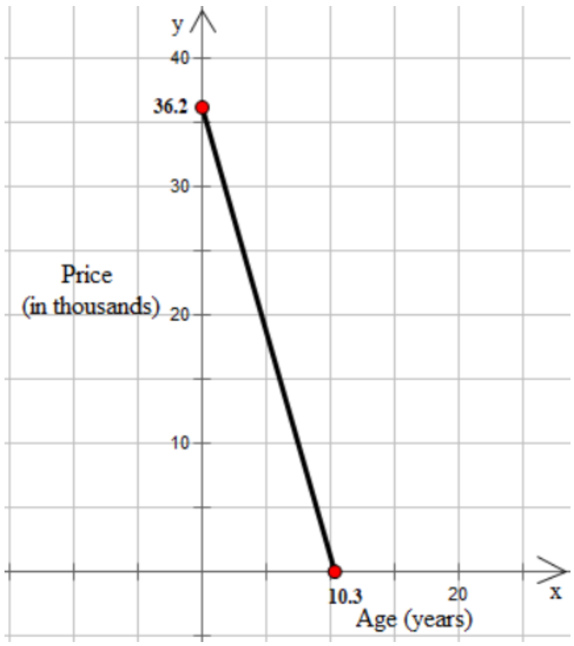
$$0 = -3.5x + 36.2$$

$$3.5x = 36.2$$

$$x = \frac{36.2}{3.5}$$

$$x = 10.3$$

Thus, the x-intercept has the coordinates (10.3, 0). The graph is sketched by connecting (0, 36.2) and (10.3, 0) in the first quadrant.



The slope of $-3.5 \, \frac{\text{thousand dollars}}{\text{year}}$ means that each year the price of the car decreases by 3.5 thousand dollars. The y-intercept is (0, 36.2). The y-intercept indicates that a brand new car (age=0) costs 36.2 thousand dollars.

Example 3 Solving a linear equation

Solve $-1.2 = \frac{x-12}{3}$.

Solution

We multiply both sides of the given equation by 3.

$$3(-1.2) = \frac{3}{1} \cdot \frac{(x-12)}{3}$$

$$-3.6 = x - 12$$

$$-3.6 + 12 = x$$

$$8.4 = x$$

Example 4 Solving an inequality

Solve $4 - 3\sqrt{\frac{50}{2}} \leq x \leq 4 + 3\sqrt{\frac{50}{2}}$. Represent the solution set in interval notation.

Solution

$$4 - 3\sqrt{\frac{50}{2}} \leq x \leq 4 + 3\sqrt{\frac{50}{2}}$$

$$4 - 3\sqrt{25} \leq x \leq 4 + 3\sqrt{25}$$

$$4 - 3(5) \leq x \leq 4 + 3(5)$$

$$4 - 15 \leq x \leq 4 + 15$$

$$-11 \leq x \leq 19$$

Therefore, the solution set is [-11, 19].

Are you Ready for Math 241?

Below are several problems and solutions to them. For each solution, identify whether:

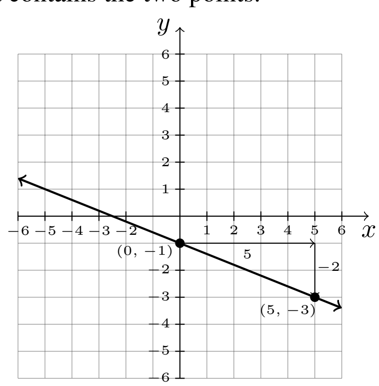
(a) You can solve such problems.
(b) You cannot solve such problems now but you have solved them before.
(c) You cannot solve such problems now and are sure you have never solved them before.

Example Number	Concept	Your Reflection Select one: (a) (b) (c)
1	Graphing a linear equation	
2	Evaluating a quadratic equation	
3	Simplifying a power expression	
4	Solving a rational equation	

Example 1 Graphing a Linear Function

Graph $f(x) = -\frac{2}{5}x - 1$.

Solution
The y-intercept is $(0, -1)$, so we plot that point. The slope is $-\frac{2}{5} = \frac{-2}{5} = \frac{\text{rise}}{\text{run}}$. So, starting from the point $(0, -1)$, we look 5 units to the right and 2 units down, and plot the point $(5, -3)$. Then we draw a straight line that contains the two points.



Example 2 Evaluating a Quadratic Function

Evaluate $g(x) = -2x^2 - 4x + 1$ at -3.

Solution
We substitute -3 for x in the equation $g(x) = -2x^2 - 4x + 1$:

$$\begin{aligned} g(-3) &= -2(-3)^2 - 4(-3) + 1 \\ &= -2(9) - 4(-3) + 1 \\ &= -18 + 12 + 1 \\ &= -5 \end{aligned}$$

Example 3 Simplifying a Power Expression

Simplify $\dfrac{(2y^2)^3}{4y^9}$.

Solution

$$\begin{aligned}\frac{(2y^2)^3}{4y^9} &= \frac{2^3(y^2)^3}{4y^9}\\ &= \frac{8y^6}{4y^9}\\ &= 2y^{6-9}\\ &= 2y^{-3}\\ &= \frac{2}{y^3}\end{aligned}$$

Example 4 Solving a Rational Equation

Solve $\dfrac{x}{x-5} + \dfrac{2}{x-6} = \dfrac{2}{x^2-11x+30}$.

Solution

$$\begin{aligned}\frac{x}{x-5} + \frac{2}{x-6} &= \frac{2}{x^2-11x+30}\\ \frac{x}{x-5} + \frac{2}{x-6} &= \frac{2}{(x-6)(x-5)}\\ (x-6)(x-5)\cdot\frac{x}{x-5} + (x-6)(x-5)\cdot\frac{2}{x-6} &= (x-6)(x-5)\cdot\frac{2}{(x-6)(x-5)}\\ (x-6)\cdot x + (x-5)\cdot 2 &= 2\\ x^2-6x+2x-10 &= 2\\ x^2-4x-12 &= 0\\ (x-6)(x+2) &= 0\\ x-6=0 \quad &\text{or} \quad x+2=0\\ x=6 \quad &\text{or} \quad x=-2\end{aligned}$$

Because 6 is an excluded value, it is not a solution. The only solution is -2.

Are you ready for Math 242?

Below are several problems and solutions to them. For each solution, identify whether:

(a) You can solve such problems.
(b) You cannot solve such problems now but you have solved them before.
(c) You cannot solve such problems now and are sure you have never solved them before.

Example Number	Concept	Your reflection Select one: (a) (b) (c)
1	Converting degrees to radians	
2	Evaluating the sine, cosine, and tangent functions	
3	Graphing a trigonometric function	
4	Proving a trigonometric identity	

Example 1 Converting degrees to radians

Convert $150°$ to radians.

Solution

$1° = \frac{\pi}{180°}$ radians

$$150° = 150°\left(\frac{\pi}{180°}\right) = \frac{150\pi}{180} = \frac{5\pi}{6}$$

Example 2 Evaluating the sine, cosine, and tangent functions

Evaluate sine, cosine, and tangent at x=$-\frac{\pi}{6}$.

Solution

$$\sin\left(-\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$\cos\left(-\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\tan\left(-\frac{\pi}{6}\right) = \frac{\sin\left(-\frac{\pi}{6}\right)}{\cos\left(-\frac{\pi}{6}\right)} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

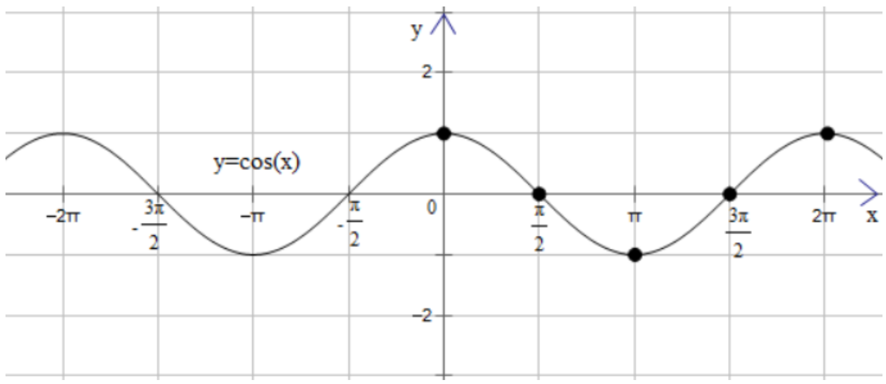
Example 3 Graphing a trigonometric function

Graph $f(\mathrm{x}) = \cos\left(\mathrm{x} - \frac{\pi}{2}\right)$ without using a calculator.

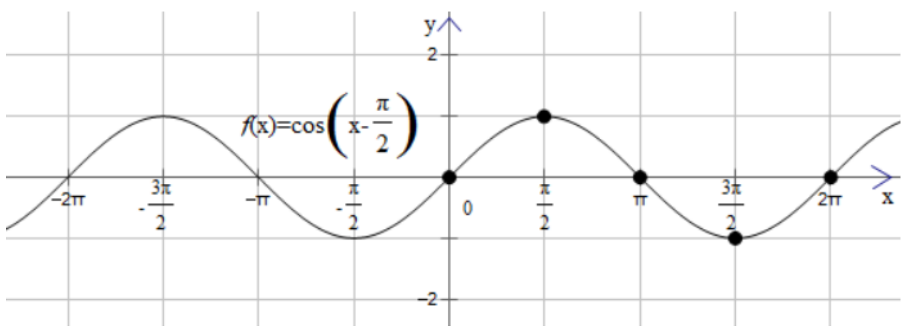
Solution

We start by graphing y= $\cos(\mathrm{x})$.

The amplitude is 1 and the period is 2π. Dividing the interval $[0,2\pi]$ into four equal parts produces the key points (0,1), $\left(\frac{\pi}{2},0\right)$, $(\pi,-1)$, $\left(\frac{3\pi}{2},0\right)$, and $(2\pi,1)$. The graph is sketched by connecting these key points and extending the curve in both directions.



Then we graph y= $\cos\left(\mathrm{x} - \frac{\pi}{2}\right)$ by translating the graph of y= $\cos(\mathrm{x})$ right $\frac{\pi}{2}$ units.



Because $\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$, the resulting graph corresponds to the graph of $y = \sin(x)$.

Example 4 Proving a trigonometric identity

Prove the identity $\cos^2(\mathrm{x}) - \sin^2(\mathrm{x}) = 2\cos^2(\mathrm{x}) - 1$.

Solution

First, we solve the Pythagorean identity $\sin^2(\mathrm{x}) + \cos^2(\mathrm{x}) = 1$ for $\sin^2(\mathrm{x})$:

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin^2(x) = 1 - \cos^2(x)$$

Then we substitute $1 - \cos^2(x)$ for $\sin^2(x)$ in $\cos^2(x) - \sin^2(x)$:

$$\cos^2(x) - \sin^2(x) = \cos^2(x) - \left(1 - \cos^2(x)\right)$$

$$= \cos^2(x) - 1 + \cos^2(x)$$

$$= 2\cos^2(x) - 1$$

Are You Ready for Math 222?

Below are several problems and solutions to them. For each solution, identify whether:

(a) You can solve such problems.
(b) You cannot solve such problems now but you have solved them before.
(c) You cannot solve such problems now and are sure you have never solved them before.

Example Number	Concept	Your Reflection Select one: (a) (b) (c)
1	Graphing a quadratic equation	
2	Solving a rational equation	
3	Solving a square root equation	
4	Solving an exponential equation	
5	Evaluating the six trigonometric functions	
6	Graphing a trigonometric function	
7	Solving a trigonometric equation	
8	Proving a trigonometric identity	

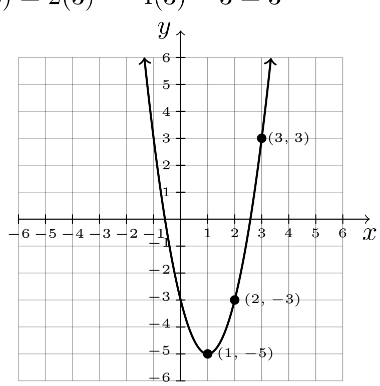
Example 1 Graphing a Quadratic Function

Graph $f(x) = 2x^2 - 4x - 3$.

Solution

$$\begin{aligned} f(x) &= 2x^2 - 4x - 3 \\ &= 2(x^2 - 2x) - 3 \\ &= 2(x^2 - 2x + 1) - 3 - 2 \\ &= 2(x-1)^2 - 5 \end{aligned}$$

The vertex is $(1, -5)$.
$f(2) = 2(2)^2 - 4(2) - 3 = -3$
$f(3) = 2(3)^2 - 4(3) - 3 = 3$



Example 2 Solving a Rational Equation

Solve $\frac{x}{x-5}-\frac{2}{x-6}=\frac{2}{x^2-11x+30}$.

Solution

$$\begin{aligned}
\frac{x}{x-5}-\frac{2}{x-6}&=\frac{2}{x^2-11x+30}\\
\frac{x}{x-5}-\frac{2}{x-6}&=\frac{2}{(x-6)(x-5)}\\
(x-6)(x-5)\cdot\frac{x}{x-5}-(x-6)(x-5)\cdot\frac{2}{x-6}&=(x-6)(x-5)\cdot\frac{2}{(x-6)(x-5)}\\
(x-6)\cdot x-(x-5)\cdot 2&=2\\
x^2-6x-2x+10&=2\\
x^2-8x+8&=0\\
x&=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(8)}}{2(1)}\\
x&=\frac{8\pm\sqrt{64-32}}{2}\\
x&=\frac{8\pm\sqrt{32}}{2}\\
x&=\frac{8\pm 4\sqrt{2}}{2}\\
x&=\frac{4(2\pm\sqrt{2})}{2}\\
x&=2(2\pm\sqrt{2})\\
x&=4\pm 2\sqrt{2}
\end{aligned}$$

Example 3 Solving a Square Root Equation

Solve $x=\sqrt{x-1}+3$.

Solution

$$\begin{aligned}
x&=\sqrt{x-1}+3\\
x-3&=\sqrt{x-1}\\
(x-3)^2&=\left(\sqrt{x-1}\right)^2\\
x^2-6x+9&=x-1\\
x^2-7x+10&=0\\
(x-2)(x-5)&=0\\
x-2=0 \quad &\text{or} \quad x-5=0\\
x=2 \quad &\text{or} \quad x=5
\end{aligned}$$

We check that 2 and 5 satisfy the original equation.

Check $x = 2$

$$x = \sqrt{x-1} + 3$$
$$2 = \sqrt{2-1} + 3$$
$$2 = 4$$

false

Check $x = 5$

$$x = \sqrt{x-1} + 3$$
$$5 = \sqrt{5-1} + 3$$
$$5 = 5$$

true

The only solution is 5.

Example 4 Solving an Exponential Equation

Solve $3(4)^x = 71$.

Solution

$$3(4)^x = 71$$
$$4^x = \frac{71}{3}$$
$$\log(4^x) = \log\left(\frac{71}{3}\right)$$
$$x\log(4) = \log\left(\frac{71}{3}\right)$$
$$x = \frac{\log\left(\frac{71}{3}\right)}{\log(4)}$$
$$x \approx 2.2824$$

See the next three pages for more examples.

Example 5 Evaluating the six trigonometric functions

Evaluate the six trigonometric functions at x= $\frac{13\pi}{6}$.

Solution

$\frac{13\pi}{6} = \frac{12\pi}{6} + \frac{\pi}{6} = 2\pi + \frac{\pi}{6}$

$$\sin\left(\frac{13\pi}{6}\right) = \sin\left(2\pi + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\frac{13\pi}{6}\right) = \cos\left(2\pi + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{13\pi}{6}\right) = \frac{\sin\left(\frac{13\pi}{6}\right)}{\cos\left(\frac{13\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\cot\left(\frac{13\pi}{6}\right) = \frac{\cos\left(\frac{13\pi}{6}\right)}{\sin\left(\frac{13\pi}{6}\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\sec\left(\frac{13\pi}{6}\right) = \frac{1}{\cos\left(\frac{13\pi}{6}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\csc\left(\frac{13\pi}{6}\right) = \frac{1}{\sin\left(\frac{13\pi}{6}\right)} = \frac{1}{\frac{1}{2}} = \frac{2}{1} = 2$$

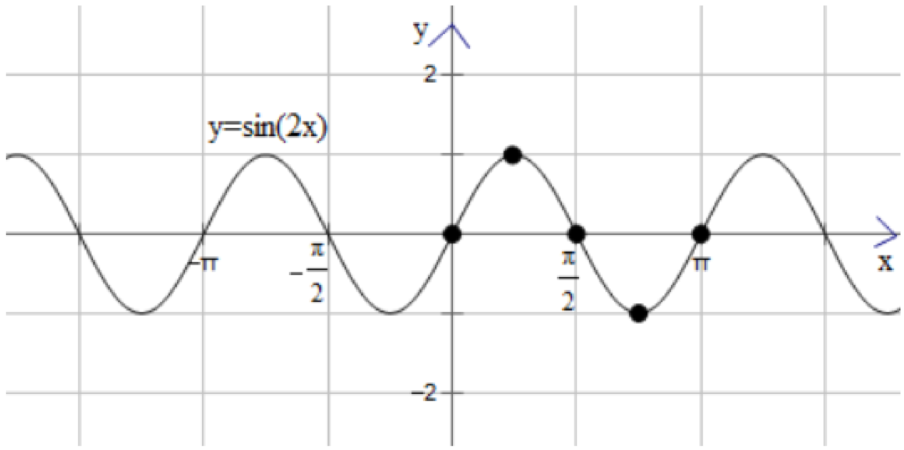
Example 6 Graphing a trigonometric function

Graph $h(x) = 1 + \sin(2x)$ without using a calculator.

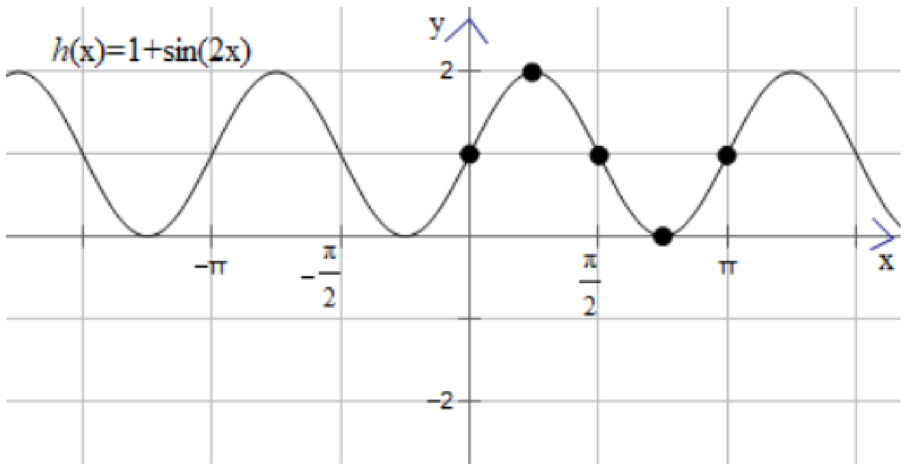
Solution

We start by graphing y= $\sin(2x)$

The amplitude is 1 and the period is $\frac{2\pi}{2} = \pi$. Dividing the interval $[0, \pi]$ into four equal parts produces the key points (0,0), $\left(\frac{\pi}{4}, 1\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{4}, -1\right)$, and $(\pi, 0)$. The graph is sketched by connecting these key points and extending the curve in both directions.



Then we graph $h(x) = 1 + \sin(2x)$ by translating the graph of y= $\sin(2x)$ up 1 unit.



Example 7 Solving a trigonometric equation

Solve $\sin(2x) = \sin(x)$ on the interval $[0, 2\pi]$.

Solution

$\sin(2x) = \sin(x)$

$\sin(2x) - \sin(x) = 0$

$2\sin(x)\cos(x) - \sin(x) = 0$

$\sin(x)(2\cos(x) - 1) = 0$

$\sin(x) = 0$ or $2\cos(x) - 1 = 0$

$\sin(x) = 0$ or $\cos(x) = \frac{1}{2}$

On $[0, 2\pi]$, $\sin(x) = 0$ when x = 0, π, or 2π; $\cos(x) = \frac{1}{2}$ when $x = \frac{\pi}{3}$ or $x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Therefore, the solutions are $x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$, and 2π.

Example 8 Proving a trigonometric identity

Prove the identity $\frac{\sec^2(x)-1}{\sec^2(x)} = \sin^2(x)$

Solution

$$\frac{\sec^2(x) - 1}{\sec^2(x)} = \frac{\sec^2(x)}{\sec^2(x)} - \frac{1}{\sec^2(x)} = 1 - \frac{1}{\frac{1}{\cos^2(x)}} = 1 - \cos^2(x) = \sin^2(x)$$

Are You Ready for Math 251?

Below are several problems and solutions to them. For each solution, identify whether:

(a) You can solve such problems.
(b) You cannot solve such problems now but you have solved them before.
(c) You cannot solve such problems now and are sure you have never solved them before.

Example Number	Concept	Your Reflection Select one: (a) (b) (c)
1	Finding a difference quotient	
2	Solving a rational equation	
3	Solving a square root equation	
4	Solving an exponential equation	
5	Solving an exponential equation	
6	Solving a quadratic inequality	
7	Solving a trigonometric equation	
8	Expressing one quantity as a function of another	

Example 1 Finding a Difference Quotient

Let $f(x) = \dfrac{5-2x}{3x}$. Find $\dfrac{f(x+h)-f(x)}{h}$.

Solution

$$
\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{\dfrac{5-2(x+h)}{3(x+h)} - \dfrac{5-2x}{3x}}{h} \\
&= \frac{\dfrac{5-2x-2h}{3(x+h)} - \dfrac{5-2x}{3x}}{h} \cdot \frac{3x(x+h)}{3x(x+h)} \\
&= \frac{\dfrac{5-2x-2h}{3(x+h)} \cdot \dfrac{3x(x+h)}{1} - \dfrac{5-2x}{3x} \cdot \dfrac{3x(x+h)}{1}}{h \cdot 3x(x+h)} \\
&= \frac{(5-2x-2h)x - (5-2x)(x+h)}{3xh(x+h)} \\
&= \frac{5x - 2x^2 - 2xh - (5x + 5h - 2x^2 - 2xh)}{3xh(x+h)} \\
&= \frac{5x - 2x^2 - 2xh - 5x - 5h + 2x^2 + 2xh}{3xh(x+h)} \\
&= \frac{-5h}{3xh(x+h)} \\
&= -\frac{5}{3x(x+h)}
\end{aligned}
$$

Example 2 Solving a Rational Equation

Solve $\dfrac{2y}{y-2} - \dfrac{2y-5}{y^2-7y+10} = \dfrac{-4}{y-5}$.

Solution

$$\begin{aligned}
\frac{2y}{y-2} - \frac{2y-5}{y^2-7y+10} &= \frac{-4}{y-5} \\
\frac{2y}{y-2} - \frac{2y-5}{(y-2)(y-5)} &= \frac{-4}{y-5} \\
(y-2)(y-5)\left(\frac{2y}{y-2} - \frac{2y-5}{(y-2)(y-5)}\right) &= (y-2)(y-5)\cdot\frac{-4}{y-5} \\
(y-2)(y-5)\cdot\frac{2y}{y-2} - (y-2)(y-5)\cdot\frac{2y-5}{(y-2)(y-5)} &= (y-2)(-4) \\
(y-5)(2y) - (2y-5) &= -4y+8 \\
2y^2 - 10y - 2y + 5 &= -4y+8 \\
2y^2 - 8y - 3 &= 0 \\
y &= \frac{-(-8)\pm\sqrt{(-8)^2-4(2)(-3)}}{2(2)} \\
y &= \frac{8\pm\sqrt{88}}{4} \\
y &= \frac{8\pm 2\sqrt{22}}{4} \\
y &= \frac{2(4\pm\sqrt{22})}{4} \\
y &= \frac{4\pm\sqrt{22}}{2}
\end{aligned}$$

Example 3 Solving a Square Root Equation

Solve $\sqrt{2p-1} + \sqrt{3p-2} = 2$

Solution

$$
\begin{aligned}
\sqrt{2p-1}+\sqrt{3p-2} &= 2 \\
\sqrt{2p-1} &= 2-\sqrt{3p-2} \\
\left(\sqrt{2p-1}\right)^2 &= \left(2-\sqrt{3p-2}\right)^2 \\
2p-1 &= 4-4\sqrt{3p-2}+3p-2 \\
4\sqrt{3p-2} &= p+3 \\
\left(4\sqrt{3p-2}\right)^2 &= (p+3)^2 \\
16(3p-2) &= p^2+6p+9 \\
48p-32 &= p^2+6p+9 \\
0 &= p^2-42p+41 \\
0 &= (p-1)(p-41) \\
p-1=0 \quad \text{or} &\quad p-41=0 \\
p=1 \quad \text{or} &\quad p=41
\end{aligned}
$$

We check that 1 and 41 satisfy the original equation.

$$
\begin{aligned}
&\text{Check } p=1 \\
\sqrt{2p-1}+\sqrt{3p-2} &= 2 \\
\sqrt{2(1)-1}+\sqrt{3(1)-2} &= 2 \\
1+1 &= 2 \\
&\text{true}
\end{aligned}
\qquad
\begin{aligned}
&\text{Check } p=41 \\
\sqrt{2p-1}+\sqrt{3p-2} &= 2 \\
\sqrt{2(41)-1}\sqrt{3(41)-2} &= 2 \\
9+11 &= 2 \\
&\text{false}
\end{aligned}
$$

The only solution is 1.

Example 4 Solving an Exponential Equation

Solve $3(4)^{2x-7}=71$.

Solution

$$
\begin{aligned}
3(4)^{2x-7} &= 71 \\
4^{2x-7} &= \frac{71}{3} \\
\log 4^{2x-7} &= \log \frac{71}{3} \\
(2x-7)\log 4 &= \log \frac{71}{3} \\
2x-7 &= \frac{\log \frac{71}{3}}{\log 4} \\
2x &= \frac{\log \frac{71}{3}}{\log 4} + 7 \\
2x &= \frac{\log \frac{71}{3}}{\log 4} + 7 \cdot \frac{\log 4}{\log 4} \\
2x &= \frac{\log \frac{71}{3} + 7\log 4}{\log 4} \\
x &= \frac{\log \frac{71}{3} + 7\log 4}{2\log 4}
\end{aligned}
$$

Example 5 Solving an Exponential Equation

Solve $\dfrac{(2+e^x)^2 e^x - 2e^x(2+e^x)e^x}{(2+e^x)^4} = 0$.

Solution

$$
\begin{aligned}
\frac{(2+e^x)^2 e^x - 2e^x(2+e^x)e^x}{(2+e^x)^4} &= 0 \\
(2+e^x)^2 e^x - 2e^x(2+e^x)e^x &= 0 \\
e^x(2+e^x)[(2+e^x) - 2e^x] &= 0 \\
e^x(2+e^x)[(2-e^x] &= 0 \\
e^x = 0 \quad \text{or} \quad 2+e^x = 0 \quad &\text{or} \quad 2-e^x = 0 \\
e^x = 0 \quad \text{or} \quad e^x = -2 \quad &\text{or} \quad e^x = 2 \\
e^x = 0 \quad \text{or} \quad e^x = -2 \quad &\text{or} \quad x = \ln 2
\end{aligned}
$$

Both of the equations $e^x = 0$ and $e^x = -2$ have no solutions. So, the only solution is $\ln 2$.

Example 6 Solving a Quadratic Inequality

Solve $12x^3 - 12x^2 - 24x < 0$.

Solution

$$
\begin{aligned}
12x^3 - 12x^2 - 24x &< 0 \\
12x(x^2 - x - 2) &< 0 \\
12x(x-2)(x+1) &< 0
\end{aligned}
$$

We perform a sign analysis of $12x(x-2)(x+1)$ in the following table.

Interval	$12x$	$x-2$	$x+1$	$12x(x-2)(x+1)$
$x < -1$	$-$	$-$	$-$	$-$
$-1 < x < 0$	$-$	$-$	$+$	$+$
$0 < x < 2$	$+$	$-$	$+$	$-$
$x > 2$	$+$	$+$	$+$	$+$

So, the solution set is $(-\infty, -1) \bigcup (0, 2)$.

Example 7 Solving a Trigonometric Equation

Solve $\dfrac{(2+\sin x)(-\sin x) - \cos x(\cos x)}{(2+\sin x)^2} = 0$.

Solution

$$
\begin{aligned}
\frac{(2+\sin x)(-\sin x) - \cos x(\cos x)}{(2+\sin x)^2} &= 0 \\
(2+\sin x)(-\sin x) - \cos x(\cos x) &= 0 \\
-2\sin x - \sin^2 x - \cos^2 x &= 0 \\
-2\sin x - (\sin^2 x + \cos^2 x) &= 0 \\
-2\sin x - 1 &= 0 \\
-2\sin x &= 1 \\
\sin x &= -\frac{1}{2}
\end{aligned}
$$

The solutions are $\dfrac{7\pi}{6} + 2\pi k$ and $\dfrac{11\pi}{6} + 2\pi k$, where k is an integer.

Example 8 Expressing One Quantity as a Function of Another

Express the surface area (in inches squared) of a cube as a function of its volume (in feet cubed).

Solution

Let x be the side length (in inches) of a cube, S be the cube's surface area (in inches squared), and V be the cube's volume (in inches cubed). Because the cube's volume is equal to the product of the length, width, and height, we have

$$
\begin{aligned}
x^3 &= V \\
x &= V^{1/3}
\end{aligned}
$$

The area of one face of the cube is x^2. So, the surface area (of the six sides) is given by $S = 6x^2$. Next, we substitute $V^{1/3}$ for x in the equation $S = 6x^2$:

$$
\begin{aligned}
S &= 6x^2 \\
&= 6\left(V^{1/3}\right)^2 \\
&= 6V^{2/3}
\end{aligned}
$$

So, we can write $S(V) = 6V^{2/3}$.