2011
MATHCOUNTS STATE COMPETITION

SPRINT ROUND

1. 12 boy scouts are accompanied by 3 scout leaders. Each person needs 3 bottles of water per day and the trip is 1 day.
   12 + 3 = 15 people
   15 × 3 = 45 bottles **Ans.**

2. Cammie has pennies, nickels, dimes and quarters and we are asked to find the least number of coins that she can use to make 93 cents.
   Let's start with quarters. 3 quarters equals 75¢. 93 – 75 = 18¢.
   We can use 1 dime. 18 – 10 = 8
   We can use 1 nickel. 8 – 5 = 3
   And we can use 3 pennies.
   3 + 1 + 1 + 3 = 8 **Ans.**

3. We have 3 non-overlapping circles. One circle has a diameter of 8 inches. The other two circles have diameters of 6 and 2 inches, respectively. We are asked to find how much larger the area of the first circle is than the total area of the other 2 circles.
   If the diameter of the first circle is 8, then its radius is 4. Its area is 16π.
   The radii of the other two circles are 3 and 1, respectively. Their areas are 9π and π, respectively.
   $16\pi - (9\pi + \pi) = 6\pi$ **Ans.**

4. Mike had 7 hits in 20 turns at bat in 5 baseball games. In the sixth game he had 5 hits in 5 turns at bat. We are asked to find the percent of Mike's turns at bat that resulted in hits.
   The total number of turns at bat is
   20 + 5 = 25 turns
   The total number of hits is
   7 + 5 = 12 hits
   $\frac{12}{25} = 48\%$ **Ans.**

5. The first term of an arithmetic sequence is –37 and the second is –30. We are asked to find the smallest positive term of the sequence.
   There is a difference of 7 between the two terms. If we look at the next 4 terms (–23, –16, –9, –2), we are adding 28 to the second term to get –2 as the sixth term.
   –2 + 7 = 5 **Ans.**

6. Fraser fir tree saplings are 8 inches tall. Blue spruce tree saplings are 5 inches tall. Fraser firs grow at a constant rate of 12 inches per year and spruce trees at a constant rate of 14 inches per year. We are asked to find after how many years these trees will be the same height.
   Let n = the number of years. Then,
   8 + 12n = 5 + 14n
   3 = 2n
   n = $\frac{3}{2}$ **Ans.**

7. From the pie chart, $\frac{1}{3}$ of my \$30 allowance was spent on movies, $\frac{3}{10}$ on music, $\frac{1}{5}$ on ice cream and the rest on burgers.
   $\frac{1}{3} + \frac{3}{10} + \frac{1}{5} = \frac{10+9+6}{30} = \frac{25}{30}$
   or \$25 out of \$30 were spent on items other than burgers.
   30 – 25 = 5 **Ans.**

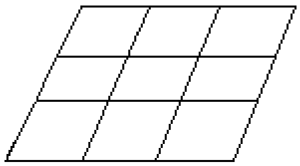
8. M and N are both perfect squares less than 100. If M – N = 27, then what is the value of $\sqrt{M} + \sqrt{N}$?
   Let's list all the squares less than 100.
   1, 4, 9, 16, 25, 36, 49, 64, 81, 100.
   If we add 27 to each square we get 28, 31, 36, 43, 52, 63, 76, 91, 108 and 127.
   36 is the only square. Therefore,
   M = 9 and N = 36
   $\sqrt{M} + \sqrt{N} = \sqrt{9} + \sqrt{36} = 3 + 6 = 9$ **Ans.**

9. John made two 120-mile trips. The second trip took him one hour less than the first trip. Total time for both trips was 9 hours. We are asked to find his average rate for the second trip.
   If the second trip took him one hour less

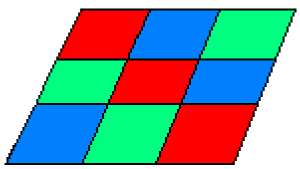
than the first and the total number of hours is 9, then the first trip was 5 hours long and the second was 4 hours long.

$\frac{120}{4} = 30$ **Ans.**

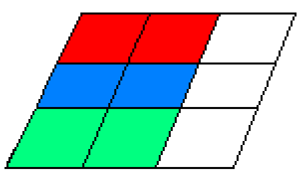
10. We are asked to find the number of parallelograms in the diagram.

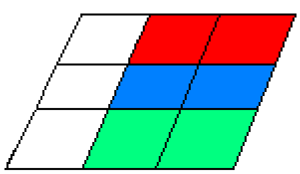


First the diagram itself contains 1 parallelogram. Each of the smallest items outlined in the parallelogram is itself a parallelogram.

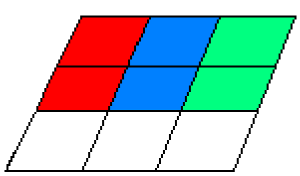


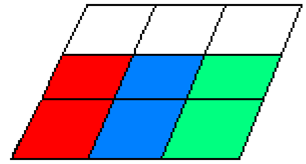
There are 9 of those for a total of 10 so far.
There are also 6 "horizontal" parallelograms that contain 2 small parallelograms.



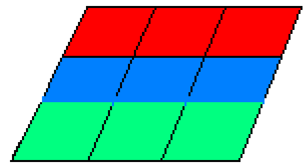


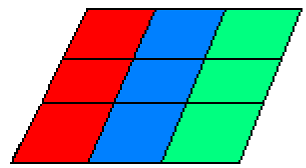
And there are 6 "vertical" parallelograms that contains 2 small parallelograms.



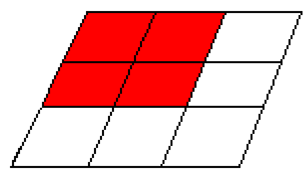


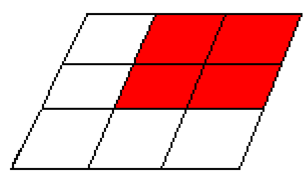
10 + 12 = 22 so far.
We can also make parallelograms from 3 small parallelograms. There are 3 "horizontal" ones and 3 "vertical" ones.

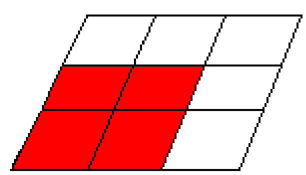


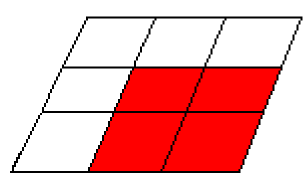


22 + 6 = 28
We can also make parallelograms from 4 parallelograms (2 adjacent and 2 directly below). There are 4 of these.

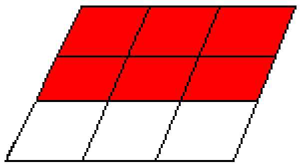


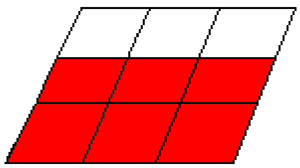


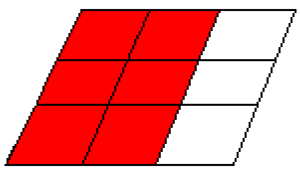


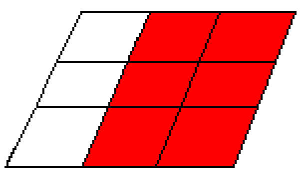


28 + 4 = 32
We can also make 2 “horizontal” parallelograms using 6 small parallelograms and 2 “vertical” ones as well.









And that’s it. 32 + 4 = 36 **Ans.**

11. 5 cm more than 3 times the length of a rectangle is less than or equal to 44 cm. It’s also greater than or equal to 20 cm. The width of the same rectangle is 10 cm. We are asked to find the positive difference between the maximum possible area of the rectangle and the minimum possible area of the rectangle.
Let l = the length of the rectangle.
Let w = the width of the rectangle.
3l + 5 <= 44
3l + 5 >= 20
w = 10
Let’s try and find the largest value of l.
3l + 5 <= 44
3l <= 39
l <= 13
Therefore, 13 is the largest possible value for the length and the area is
13 × 10 = 130
Now let’s try and find the smallest value of l.
3l + 5 >= 20
3l >= 15
l >= 5
5 is the smallest possible length of the rectangle and the smallest area is
5 × 10 = 50
130 – 50 = 80 **Ans.**

12. Javier needs to exchange a dollar bill for coins. The cashier has 2 quarters, 10 dimes and 10 nickels. We are asked to determine how many possible combinations of coins Javier could get back if he must get back at least one of the quarters.
Let’s start by assuming he uses only 1 quarter.

D	N	
7	1	
6	3	
5	5	That’s 5 combinations.
4	7	
3	9	

Now, assume he’s going to get both quarters.

D	N	
5	0	
4	2	
3	4	That’s 6 combinations.
2	6	
1	8	
0	10.	

5 + 6 = 11 **Ans.**

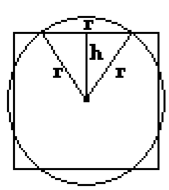
13. There are four test scores which have a median of 80 and a range of 12. We are asked to find the maximum possible score that could have been received on a test.
Since there is an even number of tests, either both middle tests were 80 or their sum was 2 × 80 = 160. The largest number we can choose for test #1 is 80 so that the range will the maximum score for test #4. Test #2 and #3 would then be 80 each. Since the range is 12, then 80 + 12 = 92 is the largest test score. 92 **Ans.**

14. RS,TUV is a five-digit integer. It is divisible by 5, $S = R^2$, and 10 × T + U = 5. R, S, T, U and V are not necessarily distinct. We must find how many five-digit integers satisfy these conditions.
If the five-digit integer is divisible by 5, then V = 0 or V = 5.

10 × T + U = 5 must mean that T = 0. and U = 5. Otherwise the expression would be a value greater than 5.
If S = R^2, then S could be 1 and R could be 1; or S could be 2 and R could be 4; or S could be 3 and R could be 9.
So what does this all tell us?
We can have:
11050, 11055,
42050, 42055,
93050, 93055 or 6 integers. 6 **Ans.**

15. y = x^2 + 10x + 21
What is the least possible value of y?
First, let's find when y = 0.
x^2 + 10x + 21 = 0
(x + 3)(x + 7) = 0
x = –3, and x = –7
What happens when x = –4?
$(–4)^2$ + (10 × –4) + 21 =
16 – 40 + 21 = –3
What happens when x = –5?
$(–5)^2$ + (10 × –5) + 21 =
25 – 50 + 21 = –4
And what happens when x = –6?
$(–6)^2$ + (10 × –6) + 21 =
36 – 60 + 21 = –3
Since x = –3 and x = –7 give us expressions that evaluate to 0, the smallest value of the expression is –4. **Ans.**

16. A square and a circle intersect so that each side of the square contains a chord of the circle which equals the circle's radius. We are asked to find the ratio of the area of the square to the area of the circle.
The area of the circle is πr^2. If we draw a line from the center of the circle to the circumference at the points where the chord is created, we have created an equilateral triangle of length r.

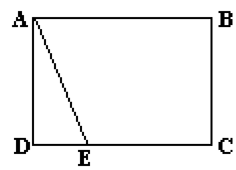


The height of that triangle is $\frac{1}{2}$ the length of the square.

$$h^2 + \left(\frac{r}{2}\right)^2 = h^2 + \frac{r^2}{4} = r^2$$
$$h^2 = \frac{3}{4}r^2$$
$$h = \frac{\sqrt{3}}{2}r$$
$$2h = \sqrt{3}r$$

The area of the square is $\left(\sqrt{3}r\right)^2 = 3r^2$
So the ratio of the area of the square to the area of the rectangle is
$\frac{3r^2}{\pi r^2} = \frac{3}{\pi}$ **Ans.**

17. In rectangle ABCD, point E is on side CD. The area of triangle ADE is $\frac{1}{5}$ of the area of quadrilateral ABCE. We are asked, what is the ratio of the length of segment DE to the length of segment DC?



Let l = AD and let w = DC.
Then lw is the area of the rectangle.
Let x = DE.
Let y = the area of quadrilateral ABCE.
Then $\frac{1}{5}y + y = lw.$

$\frac{6}{5}y = lw$ and $y = \frac{5}{6}lw$

The area of triangle ADE is $\frac{xl}{2}$

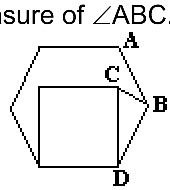
$$\frac{xl}{2} = \frac{1}{5} \times \frac{5}{6}lw = \frac{1}{6}lw$$
3xl = lw and 3x = w
Therefore, x = $\frac{1}{3}w$ and w is DC.

$\frac{1}{3}$ **Ans.**

18. The sum of the first n numbers in a sequence of numbers is given by

$n^3 + 4n$ for all positive integers n. We are asked to find the tenth number in the sequence.
The first number is $1^3 + 4 = 5$. That is actually the first number.
The sum of the first 2 numbers is 8 + 8 = 16. The second number is 11.
The sum of the first 3 numbers is 27 + 12 = 39. The third number is 23.
The sum of the first numbers is 64 + 16 = 80. The fourth number is 51.
So we have 5, 11, 23, 41. Can we see a pattern? I think so.
11 – 5 = 6
23 – 11 = 12
41 – 23 = 18
Therefore, the fifth number is 41 + 24 = 65. The sixth is 65 + 30 = 95. Seventh is 95 + 36 = 131. Eighth is 131 + 42 = 173. Ninth is 173 + 48 = 221 and the tenth number is 221 + 54 = 275. **Ans.**

19. We are given a square in the interior of a hexagon and asked to find the degree measure of ∠ABC.



Hexagons have interior angles of 120º. Let s = the side of the hexagon (and of the square).
Then CD = s and DB = s making CDB an isosceles triangle. Since ∠D = 120º and an angle of the square is 90º, ∠BDC must be 30º. ∠DCB = ∠DBC
180 – 30 = 150
Both ∠DCB and ∠DBC must then be 75º. Since ∠B is also 120º, ∠ABC = 120 – 75 = 45. **Ans.**

20. When the diameter of a pizza increased by 2 in, the area of the pizza increases by 44%. We are asked to find the area of the original pizza.
Let x = the radius of the original pizza. Then the area of the original pizza is πx^2. Increasing the diameter by 2 in is the same increasing the radius by 1 in.

$$\frac{\pi(x+1)^2}{\pi x^2} = 1.44$$

$$\frac{(x+1)^2}{x^2} = 1.44$$

$$\frac{x+1}{x} = 1.2$$

$$x + 1 = 1.2x$$

0.2x = 1
2x = 10
x = 5
The area of the original pizza is 25π.
Ans.

21. When the numerator of a fraction is increased by six, the value of the fraction increases by 1. If the denominator of the fraction is increased by 36, the value of the fraction decreases by one.
Let $\frac{x}{y}$ be the fraction. Then

$$\frac{x+6}{y} = \frac{x}{y} + 1$$ and

$$\frac{x}{y+36} = \frac{x}{y} - 1$$

$$\frac{x+6}{y} = \frac{x+y}{y}$$

x + 6 = x + y
y = 6
Substituting into the second equation:

$$\frac{x}{6+36} = \frac{x}{6} - 1$$

$$\frac{x}{42} = \frac{x-6}{6}$$

6x = 42(x – 6)
x = 7(x – 6)
x = 7x – 42
6x = 42
x = 7

The original fraction was $\frac{7}{6}$. **Ans.**

22. The 80^{th} term of an arithmetic sequence is twice the 30^{th} term. If the first term of the sequence is 7, then what is the 40^{th} term?
The second term is 7 + x, the third is 7 + 2x and so on. That makes the 30^{th} term 7 + 29x. The 80^{th} term is 7 + 79x
7 + 79x = 2 × (7 + 29x)

7 + 79x = 14 + 58x

21x = 7 and x = $\frac{1}{3}$

The 40th term is 7 + 39x.

$$7 + 39x = 7 + \left(39 \times \frac{1}{3}\right) =$$

7 + 13 = 20 **Ans.**

23. A right circular cone is sliced into four pieces which have the same height. We are asked to find the ratio of the volume of the second largest piece to the volume of the largest piece.
Let r = the radius of the cone and
h = the height of the cone.
The radius grows proportionally as does the height so that for the smallest cone the radius is $\frac{1}{4}r$ and the height is $\frac{1}{4}h$, and for the cone made up of the first two pieces the radius is $\frac{1}{2}r$ and the height is $\frac{1}{2}h$, and so on for the other two cones.
The second largest piece is the volume of the second largest cone (with radius $\frac{3}{4}r$ and height $\frac{3}{4}h$) minus the volume of the third largest cone (with radius $\frac{1}{2}r$ and height $\frac{1}{2}h$).

$$V_1 = \frac{1}{3} \times \pi \times \left(\frac{3}{4}r\right)^2 \left(\frac{3}{4}h\right) =$$

$$\frac{1}{3} \times \pi \times \frac{9}{16} \times \frac{3}{4} r^2 h = \frac{9}{64}\pi r^2 h$$

$$V_2 = \frac{1}{3} \times \pi \times \left(\frac{1}{2}r\right)^2 \left(\frac{1}{2}h\right) =$$

$$\frac{1}{3} \times \pi \times \frac{1}{4} \times \frac{1}{2} r^2 h = \frac{1}{24}\pi r^2 h$$

$$V_1 - V_2 = \frac{9}{64}\pi r^2 h - \frac{1}{24}\pi r^2 h =$$

$$\frac{27}{192}\pi r^2 h - \frac{8}{192}\pi r^2 h = \frac{19}{192}\pi r^2 h$$

The volume of the largest piece is the volume of the entire cone minus the volume of the cone with radius $\frac{3}{4}r$ and height $\frac{3}{4}h$, or V_1.

$$V_3 = \frac{1}{3}\pi r^2 h$$

$$V_3 - V_1 = \frac{1}{3}\pi r^2 h - \frac{9}{64}\pi r^2 h =$$

$$\frac{64}{192}\pi r^2 h - \frac{27}{192}\pi r^2 h = \frac{37}{192}\pi r^2 h$$

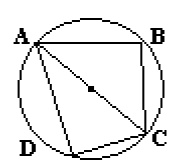
Taking the ratio:

$$\frac{\frac{19}{192}\pi r^2 h}{\frac{37}{192}\pi r^2 h} = \frac{\frac{19}{192}}{\frac{37}{192}} = \frac{19}{37}$$ **Ans.**

24. x + y + z = 7
$x^2 + y^2 + z^2 = 19$
We are asked to find the mean of the three products, xy, yz and xz.
$(x + y + z)^2 = 7^2 = 49$
$(x + y + z)^2 =$
$x^2 + xy + xz + yx + y^2 + yz + zx + zy + z^2$
$x^2 + y^2 + z^2 + 2xy + 2xz + 2yz =$
19 + 2(xy + xz + yz) = 49
2(xy + xz + yz) = 49 – 19 = 30
xy + xz + yz = 15

$$\frac{xy + xz + yz}{3} = \frac{15}{3} = 5$$ **Ans.**

25. Quadrilateral ABCD is inscribed in a circle with segment AC a diameter of the circle. ∠DAC = 30º and ∠BAC = 45º. The ratio of the area of ABCD to the area of the circle can be expressed as a common faction in the form $\frac{a + \sqrt{b}}{c\pi}$. We are asked to find the value of a + b + c.



Because ∠ADC and ∠ABC each intercept a semi-circle, each one is a 90° angle. Therefore, △ACD is a 30-60-90 triangle and △ABC is a

45-45-90 triangle. The hypotenuse of each triangle is 2r, where r is the radius of the circle. The two legs of $\triangle$ACD are r and $r\sqrt{3}$ (using 30-60-90 properties) and its area is $\frac{1}{2}r(r\sqrt{3}) = \frac{\sqrt{3}}{2}r^2$. The legs of $\triangle$ABC are each $r\sqrt{2}$ (using 45-45-90 properties) and its area is $\frac{1}{2}(r\sqrt{2})^2 = \frac{1}{2}(2r^2) = r^2$.
Therefore, the area of quadrilateral ABCD is $\frac{\sqrt{3}}{2}r^2 + r^2 = \frac{2+\sqrt{3}}{2}r^2$. The area of the circle is $\pi(r^2)$. So, the ratio of the area of the quadrilateral to the area of the circle is

$$\frac{\frac{2+\sqrt{3}}{2}r^2}{\pi r^2} = \frac{\frac{2+\sqrt{3}}{2}}{\pi} = \frac{2+\sqrt{3}}{2\pi}.$$

This makes a = 2, b = 3, and c = 2.
2 + 3 + 2 = 7 **Ans.**

26. The numbers a, b, c and d form a geometric sequence.
b = a + 3 and c = b + 9. We are asked to find the value of d.
If c = b + 9, then c = (a + 3) + 9 = a + 12.
So we actually have the sequence:
a, a + 3, a + 12, d
In a geometric sequence, each term is n times the previous term.
a + 3 = n × a

$$n = \frac{a+3}{a}$$

$$a+12 = \frac{a+3}{a} \times (a+3)$$

$$a+12 = \frac{a^2+6a+9}{a}$$

$a^2 + 12a = a^2 + 6a + 9$
12a = 6a + 9
6a = 9

$$a = \frac{9}{6} = \frac{3}{2}$$

$$n = \frac{a+3}{a} = \frac{\frac{3}{2}+3}{\frac{3}{2}} = \frac{\frac{9}{2}}{\frac{3}{2}} = \frac{9}{3} = 3$$

Finally, now that we know what n is, let's determine the value of a + 12 and multiply it by n.

$$a+12 = \frac{3}{2} + \frac{24}{2} = \frac{27}{2}$$

$$d = n \times (a+12) =$$

$$3 \times \frac{27}{2} = \frac{81}{2} = 40\frac{1}{2}$$ **Ans.**

27. If you flip an unfair coin three times, the probability of getting 3 heads is the same as the probability of getting exactly two tails. We are asked to find the ratio of the probability of flipping a tail to the probability of flipping a head.
Let h = the probability of a head coming up.
Let t = the probability of a tail coming up.
We have one possibility that gives us 3 heads, i.e., HHH and 3 possibilities of exactly two tails, TTH, THT, and HTT.

$$h^3 = 3 \times ht^2$$

$$h^2 = 3t^2$$

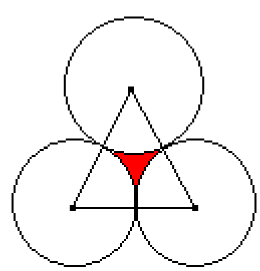
$$t^2 = \frac{h^2}{3}$$

$$t = \sqrt{\frac{h^2}{3}} = \frac{h}{\sqrt{3}} = \frac{h\sqrt{3}}{3}$$

The ratio of the probability of flipping a tail to the probability of flipping a head is

$$\frac{t}{h} = \frac{\frac{h\sqrt{3}}{3}}{h} = \frac{\sqrt{3}}{3}$$ **Ans.**

28. The region shown is bounded by the arcs of circles having radius 4 units. Each arc has a central angle measure of 60º and intersects at points of tangency. The area of the region can be expressed in the form $a\sqrt{b} + c\pi$. We are asked to find the value of a + b + c.



This turns out to be pretty simple.
If you draw lines connecting the three

centers of the circles, you end up with an equilateral triangle (that's where the 60º comes in) whose sides are twice the radius or 8. We must first find the area of the triangle and for that we must have the length of the height of the triangle.
$h^2 + 4^2 = 8^2$
$h^2 + 16 = 64$
$h^2 = 64 – 16 = 48$
$h = \sqrt{48} = 4\sqrt{3}$
Therefore, the area of the triangle is:
$\frac{1}{2} \times 8 \times 4\sqrt{3} = 16\sqrt{3}$
From this, we need to subtract the area of the sections of the circle formed by the 60º central angles. But we have 3 of those which is equivalent to the area formed by a semicircle. The area of the circle is: $\pi r^2 = 16\pi$. Half of that is 8π. So, the area of the region bounded by the arcs is $16\sqrt{3} - 8\pi$. This means:
a = 16, b = 3, c = –8
a + b + c = 16 + 3 – 8 = 11 **Ans.**

29. A bag contains red and white balls. If 5 balls will be pulled from the bag with replacement, the probability of getting exactly 3 red balls is 32 times the probably of getting exactly one red ball. We are asked to find what percent of the balls originally in the bag are red.
Let r = the number of red balls.
Let w = the number of white balls.
Then the probability of picking a red ball from the bag is $\frac{r}{r+w}$ and the probability of picking a white ball from the bag is $\frac{w}{r+w}$.
The probability of getting 3 red balls is:
$\frac{r^3 w^2}{(r+w)^5}$
There are $\frac{5!}{3!2!} = 10$ ways to do it.
The probability of getting only one red ball is: $\frac{rw^4}{(r+w)^5}$
There are $\frac{5!}{4!1!} = 5$ ways to do it.
$\frac{r^3 w^2}{(r+w)^5} \times 10 = 32 \frac{rw^4}{(r+w)^5} \times 5$
$r^3 w^2 = 16rw^4$
$r^2 = 16w^2$
$r = 4w$
So, there are 4 more times as many red balls as white balls. That's 80%. **Ans.**

30. 52,683 × 52,683 – 52,660 × 52,706 =
$(52660 + 23)^2$–(52660 × (52660 + 46)) =
$52660^2 + 23^2$ + (2 × 23 × 52660) –
(52660^2 + (52660 × 46)) =
23^2 + (46 × 52660) – (46 × 52660) =
23^2 = 529 **Ans.**

TARGET ROUND

1. We are asked to find the smallest possible integer such that none of the digits is a zero, the integer is a multiple of 3 and exactly one of the digits is a perfect square.
A multiple of 3 means that if you add the digits in the number, the sum is itself a multiple of 3.
The first perfect square is 1. We must have 2 other digits that are not perfect squares. That would have to be 2 and 3 so the smallest number is 123. **Ans.**

2. We have a truck that is driven 480 miles at 60 miles per hour. The truck averages 20 miles per gallon. The driver is paid $12/hour. Gasoline costs $3 per gallon Finally, maintenance on the truck is $0.15 per mile. We must determine what the total cost of the trip is for the company.
480 miles at 60 miles per hour means that the truck is driven for $\frac{480}{60} = 8$ hours. So the labor cost is
12 × 8 = 96. 480 miles at 20 miles per gallon means we must buy $\frac{480}{20} = 24$ gallons of gas at $3 per gallon.
24 × 3 = 72.
96 + 72 = $168, so far.
Is there anything else? Yes, the maintenance.

$480 \times 0.15 = 72$
$168 + 72 = 240$ **Ans.**

3. Jose had $\frac{1}{5}$ of the pizza. Marti had $\frac{1}{4}$ of what was left. Then Bobbi had $\frac{1}{3}$ of the remaining pizza and after that, Ms. Gauss took $\frac{1}{2}$ of that. So how much pizza is left?
After Jose had $\frac{1}{5}$ of the pizza there was $1-\frac{1}{5}=\frac{4}{5}$ of the pizza left. Marti ate $\frac{1}{4}$ of that or $\frac{1}{4}\times\frac{4}{5}=\frac{1}{5}$ of the pizza. Now $\frac{4}{5}-\frac{1}{5}=\frac{3}{5}$ of the pizza is left. Bobbi gets $\frac{1}{3}$ of that or $\frac{1}{3}\times\frac{3}{5}=\frac{1}{5}$ of the pizza which leaves $\frac{3}{5}-\frac{1}{5}=\frac{2}{5}$ of the pizza left. Finally Ms. Gauss gets her $\frac{1}{2}$ of the leftovers or $\frac{1}{2}\times\frac{2}{5}=\frac{1}{5}$ of the pizza. That leaves $\frac{2}{5}-\frac{1}{5}=\frac{1}{5}$ of the pizza left. $\frac{1}{5}$ or 20% **Ans.**

4. Marco will celebrate his Nth birthday in the year N^2, which is in the 21st century. So, in what year did he celebrate his 13th birthday?
Okay, we have to find a square that is between 2000 and 2100.
$40^2 = 1600$ and $50^2 = 2500$ We're looking for a value between 40 and 50, probably somewhere in the mid 40's range. $45^2 = 2025$ and that must be it!
So, Marco will be 45 in 2025.
45 – 13 = 32 which means 32 years before 2025, Marco was 13.
2025 – 32 = 1993 **Ans.**

5. How many distinct sums can be obtained by adding three different numbers from the set {3, 6, 9, …, 27, 30}?
Let's rewrite that as
{3×1, 3×2, 3×3, …, 3×9, 3×10}
And we can reduce this to how many values can you get adding three of the numbers 1 through 10.
There are 10×9×8 = 720 different values. But many of them are the same.
The smallest value is 1 + 2 + 3 = 6.
The largest value is 8 + 9 + 10 = 27.
And we can get every value in between.
27 – 6 = 21 But don't forget to count the 6, because there are actually 22 numbers from 6 to 27. 22 **Ans.**

6. x and y are real numbers such that
$xy = 9$ and $x^2y + xy^2 + x + y = 100$
We are asked to find the value of
$x^2 + y^2$
$x^2y + xy^2 + x + y = 100$
$x \times xy + y \times xy + x + y = 100$
$9x + 9y + x + y = 100$
$10(x + y) = 100$
$x + y = 10$
$(x + y)^2 = 100 = x^2 + 2xy + y^2$
$x^2 + y^2 + (2 \times 9) = 100$
$x^2 + y^2 = 100 – 18 = 82$ **Ans.**

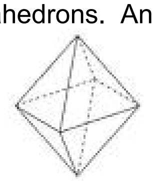
7. In a shooting competition, the object of the match is to be the first to hit the bull's eye of a target 100 feet away. Each opponent has a 40% chance of hitting the bull's-eye on a given shot. Franz shoots first and we must determine the probability that Hans will win the competition and take no more than 3 shots.
Scenario 1: Franz misses and then Hans hits the bull's-eye.
The probability of this is
0.6 × 0.4 = 0.24
Scenario 2: Franz misses, Hans misses, Franz misses and Hans hits the bull's-eye.
0.6 × 0,6 × 0.6 × 0.4 = .0864
Scenario 3: Franz misses, Hans misses, Franz misses, Hans misses, Franz misses and Hans hits the bull's-eye.
0.6 × 0.6 × 0.6 × 0.6 × 0.6 × 0.4 = 0.031104
The probability of all 3 scenarios is:
0.24 + 0.0864 + 0.031104 = 0.357504 ≈ 0.36 **Ans.**

8. The solid figure has 6 faces that are squares and 8 faces that are equilateral triangles. Each of the 24 edges has length 2 cm. The volume of the solid can be expressed in the form

$\frac{a}{b}\sqrt{c}$ where c has no perfect square factor except 1, and a and b are relatively prime. We must find the value of a + b + c.

OK. I'm not going to even try to reproduce that figure so I'm just going to talk through it. First of all, there is a rectangular solid in there whose length and width are each 2 cm and whose height is the diagonal of a square whose side is 2 cm. The diagonal must be $2\sqrt{2}$ and the volume of the solid is $2 \times 2 \times 2\sqrt{2} = 8\sqrt{2}$.

Now, if you remove that part of the object, you're left with 4 half-octahedrons. An octahedron looks like



this:

We actually, then, have 2 full octahedrons. The volume of an octahedron with side 2 is

$\frac{1}{3} \times \sqrt{2} \times 2^3 = \frac{8}{3}\sqrt{2}$

The volume of two of them is $\frac{16}{3}\sqrt{2}$.

$8\sqrt{2} + \frac{16}{3}\sqrt{2} = \frac{40}{3}\sqrt{2}$

This means that a = 40, b = 3 and c = 2.

a + b + c = 40 + 3 + 2 = 45 **Ans.**

TEAM ROUND

1. Kevin, Cindi and Marcus have 1020 widgets. Marcus has half of what Cindi has and Kevin has 219 widgets. So how many widgets does Cindi have?

Let x = the number of widgets that Marcus has. Then Cindi has 2x widgets.

1020 – 219 = 801

x + 2x = 801

3x = 801

x = 267

2x = 534 **Ans.**

2. We have a three-digit integer where all digits are prime and distinct. They also increase in order from left to right. We are asked to determine how many possibilities there are for the number.

Let's list all one-digit primes.

2, 3, 5, 7

The possibilities are:

235, 237, 257, 357 for a total of 4 **Ans.**

3. Every 25th customer receives a free Frisbee. Every 35th customer receives a free baseball hat. How many customers were there before someone gets both.

Uh, we just have to find the LCM of 25 and 35 and then remember to subtract 1.

25 = 5 × 5

35 = 5 × 7

The LCM = 5 ×5 × 7 = 25 × 7 = 175.

175 – 1 = 174 customers who came before the customer who received both.

174 **Ans.**

4. A train travels at a constant rate of 55 mph through a tunnel. 45 seconds after the front of the train enters the tunnel, the front of the train exits the tunnel. So how many feet long is the tunnel?

How far can you go at 55 mph in 45 seconds.

55 mph = $\frac{55}{60} = \frac{11}{12}$ miles per minute.

$\frac{11}{12} \times \frac{45}{60} = \frac{11}{12} \times \frac{3}{4} = \frac{11}{4} \times \frac{1}{4} = \frac{11}{16}$ miles per 45 seconds.

$\frac{11}{16} \times 5280 = 330 \times 11 = 3630$ ft **Ans.**

5. An L-shaped piece is placed on the grid so that it covers three of the unit squares of the grid. The sum of the covered squares is S. We are allowed to rotate the L-shaped piece. So we need to find the values of S for all possible placements on this grid.

1	2	3
4	5	6
7	8	9

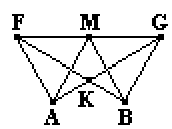
Let's look at the numbers 1, 2, 4, and 5. We can place the L-shaped piece on {1,

2, 5}, {2, 5, 4}, {1, 4, 5} and {1, 2, 4}.
The sums are 8, 11, 10 and 7.
8 + 11 + 10 + 7 = 36
Now, look at 2, 3, 5, 6. The sums are 11, 14, 13, and 10.
11 + 14 + 13 + 10 = 48
Now, look at 4, 5, 7, 8. The sums are 16, 20, 19, and 17.
16 + 20 + 19 + 17 = 72
Finally, look at 5, 6, 8, 9. The sums are 19, 23, 22, 20.
19 + 23 + 22 + 20 = 84
36 + 48 + 72 + 84 = 240 **Ans.**

6. We have a container with four colors of marbles, red, yellow, blue and green. All but 45 of the marbles are red. All but 45 are yellow. All but 45 are blue. All but 60 are green. We must determine how many marbles are green.
Let r = the number of red marbles.
Let y = the number of yellow marbles.
Let b = the number of blue marbles.
Let g = the number of green marbles.
Let t = the total number of marbles.
t = r + 45
t = y + 45
t = b + 45
t = g + 60
If r + 45 = y + 45 = b + 45, then r = y = b.
From t = g + 60, we surmise that r + y + b = 60.
3r = 60
r = y = b = 20.
t = r + 45 = 20 + 45 = 65
Therefore, g = 65 – 60 = 5. **Ans.**

7. If 3 cookies are placed in a bag, then there are 2 cookies left over.
If 5 cookies are placed in a bag, there are no cookies left over.
If 8 cookies are placed in a bag, then there are 6 cookies left over.
Determine the fewest number of cookies that could have been baked.
Obviously, the number of cookies is a multiple of 5.
One more than the number of cookies baked is divisible by 3 (because there are 2 cookies left over in the first scenario. The number of cookies must at least 15. (One bag of 8 cookies plus 6 left over and divisible by 5.)
Find the first number divisible by 3 that is one more than a multiple of 5.
That's 21 so if we baked 20 cookies and put them into bags of 8, **4** would be left over. Not there yet. What's next?
36 is the next candidate. If we baked 35 cookies and put them in bags of 8 – wait, we better be baking an even number of cookies. That means skipping every other possibility.
Next is 51. If we baked 50 cookies and put them in bags of 8, **2** would be left over.
Next is 81. If we baked 80 cookies and put them in bags of 8 there would be **0** left over.
It's got to get 111. If we baked 110 cookies and put them in bags of 8 there would be **6** left over. 110 **Ans.**

8. M is the midpoint of segment FG. A and B are points coplanar to points F and G. A and B are positioned on the same side of the line containing segment FG such that triangles FMA and MGB are equilateral. The lines FB and GA intersect at point K. So what is the measure of ∠GKB?
Okay. We have to draw this out.



Triangle FMA and triangle MGB are equilateral. Therefore, ∠FMA and ∠GMB are both 60º. That means that ∠AMB is 180º - 60º - 60º = 60º
M and K are opposite points in a quadrilateral whose other 2 points are the intersections of lines FB and MA, and lines GA and MB, respectively. The sum of two opposite angles in a quadrilateral is 180º. Therefore, ∠FKG is 120º. ∠GKB is just the supplement, or 60º. **Ans.**

9. A bag contains 5 red marbles, 3 blue marbles and 2 green marbles. We draw 6 marbles from the bag with replacement. We must find the probability that two marbles of each color will be drawn.
There are a total of 5 + 3 + 2 = 10 marbles in the bag. With all 10 in the bag, there is a $\frac{5}{10} = \frac{1}{2}$ chance that we

draw a red marble. There is a $\frac{3}{10}$ chance that we draw one blue marble. There is a $\frac{2}{10}=\frac{1}{5}$ chance that we draw 1 green marble.

$$\left(\frac{1}{2}\right)^2 \times \left(\frac{3}{10}\right)^2 \times \left(\frac{1}{5}\right)^2 =$$

$\frac{1}{4}\times\frac{9}{100}\times\frac{1}{25}=\frac{9}{10000}$ is the probability that we draw two of each.

There are

$\frac{6!}{2!2!2!}=\frac{6\times5\times4\times3}{4}=90$ possibilities.

$\frac{9}{10000}\times 90=\frac{810}{10000}=\frac{81}{1000}$ **Ans.**

10. The geometric mean of two positive numbers a and b is $\sqrt{ab}$. The third term of an arithmetic sequence of positive numbers is the geometric mean of the first and eleventh terms. We must find the ratio of the second term to the first term of the sequence.

    In an arithmetic sequence, the terms differ by some value n.

    If x is the first term, then x + n is the second, x + 2n is the third, …, and x + 10n is the 11th term.

$$x+2n=\sqrt{x\times(x+10n)}$$

$$x+2n=\sqrt{x^2+10nx}$$

$$(x+2n)^2=x^2+4nx+4n^2=x^2+10nx$$

$$4n^2=6nx$$

$$4n=6x$$

$$n=\frac{6}{4}x=\frac{3}{2}x$$

The ratio of the second term in the sequence to the first term in the sequence is

$\frac{x+n}{x}=\frac{x+\frac{3}{2}x}{x}=\frac{\frac{5}{2}x}{x}=\frac{5}{2}$ **Ans.**

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2011
■ Chapter Competition ■
Sprint Round
Problems 1–30

Name ____

School ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

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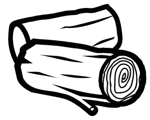
1. If a woodchuck could chuck 60 pounds of wood in 1.5 days, how many pounds of wood could a woodchuck chuck in 6 days?

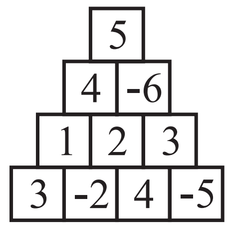


1. ______________ pounds

2. In degrees Fahrenheit, half the temperature of Papa's oatmeal is equal to 20 degrees cooler than Baby's oatmeal. If Papa's oatmeal is 180 degrees, what is the temperature of Baby's oatmeal?

2. ______________ degrees

3. Exactly one number is to be selected from each of the four rows of this Number Wall. What is the largest possible product of any such four numbers?

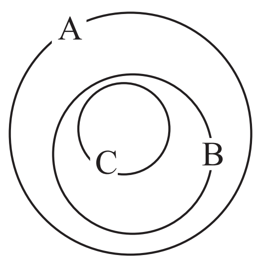


3. ______________

4. Hannah's number of runs scored in softball this season is 75% of April's number of runs scored this season. If April scored 16 runs this season, how many runs did Hannah score?

4. ______________ runs

5. A, B and C are circular regions as shown. There are 7 items in circle C. There are exactly 20 items in A and 10 of those items are not in B. How many items are in B, but not in C?



5. ______________ items

6. A signature line on a certificate is 4 inches long. If Karla wants to leave a $\frac{3}{4}$-inch blank space at each end of her signature, how long is the portion of the line on which she can sign her name? Express your answer as a mixed number.

6. ______________ inches

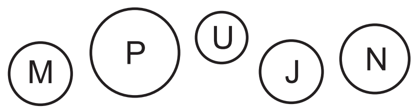
Karla Spaghetti

Karla Spaghetti, Chapter Coordinator

7. Kwanisha defined the operation $\otimes$ as $a \otimes b = a^2 + b + 1$. Using Kwanisha's definition, what is the value of $6 \otimes 5$?

7. ______________

8. Malton has twice as many moons as Planar. The number of Nero's moons is the cube of the number of Malton's moons. Ufda has 4 more moons than Jir. If you double the number of Nero's moons and add the number of Planar's moons, then you will get the number of Jir's moons. If Planar has 1 moon, how many moons does Ufda have?



8. ______________ moons

9. The sum of three consecutive prime numbers is 173. What is the largest of these numbers?

9. ______________

10. If $(3^x)(9) = 81$, what is the value of x?

10. ______________

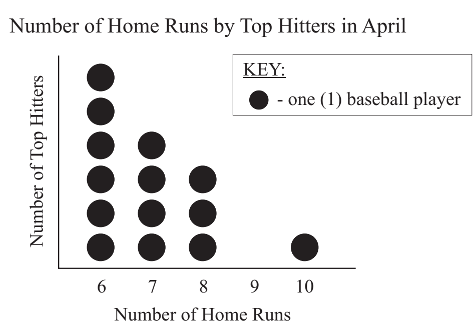
11. If Kenton walks for 60 minutes at the rate of 3 mph and then runs for 15 minutes at the rate of 8 mph, how many miles will he travel?

11. ______________ miles

12. If x and y are each integers greater than 3 and less than 20, what is the sum of the three possible values of x that satisfy the equation $\frac{x}{y} = \frac{3}{4}$?

12. ______________

13. The graph to the right shows the number of home runs in April for the top hitters in the league. What is the mean (average) number of home runs hit by these players?



13. ______________ home runs

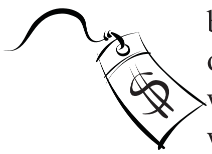
14. If $\frac{5}{33}$ is expressed in decimal form, what digit is in the 92nd place to the right of the decimal point?

14. ________________

15. In a particular game, a player can earn either 3 points or 5 points on each turn. If Capri has earned a total of 18 points, what is the fewest number of turns she could have taken?

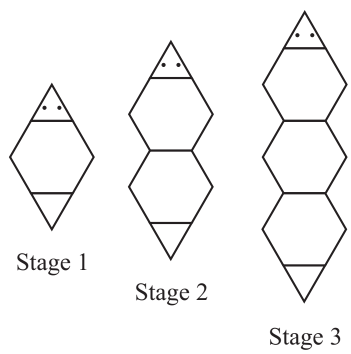
15. ________________ turns

16. A fonk originally was priced at $100 when fonks were first introduced. The price of a fonk then increased by 20% once it became popular to own a fonk. Now that fonks are out of style, their price has decreased by 30% from the price when they were popular. This current price of a fonk is what percent of the original price?



16. ________________ %

17. Growing Worms are created as shown here. Notice that each body segment is a regular hexagon and its head and tail are equilateral triangles. A Stage 1 Growing Worm has a perimeter of 8 cm. What is the perimeter of a Stage 4 Growing Worm?



17. ________________ cm

18. Each term of a sequence is one more than twice the term before it. If the first term is 1, what is the sum of the first 5 terms of the sequence?

18. ________________

19. If a fly is buzzing randomly around a room 8 ft long, 12 ft wide and 10 ft high, what is the probability that, at any given time, the fly is within 6 feet of the ceiling? Express your answer as a common fraction.

19. ________________

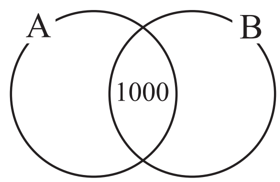
20. If five less than three-fourths of an integer is the same as five more than one-eighth of the same integer, what is the integer?

20. ________________

21. What is the sum of the negative integers that satisfy the inequality $2x - 3 \geq -11$?

21. ________________

22. Sets A and B, shown in the Venn diagram, are such that the total number of elements in set A is twice the total number of elements in set B. Altogether, there are 3011 elements in the union of A and B, and their intersection has 1000 elements. What is the total number of elements in set A?

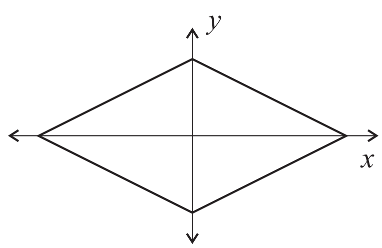


22. ________________ elements

23. The quotient of two consecutive positive integers is 1.02. What is the sum of these two integers?

23. ________________

24. What is the area enclosed by the graph of $|x| + |2y| = 10$ shown here?



24. ________________ sq units

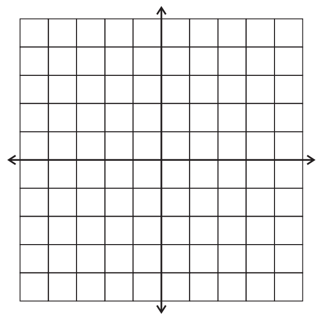
25. Two similar right triangles have areas of 6 square inches and 150 square inches. The length of the hypotenuse of the smaller triangle is 5 inches. What is the sum of the lengths of the legs of the larger triangle?

25. ________________ inches

26. If a committee of six students is chosen at random from a group of six boys and four girls, what is the probability that the committee contains the same number of boys and girls? Express your answer as a common fraction.

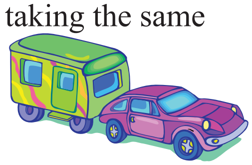
26. ________________

27. The point A(3, 4) is reflected over the x-axis to B. Then B is reflected over the line $y = x$ to C. What is the area of triangle ABC?



27. ________________ sq units

28. Tonisha leaves Maryville at 7:15 a.m. headed back to college after summer break. Since she is towing a trailer with all of her belongings, she is limited to an average speed of 45 mph. Her friend Sheila leaves Maryville an hour later taking the same route averaging the speed limit of 60 mph. At what time will Sheila pass Tonisha?

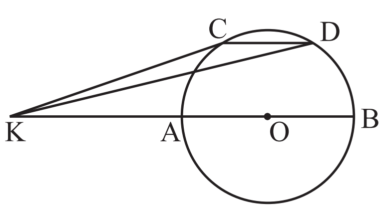


28. ______ : ______ a.m.

29. Fido's leash is tied to a stake at the center of his yard, which is in the shape of a regular hexagon. His leash is exactly long enough to reach the midpoint of each side of his yard. If the fraction of the area of Fido's yard that he is able to reach while on his leash is expressed in simplest radical form as $((\sqrt{a})/b)\pi$, what is the value of the product ab?

29. ________________

30. In the figure, circle O has radius 6 units. Chord CD has length 8 units and is parallel to segment KB. If KA = 12 units and points K, A, O and B are collinear, what is the area of triangle KDC? Express your answer in simplest radical form.



30. ________________ sq units

MATHCOUNTS®

2011
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Target Round
Problems 1 and 2

Name ____

School ____

Chapter ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

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1. Blossom is thinking of a positive three-digit integer. She gave some clues to finding it.
   - None of the digits is a zero.
   - The integer is divisible by 3.
   - Exactly one of the digits is a perfect square.

   What is the smallest possible integer Blossom could be thinking of?

1. ________________

2. An employee of a delivery company drives 480 miles at 60 miles per hour in a vehicle that averages 20 miles per gallon of gas. The employee is paid \$12 per hour, gasoline costs \$3 per gallon, and maintenance on the vehicle is \$0.15 per mile. According to these rates, what is the total cost of this trip for the company?

2. \$________________

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Target Round

Problems 3 and 4

Name ______________________________

School ______________________________

Chapter ______________________________

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3. The Math Club ordered the Super Monster pizza from Pizza Shack for their meeting. Jose took 1/5 of the pizza. Marti took 1/4 of what was left. Then Bobbi took 1/3 of the remaining pizza. Finally, Ms. Gauss, the advisor, took 1/2 of the remaining pizza. What percent of the entire pizza was left after Ms. Gauss took her share?

3. ________________ %

4. In the 21st century, Marco will celebrate his Nth birthday in the year N^2. In what year did he celebrate his 13th birthday?

4. ________________

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Target Round
Problems 5 and 6

Name ______________________________

School ______________________________

Chapter ______________________________

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5. How many distinct sums can be obtained by adding three different numbers from the set {3, 6, 9. …, 27, 30}?

5. ________ sums

6. Suppose x and y are real numbers such that $xy = 9$ and $x^2y + xy^2 + x + y = 100$. What is the integer value of $x^2 + y^2$?

6. ________

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2011
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Target Round
Problems 7 and 8

Name ____

School ____

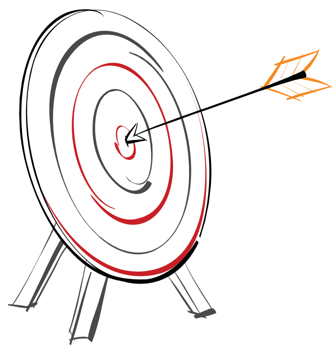
Chapter ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

Total Correct	Scorer's Initials

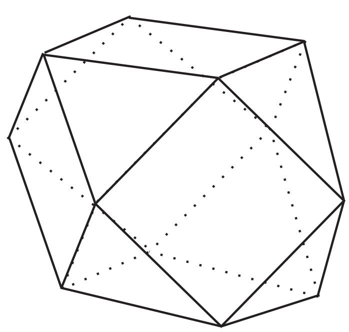
7. Hans and Franz are in a shooting competition. The object of the match is to be the first to hit the bull's-eye of a target 100 feet away. The two opponents alternate turns shooting, and each opponent has a 40% chance of hitting the bull's-eye on a given shot. If Hans graciously allows Franz to shoot first, what is the probability that Hans will win the competition and take no more than three shots? Express your answer as a decimal to the nearest hundredth.

7. ________________



8. The solid figure shown has six faces that are squares and eight faces that are equilateral triangles. Each of the 24 edges has length 2 cm. The volume of the solid can be expressed in cubic centimeters in the form $\frac{a}{b}\sqrt{c}$ where c has no perfect square factor except 1 and where a and b are relatively prime. What is the value of $a + b + c$?

8. ________________



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2011
■ State Competition ■
Sprint Round
Problems 1–30

Name ____________________

School ____________________

Chapter ____________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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1. Boy Scout Troop 324 is planning to go hiking for 1 day. There are 12 boys in the troop, and they will be accompanied by 3 scout leaders. Each person will need at least 3 bottles of water that day. What is the minimum number of bottles of water needed for the trip?

1. ________ bottles

2. Cammie has some pennies, nickels, dimes and quarters. What is the least number of coins that she can use to make 93 cents?

2. ________ coins

3. Kara has three non-overlapping circles. The area of the circle 8 inches in diameter is how much greater than the combined areas of the circle 6 inches in diameter and the circle 2 inches in diameter? Express your answer in terms of π.

3. ________ sq in

4. During the first five games of the baseball season, Mike Slugger had 7 hits in 20 turns at bat. In the next game, he had 5 hits in 5 turns at bat. For the first six games, what percent of Mike's turns at bat resulted in hits?

4. ________ %

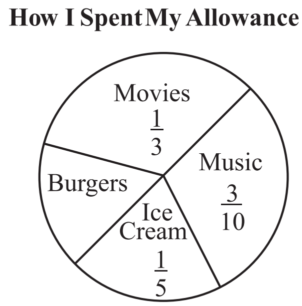
5. The first term of an arithmetic sequence is –37, and the 2nd term is –30. What is the smallest positive term of the sequence?

5. ________

6. At Paul Bunyan's tree farm they sell Fraser firs and blue spruce trees. They plant saplings of these two kinds of trees that are 8 inches and 5 inches tall, respectively. The Fraser firs grow at a constant rate of 12 inches per year, and the blue spruce trees grow at a constant rate of 14 inches per year. After how many years will these trees be the same height? Express your answer as a common fraction.

6. ________ years

7. I had $30 in allowance money and spent it as indicated in the pie graph shown. How many dollars did I spend on burgers?



7. $ ________________

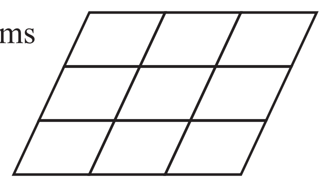
8. M and N are both perfect squares less than 100. If M – N = 27, what is the value of $\sqrt{M} + \sqrt{N}$?

8. ________________

9. John made two 120-mile trips. He made his second trip in one hour less time than his first trip. The total time for the two trips was 9 hours. What was his average rate, in miles per hour, for the second trip?

9. ________________ mph

10. In the figure shown, there are parallelograms of many sizes. How many total parallelograms are there in the diagram?



10. ________________ parallelograms

11. Five centimeters more than three times the length of a rectangle is less than or equal to 44 cm and greater than or equal to 20 cm. The width of the same rectangle is 10 cm. What is the positive difference between the maximum possible area of the rectangle and the minimum possible area of the rectangle?

11. ________________ sq cm

12. Javier needs to exchange his dollar bill for coins. The cashier has 2 quarters, 10 dimes and 10 nickels. Assuming the cashier gives Javier the correct amount and at least one quarter, how many possible combinations of coins could Javier receive?

12. ________________ combinations

13. Joan has four test scores. The median is 80 points, and the range is 12 points. What is the maximum possible score she could have received on a test?

13. ______________ points

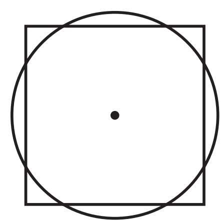
14. The five-digit integer RS,TUV is divisible by 5, S = R^2 and $10 \times T + U = 5$. If R, S, T, U and V are not necessarily distinct, how many positive five-digit integers satisfy these conditions?

14. ______________ integers

15. Given the function $y = x^2 + 10x + 21$, what is the least possible value of y?

15. ______________

16. A square and a circle intersect so that each side of the square contains a chord of the circle equal in length to the radius of the circle. What is the ratio of the area of the square to the area of the circle? Express your answer as a common fraction in terms of π.



16. ______________

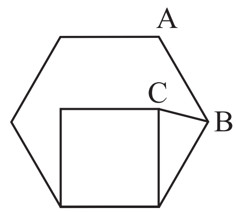
17. In a rectangle ABCD, point E is on side CD. The area of triangle ADE is one-fifth of the area of quadrilateral ABCE. What is the ratio of the length of segment DE to the length of segment DC? Express your answer as a common fraction.

17. ______________

18. For a certain sequence of numbers, the sum of the first n numbers in the sequence is given by $n^3 + 4n$ for all positive integers n. What is the tenth number in the sequence?

18. ______________

19. The figure shows a square in the interior of a regular hexagon and sharing a common side. What is the degree measure of $\angle ABC$?



19. ____________ degrees

20. When the diameter of a pizza increases by 2 inches, the area increases by 44%. What was the area, in square inches, of the original pizza? Express your answer in terms of π.

20. ____________ sq in

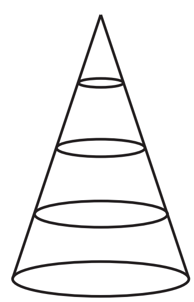
21. If the numerator of a fraction is increased by six, the value of the fraction will increase by one. If the denominator of the original fraction is increased by 36, the value of the original fraction will decrease by one. What is the original fraction? Express your answer as a common fraction.

21. ____________

22. The 80th term of an arithmetic sequence is twice the 30th term. If the first term of the sequence is 7, what is the 40th term?

22. ____________

23. A right circular cone is sliced into four pieces by planes parallel to its base, as shown in the figure. All of these pieces have the same height. What is the ratio of the volume of the second-largest piece to the volume of the largest piece? Express your answer as a common fraction.



23. ____________

24. If $x + y + z = 7$ and $x^2 + y^2 + z^2 = 19$, what is the arithmetic mean of the three products xy, yz and xz?

24. ____________

25. Quadrilateral ABCD is inscribed in a circle with segment AC a diameter of the circle. If $m\angle DAC = 30°$ and $m\angle BAC = 45°$, the ratio of the area of ABCD to the area of the circle can be expressed as a common fraction in simplest radical form in terms of π as $\frac{a+\sqrt{b}}{c\pi}$. What is the value of $a + b + c$?

25. ________________

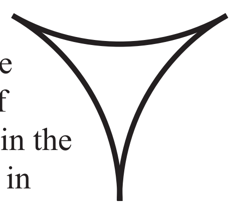
26. The numbers a, b, c and d form a geometric sequence, in that order. If b is three more than a, and c is nine more than b, what is the value of d? Express your answer as a mixed number.

26. ________________

27. In three flips of an unfair coin the probability of getting three heads is the same as the probability of getting exactly two tails. What is the ratio of the probability of flipping a tail to the probability of flipping a head? Express your answer as a common fraction in simplest radical form.

27. ________________

28. The region shown is bounded by the arcs of circles having radius 4 units, having a central angle measure of 60 degrees and intersecting at points of tangency. The area of the region can be expressed in the form $a\sqrt{b} + c\pi$ square units, where $\sqrt{b}$ is a radical in simplest form. What is the value of $a + b + c$?



28. ________________

29. A bag contains red balls and white balls. If five balls are to be pulled from the bag, with replacement, the probability of getting exactly three red balls is 32 times the probability of getting exactly one red ball. What percent of the balls originally in the bag are red?

29. ________________ %

30. What is the value of 52,683 × 52,683 – 52,660 × 52,706?

30. ________________

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Countdown Round
Problems 1–80

This section contains problems to be used in the Countdown Round.

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1. A jackpot for a contest starts at \$100,000 and increases 10% after each round. How many dollars is the jackpot worth after four rounds?

1. ____________ (dollars)

2. What is the sum of the cubes of the first three even positive integers?

2. ____________

3. What is the area, in square units, of a square whose sides are the same length as the radius of a circle with a circumference of 12π units?

3. ____________ (sq units)

4. What is the area, in square units, of the triangle bounded by $y = 0$, $y = x + 4$ and $x + 3y = 12$?

4. ____________ (sq units)

5. The population of the town of Leibnitz was 180,000 in 2002. If the population grows at a rate of 3000 people per year, what will the population be in 2020?

5. ____________ (people)

6. If x and y are positive integers such that $4x + 2y = 36$ and $3y - 2x = 14$, what is the value of the product xy?

6. ____________

7. How many integers are in the solution of the inequality $|x + 4| < 9$?

7. ____________ (integers)

8. A set of nine distinct positive integers has mean 9 and median 9. What is the greatest possible value of one of these integers?

8. ____________

9. For how many positive integers N does $\sqrt{N}$ differ from 10 by less than 2?

9. ____________ (integers)

10. What is the value of the expression $40(3000) - 50(400) + 18(20) - 20(5)$?

10. ____________

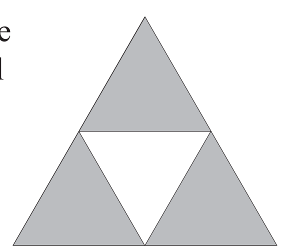
11. Jack takes twice as long to dig a hole as Ken. The two working together can dig a hole in ten minutes. How many minutes does it take Ken to dig a hole by himself?

11. ____________ (minutes)

12. Point P(–5, 12) is graphed in a coordinate plane. What is the number of units in the distance from point P to the origin?

12. ____________ (units)

13. A region consists of an equilateral triangle divided into smaller congruent equilateral triangles. What percent of the region is gray?



13. ____________ (percent)

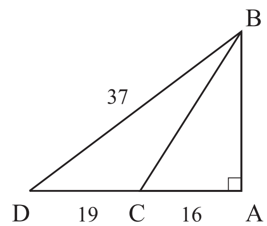
14. What is the measure, in units, of the hypotenuse of a right triangle with leg lengths of 75 and 100 units? 14. ____________ (units)

15. Mary will randomly choose an integer from the integers 1 to 100, inclusive. If Mary chooses a multiple of 4, what is the probability she will choose a perfect square? Express your answer as a common fraction. 15. ____________

16. What is the value of x if $6^x + 6^x + 6^x + 6^x + 6^x + 6^x = 6^6$? 16. ____________

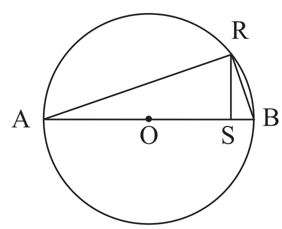
17. Amy drove from Mathtown to Mathville. Her average speed for the first two hours was 35 mph due to road construction. She then averaged 70 mph for the rest of the trip. If her average speed for the entire trip was 60 mph, how many hours did the trip take? 17. ____________ (hours)

18. Given the right triangles ABC and ABD, what is the length of segment BC, in units? 18. ____________ (units)



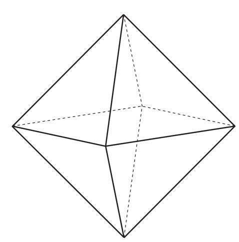
19. Given 3 consecutive positive integers, what is the positive difference between the product of the first and last integers and the square of the middle integer? 19. ____________

20. Circle O has diameter AB of length 26 units. Segment RS is perpendicular to segment AB with R on the circle and S on segment AB. If the length of segment RS is 12 units, what is the product of the lengths of segments AS and SB, in square units? 20. ____________ (sq units)



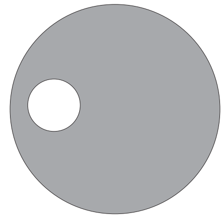
21. What is the integer value of $\left(\left(\sqrt{2}\right)^{\sqrt{2}}\right)^{\sqrt{8}}$? 21. ____________

22. Two of the vertices of a regular octahedron are to be chosen at random. What is the probability that they will be the endpoints of an edge of the octahedron? Express your answer as a common fraction. 22. ____________



23. Two successive discounts of 20% are equivalent to a single discount of what percent? 23. ____________ (percent)

24. What is the area of the gray region, in square units, if the radius of the larger circle is four times the radius of the smaller circle and the diameter of the smaller circle is 2 units? Express your answer in terms of π.



24. ____________ (sq units)

25. The sum of five consecutive integers is 185. What is the mean of these five integers?

25. ____________

26. A regular octagon is graphed in the coordinate plane. Two adjacent vertices are graphed at (4, –2) and (1, 2). What is the perimeter of this octagon, in units?

26. ____________ (units)

27. Joey and Mike live 300 miles from each other at opposite ends of State Highway 1. Driving toward each other on State Highway 1, they met in the central part of the state. Joey left home at 9:30 a.m. traveling at an average speed of 50 mph and Mike left home at 10:00 a.m. traveling at an average speed of 60 mph. At what time that afternoon did they meet?

27. ______ : ______ (p.m.)

28. Four fair coins are to be flipped. What is the probability that all four will be heads or all four will be tails? Express your answer as a common fraction.

28. ____________

29. The arithmetic mean of eight positive integers is 7. If one of the eight integers is removed, the mean becomes 6. What is the value of the integer that is removed?

29. ____________

30. What is the smallest number of people you would need to have in a room if you wanted to be certain that at least 25 of them have the same birth month?

30. ____________ (people)

31. On her previous five attempts Sarah had achieved times, in seconds, of 86, 94, 97, 88 and 96, for swimming 50 meters. After her sixth try she brought her median time down to 92 seconds. What was her time, in seconds, for her sixth attempt?

31. ____________ (seconds)

32. What is the integer value of $78^2 + 78 \times 22 + 22^2$?

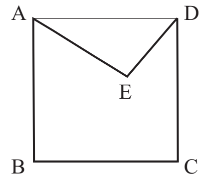
32. ____________

33. Marty bought a tie that was on sale for 20% off, and he used a coupon that took off an additional 40% from the sale price. His son Jerry bought a tie that was on sale for 60% off with no coupon. If both ties originally cost \$50, how much more did Marty spend on a tie than Jerry, in dollars?

33. ____________ (dollars)

34. What is one-half of the reciprocal of $\frac{7}{\sqrt{98}}$? Express your answer in the form $\frac{\sqrt{a}}{b}$ where $\sqrt{a}$ is in simplest radical form. 34. ________________

35. Quadrilateral ABCD is a square, and segment AE is perpendicular to segment ED. If AE = 8 units and DE = 6 units, what is the area of pentagon AEDCB, in square units? 35. ________________ (sq units)



36. A 2 × 2 × 2 cube is removed from each corner of an 8 × 8 × 8 cube. What fraction of the original cube remains? Express your answer as a common fraction. 36. ________________

37. What is the sum of the roots of $x^2 - 4x + 3 = 0$? 37. ________________

38. If the sum of $0.\overline{4} + 0.0\overline{4} + 0.00\overline{4}$ is written as a fraction with a denominator of 300, what is the numerator? 38. ________________

39. How many positive common fractions with a value less than $\frac{1}{2}$ have 37 as their denominator? 39. ________________ (fractions)

40. If $\frac{4}{3}(r + s + t) = 12$, what is the average of r, s and t? 40. ________________

41. The measure of the supplement of angle A is six times the measure of the complement of angle A. What is the measure, in degrees, of angle A? 41. ________________ (degrees)

42. In a survey of 100 students who watch television, 21 watch American Idol, 39 watch Lost, and eight watch both. How many of the students surveyed watch neither show? 42. ________________ (students)

43. The area of an equilateral triangle is numerically equal to the length of one of its sides. What is the perimeter of the triangle, in units? Express your answer in simplest radical form. 43. ________________ (units)

44. When the sum of the reciprocals of two distinct positive integers is divided by the sum of the two integers, the result is $\frac{1}{25}$. What is the sum of the two integers? 44. ________________

45. If x varies inversely with y and y varies directly with the square of z, then by what positive factor is z multiplied when x is multiplied by one-fourth? 45. ________________

46. To make orange juice from concentrate, the directions call for mixing one 12-ounce can of concentrate with five 12-ounce cans of water. A serving of orange juice is 8 ounces. If only whole cans can be used, what is the minimum number of cans of concentrate that Willy will need for at least 130 servings?

46. ____________ (cans)

47. The side length of square A is 36 cm. The side length of square B is 42 cm. What is the ratio of the area of square A to the area of square B? Express your answer as a common fraction.

47. ____________

48. If n is the smallest integer greater than the reciprocal of 0.272727, what is n?

48. ____________

49. The real numbers x, y and z satisfy the equations $x + 2y + 3z = 950$ and $3x + 2y + z = 1450$. What is the average of x, y and z?

49. ____________

50. How many integers between 211 and 2101 are multiples of 7?

50. ____________ (integers)

51. If two numbers will be randomly chosen without replacement from {3, 4, 5, 6}, what is the probability that their product will be a multiple of 9? Express your answer as a common fraction.

51. ____________

52. For every 3°C rise in temperature, the volume of a certain gas increases by 4 cubic centimeters. If 50 cubic centimeters of this gas at –8°C is heated to 28°C, by what percent does the volume increase?

52. ____________ (percent)

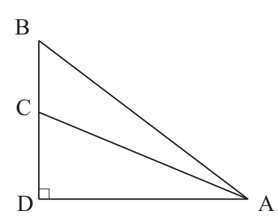
53. The three-digit integer 1AB equals 1! + A! + B!. What is A + B?

53. ____________

54. A bag contains only blue marbles, green marbles and 24 red marbles. If the probability of drawing a blue marble is $\frac{1}{2}$ and the probability of drawing a green marble is $\frac{1}{8}$, how many marbles are in the bag?

54. ____________ (marbles)

55. What is the area, in square units, of triangle ABC in the figure shown if points A, B, C and D are coplanar, angle D is a right angle, AC = 13, AB = 15 and DC = 5?



55. ____________ (sq units)

56. The sum of three consecutive integers is greater than 66. What is the smallest possible product of the largest and smallest of these integers?

56. ____________

57. If $g(x) = 3x + 7$ and $f(x) = 5x - 9$, what is the value of $f(g(8))$?

57. ____________

58. How many diagonals does a convex polygon with 23 sides have?

58. __________ (diagonals)

59. What is the sum of all real solutions to the equation: $\sqrt{x+\sqrt{9-x}} = 3$?

59. __________

60. Mr. George has 25% more students this year than he had last year. He has 120 students this year. How many students did he have last year?

60. __________ (students)

61. If $(x^2 - 5y)^3$ is written in expanded form, what is the coefficient of the x^2y^2 term?

61. __________

62. What is the smallest prime that is the sum of four different, positive composite integers?

62. __________

63. Triangle ABC is an isosceles triangle with side lengths of 25, 25 and 48 centimeters. What is the area of triangle ABC, in square centimeters?

63. __________ (sq cm)

64. A line passes through A(8, 4) and B(–12, –11). What is the value of x if (x, –5) is on the line?

64. __________

65. Let # be the relation defined by A # B = $A^2 + B^2$. If A # 5 = 169, what is the positive value of A?

65. __________

66. Triangle ABC with vertices at A(1, 1), B(1, –2) and C(5, –2) is translated up 3 units and then dilated with respect to the origin by a factor of 2. What are the new coordinates of point C? Express your answer as an ordered pair.

66. (,)

67. Major Domo wishes to donate a sum of money to each of his three favorite charities. His total donation is to be divided among the charities in a ratio of 5:3:2. If his total gift is $20,000, what is the difference, in dollars, between the largest donation and the smallest donation?

67. __________ (dollars)

68. If n is a positive integer, what is the smallest value of n for which $\sqrt{\frac{8n}{5}}$ is an integer?

68. __________

69. Triangle ABC is drawn inside regular hexagon ABCDEF. What is the ratio of the area of triangle ABC to the area of the hexagon? Express your answer as a common fraction.

69. __________

70. Jamie has 2 dimes, 4 nickels and 8 pennies. In how many different ways can she make 26¢?

70. ____________ (ways)

71. What is the product of all integer values of x for which the value of $|x^2 - 9|$ is a prime number?

71. ____________

72. If a is $\frac{2}{3}$ of b and b is $\frac{1}{4}$ of c, what fraction of c is a? Express your answer as a common fraction.

72. ____________

73. The measure of an exterior angle of a triangle is 75 degrees. If one of the non-adjacent interior angles measures 28 degrees, what is the number of degrees in the other non-adjacent interior angle?

73. ____________ (degrees)

74. If the radius of a circle is increased by 30%, by what percent is the area increased?

74. ____________ (percent)

75. Winnifred has 400 wooden blocks that are exactly 1-inch cubes. What is the length, in inches, of a face of the largest cubical box that Winnifred could fill using these blocks?

75. ____________ (inches)

76. A grocer sells sugar in 5-kg and 3-kg bags. Yesterday he sold 600 kg of sugar using the same number of 5-kg bags as 3-kg bags. What is the total number of bags of sugar that were sold by the grocer yesterday?

76. ____________ (bags)

77. If four times k is added to 8, the result is 44. What is the value of k^2?

77. ____________

78. What is the sum of the distinct prime factors of the integer 385?

78. ____________

79. If $64^5 = 32^x$, what is the value of 2^{-x}? Express your answer as a common fraction.

79. ____________

80. Given that 10 is the arithmetic mean of the set $\{6, 13, 18, 4, x\}$, what is the value of x?

80. ____________

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Answer Key

The appropriate units (or their abbreviations) are provided in the answer blanks.

Note to coordinators: Answers to the Tiebreaker Round problems appear in the Tiebreaker Round Booklet.

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Sprint Round

1. 240 pounds
2. 110 degrees
3. 450
4. 12 runs
5. 3 items
6. $2\frac{1}{2}$ inches
7. 42
8. 21 moons
9. 61
10. 2
11. 5 miles
12. 27
13. 7 home runs
14. 5
15. 4 turns
16. 84 %
17. 20 cm
18. 57
19. $\frac{3}{5}$

Sprint Round

20. 16

21. −10

22. 2674 elements

23. 101

24. 100 sq units

25. 35 inches

26. $\frac{8}{21}$

27. 28 sq units

28. 11:15 a.m.

29. 18

30. $8\sqrt{5}$ sq units

Target Round

1. 64 regions

2. 4020

3. 7.1 sq inches

4. 366 dogs

5. 47.25 sq meters

6. April 24

7. 7.5 points

8. 3

Team Round

1. (5, 3)

2. 7

3. 60 %

4. 110 minutes

5. $\frac{3}{2}$

6. 8:15 p.m.

7. 4 cm

8. 640 area codes

9. 6729

10. 4 %

Countdown Round

1. 102 (minutes)
2. 12
3. 505 (students)
4. 27 (times)
5. 131
6. 51
7. 225 (sq units)
8. $\frac{1}{2}$
9. 34 (units)
10. $\frac{1}{8}$
11. 23
12. 25
13. 21
14. 13
15. 16
16. 30
17. 6
18. 120 (degrees)
19. 42 (marbles)
20. 18 (sq units)
21. 3
22. 9 (minutes)
23. 30
24. 700
25. 120 (triangles)
26. 1000 (dollars)
27. 6 (weeks)
28. 581
29. 48 (cubes)
30. 15 (games)
31. 44 (percent)
32. 6
33. −9
34. $2222\frac{1}{5}$
35. $\frac{13}{8}$
36. 4
37. 15 (zeros)
38. 89.1
39. 10 (integers)
40. 240

Countdown Round

41. $\frac{11}{18}$
42. 96 (units)
43. 32 (years)
44. $\frac{5}{54}$
45. 142 (integers)
46. 50
47. (−1, −3)
48. 5 (units)
49. 5
50. 11 (regions)
51. 7 (people)
52. $\frac{1}{96}$
53. 65 (cm)
54. 12
55. 132 (degrees)
56. 3
57. 1006
58. 1990
59. 150
60. 225 (sq inches)
61. $\frac{4}{3}$
62. 32 (times)
63. 0.5 (units)
64. 11 (rows)
65. 41
66. 50 (units)
67. (5, 7)
68. 10 (percent)
69. 899
70. 52
71. 10 (divisors)
72. Sunday
73. 250
74. 2π (sq units)
75. 8 (factors)
76. 25
77. 7 (factors)
78. $10k$
79. $\frac{2}{3}$
80. 4 (sides)

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2011
■ Chapter Competition ■
Team Round
Problems 1–10

School ______________________________

Team Members ______________________________ , Captain

____

____

____

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This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

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1. Three vertices of square ABCD are located at the points A(5, −1), B(7, 1) and D(3, 1). What are the coordinates of point C? Express your answer as an ordered pair.

1. ______ (,) ______

2. What is the units digit when 3^{2011} is multiplied out?

2. ________________

3. An ad indicates that a store is offering a sale on carpet that is priced per square foot. Mr. Adams said that the sale price he paid for a piece of carpet measuring 10-feet by 12-feet was the same as the non-sale price for a piece of the same type of carpet measuring 6-feet by 8-feet. What percent off the original carpet price is the store's sale price?

3. ________________ %

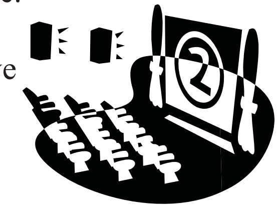
4. Zeta always runs around the track at a rate of 30 laps per 75 minutes, and Ray always runs around the track at a rate of 20 laps per 40 minutes. If they start at the same time, how many minutes will it take them to run a combined distance of 99 laps?

4. ____________ minutes

5. Line L passes through the points $(0, \frac{1}{2})$ and $(4, k)$ and is perpendicular to the line $y = -4x + 5$. What is the value of k? Express your answer as a common fraction.

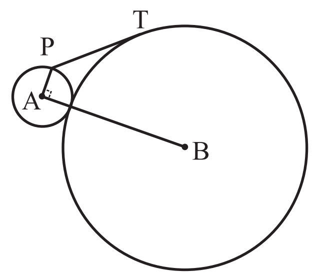
5. ________________

6. At the MC Theater, the first showing of a particular movie starts at 11:15 a.m. Each showing begins with 10 minutes of previews followed by the 105-minute featured movie. Twenty minutes are needed between showings to clean the theater and seat the audience. There are five showings of the movie prior to midnight. According to a 12-hour clock, what is the earliest possible time the last showing could begin?



6. ________ : ________ p.m.

7. Circles A and B are externally tangent. Angle PAB is a right angle. Segment PT is tangent to circle B at T. If the radius of circle A is 1 cm and the radius of circle B is 7 cm, what is the length of segment PT?



7. ____________ cm

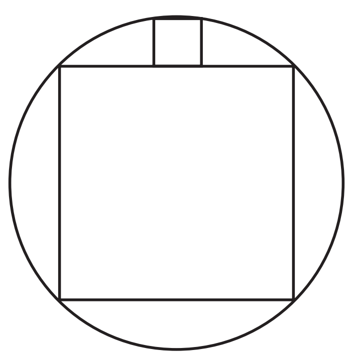
8. When 3-digit area codes were first used, the first digit could not be a 0 or a 1, and the second digit could *only* be a 0 or a 1. There were no restrictions on the third digit. In 1995 the restrictions on the second digit were lifted. How many more 3-digit area codes are possible today than were possible prior to 1995?

8. ____________ area codes

9. Suppose each of the nine digits {1, 2, 3, … , 9} is used exactly once as a digit in either the four-digit positive integer a or the five-digit positive integer b. What is the smallest possible value of a if $\frac{a}{b} = \frac{1}{2}$?

9. ____________

10. A square is inscribed in a circle. A smaller square has one side coinciding with a side of the larger square and has two vertices on the circle, as shown. What percent of the area of the larger square is the area of the smaller square?



10. ____________ %

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2011
■ Chapter Competition ■
Target Round
Problems 1 and 2

Name ____

School ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

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1. How many of the smallest, non-overlapping triangular regions are there in Figure 4 of the sequence whose first three figures are shown?

1. ______________ regions

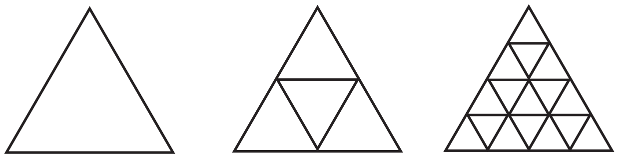

Figure 1 Figure 2 Figure 3

2. Given arithmetic sequences A and B shown, what is the positive difference between the 2011th term of sequence A and the 2011th term of sequence B?

2. ______________

Sequence A: 1, 5, 9, 13, . . .
Sequence B: 1, 7, 13, 19, . . .

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2011
■ Chapter Competition ■
Target Round
Problems 3 and 4

Name ____

School ____

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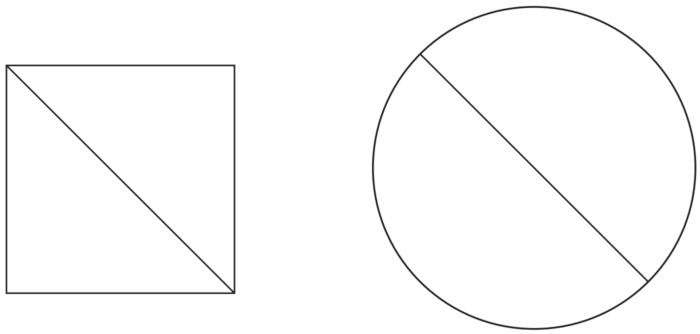
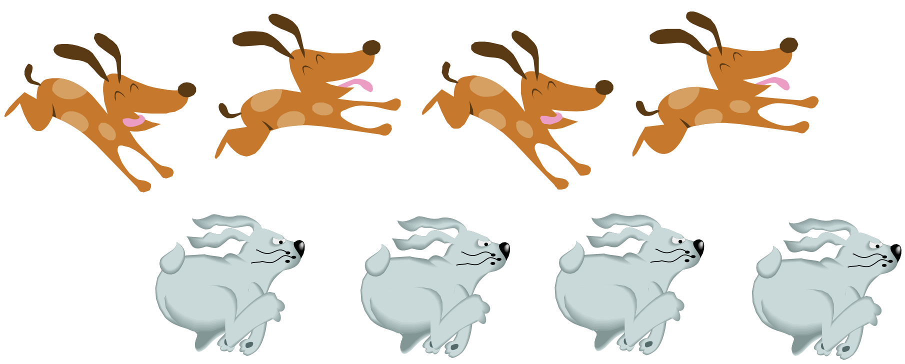
3. The diagonal of a particular square is 5 inches. The diameter of a particular circle is also 5 inches. By how many square inches is the area of the circle greater than the area of square? Express your answer as a decimal to the nearest tenth.

3. ____________ sq inches



4. In a small town all the registered pets are either rabbits or dogs. In a recent parade 65% of the 840 registered pets participated. If 180 registered rabbits participated, how many registered dogs participated?

4. ____________ dogs



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2011
■ Chapter Competition ■
Target Round
Problems 5 and 6

Name ____

School ____

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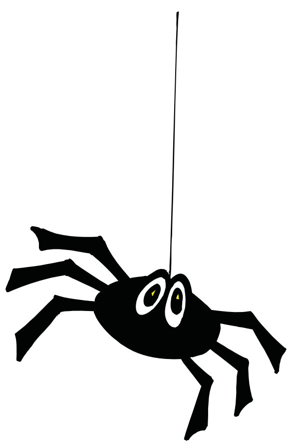
5. On a scale drawing showing the floor plan of an apartment, the rectangular living room measures 3.5 cm by 6 cm. If 1 centimeter represents 1.5 meters, what is the area of the floor of the actual living room, in square meters? Express your answer as a decimal to the nearest hundredth.

5. ____________ sq meters

6. Each day, the itsy-bitsy spider will travel 5 feet up the waterspout of a building. Each night, it will rain and wash the spider 3 feet down the waterspout. If the itsy-bitsy spider starts traveling up the 50-foot waterspout on the morning of April 1, on what date will it first reach the top of the spout?

6. April ____________



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2011
■ Chapter Competition ■
Target Round
Problems 7 and 8

Name ____

School ____

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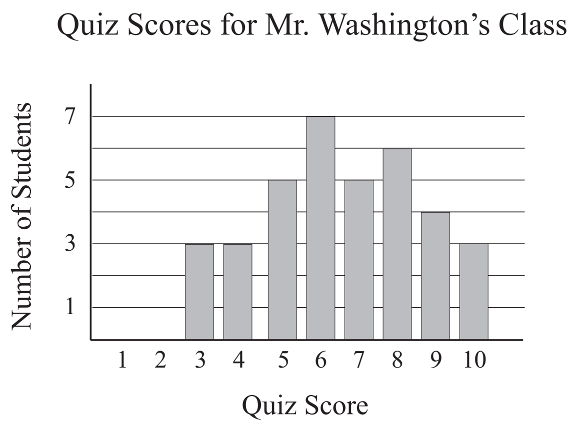
7. The following graph shows the quiz scores Mr. Washington recorded for his 10-question algebra quiz. Each question was worth one point. After looking at this graph and knowing that only three students answered #7 correctly, he decided to give full credit for #7 to every student. With this adjustment, what was the new mean quiz score? Express your answer as a decimal to the nearest tenth.

7. ______________ points



8. Square ABCD has side length 1 unit. Points E and F are on sides AB and CB, respectively, with AE = CF. When the square is folded along the lines DE and DF, sides AD and CD coincide and lie on diagonal BD. The length of segment AE can be expressed in the form $\sqrt{k} - m$ units. What is the integer value of $k + m$?

8. ______________

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2011
■ State Competition ■
Team Round
Problems 1–10

School ______________________________

Chapter ______________________________

Team Members ______________________________, Captain

____

____

____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

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1. Kevin, Cindi and Marcus have a total of 1020 widgets. Marcus has half the number of widgets that Cindi has. Kevin has 219 widgets. How many widgets does Cindi have?

1. ______________ widgets

2. Emily is thinking of a positive three-digit integer. All of the digits in her number are prime and distinct. The digits also increase in order from left to right. How many possibilities are there for Emily's number?

2. ______________ possibilities

3. As a special promotion for the grand opening of the sporting goods store, it was advertised that every 25th customer would receive a free Frisbee and every 35th customer would receive a free baseball hat. Johnny was the first customer to receive both a Frisbee and a hat. How many customers had entered the store before Johnny?

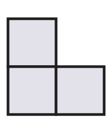
3. ______________ customers

4. A train travels at a constant rate of 55 miles per hour through a tunnel. Forty-five seconds after the front of the train enters the tunnel the front of the train exits the tunnel. How many feet long is the tunnel?

4. ______________ feet

5. The L-shaped piece shown will be placed on the grid so that it covers exactly three unit squares of the grid. The sum of the numbers in the grid's covered three unit squares will be S. If rotating the L-shaped piece is permitted, what is the sum of all the values of S for all possible placements on this grid of the L-shaped piece?

5. ______________



1	2	3
4	5	6
7	8	9

6. An urn contains marbles of four colors (red, yellow, blue and green). All but 45 of the marbles are red; all but 45 are yellow; all but 45 are blue; and all but 60 are green. How many of the marbles are green?

6. ____________ marbles

7. Mrs. Jackson baked a batch of cookies. If she makes bags of cookies with 3 cookies in each bag, 2 cookies are left over. If she makes bags with 5 cookies in a bag, no cookies are left over. If she makes bags with 8 cookies in each bag, 6 cookies are left over. What is the fewest number of cookies Mrs. Jackson could have baked?

7. ____________ cookies

8. Let M be the midpoint of the segment FG. Let A and B be points coplanar to points F and G. Points A and B are positioned on the same side of the line containing segment FG such that triangles FMA and MGB are equilateral. The lines FB and GA intersect at point K. What is the measure of angle GKB?

8. ____________ degrees

9. A bag contains five red marbles, three blue marbles and two green marbles. Six marbles are to be drawn from the bag, replacing each one after it is drawn. What is the probability that two marbles of each color will be drawn? Express your answer as a common fraction.

9. ____________

10. The geometric mean of two positive numbers a and b is $\sqrt{ab}$. The third term of an arithmetic sequence of positive numbers, in which the difference between the terms is not zero, is the geometric mean of the first and eleventh terms. What is the ratio of the second term to the first term of the sequence? Express your answer as a common fraction.

10. ____________

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Countdown Round
Problems 1–80

This section contains problems to be used in the Countdown Round.

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1. Meera began an exam at 11:37 a.m. and finished at 1:19 p.m. the same day. How many minutes did she take to complete the exam?

1. ______________ (minutes)

2. If $3x + 8 = 23$, what is the value of $3x - 3$?

2. ______________

3. Three out of every five students at Pythagoras Middle School went to the Spring Fling. If 202 students did not go to the Spring Fling, how many students attend this school?

3. ______________ (students)

4. Each edge length of a cube is tripled. How many times the volume of the original cube is the volume of the new cube?

4. ______________ (times)

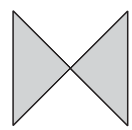
5. What is the sum of the three prime numbers between 40 and 50?

5. ______________

6. If x is an integer such that $9 > x > 2$, what is the greatest value of $5x + 11$?

6. ______________

7. Two congruent isosceles right triangles are joined to create this figure. Each leg of each triangle measures 15 units. What is the total area of the figure, in square units?



7. ______________ (sq units)

8. If the sum of the digits of a positive two-digit integer is 15, what is the probability that one of the digits is 9? Express your answer as a common fraction.

8. ______________

9. A right triangle has a hypotenuse of 26 units. If one leg is 4 more than twice the other, what is the sum of the lengths of the legs, in units?

9. ______________ (units)

10. The length of a right, rectangular prism is doubled, its width is quadrupled and its height is unchanged. What is the ratio of the original volume to the new volume? Express your answer as a common fraction.

10. ______________

11. If $n + (1/n) = 5$, what is the value of $n^2 + (1/n^2)$?

11. ______________

12. The endpoints of segment BC are B(7, 4) and C(3, 6). What is the product of the coordinates of the midpoint of segment BC?

12. ______________

13. If $m \blacktriangle n = 2m + n$ and $m \blacktriangledown n = 4m - n$, what is the value of $(2 \blacktriangle (5 \blacktriangledown 3))$?

13. ______________

14. Given $a + b = 23$, $b + c = 35$ and $a + c = 20$, what is the average of a, b and c?

14. ______________

15. What is the sum of the distinct prime divisors of $11 + 11^2$?

15. ______________

16. The product of three consecutive integers is 990. What is their sum? 16. ________________

17. If $3^n = 9^3$, what is the integer value of n? 17. ________________

18. If a 12-hour analog clock reads 8:00, what is the degree measure of the smaller angle formed by the minute and hour hands? 18. ________________ (degrees)

19. The probability that Joshua will draw a blue marble at random from a bag containing yellow marbles and blue marbles is $\frac{2}{3}$. If 14 of the marbles are yellow, how many marbles are in the bag before any marbles are removed? 19. ________________ (marbles)

20. What is the area, in square units, of a triangle whose vertices are at (4, –1), (10, 3) and (4, 5)? 20. ________________ (sq units)

21. What is the value of the digit K that will make the number 481,5K6 divisible by 2, 3, 4 and 9? 21. ________________

22. Jamie can paint a wall in 20 minutes, and Ellie can paint the same size wall in 15 minutes. If they work together, how many minutes will it take them to paint one wall of this size? Express your answer to the nearest whole minute. 22. ________________ (minutes)

23. When four consecutive integers are added, the result is –18. What is the greatest product that can be obtained by multiplying two of these four integers? 23. ________________

24. The GCF of two numbers is 70. Their product is 49,000. What is the LCM of the two numbers? 24. ________________

25. Ten distinct points are arranged on a circle. How many different triangles are there whose three vertices are among those ten points? 25. ________________ (triangles)

26. A store advertised a computer at 50% off plus an additional 20% off the sale price. On top of that, Erich had a coupon for 10% off the final price. If he paid $360 before taxes, what was the original price of the computer? 26. ________________ (dollars)

27. Mellie bought a box of dog biscuits for her golden retriever Karma. She gives Karma 3 dog biscuits a day. After ten days she counted 96 biscuits left in the box. At this rate, for how many weeks will a full box of dog biscuits feed Karma? 27. ________________ (weeks)

28. Miss Minerva challenges her class to guess her secret positive integer. She gives them these clues: The sum of the digits of this three-digit number is 14. The tens digit is the cube of a prime number. The units digit is one-fifth of the hundreds digit. What is Miss Minerva's number?

28. ______________

29. What is the maximum number of 2-inch by 2-inch by 2-inch cubes that can be placed in a box that measures 7-inches by 8-inches by 9-inches?

29. ______________ (cubes)

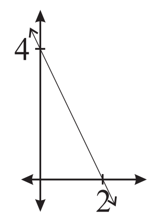
30. How many different games need to be played so that 6 teams each play each other exactly once?

30. ______________ (games)

31. Troy started with a solid 3 by 3 by 3 cube. To each face of this 3 by 3 by 3 cube, he attached one unit cube by pasting one face of the unit cube to the face of the larger cube. What is the percent of increase in the total surface area of this structure? Express your answer to the nearest whole percent.

31. ______________ (percent)

32. The graph of the line $y = mx + b$ is shown. What is the value of $b - m$?



32. ______________

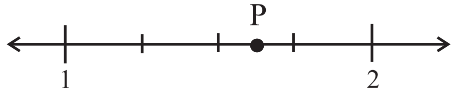
33. If $4x + y = -12$ and $x + 4y = -3$, what is the value of $3x + 3y$?

33. ______________

34. What is the arithmetic mean of the first five positive integers that are powers of 10? Express your answer as a mixed number.

34. ______________

35. The number line shown has uniformly-spaced markings. If point P is equidistant from its two closest markings, what is the coordinate of P? Express your answer as a common fraction.



35. ______________

36. What is the smallest positive integer that can be added to the sum of the consecutive integers $(1 + 2 + \ldots + 10 + 11)$ so that the resulting total is divisible by 5?

36. ______________

37. If the product $(2^2)(3^3)(4^4)(5^5)(6^6)(7^7)(8^8)(9^9)(10^{10})$ is written as an integer, how many zeros are to the right of the right-most non-zero digit?

37. ______________ (zeros)

38. When the decimal point is moved one place to the left in a two-digit positive integer, the difference between these two numbers is 72.9. What is the sum of the two numbers? Express your answer as a decimal to the nearest tenth.

38. ______________

39. How many positive two-digit integers are 27 more than the sum of their digits? 39. ____________ (integers)

40. When 60 is increased by 60% it then is equal to what number decreased by 60%? 40. ____________

41. Two fair, standard six-sided dice are rolled. What is the probability that one of the two numbers is a factor of the other number? Express your answer as a common fraction. 41. ____________

42. Given a right triangle whose side lengths are all integer multiples of 8, how many units are in the smallest possible perimeter of such a triangle? 42. ____________ (units)

43. The average age of six people in a room is 30 years. A 20-year-old person leaves the room. What is the average age of the five remaining people, in years? 43. ____________ (years)

44. If the ratio of $3x$ to y is $\frac{5}{6}$, what is the ratio of x to $3y$? Express your answer as a common fraction. 44. ____________

45. How many integers x, such that $1 \le x \le 1000$, are multiples of 7? 45. ____________ (integers)

46. What is the greatest common factor of 400 and 250? 46. ____________

47. Right triangle ABC has vertices at A(1, 3), B(5, 3) and C(5, 1). If this triangle is reflected over the x-axis and the resulting triangle is then reflected over the y-axis, what are the coordinates of the final image of point A? Express your answer as an ordered pair 47. (,)

48. The width of a rectangle is equal to the side length of a square. If the ratio of the area of the square to the area of the rectangle is 1:3, and the length of the rectangle is 15 units, what is the side length of the square, in units? 48. ____________ (units)

49. What is the product of $\sqrt[4]{25}$ and $\sqrt[6]{125}$? 49. ____________

50. What is the maximum number of non-overlapping interior regions that can be created by four chords of a circle? 50. ____________ (regions)

51. Everyone at the party shook hands with everyone else exactly once. If there were a total of 21 handshakes, how many people were at the party? 51. ____________ (people)

52. What fraction of 2 days is 30 minutes? Express your answer as a common fraction. 52. ________________

53. An isosceles triangle has side lengths 8 cm, 8 cm and 10 cm. The longest side of a similar triangle is 25 cm. What is the perimeter of the larger triangle, in centimeters? 53. ________________ (cm)

54. The integer A67,83B where A and B are the first and last digits of the integer, respectively, is divisible by 15. What is the largest possible sum of A and B? 54. ________________

55. What is the degree measure of the supplement of the complement of a 42-degree angle? 55. ________________ (degrees)

56. The prime factorization of N is $3^3 \times 7^R$. If N has exactly 16 positive integer factors, what is the value of R? 56. ________________

57. What is the value of the product $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)...\left(1+\frac{1}{2011}\right)$? 57. ________________

58. What is the sum of the two products 199 × 7 and 199 × 3? 58. ________________

59. If $\frac{2}{5}$ of Y is 36, what is $\frac{5}{3}$ of Y? 59. ________________

60. A square and a right triangle have equal perimeters. The legs of the right triangle are 20 inches and 15 inches. What is the area of the square, in square inches? 60. ________________ (sq inches)

61. If $3^{3x} = 81$, then what is the value of x? Express your answer as a common fraction. 61. ________________

62. If the numerator of a positive fraction is multiplied by 8 and the denominator is divided by 4, how many times larger does the fraction become? 62. ________________ (times)

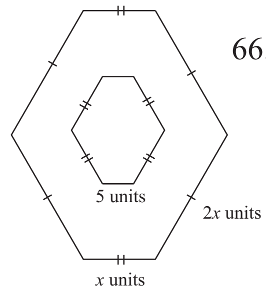
63. If the legs of a right triangle measure 0.3 units and 0.4 units in length, what is the length of the hypotenuse, in units? Express your answer as a decimal to the nearest tenth. 63. ________________ (units)

64. In a marching band, there are 68 woodwinds, 85 brass and 34 percussion players. When marching, the band must have the same number of players in each row, and each row must contain only players of one type of instrument. What is the fewest number of rows that can be formed when marching? 64. ________________ (rows)

65. The product of two consecutive integers is less than or equal to 448. What is the largest possible sum of two such integers?

65. ________________

66. The two hexagons shown are similar. Each segment marked with a double tick mark is x units long and each segment marked with a single tick mark is $2x$ units long. The two unmarked segments are each 5 units long. What is the positive difference of the perimeters of the two hexagons, in units?

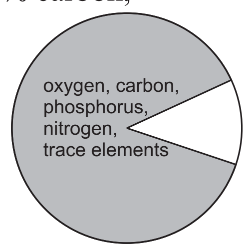


66. ________________ (units)

67. Circle O is located on the coordinate plane with center at (2, 3). One endpoint of a diameter is at (–1,–1). What are the coordinates of the other endpoint of this diameter? Express your answer as an ordered pair.

67. (,)

68. The human body is composed of 65% oxygen, 18% carbon, 1% phosphorus, 3% nitrogen and 1% trace elements. These are all shown in the gray region. Hydrogen and calcium make up the remainder of the pie graph. If there is five times as much hydrogen as calcium, what percent of the human body is hydrogen?



68. ________________ (percent)

69. What is the largest product of two primes that have a sum of 60?

69. ________________

70. The average of four consecutive odd integers is 26. What is the sum of the least and greatest of these integers?

70. ________________

71. How many positive integer divisors does 48 have?

71. ________________ (divisors)

72. One Saturday Gary noticed that his baby sister was 111 days old. On what day of the week was his baby sister born?

72. ________________

73. When a positive integer is divided by 7, the result is 35 with a remainder of 5. What is the integer?

73. ________________

74. The area of square ABCD is 8 square units. What is the area, in square units, of circle O, which is inscribed in the square? Express your answer in terms of π.

74. ________________ (sq units)

75. How many positive factors does 30 have?

75. ________________ (factors)

76. If 15 is $\frac{3}{5}$ of N, what is the value of N?

76. ________________

77. The integer 144 has 15 factors. How many factors of 144 are greater than $\sqrt{144}$?

77. ________________ (factors)

78. What is the sum of 10 consecutive positive integers that have a mean of k? Express your answer in terms of k.

78. ________________

79. At Washington Middle School there are 900 students. If $\frac{3}{5}$ of the students are boys, what is the ratio of girls to boys? Express your answer as a common fraction.

79. ________________

80. The measure of an exterior angle of a regular polygon is 90 degrees. How many sides does the polygon have?

80. ________________ (sides)

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Answer Key

The appropriate units (or their abbreviations) are provided in the answer blanks.

Note to coordinators: Answers to the Tiebreaker Round problems appear in the Tiebreaker Round Booklet.

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Sprint Round

1. 45 bottles
2. 8 coins
3. 6π sq in
4. 48 %
5. 5
6. $\frac{3}{2}$ years
7. \$5 or \$5.00
8. 9
9. 30 mph
10. 36 parallelograms
11. 80 sq cm
12. 11 combinations
13. 92 points
14. 6 integers
15. –4
16. $\frac{3}{\pi}$
17. $\frac{1}{3}$
18. 275

Sprint Round

19. 45 degrees

20. 25π sq in

21. $\frac{7}{6}$

22. 20

23. $\frac{19}{37}$

24. 5

25. 7

26. $40\frac{1}{2}$

27. $\frac{\sqrt{3}}{3}$

28. 11

29. 80 %

30. 529

Target Round

1. 123

2. \$240 or \$240.00

3. 20 %

4. 1993

5. 22 sums

6. 82

7. 0.36

8. 45

Team Round

1. 534 widgets

2. 4 possibilities

3. 174 customers

4. 3630 feet

5. 240

6. 5 marbles

7. 110 cookies

8. 60 degrees

9. $\frac{81}{1000}$

10. $\frac{5}{2}$

Countdown Round

1. 146,410 (dollars)
2. 288
3. 36 (sq units)
4. 32 (sq units)
5. 234,000 (people)
6. 40
7. 17 (integers)
8. 29
9. 79 (integers)
10. 100,260
11. 15 (minutes)
12. 13 (units)
13. 75 (percent)
14. 125 (units)
15. $\frac{1}{5}$
16. 5
17. 7 (hours)
18. 20 (units)
19. 1
20. 144 (sq units)
21. 4
22. $\frac{4}{5}$
23. 36 (percent)
24. 15π (sq units)
25. 37
26. 40 (units)
27. 12:30 (p.m.)
28. $\frac{1}{8}$
29. 14
30. 289 (people)
31. 90 (seconds)
32. 8284
33. 4 (dollars)
34. $\frac{\sqrt{2}}{2}$
35. 76 (sq units)
36. $\frac{7}{8}$
37. 4
38. 148
39. 18 (fractions)
40. 3

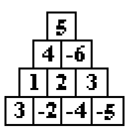
Countdown Round

41. 72 (degrees)
42. 48 (students)
43. $4\sqrt{3}$ (units)
44. 26
45. 2
46. 15 (cans)
47. $\frac{36}{49}$
48. 4
49. 200
50. 270 (integers)
51. $\frac{1}{6}$
52. 96 (percent)
53. 9
54. 64 (marbles)
55. 24 (sq units)
56. 528
57. 146
58. 230 (diagonals)
59. 17
60. 96 (students)
61. 75
62. 29
63. 168 (sq cm)
64. –4
65. 12
66. (10, 2)
67. 6000 (dollars)
68. 10
69. $\frac{1}{6}$
70. 5 (ways)
71. 64
72. $\frac{1}{6}$
73. 47 (degrees)
74. 69 (percent)
75. 7 (inches)
76. 150 (bags)
77. 81
78. 23
79. $\frac{1}{64}$
80. 9

2011
MATHCOUNTS CHAPTER
SPRINT ROUND

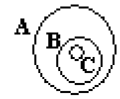
1. A woodchuck chucks 60 pounds of wood in 1.5 days. 6 days is 4 times 1.5 days. 60 × 4 = 240 **Ans.**

2. Half the temperature of Papa's oatmeal is 20º cooler than Baby's oatmeal. Papa's oatmeal is 180º.
Let P = the temperature of Papa's oatmeal.
Let B = the temperature of Baby's oatmeal
$\frac{1}{2}$P = B – 20
P = 180
$\frac{1}{2}$ × 180 = B – 20
90 = B – 20
B = 110 **Ans.**

3. One number is to be selected from each of the four rows of this Number Wall. We are asked to find the largest possible product of any such four numbers.


The first and third rows (from the top) are all positive. The second and fourth rows are mixed. The largest absolute values from the second and fourth rows come from the negative numbers. Therefore, if we choose negative numbers in the second and fourth rows, we will get a positive number.
First row: 5
Second row: -6
Third row: 3
Fourth row: -5
5 × –6 × 3 × –5 =
–30 × –15 = 450 **Ans.**

4. Hannah scored 75% of April's runs. April scored 16 runs.
75% = $\frac{3}{4}$ and $\frac{3}{4}$ × 16 = 12 **Ans.**

5. Circle C has 7 items. 20 items are in Circle A but 10 of those items are not in Circle B. We are asked to find how many items are in Circle B, but not in Circle C.


Circle A has 20 items. But 10 of those items are not in Circle B. Therefore, there are 20 – 10 = 10 items in Circle B that are not in Circle A. But Circle C has 7 items. Therefore, there are 10 – 7 = 3 items that are in Circle B, but not in Circle C. 3 **Ans.**

6. A signature line is 4 in. long.
A $\frac{3}{4}$ - in. blank space is reserved at each end of the signature. So how much space is available for the signature?
$\frac{3}{4} \times 2 = \frac{6}{4} = \frac{3}{2}$
$4 - \frac{3}{2} = 2\frac{1}{2}$ **Ans.**

7. The operation $\otimes$ is defined as:
$a \otimes b = a^2 + b + 1$
$6 \otimes 5 = 6^2 + 5 + 1 =$
36 + 6 = 42 **Ans.**

8. Malton has twice as many moons as Planar. The number of Nero's moons is the cube of Malton's moons. Ufda has 4 more moons than Jir. Jir's moons are equal to double the number of Nero's moons plus the number of Planar's moons. Planar has 1 moon. How many moons does Ufda have?
Let M = the number of Malton's moons.
Let P = the number of Planar's moons.
Let N = the number of Nero's moons.
Let U = the number of Ufda's moons.
Let J = the number of Jir's moons.

We know that P = 1.
M = 2P = 2 × 1 = 2
$N = M^3 = 2^3 = 8$
J = 2N + P = 16 + 1 = 17
U = J + 4 = 17 + 4 = 21 **ANS.**

9. The sum of 3 consecutive prime numbers is 173. We are asked to find the largest of these numbers. First, let's find the mean of the 3 numbers.
   3x = 173
   x ≈ 58
   So let's look at the primes, say between 50 and 70. They are:
   53, 59, 61, 67
   The one's value in 173 is 3. I think I'll try 53 + 59 + 61 since 3 + 9 + 1 = 13.
   53 + 59 + 61 = 173 and the largest of the primes is 61. **Ans.**

10. $(3^x)(9) = 81$
    $3^x 3^2 = 81 = 3^4$
    x + 2 = 4
    x = 2 **Ans.**

11. Kenton walks for 60 min. at the rate of 3 mph. He then runs for 15 min. at the rate of 8 mph. So how far does he travel?
    60 min. at the rate of 3 mph is 3 miles.
    15 min = $\frac{1}{4}$ hour
    $\frac{1}{4}$ × 8 = 2 miles
    3 + 2 = 5 **Ans.**

12. x and y are each integers greater than 3 and less than 20. What is the sum of the three possible values of x that satisfy the equation $\frac{x}{y} = \frac{3}{4}$?
    Since x is greater than 3 we cannot consider $\frac{3}{4}$. The next fraction that is equivalent to $\frac{3}{4}$ is $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$.
    That's a good value. Next:
    $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$
    That's good too. Next:
    $\frac{3}{4} \times \frac{4}{4} = \frac{12}{16}$
    That's also good. Are there more?
    $\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$
    This one is not good because y = 20 and y < 20. Any other values will fail the requirements. Therefore, x = 6, 9, 12.
    6 + 9 + 12 = 27 **Ans.**

13. We are asked to find the average number of home runs hit by the players.
    6 players hit 6 home runs.
    6 × 6 = 36
    4 players hit 7 home runs.
    4 × 7 = 28
    3 players hit 8 home runs.
    3 × 8 = 24
    1 player hit 10 home runs.
    1 × 10 = 10
    There were a total of
    36 + 28 + 24 + 10 = 98
    home runs hit.
    The number of players is
    6 + 4 + 3 + 1 = 14 players.
    $\frac{98}{14} = 7$ **Ans.**

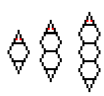
14. $\frac{5}{33} = 0.\overline{15}$
    The first digit to the right of the decimal point is 1. All odd digits to the right of the decimal point are 1. The second digit to the right of the decimal point is 5. All even digits to the right of the decimal point are 5. 92 is an even number. Therefore, 5 is the value of the 92^{nd} digit to the right of the decimal point. 5 **Ans.**

15. A player can earn either 3 points or 5 points on a turn. Capri has earned a total of 18 points. What is the fewest number of turns she could have taken?
    My first guess would be 4.
    5 + 5 + 5 + 3 = 18

3 + 3 + 3 + 3 + 3 + 3 = 18 but that's 6 turns.
4 **Ans.**

16. Fonks were originally priced at \$100. Then the price was increased by 20%. Finally, the price was decreased by 30%. We are asked to find the percentage of the original price that the fonk currently sells for. When the price was increased by 20% the fonk sold at 1.2 times the original price. When the price was decreased by 30% the fonk sold at 1.2 × .7 = 0.84 of the original price, or 84%. **Ans.**

17. A Growing Worm in Stage *n* contains *n* hexagons and two equilateral triangles. Because a side of the equilateral triangle is also a side of the hexagon, the sides of the hexagon are *s* and the sides of the equilateral triangle are *s*.



The perimeter of the Glowing Worm in Stage 1 is 8 and we are asked to find the perimeter of a Stage 4 Growing Worm.
In Stage 1, 4 sides of the hexagon are part of the perimeter as well as 2 sides of each equilateral triangle. That's a total of 8 sides.
8*s* = 8
s = 1
In the Stage 2 drawing, there are 4 × 2 = 8 sides of hexagons that are part of the perimeter and 2 sides of each equilateral triangle. That's a total of 8 + 4 = 12.
In the Stage 3 drawing, there are 4 × 3 = 12 sides of hexagons that are part of the perimeter and 2 sides of each equilateral triangle. That's a total of 12 + 4 = 16.
8, 12, 16. Obviously, the Stage 4 Glowing Worm has a perimeter of 20. **Ans.**

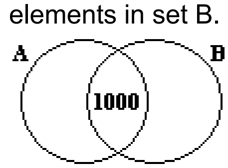
18. Each term of a sequence is one more than twice the term before it. The first term is 1. What is the sum of the first 5 terms?
The second term is (2 × 1) + 1 = 3.
The third term is (2 × 3) + 1 = 7.
The fourth term is (2 × 7) + 1 = 15.
The fifth term is (2 × 15) + 1 = 31.
31 + 15 + 7 + 3 + 1 = 57 **Ans.**

19. A fly buzzes randomly around a room 8' × 12' × 10' high. We are asked to find the probability that the fly is within 6' of the ceiling.
We can do that by dividing the volume of a room that is 8' × 12' × 6' high by the volume of the entire room.
$\frac{8\times12\times6}{8\times12\times10}=\frac{6}{10}=\frac{3}{5}$ **Ans.**

20. If 5 less than $\frac{3}{4}$ of an integer is the same as 5 more than $\frac{1}{8}$ of that integer, then what is the integer?
Let x = the integer.
$\frac{3}{4}x-5=\frac{1}{8}x+5$
$\frac{3}{4}x=\frac{1}{8}x+10$
$\frac{5}{8}x=10$
5x = 80
x = 16 **Ans.**

21. What is the sum of the negative integers that satisfy the inequality $2x-3\geq-11$?
$2x\geq-8$
$x\geq-4$
So x can be –4, –3, –2 and –1.
–4 + –3 + –2 + –1 = – 10 **Ans.**

22. The total number of elements in set A is twice the total number of elements in set B.



There are 3011 elements in the

union of A and B. The intersection of A and B is 1000 elements. We are asked to find the total number of elements in set A.
Let x + 1000 be the total number of elements in set A.
Let y + 1000 be the total number of elements in set B.
We also know that
x + y + 1000 = 3011
x + y = 2011
And, finally, we know that
2 × (y + 1000) = x + 1000
2y + 2000 = x + 1000
2y = x – 1000

$$y = \frac{x-1000}{2}$$

Substituting for y in
x + y = 2011, we get

$$x + \frac{x-1000}{2} = 2011$$

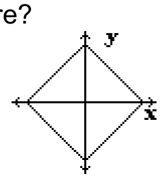
2x + x – 1000 = 4022
3x = 5022
x = 1674
1674 + 1000 = 2674 **Ans.**

23. The quotient of 2 consecutive positive integers is 1.02. So what is the sum of these 2 integers?
Since the quotient is more than 1, the numerator must be larger than the denominator.
Let x be the smaller number. Then

$$\frac{x+1}{x} = 1.02$$

x + 1 = 1.02x
1 = .02x
100 = 2x
x = 50
x + 1 = 51
50 + 51 = 101 **Ans.**

24. What is the area enclosed by the graph of $|x| + |2y| = 10$ shown here?



If x = 0, then

$$|2y| = 10$$

$$|y| = 5$$

So (0,5) and (0,-5) are the top and bottom coordinates of the area, respectively..
If y = 0 then $|x| = 10$ so
(-10,0) and (10,0) are the left and right coordinates of the area, respectively. Thus, the area is made up of 4 right triangles with legs of 5 and 10.

$$4 \times \left(\frac{1}{2} \times 5 \times 10\right) = 100$$ **Ans.**

25. Two similar right triangles have areas of 6 square inches and 150 square inches. The hypotenuse of the smaller triangle is 5 inches. We are asked to find the sum of the lengths of the legs of the larger triangle.
Clearly, the smaller triangle is a 3,4,5 right triangle.

$$\frac{1}{2} \times 3 \times 4 = 6$$

$\frac{1}{2}$ x = 150
x = 300
This is the product of the 2 legs of the larger triangle that are not the hypotenuse of the triangle.
The product of the 2 legs is 25 times the product of the two corresponding legs of the smaller triangle.
$25 = 5^2$ and each leg is 5 times the corresponding leg of the smaller triangle.
5 × 3 = 15
5 × 4 = 20
5 × 5 = 25
This is a 15, 20, 25 right triangle.
15 + 20 = 35 **Ans.**

26. A committee of 6 students is chosen at random from a group of 6 boys and 4 girls. What is the probability that the committee contains the same number of boys and girls?
Since the committee is composed of 6 people, to have the same number of each sex means 3 boys and 3 girls.

The number of combinations of 3 boys is

$$\frac{6!}{3!3!} = \frac{6\times5\times4}{3\times2\times1} = 20$$

The number of combinations of 3 girls is

$$\frac{4!}{3!1!} = 4$$

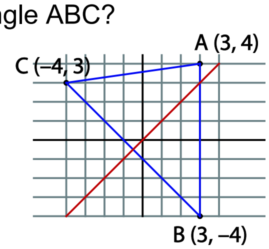
So, the number of combinations of 3 boys and 3 girls is 20 × 4 = 80 combinations.

The total number of combinations is

$$\frac{10!}{6!4!} = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$$

$$\frac{80}{210} = \frac{8}{21}$$ **Ans.**

27. The point A(3,4) is reflected over the x-axis to B. B is reflected over the line y = x to C. What is the area of triangle ABC?



When a point is reflected over the x axis, its y coordinate is multiplied by -1. Therefore, B is (3,-4) and the length of AB is 8.

The red line shows the plot of y = x. When the point (a,b) is reflected over the line y = x, then the coordinates are flipped, e.g., the new point is (b,a). In this case C becomes (-4,3).

The easiest way to find the area of triangle ABC is to form a rectangle around ABC. This consists of the points A, B, (-4,4) and (-4,-4). This is a 7 × 8 rectangle so its area is 56.

The area of the triangle with points A, C, and (-4,4) is $\frac{1}{2}\times7\times1 = \frac{7}{2}$

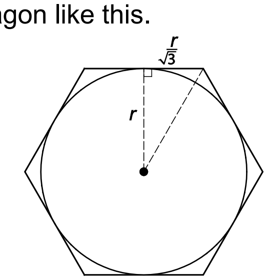
The area of the other triangle within the rectangle is formed by the points C, B, and (-4,-4). The area of this triangle is $\frac{1}{2}\times7\times7 = \frac{49}{2}$

The total area of these two triangles is $\frac{7}{2}+\frac{49}{2} = \frac{56}{2} = 28$

56 – 28 = 28 **Ans.**

28. Tonisha leaves Maryville at 7:15 AM. She travels at an average speed of 45 mph. Sheila leaves an hour later averaging 60 mph. We are asked to find out at what time Sheila passes Tonisha.
Let x = the number of hours that Tonisha travels. Then
45 + 45(x – 1) = 60(x – 1)
45 + 45x – 45 = 60x – 60
45x = 60x – 60
15x = 60
x = 4
Thus, Sheila will pass Tonisha at 11:15 AM. **Ans.**

29. If Fido goes as far from the center of the yard as his leash permits, then he can create a circle. So, we have a circle inscribed in a regular hexagon like this.



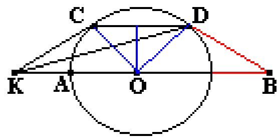
We have drawn in one of the twelve 30-60-90 triangles that exists. Letting the radius of the circle (and the long leg of the 30-60-90 triangle) be *r*, we can see that the short leg is $\frac{r}{\sqrt{3}}$, using the relationships of the 30-60-90 triangle. Therefore, Fido can reach πr^2 of his yard that measures $(12)(1/2)(\frac{r}{\sqrt{3}})(r)$. (Note that the area of the hexagon is the area of 12 of these triangles.) This is

$$\frac{\pi r^2}{(12)(1/2)(\frac{r}{\sqrt{3}})(r)} = \frac{\pi r^2}{\frac{6r^2}{\sqrt{3}}} = \frac{\sqrt{3}}{6}\pi$$

of the yard. Thus, *a* = 3, *b* = 6 and *ab* = 18. **Ans.**

30. In the figure below, circle O has radius 6 units. The length of chord CD is 8 and KA = 12 units. We are

asked to find the area of triangle KDC. The figure looks like an unfinished trapezoid and we can draw that in. (See the red lines in the figure below.)



So we will be able to find the area of triangle KDC by figuring out the area of the trapezoid CDBK and then subtracting the area of triangle DKB. First of all, how long is KB? We know that KA = 12 and since the radius is 6, we also know that AO is 6. So KO = 18. Similarly, OB is 18 and KB = 36.
Now what is the height of the trapezoid? Using the blue lines we create triangle COD. CO = OD = 6 and CD is broken up into 2 equal line segments each of which is 4.
Let h = the height of triangle COD.
Then $4^2 + h^2 = 6^2$
$16 + h^2 = 36$ and $h^2 = 20$.
Therefore, h = $\sqrt{20} = 2\sqrt{5}$
The area of the trapezoid is
$\frac{1}{2}(8+36)2\sqrt{5} = 44\sqrt{5}$
Now we need to know the height of triangle DKB. But that's the same as the height of triangle COD or $2\sqrt{5}$. So the area of triangle DKB is $\frac{1}{2} \times 2\sqrt{5} \times 36 = 36\sqrt{5}$
Subtracting the area of triangle DKB from the area of the trapezoid gives us $44\sqrt{5} - 36\sqrt{5} = 8\sqrt{5}$ **Ans.**

TARGET ROUND

1. How many of the smallest triangular regions are there in Figure 4 of the sequence whose first three figures are shown here?

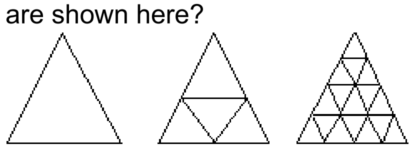


Figure 1 has 1 triangle in 1 row.
Figure 2 has 1 + 3 = 4 triangles in 2 rows.
Figure 3 has 1 + 3 + 5 + 7 = 16 triangles in 4 rows.
The number of rows doubles and each row has 2 more than the one above it.
Figure 4 must have 8 rows or
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64 triangles.
The sequence is actually
$4^0, 4^1, 4^2, 4^3, \ldots$ 64 **Ans.**

2. We have Sequence A which is
1, 5, 9, 13, …
and Sequence B which is
1, 7, 13, 19, …
We are asked to find the positive difference between the 2011th term of sequence A and the 2011th term of sequence B.
We can represent Sequence A by $a_n = 4n - 3$ where n is the number of the term in the sequence.
Therefore, the 2011th term is
$a_{2011} = 4 \times 2011 - 3 =$
$8044 - 3 = 8041$
We can represent Sequence B by $b_n = 6n - 5$ where n is the number of the term in the sequence.
Therefore, the 2011th term is
$b_{2011} = 6 \times 2011 - 5 =$
$12066 - 5 = 12061$
The positive difference is
12061 – 8041 = 4020 **Ans.**

3. The diagonal of a square is 5. The diameter of a circle is also 5. We are asked to find by how much greater the area of the square is than the area of the rectangle.
Let s = the side of the square.
Then $s^2 + s^2 = 5^2 = 25$ or
$2s^2 = 25$ and $s^2 = 12.5$
If the diameter of the circle is 5, the radius is 2.5 and the area of the circle is $\pi \times 2.5 \times 2.5 =$
$3.14159 \times 2.5 \times 2.5 \approx 19.6349375$
$19.6349375 - 12.5 \approx 7.1349375 \approx$
7.1 **Ans.**

4. Pets are either rabbits or dogs. 65% of the 840 pets participated in a parade. 180 rabbits participated and we must find out how many

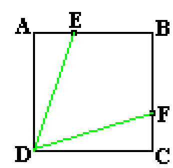
dogs participated.
840 × 0.65 = 546 pets participated.
546 – 180 = 366 pets that were not rabbits. They must be the dogs!
366 **Ans.**

5. On a scale drawing, a room measures 3.5 cm by 6 cm. 1 cm represents 1.5 m. We are asked to find the area of the room in square meters.
3.5 cm = 3.5 × 1.5 = 5.25 m
6 cm = 6 × 1.5 = 9 m
5.25 × 9 = 47.25 **Ans.**

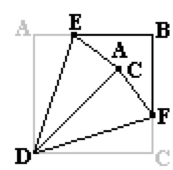
6. Each day a spider travels 5 feet up the waterspout. Each night it is washed 3 feet down the water spout. The waterspout is 50 feet long. The spider starts up the waterspout on April 1 and we are asked to find when it gets to the top.
Travelling up 5 feet and dropping 3 feet each day means that every day it actually makes 2 feet of headway. The simplest thing is to say
$\frac{50}{2} = 25$ days.
But that would be wrong because we have to find when it will **first** reach the top. So to make sure we do it right, after 22 days (April 1 – April 22) it's 44 feet up the spout.
On April 23 it gets up to 49 feet and slips back to 46 feet.
On April 24 it gets up to 51 feet before slipping back to 48 feet. But that's enough. The answer is actually April 24!!!! **Ans.**

7. A graph shows quiz scores for a 10-question quiz. Each question is worth one point. Only 3 students answered #7 correctly but credit is given for #7 to every student. Find the new mean quiz score.
According to the graph:
3 students got 3 correct
3 students got 4 correct
5 students got 5 correct
7 students got 6 correct
5 students got 7 correct
6 students got 8 correct
4 students got 9 correct
3 students got all 10 correct – they must be the ones that answered #7 correctly, obviously.
First, figure out the number of students.
3 + 3 + 5 + 7 + 5 + 6 + 4 + 3 = 36
Now, figure out how many total points there were.
(3 × 3) + (3 × 4) + (5 × 5) + (7 × 6) + (5 × 7) + (6 × 8) + (4 × 9) + (3 × 10) = 237 points
Since there are 36 students and 3 already have #7 right, we need to add 33 points for #7 to the total.
237 + 33 = 270
$\frac{270}{36} = 7.5$ **Ans.**

8. Square ABCD has sides of length 1. Points E and F are on sides AB and CB, respectively, with AE = CF. When the square is folded along the lines DE and DF, sides AD and CD coincide and line up on diagonal BD. Given that the length of segment AE is expressed in the form $\sqrt{k} - m$, then what is the value of k + m?
Before we make the fold the square looks like this.



After the fold the square looks like this:



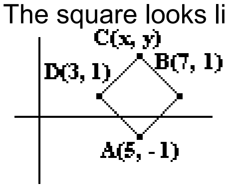
Let x = AE. The angle at DAE is 90º. DA = 1. And we know that DB, the diagonal, is $\sqrt{2}$. In the second image, if we continue the line through point A (also point C) to B, then we have the triangle EAB, where angle EAB is also 90º.
EB = 1 – x
AB = $\sqrt{2}$ – 1
Thus, we can write

$$x^2 + (\sqrt{2} - 1)^2 = (1 - x)^2$$
$$x^2 + 2 - 2\sqrt{2} + 1 = 1 - 2x + x^2$$
$$3 - 2\sqrt{2} = 1 - 2x$$
$$2x = 1 - 3 + 2\sqrt{2} = -2 + 2\sqrt{2}$$
$$x = \frac{2\sqrt{2} - 2}{2} = \sqrt{2} - 1$$

Thus, k = 2 and m = 1.
k + m = 2 + 1 = 3 **Ans.**

TEAM ROUND

1. 3 vertices of square ABCD are located at A(5, -1), B(7, 1) and D(3,1). We are asked to find the coordinates of point C.
   The square looks like this:



   The difference between the x coordinate of B and the x coordinate of A is 7 – 5 = 2.
   The difference between the y coordinate of B and the y coordinate of A is 1 – (–1) = 2.
   The coordinates of D are (3, 1)
   If x is the coordinate of point C, then x = 3 + 2 = 5.
   If y is the coordinate of point C, then y = 1 + 2 = 3.
   Therefore, the coordinates of point C are (5, 3). **Ans.**

2. What is the units digit of 3^{2011}?
   If you look at the units digits of the powers of 3, you see a pattern.
   3, **9**, 2**7**, 8**1**, 24**3**, 72**9**, 218**7**, 656**1** or 3, 9, 7, 1.
   When the power is divisible by 4 and has a remainder of 1, then the units digit is 3. If it's divisible by 4 and has a remainder of 2, then the units digit is 9. If the remainder is 3, then the units digit is 7 and if the remainder is 0, then the units digit is 1. All we have to do is divide 2011 by 4.
   $\frac{2011}{4} = 502\frac{3}{4}$ so the remainder is 3. Therefore the units digit is 7.
   Ans.

3. The sale price that Mr. Adams paid for a 10-ft by 12-ft piece of carpet was the same as the non-sale price of a piece of carpet measuring 6-ft by 8-ft. We are asked to find the percent off the original carpet price that was taken to create the sale price.
   Let x = the regular price.
   Let y = the sale price.
   Then (6 × 8)x = (10 × 12)y
   48x = 120y
   $$\frac{y}{x} = \frac{48}{120} = \frac{4}{10}$$
   So y, the sale price, was 40% of x, the original price. Therefore, the sale price was reduced by 100 – 40 = 60% **Ans.**

4. Zeta runs around a track at a rate of 30 laps per 75 min. Ray runs around the track at a rate of 20 laps per 40 min. We must find how many minutes it takes them to run a combined distance of 99 laps.
   30 laps per 75 minutes means that Zeta runs $\frac{30}{75} = \frac{6}{15} = \frac{2}{5}$ lap each minute.
   20 laps per 40 minutes means that Ray runs $\frac{20}{40} = \frac{1}{2}$ lap each minute.
   Together they run $\frac{2}{5} + \frac{1}{2} = \frac{4}{10} + \frac{5}{10} = \frac{9}{10}$ lap each minute.
   $$\frac{99}{\frac{9}{10}} = 99 \times \frac{10}{9} = 11 \times 10 = 110$$
   minutes. 110 **Ans.**

5. Line L passes through the points $\left(0, \frac{1}{2}\right)$ and $(4, k)$. Line L is perpendicular to the line

y = –4x + 5. We are asked to find the value of k.
First, let's come up with line L.
y = mx + b

$$\frac{1}{2} = 0m + b$$

$$b = \frac{1}{2}$$

Using $(4, k)$,

$$k = 4m + \frac{1}{2}$$
$$2k = 8m + 1$$
$$8m = 2k - 1$$
$$m = \frac{2k-1}{8}$$

We know that the slope of a line perpendicular to line L must be $-\frac{1}{m}$ and we know that the slope of the perpendicular line is –4. Therefore,

$$-\frac{1}{m} = -\frac{1}{\frac{2k-1}{8}} = -4$$

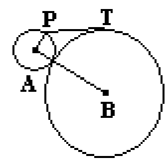
$$\frac{1}{\frac{2k-1}{8}} = 4$$

$$\frac{8}{2k-1} = 4$$

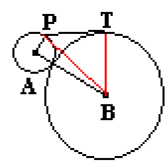
8 = 8k – 4
8k = 12
k = $\frac{12}{8} = \frac{3}{2}$ **Ans.**

6. Movie showings begin at 11:15 a.m. Each showing consists of 10 minutes of previews and 105 minutes of the movie. It takes 20 minutes to get the theater ready for the next showing and there are 5 showings of the movie prior to midnight. We are asked to find the earliest possible time for the last showing to begin.
Since there are 5 showings, the last showing begins after 4 sequences of previews, movie and cleanup occur.
10 + 105 + 20 = 135 minutes.
135 × 4 = 540 minutes
$\frac{540}{60} = 9$ hours
11:15 a.m. + 9 hours = 8:15 p.m. **Ans.**

7. Circles A and B are externally tangent. Angle PAB is a right angle. Segment PT is tangent to circle B at T. The radius of circle A is 1 cm and the radius of circle B is 7 cm. We are asked to find the length of segment PT.



Let's draw lines from B to T and B to P.



Angle PAB and Angle PTB are 90º angles. We know that the length of AP is 1 and the length of AB is 1 + 7 = 8. Therefore, the length of PB is:

$$\sqrt{1^2 + 8^2} = \sqrt{65}$$

The length of TB is 7. Let x = the length of PT. Then

$$x^2 + 7^2 = \sqrt{65}^2$$
$$x^2 + 49 = 65$$
$$x^2 = 65 - 49 = 16$$

x = 4 **Ans.**

8. Originally, the first digit in a 3-digit area code could not be a 0 or 1. The second digit could only be a 0 or 1. The third digit could be anything. The restrictions on the second digit were lifted and we are asked to determine how many more 3-digit area codes are possible now. Before the restrictions were lifted, we could have 8 possibilities for the

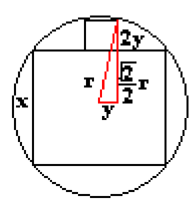
first digit, 2 for the second and 10 for the third. That's a total of
8 × 2 × 10 possibilities.
Now, instead of 2 for the middle digit we have 10 possibilities or
8 × 10 × 10 possibilities.
8 × 10 × 10 – (8 × 2 × 10) =
(8 × 10) × (10 – 2) =
80 × 8 = 640 **Ans.**

9. Each of the nine digits {1, 2, 3,…, 9} is used exactly once as a digit in either a four-digit positive integer *a* or the five-digit positive integer *b*. We must find the smallest possible value of *a* if

$$\frac{a}{b} = \frac{1}{2}$$

So we know that *b* = 2*a* and that means *b* must end in 2, 4, 6, or 8.
How big can *b* be?
9876 × 2 =19752. Therefore, since *b* is a 5-digit integer, the ten-thousandth's digit must be 1.
What is the smallest 4-digit number that multiplied by 2 gives a 5-digit number? That must be 5000 because 5000 × 2 = 10000.
So the thousandth's digit of *a* $\geq$ 5.
And since 1 is now reserved and 0 cannot be used, we have to result in a number that is $\geq$ 12000. So *a* $\geq$ 6000.
Let us reserve 6, 1 and 2. That leaves 3, 4, 5, 7, 8, 9 for the smallest value of *a*. 5 cannot be in the one's column because 5 × 2 = 10. Neither can 3 because 3 × 2 = 6 or 8 because 8 × 2 = 16.
6300 × 2 = 12600 which uses 6 twice. Therefore, we have to start at 6350 × 2 = 12700.
Is there anything else we can remove before trying this?
If the one's column of *a* is 4, then 8 cannot be in *a*.
If the one's column of *a* is 7, then 4 cannot be in *a*.
If the one's column of *a* is 9, then 8 cannot be in *a*. Neither can the last two digits of *a* be between 50 and 60 because, at minimum, that will make the ten's column of *b* be 0 or 1. So, we start at 6350 but immediately move up to 6360 which we can't do and then up to 6370. Can't use 1, 2, 3 in the one's column.
6374 × 2 = 12748 No good.
6379 × 2 = 12758 No good.
6380 × 2 = 12760 so we have to look greater than 6384.
6387 × 2 = 12774 No good.
6389 × 2 = 12778 No good.
6394 × 2 = 12788 No good.
6397 × 2 = 12794 No good.
So we're now looking at 6400 and up.
Remember, 0, 1, and 2 are out for the ten's column.
6430 × 2 = 12860 so it must be greater than 6434.
6439 × 2 = 12878 No good.
6479 × 2 = 12958 No good.
6480 – 6484 is out.
6489 × 2 = 12978 No good.
Nothing in the 6490's will work either. Where does that leave us?
6500 × 2 = 13000
That means that 1, 3, and 6 are now reserved. But 6500 – 6549 are out because there will be a 0 in *b*.
So are 6550-6599 because there will be a 1 in *b.*
Next up is 6600 but that's 2 6's.
So now we're at 6700 × 2 = 13400.
6729 × 2 = 13458 !!!!! **Ans.**

10. A square is inscribed in a circle and there is a smaller square with one side coinciding with a side of the larger square and two vertices on the circle. We are asked to find the percentage of the larger square's area that the smaller square's area has.



Let x = the side of the larger square.
Let y = half the side of the smaller square.
Let r = the radius of the circle.
The diagonal of the larger square is 2r. Find the area of the larger square.

$$x^2 + x^2 = (2r)^2$$
$$2x^2 = 4r^2$$
$$x^2 = 2r^2$$
$$x = \sqrt{2}r$$

The area of the smaller square is

$$(2y)^2 = 4y^2$$

The ratio of the area of the smaller square to the area of the larger square is:

$$\frac{4y^2}{x^2} = \frac{4y^2}{2r^2} = 2\frac{y^2}{r^2}$$

We can construct a triangle whose legs are y and $\frac{\sqrt{2}}{2}r + 2y$. The hypotenuse of this right triangle is r.

$$y^2 + \left(\frac{\sqrt{2}}{2}r + 2y\right)^2 = r^2$$

$$y^2 + \frac{1}{2}r^2 + 2\left(2y \times \frac{\sqrt{2}}{2}r\right) + 4y^2 =$$

$$5y^2 + 2\sqrt{2}ry + \frac{1}{2}r^2 = r^2$$

Divide by r^2.

$$5\frac{y^2}{r^2} + 2\sqrt{2}\frac{y}{r} + \frac{1}{2} = 1$$

$$5\frac{y^2}{r^2} + 2\sqrt{2}\frac{y}{r} - \frac{1}{2} = 0$$

Remember that we would really like to find out what $2\frac{y^2}{r^2}$ is so define z = $\frac{y}{r}$ and rewrite the equation.

$$5z^2 + 2\sqrt{2}z - \frac{1}{2} = 0$$

$$10z^2 + 4\sqrt{2}z - 1 = 0$$

This can be solved using the quadratic formula

$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where

$a = 10$, $b = 4\sqrt{2}$ and $c = -1$.

$$z = \frac{-4\sqrt{2} \pm \sqrt{\left(4\sqrt{2}\right)^2 - 4 \times 10 \times -1}}{20}$$

$$z = \frac{-4\sqrt{2} \pm \sqrt{32 + 40}}{20}$$

$$z = \frac{-4\sqrt{2} \pm \sqrt{72}}{20} = \frac{-4\sqrt{2} \pm 6\sqrt{2}}{20}$$

Remember that z must be positive.

$$z = \frac{y}{r} = \frac{2\sqrt{2}}{20} = \frac{\sqrt{2}}{10}$$

$$2\frac{y^2}{r^2} = 2\left(\frac{\sqrt{2}}{10}\right)^2 = 2 \times \frac{2}{100} = \frac{4}{100}$$

$\frac{4}{100} = 4\%$ **Ans.**