Mathematical Modeling in Grades 3-5

High Five Institute

August, 2013

Tripling the Perimeter

- The perimeter of a rectangle is tripled and the new perimeter is between 65 and 73 inches. The original length is at least 3 more than the original width. What are the possible dimensions of the original rectangle? Use a picture and equations to justify your answer.

Tripling the Perimeter

- Approaches?
- Representations?
- Mathematical concepts?

Tripling the Perimeter

- How do you support this task before students start?
- As students are exploring?
- After students have done the task?

Perimeter

- Compare this task to:

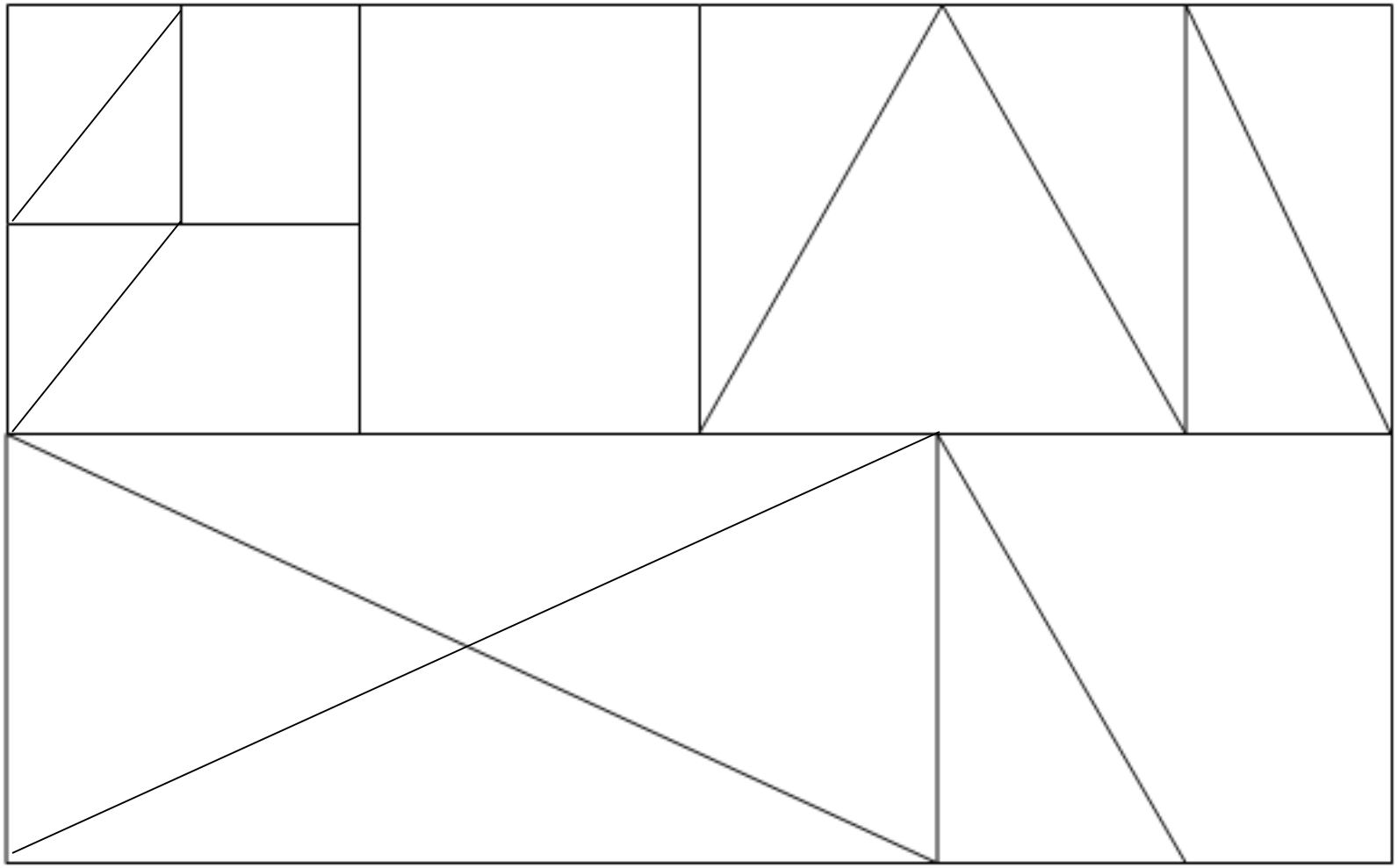
- What is the perimeter of a 7” x 5” rectangle?

- What is the perimeter of a square with an area of 64 square inches?

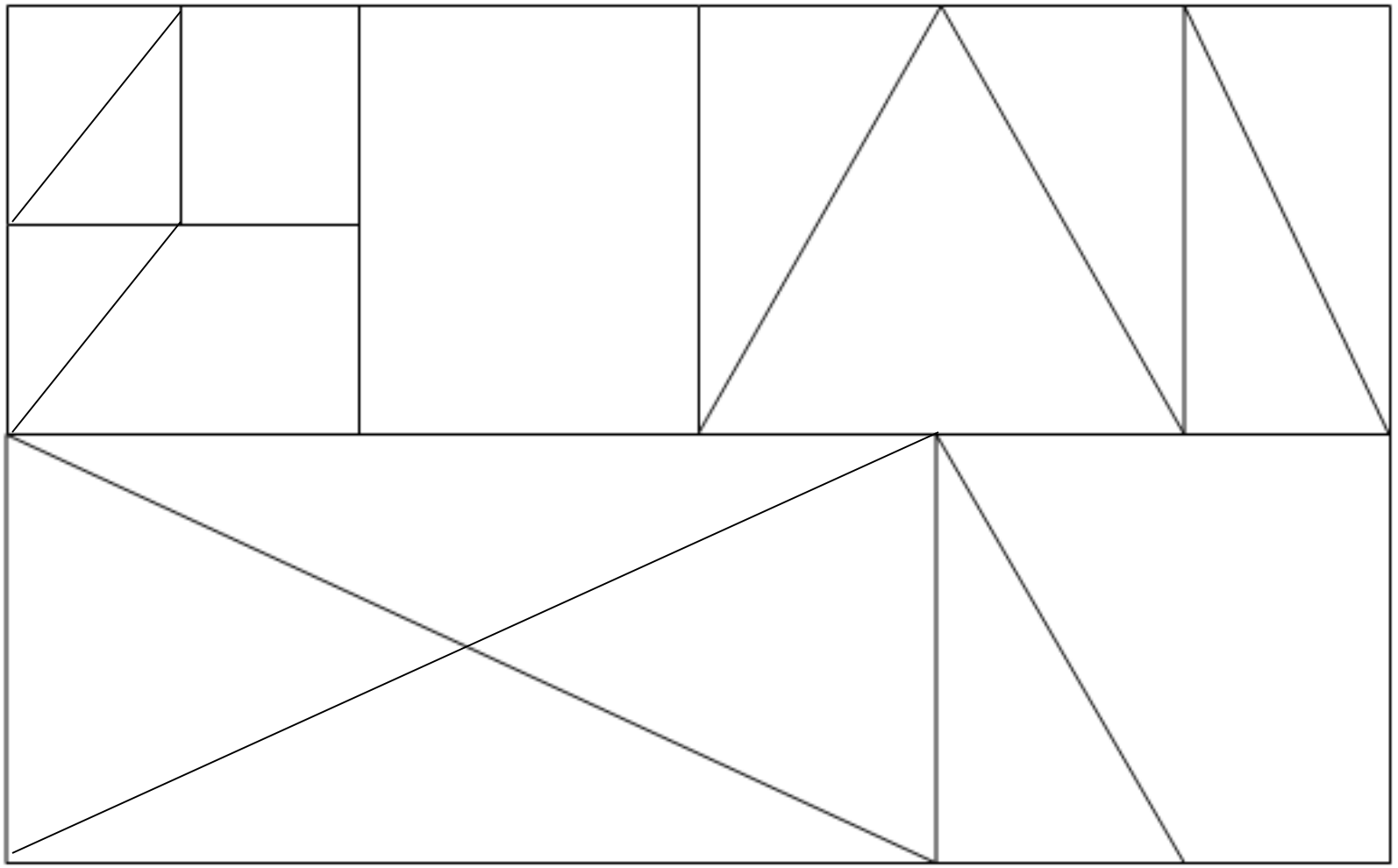
Tasks

- Open-ended
  - Multiple answers, multiple ways to solve them
- Closed
  - One answer, multiple ways to solve them
- What is the benefit of each type of task?

The picture is a neighborhood. For each section find the fractional value of the whole neighborhood.



What did you start with?



Neighborhood Task

- How would you modify this for your students?

Perimeter vs Neighborhood

- Modeling… how?
- Attending to precision…how?

Pattern Block Fractions- you will need pattern blocks

- If 2 joint hexagons =1 what is the value of….

- A triangle
- 2 triangles
- 2 blue rhombi and a triangle
- A trapezoid and a rhombus
- A hexagon joined with a triangle
- A hexagon joined with a rhombus
- A hexagon joined with a rhombus

- Write equations to go with your picture

More Pattern Block Explorations

Write equations

- If the hexagon joined with the triangle = 1, come up with 3 shapes that are greater than ½ but less than 1.
- If the hexagon joined with the blue rhombus = 1, find the sum of 2 joint hexagons.
- If the blue rhombus = 1/3, what is the value of the red trapezoid?
- If the red trapezoid = 1/3, what is the value of 2 connected blue rhombi?

Pattern Blocks

- How did you use the blocks?
- Where did the equations come from?
- Pattern blocks versus pre-marked fraction tiles/strips?

Partitioning…

- Your friends come to a party with different lengths of long strips of gum. There is an argument over who has more.

- Sara has 5/6 of a foot
- Tomas has 3/4 of a foot
- Jillian has 5/7 of a foot
- Miguel has 7/9 of a foot

- Draw a picture of each strip of gum and determine who has more.

Partitioning

- Approaches?
- Representations- how did you make them?
- Reasoning- how did this help you?

Partitioning… 3rd grade version

- Your friends come to a party with different lengths of long strips of gum. There is an argument over who has more.

- Sara has 1/3 of a foot
- Tomas has 1/4 of a foot
- Jillian has 1/6 of a foot

- Draw a picture of each strip of gum and determine who has more.

3.NF.3d

- Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole.

3.NF.2

- Understand a fraction as a number on the number line; represent fractions on a number line diagram.
- Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

Pizzas

- A pizza has 2/3 of it left.

- How many slices are left if the original pizza had:
  - 3 slices
  - 6 slices
  - 12 slices
  - 24 slices
  - 300 slices

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Why these numbers?

Pizza Joint…

- A pizza place cuts their pizza in 8 slices. At the end of the lunch buffet there is the equivalent of 2 pizzas left. Here are the types:
- 9 slices have pepperoni. 4 slices have only cheese. 2 slices have only peppers. 1 slice has only pineapple.

- Draw a picture and determine how much of a pizza there is of each type of pizza.

4.NF.3

- 4.NF.3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

4.NF.3

- Easier with a region (pizza) or a number line?
- Why?
- When we add and subtraction fractions with unlike denominators (5.G.1 and 5.G.2), how can partitioning support this work?
- ½ + ¼ + 1/3

Multiplying and Dividing Fractions

- Problem Types
- What does “Cognitively Guided Instruction” mean to you?
- Glossary, Table 1 and Table 2

Problem types

- In Glossary, Table 1 and Table 2
- Different types of problems
  - Context and ‘action’ of the problem determines how we represent them and how we solve them
- Similar ideas for fractions and decimals

Sharing Tasks

- How many brownies does a person get if:

- a) You have 1 brownie that you want to share 5 ways.
- b) You have 3 brownies that you want to share 5 ways.
- c) You have 10 brownies that you want to share 5 ways.
- d) You have 11 brownies that you want to share 5 ways.

Sharing Tasks

- How did you solve them ?

Types of Strategies- 11 brownies 5 ways

- No coordination
  - Random sharing, usually incomplete or not accurate
- Non-anticipatory coordination
  - No relationship between number of shares and pieces– repeated halving
- Additive
  - Every brownie is split 5 ways- each person gets 11 sections that are 1/5 of a brownie or 11 fifths
- Multiplicative
  - Recognizes 11 brownies will be numerator (number of items) and 5 will be denominator (number of groups)

5.NF.3

- Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

- Does this work also start earlier in Grade 3 or Grade 4?

Multiple Groups

- a) We need to make sandwiches for 12 children. Each child will receive 2 sandwiches. How many sandwiches do we need to make?
- b) We need to make sandwiches for 12 children. Each child will receive 2/3 of a sandwich. How many whole sandwiches do we need to make?
- c) We have 12 sandwiches. How many children can we feed if we give each child 3 sandwiches?
- d) We have 12 sandwiches. How many children can we feed if we give each child 2/3 of a sandwich?

Let’s look at some of your work

Connection between Tasks

- What similarities and differences were there between Tasks A and B?
  - Between tasks C and D?
- When we do Multiple Groups tasks what operations with fractions are we doing?
- Where are these Standards in the Common Core?

Apartment Demographics

- In an apartment complex…
  - 3/4 of the men are married to women
  - 2/3 of the women are married to men
  - The rest of the men are single
  - The rest of the women are single
  - There are twice as many single women than single men
- How many of each demographic (single/married, men/women) could there be?
- Find as many possible solutions as you can.

Apartment Demographics

- Approaches?

Apartment Demographics

- Solutions?

Creating Tasks

- Equal Sharing
- Multiple Groups

- Create 2 tasks of each type
  - Progressing in levels of difficulty
- Be careful with the numbers that you select

Multiplying Fractions

- Area models… graph paper!
- You are covering a rectangular play area with rectangular carpet pieces. Each piece is 1 yard wide and ¼ of a yard long. If you cover the area by putting 10 pieces together, draw a picture of the play area. Find the area of each piece and the total play area.
- Write 2 equations to match this task: one with only addition and one with only multiplication.

More area models

- What is the total area of a pamphlet that is ¼ of a foot long and a 1/3 of a foot tall?

- Solve it in the following ways:
  - Open array
  - Computing

Show your work…

More Area Models

- How many 1/2” by 1/2” square tiles do we need to fill a region that is 2” by 3” ?

- Solve it 2 ways. Draw a picture.
- Use numbers and an equation

- Let’s look at the 5^{th} grade NF standards

Area Models

- What are some implications for working with teachers on supporting their use of area models to multiply fractions?

Summing it up

- What would you say to teachers at your school about how the Common Core Fractions concepts build across grade levels?

Tasks → Mathematical Practices

- Think about the tasks we explored
- How could we help students to…
  - Model with an equation or representation?
  - Attend to precision through communicating about strategies and mathematical ideas?
  - Construct a viable argument or critique the reasoning of others?

Mathematical Practices

- How do we get these into classrooms?

Questions?

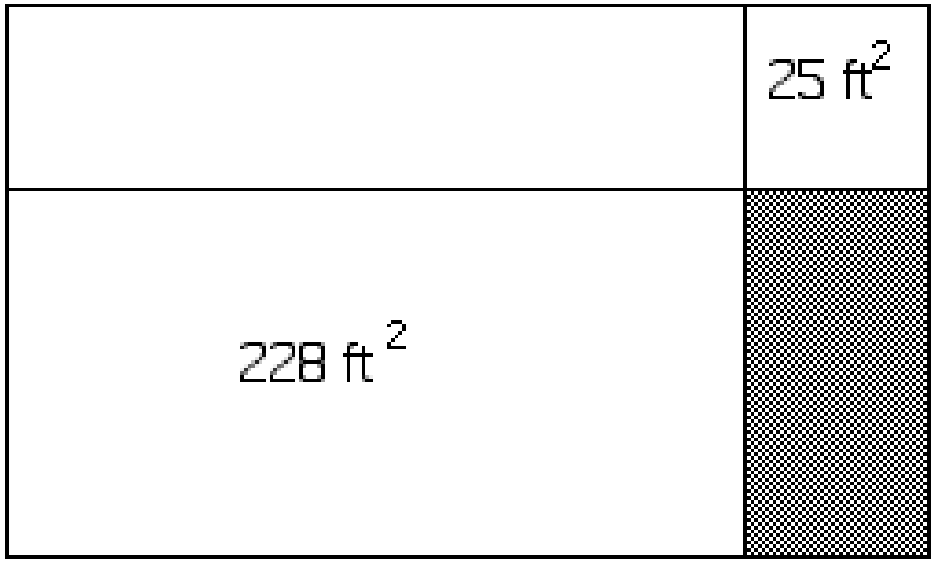
- Drew.Polly@uncc.edu
- NC DPI Wiki….
- http://elemath.pbworks.com

Resources….

- DPI Math wiki
  - Unpacking document
    - Standards for Math Practice explanation
  - Formative assessment tasks
  - Lessons for Learning
  - Coming… math games

Area of the missing rectangles

A rectangle with integral side lengths and area less than 400 ft2 is divided into four smaller rectangles with the upper-right corner being a square. Find a possible area of the shaded rectangle if one of its side lengths is greater than the square's side lengths. The area of two of the rectangles is given in the diagram.



Area of missing rectangles

- Approaches?