enquiries@examfactor.com.au

EXAM FACTOR

ABN: 15568488769

www.examfactor.com.au

Student Name:

MATHEMATICAL METHODS UNITS 3 & 4

Trial examination 2

2016

Reading time: 15 minutes

Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 26 pages with a detachable sheet of miscellaneous formulas.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet.
- Write your name in the space provided above on this page.
- Write your name on your answer sheet for multiple-choice,
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book

Students are NOT permitted to bring mobile phones and/or any other unauthorized electronic devices into the examination room.

SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple–choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any questions.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The range of the function $f:[-2,\ 4] \to R, f(x) = (5-x)(x+1)$ is

A. $[-7,\ 5]$

B. $(-\infty,\ 9]$

C. $[-7,\ \infty)$

D. $[-2,\ 4]$

E. $[-7,\ 9]$

Question 2

The function $f:\left[\frac{\pi}{6}, a\right] \to R$, $f(x) = 2\sin\left(3x\right)$ will have an inverse if

A. $a \le \frac{\pi}{3}$

B. $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$

C. $a \ge \frac{\pi}{3}$

D. $a \le \frac{\pi}{3}$

E. $\frac{\pi}{6} < x \le \frac{\pi}{3}$

Question 3

The simultaneous linear equations $kx - 4y = 3$ and $4x - ky = 7 - k$ have an **infinite set** of solutions for

A. $k \in R \setminus \{-4, 4\}$

B. $k = 4$

C. $k \in R \setminus \{4\}$

D. $k = -4$

E. $k = -4$ and $k = 4$

Question 4

If $x + a$ is a factor of $3a^3 - 8x^2 - 2ax$ where $a \in R \setminus \{0\}$, then the value of a is

A. $\frac{10}{3}$

B. -2

C. $\frac{10}{3}$

D. 2

E. 3

Question 5

A cosine function *f,* has an amplitude of 3 and a period of 6. The rule for *f* could be

A. $f(x) = 3\cos\left(\frac{\pi x}{6}\right)$

B. $f(x) = 3\cos\left(\frac{\pi x}{3}\right)$

C. $f(x) = 6\cos\left(\frac{\pi x}{2}\right)$

D. $f(x) = 3\cos(6\pi x)$

E. $f(x) = 3\cos(3\pi x)$

SECTION A – continued
TURN OVER

Question 6

The transformation $T: R^2 \to R^2$ with rule

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

maps the line with equation $2x - y = 5$ onto the line with equation.

A. $3x + y = 10$

B. $3x - y = 10$

C. $3x + y = -10$

D. $-3x + y = 10$

E. $3x - y = -10$

Question 7

Which one of the following functions satisfies the functional equation

$$f(x - y) = \frac{f(x)}{f(y)}$$

where x and y are non–zero real numbers?

A. $f(x) = \cos(x)$

B. $f(x) = x$

C. $f(x) = e^{2x}$

D. $f(x) = \ln(x)$

E. $f(x) = \sqrt{x}$

Question 8

If $\log_e(x) = \log_e(x - 2) + b$ then x is equal to

A. $\dfrac{e^b}{2 - e^b}$

B. $\dfrac{2}{e^b - 1}$

C. $\log_e \dfrac{x}{x - 2}$

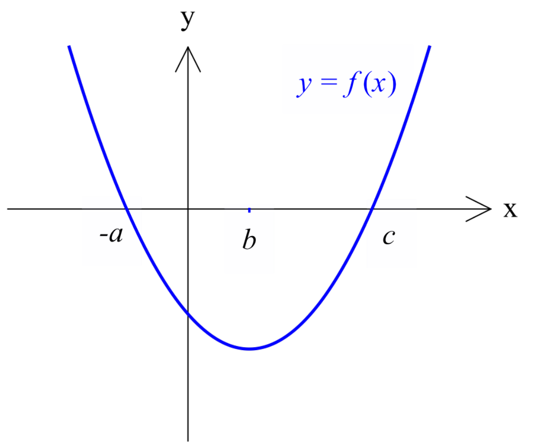
D. $\dfrac{2}{1 - e^b}$

E. $\dfrac{2}{1 - e^{-b}}$

SECTION A – continued

Question 9

The graph of the quadratic function $y = f(x)$ is shown below.



Which one of the following statements is true?

A. $f'(b) < 0$

B. $f'(-a) > 0$

C. $f(0) > 0$

D. $f(b) > 0$

E. $f'(c) > 0$

Question 10

If X is a random variable such that $\Pr(X > 3) = m$ and $\Pr(X > 7) = n$ then $\Pr\left(X < 3 \mid X < 7\right)$ is

A. $\dfrac{n-1}{m-1}$

B. $\dfrac{m}{n}$

C. $\dfrac{m}{m+n}$

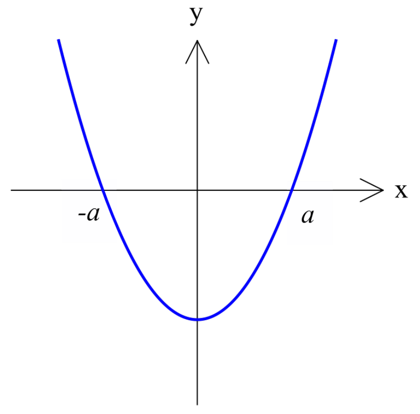
D. $\dfrac{m-1}{n-1}$

E. $\dfrac{1}{m-n}$

SECTION A – continued
TURN OVER

Question 11

The graph of a quadratic function is shown below.



The average rate of change of y against x between $x = -a$ and $x = 0$ is more than the instantaneous rate of change of y against x at

A. $x = -a$

B. $x = 0$

C. $x = \frac{a}{3}$ and $x = \frac{a}{2}$

D. $x = \frac{a}{2}$ and $x = a$

E. $x = a$

Question 12

Let $f : R \to R$ be a differentiable function.

Then for all $x \in R$, the derivative of $f(e^{2x-1})$ with respect to x is equal to

A. $2e^{2x-1} f'(x)$

B. $2e^{2x-1} f'(e^{2x-1})$

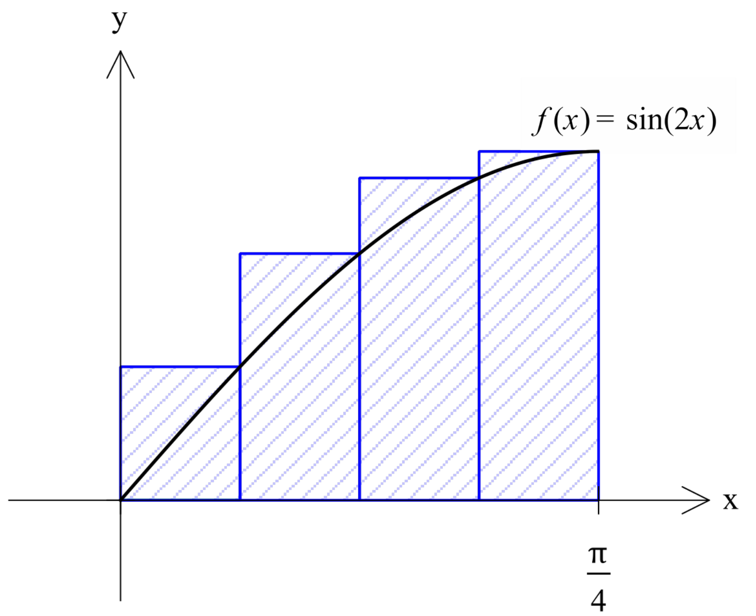
C. $2f'(e^{2x-1})$

D. $(2x-1)e^{2x-1}$

E. $f'(e^{2x})$

Question 13

Part of the function $f(x) = \sin(2x)$ is shown below. The area bounded by the graph of f, the x–axis and the lines $x = 0$ and $\frac{\pi}{4}$ can be approximated by the area of the four shaded rectangles. The area of the rectangles is approximately equal to



A. $\frac{\pi}{16}$
B. 0.5917
C. 3.02
D. 0.395
E. 0.188

Question 14

The random variable X has a normal distribution with mean 9 and standard deviation 3. If Z has the standard normal distribution then the probability that X is greater than 15 is equal to

A. $\Pr(Z > -2)$
B. $1 - \Pr(Z < -2)$
C. $\Pr(Z < 2)$
D. $1 - \Pr(Z > 2)$
E. $1 - \Pr(Z < 2)$

SECTION A – continued
TURN OVER

Question 15

Over a two–week period Kristy recorded the number of hours that she spent studying.

Number of hours spent each day studying	0	1	2	3	4	5
Proportion of days on which *x* hours were spent studying	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{3}{14}$	$\frac{1}{14}$	$\frac{3}{14}$	$\frac{5}{14}$

During the two-week period the mean number of hours each day that Kristy spent studying was:

A. $\frac{13}{7}$

B. $\frac{25}{14}$

C. $\frac{23}{14}$

D. $\frac{13}{14}$

E. $\frac{47}{14}$

Question 16

It is known that $\int_0^5 f(x)dx = 7$. The value $\int_2^7 f(x-2) + 3\ dx$

A. 12
B. 10
C. 22
D. 15
E. 18

SECTION A – continued

Question 17

The continuous random variable X has a probability density function given by:

$$f(x)=\begin{cases}\dfrac{3\sqrt{x}}{2}, & \text{for } 0 < x < 1\\ 0\,, & \text{elsewhere}\end{cases}$$

The value of k such that $\Pr(X>k)=\dfrac{19}{27}$.

A. $\dfrac{3}{2}$

B. $\dfrac{4}{9}$

C. $\dfrac{1}{4}$

D. $\dfrac{9}{4}$

E. $\dfrac{2}{3}$

Question 18

If the average value of the function $y=x^3$ over the interval $[1,b]$ is 10 the value of b is

A. $41^{\frac{1}{4}}$

B. 0.025

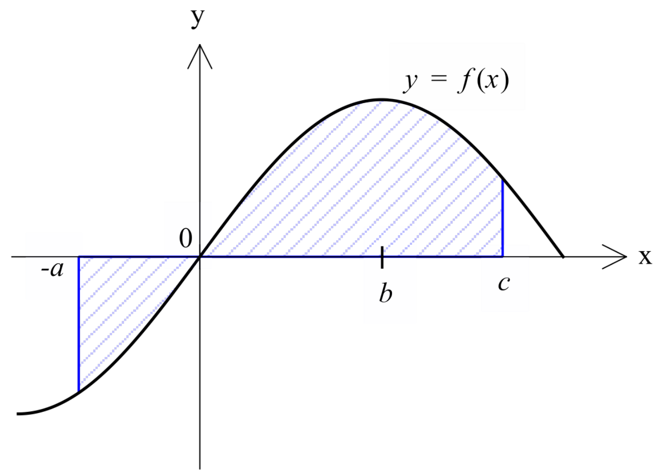
C. 3.43

D. 1

E. 3

Question 19

The graph of the function f with rule $y = f(x)$ is shown below:



The total area bounced by the curve $y = f(x)$, the x –axis on the interval $\left[-a, c\right]$ is given by:

A. $\int_0^a f(x)\,dx + \int_0^c f(x)\,dx$

B. $\int_a^c f(x)\,dx$

C. $\int_{-a}^0 f(x)\,dx - \int_0^c f(x)\,dx$

D. $\int_0^c f(x)\,dx - \int_a^0 f(x)\,dx$

E. $\int_0^{-a} f(x)\,dx + \int_0^c f(x)\,dx$

Question 20

Leonie regurlarly attends boxing classes at her gym. If she attends a boxing class on a particular day, the probability that she attends one the next day is 0.7. If she does not attend a boxing class one day the probability that she attends one the next day is 0.9. If she attends a boxing class on Monday the probability that she attends a boxing class on Wednesday is

A. 0.733
B. 0.76
C. 0.559
D. 0.21
E. 0.49

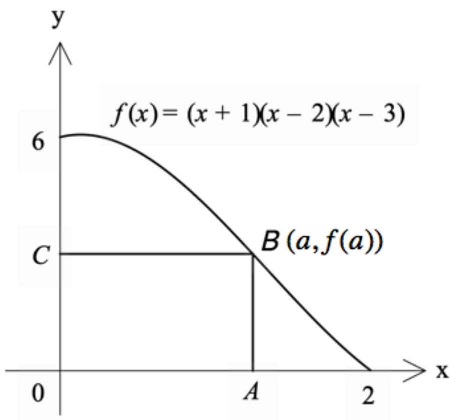
END OF SECTION A

SECTION B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (6 marks)

The graph of $f(x)=(x+1)(x-2)(x-3);\ 0\le x\le 2$ is shown in the diagram below



The rectangle OABC is inscribed in the region bounded by the graph of $f(x)=(x+1)(x-2)(x-3)$, the x-axis and y-axis.

a. Show that the area, A of the rectangle can be written as 1 mark

$$A=a(a+1)(a-2)(a-3)$$

b. Determine an appropriate domain for A. 1 mark

SECTION B – Question 1 – continued
TURN OVER

c. Find the derivative of A. 1 mark

d. Find the length and width for which the rectangle has a maximum area. 2 marks

e. Determine the maximum possible area of the inscribed rectangle. 1 mark

Question 2 (12 marks)

Consider the function $f(x) = \log_e(x+2)$.

a. Find the maximal domain of this function. 1 mark

b. Find the rule and domain of the inverse function $y = f^{-1}(x)$. 2 marks

c. Determine the coordinates of the point(s) of intersection of $f(x)$ and $f^{-1}(x)$.

Give your answer correct to two decimal places. 2 marks

d. Find correct to four decimal places, the area of the region bounded by the graphs of $f(x)$ and $f^{-1}(x)$. 2 marks

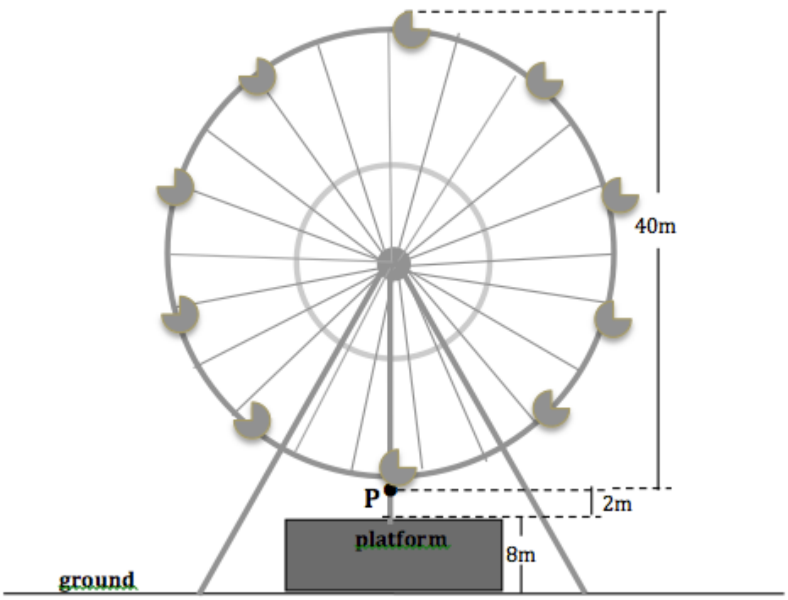
e. Find the equation of the tangent to the graph of $y = f(x)$ that passes through the point with coordinates $(-1, 0)$. 2 marks

f. Find the area of the region bounded by $f(x), f^{-1}(x)$ and the tangent found in part **e**. Give your answer correct to four decimal places. 3 marks

SECTION B – continued

Question 3 (14 marks)

Mel is designing a new ferris wheel to be the centre of attraction at a theme park. The circular ferris wheel will have a diameter of 40 metres and rise 50 metres above the ground at its highest point. Customers will enter the cabins of the ferris wheel from a platform 8 metres above ground level.



The height above ground level of a point, P on the floor of one of the cabins will be at its lowest point at the start of the ride. The height, h metres of the point, P, at after ***t* minutes** is given by the equation:

$$h(t) = b + a\sin c(t+d)$$

a. Show that the value of the amplitude, a is 20. 1 mark

b. Show that the value of b is 30. 1 mark

SECTION B – Question 3 – continued
TURN OVER

The wheel will rotate clockwise at a constant speed and not stop at any time during the 18 minutes the ride lasts. It takes 40 seconds for point, P to complete one full rotation.

c. Show that the value of c is 3π. 2 marks

d. Initially point, P is at its lowest point above the ground.

Show that $d = \frac{1}{2}$. 2 marks

e. How many times during the **18 minute** ride will point, P reach the maximum height? 1 mark

SECTION B – Question 3 – continued

f. Find correct to two decimal places the time when P first reaches a height of at least 40 metres above ground level. 2 marks

g. Find the average rate of change of the height of the point, P over the interval $\left[0, \frac{1}{6}\right]$.

Give your answer correct to two decimal places. 2 marks

h. Mel will feel sick if the rate of change of the height of the point, P is greater than 100 m/s for more than 12 seconds at any time. Explain whether Mel feel sick on the ferris wheel.

3 marks

Question 4 (8 marks)

BK is a viral infection that causes flu like symptoms. The virus is contagious and can be spread from person to person.

The number of new BK cases (M) after t days in an isolated population of 2800 people previously unexposed to the virus can be modelled by

$$M = \frac{at}{2t+3}, \; 0 \le t \le 10$$

where a is a positive constant.

a. After two days, 1600 people previously **unexposed** to the virus will have BK.
Show that $a = 5600$. 1 mark

The number of new BK cases (N) after t days in an isolated population of 2800 people previously **exposed** to the virus can be modelled by:

$$N = \frac{bt}{t+2}, \; 0 \le t \le 10$$

where b is a positive constant.

b. After five days, 2000 people previously **exposed** to the virus will have BK.
Show that $b = 2800$. 1 mark

SECTION B – Question 4 - continued

c. The difference (D) in new BK cases between those previously unexposed and those previously exposed is given by

$$D = \frac{ct}{(2t+3)(t+2)}$$

Show that the value of c is 2800. 3 marks

d. Write a rule for the rate at which the difference in new BK cases between those previously unexposed and those exposed to the virus is changing with time. 1 mark

e. **Hence** find the maximum difference in new BK cases between those previously unexposed and those exposed to the virus. 2 marks

Question 5 (20 marks)

On any one day thousands of passengers wait for planes at Melbourne Airport. Planes with a particular company flying out of Melbourne can leave on time or be late. The probability that a plane leaves on time is $\frac{3}{5}$.

a. A random sample of 20 passengers is taken.
Let X be the random variable that represents the number of passengers whose plane leaves on time.

i Find the $\Pr(X \geq 5)$ correct to four decimal places. 2 marks

ii Find the $\Pr(X \geq 10 \mid X \geq 5)$ correct to four decimal places. 2 marks

For samples of 20 passengers, $\hat{P}$ is the random variable of the distribution of sample proportions of people whose plane arrives on time.

iii Find the expected value and variance of $\hat{P}$. 3 marks

SECTION B – Question 5 continued

iv Find the probability that the sample proportion lies within two standard deviation of $\frac{3}{5}$.

Give you answer correct to three decimal places. Do not use a normal approximation.

3 marks

v Find $\Pr\left(\hat{P} \geq \frac{3}{4} \mid \hat{P} \geq \frac{3}{5}\right)$ correct to four decimal places. 2 marks

b. Assume that the number of minutes, T, that a plane is late by is a normally distributed random variable with a mean of 14 and a standard deviation of 4.
The table below shows the number of minutes a plane may be late by and whether a passenger will consequently arrive on time or be late to their destination.

Number of minutes a plane is late by	Arrival at destination
More than 30 minutes	Late
Between 30 and 60 minutes	On time
Less than 15 minutes	Early

i Find correct to four decimal places the probability that a passenger arrives early to their destination. 1 mark

SECTION B – Question 5 continued
TURN OVER

At 8 a.m. on a Monday morning there are 10 planes due to arrive at Melbourne Airport.
The number of minutes each plane is late by is independent of each other.

ii Find correct to 4 decimal places the probability that 3 planes are less than 15 minutes late. 1 mark

iii Find correct to 4 decimal places the probability that the first two planes are less than 15 minutes late and the rest are not. 1 mark

c. The number of minutes that Billie will be late to her destination due a plane being late is given by the probability density function:

$$f(t)=\begin{cases}\dfrac{e^{-\frac{t}{5}}}{5}, & x>0\\ 0, & \text{elsewhere}\end{cases}$$

i Find correct to 2 decimal places the probability that Billie is arrives at her destination more than10 minutes late . 1 mark

SECTION B – Question 5 continued

ii Find the mean number of minutes that Billie arrives late to her destination.

1 mark

iii Show that the median number of minutes that Billie arrives late her destination is exactly $\log_e 32$.

3 marks

END OF QUESTION AND ANSWER BOOK

Mathematical Methods Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n\,dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$		$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}\,dx=\log_e(x)+c, x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int \cos\ (ax)\,dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule:	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule:	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability Distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x\, p(x)$	$\sigma^2 = \Sigma (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\, f(x)\, dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$sd(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

Student Name:

MATHEMATICAL METHODS UNITS 3 & 4

Trial examination 2

2016

MULTIPLE CHOICE QUESTIONS ANSWER SHEET

SHADE THE CORRECT RESPONSE

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E