TEAL

Teaching Excellence in Adult Literacy

Math Works! GUIDE

March 2014

AMERICAN INSTITUTES FOR RESEARCH

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1. INTRODUCTION

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Teaching Excellence in Adult Literacy

https://teal.ed.gov/

Mary Ann Corley, Ph.D., TEAL Director
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March 15, 2014

It is my pleasure to introduce the Teaching Excellence in Adult Literacy (TEAL) *Math Works! Guide.* A companion piece to the TEAL *Just Write! Guide* published in February 2012, the *Math Works! Guide* represents the culmination of 2 years' work in identifying research-based instructional practices in the content areas of mathematics and numeracy. It synthesizes important practices and competencies represented in current research on math instruction. The principles represented for preparing adult learners for college and careers are rooted in the work of the National Research Council, the U.S. Department of Education, and the National Council for Teachers of Mathematics, as well as in the Common Core State Standards, a product of the National Governors' Association Center for Best Practices and the Council of Chief State School Officers. Many of the activities are based on content from Adult Numeracy Instruction–Professional Development (ANI-PD), which was field tested in 2010–2011 through the U.S. Department of Education project, *Strengthening America's Competitiveness through Adult Math Instruction*. The TEAL *Math Works! Guide* is a resource for ABE teachers, intended to increase their familiarity with evidence-based mathematics and numeracy instruction and to facilitate the translation of research findings into teaching practices that will enhance the quality of instruction delivered to adult learners.

American Institutes for Research, through contracts awarded by the U.S. Department of Education Office of Career, Technical, and Adult Education (OCTAE), assists OCTAE in its efforts to enhance the capacity of state and local adult education practitioners to understand and apply evidence-based instructional practices that promote student learning across content areas. Recognizing that *content* knowledge is only one important facet of quality instruction, TEAL staff developed a series of eight online courses to familiarize teachers with the equally important elements of quality teaching: research-based *processes* and approaches. Following are the titles of the eight TEAL online courses, which are offered through the U.S. Department of Education's Literacy Information and Communication System (LINCS):

1. Universal Design for Learning
2. Self-Regulated Learning
3. Differentiated Instruction
4. Effective Lesson Planning
5. Formative Assessment
6. Strategy Instruction
7. Deeper Learning through Questioning
8. Student-Centered Learning: Keys to Motivation and Persistence

Many people helped with the development of this guide and the online courses. First and foremost is Ruth Sugar from RTI International, who ably led the conceptualization and development of the guide, assisted by Kristen Kulongoski. Reviewing the guide and offering valuable input were Donna Curry, a TEAL subject matter expert, and Steve Leinwand, an AIR math content specialist and principal researcher. Also on behalf of AIR and OCTAE, I offer sincere appreciation to the TEAL subject matter experts who willingly reviewed and provided guidance to TEAL staff as we fashioned the guide and the courses: Aydin Durgunoglu, Daphne Greenberg, Noel Gregg, Steve Hinds, Glynda Hull, Charles "Skip" MacArthur, Sharan Merriam, Elizabeth Birr Moje, and Dolores Perin. Finally, I extend appreciation to OCTAE staff members who provided constant support throughout the development of the online courses and guide: Diane McCauley, Mary Jo Maralit, Heidi Silver-Pacuilla, and Chris Coro, as well as to AIR staff members who have worked tirelessly to produce a quality effort: Delphinia Brown, Darren Cambridge, Mariesa Cash, Larry Condelli, Marshal Conley, Steve Leinwand, Beth Ratway, and W. Christine Rauscher.

For additional information, visit the TEAL website, https://teal.ed.gov, or contact the TEAL staff at teal@air.org.

Sincerely,

Mary Ann Corley

Mary Ann Corley, PhD
Director, TEAL Center and
Principal Researcher, American Institutes for Research

1.1 Why Is Numeracy Important?

The Role of Numeracy

Mathematics skills are critical for adults to engage in the activities of daily life, education and training, and the workplace. In the recent Survey of Adult Skills (2013), the Organization for Economic Cooperation and Development (OECD)[1] defines a numerate adult as one who "responds appropriately to mathematical content, information, and ideas represented in various ways in order to manage situations and solve problems in a real-life context." Whether consuming information or goods for personal or job-related responsibilities, adults need to develop mathematical proficiency to make informed decisions. Making sense of numbers and mathematical concepts is necessary for tasks such as computing and comparing interest rates on a loan, assessing a candidate's economic platform, buying groceries, evaluating bids on office equipment, or managing a project budget. To operate effectively as participants and producers in everyday activities, adults must be able to manipulate numbers and interpret quantitative information competently.

Many adults, however, struggle with problem solving, expressing ideas mathematically, making connections among math concepts, applying math to other contexts, and reasoning analytically. Adult learners also often lack competency in essential concepts such as proportional reasoning, algebraic thinking, and statistical understanding (U.S. Department of Education, 2011). These gaps in knowledge make it difficult for them to be successful in daily life, education and training, and the workforce.

The Impact of Low Numeracy Skills

A population with low numeracy skills faces wide-ranging consequences. It has been argued that, for the citizenry to participate in the democratic process, to ensure the strength and security of the nation, and for the United States to remain globally competitive, adults need deeper and broader understanding of math and science concepts. Research shows that those with higher literacy and numeracy skills have better employment outcomes and greater earning potential. As documented by the Survey of Adult Skills (OECD, 2013), adults with better numeracy skills tend to earn more, even when accounting for other factors such as education, age, gender, immigrant status, and job tenure. Regrettably, this study also documented that nearly one in three adults in the United States scores below Level 2[2] in numeracy. This ability threshold covers skills such as basic multistep problem solving, calculations with whole numbers, common fractions, percentages, decimals, and simple measurement and estimation. Further still, the majority of adults in the United States performs at Level 2 and below, and only

[1] This study of 23 countries is a data collection and analysis initiative that is part of the Programme for the International Assessment of Adult Competencies (PIAAC); therefore, the Survey of Adult Skills is often referred to as the PIAAC. The Survey of Adult Skills was conducted in the United States from August 1, 2011, to March 31, 2012. Approximately 5,010 adults aged 16–65 were surveyed in the United States.

[2] Level 2 required the respondent to complete successfully the following types of tasks: "…identify and act on mathematical information and ideas embedded in a range of common contexts where the mathematical content is fairly explicit or visual with relatively few distracters. Tasks tend to require the application of two or more steps or processes involving calculations with whole numbers and common decimals, percents and fractions; simple measurement and spatial representation; estimation; and interpretation of relatively simple data and statistics in texts, tables, and graphs."

9%—a rate lower than all other participating countries—scores at Levels 4 and 5. These two highest levels of numeracy measured by the survey require skills such as analysis and reasoning, understanding data and statistics, translating or interpreting various types of quantitative information, and reflecting critically on mathematical solutions. The American Council on Education further supports the need to improve the numeracy skills of U.S. adults, noting that labor market demand for postsecondary education and higher-level mathematical understanding is growing at a rapid rate. By 2018, nearly two-thirds of new positions will require some postsecondary education (Pulley, 2011). However, Complete College America has found that, although approximately 70% of community college students require at least one remedial course upon entry, only one-third of these students completes these required courses (Charles A. Dana Center; Complete College America, Inc.; Education Commission of the States & Jobs for the Future, 2012). As a result, too many students are unable to complete postsecondary education and earn a credential because of limited mathematical skills.

Employers are targeting this low level of numeracy skills among the adult U.S. population. In 2006, human resources and executive staff from more than 400 employers were interviewed and surveyed about the work readiness of recent job entrants, and more than half of respondents rated high school graduates as “deficient” in mathematics.[3] The study also noted that employers have even higher mathematics expectations for adults who have completed some postsecondary education (Casner-Lotto & Barrington, 2006).

As these studies illustrate, students are not achieving the level of mathematics skills needed to meet the demands of school and work. In fact, researchers from the Harvard Kennedy School contend that the United States would experience a significant increase in gross domestic product per capita by enhancing the math proficiency of its K–12 students. It is evident that low numeracy ability negatively affects one’s ability to navigate daily tasks and earning potential, causing both individual and national economic repercussions. For these reasons, it is imperative that mathematics instruction and learning outcomes be improved to significantly increase the success of adult learners.

How This Guide Addresses Numeracy Instruction

A number of research-based instructional practices can be applied across many content areas and serve to enhance math instruction. A group of such approaches is explained in detail In the TEAL *Just Write!* Guide, and a subset of these is highlighted in this mathematics guide. Excerpts from fact sheets in the *Just Write!* Guide are used as background information in Section 1.2, and they inform the introductory material for most of Section 2.

[3] This study was conducted by the Conference Board, Partnership for 21st Century Skills, Corporate Voices, and the Society for Human Resource Management.

1.2 Instructional Practices That Make a Difference

Self-Regulated Learning (Fact Sheet 3)

About Self-Regulated Learning

Self-regulated learning refers to one's ability to understand and control one's learning environment. Self-regulation abilities include goal setting, self-monitoring, self-instruction, and self-reinforcement (Harris & Graham, 1999; Schraw, Crippen, & Hartley, 2006; Shunk, 1996). Self-regulation should not be confused with a mental ability or an academic performance skill. Instead, self-regulation is a self-directive process and set of behaviors whereby learners transform their mental abilities into skills (Zimmerman, Bonnor, & Kovach, 2002) and habits through a developmental process (Butler, 2002) that emerges from guided practice and feedback (Paris & Paris, 2001).

Self-Regulated Learners (Paris & Paris, 2001):

- Analyze task requirements
- Set goals
- Select strategies to achieve objectives
- Monitor their work on a task
- Manage affective variables
- Ask questions
- Take notes
- Manage time and resources

Self-regulated learning strategies help to prepare learners for lifelong learning and the important capacity to transfer skills, knowledge, and abilities from one domain or setting to another.

Universal Design for Learning (Fact Sheet 2)

About Universal Design for Learning

Universal Design for Learning (UDL) is a set of principles for designing a curriculum that provides all individuals with equal opportunities to learn. UDL is designed to serve all learners, regardless of ability, disability, age, gender, or cultural and linguistic background. UDL provides a blueprint for designing goals, methods, materials, and assessments to reach all students including those with diverse needs. Grounded in research of learner differences and effective instructional settings, UDL principles call for varied and flexible ways to

- Present or access information, concepts, and ideas (the "what" of learning)
- Plan and execute learning tasks (the "how" of learning)
- Get engaged—and stay engaged—in learning (the "why" of learning)

"I was most surprised [in the course] at how well the UDL concepts fit into my teaching beliefs that all students learn when given information in a form that relates to their needs."

Sherri Soluri
Florida TEAL Team

UDL is different from other approaches to curriculum design in that educators begin the design process expecting the curriculum to be used by a diverse set of students with varying skills and abilities. UDL is an approach to learning that addresses and redresses the primary barrier to learning: inflexible, one-size-fits-all curricula that raise unintentional barriers. Learners with disabilities are the most vulnerable to such barriers, but many students without disabilities also find that curricula are poorly designed to meet their learning needs. UDL helps meet the challenges of diversity by recommending the use of flexible instructional materials, techniques, and strategies that empower educators to meet students' diverse needs. A universally designed curriculum is shaped from the outset to meet the needs of the greatest number of users, making costly, time-consuming, and after-the-fact changes to the curriculum unnecessary.

The UDL framework is grounded in the following three principles:

1. **Multiple means of representation—**using a variety of methods to present information, provide a range of means to support learning
2. **Multiple means of action and expression—**providing learners with alternative ways to act skillfully and demonstrate what they know
3. **Multiple means of engagement—**tapping into learners' interests by offering choices of content and tools; motivating learners by offering adjustable levels of challenge

Formative Assessment (Fact Sheet 9)

What Is Formative Assessment?

The use of assessment to provide feedback to teachers and students in the course of learning is called *formative assessment.* Information gained through informal assessments provides opportunities for teachers to make adjustments to the ways they deliver instruction. For example, based on how the students are performing in the classroom, they may reteach a concept, use alternative instructional approaches, or offer more opportunities for practice and reinforcement. These activities can lead to greater student success.

Formative assessment centers on active feedback loops that assist learning (Black & Wiliam, 2004; Sadler, 1989; Shavelson, 2006). Teachers use formative assessments both to provide feedback to students about their progress and to guide decisions about next steps in the learning process, thereby closing the gap between the learner's current and desired states. Popham (2008) defines formative assessment as "a planned process in which teachers or students use assessment-based evidence to adjust what they are currently doing" (p. 15). The operative word in this definition is process, in that formative assessment is happening throughout learning, as opposed to summative assessment, which is often a one-time event that occurs at the end of a learning unit and is used to make judgments about student competence.

Elements of the Formative Assessment Process

Several researchers (e.g., Black & Wiliam, 1998; Sadler, 1989) have identified essential elements of formative assessment. These include (1) identifying the gap, (2) feedback, (3) learning progressions, and (4) student involvement, which are described below.

1. **Identifying the gap** is the process of defining the difference (the "gap") between what students know and what they need to know; it includes collaboration between teacher and learner to identify learning goals and outcomes and criteria for achieving these.

2. **Feedback** (i.e., rich conversations between the teacher and student) gives the teacher information needed to identify the current status of a student's learning as well as the specific next steps that he or she can take to improve. Teacher feedback to students must be both constructive and timely to enable students to advance their learning. It must include a description of how their response differed from that reflected in the desired learning goal and how they can move forward. Student feedback and reflection can alert the teacher to the need for modifying instructional approaches.
3. **Learning progressions** are used by the teacher to break a learning goal into smaller, more manageable subgoals. The teacher identifies a student's location on the learning continuum and works collaboratively with the student to set a series of smaller goals.
4. **Student involvement** in decisions about their own learning and in self-assessment helps students engage in reflection and build their metacognitive skills. See the TEAL Fact Sheet 4 on Metacognitive Processes. Student involvement in self-assessments and understanding of how to improve have a profound influence on student motivation and self-esteem.

Differentiated Instruction (Fact Sheet 5)

About Differentiated Instruction

Differentiated instruction is an approach that enables instructors to plan strategically to meet the needs of every learner. It is rooted in the belief that there is variability among any group of learners and that instructors should adjust instruction accordingly (Tomlinson, 1999, 2001, 2003). The approach encompasses the planning and delivery of instruction, classroom management techniques, and expectations of learners' performance that take into consideration learners' diversity and varied levels of readiness, interests, and learning profiles. Differentiated instruction can be viewed as an instructor's response to learner differences by adapting curriculum and instruction in six dimensions, including how the instructor approaches the (1) content (the what of the lesson); (2) process (the how of the lesson); (3) expected product (the learner-produced result); (4) considerations of the learner's interest; (5) profile (learning strengths, weaknesses, and gaps); and (6) readiness. These adaptations can occur simultaneously, in sequence, or as needed, depending on the circumstance and goals of instruction. Teaching small groups of learners, grouped based on instructional approach and learner profile; using multiple representations; and valuing alternative approaches to solving problems are cornerstones of differentiated instruction.

> *"[ANI is strong because of] the different ways and methods math can be taught to help a variety of students, because everyone doesn't learn the same way."*
>
> Georgia Instructor
> ANI Institute, 2011

Effective Lesson Planning (Fact Sheet 8)

About Effective Lesson Planning

Planning ahead to identify a course of action that can effectively reach goals and objectives is an important first step in any process, and education is no exception. In education, the planning tool is the lesson plan, which is a detailed description of the elements of an individual lesson intended to help learners achieve a particular learning objective. Lesson plans communicate to learners what they will learn and how they will be assessed, and they help instructors organize content, tasks, problems, activities, materials, time, instructional strategies, and assistance in the classroom. Lesson planning helps English as a second language (ESL), adult basic education (ABE), adult secondary education (ASE), and other instructors create a smooth instructional flow and scaffold instruction for learners.

The Lesson Planning Process

Before the actual delivery of a lesson, instructors must engage in a planning process, which begins with the determination of a learning goal (or goals) for the lesson. If states have implemented content standards, the learning goals should be derived from them. Learning goals identify the concepts and ideas that learners are expected to develop and the specific knowledge and skills that learners are expected to acquire and use at the end of the lesson. These goals or objectives are critical to effective instruction because they help instructors plan the instructional strategies and activities they will use, including the materials and resources to support learning. It is essential that the objective be clear and describe the intended learning outcome. Objectives can communicate to learners what is expected of them—but only if they are shared with learners in an accessible manner. Instructional objectives must be specific, outcome-based, and measurable, and they must describe learner behavior. Heinich, Molenda, Russell, and Smaldino (2001) refer to the ABCD's of writing objectives:

> *"Teachers and students like [setting goals] because it keeps the learning goal in focus and is transparent for students as to why the lesson is relevant to their personal goals. It helps with the 'why are we doing this' issue…"*
>
> Pam Blundell
> Oklahoma TEAL Team

- **Audience—**learners for whom the objective is written (e.g., ESL, ABE, GED)
- **Behavior—**the verb that describes what the audience will be able to do (e.g., describe, explain, locate, synthesize, argue, communicate)
- **Condition—**the circumstances under which the audience will perform the behavior (e.g., when a learner obtains medicine from the pharmacy, he or she will be able to read the dosage)
- **Degree—**acceptable performance of the behavior (i.e., how well the learner performs the behavior)

Learner assessment follows from the objectives. Based on the principles of backward design developed by Wiggins and McTighe (1998), instructors identify the lesson objective or desired results and then decide what they will accept as evidence of learners' knowledge and skills. The concept of backward design holds that the instructor must begin with the end in mind (i.e., what the student should be able to know, understand, or do) and then map backward from the desired result to the current time and the students' current ability/skill levels to determine the best way to reach the performance goal.

1.3 Math Proficiencies: Guiding Lesson Planning and Instruction

Additional guidance to assist in designing effective mathematics instruction includes essential math competencies identified by experts as necessary for college and career preparation. These include: (1) Five Strands of Mathematical Proficiency identified by the National Research Council (NRC) in *Adding It Up: Helping Children Learn Mathematics* (2001); (2) Guidelines on Instructional Practice from the Office of Career, Technical, and Adult Education report, *Building on Foundations for Success* (U.S. Department of Education, 2011); and (3) the Common Core State Standards, Standards for Mathematical Practice, developed under the auspices of the National Governors Association Center for Best Practices and the Council of Chief State School Officers (2010). See Appendix A for details.

1.4 Interventions and Teaching Practices

The following two sections, Research-Based Interventions and Enhancing Teaching Practice, illustrate effective adult education and mathematical teaching practices through narrative descriptions of classroom examples. The vignettes serve to put the approaches in context and provide readers with the opportunity to become familiar with the concepts by analyzing the narrative examples and identifying the various instructional and mathematical principles they represent. As Steen (2000) notes:

> Quantitatively literate citizens need to know more than formulas and equations. They need a predisposition to look at the world through mathematical eyes, to see the benefits (and risks) of thinking quantitatively about commonplace issues, and to approach complex problems with confidence in the value of careful reasoning. Quantitative literacy empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are skills required to thrive in the modern world.

This statement still holds true and is borne out in the skills and proficiencies currently identified and promoted by the National Research Council, the National Council of Teachers of Mathematics in the Principles & Standards for School Mathematics, the Common Core State Standards (CCSS) (National Governors Association Center for Best Practices & Council on Chief States School Officers, 2010), and college faculty and employers: The goal is to develop mathematical thinkers who make reasoned and informed decisions using quantitative information in their daily lives at home, school, and work.

2. RESEARCH-BASED INTERVENTIONS

2.1 Increase Communication in the Math Classroom

Adult learning theories encompass student-centered learning, increased student autonomy, collaboration among students, strategy development, and increased metacognitive practices by learners. (Refer to TEAL Fact Sheets 11, 6, and 3 for more information.) Increased communication among students and between instructor and students is one instructional technique that incorporates all of these principles and facilitates math learning.

> *"… I feel students (especially adults) can learn from one another. Maybe they feel more comfortable or maybe other students can speak the same 'language' as another student….I feel the cooperative learning groups can be a very efficient and effective part of the learning process."*
>
> Mary Ellen Davis
> Oklahoma TEAL Team

Building on Foundations for Success: *Guidelines for Improving Adult Math Instruction* identifies communication as a critical instructional and learning practice in the math classroom, which helps students develop problem-solving skills and integrate new information into previous knowledge. It also allows students to expand on their own thinking and the ideas of others by engaging in discourse about problem-solving methods and math concepts. The National Council on Teachers of Mathematics contends that communication and collaboration among students and the instructor in the math classroom foster learning from one another, increase active participation by learners, and facilitate better understanding of concepts as students integrate others' ideas into their own understanding.

The following narrative illustrates how communication fosters learning in the classroom.

Narrative Example

[Source: ANI–1.2, About How Many]

Instructor description: One of my students went to a yard sale last weekend. She saw several pieces of furniture she wanted to buy, but she didn't have a measuring tape with her and wasn't sure whether the items would fit in her living room. She decided to pass on purchasing them. In class today, we completed an activity to help us think in different ways about what we as a group know about numbers and measurement.

I cut up strings of different lengths and placed a bunch at each of the four activity station tables. In groups of three, the learners were asked to go to a station and figure out how long each string was—without using a ruler, measuring tape, yard stick, or any other standard measuring tool. I asked for at least one person in each group to document their thinking and calculations while they worked through the problems together, and for the whole group to be able to explain their answers to the rest of the class when we were finished.

Several students seemed stumped at first, but I asked the group if anyone had ideas on how to approach solving these problems. One student offered that her husband uses his foot size to measure the length of a room because his shoe size is about 12 inches. This got the students thinking about other ways they could approximate length.

Students presented their work as follows:

Group 1

James: *Janice held the string in front of Kent so that it dangled down to the ground. She said that it was about half as long as he was tall. Kent said his height was 5 feet 10 inches.*

Instructor: *What did you do with that information?*

James: *We agreed that we needed to find out what half of Kent's height was to get a sense of how long the string was.*

Instructor: *Why did you want to find half of Kent's height?*

Janice: *Because in estimating the length of the string, we compared it to Kent's height and it seemed like the string length could be held up to his height about two times. We had him lie down on the ground and the string was almost two times his height exactly.*

Instructor: *Okay, so how did you approach that calculation?*

Janice: *I suggested dividing the 5 feet of his height by 2, and then the 10 inches of his height by 2.*

Instructor: *Why did you decide to do it that way?*

Janice: *I thought this would be an easy way to do it because it was small quantities, and it's the same as when I take a large number like 86 and break it down. I think, "What's 80 divided by 2?" and "What's 6 divided by 2?" and then I add the two answers together. For the string, I figured it was 2 ½ feet plus 5 inches. That is 24 inches + 6 inches + 5 inches = 35 inches.*

Instructor: *Are there other ways to figure out half of Kent's height?*

Kent: *I suggested converting my height into inches first, and then dividing by 2. This seemed less confusing to me because it was all the same unit. So, I figured 5 feet is 60 inches, plus 10 inches, is 70 inches. I divided 70 by 2 and got 35 inches.*

Instructor: *What did you discover when you tried different approaches?*

James: *We did both of the calculations and found that both methods resulted in the same answer.*

Instructor: *Why do you think that you got the same answers using different methods?*

Group 2

Group 2 measured a string of a different length.

Margaret: *I was thinking about things in the room I could use to compare with the string. I remembered that the paper used in the computer printer is 8½ inches wide by 11 inches long.*

Nick: *We discussed which length would be easier to work with, and decided to use the long end of the paper to avoid dealing with a fraction.*

Margaret: Next, we laid a sheet down along the string and counted the number of paper lengths that fit along the string.

Instructor: So, it seems you had several units you were working with. What decisions did you make next?

Shawn: I wondered if we should answer in the units of paper lengths or inches. We decided we could give the answer both ways, as long as everyone was clear about what the unit size was and we used the same one.

Margaret: The paper could be laid along the string four times. So, taking the length of the paper—11 inches—we then multiplied that number by 4 and got 44 inches.

Instructor: Does this approach make sense? Can you explain how you knew this would work?

Shawn: It does make sense because we used the paper as a unit of measure, kind of like a ruler.

Instructor: Oh, I see. Did anyone think of other ways to get an answer?

Nick: After we did all these calculations, I realized that since 11 inches is pretty close to 12 inches, which is a foot, we could have considered each paper length about a foot, and just subtracted a few inches.

Instructor: Group A, does that make sense? Does it come close to their other answer of 44 inches?

Janice: Yes, because four feet would be 48 inches, and if you take away an inch from each foot, you take away four inches, which leaves you with 44 inches. That's a good way to calculate it, Nick.

After the students determined the approximate length of different strings, they started using the strings to measure other items in the room.

Instructor: So you came up with many different ways to figure out the length of the strings. How would these informal measuring strategies work when you're at the store and you don't have access to standard measuring tools? What other situations have you found yourself in where you have needed to measure items and these approaches would be useful?

Check for Understanding

Identify where and how the different instructional strategies and math principles are reflected in the example. Use the questions below to guide your thinking. After you answer these questions on your own, refer to the analysis below the questions to check for understanding.

1. What did the teacher do to encourage communication and collaboration in the classroom?
2. What principles of adult learning theory, student-centered learning, and self-regulated learning are embedded in this activity?
3. Which strands of mathematical proficiency are reflected? (See Appendix A.)
4. Which Common Core State Standards for Mathematical Practice are reflected? (See Appendix A.)

Analysis

1. The teacher encouraged communication in the classroom in the following ways:
    - Provided a small-group activity allowing for more participation by each student
    - Created an interactive environment in which each participant could contribute based on his/her knowledge of measurement or understanding of converting from one unit to another
    - Used "why" to ask students to justify and explain their thinking
    - Encouraged students to provide guidance to one another on how to make comparisons and calculate the answers
    - Provided opportunities to make decisions together as to how to approach solving the problem or presenting the answer
    - Allowed students to share their strategies and judge the rationale of others
    - Encouraged students to explain their reasoning and justify their answers
2. The example reflects adult learning theory, student-centered learning, and self-regulated learning in the following ways:
    - The activity addresses a need that students have to understand size and measurement so they can operate in daily life more effectively; they learned that they could find alternative ways to measure objects and space when they do not have access to the standard measurement tools they would normally use.
    - The task elicits knowledge the students already have and connects it to the current context; by using benchmarks of known measurements, the students were able to problem solve to find new measurements.
    - The activity fosters student autonomy and encourages them to think together and ask questions of one another rather than rely on the instructor for all the answers.
    - The activity provides the opportunity to work collaboratively and learn from their peers.
    - The activity supports different roles and contributions by various students.
3. The following Strands of Mathematical Proficiency are reflected, as follows:
    - Conceptual understanding
        - Students further developed their understanding of linear measurement, and they learned that there are many ways to measure using different "units" of measure, whether standard units such as inches and feet or nonstandard units such as paper length or a person's height.
        - Students connected previous knowledge to new situations when they used their knowledge of some units of measure along with their familiarity with personal or everyday objects.
        - Students acquired knowledge from one another that will generate more knowledge and understanding as they repeat the task.
        - Students further developed their sense of "benchmarks," which is key to being able to think critically about the reasonableness of answers.

- Strategic competence
    - Students represented the problem both verbally by describing how the measurements are similar or different to known quantities, and numerically by computing the answers.
    - Students showed flexibility in the approaches they used by drawing on known measurements for comparison as well as estimation techniques.
    - Students gained skills in estimation by gauging their answers, based on the lengths of known quantities.
- Adaptive reasoning
    - Students thought logically about the relationships and concepts they were faced with, connecting knowledge of some measurements and spatial benchmarks with which they were familiar.
    - Students explained and justified their conclusions to one another as they figured out the answers in small groups and presented their arguments to the class.

4. The following CCSS Standards for Mathematical Practice are reflected, as follows:
    - Make sense of problems and persevere in solving them (MP1).
        - Students found entry points for finding the length of the strings by relying on their prior knowledge and experiences.
        - Students assessed their answers and determined whether they made sense.
        - Students identified multiple ways to state the solution to the problem, using different unit measurements and estimation techniques.
    - Construct viable arguments and critique the reasoning of others (MP3):
        - Students explained their approaches to one another (such as how to break down larger numbers, estimation techniques, how to convert units), and they reached consensus.
        - Students responded to questions from the instructor, explaining why they proceeded as they did and the math they used to find the answers.
        - Students listened to the reasoning of their peers and were able to evaluate their thinking.

Lower-Level Instruction

- Use fewer pieces of string and shorter lengths, cutting them closer to familiar benchmarks (such as 6 inches or 10 inches rather than 3.5 inches).
- Instead of using the strings, begin by asking the class to estimate what they think the length of the room is, and put the estimates on the board. Then ask students to estimate how long their own foot is. Ask them to consider whether their own foot length would be more or less than the actual 1-foot measurement of 12 inches. Using rulers, have them measure their own feet. Record these measurements on the board. Then ask students which foot would be best to use to measure the length of the room.

Higher-Level Instruction

- Limit the resources available for measuring; for example, tell students that they cannot use sheets of paper.
- Provide a wide variety of lengths of string that are less easy to estimate, such as 17 inches or 32.5 inches.
- Ask students to determine lengths using metric measurements (such as meters or centimeters).
- Challenge students to estimate the height of the classroom or the height of the building.

2.2 Metacognition

Metacognitive skills allow learners to understand what they know and how they process and learn new information. Many adult learners need assistance in identifying these personal characteristics and how to use them to their advantage. Students may not be aware of alternative ways to organize and integrate new information, particularly if they have experienced a limited range of teaching approaches.

In mathematics, metacognitive skills are represented in students' ability to monitor their use of strategies; apply alternate approaches, if necessary; and assess the reasonableness of their answers and conclusions. As is the case when acquiring reading or writing skills, they should be exposed to the thinking processes needed to perform these functions. It is critical for students to learn what questions they need to ask of themselves and others to pursue solutions. (Refer to Fact Sheet 1 for more information.)

Adult education researchers have argued that it is important for students to develop estimation strategies and to learn that multiple approaches can be used to approximate exact values (Ginsburg, 2008). Estimation involves using accepted benchmarks and developing personal benchmarks for determining when numbers, measurements, and calculations make sense in a given situation. As students develop number sense, it helps them as consumers, on the job, and in everyday life. The National Mathematics Advisory Panel (2008) findings align with this theory as well, stating that the purpose of estimation is to determine "appropriate approximation" and should expand beyond rounding (p. 27).

The narrative below illustrates the development of metacognitive understanding in relation to using estimation strategies.

Narrative Example

[Source: ANI–1.2, About How Many? Number and Operation Sense]

Estimation can be used for the following problems:

(A) Student: When I go to the store, I want to make sure I have enough cash to pay for my items so that I don't get to the checkout and then have to put things back because I don't have the money to pay for them.

(B) Student: When I'm planning summer barbeques, I often struggle to get the right amount of food—I either get too much or too little.

Activity A:

Instructor: I've given each of your groups "$35.00" to spend on items of your choice. Also, I've given you circulars that list the items and their prices. Select as many items as you can buy with your money, without going over. Don't use a calculator or do any calculations and still find ways to come up with the answer. For those of you who feel more comfortable shopping without a calculator, how do you do it? Share your approaches with your classmates.

After the students worked in their small groups for 40 minutes, selecting the items they would buy, they started to explain their approaches to one another.

Tanya: We picked aspirin for $8.95, a notebook for $3.65, two boxes of tissues for $1.50 each, 2 boxes of crackers for $2.25 each, shampoo for $3.45, deodorant for $2.99, and a comb for $2.50.

Instructor: How did you go about figuring out if you would have enough money?

Tanya: The aspirin and deodorant were just a few cents lower than the next dollar amount, so they were close to $9.00 and $3.00. So we thought of those as 9 + 3. When we picked the notebook and the shampoo, we thought that 65 cents is sort of close to 50 cents, and 45 cents is really close to 50 cents, so both of those amounts we thought of as half a dollar. Twenty-five cents and 50 cents together are 75 cents, which we also thought of as close to a dollar—although not as close as 95 cents, it is closer than 50 cents. Our rough estimate came to about $31.00. So, if we had been at the store, we would have had just enough money.

Instructor: Can someone paraphrase Tanya's thinking? Is Tanya's thinking reasonable? Why do you think so?

Corrine: I think they used benchmarks, like there are 100 cents in a dollar, so 95 cents and 99 cents are almost 1 dollar, and 50 cents is half of a dollar. They estimated the dollar amounts instead of adding up each dollar and cent.

Instructor: You both described the approach well. For those of you who don't typically do it this way, how do you figure it out?

Later, the class did the following activity...

Activity B:

Instructor: When planning a party, how do you usually approach it?

Alice: First, I think about how many people usually come to a particular type of gathering—a holiday dinner is going to be different than a barbeque.

Instructor: Okay, so Tina, about how many people would you say usually come to your barbeques?

Tina: Hmm, probably about 25 to 30 people.

Instructor: Okay, so what do you all think about next?

James: The type of food I want to have. In the summer, I like to serve hot dogs, hamburgers, chips, potato salad, some fruit, cookies, that type of thing.

Instructor: That sounds like a nice spread. So, Tina, if you were serving that food, how would you decide how much of each item you should have?

Tina: On the Fourth of July, I will invite my brother and his kids, a few neighbors and their kids, and other friends and their kids. I figure that most of the adults will want a hotdog and a hamburger. Most of the kids want a hotdog, but I would have some hamburgers for the kids, too. All the adults could have two cookies, and the kids one cookie. So, I'm thinking that most adults will eat about twice as much as each kid. The bags of chips say how many servings you should get out of them. So, I could look at that number, and compare it to how many people are coming, and see how many bags I should get. But I also know that my family loves chips, so I would plan for double servings for us!

At the end of the activity, the Instructor asked the students the following questions:

1. What do you notice about how you figured out the two problems? Did you use the same approach or different approaches?
2. What benchmarks or familiar numbers did you use to help you make decisions?
3. In what other situations would you use these approaches?
4. How would you decide which approach to use?

Check for Understanding

Identify where and how the different instructional strategies and math principles are reflected in the example. Use the questions below to guide your thinking. After you answer these questions on your own, refer to the analysis below the questions to check for understanding.

1. What did the instructor do to encourage metacognitive strategy development?
2. Which Strands of Mathematical Proficiency are reflected? (See reference sheets.)
3. Which CCSS Standards for Mathematical Practice are reflected? (See reference sheets.)

Analysis

1. What did the instructor do to encourage metacognitive strategy development?
    - The instructor asked prompting questions, so the students will reflect on how they usually approach similar problems, and share their strategies with the class.
    - The instructor asked them to think of which approach is more effective in a given situation.
    - The instructor asked students to think about what they know about numbers, and what they know about the world.
    - The instructor asked students to verbalize their approach when they solved the shopping problem and asked the students to verbalize their approach when considering how to plan for the party.
2. Which Strands of Mathematical Proficiency are reflected?
    - Conceptual understanding
        - Students show an understanding of why estimating is important and that it can be used in different contexts.

   - Students show an understanding of multiplication in that, depending on the size of the group, amounts may need to be doubled or tripled.
- Procedural fluency
   - Students know how and when to use procedures appropriately.
   - Students can analyze different methods of calculating.
   - Students can estimate the results of procedures.
- Strategic competency
   - Students drew from multiple strategies; using benchmarks in Activity A and using context to assess the component parts in Activity B.
   - Students reflected on their own personal strategies for solving similar problems.
   - Students detected mathematical relationships between the parts of dollars in Activity A, and used the repeating *n* in Activity B.

3. Which CCSS Standards for Mathematical Practice are reflected?
   - Reason abstractly and quantitatively (MP2).
      - Students created coherent representations of the problems, attending to the meaning of the quantities: money in Activity A and serving sizes in Activity B.
   - Look for and express regularity in repeated reasoning (MP8),
      - Activity B provided a pattern in which most food items were doubled for adults, resulting in (2 × number of adults) and (1 × number of children) = 2:1

Lower-Level Instruction

Provide opportunities to estimate using less complex concepts, such as the following:

- Use concrete objects, e.g., marbles in a jar or blocks in a box. Provide students with one or a few layers of the marbles or blocks and ask them to estimate how many more of each would be required to fill either the jar or the box.
- Keep numbers friendly; for example, in the first situation, provide items that are all close to whole-dollar amounts (such as $6.97, $.98, etc.) and use a smaller total target amounts (such as $20 vs. $35). Also, provide fewer items to choose from, if necessary.

Higher-Level Instruction

Provide opportunities to estimate using more complex quantities, such as the following:

- Ask students to write generalizations using symbols (such as $2A + C$ = total hamburgers needed).
- Use more complex numbers or larger amounts (rather than a barbeque, perhaps a wedding party) and have students research some estimated costs to throw a large function like that.
- Instead of using monetary values for the problem, use a variety of practical measurements including distance, volume, angle, time, temperature, mass, speed, and density.
- Ask students to consider how changes in one measure may affect other measures (e.g., what happens to the volume and surface area of a cube when the side of the cube is halved).

2.3 Strategy Instruction and Making Connections

It is a great service to students to help them develop the cognitive skills and habits to be self-directed learners. A large body of research from both secondary and postsecondary settings suggests that strategy instruction strengthens students' ability to engage with learning, benefit from instruction, and succeed. Strategy instruction is an approach that teaches the tools and techniques necessary for understanding, learning, and retaining new content and skills. It involves teaching strategies that are both effective in assisting learners with acquiring, retaining, and generalizing information, and efficient, helping them acquire information in the least amount of time (Lenz, Ellis, & Scanlon, 1996).

There is a range of approaches and a range of uses for strategy instruction in all content areas. But one underlying key in math instruction is to give students tools to see how math fits their needs and those of the larger world around them. When students make connections between math concepts and contexts that are important to them, their understanding is deepened. For example, using a variety of math strategies to tackle a relevant and timely problem makes the concepts meaningful, helps students make connections to their own lives, and provides more scaffolding than is possible using rote exercises that are out of context. The What Works Clearinghouse produced the educators' practice guide, *Improving Mathematical Problem Solving in Grades 4–8,* in which a panel of experts found that students who are introduced to multiple strategies will develop facility and flexibility in problem solving. The panel recommends

> …teachers instruct students in a variety of strategies for solving problems and provide opportunities for students to use, share, and compare the strategies. Teachers should consider emphasizing the clarity and efficiency of different strategies when they are compared as part of a classroom discussion. (p. 32)

> *"Sometimes, I'll stop and say, 'So why does this matter?' Sometimes the confusion is because the 'big picture' is getting bogged down in details…Taking a more explicit problem-solving approach can help in breaking things down step-by-step and making each step seem more relevant…"*
>
> Hillary Major
> Virginia TEAL Team

Additionally, the process standards within the Mathematics Principles and Standards for School Mathematics produced by the National Council for Teachers of Mathematics (NCTM) emphasize the importance of students making connections while they engage in problem solving and learning. The principles include encouraging students to

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.[4]

And finally, the report, *Building on* Foundations for Success: *Guidelines for Improving Adult Math Instruction,* supports the assertion that making connections while exploring math is essential to positive learning outcomes. In the report's seven Guidelines for Instructional Strategies, Strategy 4 reads as follows:

[4] National Council of Teachers of Mathematics. Process standards. Retrieved from http://www.nctm.org/standards/content.aspx?id=322

Adults' goals and experiences offer opportunities to embed instruction in meaningful contexts. Instruction should include connections to student interests, work situations, and everyday life (e.g., following recipes, basic accounting required on the job or at home) to stimulate engagement and promote applicability. (p. 24)

The following narrative illustrates students using strategies to address a work-related task.

Narrative Example

The instructor has a student, Sam, who works at a small restaurant. Sam has told the class about the tasks he does on his job, so the instructor used that information to provide an activity for the class to explore and expand upon patterns and to connect patterns with rules.

The Problem

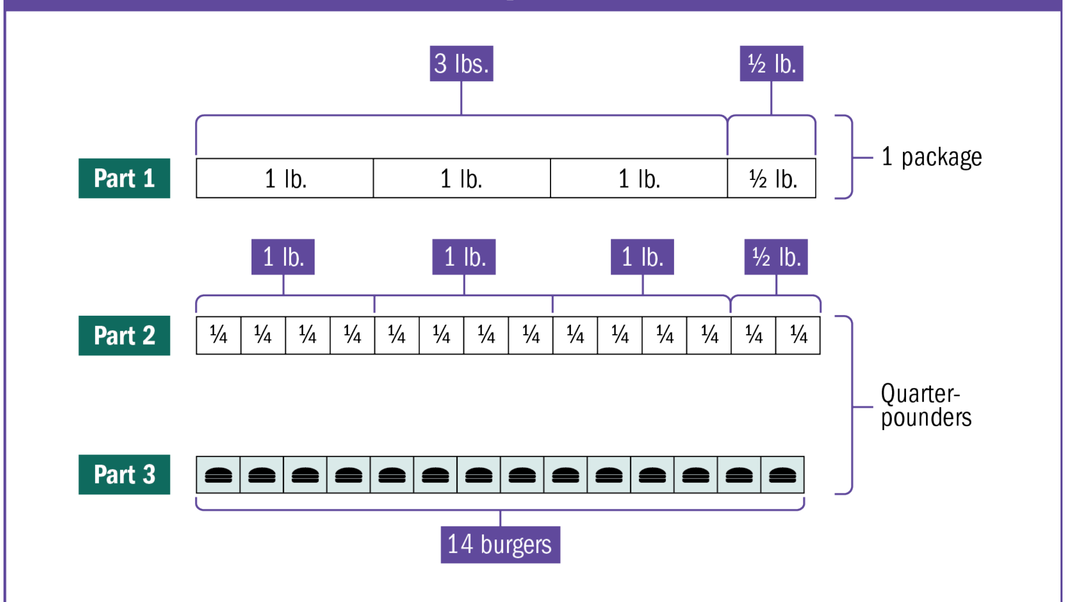
Sam has to make 50 hamburgers for the lunch run. Each burger should be a quarter pound (lb.). The ground beef comes in 3.5-lb. packages. He needs to figure out how much ground beef he needs to take out of the freezer to make 50 burgers.

Instructor:

- *What exactly are we trying to figure out in this problem? Do we need to find just one answer or multiple answers to solve the problem?*
- *Is this similar to problems we have worked on before? What approach did we use in those other problems?*
- *Can you think of ways to represent the information we have in front of us other than using words?*
- *Can anyone predict what they think a reasonable answer might be? We'll compare that to the final solution later.*

The students used a visual strategy, developing the visual representation shown below:

Visual Representation



Instructor: What does each part of the diagram represent?

Andrea:

- *Part 1 shows that each package contains 3½ lbs. of ground beef.*
- *In Part 2, it shows that we know each burger has to be ¼ of a pound. And so, each pound can be divided into 4 equal parts that equals ¼ lb. of beef. Here we show the breakdown of each pound. You can get 4 quarter-pound patties out of each pound.*
- *Finally, in Part 3, by counting them out on the drawing, you can see that each package will make 14 burgers.*

Instructor:

- *Does anyone have ideas about other ways we could represent this information visually?*
- *Does it make sense that the number of burgers in a package would be higher than the number of pounds of beef in a package? Why or why not?*
- *Now that we have this information, do we have the answer to our problem? If not, what do we need to do next?*

Andrea: Next we need to figure out how many packages are needed to make 50 burgers. Let's make a chart to show the ratio of packages to burgers.

She designed and populated the chart below:

Packages of Ground Beef	Number of Burgers
1	14
2	28
3	42
4	56

Instructor:

- *Based on the chart, how many packages should Sam get out of the freezer? Why?*
- *Were you surprised that he would need this number of packages? Why or why not?*
- *Before you started to figure it out, did you think he would need more or less?*
- *So, the problem was represented in words first, and then with diagrams. What would it look like in symbols?*

Some of the equations that students developed are shown below.

Equations for the number of hamburgers *(H)* in a package:

$3\frac{1}{2} \div \frac{1}{4} = H$

$3.5 \div .25 = H$

$\frac{1}{4} \times H = 3\frac{1}{2}$

Equations showing the relationship between packages and the number of hamburgers that can be made with each package:

$1 \times P = 14$

$2 \times P = 28$

$3 \times P = 42$

$4 \times P = 56$

$(3\frac{1}{2} \div \frac{1}{4}) \times 4 = 56$

$4P > 50$

Check for Understanding

Identify where and how different instructional strategies and math principles are reflected in the example. Use the questions below to guide your thinking. After you answer these questions on your own, refer to the analysis below the questions to check your understanding.

1. What did the instructor do to encourage the use of strategies and making connections?
2. Which Strands of Mathematical Proficiency are reflected? (See reference sheets.)
3. Which CCSS Standards for Mathematical Practice are reflected? (See reference sheets.)

Analysis

1. The instructor encouraged the use of strategies and helped students make connections in the following ways:
   - The instructor encouraged the students to use the following strategies: using visual representations, think-alouds, and converting visual information into mathematical notation.
   - The instructor posed questions that prompted students to consider which strategies they could use, indicating that there is more than one way to solve the problem.
   - The instructor helped connect ratios and proportions to a topical issue for the students: how to figure out quantities on the job.
2. The following Strands of Mathematical Proficiency are reflected:
   - Conceptual understanding
     - Students showed understanding of why the mathematical idea of ratios is important and the kinds of contexts in which it can be useful.
     - Students showed they can learn new ideas by connecting them to those they already know.
     - Students are able to represent mathematical situations in different ways (e.g., using pictures, in-out tables).
     - Students grasped the concept that there are many smaller parts (1/4-pound burgers) in a whole (1 lb., 3½ lbs.).
   - Procedural fluency
     - Students knew when and how to use procedures, such as graphics or symbolic representations.
     - Students understood that mathematics is organized, includes patterns, and is predictable.

- Strategic competence
  - Students had multiple strategies on which they could draw to approach the task.
  - Students represented problems numerically, symbolically, verbally, and graphically.
  - Students detected mathematical relationships.
- Adaptive reasoning
  - Students checked whether their reasoning was valid by communicating with teacher and classmates.
  - Students justified their mathematical claims and made them clear to others.
- Productive reasoning
  - Students exhibited interest in attempting challenging problems.
  - Students wanted to share their mathematical ideas with others.

3. The following CCSS Standards for Mathematical Practice are reflected:
   - Make sense of problems and persevere in solving them (MP1).
     - The instructor asked the students to explain the meaning of the problem and identify entry points (such as using visual representations).
     - The students could explain correspondences between verbal descriptions, tables and graphs, and equations.
     - The teacher probed to see whether it made sense.
   - Model with mathematics (MP4)
     - Students were able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, and formulas.
     - Students could analyze the relationships mathematically to draw conclusions.
     - Students reflected on whether the results made sense.

Lower-Level Instruction

- Use less complex units (e.g., number of chicken legs instead of weighted packages, or use 1- or 2-lb. packages rather than 3.5 lbs. Or students could be asked to determine the number of ½-pound items rather than ¼-pound items).
- Rather than computing packages, figure out how many units per platter are required to serve customers.
- Provide concrete manipulatives for students, such as chips or strips of paper, to represent the hamburgers and packages. The strips of paper could be represented at first using a whole strip for 1 lb. and then subdivided into fourths.

Higher-Level Instruction

- Use more complex numbers for the quantities.
- Increase the scope of the problem (e.g., make more burgers).
- Include threshold points where the relationships will change (e.g., after 300 burgers have been made, the burger size will be reduced by 1/10).
- Ask students to create graphs to show the relationships between the number of packages and burgers.

2.4 Move from the Concrete to Representational to Abstract (C-R-A)

In some content areas, such as reading and writing, teachers are encouraged to begin instruction by modeling behaviors and procedures, and eventually releasing control so that students can work independently. In math, there are many concepts as well as processes to learn. Therefore, it is important to stress the teacher's role in helping students understand ideas as well as develop a facility with methods of problem solving so students can also gain independence in this domain. To accomplish this in math instruction, research by educational psychologist Jerome Bruner strongly recommends moving from the concrete to the representational to the abstract for learning at all ages. Students can actively engage with hands-on materials, then represent ideas with pictures and words, and finally produce the concepts symbolically or abstractly. Most adult education students previously were introduced to math concepts primarily using abstract representations, such as equations and notations. By learning with concrete representations first, students can gain the foundation they missed and progress in their mathematical understanding.

The following narrative illustrates the use of the concrete, representational, and abstract presentation of math concepts.

Narrative Example

[Source: ANI-1.13, Comparing Volumes]

An instructor described a classroom activity as follows: One student mentioned that when she's preparing breakfast for her kids, she sometimes uses different-sized glasses for their juice. One is tall and skinny, and the other is shorter and wider. The kids always fight over the tall glass, thinking it has more juice.

> *"Although I know discovery is best, I often forget to use real-life connections. [ANI] will make me strive to use more hands-on, real-life math in class."*
>
> Arkansas Instructor
> ANI Institute, 2011

We did the following activity to discuss volume:

Using glasses of different heights and circumferences, we predicted which ones we thought would hold more water. We filled glasses with the same amount of water and compared how full they got and whether glasses that appeared small would hold the same amount as glasses that seemed big. We measured the heights and the diameters of the glasses and drew diagrams to record the dimensions. We discussed how the dimensions of the glasses might influence how much water they would hold. Students asked questions and conjectured based on the numbers we found in our measurements.

Next, students worked together in groups of three. I gave each group two pieces of paper, each 8½ × 11 inches. I asked them to make cylinders out of each, one with the long edge as the height, and one with the short edge as the height. I also gave each group a bag of popped popcorn. The activity went like this:

Instructor: *Before you do anything, I want you to think about and discuss in your group which of the cylinders will hold more popcorn; think about our discussion and what we learned in the water glasses activity we just completed. Write down your prediction and why you think as you do. Next, I want you to pour popcorn all the way to the top of one cylinder. Using that same popcorn, pour it into the other cylinder.*

Instructor: Now, draw representations of your cylinders, labeling them with their respective dimensions, citing the units.

After the students conducted the experiment, the teacher asked the following questions, which led to in-depth discussion and conflicting ideas:

Instructor:

1. *Was your prediction on target?*
2. *Which cylinder held more popcorn?*
3. *Why do you think it worked out that way? What does this tell us about the dimensions and volume? Does a taller shape always mean that it holds more than a shorter, fatter shape?*
4. *Why is this useful to know? In what other situations would this information come in handy?*
5. *What information do we have? How can we write equations to represent the volumes of each of the cylinders?*

Check for Understanding

Identify where and how the different instructional strategies and math principles are reflected in the example. Use the questions below to guide your thinking. After you answer these questions on your own, refer to the analysis below the questions to check your understanding.

1. What did the teacher do to encourage understanding of the concept of volume through concrete experiences?
2. Which Strands of Mathematical Proficiency are reflected? (See reference sheets.)
3. Which CCSS Standards for Mathematical Practice are reflected? (See reference sheets.)

> *"I will seek more information or feedback from students to ask, "What were you thinking—how did you arrive at that answer?"*
>
> Arkansas Instructor
> ANI Institute, 2010

Analysis

1. What did the teacher do to encourage understanding of the concept of volume through concrete experiences?
   - The teacher provided hands-on activities, using the glasses and the cylinders, so that students could explore the concept of volume.
   - Students conducted the experiments and comparisons themselves, using substances such as water or popcorn to see how the quantities fit into different-sized containers.
   - Students measured the containers, using those data in their analyses.
2. Which Strands of Mathematical Proficiency are reflected?
   - Conceptual understanding
     - Students are guided through making connections between the concrete glass and cylinder examples.
     - The concept of volume is introduced through the concrete examples, then supported through drawing and labeling diagrams, and then translated into the formula for volume.

- Procedural fluency
  - Students are asked to estimate and predict the results of the procedures before doing calculations of the different-sized cylinders.
- Adaptive reasoning
  - Students try to determine whether like containers hold the same amount. They can use different strategies to do so; if one doesn't work, they can use another way to test out their hypothesis.
  - Students are given the opportunity to see how facts, procedures, concepts, and solution methods fit together.
- Productive disposition
  - Students see firsthand that a formula has a real-life representation and serves a purpose.
  - Students discuss the various settings in life in which this knowledge will be useful.

3. Which CCSS Standards for Mathematical Practice are reflected? (See reference sheets.)
   - Make sense of problems and persevere in solving them (MP1).
     - Students predicted possible outcomes.
     - Students discussed why they were getting the results they did during each experiment.
     - Students used information they learned from the first experiment and applied it to the second experiment.
   - Construct viable arguments and critique the reasoning of others (MP3).
     - Students engaged in discussion, debating their hypotheses and results.
   - Model with mathematics (MP4).
     - Students identified important quantities and diagramed their relationships,

2.5 Provide Constructive Feedback

Descriptive, specific, timely, and measured feedback allows students to understand their errors and consider adjusting their processes and thinking so they can reach accurate solutions. The National Mathematics Advisory Panel (2008) recommends reviewing student work and providing extensive feedback to students. Along with teaching varied models of problem solving, providing opportunities to verbalize their thinking, and cultivating a communicative classroom, teachers should provide constructive feedback so that students can learn the "why" underlying their confusion and how to get back on track.

The following narrative illustrates a teacher providing guidance to a student who has provided an incorrect answer.

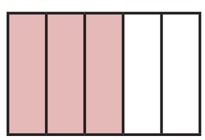
Narrative Example[5]

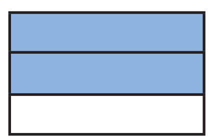
Allison found a pattern for a dress that she wanted to make. In the instructions, there were a number of places where fractions were used. One section of the dress required 3/5 of a yard of one fabric, and one section of the dress required 2/3 of a yard of the same fabric. She needed to figure out how much she should buy.

When she set up the problem, she knew that since the fabric was the same, she could ask for it all from the same bolt. So, she added the two fractions together → 2/3 + 3/5. The answer she came up with was 5/8. Rather than telling her she was incorrect, the instructor posed the following questions to her and the class to see if they could uncover what was confusing Allison and possibly others:

1. How did you come to that answer?
2. Can you explain it to us, or show us with the diagrams?
3. Do you think the answer makes sense?
4. Let's look at each of the fractions and what they mean.
   - Is 2/3 closer to 0, 1/2, or 1?
   - How about 3/5? Is that closer to 0, 1/2, or 1?
   - How about 5/8? Is that closer to 0, 1/2, or 1?
5. Okay, so both of the parts you are starting with are more than half. Do you think your answer will be greater than or less than 1?
6. Is your answer greater than or less than 1?
7. I brought in some fabric and a yard stick. Let's cut the fabric and measure it and see how our answers compare.

We also used the following diagrams to facilitate our discussion:





Check for Understanding

Use the question below to guide your thinking. After you answer the question on your own, refer to the analysis below to check for understanding.

What did the instructor do to provide constructive feedback?

Analysis

- The instructor asked the student to verbalize her thinking and use a visual representation to explain how she got her answer.
- Rather than provide the correct answer, the teacher guided the student to think about what she knows about fractions and quantities, and to use that information to determine whether her answer makes sense.

[5] The questions in this activity were adapted with permission of the Massachusetts System for Adult Basic Education Support (SABES) from Donovan, T. (2009). Questioning: Motivating discourse in the mathematics classroom. *SABES Math Bulletin, 4*(1). SABES is funded by the Massachusetts Dept. of Elementary and Secondary Education.

- The teacher used a model to support visualization of the fractions.
- By soliciting information from the student, the teacher collected valuable information: Did the student just make a computation error, or is there conceptual misunderstanding that can be cleared up in the moment or embedded in future lessons?

Lower-Level Instruction

If students still struggle with this idea, go back to the benchmark fraction ½ and use manipulatives to explain and solidify the concept.

Higher-Level Instruction

Frame the exercise using decimals or larger quantities.

3. ENHANCING TEACHING PRACTICE

3.1 Apply Universal Design for Learning

The Universal Design for Learning framework encourages using a variety of presentation methods to engage students and accommodate different competency levels. Originally conceived to make information more accessible to those with disabilities, the principles it embodies makes this approach conducive to helping adults learn math—providing information in multiple forms and allowing student choice in delivery, content, and how they can demonstrate their abilities.

Many adult learners have faltered when presented with abstract and symbolic representations of math concepts. They may memorize a formula or know how to compute a common denominator, but they don't understand the concepts that underlie the procedures. Universal Design for Learning promotes the use of manipulatives, graphic representations, music, video, movement, etc., to present information and for students to process it and express their knowledge.

The following narrative illustrates an activity that incorporates Universal Design.

Narrative Example[6]

One instructor described an activity in her class as follows: My student Jennifer works in food preparation at a restaurant. Her newest task there is to peel potatoes for their world-famous French fries. As a class, we wanted to get a sense of approximately how much time she needs to allow herself to prepare the 50 lbs. of potatoes for a day's lunch. I provided a bag of potatoes, a scale, and timers to the class so we could figure it out.

At first, the class was thinking that they needed to figure out how long it would take to peel 50 potatoes, making the assumption that the number of potatoes and number of pounds were the same. But Nadia disagreed, and said that when she makes potato salad, and the recipe calls for 1 pound of potatoes, she usually uses 2 or 3 potatoes. We decided we should weigh some of the potatoes I brought to see about how many it took to get 1 pound. We determined that approximately two potatoes make 1 pound. I asked them whether they had ideas on how to determine how long it would take to peel all the potatoes without having to actually peel 100 potatoes. One of the students suggested we conduct a sample peeling to get a sense of how long it takes to peel 1 lb. Using the property of equal fractions, we timed the peeling of two potatoes, or 1 lb., and applied that result to peeling 50 lbs. of potatoes.

This is how the activity went:

In groups of three, the students worked on figuring out this information for their classmate. The work of one group is found below. They peeled one potato and found that it took 40 seconds.

The class reasoned that if there are two potatoes per pound, there must be 100 potatoes in 50 lbs.

1 lb. of potatoes: 2 potatoes = 50 lbs. of potatoes: 100 potatoes.

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[6] The questions in this activity were adapted with permission of the Massachusetts System for Adult Basic Education Support (SABES) from Donovan, T. (2009). Questioning: Motivating discourse in the mathematics classroom. *SABES Math Bulletin, 4*(1). SABES is funded by the Massachusetts Dept. of Elementary and Secondary Education.

Lbs.	Potatoes
1	2
2	4
3	6
4	8
skip forward	*skip forward*
40	80
50	100

After determining how many potatoes Jennifer would need to peel in total, the students felt they could focus on a peeling sample. Alex peeled one potato by the class's timekeeping; it took him 40 seconds. They charted it below:

Potatoes	Seconds
1	40
2	80

2 potatoes = 1 lb.

They determined that 1 lb. of potatoes takes 80 seconds to peel. Next, they wanted to figure out how many seconds it would take to peel 50 lbs. of potatoes. They charted it as follows:

Lbs. of Potatoes	Seconds
1	80
2	160
3	240
4	320
skip forward	skip forward
40	3,200
50	4,000

1 lb. of potatoes: 80 seconds

50 lbs. of potato: 4,000 seconds

They divided 4,000 seconds by 60 to find out how many minutes it would take to peel the potatoes. They arrived at 66.667 minutes. They looked at the decimal and knew that .5 would be half a minute, and .667 is more than that. So they rounded up to 67 minutes. They thought about how many minutes are in an hour and determined that it would take a little more than an hour to peel 50 lbs. of potatoes.

1 min.: 60 seconds = 67: 4,000 seconds

One student mentioned that Jennifer's hands might get sore and tired while peeling so many potatoes. So the class figured it might take a little longer than 67 minutes to peel this amount. They thought about how many breaks she might need to take and for how long. They proposed that she should take a 30-second break after peeling 10 lbs. to stretch her hands.

Lbs.	Break time	
10	30	
20	60	
30	90	2 minutes
40	120	
~~50~~	~~150~~	

Students realized that Jennifer would not need a break after 50 lbs., because she would be done with the task at that point. So they determined that 2 minutes should be added to the total peeling time, and suggested that Jennifer allow herself 1 hour and 9 minutes to complete the task for the chef.

Check for Understanding

Identify where and how the different instructional strategies and math principles are reflected in the example. Use the questions below to guide your thinking. After you answer these questions on your own, refer to the analysis below the questions to check for understanding.

1. What did the teacher do to incorporate Universal Design principles?
2. Which Strands of Mathematical Proficiency are reflected? (See reference sheets.)
3. Which CCSS Standards for Mathematical Practice are reflected? (See reference sheets.)

Analysis

1. What did the teacher do to incorporate Universal Design principles?
    - Provided an active, participatory task in which students perform functions.
    - Presented the class with tools such as a scale and timers to collect data.
    - Provided an opportunity for collaboration among students.
    - Encouraged discussion and communication throughout the problem-solving process.
    - Encouraged the use of charts and in-out tables to organize information and show patterns and connections.
2. Which Strands of Mathematical Proficiency are reflected?
    - Conceptual understanding
        - Students learned new ideas by connecting them to those they already knew, such as referring back to other instances when cooking with potatoes for reference on how many potatoes might make a pound.
        - Students represented mathematical situations in charts and equations.
        - Students explored the idea of sampling as a way to predict the total.
    - Procedural fluency
        - Students computed multidigit numbers mentally and with pencil and paper using strategies such as in-out charts.

- Students recognized patterns and relationships as they determined the rate at which they could peel potatoes.
- Adaptive reasoning
    - Students communicated about what the mathematical result would be and what other variables might influence how long the task may take, i.e., factoring in fatigue.

3. Which CCSS Standards for Mathematical Practice are reflected?
    - Make sense of problems and persevere in solving them (MP1).
        - Students looked at the information they were given, such as the number of pounds of potatoes.
        - Students determined whether 50 lbs. of potatoes and 50 potatoes were the same, both by referring to experience and by collecting measurements.
        - Students created charts and explained how they represented equations.
    - Model with mathematics (MP4).
        - Students used the math they knew to solve a workplace problem.
        - Students used two-way tables to map the relationships between quantities and times.
        - Students interpreted their results based on the context of the situation, factoring in additional variables that were not provided in the problem set.
    - Attend to precision (MP6).
        - Students labeled their charts and tables clearly and precisely.
        - Students converted between measurements such as seconds and minutes.
        - Students verbally communicated their ideas to one another, explaining their suggestions for how to approach the problem.

Lower-Level Instruction

- When timing the sample peeling, if the result is a less manageable number, such as 47 seconds, ask the students to round up.
- Have students convert the sample time to minutes as soon as possible so they do not have to deal with large numbers such as 4,000 seconds.

Higher-Level Instruction

- Instead of doing the sample peeling as a group, which results in only one speed, have all students peel a sample and collect the differing times. Then have students determine the mean, median, and mode from all the times collected. Use those figures as you proceed with the exercise.
- Have students graph the ratio and develop an equation of the situation.
- Using the same sampling, ask students to predict the time it would take to peel 250 potatoes, or *N* (number of) potatoes.
- Using the results of this activity and adding work tasks, have each student create a work schedule that outlines how he or she will accomplish work throughout the day.

3.2 Learn Through Exploration: Increasing Learner Autonomy

Many approaches meet students' individual needs and cultivate independent learners. Differentiated instruction is one method that recognizes that students differ by their previous educational experiences, readiness to learn new material, immediate life needs, and what they find engaging. This requires balancing many elements: students' individuality, the current standards of performance, and the demands of school, training, and the workplace—all with the intent to develop active learners who can pursue answers and information independently. The National Council for Teachers of Mathematics promotes the following teaching methods, described under Differentiated Instruction:

- Be open to students using different approaches and strategies, and expect them to explain their reasoning.
- Use whole-class, group, and individual instruction; when creating small groups, consider the knowledge, interests, and learning preferences of the group members.
- Use multiple instructional strategies, providing variation in content, process, and student products.
- Offer choice and flexibility in how students can engage with material: allow for different pacing; give options in work assignments.

Following these instructional guidelines supports learner autonomy in the classroom, which will also translate to confidence and independence in the tasks of daily life.

However, currently, many adult math classrooms focus on demonstration and guided practice as the primary instructional method. Instructors and learners often act in the following ways:

Instructor Behaviors	Learner Behaviors
• Initiates • Leads • Models • Explains • Thinks aloud • Demonstrates how to do it	• Listens • Observes • Tries out after the demonstration

Although this approach can be useful for some students, it is also effective to shift responsibility to the adult learners in the classroom, focusing on how they can initiate, lead, and discover new information and make connections. The chart below outlines the increased and active role that students take on when instruction moves beyond demonstration and guided practice. The goal is to increase student engagement, stimulate thinking and inquiry, and build a foundation for further learning.

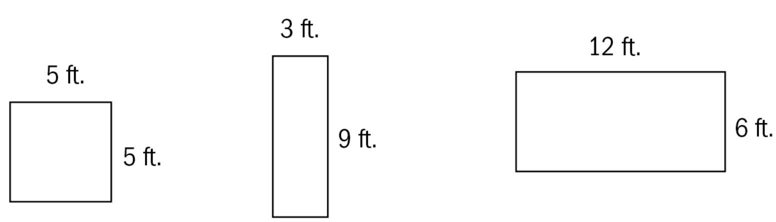
Instructor Behaviors	Learner Behaviors
• Demonstrates, explains	• Participates
• Scaffolds	• Listens
• Questions	• Questions
• Responds	• Responds
• Suggests	• Initiates
• Validates	• Tries out
• Suggests	• Approximates
• Acknowledges	• Applies learning
• Observes	• Takes charge
• Evaluates	• Practices
• Encourages	• Problem solves
• Confirms	• Self-corrects
• Affirms	• Self-directs
	• Confirms
	• Self-evaluates
	• Sets goals

The following examples illustrate two teaching approaches: (1) guided and explicit, and (2) hands-on and investigative.

Narrative Example

Example 1: A guided and explicit approach to teaching area and perimeter[7]

One instructor explained how to find perimeter and area in the following way: "To get the perimeter, you add the length of the sides; and to get the area, you multiply the sides." He then drew a few squares and rectangles on the board, labeled them with dimensions, and did the computations. The students were then given handouts with the following images and asked to determine the perimeter and area of all the rectangles.



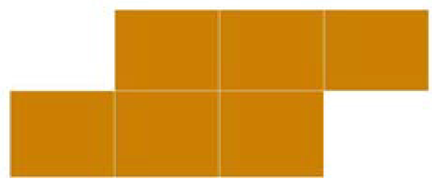
[7] Adapted from the section in Schmitt, M. J., & Curry, D. (2011). Combining squares: Geometry and Measurement. In *Adult numeracy instruction-Professional development* (ANI-PD) (p. 1.11). This activity was based on Malloy, C. E. (1999). Perimeter and area through the van Hiele Model. In *Mathematics teaching in the middle school: Reflections on practice* (pp. 87–90). Urbana, IL: National Council for Teachers of Mathematics. Permission is granted by TERC for use by participants in the TEAL project/course offered by LINCS in the course and in their classrooms. Unauthorized copying constitutes copyright infringement and is a violation of federal law.

Example 2: A hands-on and investigative approach to teaching area and perimeter[8]

Another instructor presented the following problem to demonstrate how to figure out perimeter and area.

He asked three students to walk around the rectangular classroom, one foot directly in front of the other, counting how many lengths of their own feet went into each wall length. Using the three different amounts, the teacher asked the students to write on large chart paper the dimensions they measured. He asked each student to use hash marks and lines to make up squares on the diagrams. They counted the sides and the squares to find the perimeter and the area.

Next the instructor showed the students the image below. Some students asked why the shape was not like a square or a rectangle. They wondered how you could find the area of a shape like the one in the diagram. One student mentioned that there are many rooms that are not perfectly square, and you still have to find out how to put a carpet down. So, after it was decided that this was a useful thing to think about, the class moved on.



The class discussed that there is length on the outside of the shape that can be measured, and there is space on the inside of the shape that can be measured, too, just as in the chart paper drawings. Using 1-inch square tiles, the instructor asked them to recreate the shape. She checked in with everyone to see how long they thought the side of each tile was. It was established that each was 1 inch. Then she asked each student to find the perimeter of the shape. She roamed the room to check on each student. When she determined that each one had gotten the correct answer, she divided the students into small groups. She considered the learning styles and confidence level of each student as she formed the groups.

On a projector in the front of the room, she asked one student to come up and label the image. She had another student provide in writing the rules they had just established together as a group.

When the small groups assembled, she asked them to build on the shape they already made so that the next shape had a perimeter of 16.

After each group drew its image and reported out on how it had arrived at the answer, the teacher asked students to think about their own kitchens, sketch them, and estimate their dimensions in feet. Next, within the group, they would work on finding the perimeters and

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[8] Adapted from the section in Schmitt, M. J., & Curry, D. (2011). Combining squares: Geometry and Measurement. In *Adult numeracy instruction-Professional development* (ANI-PD) (p. 1.11). This activity was based on Malloy, C. E. (1999). Perimeter and area through the van Hiele Model. In *Mathematics teaching in the middle school: Reflections on practice* (pp. 87–90). Urbana, IL: National Council for Teachers of Mathematics. Permission is granted by TERC for use by participants in the TEAL project/course offered by LINCS in the course and in their classrooms. Unauthorized copying constitutes copyright infringement and is a violation of federal law.

areas of their kitchens so that, in the next lesson, they would figure out how to tile the room: they would research how to do it, the materials that would be needed, how much it would cost, etc.

The instructor asked the students:

1. Do you see relationships between the chart drawings and your tile figures?
2. What patterns do you see as you create more figures and count the perimeters and the area?
3. What are you noticing about the connection between the length and the width?
4. What happens when you place a tile in the corner vs. along the edge of a side? What happens to the area? The perimeter?

Check for Understanding

Identify where and how the different instructional strategies and math principles are reflected in the example. Use the question below to guide your thinking. After you answer this question on your own, refer to the analysis below to check for understanding.

How did the instructor in Example 2 foster independent learning using a differentiated approach?

Analysis

- The instructor used manipulatives to introduce the concepts of perimeter and area.
- The instructor provided opportunities for the students to solve problems with one another in large groups and small groups.
- The instructor divided the class up, keeping in mind ability levels and personalities.
- The instructor let students who felt confident about their understanding come up to the front of the room and provide supports for their classmates, using the overhead projector.
- The instructor provided an individualized activity related to a room in students' homes.
- The instructor provided opportunities for movement, discussion, the use of visuals, and drawing activities.

3.3 Look at Student Work Regularly (Lesson Planning)

The work students produce, as well as their questions and comments, is critical in the lesson planning process. Below are several methods for assessing individual student performance and looking across the classroom to identify areas where students are solid or weak in their understanding of mathematics. Based on the data you gather when using the tools below, you can adjust your instruction on the spot, or as you plan future lessons.

Describe, Evaluate, Next Steps—D.E.N.

One way to determine the feedback you will give is a method called DEN, which is promoted in the Adult Numeracy Instruction—Professional Development institute training. DEN is an internal thought process the teacher uses to assess students' work and performance and to make decisions about how to address areas of confusion with students.

The teacher goes through the following steps:

- Describe what you see—without any evaluation or judgment.
- Evaluate what the student knows or understands, and what the student doesn't know or understand.
- Next steps are based on what the student does/does not know or understand; what you will do next to help the student (Source: ANI).

Using Rubrics

Rubrics are another tool that can be used to review and assess student work. Following is a sample rubric that captures information on individual students' performance in relation to the five Strands of Mathematical Proficiency. The instructor cites evidence of the demonstrated behaviors and rates how prevalent the behavior is during a session, during the week, or another time period; the more consistently the student displays the behavior, the more likely that he or she is reaching proficiency.

Mathematical Proficiency Rubric

Strand	Behaviors That Lead to Proficiency	Evidence Observed	Extent to Which Evidence Is Present 1= Slightly 2 = Somewhat 3 = Very 4 = Extremely
Conceptual understanding: Comprehension of mathematical concepts, operations, and relations	Students use multiple representations		
	Students connect mathematical ideas to what they already know		
	Students connect the math ideas to real contexts		
Procedural fluency: Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately	Students understand when to use a particular procedure		
	Students learn strategies		
	Students are accurate and efficient in the procedures they use		

Strand	Behaviors That Lead to Proficiency	Evidence Observed	Extent to Which Evidence Is Present 1= Slightly 2 = Somewhat 3 = Very 4 = Extremely
Strategic competence: Ability to formulate, represent, and solve problems	Students approach solving problems using a wide variety of problems		
	Students use strategies appropriate to the situation		
	Students build their knowledge base by creating links between what is known and unknown		
Adaptive reasoning: Capacity for logical thought, reflection, explanation, and justification	Students explain their processes for solving problems		
	Students use analogies to make connections		
	Students use different approaches to justify answers		
Productive disposition: Habitual inclination to see mathematics as sensible, useful, and worthwhile, with a belief in diligence and one's own efficacy	Students see the usefulness of math in their lives		
	Students are able to articulate how they use math in their daily lives		
	Students are willing to accept mathematically challenging problems		
	Students stick with hard problems until they are solved		

Source: Material adapted from Schmitt, M. J., & Curry, D. (2011). *Adult numeracy instruction-Professional development (ANI-PD)* (p. 1.15), which was originally adapted from Teachers Investigating Adult Numeracy (TIAN). © 2008 by Center for Literacy Studies, University of Tennessee and TERC. Permission is granted by TERC for use by participants in the TEAL project/course offered by LINCS in the course and in their classrooms. Unauthorized copying constitutes copyright infringement and is a violation of federal law.

Critical Friends

Critical Friends is a group process that instructors can use to evaluate the design of assignments by reviewing students' work completed in response to the tasks associated with assignments. Teachers examine student work to glean what aspects of an assignment are working to help students learn and what needs improvement.

The Critical Friends learning community model is a professional development approach based on dialogue and reflection. Developed by the Annenberg Institute for School Reform at Brown University in 1994, its principles are rooted in K–12 school reform, but they apply to all levels of education. This model examines both the curriculum and outcomes reflected in student work to improve classroom instruction. Using a set of guiding and thought-provoking questions, group members provide tailored feedback to an individual teacher seeking assistance. Critical Friends groups provide allocated time and formal structures to promote adult professional growth, which in turn supports student learning.).

Some practitioners have expressed concern that the "critical" in Critical Friends will lead to the disparagement of colleagues' work. In this context, however, critical connotes "important," "essential," or "urgent." In other words, teachers participating in this process are meant to provide crucial assistance to their colleagues: through critique and analysis, teachers collectively develop strategies to improve their students' learning. Costa and Kallick (1993) describe a critical friend as "a trusted person who asks provocative questions, provides data to be examined through another lens, offers critiques of a person's work as a friend, … [someone] who takes the time to fully understand the context of the work presented and the outcomes that the person or group is working toward [and who] is an advocate for the success of that work" (p. 50).

Using structured discussions, this model provides a collegial, collaborative approach to professional development for practitioners. Rather than attend 1-day workshops focused on general classroom applications, teachers working with the Critical Friends model engage in regularly scheduled group conversations to discover solutions targeted to their students' needs. The collegial exchange of ideas is designed to expand participants' knowledge. Through honest, open reflection on their own instructional practices, teachers are encouraged to be innovative and improve the quality of their teaching. Critical friends listen and ask incisive questions that encourage "presenting teachers" (those seeking guidance) to define and articulate the rationale and intended outcomes of their work. This refining technique has been called a "tuning process," in which teachers make adjustments to assignments to promote optimal learning gains for their students, much as musicians tune their instruments to achieve the desired sound quality.

The Annenberg Institute used adult learning theory as the basis for the Critical Friends process, especially the principle that adults can successfully engage in autonomous and self-directed group learning. To promote such learning:

- The teacher seeking guidance poses a question or presents a challenge to the group and describes desired outcomes to guide the group's work.
- The other teachers in the group raise questions and provide feedback, encouraging all members to gain new perspectives on their instructional practice; they
  - Introduce and determine the purpose of the assignment
  - Analyze the demands of the assignment

- Diagnose student work together
- Increase the rigor and redesign the assignment using input from your colleagues

Finally, a set of principles, features, and questions guides the use of critical friends:

Principles

- Examine teaching and student learning.
- Use data to inform the process.
- Share work so that colleagues can learn from one another.
- Commit time and energy to the group process.
- Be honest, reflective, and open to input from group members.
- Develop trust in, and respect and personal regard for, fellow group members.
- Recognize the competence and expertise that each group member brings to the process.
- Honor the norms established by the group.

Structural Features for Success

- Groups are small (6–10 members to permit open discussion and sharing) to foster equal participation among the members.
- Meetings are facilitated.
- Meetings are held regularly (at least once monthly, scheduled in advance) and for a substantial duration to maintain momentum and address pressing needs.
- Space is designated in a place convenient for teachers to hold group meetings.

Guiding Questions for All Group Members

- What am I thinking now about my classroom and my teaching? What do I want to do to improve both?
- What am I learning about my teaching practice today?
- What strategies will I try in my classroom?

Overview of the Process

Instructors will be asked to bring their classroom assignments and student work to a group meeting and to work through a five-step process challenging them to inject more rigor into their assignments. With the guidance of a facilitator and structured discussion questions, instructors will assist each other with techniques for strengthening assignments and, in turn, improving instruction. Specifically, they will:

1. Discuss the purpose of the assignment and ask what students were expected to learn from it.
2. Analyze the demands of the assignment, i.e., determine what someone needs to know and be able to do to complete the task successfully.
3. Identify the standards that best apply to the assignment. Diagnose student work to determine what the work suggests about how the assignment might be re-envisioned.
4. Redesign the assignment and plan new instructional strategies to match.

3.4 Use Technology Effectively

Technology can be used in any math classroom. The tools can be as simple as a basic calculator or as complex as mathematical simulation software. Currently, teachers are using a range of technologies, including 3-D graphics, animation, video, Smart Boards, LCD projectors, overhead projectors, talking calculators, mobile technologies, games, and massive open online courses (MOOCs). These technologies allow teachers to depict concepts in ways that may be easier for learners to comprehend than hand-drawn renderings or verbal explanations. Incorporating technology in the classroom also may help to strengthen students' computer and digital literacy skills—which are increasingly required in the workforce—and to access a growing number of public and commercial services that are moving to an online-only platform. Moreover, as documented by the Survey of Adult Skills (OECD, 2013), about two-thirds of adults in the United States perform at Level 1 or below in accessing information and problem solving in digital environments.[9] This finding demonstrates that the majority of adults are challenged in using basic email functions, conducting straightforward Internet searches, and following simple digital commands. Therefore, exposing students to technology through math instruction will serve them in numerous ways.

> *"I find our class uses a variety as well including voice recorders, video recorders, laptops, PowerPoint, I-pods, zip drives, and projectors. It really seems to keep the twinkle in students' eyes when they are utilized…"*
>
> Teacher
> New York TEAL Team

The use of technology also can mitigate time and location issues inhibiting student participation, and it can address staffing shortages within programs. In conjunction with in-class instruction, technology can improve learning outcomes. The National Research Council (2012) notes that technology allows instructors to tailor content and skill level to individual student needs, abilities, and interests, which leads to learning gains by adults in mathematics. A growing number of computer programs are being designed to track student work and provide instantaneous feedback that is shared with both students and instructors. Using technology in instruction also supports the practices outlined in the units on Strategy Instruction, Universal Design for Learning, Formative Assessment, Differentiated Instruction, and Effective Lesson Planning. Research shows that blended learning, a combination of classroom and digital learning, seems to be most effective (Russell, Lippincott, & Getman, 2013).

Websites to Support Instruction and Provide Computer-Assisted Learning for Students

Annenberg Learner: Teacher resources and professional development across the curriculum

This multimedia website includes video clips, lesson plans, sample student work, teacher reflections and more. Mathematics is one of many topics covered for K–12 and college-level instruction. There are activities for learners and materials for instructors to use in the classroom as well as professional development content.

http://www.learner.org/interactives/index.html

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[9] Level 1 in Proficiency in problem solving in technology-rich environments is defined as follows: At this level, tasks typically require the use of widely available and familiar technology applications, such as email software or a Web browser. There is little or no navigation required to access the information or commands required to solve the problem. The tasks involve few steps and a minimal number of operators. Only simple forms of reasoning, such as assigning items to categories, are required; there is no need to contrast or integrate information.

Collected Learning Units in Mathematics

These instructional units were developed by instructors who attended National Security Agency summer institutes. The topics covered include but are not limited to fractions, pre-algebra, algebra, geometry, and data analysis.

http://free.ed.gov/?page_id=6&subject=3&lr_resource=17a301c90e51f77d9e258afe2758a617

Federal Registry for Education Excellence (FREE)

Math resources section

Links to thousands of websites, many of which are sponsored in full or in part by federal agencies, including the U.S. Department Education, the National Science Foundation, and the National Academy of Sciences.

http://free.ed.gov/?page_id=6&query=Math&subject=3

Illuminations: Resources for Teaching Math

Sponsored by the National Council of Teachers of Mathematics, this website includes activities and lesson plans for K–12 standards-based mathematics instruction.

http://illuminations.nctm.org/

Math Open Reference

This site covers K–12 math content, including an extensive encyclopedia of terms with graphics that can be manipulated to represent different quantities. For example, by dragging the cursor at particular points on the images, the user can change the radius of a circle, move coordinates on a grid or number line, or move the lines of a square and the equation automatically updates based on the new positions.

http://www.mathopenref.com/

Math TV

This website presents short video clips, each demonstrating how to solve a single problem from basic to advanced math. Some of these videos can also be found on YouTube.

http://www.mathtv.com/

MathVids

This interactive website includes video clips of teachers modeling instruction, and written descriptions and scripts that accompany the videos, and teaching plans.

http://www.coedu.usf.edu/main/departments/sped/mathvids/index.html

Mathematics Across the Curriculum

The Center for Mathematics and Quantitative Education at Dartmouth College hosts this website, providing K–12 interdisciplinary instructional modules for teaching mathematics. This resource is funded in part by the National Science Foundation.

http://www.math.dartmouth.edu/~matc/eBookshelf/index.html

Online Chart Tool

Input parameters to create charts: bar, line, pie, area, scatter plots, and more.

http://www.onlinecharttool.com/

Real World Math

This website provides lesson plans, activities, videos, and more for students and teachers, using Google Earth.

http://www.realworldmath.org/

More Websites for Integrating Content Standards Into Instruction

The websites below provide lesson plans, curriculum maps, peer-reviewed problems, videos, standard alignment to assessments and curricula, and other resources for instruction using the Common Core State Standards.

Dan Meyer's Three-Act Math Tasks

https://docs.google.com/spreadsheet/pub?key=0AjIqyKM9d7ZYdEhtR3BJMmdBWnM2YWxWYVM1UWowTEE&output=html

Emergent Math

http://emergentmath.com/my-problem-based-curriculum-maps

Engage New York

www.engageny.org/mathematics

Illustrative Mathematics

http://www.illustrativemathematics.org

Inside Mathematics

http://insidemathematics.org

Khan Academy

https://www.khanacademy.org

LearnNC

http://www.learnnc.org/lp/editions/ccss2010-mathematics

Mathematics Assessment Project

http://map.mathshell.org/materials/background.php

Mathematics Common Core Toolkit

http://www.ccsstoolbox.org/

National Science Digital Library

http://nsdl.org/search/standards/D10003FB

OER Commons

http://www.oercommons.org

Ohio Resources Center

http://ohiorc.org/standards/commoncore/mathematics

Southeastern Comprehensive Center

http://secc.sedl.org/common_core_videos/index.php

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APPENDIX A.

Evidence of and Guidance for High-Quality Math Instruction: A Reference Document to Accompany The TEAL Math Works! Guide Check for Understanding Sections

Several bodies of research promote similar skills, knowledge, and behaviors that students should develop to reach mathematical proficiency. The National Research Council (NRC) [1] contends that mathematical proficiency is identified in five interwoven strands, which are outlined in Section 1 below. The findings in the U.S. Department of Education report, *Building on* Foundations for Success: *Guidelines for Improving Adult Mathematics Instruction,*[2] are based on the NRC model of mathematical proficiency and provide additional direction on instructional practices, highlighted in Section 2 below. The developers of the Common Core State Standards for Mathematics[3] also identified what students need to be able to do and understand to be mathematically proficient and prepared for college and careers; these are presented in the list of Standards for Mathematical Practice in Section 3 below. These practices are also based on the NRC findings as well as on process standards identified by the National Council for Teachers of Mathematics.[4]

When responding to the *Check for Understanding* sections of the Teachers Guide, please refer to all three lists to identify which aspects of the narrative examples will lead students to develop the necessary skills and knowledge, and demonstrate the behaviors presented below.

I. Mathematical Proficiency Strands

- **Conceptual Understanding—**comprehension of mathematical concepts, operations, and relations

  Mathematically proficient students:

  - Understand why a mathematical idea is important and the kinds of contexts in which it is useful
  - Can learn new ideas by connecting them to those they already know
  - Monitor what they remember and try to figure out whether it makes sense
  - Often can understand before they can verbalize that understanding
  - Are able to represent mathematical situations in different ways (e.g., using pictures, manipulatives, number lines) and know how different representations can be useful in different situations
  - Understand the connections between representations: how they are similar and how they are different

- **Procedural Fluency—**skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

  Mathematically proficient students:

  - Know when and how to use procedures appropriately
  - Analyze different methods of calculating
  - Can use calculators, computers, blocks, counters, beads, etc., to solve problems
  - Compute multidigit numbers, mentally and with pencil and paper
  - Estimate the results of procedures
  - Understand that mathematics is organized, includes patterns, and is predictable

- **Strategic Competence—**ability to formulate, represent, and solve mathematical problems

  Mathematically proficient students:

  - Have multiple strategies to draw from
  - Know which strategies may be useful in certain contexts
  - Can represent problems numerically, symbolically, verbally, or graphically
  - Visualize the components of the problem; create a mental model
  - Detect mathematical relationships
  - Exhibit flexibility in solving nonroutine problems; can use reasoning, guess-and check, algebraic, and other approaches

- **Adaptive Reasoning—**capacity for logical thought, reflection, explanation, and justification

  Mathematically proficient students:

  - Consider alternative solutions and can justify which ones are sound
  - Understand how facts, procedures, concepts, and solution methods fit together
  - Check that their reasoning is valid, through communication with teacher and classmates and other data
  - Use intuitive and inductive reasoning based on pattern, analogy, and metaphor
  - Justify their mathematical claims and make them clear to others

- **Productive Disposition—**habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

  Mathematically proficient students:

  - Believe mathematics is understandable and has a purpose
  - Believe that with diligent effort mathematics can be learned and used
  - Gain confidence through attempting increasingly challenging work
  - Exhibit interest in attempting challenging problems
  - Share their mathematical ideas with others

II. Guidelines on Instructional Practice

1. Adult students should be able to demonstrate all aspects of mathematical proficiency: conceptual understanding, procedural fluency, strategic competency, and adaptive reasoning. Through their learning experiences, they also should be developing a productive disposition toward learning and using mathematics.
2. Computational fluency requires not only knowledge of efficient procedures, but also understanding why they work. Both aspects should be part of adult mathematics instruction.
3. Adult mathematics instruction should recognize and address negative affective factors, including both beliefs and emotions that can interfere with learning.
4. Adults' goals and experiences offer opportunities to embed instruction in meaningful contexts. Instruction should include connections to student interests,

work situations, and everyday life (e.g., following recipes, basic accounting required on the job or at home) to stimulate engagement and promote applicability.

5. Formative assessment exposes student thinking and monitors progress and should be common practice in adult education.
6. A variety of student grouping formats should be implemented to enhance learning through communication and collaboration.
7. Mathematics instruction should include the technology used in the contexts for which students are preparing.

III. Common Core State Standards Mathematical Practices

Math.Practice.1 (MP1) Make sense of problems and persevere in solving them.

Mathematically proficient students:

- Start by explaining to themselves the meaning of a problem and looking for entry points to its solution
- Analyze givens, constraints, relationships, and goals
- Make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt
- Consider analogous problems and try special cases and simpler forms of the original problem to gain insight into its solution
- Monitor and evaluate their progress and change course, if necessary
- At more advanced levels may, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need
- Can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends
- At lower levels might rely on using concrete objects or pictures to help conceptualize and solve a problem
- Check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”
- Can understand others’ approaches to solving complex problems and identify correspondences between different approaches

Math.Practice.2 (MP2) Reason abstractly and quantitatively.

Mathematically proficient students:

- Make sense of quantities and their relationships in problem situations
- Have the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents
- Have the ability to *contextualize,* to pause as needed during the manipulation process to probe into the referents for the symbols involved

- Create coherent representations of the problem at hand by considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects

Math.Practice.3 (MP3) Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- Understand and use stated assumptions, definitions, and previously established results in constructing arguments
- Make conjectures and build a logical progression of statements to explore the truth of their conjectures
- Can analyze situations by breaking them into cases, and can recognize and use counterexamples
- Justify their conclusions, communicate them to others, and respond to the arguments of others
- Reason inductively about data, making plausible arguments that take into account the context from which the data arose
- Can compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is
- Can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments

Math.Practice.4 (MP4) Model with mathematics.

Mathematically proficient students:

- Can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace; in beginning levels, this might be as simple as writing an addition equation to describe a situation; at higher levels, a student might apply proportional reasoning to plan an event or analyze a problem in the community
- At higher levels, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another
- Can apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later
- Can identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas
- Can analyze those relationships mathematically to draw conclusions
- Can routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose

Math.Practice.5 (MP5) Use appropriate tools strategically.

Mathematically proficient students:

- Consider the available tools when solving a mathematical problem (including pencil and paper concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software)
- Are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and the limitations of those materials
- At higher levels, analyze graphs of functions and solutions generated by a graphing calculator; detect possible errors by strategically using estimation and other mathematical knowledge; when making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data
- At various levels, can identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems; can use technological tools to explore and deepen their understanding of concepts

Math.Practice.6 (MP6) Attend to precision.

Mathematically proficient students:

- Try to communicate precisely to others
- Try to use clear definitions in discussion with others and in their own reasoning
- State the meaning of the symbols they choose, including using the equal sign consistently and appropriately
- Are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem
- Calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context
- At lower levels, give carefully formulated explanations to each other
- At higher levels, have learned to examine claims and make explicit use of definitions

Math.Practice.7 (MP7) Look for and make use of structure.

Mathematically proficient students:

- Look closely to discern a pattern or structure: beginning students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have; more advanced students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, more advanced students can see the 14 as 2×7 and the 9 as $2 + 7$
- Recognize the significance of an existing line in a geometric figure and use the strategy of drawing an auxiliary line for solving problems; also can step back for an overview and shift perspective

- See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Math.Practice.8 (MP8) Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- Notice whether calculations are repeated and look both for general methods and for shortcuts
- At higher levels, may notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal
- Pay attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3
- At higher levels, might abstract the equation $(y - 2)/(x - 1) = 3$, noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series
- Maintain oversight of the process, while attending to the details
- Continually evaluate the reasonableness of their intermediate results

Endnotes

1. National Research Council. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academies Press.

2. U.S. Department of Education, Office of Vocational and Adult Education. (2011). *Building on Foundations for Success: Guidelines for improving adult mathematics instruction*. Washington, DC: Author. Retrieved from http://lincs.ed.gov/publications/pdf/AdultNumeracyReportFinal2011.pdf

3. National Governors Center Association for Best Practices and the Council of Chief State School Officers. (2010). Common core state standards for mathematics. National Governors Association Center for Best Practices and the Council of Chief State School Officers. Retrieved from http://www.corestandards.org/Math/Practice

4. National Council of Teachers of Mathematics. Principles and standards for school mathematics: Process standards. Retrieved from http://www.nctm.org/standards/content.aspx?id=322

GLOSSARY

A

absolute value—A number's distance from zero on the number line. Example: –6 and 6 both have the absolute value of 6.

abstraction—The ability to distill an essential message or meaning from a text. Abstraction is an essential skill for summarizing and many kinds of analysis.

accommodations—Techniques and materials that are legally required to reduce or eliminate the impact of a learning disability on successful learning and performance. Examples of accommodations include spell-checkers, tape recorders, and expanded time for completing assignments.

adaptations—Alternative techniques and/or materials provided by a teacher for individual learners to increase the effectiveness of instruction. Unlike accommodations, which are legally required for persons with documented disabilities, adaptations may be made for any learner, on an as-needed basis, and do not require the documentation of a disability. Two examples of adaptations are as follows: providing large-print materials for older learners with diminished visual acuity or allowing the use of native language dictionaries for learners whose primary language is not English.

- **adapted materials**—Authentic texts and other materials that have been modified for lower-level learners. The format, vocabulary, grammatical forms, or sentence structure of authentic materials can be adapted. (*See also authentic materials*.)

addend—A number that is added to another number.

advance organizers—*See graphic organizers*.

algebraic expression—A mathematical phrase that includes at least one variable and sometimes numbers and operation symbols.

algorithm—A set of rules that when followed in proper sequence will provide accurate calculations.

analysis—An examination of the elements of a text to reveal how the text operates as a whole. In writing, analysis is a method of idea development in which the writer closely breaks apart a subject or topic to examine its parts and the relationships of the parts to reach new conclusions about the whole.

anchor activities—Activities that extend the curriculum and in which learners may participate if they have spare time while waiting for the teacher's help or after they have completed a task.

area—Amount of a figure's surface, measured in square units.

assessment—The process of gathering, describing, or quantifying information about performance. It is a general term that refers to tests and other measures, such as oral reading performances, collections of writings and other work products, teacher observations, and self-evaluations. It is the process of collecting and analyzing data to make educational decisions. (*See also portfolio*.)

- **alternative assessment (also called authentic or performance assessment)**—Any of a variety of assessments that allow teachers to evaluate learners' understanding or performance. It is an assessment that requires learners to generate a response to a question rather than choose from a set of responses provided to them. Examples include exhibitions, investigations, demonstrations, written or oral responses, journals, performance assessments, portfolios, journals, and authentic assessments.
- **assessment portfolio**—A selection of a learner's writing submitted for assessment purposes. The learner, in conferences with teachers, chooses the entries for this

portfolio. Ideally, the writings will grow naturally out of instruction rather than being created solely for the portfolio.

- **authentic assessment**—*See alternative assessment*.
- **formative assessment—**A part of the instructional process. When incorporated into classroom practice, it provides the information needed to adjust teaching and learning while they are happening. In this sense, formative assessment informs both teachers and learners about learner understanding at a point when timely adjustments can be made. These adjustments help to ensure learners achieve targeted standards-based learning goals within a set time frame. Although formative assessment strategies appear in a variety of formats, there are some distinct ways to distinguish them from summative assessments.
- **performance assessment**—*See alternative assessment*.
- **summative assessment**—An assessment given periodically to determine at a particular point in time what learners know and do not know. Summative assessments may take the form of standardized tests, such as state assessments or assessments at the district or classroom level for accountability purposes, and generally are used as part of the grading process. They provide a means to gauge learning relative to content standards. Summative assessments happen too far down the learning path to provide information at the classroom level and make instructional adjustments and interventions *during* the learning process. Formative assessment accomplishes this.

assistive technology—Equipment that enhances learners' ability to access instruction and be more efficient and successful. Examples include computer grammar checkers, teacher use of an overhead projector, or audiovisual information delivered through a CD-ROM.

associative property of addition—Groupings of addends do not affect the solution. Example: $(a + b) + c = a + (b + c)$.

associative property of multiplication—Groupings of factors do not affect the solution. Example: $(a \times b) \times c = a \times (b \times c)$.

attention deficit disorder (ADD)—A disorder characterized by severe and persistent difficulties in one or more of the following areas: attention, impulsivity, and motor behaviors. These difficulties can lead to learning and behavior problems at home, school, or work. (*See also attention deficit hyperactivity disorder [ADHD]*.)

attention deficit hyperactivity disorder (ADHD)—Attention deficit disorder with hyperactivity or excessive and exaggerated motor activity.

auditory—Having to do with the sense of hearing.

- **auditory discrimination**—The ability to differentiate among speech sounds.
- **auditory memory**—The ability to remember information that has been presented orally.
- **auditory perception**—The ability to recognize sounds.

authentic—Original, realistic, and genuine. When applied to writing, authentic means that the work is the learner's own, done for a realistic purpose and readership and in a realistic form that logically fits the purpose and audience or situation. The writing reveals a genuine effort to communicate with others; it is not merely an academic exercise.

- **authentic assessment**—*See assessment*.

- **authentic materials**—Actual reading or listening materials, not modified or simplified, from the real world (e.g., newspaper articles, pamphlets, and radio broadcasts). (*See also adapted materials*.)
- **authentic task**—An assignment given to learners designed to assess their ability to apply standard-driven knowledge and skills to real-world challenges.

automaticity—Automatic and correct responses to stimuli without conscious effort.

average—A single number that can represent all other values in a data set.

axes—The vertical line along the left side of the graph (the y axis) and the horizontal line along the bottom of the graph (the x axis).

B

backward design—A method of designing curriculum by setting goals before choosing activities or content to teach. The idea is to teach toward those goals, which ensures that the content taught remains focused and organized, promoting a better understanding for learners. Backward design challenges traditional methods of curriculum planning, in which the teacher lists the content to be taught and proceeds to assessment to measure the extent to which learners have mastered the content. With backward design, the teacher starts with goals, then assessments, and finally lesson plans.

background knowledge—Information that is essential to understanding a situation or a problem. It is what one already knows about a subject.

benchmarks—*See standards*.

bivariate data—Pairs of linked numerical observations. Example: years of education and amount of money earned per year.

Bloom's taxonomy—A hierarchy of six levels within the cognitive domain, from the simple recall or recognition of facts (knowledge) as the lowest level through increasingly more complex and abstract mental levels (understanding, application, analysis, synthesis) to the highest order (evaluation). Benjamin Bloom created this taxonomy for categorizing levels of abstraction in questions that commonly occur in educational settings.

brainstorming—A group thinking process for developing creative solutions to problems. The group thinks open-mindedly about a topic or a problem and generates a written list of possibilities without worrying if any possibility is reasonable or not. One of the reasons this technique is effective is that the brainstormers not only come up with new ideas but also spark associations from other people's ideas by developing and refining them.

C

carousel brainstorming—A strategy in which learners brainstorm responses to prompts or questions written on a flipchart page and placed at five different stations around the room. Learners rotate from station to station and discuss their responses with others in their group. Teachers may use carousel brainstorming as a preassessment tool or as a review opportunity.

chunking—The working memory has a capacity for immediate recall that is limited to five to nine pieces of unrelated items. If information is separated into chunks of that size, learners can remember it more successfully.

circle graph—A graph of data where the entire circle represents the whole or 100%.

clustering—An estimation strategy in which a group of numbers that are close in value are rounded to the same value. Example: 424 + 410 + 390 + 403. All of these values are close to 400, so the answer can be estimated by computing 400 × 4.

coefficient—Numerical factor of a mathematical expression that contains a variable. Example: 5*x* +2. The 5 is a coefficient.

cognitive learning—Learning that is concerned with acquisition of problem-solving abilities and with intelligence and conscious thought; it is demonstrated by knowledge recall and skills such as comprehension; ability to organize, analyze, and synthesize ideas and data; applying knowledge; and evaluating ideas or actions.

cognitive map—The psychological definition of a cognitive map is the framework in the human mind through which we interpret objects, events, and concepts. The phrase *cognitive mapping* also has been used to describe concept maps.

cognitive skills—Skills that are used for thinking, comprehending, analyzing, or evaluating.

cognitive strategy—A strategy or group of strategies or procedures that a learner uses to perform academic tasks or improve social skills. Often, more than one cognitive strategy is used with others, depending on the learners and their schema for learning. In fact, research indicates that successful learners use numerous strategies. Some of these strategies include visualization, verbalization, making associations, chunking, questioning, scanning, underlining, accessing cues, using mnemonics, sounding out words, and self-checking and monitoring.

cognitive strategy instruction (CSI)—An instructional approach that emphasizes the development of thinking skills and processes as a means to enhance learning. The objective of CSI is to enable all learners to become more strategic, self-reliant, flexible, and productive in their learning endeavors. CSI is based on the assumption that there are identifiable cognitive strategies, previously believed to be used by only the best and the brightest learners, which can be taught to most learners. The use of these strategies has been associated with successful learning.

collaborative learning—Any kind of work that involves two or more learners.

compacting—A strategy that allows learners to demonstrate what they already know or can do, provides an opportunity to learn or master what they do not already know, and then allows them to spend the time earned from compacting to participate in enrichment, or extension activities, and/or accelerated study.

complex fraction—a fraction a/b where a and/or b are fractions.

computation algorithm—A set of steps used to solve a category of problems that gives the correct result in every case when the steps are applied correctly.

concept map—*See graphic organizers*.

congruent—Shapes or figures are congruent in size if one can be obtained from

connected instruction—As a key principle of learning disabilities–appropriate instruction, it involves showing the adult learner how information in and between units and lessons is linked to learning and to the adult's goals.

connectionism—Edward L. Thorndike's behavioral theory that learning occurs as the result of connections made in the mind between stimuli and responses.

constructivist models—Models based on the philosophy that knowledge cannot be transferred from the teacher to the learner but must be constructed by each individual. Connections must be made between the learner's existing conceptual network and the new material to be learned.

content-based instruction—Using subject matter such as life-skills topics (e.g., housing and work), themes, or academic course materials (e.g., math, science, and social studies) as a basis for language teaching.

content mastery approach—A teaching method wherein a learner receives intensive instruction in topics that are needed for daily living, such as obtaining insurance, getting a driver's license, completing tax forms, and procuring health care services.

content standards—*See standards*.

contextualized instruction—Instruction that is presented within a meaningful context to facilitate learning (e.g., the grammatical structure of commands taught within the context of a doctor's visit: "Open your mouth." "Raise your arms.").

conventions—Features of standard written English that usually include sentence formation, grammar, spelling, usage, punctuation, and capitalization.

cooperative learning—A range of team-based learning approaches in which learners share knowledge and work together to complete a task. Cooperative learning originated in the 1960s with the work of David Johnson and Roger Johnson. True cooperative learning includes five essential elements: positive interdependence, face-to-face interactions, individual accountability, some structured activity, and team-building (group processing) skills.

cooperative planning—The teacher announces a topic or a problem and asks learners to help think about the best ways to deal with it.

coordinate grid—A grid formed by a horizontal line called the *x* axis and a vertical line called the *y* axis.

coping strategy—A method or behavioral strategy that helps an individual succeed despite learning or other disabilities.

criteria—Guidelines, rules, characteristics, or dimensions used to judge the quality of learner performance. Criteria indicate what we value in learner responses, products, or performances. They may be holistic, analytic, general, or specific.

critical content—Specific information that a learner needs to master for a given task, such as the skills needed to pass a driver's test.

critical thinking—Skilled and active interpretation and evaluation of observations and communications; reasonable, reflective thinking that focuses on deciding what to believe or do. It is relevant not only to the formation and checking of beliefs but also to deciding on and evaluating actions. It involves creative activities such as formulating hypotheses, plans, and counterexamples; planning experiments; and seeing alternatives.

cubing—As a strategy for differentiating instruction, it allows a teacher to plan diverse activities for different learners or groups of learners based on readiness, learning style, and/or interests. The teacher creates a cube for a group of learners. On each of the cube's six faces, the teacher describes a different task related to the subject and/or concept being learned.

cue-do-review—A teaching strategy to help ensure learning: the teacher should *cue* the learner, explaining the level of instruction, *do* the activities in partnership with the learner, and *review* the learning at the end of each level.

cues—Visual or verbal prompts to either remind the learner what has already been learned or provide an opportunity to learn something new. Cues can also be employed to prompt learner use of a strategy.

cultural competence—An ability to interact effectively with people of different cultures. It consists of four components: (1) awareness of one's own cultural worldview, (2) attitude toward cultural differences, (3) knowledge of different cultural practices and worldviews, and (4) cross-cultural skills. Developing cultural competence results in an ability to understand, communicate with, and effectively interact with people across cultures.

cultural dissonance—A phenomenon that may present itself when someone who participates in multiple cultures is faced with situations where there are perceived conflicts between a set of rules from one culture and the rules of another. May also refer to an uncomfortable sense of discord, disharmony, confusion, or conflict experienced by people in the midst of change in their cultural environments. The changes are often unexpected, unexplained, or not understandable as a result of various types of cultural dynamics.

culture—Shared values, attitudes, beliefs, behaviors, and language use within a social group. These cultural values, beliefs, and practices are at the core of group life and identity and are powerful forces that shape or influence individual attitudes, beliefs, and behaviors.

curriculum—What should take place in the classroom. It describes the topics, themes, units, and questions contained within the content standards, which are the framework for the curriculum.

D

data—Facts and statistics gathered for reference or analysis. Quantitative data can be discrete (counted) such as “5 people” or continuous (measured) such as “33%.”

diagnosis—The confirmation of the existence of a condition by someone qualified to reach such a conclusion. For example, a licensed psychologist can make a diagnosis of a learning disability.

diagnostic test—An aid to assessment that yields information concerning a learner’s weaknesses in areas such as reading or math. It is composed of several parts, including personal history and psychoeducational tests.

differentiated instruction—Occurs when the teacher provides multiple options for learners to take information, make sense of ideas, and express what they learn. In providing diverse avenues for learners to access information, the teacher ensures that each learner learns effectively. The components of differentiated instruction include the following: (1) what to teach, or content; (2) how to teach it, or process; (3) how to find out whether learners have learned it; and (4) the environment in which the instruction occurs.

direct instruction—A teacher-centered instructional approach that emphasizes the use of carefully sequenced steps that include demonstration, modeling, guided practice, and independent application. It is characterized by high rates of teacher control during the initial stages of information acquisition, followed by careful performance monitoring as the learner gradually assumes control over application. The instruction is structured, modular, and sequential (simple to complex and concrete to abstract). The teacher provides the learners with much of the information they need, often through lectures, explanations, examples, and problem solving. Most direct instruction techniques allow for only minimal learner-teacher interaction, and they need to be supplemented by review, practice, and group discussions.

distributive property—Multiplying a sum by a number is the same as multiplying each addend by that number and then adding the two products.
Example: $a \times (b + c) = a \times b + a \times c$.

dividend—A number to be divided by another number.

divisor—Number by which another number (the dividend) is divided.

E

editing—Checking for and correcting errors in spelling, punctuation, capitalization, grammar, and usage. Editing becomes a concern only after writers are satisfied that the writing clearly says what they want it to say; editing is the final stage of document preparation.

emergent literacy—The concept that learning to read or write does not happen quickly but is built on many small steps that occur over the course of a child's early childhood. The process begins with activities that happen naturally in the home, such as talking with and reading with the child and then continues in the classroom with more formalized strategies to encourage reading and writing.

executive function—Cognitive processing of information that takes place in areas in the left frontal lobe and prefrontal cortex that exercise conscious control over one's emotions and thoughts. This control allows for patterned information to be used for analyzing, connecting, organizing, planning, prioritizing, sequencing, self-monitoring, self-correcting, and sorting, as well as abstracting, assessing, focusing attention, problem solving, and linking information to appropriate actions.

exit cards—A formative assessment technique whereby learners fill out a 3-by-5 card at the end of class and respond to open-ended questions posed by the teacher. It is a great way for a teacher to assess learner understanding and readiness for the next lesson.

experiential learning—Carl Roger's theory that there are two types of learning: cognitive (memorizing or studying simply because work is assigned) and experiential (learning to satisfy the needs and wants of the learner). Studying a book with commonly used phrases in Norwegian is experiential if you are planning a trip to Norway, but the same activity is cognitive if you are taking a language class and the teacher assigns reading from the book.

explicit instruction—The intentional design and delivery of information by a teacher to learners. It involves presenting content clearly and directly, providing detailed explanations and models about how to approach, think about, perform, and evaluate learning and performance. It includes three steps: (1) the teacher's modeling or demonstration of the skill or strategy; (2) a structured and substantial opportunity for learners to practice and apply newly taught skills and knowledge under the teacher's direction and guidance; and (3) an opportunity for feedback.

expository text—Expository text presents and explains facts and information about a topic. It is distinguished from narrative text, which tells a story or relates a series of events.

F

facilitative questioning—To facilitate means to help another person accomplish something. Facilitative questioning is an approach whereby a teacher or a counselor poses open-ended questions to learners to allow them to explore ideas that may be complex or emotionally difficult. In writing classes, the purpose of facilitative questions is to allow a teacher to give assistance to learners without actually contributing new ideas to the work being written. In counseling, the purpose of facilitative questions is to allow learners to generate their own solutions to problems or tasks without being unduly influenced by the counselor's ideas. Facilitative questioning is used most often in situations where there is no right answer, but the solution is dependent on what is best for the individual.

factor—A number that is multiplied by another number.

feedback—The means by which a teacher informs a learner about the quality or correctness of the learner's products or actions.

flexible grouping—A method of grouping and regrouping learners according to differences in readiness/performance, interests, and learning profiles. Learners may work in groups with different learners several times a day or in a week.

formative assessment—*See assessment*.

formula—An equation that states a rule or a fact.

fraction—A number expressible in the form a/b where a is a whole number and b is a positive whole number, representing parts of a whole or group.

front-end estimation—a strategy of estimating a sum by adding the front-end digits. Example: 3,452 + 5,912 would be estimated as: 3,400 + 6,000.

function—A formula that indicates how to compute an output based on a given input thereby showing the relationship between the input and the output. Example: $f(x) = x + 4$, if the input is 3, the output is 7.

G

generalizable instruction—Activities before, during, and after information has been mastered that ensures the continued application of the information by learners to increase life success outside the literacy setting. It refers to how well learners use information outside the classroom to increase their success in life.

general-to-specific sequencing—An instructional approach in which objectives are presented to learners, beginning with general principles and proceeding to specific concepts.

goal setting—An effective tool for making progress by ensuring that learners are clearly aware of what is expected from them if an objective is to be achieved. It involves establishing specific, measurable, and time-targeted objectives.

gradual release—An evidence-based instructional model in which a teacher purposefully transitions from performing a task to mentoring learners to gradually assume responsibility until they can perform the task independently. It is a means of mentoring learners to become capable thinkers and learners in performing tasks they have not yet mastered. This model of instruction has been documented as an effective approach for improving writing achievement, reading comprehension, and literacy outcomes for English language learners.

graphic organizers—Visuals used to organize information so it can be more easily represented, recalled, or understood (e.g., word webs, flowcharts, Venn diagrams, charts, and tables). These mental maps help learners make connections between concepts. Graphic organizers represent key skills, such as sequencing, comparing and contrasting, and classifying. They involve learners actively in the thinking process, and they provide tools to help learners organize and structure information. They can be used in advance, during, and/or after presentation of information: those used before learning (an *advance organizer*) help remind learners of what they already know about a subject; those used during learning provide cues for what to look for in the resources or information; and those used during review activities help remind learners of the number and variety of components to remember. (*See also* *Venn diagram*.)

- **advance organizer**—A concise overview or summary of a larger body of information that is used to gain prior knowledge before reading or listening to the larger body of information.
- **concept map**—Any of several forms of graphic organizers that allow learners to perceive relationships between concepts through diagramming keywords representing those concepts. Joseph Novak originated the concept map in the 1960s.
- **KWL chart**—A graphic organizer or preassessment tool consisting of three columns: "know," "want to know," and "learn." Learners list in the left column what they know about a topic or idea and in the center column what they want to know about the topic or idea. Then, after reading or instruction, they return to the chart to list in the right column what they learned about the topic or idea or what they still would like to learn. KWL charts can be completed as a class with the teacher or independently.
- **mind map**—A graphic organizer that is used to develop ideas and organize information. Mind mapping helps to identify central ideas, the relative importance of

other ideas, and how they are connected. A main or central word or image is placed in the center of a piece of paper and then key words, symbols, images, and abbreviations are added as subideas. Subideas should be on lines that ultimately connect to the center. Each new line should be open, allowing space for more connections to subideas farther from the center. Mind maps are used for a prewriting activity, note taking, developing grocery lists, brainstorming sessions, and other related tasks.

- **Venn diagram**—A graphic organizer (interconnected circles) used to demonstrate how two subjects or topics overlap and how they are unique.

guided discovery—A teaching model in which learners are taught through explorations but with directions from teacher.

guided practice—Learners practice a skill with teacher guidance so that they gradually move toward excellence. Guided practice should be conducted in small steps and should be intensely supervised. It should prevent the development of consistent error patterns and inappropriate practices. This means that guided practice must be designed and implemented so that errors are identified and reteaching is conducted immediately. The important element seems to be the provision of controlled practice with *positive* teacher feedback. (*See also gradual release*.) The effectiveness of guided practice can be evaluated by measures of learner success in independent practice. If learners are at least 80 percent successful when they begin the subsequent independent practice, then guided practice has been appropriately conducted. (*See also scaffolding/scaffolded instruction* as a vehicle to provide guided practice.)

H

habits of mind—A process that centers on the idea that learners can learn more effectively if they regulate their own thought processes.

higher-order thinking skills (HOTS)—In the simplest sense, higher-order thinking is any thinking that goes beyond the recall of basic facts. The two key reasons to improve HOTS are to enable learners to apply facts to solve real-world problems and improve the retention of facts. In addition to its basic meaning, HOTS is also used to refer to a specific program designed to teach higher-order thinking skills through the use of computers and the Socratic method for teaching thinking skills.

I

improper fraction—A fraction whose numerator is larger than the denominator.

independent practice—One learner works independently or with other learners, without teacher intervention, to practice new skills or strategies. This approach includes many activities performed on a computer.

independent study—Opportunities for learners at all readiness levels to pursue topics that interest them.

indirect instruction—This approach to teaching presents learners with instructional stimuli in the form of materials, objects, and events and requires them to go beyond the basic information they are given to make their own conclusions and generalizations. Indirect instruction allows teachers to engage learners in activities that require the learners to learn independently. The role of the teacher is facilitator, supporter, and resource person. The teacher arranges the learning environment, provides opportunity for learner involvement, and, when appropriate, provides feedback to learners while they conduct the inquiry.

individualized education plan (IEP)—A specifically tailored program designed to meet the distinctive needs of learners diagnosed with a disability.

individualized instruction—As a key principle of learning disabilities–appropriate instruction, it involves maintaining a high degree of learner attention and response during ongoing instructional interactions that are scheduled as frequently and as close together as possible.

inductive thinking—Analyzing individual observations to come to general conclusions and proceed from facts to the "big picture."

inquiry—The process by which learners investigate subjects that interest them. Embedded across all content areas and grade levels, inquiry promotes learner ownership and authenticity in their work.

integer—a number that can be expressed in the form *a* or –*a* for the whole number *a*.

intensive instruction—Learner engagement and time are the defining factors in intensive instruction. In intensive instruction, learners are paying attention and actively engaged in learning tasks—listening, thinking, responding, creating, or otherwise working—and doing so frequently for *significant* amounts of time.

interest centers—A classroom area that contains a collection of exploration activities related to the specific interests of learners.

interest inventory—An assessment tool designed to help a teacher determine learner interests. These may be open-ended or very controlled and specific.

irrational number—A number that cannot be represented by a fraction or ratio. Example: 1.23.

J

jigsaw—A type of collaborative work in which learners read and examine a portion of a reading assignment and report what they have learned to the entire group. It is an effective way to vary content according to complexity or depth of content to match reading readiness levels. It is also a great way to involve learners in subject matter presented in text.

K

kinesthetic—Learning by doing.

KWHL—A graphic organizer or pre-assessment tool consisting of four columns, "know," "want to know," "how to find out," and "learn."

KWL chart—*See graphic organizers*.

L

learning centers—A classroom area that contains a collection of activities or materials designed to teach, reinforce, or extend a particular skill or concept.

learning disabilities—A variety of neurological disorders, including differences in one or more of the basic processes involved in understanding or using spoken or written language. Learning disabilities are lifelong conditions not related to visual or auditory deficiencies and are not the result of delays in mental development. Specific learning disabilities include the following:

- **developmental aphasia**—A severe language disorder that is presumed to be due to a brain injury rather than a developmental delay in the normal acquisition of language.
- **dyscalculia**—A severe difficulty in understanding and using symbols or functions needed for success in mathematics.
- **dysgraphia**—A severe difficulty in producing handwriting that is legible and written at an age-appropriate speed.

- **dyslexia**—A severe difficulty in understanding or using one or more areas of language, including listening, speaking, reading, writing, and spelling.
- **dysnomia**—A marked difficulty in remembering names or recalling words needed for oral or written language.
- **dyspraxia**—A severe difficulty in performing drawing, writing, buttoning, and other tasks requiring fine motor skills or in sequencing the necessary movements.

learning modalities—The means through which information is perceived, such as visual, auditory, or tactile/kinesthetic means.

learning stations or centers—Different spots in the classroom where learners work on various tasks simultaneously. They invite flexible grouping because not all learners need to go to *all* the stations *all* the time, and not all learners spend the same amount of time at each station. Stations work in concert with one another, and there are usually several stations related to the same subject. (*See also* *interest centers*.)

learning strategy—A set of specific actions, behaviors, steps, or techniques used by learners to accomplish a particular task, such as taking a test, comprehending text, and writing a story, and improve learning (e.g., using a graphic organizer and using context clues). A first-letter mnemonic is often used to help learners follow the steps of the strategy. Also, it is how a person approaches learning, including how that person thinks and acts before, during, and after a task, and how a person evaluates the impact of the strategy on learning and performance.

learning strategy approaches—Instructional approaches that focus on efficient ways to learn, rather than curriculum. They include specific techniques for organizing, actively interacting with material, memorizing, and monitoring any content or subject.

lesson planning—A written plan that notes the method of delivery, the activities, and the specific goals and timelines associated with the delivery of lesson content. It is like a roadmap that helps a teacher know what to do in a class, including the sequence of activities, to help learners achieve the stated learning goals.

linear function—A function in which the graph of the solutions forms a line.

locus of control—The tendency to attribute success and difficulties either to internal factors, such as effort, or external factors, such as chance. Individuals with learning disabilities tend to blame failure on themselves and achievement on luck, which leads to frustration and passivity.

logical reasoning—A catchall term for inductive and deductive thinking that is frequently used as a support for an argument or a thesis. It is frequently used by writers to point out flaws (logical fallacies) in an opposing point of view.

M

mastery learning—An instructional method that presumes all learners can learn if they are provided with the appropriate learning conditions. Specifically, mastery learning is a method whereby learners are not advanced to a subsequent learning objective until they demonstrate proficiency with the current one. Objectives for learning are established and communicated to learners. Learners progress at their own speed and continue to work until their performance indicates they have mastered each set of objectives.

mean—The sum of the values in a data set divided by the number of values in that set.

median—The central number in a set of numerical data when they are arranged in order. If the data set has an even number of values, then the median is the mean of the two middle numbers.

mental models—Learners enter learning situations with existing knowledge. This knowledge is organized into patterns or models that help them explain phenomena. Learning involves adding to or altering a learner's existing mental models.

metacognition—Refers to higher-order thinking that involves active control over the cognitive processes engaged in learning: knowledge about one's own information processing and strategies that influence one's learning. By prompting learners to reflect on and identify the successful learning strategies that they used to solve a problem, teachers encourage learners to act on this awareness to choose appropriate learning strategies that optimize future learning. Successful learners monitor their own thought processes to decide whether they are learning effectively. Metacognitive activities include planning how to approach a given learning task, monitoring comprehension, and evaluating progress toward the completion of a task.

- **metacognitive learning**—An instructional approach that emphasizes awareness of the cognitive processes that facilitate one's own learning and its application to academic and work assignments. Typical metacognitive techniques include the systematic rehearsal of steps or the conscious selection among strategies for completing a task.

metaphorical thinking—Thought in which unlike objects are compared to one another, frequently for an aesthetic effect, such as using figurative language to express a point of view.

mind map—*See graphic organizers*.

mixed number—A value that includes a whole number and a fractional part.

mnemonic/mnemonic device—Pertaining to memory. It is a device for remembering information. Association techniques are used to remember specific information by linking the information to a word or a phrase, such as using the first-letter mnemonic HOMES to remember the names of the Great Lakes: Huron, Ontario, Michigan, Erie, and Superior or the first-letter mnemonic PLAN for writing: pay attention to the prompt, list main ideas, add supporting ideas, and number your ideas.

mode—The number seen most often in a list of values. It is possible that there will be no mode, one mode, or more than one mode in a data set.

modeling—Showing learners how to accomplish a task or use a strategy by demonstrating it explicitly while using think-alouds. (*See also think-aloud*.)

multicultural awareness—An understanding, sensitivity, and appreciation of the history, values, experiences, and lifestyles of groups that include but are not limited to race, ethnicity, gender, sexual orientation, religious affiliation, socioeconomic status, and mental/physical abilities.

multisensory learning—An instructional approach that combines auditory, visual, and tactile elements into a learning task (e.g., moving one's finger under each syllable of a word as the word is read and sounded out or tracing sandpaper numbers while saying a number fact aloud).

N

National Reporting System (NRS)—The National Reporting System for Adult Education is an outcome-based reporting system for the state-administered, federally funded adult education program. Developed by the U.S. Department of Education's Division of Adult Education and Literacy (DAEL), the NRS continues a cooperative process through which state adult education directors and DAEL manage a reporting system that demonstrates learner outcomes for adult education. The project is conducted by the American Institutes for Research (AIR) in Washington, DC.

numeracy—Mathematical competence or skills required to perform quantitative tasks.

numerator—The top number in a fraction.

O

Online Lesson Planner—The TEAL Lesson Planner is a tool to help adult educators design quality lesson plans in any content area. The Lesson Planner provides a template to help users develop lesson plans for adult education classes. It allows users to "backward" design lessons so that desired learner outcomes provide the foundation for the lesson design, stores lessons for future use, and allows sharing lessons with others. (*See also backward design*.)

organization—Clear evidence of a plan or a foundation on which writing is built. It includes an intentional introduction, internal/external transitions to connect ideas, and a conclusion.

ordered pair—A pair of numbers that provides the location of a point on a grid.

P

peer tutoring—Having learners work in pairs, with one learner tutoring the other on a particular concept.

perception—A process involving the reception, selection, differentiation, and integration of sensory stimuli. The teacher of learners with dyslexia must teach such learners to attend actively and consciously to aspects of the perception process until it becomes automatic.

performance standards—*See standards*.

plus-minus-interesting (PMI) chart—A device developed by Edward DeBono in which learners summarize their findings about a particular topic or idea by listing what is good about it, what is possibly negative about it, and what is interesting about it.

portfolio—A collection of learner work gathered to exhibit or demonstrate a learner's efforts, progress, or achievement in one or more areas.

- **portfolio assessment**—A portfolio becomes a portfolio assessment when (1) the assessment purpose is defined; (2) criteria or methods are made clear for determining what is put into the portfolio, by whom, and when; and (3) criteria for assessing either the collection or individual pieces of work are identified and used to make judgments about performance. Portfolios can be designed to assess learner progress, effort, and/or achievement and encourage learners to reflect on their learning. (*See also assessment*.)

prime number—An integer that has no integral factors but itself and 1.

prior knowledge—*See background knowledge*.

probability—A number between 0 and 1 used to express the likelihood of results for processes that have uncertain outcomes, such as winning the lottery.

problem-based learning—An approach to learning that places learners in the active role of solving problems in much the same way that adult professionals perform their jobs.

product—The sum of any two or more numbers that are multiplied together.

proficiency level—Portrays what learners at a particular level know and can do in relation to what is being measured. Proficiency levels are not to be confused with a program's class design levels. Programs should use proficiency levels, though, to closely crosswalk with their program class design levels.

progress monitoring—As a type of formative assessment, it is a scientifically based practice used to assess learners' academic performance and evaluate the effectiveness of instruction. It is a set of assessment procedures for determining the extent to which learners are benefiting from classroom instruction and for monitoring the effectiveness of curriculum. Progress monitoring can be implemented either with individual learners or an entire class. (*See also* *assessment*.)

proper fraction—A fraction with a numerator smaller than the denominator.

proportion—An equation stating that two ratios are equivalent.

Q

quantitative—something that is expressed as a quantity or can be measured.

questioning strategies—Effective teaching uses questioning strategies that assist learners in the development of thinking skills and increase comprehension. Teachers pose questions to learners to elicit deeper-level thinking about the subject under discussion.

- **adjusting questions**—A strategy for differentiating instruction in which a teacher adjusts questions posed to learners based on their readiness, interests, and learning profile. This strategy is an excellent strategy for teachers new to differentiated instruction because it builds on strengths and abilities readily used by most teachers.
- **critical questions**—Questions that an instructor should pose that will lead to discourse on learning and help the learner identify goals.

questions—There are two general types of questions: closed-ended, such as yes/no questions, and informational (often open-ended) questions. Informational questions begin with who, whom, what, where, when, why, how, and which. For example, “Do you like this class?” is a closed-ended question that requires a yes/no answer; “What do you like about this class?” is an open-ended question that requires learners to provide information.

quotient—The result of a division problem.

R

ratio—The proportional relationship between two numbers or quantities.

rational number—A number that is derived by dividing one integer by another, representing a ratio. Example: 1.5 is rational because it can be represented by 3/2.

real numbers—The set of numbers that includes rational and irrational numbers.

reciprocal teaching—Learners take turns being the teacher for a pair or a small group. The teacher role may be to clarify, ask questions, ask for predictions, and other related tasks.

reflection—A metacognitive activity. A learner pauses to think about and organize information gathered from reading, discussions, or other activities. It is an exploration of an idea, a text, a topic, a writing process, or one’s general state of mind that is frequently written in a journal or in freewriting.

response cards—A teacher asks a question, student learners write brief answers on the cards, and then all students learners hold up their cards. The teacher can scan the answers of all students learners for understanding. Sometimes cards just have “yes” or “no” on them and can also be prepared by the teacher.

response to intervention—A method of academic intervention designed to provide early, effective assistance to school-age children who are having difficulty learning. It includes frequent progress measurement and increasingly intensive research-based instructional interventions for children who continue to have difficulty.

rote memory—This type of memorization is usually the most commonly required memory task for students in Grades K–12. It involves memorizing, and soon forgetting, facts that often are of little primary interest or emotional value to a student, such as a list of words.

Having nothing to give these words context or relationship either to each other or to students' lives, these facts are stored in remoter areas of the brain. These isolated bits of information are more difficult to locate and retrieve later because there are fewer nerve pathways leading to these remote storage systems in the brain.

S

scaffolding/scaffolded instruction—A process that involves the frequent use of connected questions and collaboratively constructed explanations to create a context for learning based on a learner's prior knowledge. Broad terms refer to various methods of supporting learners as they learn; gradually, these supports are withdrawn as they become capable of independent performance of a task or a skill. Supports may include clues, clarifying questions, reminders, encouragement, or breaking the problem down into steps. This temporary support from a teacher enables learners to take on and understand new material and tasks that they are not quite ready to do independently. The teacher models, assists, or provides necessary information, building on what learners already know; this should eventually lead to independence.

scatter plot—Bivariate data graphed on a coordinate plane.

screening—A process of collecting information through a variety of sources over time that would lead to the conclusion that an individual might be significantly at risk for a specific condition, such as a learning disability.

screening instrument—An initial test in a sequence of tests that is usually quickly administered. The results are used to determine whether further testing is necessary and may possibly guide the selection of other tests to be administered.

self-advocacy—The ability of individuals with learning disabilities to explain their disabilities effectively to others, request legal accommodations, act independently, and cope positively with the attitudes of others.

self-assessment—Learners reflect on their performance and assess themselves.

self-directed learning (SDL)—Learners take the initiative and the responsibility for selecting, managing, and assessing their own learning activities, which can be pursued at any time, in any place, through any means, and at any age. SDL involves initiating personal challenge activities and developing the personal qualities to pursue them successfully. Adult education teachers can emphasize skills, processes, and systems to help learners become self-directed.

self-monitoring strategies—Plans used to increase independence in academic, behavioral, self-help, and social areas. When used in reading, the ability to self-monitor the meaning of words enables learners to select and use strategies to improve comprehension.

self-regulation—The understanding learners have about how they learn, including the strategies used to accomplish tasks, and the process by which they oversee and monitor their use of strategies and make adjustments, as needed.

short-term memory—*See working memory*.

small-group discussion—The discussion of an assignment, an assigned text, or learner writing by groups of usually two to six learners. The groups then usually report back to the instructor and the entire class in a full class discussion.

social development theory—A theory based on Lev Vygotsky's philosophy that learning occurs through social interactions. It emphasizes the importance of cooperative learning groups, motivation, the observation of models, and learner attitudes.

specific learning disability (SLD)—The official term used in federal legislation to refer to difficulty in certain areas of learning, rather than in all areas of learning. (*See also learning disabilities*.)

standardization—A consistent set of procedures for designing, administering, and scoring an assessment. The purpose of standardization is to assure that all learners are assessed under the same conditions so that their scores have the same meaning and are not influenced by differing conditions. Standardized procedures are very important when scores will be used to compare individuals or groups.

standardized test—Any test that is administered and scored in a consistent, or standard, manner to all test takers. These tests are designed so that the questions, the conditions for administering, scoring procedures, and interpretations are consistent and administered and scored in a predetermined, standard manner.

standards—Expectations that describe what learners should know and be able to do within a specific content area. It is the broadest of a family of terms referring to statements of expectations for learning, including benchmarks, content standards, and performance standards.

- **benchmarks**—Descriptions of the set of skills learners need to develop and achieve to meet the more broadly stated content standards.
- **content standards**—Broadly stated expectations of what learners should know and be able to do in particular subjects and grade levels. They define the knowledge, skills, processes, and other understandings that guide curriculum for learners to attain high levels of competency in various subjects, and they reflect what stakeholders of educational systems recognize as essential to be taught and learned.
- **performance standards**—Explicit definitions of what learners must do to demonstrate proficiency at a specific level on the content standards. For example, the performance level "exceptional achievement" on a dimension "communication of ideas" is reached when the learner examines the problem from several different positions and provides adequate evidence to support each position.

standards-based reform—A program of school improvement involving setting high standards for all learners and a process for adapting instruction and assessment to make sure all learners can achieve the standards.

strategy instruction—Teaching learners tools for learning how and when to use strategies. The focus is on teaching learners how to learn effectively by applying principles, rules, or multistep processes to solve problems or accomplish learning tasks. Included in strategy instruction is the process of helping learners identify personally effective strategies and encouraging them to make strategic behaviors part of their learning schema.

student-centered learning—An instructional approach that focuses on the needs of students rather than the needs of teachers or administrators. Students might not only choose what to study but also how and why that topic might be an interesting one to study. This approach centers on student responsibility and activity, in contrast to more conventional teaching approaches in which the emphasis is on teacher control and the coverage of academic content.

students with disabilities (SWD)—A broadly defined group of students with physical and/or mental impairments, such as blindness or learning disabilities, that might make it more difficult for them to do well on assessments without accommodations or adaptations. (*See also accommodations* and *adaptations*.)

summative assessment—*See assessment*.

T

think-aloud—A metacognitive strategy in which a teacher models his or her thinking, describing thoughts while reading aloud to the class or completing a task. By demonstrating metacognitive thought, a teacher explicitly gives learners a model of how the teacher's thinking proceeded. Examples where think-alouds are helpful include demonstrating the steps in solving a math problem, reading a story aloud and stopping at points to think aloud about reading strategies, or crafting a response to an essay prompt.

think-pair-share—Learners think individually, pair (discuss with partner), and then share ideas with the class.

tic-tac-toe extension menu or choice board—A collection of activities from which a learner can choose. It is generally presented in the form of a 3-by-3 or a 5-by-5 grid, similar to a tic-tac-toe board, with the center square often allowing for learner choice. This format can be applied to extension activities, contracts, study guides, or independent studies. Such boards allow a teacher to differentiate content, process, and product according to different levels of learner performance and readiness, interests, and learning styles.

tiered assignments—Parallel tasks at varied levels of complexity, depth, and abstractness with various degrees of scaffolding, support, or direction. Learners work on different levels of activities, all with the same essential understanding or goal in mind. Tiered assignments accommodate mainly for differences in learner readiness and performance levels and allow learners to work toward a goal or objective at a level that builds on their prior knowledge and encourages continued growth.

transition—A device commonly used to refer to the change from secondary school to postsecondary programs, work, and independent living typical of young adults. It is also used to describe other periods of major change, such as from early childhood to school or from more specialized to mainstreamed settings.

U

universal design for learning (UDL)—A framework for designing the educational environment so that it offers flexible learning environments that can accommodate individual learning differences. It is a key to helping *all* learners achieve. This environment is accomplished by simultaneously reducing or removing barriers from teaching methods and curriculum and providing rich supports for learning.

V

variable—A symbol that represents a value.

Venn diagram—*See graphic organizers*.

visual discrimination—Assuming normal visual acuity, the ability to distinguish slight differences in stimuli, especially in letters and words, that have graphic similarities.

visual perception—The ability to recognize visual stimuli. Individuals with this learning disability may have problems with such activities as reading, writing, tracking, recognizing people or items, or reading a map or a graphic display.

W

whole number—A number that does not include a fraction or decimal and is not negative.

working folder—A collection of a learner's work in which a learner can see evidence of growth in writing. It should include some dated samples that address a variety of writing tasks and allow learners and teachers to use past writing experiences as teaching tools for current and projected instruction. Most often, this folder contains all the drafts of a piece of writing. On a regular basis, learners should review and reflect on what has been placed in the folder to make decisions about what to keep for further development. The pieces in the working folder are springboards for the generation of possible portfolio entries.

working memory (short-term memory)—This memory can hold and manipulate information for use in the immediate future. Information is held in working memory for only about a minute. For example, looking up and repeating aloud a phone number long enough to place a call is using short-term memory; to place the number into long-term memory, one would need to make some association between the number and the person being called.

Z

zone of proximal development (ZPD)—Lev Vygotsky's "zone of readiness," including the actions or topics a learner is ready to learn. It refers to the gap between a learner's current and potential levels of development. This is the set of knowledge that the learner does not yet understand but has the ability to learn with guidance.

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