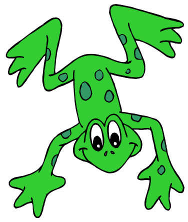


Math in Motion:
Experiencing Proportional Relationships

Through a series of hands-on activities, students develop proportional reasoning and are able to create tables, graphs, and equations.

Grade Level: 6th – 8th

Topics: ratios, rates, unit rates, measurement conversion, writing and solving real life equations, proportions, tables, graphing

Common Core Math Standards:

6.RP.1	7.RP.2	8.EE.5
6.RP.2	7.EE.4	
6.RP.3		

Standards for Math Practice: 1.Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique reasoning.
8. Look for and express regularity in repeated reasoning.

Goals:

- Students will be able to determine a unit rate.
- Students will be able to use proportional reasoning to create tables, graphs, and equations.
- Students will be able to use the $d = rt$ equation to find distances, rates, or times given two of the variables.
- Students will use proportional reasoning to convert measurements.
- Students will be able to analyze graphs for information including rates of change.
- Students will be able to identify proportional and non-proportional relationships through tables, graphs, and/or equations.

Prerequisite Knowledge:

- Students are able to solve one step equations
- Students are able to graph coordinates on the Cartesian Plane
- Students can express ratios and verify equivalent ratios.

Materials:

- *If You Hopped Like a Frog* by David M. Schwartz
- 3" x 5" cards
- pre-measured distance (in hallway, gym, etc.)
- stopwatch
- meter sticks and/or tape measures (with metric and English standard units)

- “T-table and Grid” paper or regular grid paper
- Origami Jumping Frog instructions
- large gridded chart paper
- scale (to weigh students)
- student activity sheets
  - “If You Could...”
  - “What's My Rate?”
  - “The Story of Two Frogs”
  - “More Frog Tales”
- standard calculators
- math Journals

Preparation Time: 1 - 2 hours

Activity Time: 6 lessons

Extensions:
The following NCTM Illuminations website lessons extend the concepts of this module:
- *Barbie Bungee* (linear functions and slope)
- *Constant Dimensions* (ratio of length to width)
- *Bouncing Tennis Balls* (linear relationships between time and bounces)
- *Pedal Power* (slope and rate of change)
- *Representing Data* (create graphs from a table)
- *Taking Its Toll* (exploring toll/distance relationships)
- *The Line Runner* (slope and linear equations with a running model).

Lesson Plans

Lesson One: Students explore ratios and are introduced to a special ratio known as rate.

Tell students that we want to find out how far six classroom volunteers can jump. Create teams to measure each distance in both inches and centimeters . Chart the results and ask students if they think the volunteers' distances were more than their heights or less. Measure the height of each student, and write the height/distance ratios for inches and centimeters on the chart. Have students compare each volunteer's ratios. They will discover that the ratios are the same even though the units of measurement are different.

Read aloud from the book *If You Hopped Like a Frog.* As you read through each "if you.." page, ask students what the author is saying about humans compared to various animals. After reading the text, pair students and have them complete the first section of the activity sheet "If You Could…" that requires them to write ratios and answer the question, "How far could you jump, if you could jump like a frog?" Students will need to measure their height in inches and centimeters.

Discuss the results that students created and make a class chart of height and "froggie" jumps.

Have each student measure their weight and write it on a 3" x 5" card. Collect the cards. As you are writing the weights on chart paper, have students work on the second section of the activity sheet in order to find out how much more 1/5 is than 1/250. If students are struggling, remind them that 1/250 ounces + 1/250 ounces would make an amount that is twice the weight.

Invite students to share their solutions to the question and their strategies for finding it.

Once the class agrees that 1/5 is 50 times more than 1/250, use that ratio to help students complete the chart in order to answer the question: "How many pounds could we lift if we were as strong as ants?" Have information on various weights for comparison. For example, if a 120 pound human could lift 50 times her weight, she could lift 6000 pounds which is the weight of two cars.

For a final activity, students will be exploring a rate - speed. Each student should work independently to determine the answer to "How far could you travel in one second, if you were as fast as a spider?" You may want to allow calculators. Have students compare their results with classmates. For 7^{th} and 8^{th} grade students, use dimensional analysis to convert feet per second to miles per hour. An example is below:

A 64 inch person would be able to travel 2,112 inches in one second if that person could scurry like a spider. 2,112 inches is 180 feet.

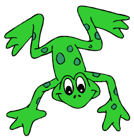
$$\frac{180\,\cancel{feet}}{1\,\cancel{second}} \times \frac{60\,\cancel{seconds}}{1\,\cancel{minute}} \times \frac{60\,\cancel{minutes}}{1\,hour} \times \frac{1\,miles}{5{,}280\,\cancel{feet}} = \frac{180\,(60)(60)miles}{5280\,hours} = 120 \text{ miles per hour}$$

If You Could…..

Before completing this worksheet, find your height in inches and centimeters.

Name:____________________ Height in inches:________________

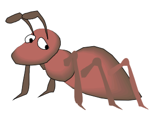
Height in centimeters:___________

Part One  **Hop Like a Frog**

The frog is an excellent jumper. On average, a 3-inch frog can hop 60 inches. Write that as a ratio:

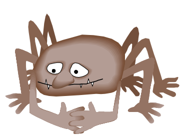
A frog can jump ___________times farther than its length. How many inches could you jump ***if*** you could jump like a frog?

How many centimeters could you jump ***if*** you could jump like a frog?

Part Two  **Be as Strong as an Ant**

Ants may be tiny, but they are great weight lifters. For example, an ant that weights $\frac{1}{250}$ of an ounce, can lift a bread crumb that weights $\frac{1}{5}$ of an ounce. How much larger is $\frac{1}{5}$ than $\frac{1}{250}$?

If you were as strong as an ant, how much weight could you lift?

Part Three  **Scurry Like a Spider**

Considering its length, a female spider is incredibly fast. It can move 33 times its body length in only 1 second! How many inches could you travel in one second ***if*** you were as fast as a spider?

How many feet could you travel in one second?

How many miles could you travel in one hour?

Lesson Two: Students explore their own rates and compare them. Students find unit rates and use them in proportions.

Help students find their pulse so they can count the number of heart beats. Time students for ten seconds and have them record the number of beats . Have students use this ratio of beats to seconds, which is a heart ***rate***, to determine beats per minute. Collect the information and chart the results in a table labeled “Student Heart Rates”.

Have students hop on one foot for 20 seconds and record their hopping rate and use that information to find the number of hops they could do in one minute *if they continued at the same rate*. Also, have students find their ***unit rate*** (hops per second). Ask students to find the person in the class that can hop the fastest and then explain how they know.

Time students for 30 to 45 seconds doing jumping jacks. Students then find a pulse for another heart rate check. Each student will count his or her own heartbeats as you time the class for 15 seconds. With a partner, students should complete the activity sheet “What’s My Rate?” and answer questions regarding the rates they found.

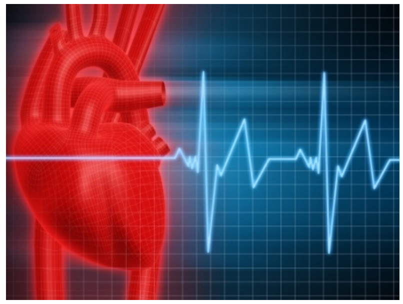
As teams complete the activity sheet, they should compare their results with other partners. Discuss the results with the class and add the new data to the “Student Heart Rates” chart.

Hand out graph paper and have students select the two heart rate tables to graph as a set of ordered pairs. It would be helpful to begin by discussing which quadrants are needed for the data (only quadrant I because all data is positive), where to place time (x-axis) and beats/jumps/hops (y-axis), and how to scale each axis.

Students should make observations about the points on the graph (for example, they fall in a straight line) and then compare the two sets of data (one set of points should be increasing more quickly because the rate is greater). Have students verify that points on each graph are *proportional.* For example, (12,10) which represents 12 beats in 10 seconds and (36,30) for 36 beats in 30 seconds, i.e.

$$\frac{12\ beats}{10\ seconds} = \frac{36\ beats}{30\ seconds}$$

A student should explain why these two rates are equivalent in at least two ways. One way might be to change both to the unit rate of 1.2 beats per second.



What's My Rate?

How many beats did you count in ten seconds? Imagine that this *heart rate* will remain unchanged. Complete the table below using this *rate* until you are able to identify your *beats per minute* .

Heart Rate Trial 1

Time (seconds)	**Beats**
10	
20	
30	

My first heart rate is ______________________.

How many beats did you count in fifteen seconds? Imagine that this *heart rate* will remain unchanged. Complete the table below using this second *rate* until you are able to identify your *beats per minute.* Explain how your heart rate changed.

Heart Rate Trial 2

Time (seconds)	**Beats**
15	
30	

My second heart rate is ______________________.

Lesson Three: Students fold origami jumping frogs and determine their frog's jumping rate. Students create tables, graphs, and equations based on their frog's jumping rate.

Each students should be given a 3" x 5" card and instructions for folding an origami jumping frog. Students should be allowed to help each other with the folding. It is also fun to have students name and decorate their paper frogs.

Have students form teams of two or three. Each team will need a meter stick. Time each student for ten seconds (if there are eleven teams, eleven students will be timed at once) and have them record the distance their frog traveled in that amount of time (to the nearest whole centimeter). Continue until all students have found their frog's distance.

Ask students what two variables are being considered and which one is the independent and which one is the dependent variable (time - independent; distance – dependent). Have students create a table for this data and, assuming the frog could continue at the same *rate*, add distances for ten-second intervals up to sixty seconds. Have students compute the centimeters per second *unit rate* for their frog and the centimeters per minute *unit rate.*

Have teams discuss the following question: "What is the relationship between the *rate, time and distance* as you look at the data?" Discuss ideas as a class and write these ideas on chart paper. Include all ideas, even if some of them are not correct. These ideas will be revisited later.

Each student should be given grid paper in order to graph the data in the table. Prior to graphing, discuss qualities of a good graph, determining appropriate scales, etc. Each team should use the same scale so they can accurately compare their graphs. Since students are graphing continuous data, a line should be drawn through the points on the graph.

Have students write observations about their graphs and team member's graphs.

Have a class discussion about these observations. Follow up questions should include:

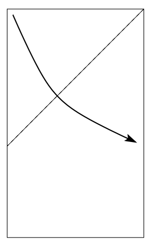
- "Why do the coordinates on the graph form a straight line?"
- "What would the coordinate (10,40) mean on this graph?"
- "Why are some lines steeper than others?"
- "What is the relationship between the steepness of the line and the frog jumping rate?"

Refer back to the chart ideas for the relationship between *rate, time and distance.* Ask students to test each idea using data from their table. The class should conclude that *distance equals the rate multiplied by the time* is one true relationship. Introduce the formula $d = rt$. Using that formula, have students write an equation for their own frog and include that with the graph and T table.

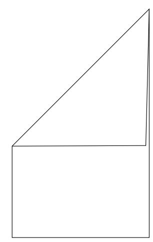
Using their own equation, have students determine the distance their frog would jump in 15 seconds. Students should then be able to locate that coordinate on the graph. Ask students how they could find it in the table.

Extension: What if students knew the distance and the rate? Could they find the time? Have students challenge other classmates using their own data on 3'' x 5'' cards posted around the room.

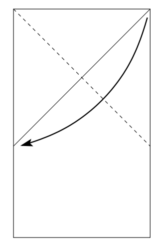
Jumping Frog



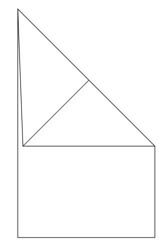
Fold the top edge to meet the right edge.



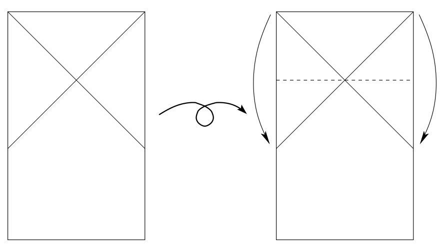
Undo the last fold.



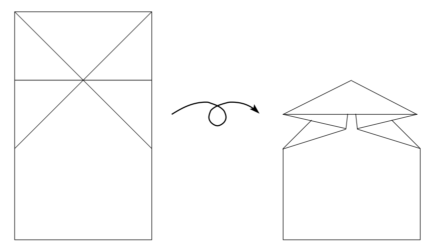
Fold the top edge to meet the left edge.



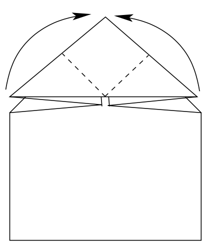
Undo the last fold.



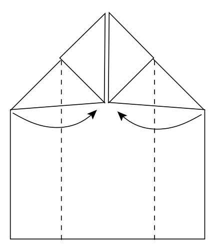
Turn the card over and fold top corners down to the bottom points of the X. Undo this fold.



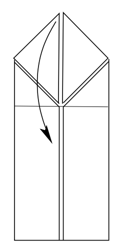
Turn the card over. Leaving the middle of the X on the table, bring the left and right edges in to the middle until they meet. Collapse the upper corners onto the bottom points of the X.



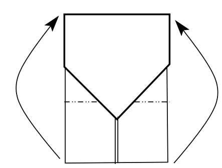
Fold the tips of the triangle up to the top.



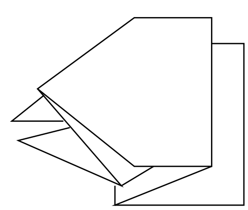
Fold the left and right edges in to the middle.



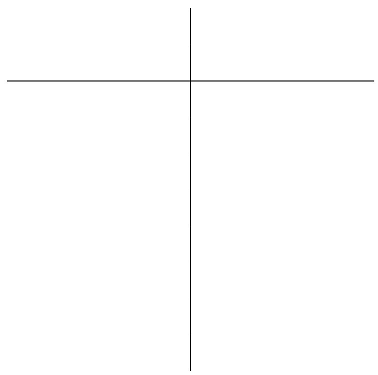
Fold the head down.

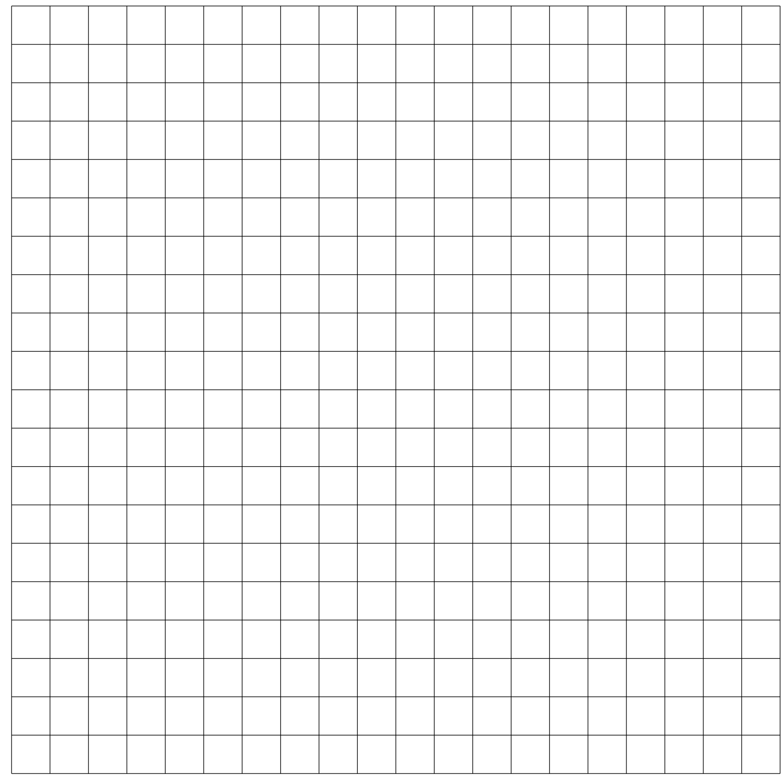


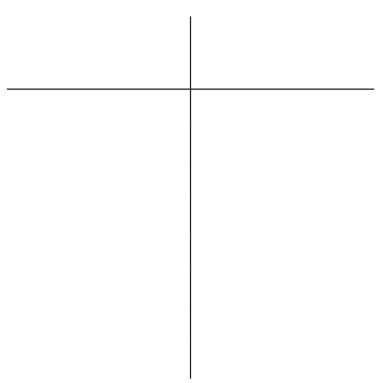
Fold the legs back.

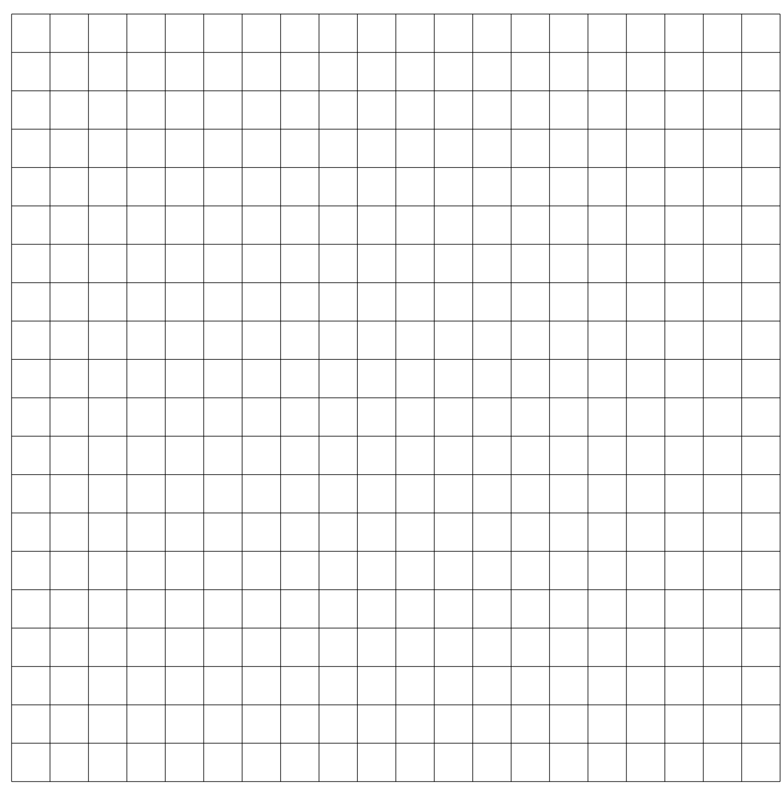


The frog is done. Pet its back to make it jump!









Lesson Four: Students recognize the proportional relationship of the distance, rate, and time graphs and are introduced to rate of change.

Note: Prior to class, prepare a large display graph for Hip, Hop and Frogger using the data from the activity sheet "The Story of Two Frogs."

Each student should have their table and graph from the previous lesson. Spend a few minutes having students pair/share the information on their tables and graphs and compare their equations. As a whole class, discuss the similarities and differences of various tables and graphs. Students should observe that all graphs include the origin (0,0). Ask students why this coordinate makes sense in the context of the activity of lesson three.

Place students in pairs and hand out the activity sheet, "The Story of Two Frogs". Allow the use of calculators. Students will be asked to determine hopping rates, as well as write equations. After students have completed the "Story of Hip Hop" section of the activity sheet, discuss results as a class.

Have students work independently on the "Story of Frogger" section of the activity sheet and then have them compare results with their partners. Discuss results as a class.

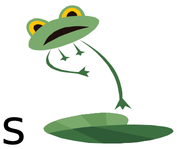
Using their own graphs and tables, have students determine whether or not their graph shows a ***proportional relationship***. Explain that in a proportional relationship, every coordinate forms an equal ratio with every other coordinate. For example, if one coordinate was (10,45) and another coordinate was (40,180), the ratios $\frac{45}{10} = \frac{180}{40}$ which means they form a *proportion.* Have students test various coordinates in their table or on their graph. Ask students how they verified that the two ratios were equal. **Note: Students in eighth grade could be introduced to the concept of a *direct variation*.**

Earlier in the lesson, students were asked to identify similarities and differences between their graphs and their partner's graphs. One observation may be that one graph is steeper than another. Ask students what affects the steepness of each graph. Some students may respond that the steeper graph was because the frog jumped "faster". Connect the idea of jumping speed to the jumping rate. Miles per hour is a familiar rate that is commonly used to describe the speed of a car, bus, bike, etc. Challenge students to consider how the speed or rate could be found just using the graphs students drew (the rate is the slope of their graph).

Allow students to work with partners to find a way to use their graphs to find the frog hopping rate that they were able to determine using the tables. As a helpful hint, remind students that the rate is comparing distance and time.

Have teams share their strategies using the large display graphs for "Hip Hop" and "Frogger." Introduce the idea of *rate of change* to students. As an added challenge, ask students to find the *rate of change* for a third frog named Greenie and only give the following coordinates: (0,0) and (30, 75). Encourage students to consider that the distance changed 75 centimeters and the time changed 30 seconds. That is a rate of 75cm/30 seconds which is the same as 25 cm per 10 seconds which becomes a unit rate of 2.5 cm per second.

As an independent follow up, provide "T-table and Grid" paper to students and have them create a table of values and a graph for Greenie, assuming it has an average hopping rate of 2.5 cm per second.

 A Tale of Two Frogs 

The Story of Hip Hop

Marty's frog Hip Hop jumped for several seconds. After measuring the time and distance, Marty wrote this equation: d = 5.4t. Using this equation, help Marty complete the table below.

Time in seconds	Distance in centimeters
10	________
15	________
18	________
30	________
__	270

1. Explain how the rate of 5.4 centimeters per second can be found using the data in the table.

2. Explain how the table demonstrates that the graph of this relationship would be a straight line.

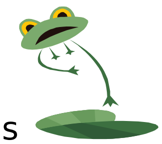
The Story of Frogger

Gisella's frog Frogger jumped for 20 seconds and reached a distance of 140 cm. Gisella created a table of values using Frogger's average jumping rate. Help her complete the table below.

Time in seconds	Distance in centimeters
20	140
40	280
50	____
55	____
___	420

3. What is Frogger's jumping rate in centimeters per second? Show the work done to determine this rate.

4. What equation would Gisella write for Frogger?_____________________

 A Tale of Two Frogs 

TEACHER KEY

The Story of Hip Hop

Marty's frog Hip Hop jumped for several seconds. After measuring the time and distance, Marty wrote this equation: d = 5.4t. Using this equation, help Marty complete the table below.

Time in seconds	Distance in centimeters
10	**54**
15	**81**
18	**97.2**
30	**162**
50	270

1. Explain how the rate of 5.4 centimeters per second can be found using the data in the table.

ANSWERS VARY: divide the distance by the time; for example, 54/10 = 5.4cm/sec

2. Explain how the table demonstrates that the graph of this relationship would be a straight line.

ANSWERS VARY: when the time in seconds tripled, so did the distance; d/t is always 5.4.

The Story of Frogger

Gisella's frog Frogger jumped for 20 seconds and reached a distance of 140 cm. Gisella created a table of values using Frogger's average jumping rate. Help her complete the table below.

Time in seconds	Distance in centimeters
20	140
40	280
50	**350**
55	**385**
60	420

3. What is Frogger's jumping rate in centimeters per second? Show the work done to determine this rate.

Frogger has a rate of 7 cm per second. Divide the distance by the time.

4. What equation would Gisella write for Frogger ? ***d = 7t***

Lesson Five: Students explore how a head start affects the table, graph, and equation for their frog's jumping data and how the head start creates a relationship that is no longer proportional.

Have students look at the work they did in *Lesson Three*. Place students in teams of three. Distribute new grid paper to each team. Each team should now use all three classmates' T-tables as data to make one graph. Students should use a different color for each set of data and create a key. Have students discuss how they know which frog is the "fastest" based on the table and how they know based on the graphs. Have each team identify which frog has the lowest jumping rate and which frog has the highest jumping rate for their team. Post the team data on chart paper. For example, Team One might post that Green Guy has a rate of 11 cm/sec and Marvin has a rate of 6 cm/sec.

Tell each group that the "slowest" frog will now be given a head start of 20 centimeters. Have each group member discuss how this head start will change the table (add 20 cm to each distance) and how it will change the graph (the graph now has the coordinate (0,20) instead of (0,0) and has been shifted up 20 units). Have teams make observations about how the graph with the head start compares to the one without. Encourage students to notice that the two lines are parallel. Ask students why the two lines would be parallel (the steepness of the line is determined by the rate; the head start does not change the rate or speed of the frog). Students should have to verify that the rate of the new graph is the same as the rate of the first graph using coordinates as discussed in lesson four.

Ask students if any of the graphs now intersect and have students discuss what the points of intersection are and what they mean (at the point of intersection, the two frogs have traveled the same distance at the given time). Have students determine the new equation for the frog with the head start. For example: Green Guy's equation is $d = 11t$ and Marvin's would be $d = 6t$. If Marvin is given a 20 cm head start, its equation becomes $d = 6t + 20$. As students look at the graphs and tables, ask them to prove whether or not a head start of 20 centimeters will allow the "slow" frog to win if the frogs jump for one minute.

Note: For 8th graders, this point of intersection can be discussed as a solution to a system of equations; it is the point where ***$11t = 6t + 20$ because of the same distance;*** **the distances are the same at four seconds.**

Have students determine whether or not the equation, table, and graph with a head start is a *proportional relationship* as discussed in lesson four. Discuss the ideas with the class. Students should observe that, in the case of the Green Guy example, two coordinates would be (10,80) and (20, 140). These do not form equal ratios, so they do not form a proportion. 8th graders should have discussed the idea of a direct variation which can be written as $\boldsymbol{y = kx}$ with $\boldsymbol{k}$ being the constant of variation. Students should be able to see that this formula can be rewritten as $\boldsymbol{k = y / x}$. For the frog jumping equations, the constant of variation is the jumping rate which is also the slope of the line. It might be helpful to relate the equation to the slope-intercept form if it has already been introduced.

For the math journal, have students answer the following question: "What is the smallest head start the slow frog would need to be given in order to win a 60 second race and how did you find the answer?"

Lesson Six: Students are able to recognize proportional and non-proportional relationships with tables, graphs, and equations.

Have students pair/share their journal responses to the question posed in lesson five. Have a whole class discussion of strategies students used to determine a correct head start for the slowest frog. Strategies may have included extending the table of values for the two frogs for various head starts, using a system of equations, guess and test, drawing a line parallel to the slow frog's graph that will allow it to pass the fast frog and finding the intersection with the y-axis, etc.

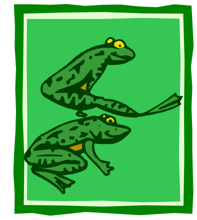
Distribute the activity sheet "More Frog Tales" and grid paper to each student. Have students work in teams for part one and then discuss the results as a class.

Students should work independently on part two.

Have students pair/share their results of part two as time allows and then discuss strategies as a whole class.

Extension: Using the measured hallway or gym distance, time students walking for a given period of time and have students determine their walking rate. Students create tables, graphs, and equations. Post the various rates and have students graph several results on the same grid. Have students add head starts of various distances and discuss the outcomes.

More Frog Tales



Part One: Patrick's frog Jumper was able to hop at an average rate of 4 centimeters per second. Create a table, equation, and graph for Patrick's frog. Show all the work done to answer the five questions below. Be prepared to explain your reasoning.

Time in seconds	Distance in centimeters
0	______
10	______
20	______
30	______
40	______
50	______
60	______

equation:____________________

1. If Jumper could continue at this rate, how far could Jumper travel in an hour?

How far would this distance be in meters?

2. How long would it take Jumper to travel 300 centimeters?

3. Patrick decided to change Jumper's equation so that ***t*** represents time in minutes and ***d*** represented distance in meters. What equation did Patrick write?

4. Using Patrick's new equation, how long would it take Jumper to travel 12 meters?

5. Verify whether or not the relationship shown in the table is proportional.

Part Two: Dana's frog Cutie averaged 2 centimeters per second. Create a table, equation, and graph for Dana's frog. Graph Cutie's results on the same grid used for Jumper.

Time in seconds	Distance in centimeters
0	______
10	______
20	______
30	______
40	______
50	______
60	______

equation:__________________

Patrick realized that Dana's frog Cutie was never going to beat Jumper in a race. He wanted to give Cutie a head start to make the race more exciting. He decided to give Cutie a 50 cm head start. Create a new table, equation, and graph for Cutie with the head start.

Time in seconds	Distance in centimeters
0	______
10	______
20	______
30	______
40	______
50	______
60	______

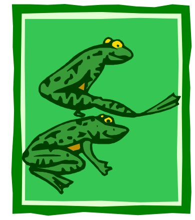
equation:__________________

6. If the race lasts one minute, who will win? Prove your conclusion in at least two ways.

7. Did any graphs intersect?___ If so, what was the point of intersection and what did it mean?

8. Which of the equations for Cutie was NOT proportional?_________________
Explain why it does not form a proportional relationship.

More Frog Tales



KEY

Part One: Patrick's frog Jumper was able to hop at an average rate of 4 centimeters per second. Create a table, equation, and graph for Patrick's frog. Show all the work done to answer the five questions below. Be prepared to explain your reasoning.

Time in seconds	Distance in centimeters
0	0
10	40
20	80
30	120
40	160
50	200
60	240

equation: ***d = 4t***

1. If Jumper could continue at this rate, how far could Jumper travel in an hour?
240 cm in a minute x 60 minutes = 14,400 centimeters

How far would this distance be in meters? **100cm/1m = 14400 cm/144m; 144m**

2. How long would it take Jumper to travel 300 centimeters?
300 = 4t; 300/4 = 75; 75 seconds or 1 minute and 15 seconds (students could also use the graph to try to find the point where d is 300)

3. Patrick decided to change Jumper's equation so that ***t*** represents time in minutes and ***d*** represented distance in meters. What equation did Patrick write?
240 cm = 2.4 meters; Jumper travels 2.4 meters in one minute. The new equation is *d* = *2.4t*

4. Using Patrick's new equation, how long would it take Jumper to travel 12 meters?
12 = 2.4*t; 12/2.4* = *5;* it would take 5 minutes which can be verified by a Table.

5. Verify whether or not the relationship shown in the table is proportional.
Answers vary: students could show that all the ratios of d/t are equal.

Part Two: Dana's frog Cutie averaged 2 centimeters per second. Create a table, equation, and graph for Dana's frog. Graph Cutie's results on the same grid used for Jumper.

Time in seconds	Distance in centimeters
0	0
10	20
20	40
30	60
40	80
50	100
60	120

equation: ***d* = 2*t***

Patrick realized that Dana's frog Cutie was never going to beat Jumper in a race. He wanted to give Cutie a head start to make the race more exciting. He decided to give Cutie a 50 cm head start. Create a new table, equation, and graph for Cutie with the head start.

Time in seconds	Distance in centimeters
0	50
10	70
20	90
30	110
40	130
50	150
60	170

equation: ***d* = 2*t* + 50**

6. If the race lasts one minute, who will win? Prove your conclusion in at least two ways. **Jumper will win. Looking at the table, Cutie's distance is 170 cm in one minute and Jumper's distance is 240 cm. Also, looking at the graphs, Jumper's distance at one minute is greater than Cutie's.**

7. Did any graphs intersect? **yes** If so, what was the point of intersection and what did it mean? **(25,100) It means that at 25 seconds, the two frogs traveled the same distance. They would be side by side.**

8. Which of the equations for Cutie was NOT proportional? ***d* = 2*t* +50** Explain why it does not form a proportional relationship. **answers vary: d/t ratios are not equal; for example 150/50 is not equal to 90/20.**