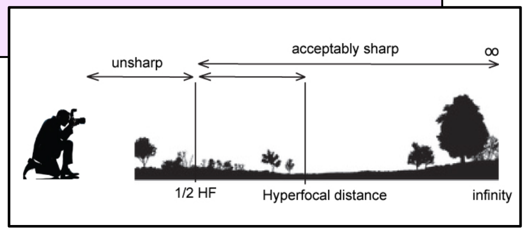
Math in Film

Math plays a crucial role in creating a film. Film production and animation involve a variety of math techniques such as statistics, trigonometry, algebra, geometry, subdivision surfaces, and more.

Tony DeRose

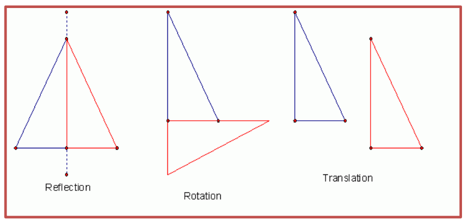
"We typically end up with about 300 degrees of freedom in these characters and perhaps 700 controls."

Hyperfocal distance: affects the camera's depth of field (which objects are in focus in a shot)
To determine depth of field, one must determine the hyperfocal distance, which is the smallest distance where all the objects from half this distance to infinity are in focus.

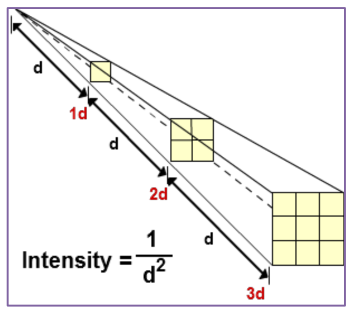


Linear equations:
For all the points in a frame, it must be determined how much light is traveling between them. With typically 1 million to 10 million pieces and one equation per piece, that's a lot of equations to solve to create an entire film!

Coordinate geometry:
used to move or change the size of objects
- -Translation-addition
- -Scaling-multiplication
- -Rotation-trigonometry



Inverse square law:
Influences how the crew chooses to light a scene so it can be shot with desired effects



2D to 3D animation:
3D animation exists on a x,y,z plane and can do things that are impossible for 2D characters to do. 2D animation was usually drawn frame by frame but 3D animation uses computer controls to move characters like a puppet with digital strings.

Harmonic Coordinates: helps simplify the controls needed to animate/move a character
Harmonic coordinates-describe how interior points move within polygons in the plane
To obtain harmonic coordinates that behave just the way they want, animators solve Laplace's equation:

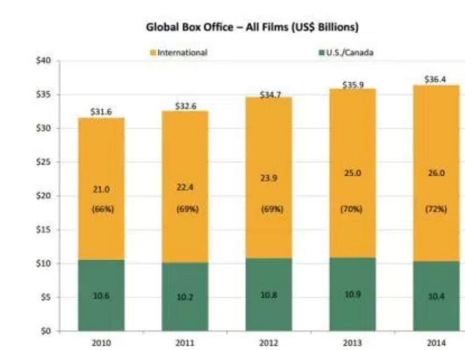
$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Statistics: used to gather data about production costs, ratings, moviegoers, and predict the future of the film industry

Global box office revenue predicted to increase to nearly $50 billion in 2020

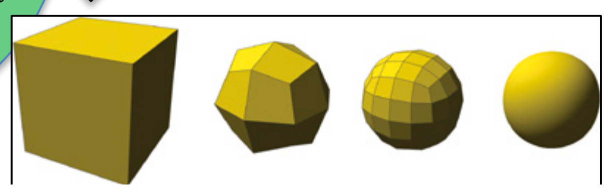
More than 1.2 billion movie tickets sold in the U.S. in 2015

Global box office revenue is about $38.3 billion

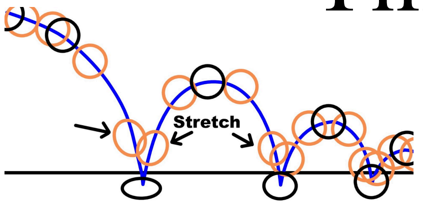


Subdivision surfaces:
Transforming a complex shape/surface on a computer into a smooth surface by repeatedly splitting and averaging a midpoint of a line

<u>Smoothing out a 4 point polygon using midpoints</u>:
1. Split - add midpoints (making 8 points)
2. Average - move those points to the midpoint of their clockwise neighbor and repeat split (making 16 points)
3. Put those two steps together (average and split) creates subdivision
4. Repeat until it is smooth



Timeline:



1832

Phenakistoscope: Invented by Belgian physicist Joseph Plateau, this early animation device consisted of two spinning disks that when reflected in mirrors, it appeared as if the drawings were moving.

1930s

Animators at Walt Disney Studios developed the principles of animation which include:

-The arc principle-almost all actions follow some form of arc like a bouncing ball following a parabolic path

-Squash and stretch principle-dilation is used for stretching and shrinking an object like how showing a slight stretch when a ball falls infers speed and a squash as it bounces infers weight

1995

Toy Story is known to be the first feature length film that was fully computer animated.

2006

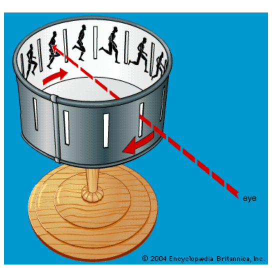
Tony DeRose won a scientific and technical Academy Award for his work with subdivision surfaces. After receiving a Ph.D. in computer science from Berkeley, he became a professor of computer science and engineering at the University of Washington and then went on to become a Senior Scientist at Pixar.

2014

Tony DeRose gave his TED talk about how math is used in movies, and encouraged his audience, especially kids, to learn as much math as they can so they can help create the technologies of the future.

1834

Zoetrope: This was another early animation device that was invented by British mathematician William Horner and it involved a cylinder with vertical slits and pictures on the inside. While the cylinder was rotating, the images would appear to be moving.

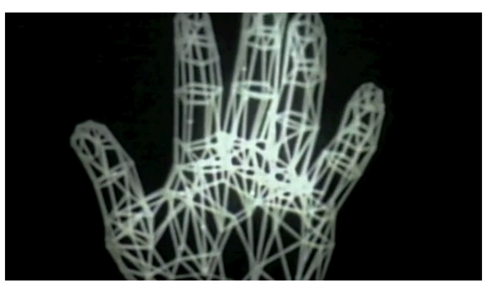


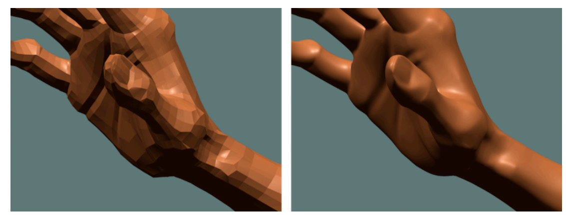
1970s

Pixar Animation Studios created a 3D rendering computer program, called Renderman, which has been used to create numerous films.

1972

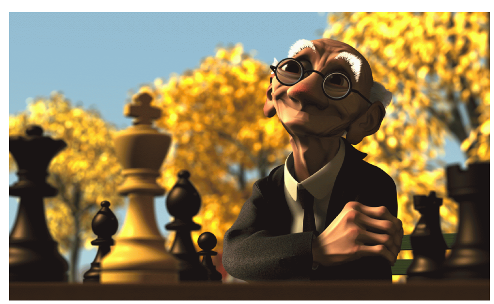
Ed Catmull made a short film called *A Computer Animated Hand*, which is considered the first example of 3D digital rendering.





1997

The technique of subdivision surfaces was first used in animation to create the character Geri in the Pixar short called *Geri's Game*, which won an Academy Award.



2012

Brave was released, which demonstrates how Pixar has continued to use and develop subdivided surfaces and their animation skills. Merida's hair alone consists of 100,000 individual elements!

Annotated Bibliography

Carmody, Tim. "Pixar's Senior Scientist explains how math makes the movies and games we love." *The Verge.* Vox Media, Inc. 7 March 2013. Web. 30 Nov. 2016. http://www.theverge.com/2013/3/7/4074956/pixar-senior-scientist-derose-explains-how-math-makes-movies-games.

Similar to the other articles I looked at, it provided reliable information about Tony DeRose and what he does as the Senior Scientist working with Pixar Animation Studios. What makes this article unique is the specific mathematical techniques examples it provides. The article describes how complicated it can be to create a character or setting that has many points and curves that the animator needs to pay attention to. DeRose's specific examples include the 100,000 individual elements of Merida's hair in *Brave* and the details generated by subdivision surfaces seen in *The Incredibles.* I found it very useful to include those examples because it allowed me to connect the mathematical techniques I was learning about to the final products that are created by these animators in movies I have watched myself.

DeRose, Tony. "Pixar: The math behind the movies." *TED Talks Education.* N.d. Web. 30 Nov. 2016. http://ed.ted.com/lessons/pixar-the-math-behind-the-movies-tony-derose.

This TED talk video given by Tony DeRose himself was one of the most helpful sources I found. While other sources provided a lot of important information, often times it was difficult for me to completely understand due to the complex mathematical terms used. On the other hand, this TED talk was directed to an audience who might include those unfamiliar with complex math and younger children. Therefore, the talk was easy to understand and included many visuals. The visual that was especially informative was when DeRose actually step by step simulated the transformations he was talking about on his computer. I was able to visually see how subdivision surfaces work and how an animator is able to smooth out a surface using mathematical tools.

DeRose, Tony, Kass, Michael and Truong, Tien. "Subdivision Surfaces in Character Animation." *Pixar Online LIbrary.* Pixar Animation Studios. Aug. 1998. Web. 1 Dec. 2016. http://graphics.pixar.com/library/Geri/paper.pdf.

This source is a scholarly paper written by three very well educated and experienced animators. Tony DeRose and Michael Kass work as Senior Scientists at Pixar Animation Studios where they have developed new animation techniques and worked on numerous Pixar films. Kass received his B.A. from Princeton in 1982, his M.S. from M.I.T., and his Ph.D. from Stanford, and joined Pixar in 1995. DeRose received a B.S. in physics and a Ph.D. in computer science from Berkeley, and became a professor of computer science and engineering at the University of Washington before working at Pixar. Tien Truong also attended Berkeley and works as the lead software engineer at OpenX, but has previously worked as a software developer and artist in computer animation. The paper that they wrote includes their research and work with subdivision surfaces as well as a variety of equations and images that show the mathematics behind the process. Although the paper was somewhat hard to for me to understand due to the complexity of their research, I was still able to gain valuable information from the key points, equations, and images that were included.

Griffiths, William. "Moves, Making of." *Encyclopedia of Mathematics & Society.* Ed. Sarah J. Greenwald and Jill E. Thomley. Ipswich, Mass: Salem Press, 2012. *Salem Online.* Web. 1 Dec. 2016. http://0online.salempress.com.wncln.wncln.org/articleDetails.do?bookId=300&articleName=Math_128536901285&searchText=film&searchOperators=exact&category=Science.

Found in the Encyclopedia of Mathematics and Society, this scholarly article provided a lot of reliable information about how math is specifically used in the camera work and audio and visual signal processing aspects of film production. I was able to learn about the specific mathematical techniques and equations that are crucial to film production. This included signal processing, lighting with f-stops and focal length, inverse square law, hyperfocal distance, Fourier transform, and Nyquist frequency. Much of this information I did not know before, so this article was helpful in taking an unfamiliar concept and making it easy to understand. Also, I was able to relate some of this information to class discussions such as the inverse square law, which we discussed in class, and the Fourier transform, which was described as transforming the model of audio signal into simpler trigonometric components. It made me think of the Jeff Weeks concept of turning a complex process into simpler, more manageable elements so that we are better able to understand it.

Jones, D. Keith and Moore-Russo, Deborah. "Animation and CGI." *Encyclopedia of Mathematics & Society.* Ed. Sarah J. Greenwald and Jill E. Thomley. Ipswich, Mass: Salem Press, 2012. *Salem Online.* Web. 1 Dec. 2016. http://0online.salempress.com.wncln.wncln.org/articleDetails.do?bookId=300&articleName=Math_101836901018&searchText=film&searchOperators=exact&category=Science.

Also found within the Encyclopedia of Mathematics and Society, this article focuses more on animation and CGI. By using math, animators have been able to create an animated character/setting/object and make it appear more realistic on screen. On top of providing a brief history of early animation devices, the source also goes into detail about the principles of animation which include parabolic arcs, dilation, and three dimensional shapes. Furthermore, a large part of the article describes CGI and how math plays a huge role in its creation. The authors discuss trigonometry, vector algebra, matrices, interpolation, transformations, and differential equations. Another positive aspect of this source is that it includes further reading citations at the bottom which can be useful if I wanted to find more credible related research.

"Math in the Movies." *Mathematical Association of America.* MAA. 2016. Web. 1 Dec. 2016. http://www.maa.org/meetings/calendar-events/math-in-the-movies.

This source was found through the Mathematical Association of America, and provides credible information about how math can be used to help create movies. The Mathematical Association of America is the largest professional society that focuses on collegiate math, and is a great resource for advanced research and studies about numerous mathematical topics. This page in particular was helpful for my project because it introduced me to Tony DeRose and his talk that was a part of the MAA's Distinguished Lecture Series. It provided a lot of direct quotes from the mathematician as well as more specific information about the use of certain mathematical equations that can be used within the field such as subdivision surfaces, harmonic coordinates, and Laplace's equation. Overall, I believe this source provided me with a strong foundation of information in which I was further able to explore in detail through other readings.

Peterson, Ivars. "An Interview with Tony DeRose." *Mathematical Association of America.* MAA. 15 Oct. 2009. Web. 2 Dec. 2016. http://www.maa.org/ news/interview-tony-derose.

This source was really helpful because it provided direct quotes from Tony DeRose, who became a major part of my project due to his research on how mathematics impacts the making of movies. The first few questions of the interview gave me some background information about who Tony DeRose is and how he became interested in math from a young age. The other questions allow me more insight on what exactly DeRose studies, and how that information has helped him create popular Pixar films and speak to larger audiences about his work. The source is reliable as the interview was conducted through the Mathematical Association of America and displays the entire interview with the exact answers that Tony DeRose gave.

Soffe, Emilie. "The math behind the movies: An interview with Tony DeRose of Pixar." *TEDBlog.* 3 May 2014. Web. 2 Dec. 2016. http://blog.ted.com/the-math-behind-the-movies-an-interview- with-tony-derose-of-pixar/.

This source is another interview with Tony DeRose, which I found to be useful because it provided more information about him as a person and his interests. With his research being a large part of my research and project, I liked being able to read more about how he became interested in the mathematics field from a young age, and how he became involved with Pixar in the first place. Also, it includes advice he would give to someone who wanted to use math to make movies in which he encourages the learning of math throughout school and thinking outside of the box to discover new mathematical techniques and breakthroughs that have never been thought of before. Overall, I think this interview allows readers to get a sense of the type of person and mathematician Tony DeRose and how he has strongly impacted animation as well as other people including kids to continue to learn math and think creatively.

"The History of Animation." N.p. N.d. Web. 1 Dec. 2016. http://history-of-animation.webflow.io/.

Although I could not identify an author, publisher or date associated with this website, I still think it can be considered credible source due to its set up and the information provided. For my project I decided to look at mathematical techniques found in film production, and a large part of the research I came across involved how animation is created. Therefore, I thought it would be beneficial for me to also gain a better understanding of the history of animation. Within this one website, I have learned about animation that dates back to 3,000 BC all the way to the present day. It also includes numerous dates and inventions/breakthroughs that was very helpful to me when creating my timeline. Regarding the setup of the source itself, it's like an interactive timeline that includes important dates for animation, great visuals, and relevant videos.

"Theatrical Market Statistics 2015." *Motion Picture Association of America.* 31 Dec. 2015. Web. 1 Dec. 2016. http://www.mpaa.org/wp-content/uploads/2016/04/MPAA-Theatrical-Market- Statistics-2015_Final.pdf.

Conducted through the Motion Picture Association of America, this source provides tons of statistics related to film production worldwide, such as global box office revenues, production costs, and attendance to movie theaters. Although this math is quite different from the math mentioned in the other articles, I thought it would be interesting to also include a little bit of the statistics side of the film industry in my project. I think these statistics can also relate to the modern significance aspect of the project as it shows how much money and effort goes into making a film and how many people depend on them for entertainment on a daily basis. The source displays their data through a variety of bar graphs, scatter plots, and pie charts, all of which were easy to read. On top of the statistics themselves, the source also provides a methodology section, which gave more helpful information about how the data was actually found. It discusses how the association conducted their surveys including the exact sample sizes, the demographics within those samples, possible sampling errors, and the exact wording of a few of the questions that were asked to those who were surveyed. Because of this, I believe this source to be very reliable and the statistics to be accurate in their data research.

Images:

http://biginmicrostock.com/pages/photography/Focusing-on-hyperfocal-distance.html

I chose this image because it displays what hyperfocal distance is in a simple way for anyone to understand. It helped me grasp the idea of what exactly hyperfocal distance means and what objects would therefore be in focus in the shot.

http://www.xpmath.com/careers/topicsresult.php?subjectID=3&topicID=15

This image displays the mathematical process of transformation, specifically reflection, rotation, and translation. One of the topics I discuss relating to film is coordinate geometry and transformation allowing animators to move and change their characters/objects. Therefore, this image shows what transformation is, and how math can be used to move an object around a space.

https://www.ndeed.org/GeneralResources/Formula/RTFormula/InverseSquare/InverseSquareLaw.htm

To provide the equation for the inverse square law, I decided to add this image because I like how it states the equation but also shows how the equation works so that viewers will be able to understand it better.

http://http.developer.nvidia.com/GPUGems2/gpugems2_chapter07.html

I chose this image because I think it displays a great representation of what subdivision surfaces are. After reading about the steps of smoothing out a surface, viewers can see from the image what happens to the surfaces visually. As the process starts with a cube, it can be seen how splitting and averaging can lead to the addition of more points until it eventually appears smooth. Reading about the process may make it seem complicated, but I think the image makes it easier to understand.

http://www.mpaa.org/wp-content/uploads/2015/03/MPAA-Theatrical-Market-Statistics-2014.pdf

I included this image to show how statistics also plays an important role in the film industry. After including an example of three statistics I found, I thought it would also be helpful to give an example of a bar chart displaying film statistics. I wanted to show that graphs we learned about in class are also used when studying film revenues, ratings, attendees, as well as used to predict where the film industry might be heading in the future. Although this chart is from 2014 instead of 2015 like my other statistics, I wanted it more to show how the data can be displayed instead of the actual data that is included in the graph.

http://www.mhsgent.ugent.be/engl-plat5.html

I chose to use this image because whenever I read about older inventions that are not still used today, it's hard for me to comprehend exactly what it looks like or how it works. This image shows what a phenakistoscope looks like and also how someone can use it.

https://zoetropic.wordpress.com/about-zoetropes/

Similar to my reasoning for the phenakistoscope image, I wanted to include an image that shows what a zoetrope looks like and how it is used. I think this image displays to the viewer what it looks like and also how the vertical slits and the pictures on the inside of the device can create the illusion of movement while the device is rotating.

http://animateducated.blogspot.com/p/blog-page_31.html?m=0

I chose this image because it displays what I talk about relating to the principles of animation. It easily shows how animation of a bouncing ball involves the use of math by stretching as it falls and squashing as it bounces.

http://www.digitaltrends.com/gaming/give-hand-first-3d-computer-animation-1972/

I included this picture on timeline because I wanted people to see animation that dates back to the 1970s. The short created by Ed Catmull played an important role in the history of animation and this image shows the first step in creating 3D animation.

https://en.wikipedia.org/wiki/Toy_Story_(franchise)

I included this picture on the timeline because it's a familiar logo to everyone who has seen the movie *Toy Story* and it's a fun, colorful image to add that will capture people's attention.

https://54disneyreviews.com/2015/05/12/pixar-review-8-geris-game/

This image shows the character Geri from the short *Geri's Game,* which I mention on my timeline. I think it is a simple, cute image to add but it also shows how much animation goes into creating something like this. Every aspect of the image used subdivided surfaces like his face, his hands, his jacket, and his clothes. Therefore, I wanted people to see how the final product can look after all the behind the scenes math and animation techniques are used.

http://www.huffingtonpost.com/tim-chartier/pixar-math-inside-and-out_b_7756552.html

I chose to also include this image of Geri's hands because it shows what the animation looks like before and after subdivision occurs. In the first image, viewers can get a good sense of how many points and faces there are but the second image shows that one can obtain a smooth looking hand after subdivision occurs.

http://www.momtrends.com/2012/06/exclusive-interviews-with-the-cast-and-director-of-brave/

This image of Merida from the film *Brave* is included because I wanted to include another popular Pixar movie that is more recent to show how animation has developed from how it began to where it is now. Also, this specific picture of Merida shows how complex her hair is, which also infers how much behind the scenes work went in to creating this movie.

https://www.youtube.com/watch?v=_IZMVMf4NQ0

This image is a still shot from the video in which Tony DeRose gives his TED talk. I decided to include this image of Tony DeRose because I think it provides viewers with a good sense of who he really is. He is presenting to a large audience about his research and his love for math, and he really is able to inspire others and I think that can be seen through the image. It also shows him holding a model of Geri's hand, which was a part of his first project working with Pixar. Geri was the first character he worked on, and is therefore his favorite character.