Mathematical Modeling and Simulation of Photovoltaic Solar Module in Matlab-Mathworks environment

Yamina Khlifi

Abstract — This paper presents a mathematical modeling and simulation of a photovoltaic solar module. Mainly an accurate mathematical model for computing Maximum Power output of a photovoltaic PV module is presented. The model for PV panel is developed based on the single-diode photovoltaic model, found in the literature, including the effect of the series resistance.

A typical 60 W photovoltaic panel is selected for simulation in Matlab-Mathworks environment. The essential parameters, required for modeling the photovoltaic module, are taken from datasheets provided from manufactures'. Current-Voltage and Power-Voltage characteristics are obtained, for the selected module, from simulation and compared with the experimental curves.

On the other hand, this model is used to compute and to investigate the variation of Maximum Power output of a PV module with temperature and irradiance intensity levels.

A validation of the proposed mathematical model is performed by an interactive analysis and comparison between simulation results and the typical PV module datasheet. The simulation results are well matched with the experimental data.

Index Terms — Mathematical Modeling, Maximum Power PV module, Newton-Raphson's method, Photovoltaic module.

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1 INTRODUCTION

RENEWABLE sources of energy acquire increasing importance due to the enormous consumption and exhaustion of fossil fuels such as coal, oil and natural gas. Renewable energy is abundant, free, sustainable, clean, and can be harnessed from different sources in the form of wind, solar, tidal, hydro, geothermal, and biomass [1]. Energy supplied by the sun in one hour is equal to the amount of energy required by the human in one year [2]. Under these circumstances, interest in photovoltaic (PV) solar cell is increasing rapidly as an alternative and clean energy source [3].

Knowledge of the characteristic of photovoltaic module is essential for designing and dimensioning a PV power supply. This is the reason for the development of PV panel models.

This paper presented a simple method of modeling and simulation of photovoltaic panels using MATLAB-Mathworks. Taking the effect of irradiance and temperature into consideration, the output current and power characteristics of photovoltaic module are simulated using the proposed model.

On the other hand predicting the performance of PV panels is important for design engineers. The manufacturer provides information about the electrical characteristics of PV panel by specifying certain points in its current-voltage characteristics which are called remarkable points [4]: short circuit (0, I_{sc}), maximum power point (V_m, I_m) and open-circuit (V_{oc}, 0).

The determination of these points, mainly the Maximum Power Point (MPP), is essential for the development of appropriate PV models. Furthermore, most of these parameters depend on both the cell temperature and the solar irradiance; therefore, the knowledge of their behavior is crucial to correctly predict the performance of PV panel.

The aim of this paper is to present a simple mathematical model for computing Maximum Power output of the PV panel as function of irradiation levels and cell temperature.

2 MODELING OF PHOTOVOLTAIC MODULE

2.1 Solar cell

Solar cell is a key device that converts the solar energy into the electrical energy. In most cases, semiconductor is used for solar cell material. The energy conversion consists of absorption of photon energy producing electron-hole pairs in a semiconductor and charge carrier separation. A PN junction is used for charge carrier separation in most cases [3].

In the dark, the current-voltage I(V) characteristic of a solar cell has an exponential characteristic similar to that of a diode.

When the cell is illuminated, electron-hole pair is generated by the photons that have energy greater than the band gap. These carriers are separated under the effect of the electric field in the depletion region due to the ionized impurity atoms. This charge separation creates a current proportional to the incident radiation.

When the cell illuminated is open-circuited, the voltage is generated due to the charge carrier separation. The voltage developed is called the open-circuit voltage V_{oc}.

When the p and n-side of the PN junction are short-circuited, the current generated is called short-circuit current I_{sc} and equals to the photo-generated current I_{PH}.

2.2 Mathematical modeling of PV cell

Different models have been presented in literature to accomplish better accuracy and serve for different purposes [5], [6], [7], [8], [9]. For simplicity, the single diode model of figure 1, with the basic structure composed of a current source and a parallel diode [10], [11], [12] including also the effects of a series resistance is considered for this work.

- *KHLIFI is a Prof in Micro-Nanoelectronics Systems and Dispositifs (SDMN) Laboratory, Department of Physics, National School of Applied Sciences(ENSAO) in Mohamed First University, Morocco. E-mail: khlifi_yamina@yahoo.fr*

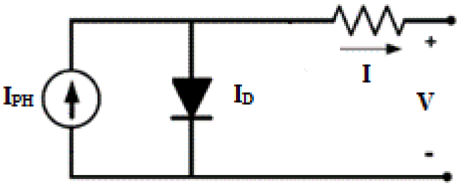

Fig.1 Circuit Diagram for a photovoltaic cell

The mathematical model of photovoltaic cell varies with the short circuit current (I_{sc}) and the open circuit voltage (V_{oc}), which are gleaned from the cell manufacturer's data sheet.
Increasing accuracy can be introduced to the model by adding the temperature dependence of the semiconductor Band Gap energy.

Using the general model, while applying Kirchhoff's law on the common node of the current source, diode and series resistance (R_S), the PV current, I, can be derived by :

$$I = I_{PH} - I_D \quad (1)$$

$$I = I_{PH} - I_s(exp[(V + R_s I)/U_T] - 1) \quad (2)$$

where :
I : Solar module output current (A),
V : Solar module output voltage (V),
I_{PH}: Photo-current of the PV module (A),
I_S : Diode reverse saturation current (μA),
R_S : Series resistance associated with the cell (mΩ),
U_T : Thermal voltage [$U_T = nK_B\ T/q$],
n : Diode ideality factor, its value ranges between 1 and 2,
q : Electron charge [$q = 1.602 \times 10^{-19}$C],
K_B : Boltzman's constant [= 1.38×10^{-23} J/K],
T : Temperature (K).

The photocurrent depends of temperature [13], [14], [15], [16] :

$$I_{PH} = I_{PH0}\left(1 + K_0\ (T - T_0)\right) \quad (3)$$

where:

$$I_{PH0} = \left(I_{sc.0}/G_0\right) G \quad (4)$$
$$K_0 = \left(\left(I_{sc,T_1} - I_{sc,0}\right)/(T_1 - T_0)\right) \quad (5)$$

where :

G : Irradiance level (W/m²),
G_0: Irradiance level at Standard Test Conditions (STC) where an average solar spectrum at AM 1.5 is used, the irradiance G_0 is normalized to 1 sun = 1000 W/m², and the cell temperature T_0 is equal to 25°C.
I_{sc}: Short-circuit current of the module,
$I_{sc,0}$: Short-circuit current of the module at T_0.

The saturation current depends on temperature [14], [15], [16], [17]. It's calculated using the following equation:

$$I_s = I_{s,0}\ (T/T_0)^{3/n} \exp\left(-\left[\ (qE_g/nK_B)[(1/T) - (1/T_0)]\right]\right) \quad (6)$$

where:

$$I_{s,0} = I_{sc,0}/\left[\exp\left(V_{oc,0}\ /U_{T0}\right)\ - 1\right] \quad (7)$$

$I_{S,0}$: Diode reverse saturation current at STC,
$V_{oc,0}$: Open-circuit voltage of the module at T_0,
E_g : Band-Gap energy of the semiconductor used in the cell.

The Band Gap energy E_g of the semiconductor decreases with temperature and its temperature dependence is well modeled by [18]:

$$E_g(T) = E_{g0}\ - \left[\alpha T^2/(T + \beta)\right] \quad (8)$$

where E_{g0} is the Band-Gap energy at absolute zero on the Kelvin scale in the given semiconductor and α and β are material-specific constants [18], [19]. Table 1 [19] provides these

TABLE 1
VARSHNI EQUATION CONSTANTS FOR GAAS, SI, AND GE [19]

Material	E_{g0} (eV)	α (eV/K)	β (K)
GaAs	1.519	5.41×10^{-4}	204
Si	1.170	4.73×10^{-4}	636
Ge	0.7437	4.77×10^{-4}	235

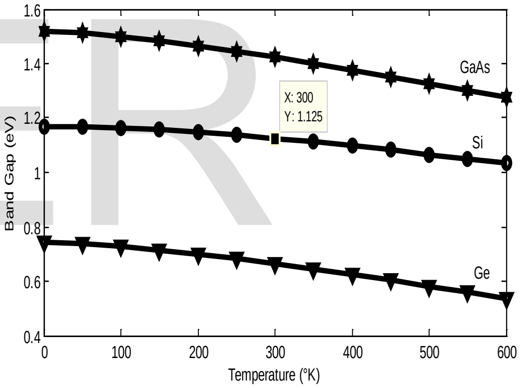

Fig.2 Energy Band-Gap temperature dependence of GaAs, Si, and Ge.

Figure 2 shows how the band gaps of the three materials decrease as temperature increases (the labeled point is the band gap of Silicium at room temperature).

The ideal factor n is dependent on PV technology [20] as shown in Table 2.

TABLE 2
DIODE IDEALITY FACTOR

Technology	n
Si Mono	1.2
Si-Poly	1.3
a-Si:H	1.8
a-Si:H tandem	3.3
a-Si:H triple	5
CdTe	1.5
CTS	1.5
GaAs	1.3

The series resistance of the module has a large impact on the slope of the current-voltage I(V) curve at V = V_{OC}. Equations (9) and (10) are found by differentiating equation (2), evaluating at V = V_{OC}, and rearranging in terms of R_S [5].

$$dI/dV = -(I_s/U_T)\ [1 + R_s(dI/dV)]\ exp((V + R_sI)/U_T) \quad (9)$$

$$R_s = -\ \ dV/dI)_{V_{oc,0}} - (U_T/I_s)\ exp\left(-\ V_{oc,0}\ /U_T\right) \quad (10)$$

dV/dI at V_{oc} per cell determined from manufacturers graph.

2.3 Matlab model of the photovoltaic module

In order to apply these concepts to developments of a solar cell model, The Solarex MSX60 PV [21], a typical 60W PV module, has been chosen for modeling. The module consists of 36 polycrystalline silicon solar cells electrically configured as two series strings of 18 cells each. The key specifications are shown in Table 3.

TABLE 3

KEY SPECIFICATIONS OF THE SOLAREX MSX60 PV MODULE AT STANDARD TEST CONDITIONS (STC) WHERE AN AVERAGE SOLAR SPECTRUM AT AM 1.5 IS USED, THE IRRADIANCE IS NORMALIZED TO 1 SUN =1000 W/M², AND THE CELL TEMPERATURE IS EQUAL TO 25°C.

Model	MSX60
Cell type	Polycrystalline silicon
Maximum power [P_{max}] (W)	60
Voltage at Maximum Power [V_m] (V)	17.1
Current at Maximum Power [I_m] (A)	3.5
Short-circuit current [I_{sc}] (A)	3.8
Open-circuit voltage [V_{oc}](V)	21.1
Number of series cells (N_s)	36
Number of parallels cells (N_P)	1

The model was evaluated using Matlab environment. The current I is then evaluated using these parameters, and the variables Voltage (V), Irradiation (G), and module Temperature (T).

The model of simulation of the PV cells behavior is based on the simple equivalent electrical model presented in figure 1 and modeled by equation (11):

$$f(I) = I - I_{PH} + I_s\left(exp\left((V + R_sI)/U_T\right) - 1\right) \quad (11)$$

The inclusion of a series resistance in the model makes the solution for current a recurrent equation. A Newton Raphson's method is used to solve it. This simple iterative technique converges much more rapidly towards solution; the newton method is described as :

$$I_{n+1} = I_n - [f(I_n)/f'(I_n)] \quad (12)$$

$$f'(I_n) = 1 + (R_sI_s/U_T)\ exp\left((V + R_sI_n)/U_T\right) \quad (13)$$

2.4 Simulation Results of PV module model

A Matlab function has been implanted in Matlab environment, based on mathematical equations (2) to (13) that characterize the photovoltaic module. The current-voltage I(V) and power voltage P(V) characteristics are presented for various temperatures, irradiation levels, and then for various ideality factors and series resistance.

Effect of temperature and irradiation variation

The solar irradiation G and the operating cell temperature T have a crucial influence on the output characteristics of a photovoltaic module.

Figures 3 to 6 show the I(V) and P(V) curves first for various temperatures (T is a vector of value (0, 25, 50, 75°C)) at a constant irradiation (G =1 sun), and then for various irradiation levels (G takes the following values 0.25, 0.5, 0.75, 1 sun) at a constant cell temperature (T = 25°C).

The voltage V is considered varying from 0 to open circuit voltage V_{oc} corresponding to the variation in current from short circuit current I_{sc} to 0.

Increase in temperature at constant irradiance is accompanied by a decrease in the open circuit voltage value and a significant reduction of the power output as shown in figures 3 and 4. In fact, increase in temperature causes increase in the band-gap of semiconductor and thus the efficiency of the solar cell is reduced [22].

A number of discrete data points are shown on the curves in figures 3 and 4. These points are taken directly from the manufacturer's published curves, and show a good agreement between the experimental I(V) and theoretical characteristics.

Based on figures (5, 6), it's clear that the I(V) and P(V) characteristics of a solar module are greatly dependent on the solar irradiance levels. At constant module temperature, it can be seen that the short circuit current as well as the power increase with the increase in irradiation level, while very little variation in the open-circuit voltage is observed. The reason is that the open-circuit voltage (V_{oc}) is logarithmically dependent on the solar irradiance, yet the short-circuit current (I_{sc}) is a linear function of the illumination [23].

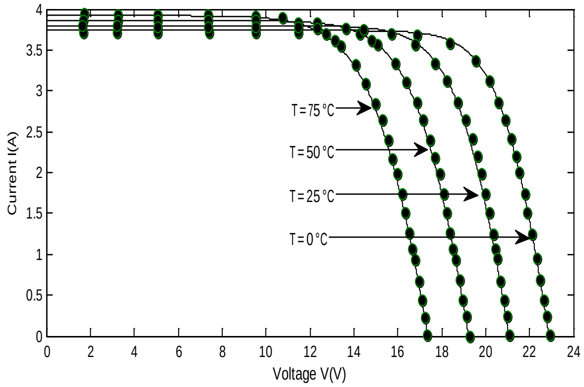

Fig.3 Current-voltage I(V) characteristics for various temperature (T = 0, 25, 50 ,75°C) and constant irradiance G = 1Sun. The discrete data points shown are taken from the manufacturer's curves.

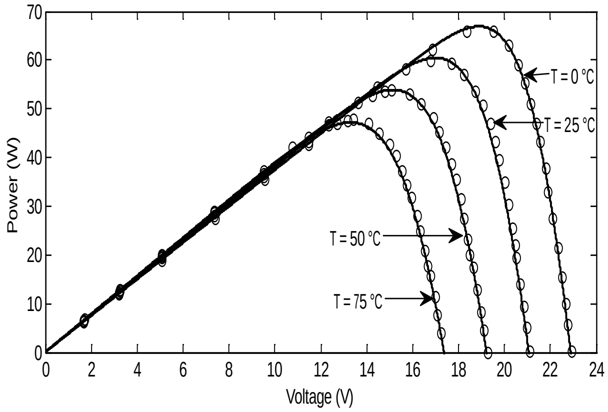

Fig.4 Power-voltage P(V) characteristics for various temperature (T = 0, 25, 50 75°C) and constant irradiance G = 1Sun. The discrete data points shown are taken from the manufacturer's curves.

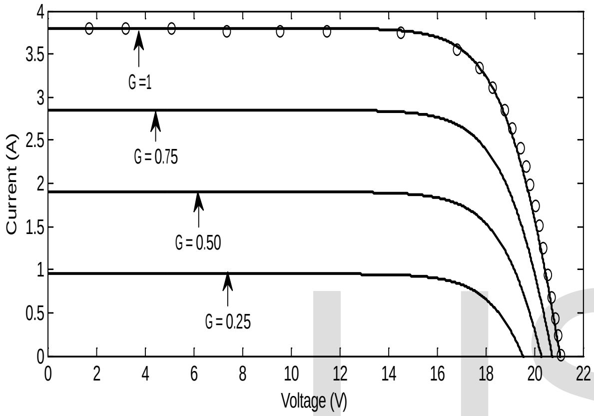

Fig.5 Current-voltage I(V) characteristics for variousirradiance levels (G = 0.25, 0.5, 0.75,1) and constant temperature T = 25°C. The discrete data points shown are taken from the manufacturer's curves.

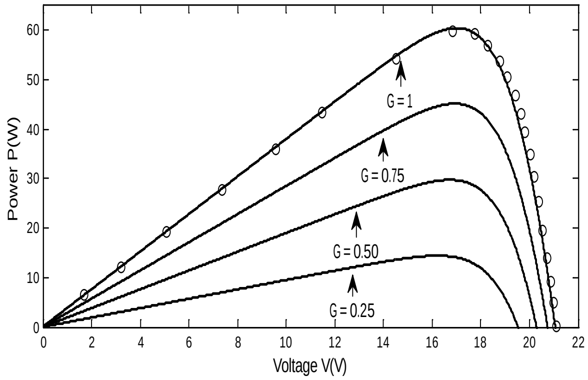

Fig.6 Power-voltage I(V) characteristics for various irradiance levels(G = 0.25, 0.5, 0.75,1) and constant temperature T = 25°C.The discrete data points shown are taken from the manufacturer's curves.

3 MODEL OF MAXIMUM POWER OUTPUT

The peak value of the product of V and I represent the maximum power point (MPP), P_m, of the solar module. The current and voltage of PV module at this MPP are denoted by I_m and V_m, respectively.

$$P_m = I_m V_m \tag{14}$$

The rectangle-defined by V_{OC} and I_{SC} provides a convenient means for characterizing the maximum power point. The fill factor, FF, is a measure of the squareness of the I(V) characteristic and is always less than one.

$$FF = P_m / (I_{SC} V_{OC}) \tag{15}$$

The solar module should always be operated at the maximum power for a given input conditions. Then, determination and knowledge of MPP behavior is crucial to correctly predict the performance of PV panel.

3.1 Theoritical model

Maximum power point is found by solving:

$$\partial P / \partial V)_{V=Vm} = [I + V(\partial I/\partial V)]|_{V=Vm} = 0 \tag{16}$$

where:

$$V = U_T \, ln((I_s + I_{PH} - I)/I_s) - R_s I \tag{17}$$

$$g(I) = I + \left(\begin{array}{c} (I - I_s - I_{PH}) \\ [ln((I_s + I_{PH} - I)/I_s) - (R_s I/U_T)] \\ /\left(1 + (I_s + I_{ph} - I)(R_s/U_T)\right) \end{array} \right) = 0 \tag{18}$$

The current at the maximum power point, I_m, is then found by evaluating equation (18) at $V = V_m$.

$$g(I_m) = \begin{array}{c} I_m + 2(R_s I_m/U_T)(I_s + I_{ph} - I_m) + \\ (I_m - I_s - I_{ph}).\left(ln(I_s + I_{ph} - I_m) - ln(I_s)\right) \end{array} \tag{19}$$

$$g'(I_m) = \begin{array}{c} I_m + 2(R_s/U_T)(I_s + I_{ph} - 2I_m) + \\ \left(ln(I_s + I_{ph} - I_m) - ln(I_s)\right) \end{array} \tag{20}$$

The current I_m is then evaluated using Newton Raphson method. Thus, we can deduce voltage at Maximum Power Point (V_m) and the MPP (P_m).

3.2 Simulation results

Simulations were performed using the previously Solarex MSX60 60W PV module model.

A Matlab function, based on mathematical equations (16) to (20), which calculates the Maximum Power Point of PV module, has been developed. This MPP depends on cell temperature, irradiantion levels, diode ideality factor (n), and series resistance.

The I(V) and P(V) characteristics (figures (7) and (8)) show a Maximum Power Point known as remarkable point at STC. The cell or PV module must operate at this point for an efficient use.

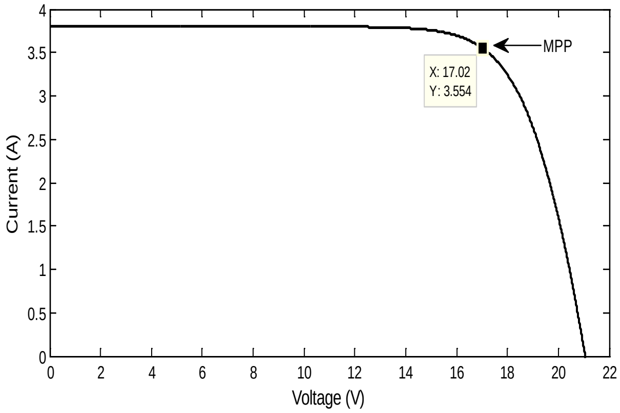

Fig.7 Current-voltage I(V) characteristics at STC (T= 25 °C; G = 1 sun).

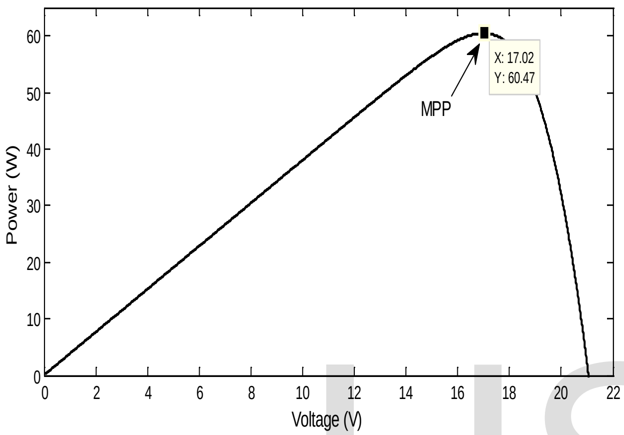

Fig.8 Power-voltage I(V) characteristics at STC (T = 25 °C; G = 1 sun).

Effect of variation of solar irradiance

Figures (9) and (10) represent the I(V) and P(V) characteristics for various irradiation levels value (0.25, 0.50, 0.75, 1 sun) at constant cell temperature (25 °C). The Maximum Power Point, computed using the developed model, is presented by a discrete point for each curve. It shows that increasing insolation levels increases the MPP and shows an excellent correspondence with the model.

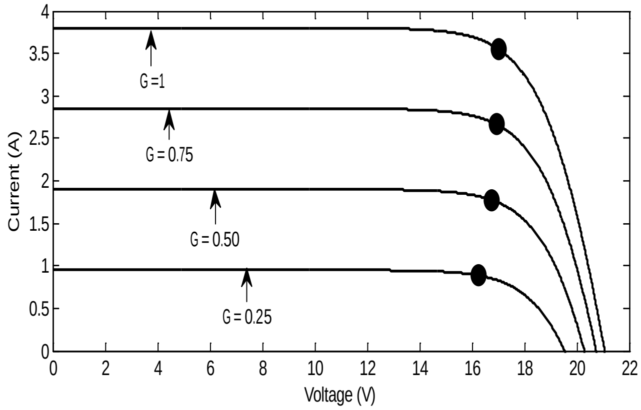

Fig.9 Current-voltage I(V) curve at various irradiance and constant temperature. The discrete points represent the calculated maximum power MPP.

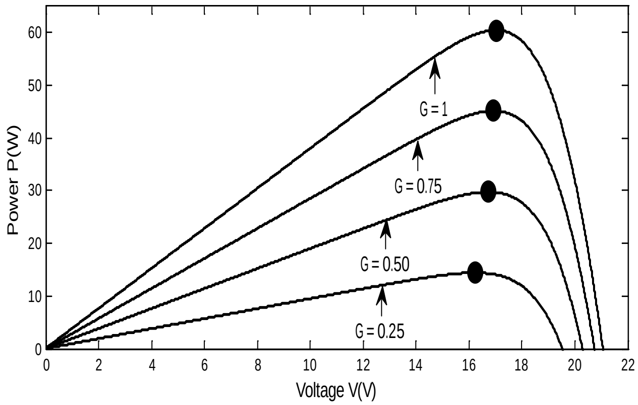

Fig.10 Power-Voltage P(V) curve at various irradiance and constant temperature. The discrete points represent the calculated maximum power MPP.

Effect of diode factor ideality and series resistance

Figures (11) and (12), show power-voltage curves for several diode factor ideality (n = 1, 1.25, 1.5, 1.75, 2), and then for various series resistance (R_s(mΩ) = 0, 8, 16) at STC (G = 1sun and cell temperature (T = 25°C)). Discrete points witch represent the Maximum Power Point, evaluated using the MPP model, are shown on the same figures (11) and (12).

The effect of varying the ideality factor (n), as can be seen in figure 11, shows that higher values of n soften the knee of the curve mainly reduces the Maximum Power Point.

The series resistance R_S affect significantly the Photovoltaic output power. It has a large impact on the slope of the P(V) curves at V_{OC} as seen in figure 12.

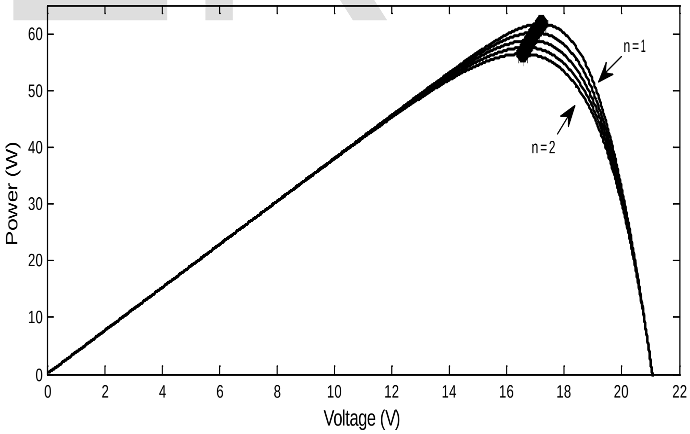

Fig.11 Power-voltage P(V) characteristics for various diode factor ideality (n = 1, 1.25, 1.5, 1.75, 2), G = 1sun and T = 25°C The discrete points represent the calculated maximum power MPP.

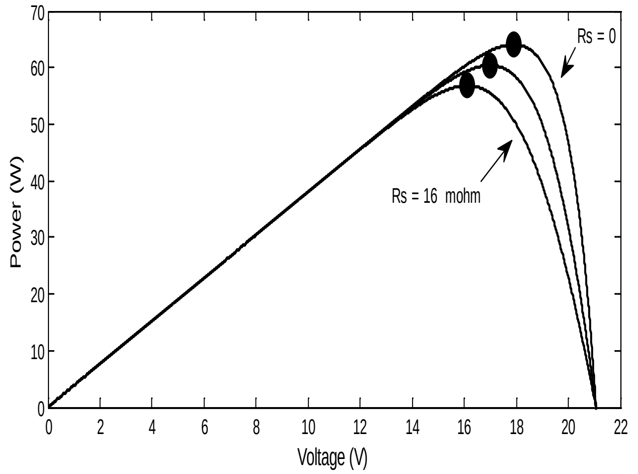

Fig.12 Power-voltage P(V) characteristics for various Series resistance (R_s(mΩ) = 0, 8, 16), G = 1sun and T = 25°C. The discrete points represent the calculated maximum power MPP.

Effect of temperature variation

Figures (13) and (14) represent the I(V) and P(V) characteristics for several temperature value (0, 25, 50, 75 °C) at constant irradiance levels (1000 W/m²). Discrete points, representing the MPP of the theoretical model, are shown in the same figures for various temperatures. It decreases with increasing temperature and shows an excellent correspondence with the model and experimental data.

4 VALIDATION OF THE MODEL

The accuracy of the proposed MPP method is successfully demonstrated by simulations and experimental evaluations of MSX60 PV module, as shown in figure 14.

The relative errors estimated on current (I_m), voltage (V_m) and MPP (P_m) at STC, according to tables 4 and 5, show a good agreement between the data taken from datasheet and the theoretical values obtained from the MPP model. It appears that the relative error on MPP is less than 0.8% at standard test conditions.

TABLE 4

THEORETICAL AND EXPERIMENTAL CURRENT, VOLTAGE AND POWER AT MPP UNDER STC (G = 1000 W/M² AND T = 25 °C).

	Im(A	Vm(V)	Pm(W)
Theoritical values	3.554	17.02	60.47
Experimental values	3.5	17.1	60

TABLE 5

ESTIMATED ERROR ON CURRENT, VOLTAGE AND POWER AT MPP UNDER STC (G = 1000 W/M² AND T = 25 °C).

Relative error on I_m (%)	Relative error on V_m (%)	Relative error on P_m (%)
1.54	0.46	0.78

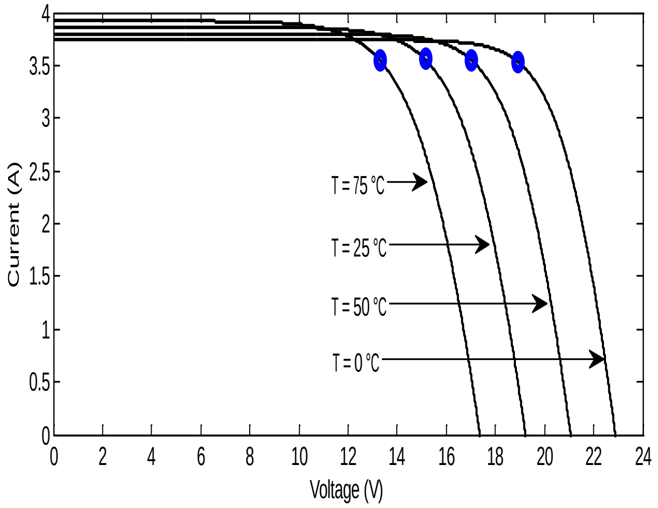

Fig.13 Current-Voltage I(V) curve at various temperature and constant irradiance. The discrete points represent the calculated maximum power MPP.

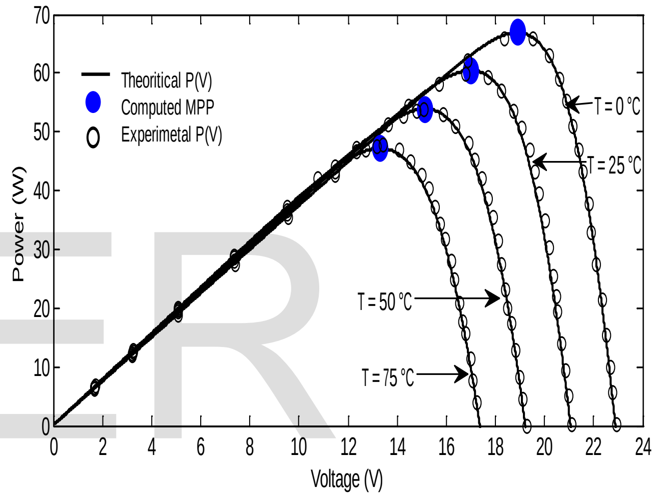

Fig.14 Power-Voltage P(V) curve at various temperature and constant irradiance. The discrete points represent the calculated maximum power MPP.

5 CONCLUSION

An accurate model of PV module was presented and tested using Matlab-Mathwors for a typical 60 W selected solar module. Based on this model, an accurate mathematical method for computing Maximum Power output of a PV module is developed. Simulation results show good agreements with the datasheets. The relative error between theoretical and experimental values of MPP output is about 0.8 %, thus proving the accuracy of the developed model.

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