MATHCOUNTS®

2005
■ State Competition ■
Sprint Round
Problems 1–30

Name _____

School _____

Chapter _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round of the competition consists of 30 problems. You will have 40 minutes to complete the problems. You are not allowed to use calculators, books or any other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

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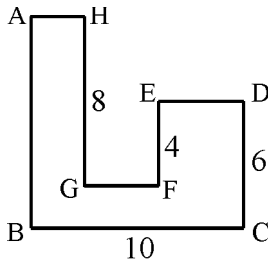
1. In octagon ABCDEFGH, every side is perpendicular to each of its adjacent sides. What is the perimeter of ABCDEFGH?



1. _____ units

2. The difference between two prime numbers is 17. What is their sum?

2. _____

3. What is the sum of the two perfect square integers that are closest to 2005?

3. _____

4. In February 2005, the Smiths watched TV for three hours every Monday, Wednesday and Friday; they watched TV for four hours every Tuesday and Thursday; and they never watched TV on weekends. How many hours of TV did they watch that month?

4. _____ hours

5. The chart below gives the air distance in miles between selected world cities. If two different cities from the chart are chosen at random, what is the probability that the distance between them is less than 7000 miles? Express your answer as a common fraction.

5. _____

	Bangkok	Cape Town	Honolulu	London
Bangkok		6300	6609	5944
Cape Town	6300		11,535	5989
Honolulu	6609	11,535		7240
London	5944	5989	7240	

6. When a water tank is 30% full, it contains 27 gallons less than when it is 20% empty. How many gallons of water does the tank hold when it is full?

6. _____ gallons

7. Each of the letters A, B, C and D represents a different odd integer between two and ten. What is the least possible value of $\frac{A \times B - C}{D}$? Express your answer as a common fraction.

7. _____

8. At a mall's food court, Crystal has $7.50 to buy a meal (one entrée, one drink and one dessert). The table below lists Crystal's choices and their prices including sales tax. How many distinct possible meals can she afford to buy?

8. _____ meals

Entrées		**Drinks**		**Desserts**	
Pizza	$3.50	Lemonade	$1.50	Frozen Yogurt	$3.00
Corn Dog	$2.50	Soda	$1.25	Cookies	$2.00
Fish & Chips	$3.50				
Fried Rice	$4.75				

9. The points (x, y) represented in this table lie on a straight line. When the equation of this line is written in the form $y = Ax + B$, what is the value of A + B?

x	y
2	7
$t-2$	v
t	$v+6$

9. _____

10. When Roslyn writes her name repeatedly in a 5 by 5 table as shown, the bottom right corner contains the letter **R**. If she repeats the process in a 20 by 20 table, what letter would occupy the bottom right corner?

R	O	S	L	Y
N	R	O	S	L
Y	N	R	O	S
L	Y	N	R	O
S	L	Y	N	**R**

10. _____

11. The number 210 is the product of two consecutive positive integers and is also the product of three consecutive integers. What is the sum of those five integers?

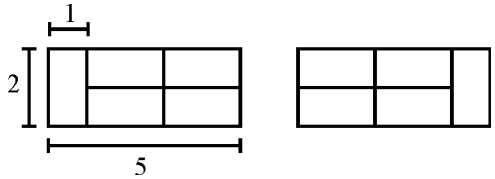
11. _____

12. A 12"-diameter pizza and a 16"-diameter pizza are each cut into eight congruent slices. Jane ate three slices of the 12" pizza. Mark ate three slices of the 16" pizza. How many more square inches of pizza than Jane did Mark eat? Express your answer as a common fraction in terms of π.

12. _____ sq inches

13. In how many patterns can a 2-by-5 rectangle be tiled with five white rectangular 2-by-1 tiles? The two examples shown below are considered to be different tiling patterns.

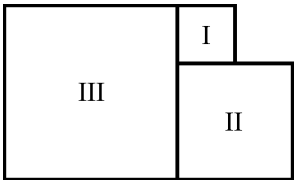
13. _____ patterns



14. For each plumbing repair job, Mr. Wrench charges N dollars for coming out to the house plus x dollars per hour that he works at the house. He charged \$97 for a one-hour repair job and \$265 for a five-hour repair job. What is his charge for a two-hour repair job?

14. \$ _____

15. Regions I, II and III are bounded by squares. The perimeter of region I is 12 units and the perimeter of region II is 24 units. What is the ratio of the area of region I to the area of region III? Express your answer as a common fraction.



15. _____

16. What is the positive difference between the 2000^{th} term and the 2005^{th} term of the arithmetic sequence $-8, -2, 4, 10, \ldots$?

16. _____

17. The ratio of teachers to students in a particular school is 1 to 11. The ratio of female students to the total number of students is 4 to 9. If there are 396 female students, how many teachers are there?

17. _____ teachers

18. A juice company sells its product in either a 48-ounce size or a 32-ounce size. It charges \$3.90 for the 48-ounce size. How much should it charge for the smaller size if it wants the price per ounce to be 25% more than the price per ounce of the larger size?

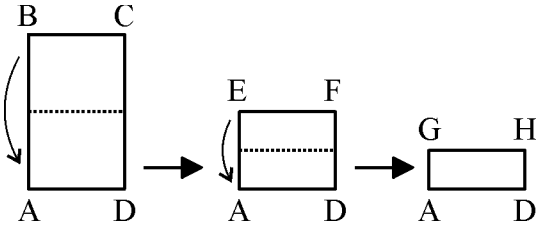
18. \$ _____

19. A stack of 45 dimes is divided into three piles in the ratio $\frac{1}{6} : \frac{1}{3} : \frac{1}{4}$. How many dimes are in the pile with the least number of dimes?

19. _____ dimes

20. Rectangle ABCD is folded in half to form rectangle AEFD, which is then folded in half to form rectangle AGHD. The perimeter of rectangle AGHD is 23 cm and the perimeter of ABCD is 47 cm. What is the area of rectangle ABCD?

20. _____ sq cm



21. How many positive three-digit integers with each digit greater than 4 are divisible by 6?

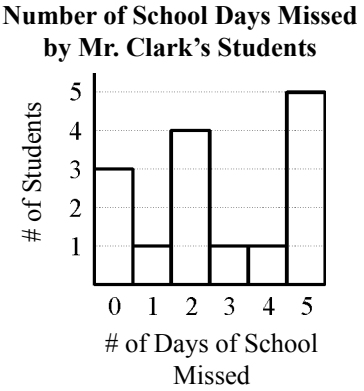
21. _____ integers

22. The sum of four positive integers that form an arithmetic sequence is 46. Of all such possible sequences, what is the greatest possible third term?

22. _____

23. For the data whose frequency histogram is shown, by how many days is the mean number of days missed per student greater than the median number of days missed per student for the 15 students? Express your answer as a common fraction.

23. _____ days



24. There are only red marbles and green marbles in a bag. The ratio of red marbles to green marbles in the bag is 4:7. Julia then adds 90 red marbles and 36 green marbles to the bag, which makes the probability of selecting a red marble from the bag on a random draw equal to $\frac{1}{2}$. How many total marbles are in the bag after Julia has added the 126 marbles?

24. _____ marbles

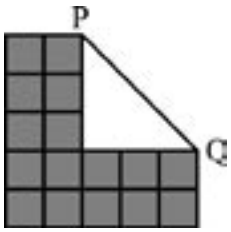
25. What is the area of the region bounded by the three lines with equations $2x + y = 8$, $2x - 5y = 20$ and $x + y = 10$?

25. _____ sq units

26. Yesterday, 28 students took a test. The arithmetic mean of those 28 scores was 72 points. Two students who were absent yesterday took the test this morning, and the arithmetic mean of all 30 test scores is 73 points. If the difference of the two scores from this morning is 22 points, what is the lower score from this morning?

26. _____ points

27. The shaded region consists of 16 congruent squares. If PQ = 6 cm, what is the area of the entire shaded region?

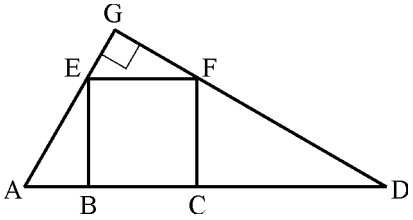


27. _____ sq cm

28. The Fibonacci sequence is the sequence 1, 1, 2, 3, 5, ... where each term is the sum of the previous two terms. What is the remainder when the 100th term of the sequence is divided by 4?

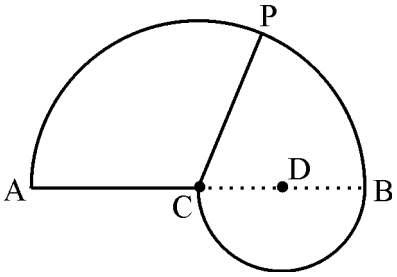
28. _____

29. Square BCFE is inscribed in right triangle AGD, as shown below. If AB = 28 units and CD = 58 units, what is the area of square BCFE?



29. _____ sq units

30. Segment AB has midpoint C, and segment BC has midpoint D. Semi-circles are constructed with diameters $\overline{AB}$ and $\overline{BC}$ to form the entire region shown. Segment CP splits the region into two sections of equal area. What is the degree measure of angle ACP? Express your answer as a decimal to the nearest tenth.



30. _____ degrees