Mathematical Approaches To Toys

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What is a toy, and what has that got to do with maths?

Some dictionary definitions

An object for a child to play with

An object regarded as providing amusement for an adult

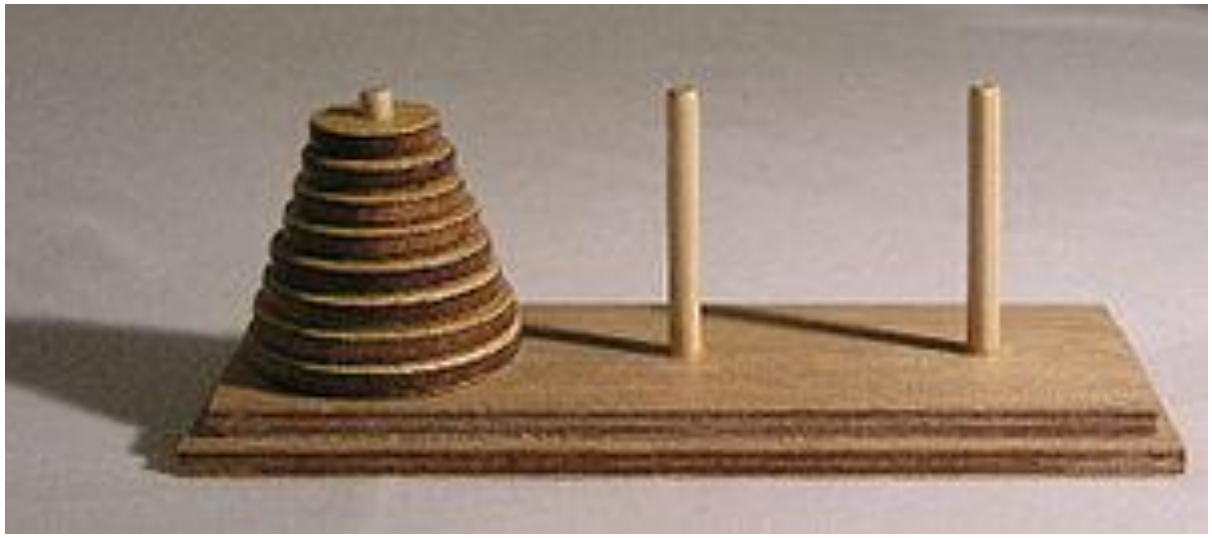
An object, often a small representation of something familiar

A thing or matter of little or no value or importance; a trifle.

Something that serves for or as if for diversion, rather than for serious practical use.

Mathematics and toys

1. A lot of mathematics can be learned through play! Toys are an excellent way of teaching, learning and discovering, mathematics



2. Many toys involve solving puzzles

This often involves maths!

Topology

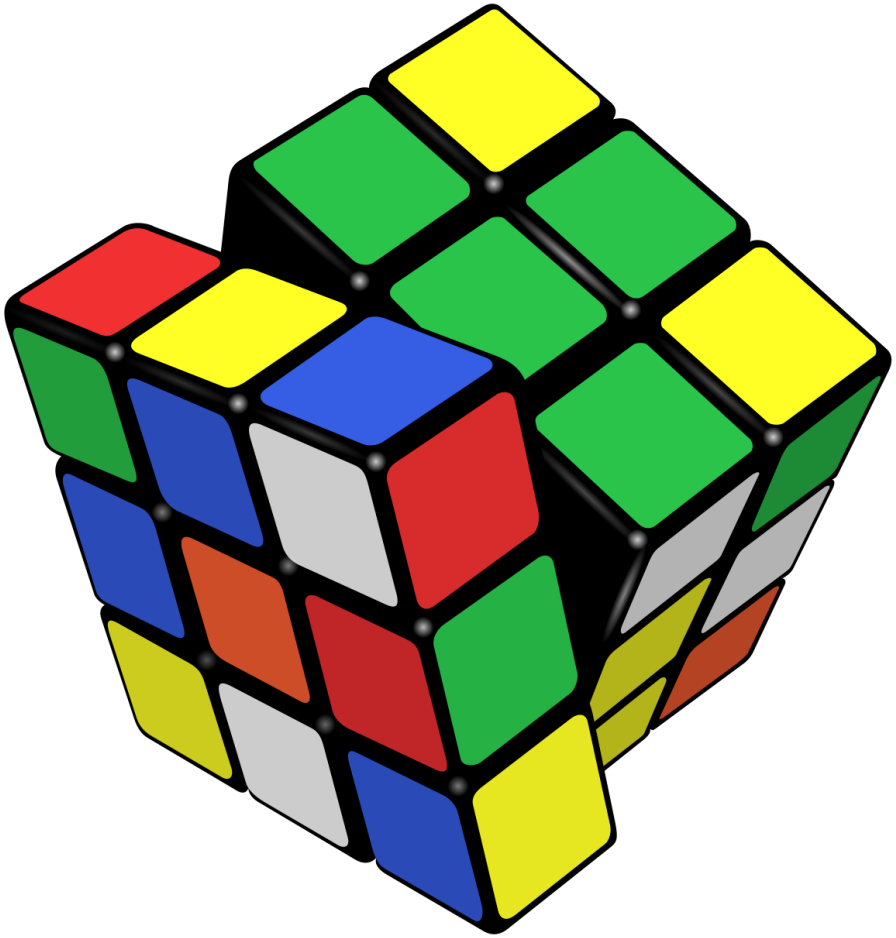
Sudoku 9x9 - Easy (135583221)

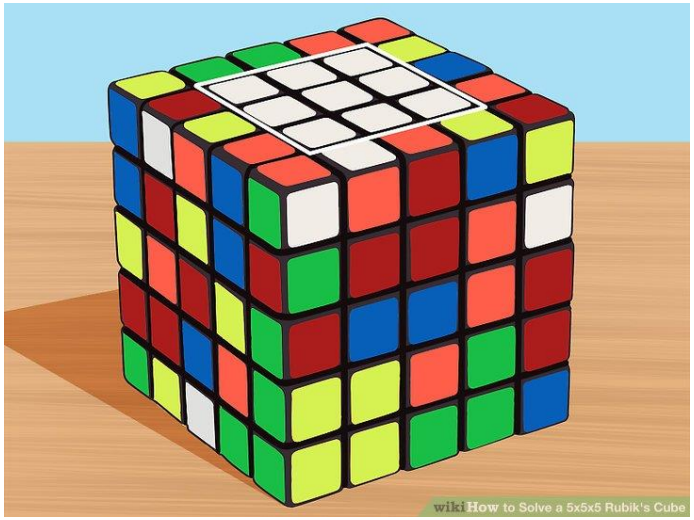
9	2						1	7
		4	6		7	8		
8								2
		1	2		3	7		
7								6
		5	4		9	3		
5								8
		6	8		5	2		
2	4						9	3

www.sudoku-puzzles.net

Logic

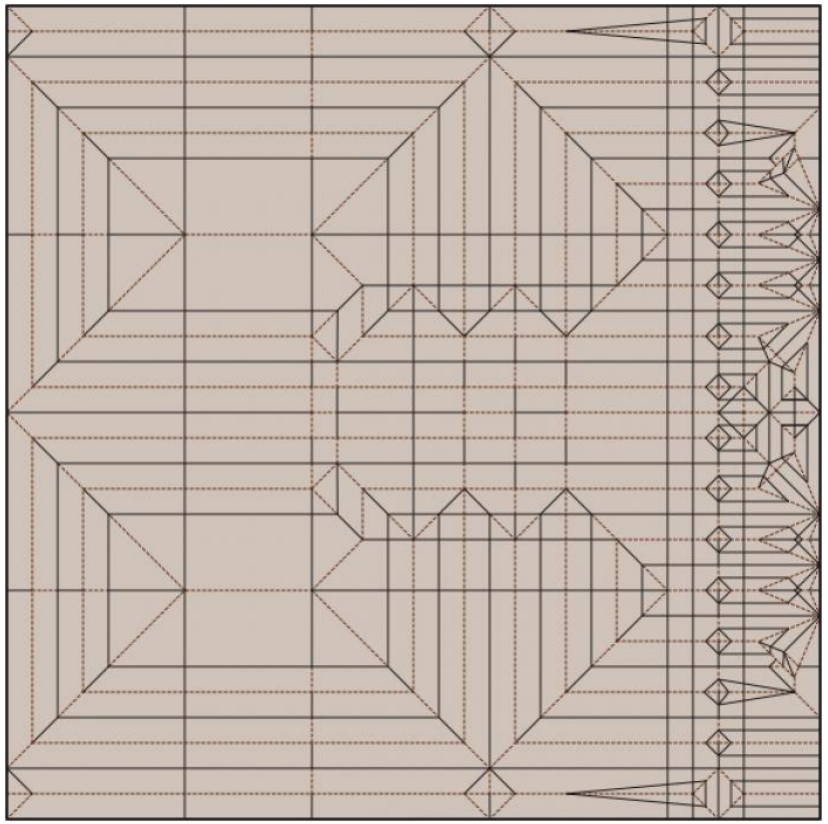
Rubik's cube





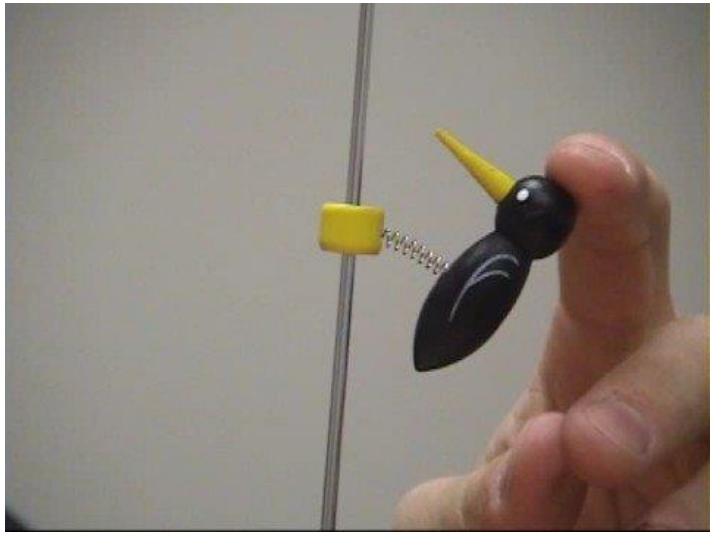
group theory (alternating groups). God's number!

3. Mathematics and toys can be almost indistinguishable



Origami is now done by mathematicians eg. Robert Lang

4. Maths is important in the design and study of toys



https://www.youtube.com/watch?v=s3YSnNAIHDg

5. Toys can be very important in applications of maths

Toy: An object, often a small representation of something familiar

We use ‘toy models’ to help us to understand much more complex systems

Eg. Climate models,

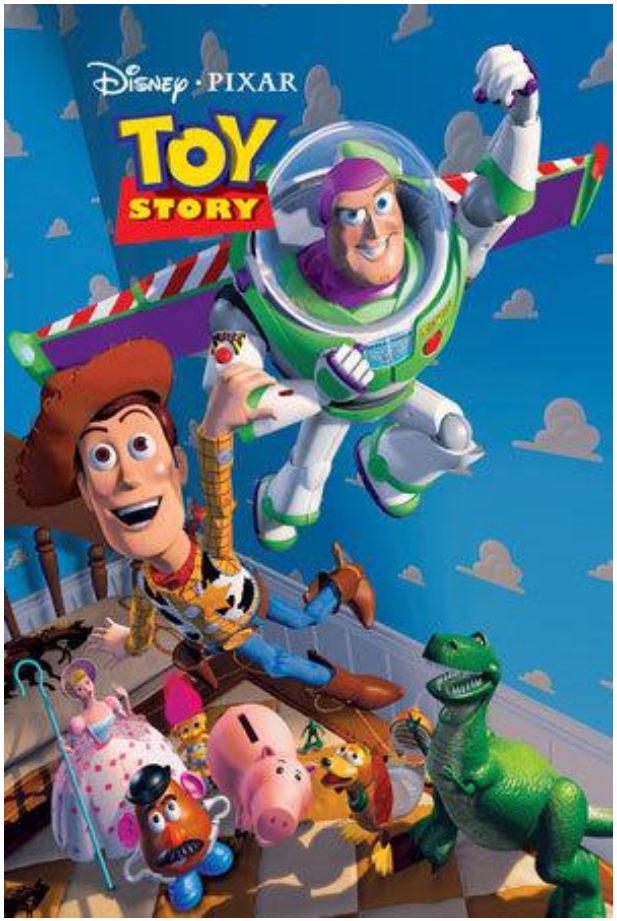
models of the solar system,

models of human behaviour

Figure 1. The rotating tank experimental apparatus and wind system is shown. The tank is 2 m in diameter and mounted on an axis which is vertical to 10^{-3} radian. It spins at about 8 rpm during the experiment. The continents are modeled in 5.0 cm foam rubber and the continental shelves in 0.63 cm plywood. Jets suspended from the wind ring above the tank simulate the zonal westerlies.

The rest of the talk ….

- Toys in teaching maths
- Art, maths and toys
- Big toys
- Chaotic toys
- Toy models



Made using maths!!!

Show how toys can awaken our mathematical curiosity

Toys in teaching maths

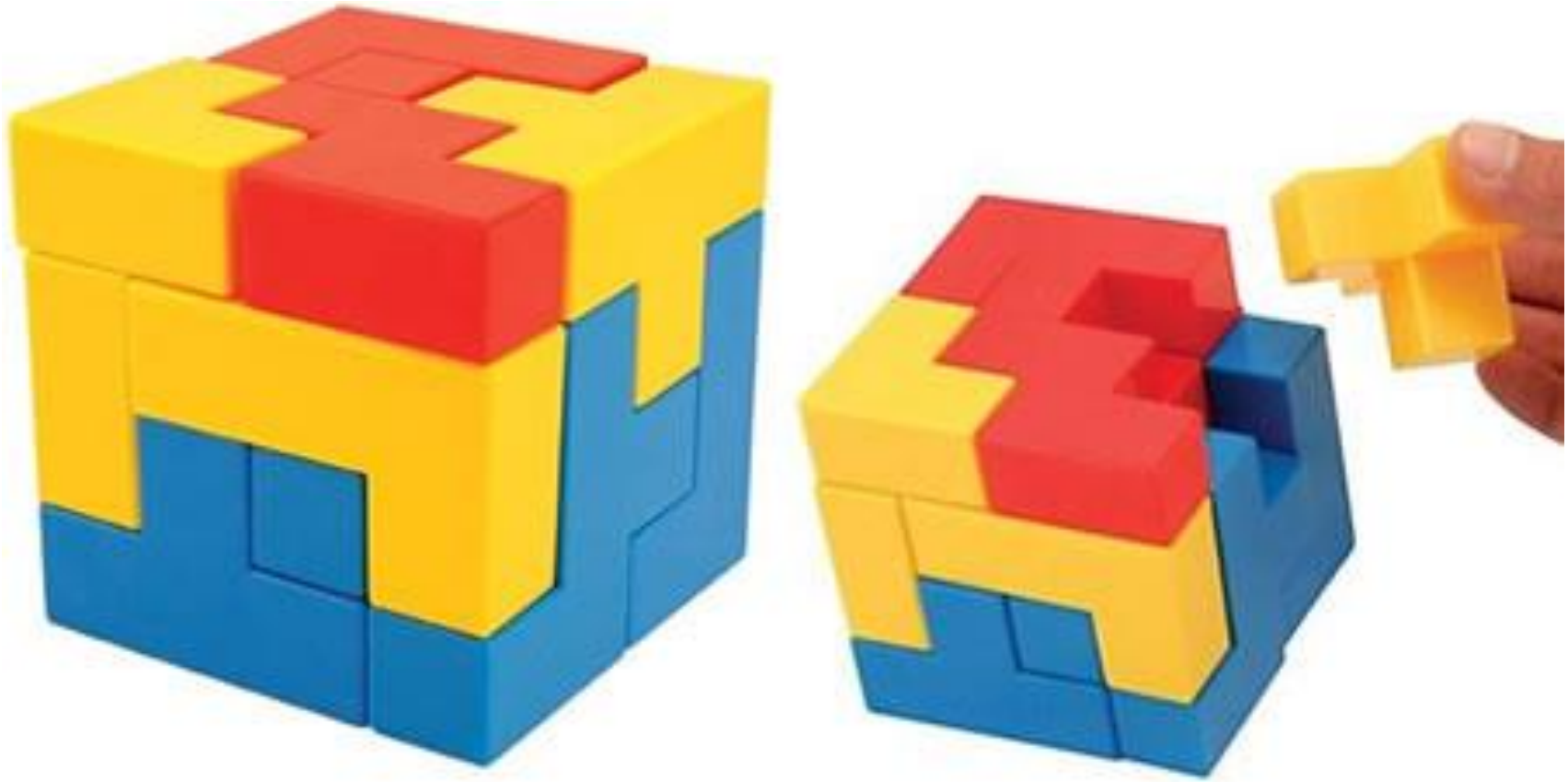
Big Bang
Fair
NEC

- Capture, Captivate, build wonder in, and hold an audience
- Explore ideas in a hands on way. Develop a sense of enquiry
- Teach them maths at the same time

Examples

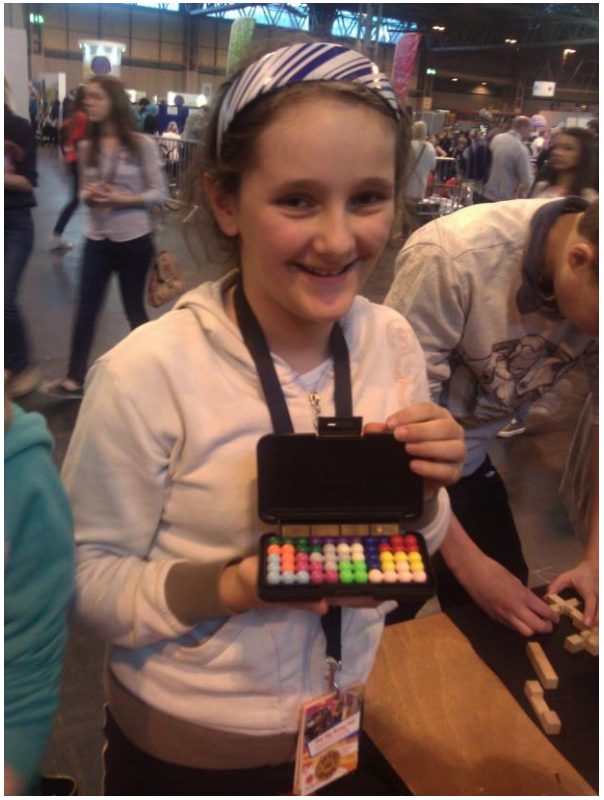
1. Packing problems (2D and 3D)

Knapsack problem, Optimisation, NP Hard, Sat-Nav



How to solve the 'Bedlam Cube' by Jasmine Age 8

https://www.youtube.com/watch?v=jhVgG_0-ln0



Huge sense of achievement on solving!!

Often spend an hour at the stall

Future mathematicians?

2. Topological toys

Zeeman's ropes

Elementary topology, knot theory

3. A touch of Magic

Number theory, fixed point theorems, probability, modular arithmetic

Cambridge Science Festival

Bath Taps Into Science

4. Mechanical toys

Stirling engine toys used to teach fluid mechanics and thermodynamics at the Big Bang Fair

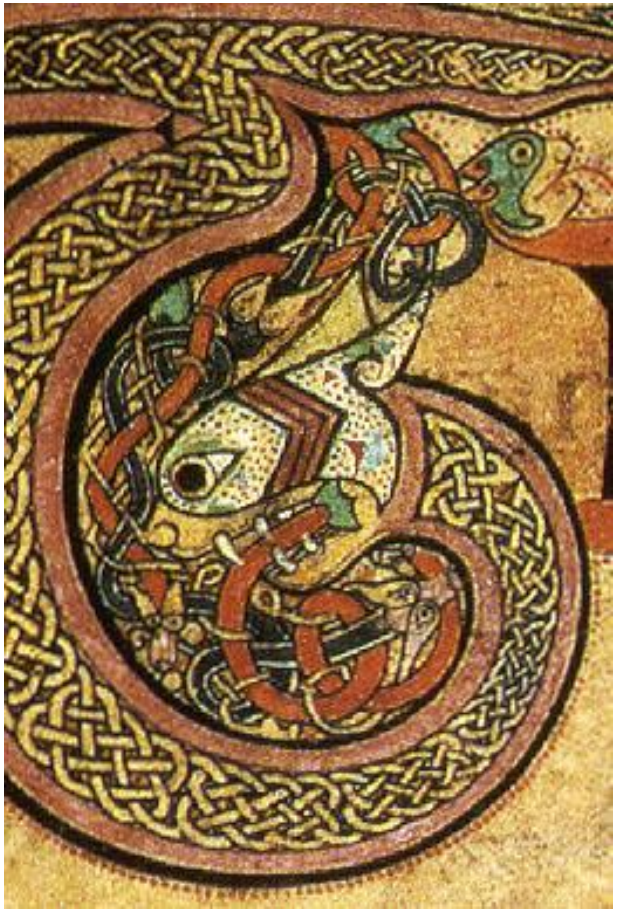
Also …

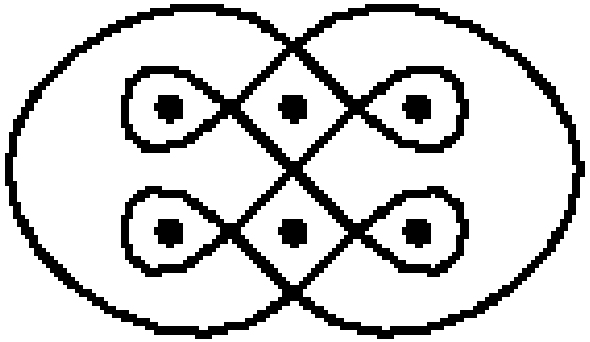
Bath taps construction

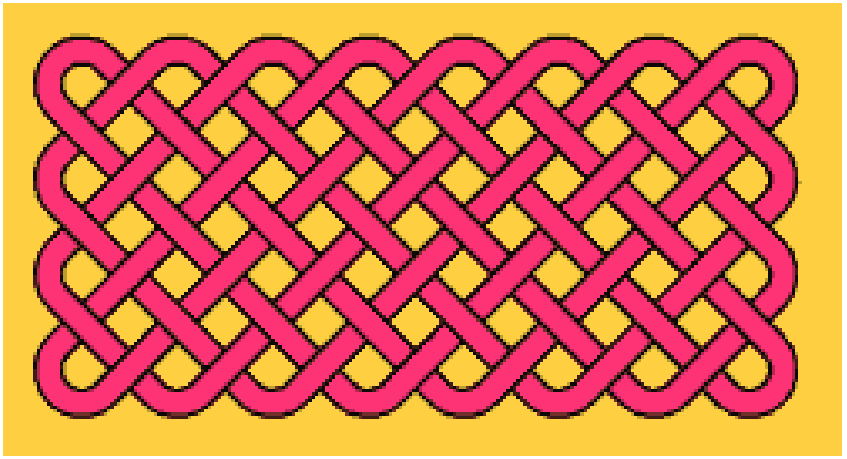
Jenga: Mechanics, moments, friction

Euler’s disc, Rattle back, gyroscopes, Newton’s cradle, ……

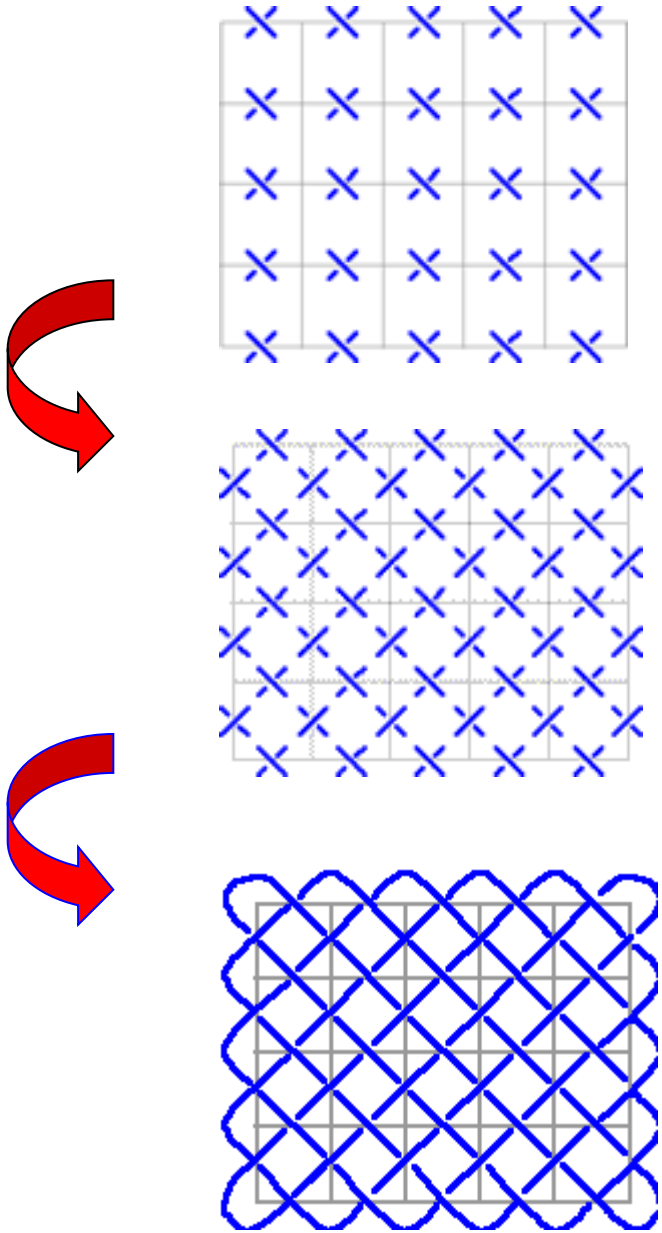
Art, maths and toys



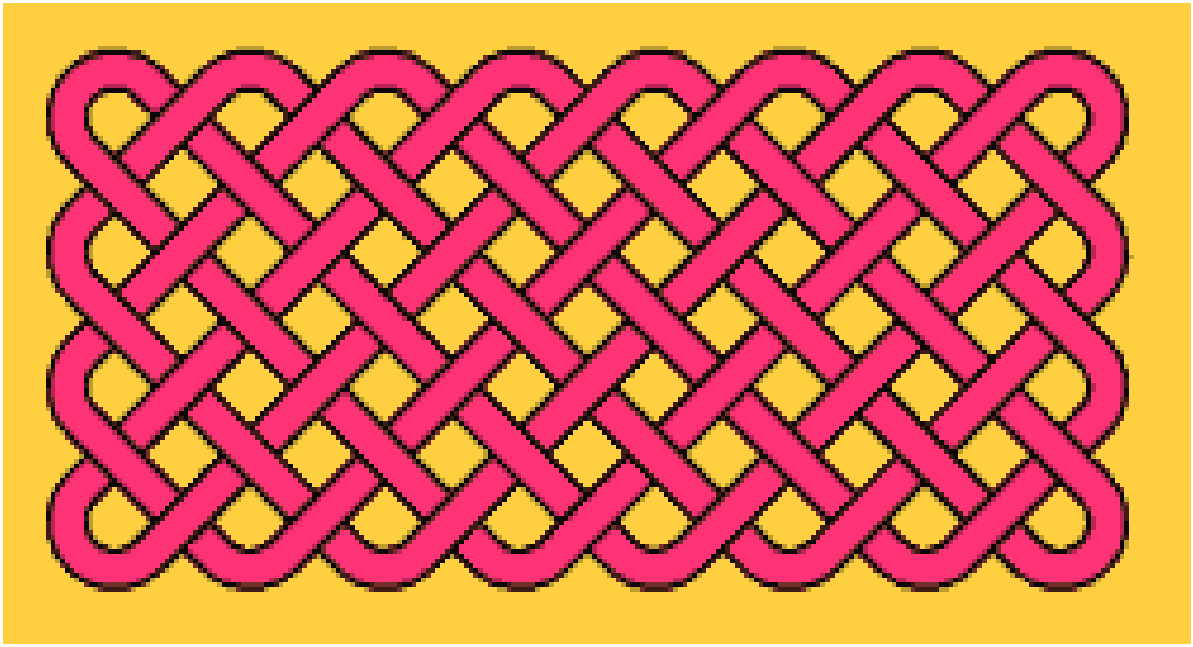




Show that maths is a highly creative subject



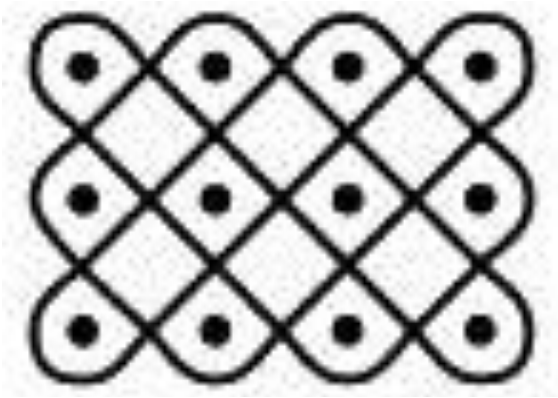
(5,6) Celtic Knot



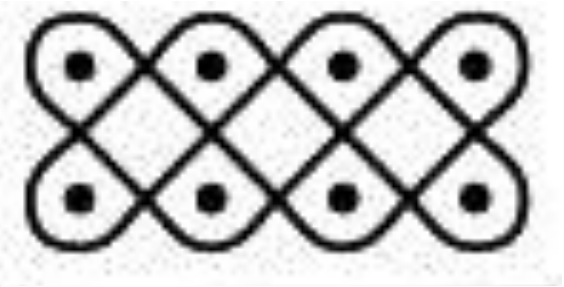
(4,8) Celtic Knot

How many pieces of string are needed?

Simple Celtic Knots are very similar to plaited mat sona drawings



(3,4)



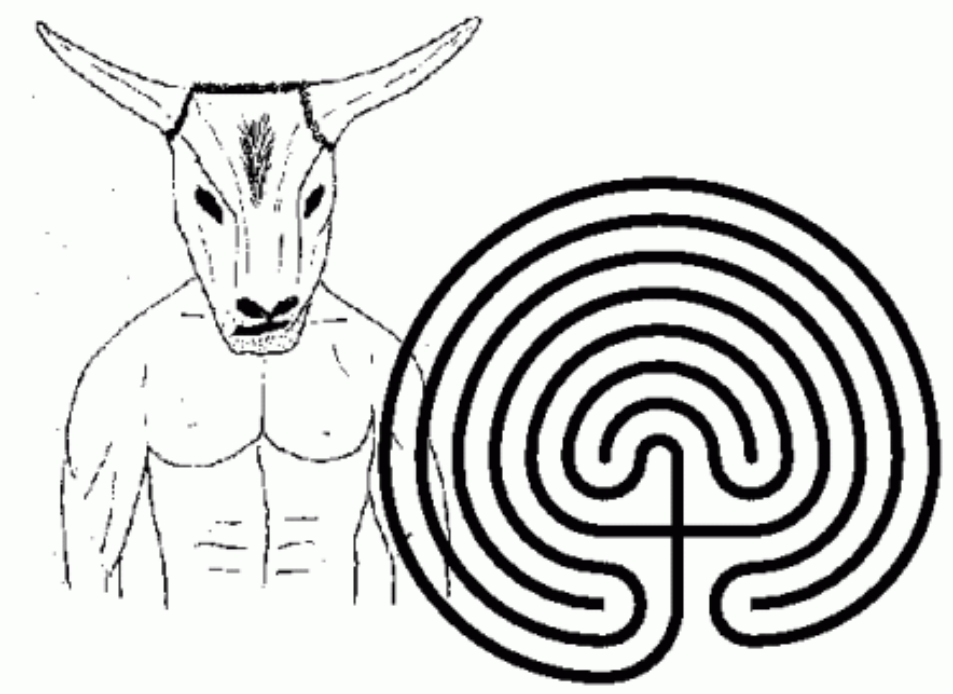
(2,4)

How many lines?

String and lines equal the Highest Common Factor of the two numbers

But why?

Big Toys Mazes and sundials



Cretan Labyrinth:

History of maths

Unicursal diagrams

Fractals

Hampton Court Maze:

Networks

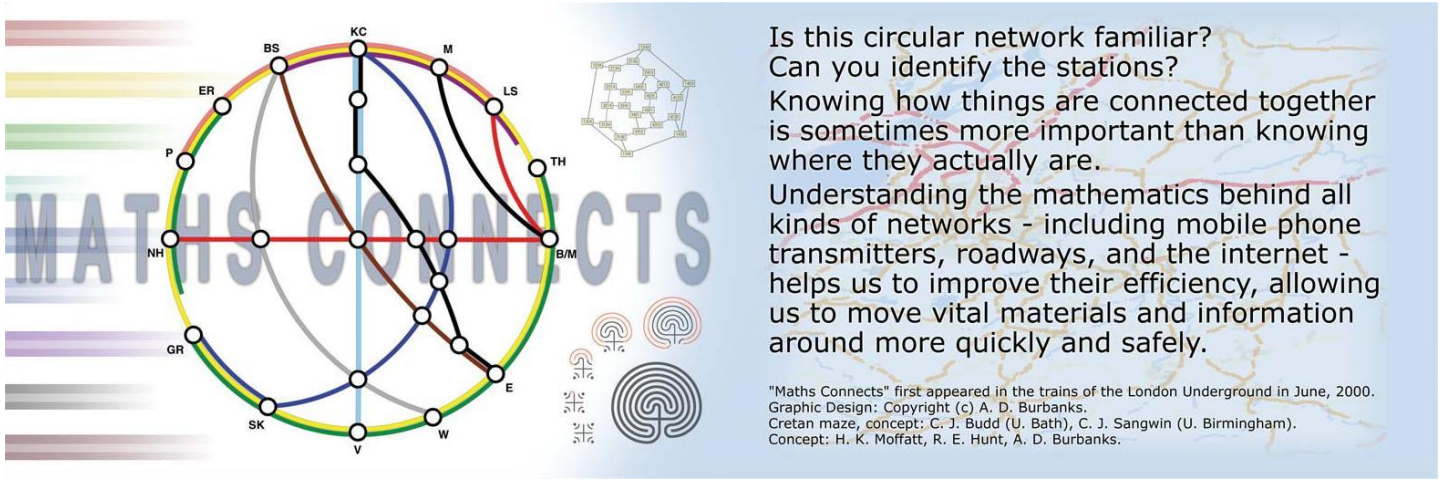
Internet

Hamilton Circuits

Large group projects: school/university Labyrinths

London Underground

Mark Wallinger Turner prize winning artist



Sundials: Toys to tell the time

Traditional sundial

Corrected sundial

Analemmatic sundial

Heavy use of trigonometry and geometry. Great to build. Useful too!

Chaotic Toys

The discovery of chaos has led to a whole new industry in toy design

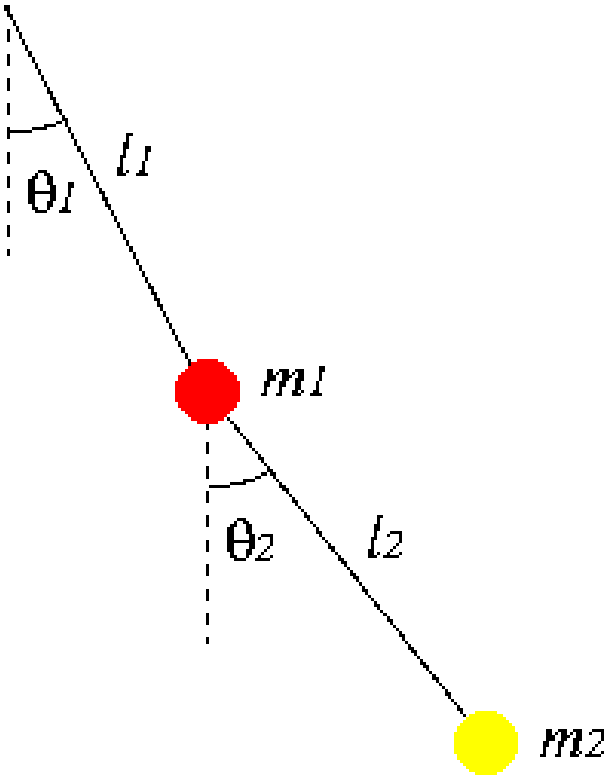
Chaos: *Seemingly complex and unpredictable behaviour arising from predictable systems*

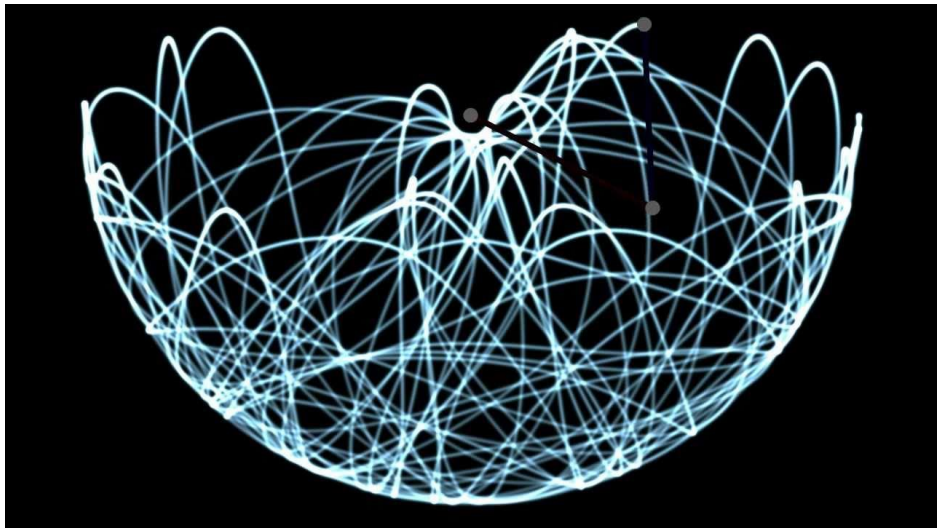
VERY important in maths, physics and engineering:

Weather forecasting
Astronomy
Mechanics
Electronics
Optics

Also seems to resonate with the human psyche and gives us great toys

Do the math: Chaotic Double Pendulum





https://www.youtube.com/watch?v=U39RMUzCjiU

Equations of motion :)

$$\theta_1'' = \frac{-g(2m_1 + m_2)\sin(\theta_1) - m_2 g \sin(\theta_1 - 2\theta_2) - 2\sin(\theta_1 - \theta_2) m_2 \left(\theta_2'^2 L_2 + \theta_1^2 L_1 \cos(\theta_1 - \theta_2)\right)}{L_1(2m_1 + m_2 - m_2 \cos(2\theta_1 - 2\theta_2))}$$

$$\theta_2'' = \frac{2\sin(\theta_1 - \theta_2)\left(\theta_1^2 L_1(m_1 + m_2) + g(m_1 + m_2)\cos(\theta_1) + \theta_2'^2 L_2 m_2 \cos(\theta_1 - \theta_2)\right)}{L_2(2m_1 + m_2 - m_2 \cos(2\theta_1 - 2\theta_2))}$$

https://www.myphysicslab.com/pendulum/double-pendulum-en.html

Chaotic water pendulum at St Mary Redcliffe Church, Bristol

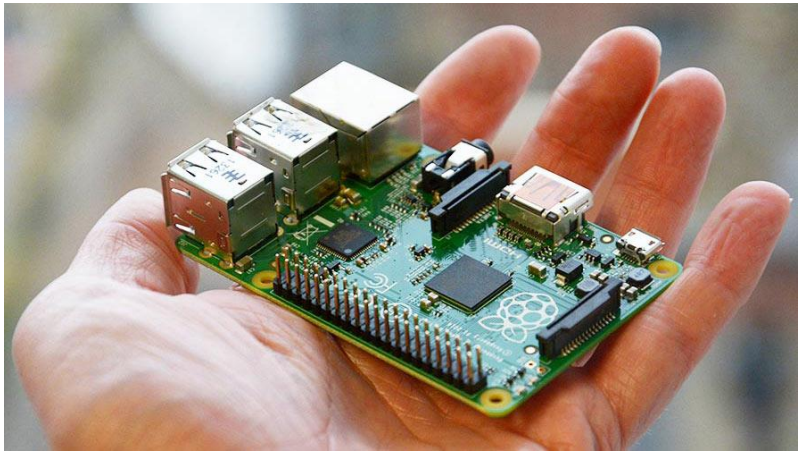
Designed by Brian Pippard and Eric Albone

https://www.youtube.com/watch?v=hBLdBbmeWxE

Mathematical Toys

An object, often a small representation of something familiar

The Climate-Pi Project (CliMathNet)



How can we understand, and teach, climate change?

Basic equations for weather were derived by Navier/Stokes and Kelvin

$$\frac{Du}{Dt} + 2f \times u + \frac{1}{\rho}\nabla p + g = \nu\nabla^2 u,$$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0,$$

$$C\frac{DT}{Dt} - \frac{RT}{\rho}\frac{D\rho}{Dt} = \kappa_h \nabla^2 T + S_h + LP,$$

$$\frac{Dq}{Dt} = \kappa_q \nabla^2 q + S_q - P,$$

$$p = \rho RT.$$

Motion

Density

Temperature

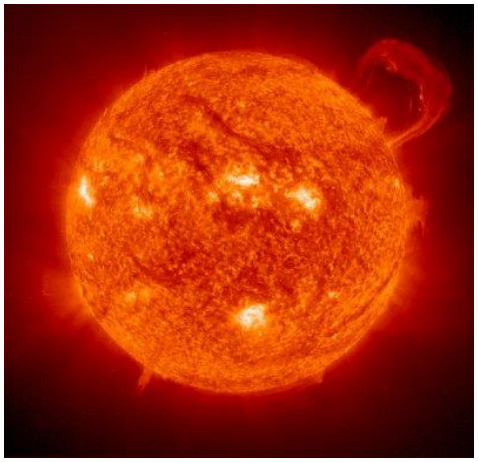
Moisture

Pressure

For climate add in ice, CO2, ocean currents, …

Tough to solve, understand and teach

Toy model: Energy Balance

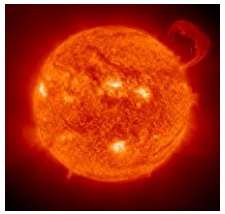


Heat into space

Heat from Sun: S

Earth's mean temperature: T

Heat absorbed $\longrightarrow (1-a)S$



a Albedo: How well the earth reflects the Sun's rays

Heat radiated away $\longrightarrow e\sigma T^4$

e emissivity: How much energy is radiated into space

Balance these to give a steady state

$$e\sigma T^4 = (1-a)S$$

Toy model is simple enough to be implemented on any computer

Useful for predictions:

Can give the correct mean temperature for the Earth

AND for the moon

Easy for students to play with AND allows effects of climate change to be assessed

$$T = \left(\frac{(1-a)\ S}{e\ \sigma} \right)^{1/4}$$

In Conclusion

Toys are wonderful for awakening our sense of mathematical curiosity, and for learning and doing maths

At all levels

And for all ages

Have fun!!!