

**MATH 151, FALL SEMESTER 2014
COMMON EXAMINATION I - VERSION A**

Last name, First name (print): _____

Signature: _____

Instructor's name: _____

Section No: _____

GUIDELINES

- In Part 1 (Problems 1–13), mark your responses on your ScanTron form using a No. 2 pencil. *For your own record, mark your choices on the examination paper as well.* ScanTrons will be collected at the conclusion of the examination; they will *not* be returned.
- In Part 2 (Problems 14–17), present your solutions **in the space provided**. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.
- Be sure to **write your name, section number, and version letter of the examination on the ScanTron form**.
- Calculators **should not be used** throughout the examination. Mobile phones **must be switched off**. Examinees found using a mobile phone during the course of the test are subject to being ejected from the examination hall, and to being given a zero on the examination.

PART 1 (52 points)

Each question is worth **4 points**. Mark your responses on the ScanTron form and on the examination paper itself.

1. If $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} - \mathbf{j}$, what is $2\mathbf{a} - 3\mathbf{b}$?
 - (a) $-5\mathbf{i} + 9\mathbf{j}$
 - (b) $5\mathbf{i} - 9\mathbf{j}$
 - (c) $-2\mathbf{i} + 9\mathbf{j}$
 - (d) $-5\mathbf{i} - 3\mathbf{j}$
 - (e) $-2\mathbf{i} + 3\mathbf{j}$
2. Which of the following is a unit vector parallel to the vector joining the points $(1, 3)$ and $(4, -1)$?
 - (a) $\langle 3/5, 4/5 \rangle$
 - (b) $\langle 3, -4 \rangle$
 - (c) $\langle -3/5, 4/5 \rangle$
 - (d) $\langle -4/5, 3/5 \rangle$
 - (e) $\langle 4/5, -3/5 \rangle$
3. Compute the scalar projection of the vector $\mathbf{b} = 2\mathbf{i} + \mathbf{j}$ onto the vector $\mathbf{a} = \mathbf{i} - \mathbf{j}$.
 - (a) $\frac{1}{2}$
 - (b) $\frac{\sqrt{2}}{2}$
 - (c) $\frac{\sqrt{5}}{5}$
 - (d) $\frac{1}{5}$
 - (e) $\frac{3\sqrt{2}}{2}$

4. Obtain a vector equation of the straight line which passes through the point $(1, 1)$, and is parallel to the line given by the Cartesian equation $2x - y + 3 = 0$.

(a) $\mathbf{r}(t) = \langle 1, 2 \rangle + t\langle 1, 1 \rangle$, for $-\infty < t < \infty$.

(b) $\mathbf{r}(t) = \langle 1, 1 \rangle + t\langle 2, 1 \rangle$, for $-\infty < t < \infty$.

(c) $\mathbf{r}(t) = \langle 1, 2 \rangle + t\langle 1, 2 \rangle$, for $-\infty < t < \infty$.

(d) $\mathbf{r}(t) = \langle 1, 1 \rangle + t\langle 1, 2 \rangle$, for $-\infty < t < \infty$.

(e) $\mathbf{r}(t) = \langle 1, 1 \rangle + t\langle 2, -1 \rangle$, for $-\infty < t < \infty$.

5. The graph of the curve determined by the parametric equations

$$x(t) = t^2, \quad y(t) = 2t + 1, \quad \text{for } -\infty < t < \infty,$$

is

(a) a straight line

(b) a circle

(c) a noncircular ellipse

(d) a parabola

(e) none of the above

6. Which of the following is the correct parametrization of the circle determined by the Cartesian equation $x^2 - 2x + y^2 = 3$? (Suggestion: Begin by completing a square.)

(a) $x(t) = 1 + \cos t$, $y(t) = 2 \sin t$

(b) $x(t) = 1 + \sqrt{3} \cos t$, $y(t) = \sqrt{3} \sin t$

(c) $x(t) = 1 + 2 \cos t$, $y(t) = 2 \sin t$

(d) $x(t) = \cos(2t)$, $y(t) = \sin(2t)$

(e) $x(t) = 1 + \cos(2t)$, $y(t) = \sin(2t)$

7. Compute $\lim_{x \rightarrow -1} \frac{2x^2 - x + 1}{x^3 - x^2 + x - 1}$, provided it exists.
- (a) $\frac{1}{4}$
 - (b) 0
 - (c) -1
 - (d) $-\frac{1}{4}$
 - (e) does not exist
8. Calculate $\lim_{x \rightarrow -1^+} \frac{1+x}{|x^2-1|}$, provided it exists.
- (a) $-\frac{1}{2}$
 - (b) $\frac{1}{2}$
 - (c) 0
 - (d) 1
 - (e) does not exist
9. Compute $\lim_{t \rightarrow -\infty} \frac{3t-2}{\sqrt{4t^2+t+1}}$.
- (a) $\frac{3}{4}$
 - (b) $\frac{3}{2}$
 - (c) $-\frac{3}{4}$
 - (d) $-\frac{3}{2}$
 - (e) 0
10. Calculate $\lim_{x \rightarrow 2^-} \frac{1}{x^2 - 5x + 6}$.
- (a) $-\infty$
 - (b) $+\infty$
 - (c) 0
 - (d) 1
 - (e) $\frac{1}{20}$

11. Let f be a function defined in some open interval containing the real number c . Which of the following statements are true? (i) $\lim_{x \rightarrow c} f(x)$ does not exist if $\lim_{x \rightarrow c^+} f(x)$ does not exist, or if $\lim_{x \rightarrow c^-} f(x)$ does not exist. (ii) If $\lim_{x \rightarrow c} f(x)$ does not exist, then $\lim_{x \rightarrow c^+} f(x)$ does not exist, or $\lim_{x \rightarrow c^-} f(x)$ does not exist. (iii) If $\lim_{x \rightarrow c} f(x)$ does not exist, then f is discontinuous at c . (iv) If f is discontinuous at c , then $\lim_{x \rightarrow c} f(x)$ does not exist.
- (a) (ii) and (iv) only
- (b) (i) and (ii) only
- (c) (i) and (iv) only
- (d) (i) and (iii) only
- (e) all four statements are true

12. Consider the function f given by

$$f(x) = \frac{(x-1)^2(x^2+x+1)}{x^3-1}, \quad \text{for } x \neq 1.$$

Which, if any, of the following statements is/are true? (i) The graph of f has a vertical asymptote at $x = 1$. (ii) The line $y = 1$ is a horizontal asymptote to the graph of f . (iii) The graph of f has no vertical asymptotes. (iv) The graph of f has no horizontal asymptotes. (v) The x -axis is a horizontal asymptote to the graph of f .

- (a) (iii) and (iv) only
- (b) (iii) only
- (c) (iv) only
- (d) none of the five statements is true
- (e) (i) only
13. Suppose that the function f is differentiable at 3, and that $\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x^2 - x - 6} = \frac{1}{4}$. Determine the value of $f'(3)$.
- (a) $\frac{1}{4}$
- (b) $\frac{4}{5}$
- (c) 4
- (d) insufficient information to make a determination
- (e) $\frac{5}{4}$

PART 2 (48 points)

Present your solutions to the following problems (14–17) in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

14. Let A denote the point with co-ordinates $(0, 0)$. Let $ABCD$ be the parallelogram whose adjacent sides AB and AD are determined by $\overrightarrow{AB} = \langle 4, 1 \rangle$ and $\overrightarrow{AD} = \langle 1, 2 \rangle$, respectively.
- (i) (4 points) Calculate the magnitude of the diagonal AC .

(ii) (6 points) Compute the cosine of the angle $\angle ADB$.

(iii) (4 points) Use (ii) to ascertain whether $\angle ADB$ is acute or obtuse. Explain your reasoning clearly but concisely.

15. Let L denote the straight line given by the following parametric equations:

$$x(t) = -2 + t, \quad y(t) = -3 + 2t, \quad \text{for } -\infty < t < \infty.$$

- (i) (6 points) Determine the values of t corresponding to the points of intersection of L and the parabola given by the equation $y = 4 - x^2$.

- (ii) (4 points) Use the findings from (i) to determine the co-ordinates of the points of intersection.

16. Let a be a fixed real number, and let f be the function defined as follows:

$$f(x) = \begin{cases} 2x + a, & \text{if } x < 1; \\ -4, & \text{if } x = 1; \\ a^2x + 2ax - 4, & \text{if } x > 1. \end{cases}$$

(i) (4 points) Compute $\lim_{x \rightarrow 1^-} f(x)$ and $\lim_{x \rightarrow 1^+} f(x)$.

(ii) (6 points) Determine the values of a for which $\lim_{x \rightarrow 1} f(x)$ exists.

(iii) (4 points) For which values of a from (ii) is f continuous at 1? Explain your reasoning briefly and clearly.

16. (10 points) Consider the function f defined below:

$$f(x) = \frac{1}{\sqrt{x-1}}, \quad \text{for } x > 1.$$

Use the **definition of the derivative** to compute $f'(2)$. (Note: No credit will be given for using any other method, correct answer notwithstanding.)

LAST NAME (print): _____

FIRST NAME (print): _____

<u>QN</u>	<u>PTS</u>
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<u>1-13</u>	_____
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<u>14</u>	_____
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<u>15</u>	_____
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<u>16</u>	_____
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<u>17</u>	_____
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TOTAL	
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