Mathematical Modelling and Mathematical Competencies: The case of Biology students.

Yannis Liakos
University of Agder, Norway

The research aims at introducing modelling tasks in order to engage students more actively into learning mathematics through tasks that are biologically 'colored'. My focus is on the individual progression (if there is any) of students' mathematical competencies during a sequence of modelling sessions that will be part of a regular course of their first year calculus. My ultimate goal is to construct a dynamic competence profile for every student that will participate in the project. Taking the above into consideration, my research suggests a number of interventions in a standard freshmen mathematics course for biology students, interventions that offer a fruitful didactical environment where students can sharpen their mathematical competencies.

Key words: mathematical modelling, mathematical competencies, tasks, progression

Introduction

There is an increasing amount of literature which provides documentation for the learning benefits associated with engaging students in mathematical modeling. There is a 'red thread' among many researchers who, through the description of mathematical modelling processes, displayed the variety of many opportunities for educational benefits (Kaiser et al., 2006). Students engaged in modeling may develop a deep understanding of the content and an ability to solve novel problems (e.g. Wynne et al. 2001, Lehrer & Schauble 2005). Other studies (Schwarz & White 2005; Windschitl et al. 2008) have shown that modeling curricula can bring students into alignment with the epistemic aims of science and help them develop more sophisticated ideas about the scientific enterprise as a whole. Sriraman et al. (2009) blended the notion of interdisciplinarity with modelling, highlighting the necessity for creativity and giftedness across disciplines. It comes as no surprise that

> Both the National Research Council (NRC) and the National Science Foundation (NSF) in the U.S is increasingly funding universities to initiate inter-disciplinary doctoral programs between mathematics and the other sciences with the goal of producing design scientists adept at using mathematical modeling in interdisciplinary fields (Sriraman & Lesh, 2006, p.247).

Theoretical Stance

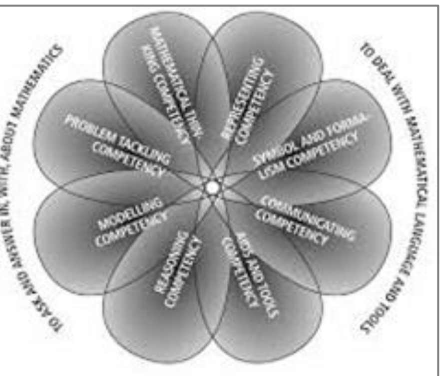

Figure 1. The KOM model (Blomhøj & Jensen, 2007)

A long-lasting and ongoing discussion among researchers and members of Educational Institutes centers on students' assessment and the need for a solid and valid evaluation (e.g. Galbraith 2007, Haines and Crouch 2007, Vos 2007). A different approach though occurred by an important number of researchers when they turned their view on students' competencies and mathematical competencies (e.g. Greer and Verschaffel 2007, Henning and Keune 2007, De Bock et al. 2007, Houston 2007, Blomhøj and Jensen 2007).

The Danish KOM (Kompetencer og matematiklæring) project (Fig. 1) Niss (2003) focused on basing the description of mathematics curricula primarily on the notion of mathematical competence. The framework they proposed could apply at any educational level. Niss and Højgaard (2011) managed to combine assessment and competence by introducing a three-dimensional model of progression of each competence, which I describe in the *Data Analysis* section. This model will be part of my set of tools for my data analysis since it focuses on the progression of students' development of a certain competence. It is possible that one mental, verbal or written action may describe two different competencies (overlapping); therefore it is important to locate discrete elements that characterize every competence even though in mathematical modelling activities students need to combine mental process in terms of combined competency profiles. We can find some attempts towards this direction from Andersen et al. (2001) and OECD (1999, 2001) focusing on the PISA investigations. These studies include an international comparison of secondary school students' competence profiles. The research reported here contributes to this research program by extending such analyses to university students.

The competence framework in my research will be based on the general term of mathematical literacy which combines the development of mathematical concepts and terms while dealing with real-world (realistic) tasks.

In my research I will use the notion of mathematical competence as something that students must bring into action in order to meet the challenges of the future. I consider this future-directedness rather important in educational terms because I am interested in the ways in which a student puts his or her mathematical knowledge to functional use. This will also give a strong connection to what Blum et al. (2002) considered as vital elements of modelling competences. He described a student, who is competent in modelling, as one who is able to structure, mathematize and solve problems. Furthermore in line with what Maaß (2004) considered as modelling competencies it is important to understand that knowledge alone is not sufficient for a student to develop his/her mathematical competencies. A student has to use and direct his knowledge with a suitable and specific way in order to be successful in modelling and this is where my study focuses on: observing, monitoring and analyzing that process.

Besides the competence theoretical framework I will adopt the theory of Didactical Transposition. Bosch and Gascón (2014) refer to four different bodies of knowledge (see Figure 2) where the transformations applied to a "content" or a body of knowledge since it is produced and put into use, until it is actually taught and learned in a given educational institution.

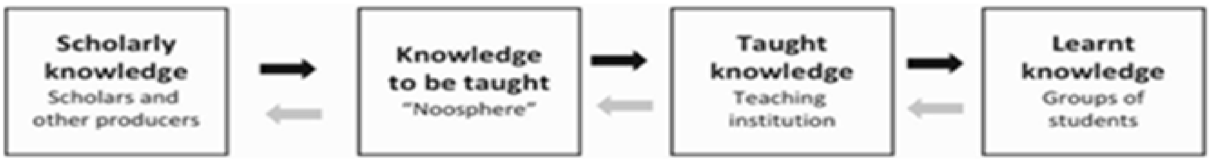

Figure 2. The didactical transposition (Bosch & Gascón)

Research Questions and Design

In this report I address the following two research questions:

1) What is the dynamic of a student's competence profile through the course of the mathematical modelling unit?
2) What competencies are deemed necessary for a student in a biology department?

By the term dynamic in the first research question I include the notion of progression (of a student's competence) from the very beginning of the project and also focus on identifying the initial set of competencies that a student brings when he or she enters the tertiary level.

In this study I make use of a design based research approach (Kelly & Lesh, 2000) in which an iterative process of design, implementation, and analysis takes place. More specifically, this study takes place in two phases. In Phase 1 (already completed), I investigated students' mathematical competencies during their engagement with a series of modelling tasks. These students were on their first year in a university's Department of Biology and the modelling session followed up a regular first year calculus course.

Phase 2 is ongoing and takes place with first year students in biology at the Norwegian university where this study takes place begin with a standard 10 week mathematics course on calculus. The modelling sessions occur weekly during their first semester. Approximately 100 students are organized in 3 separate classrooms where there is a 50 minute modelling session where students engage with modelling tasks. The students in each classroom are organized in small groups of three or four. One small group from every classroom is chosen to be monitored with audio and video devices. Every two sessions are considered to be a single modelling block where a new mathematical tool will be introduced. By the end of every session, a modelling task will be assigned to the students as part of their obligatory assignments for their mathematics course. Every modelling block will be designed in respect to the competence theoretical framework that I adopted. In addition the sessions will provide new knowledge that is also necessary for the successful engagement of students with the home assignments.

Methods for Data Generation

Data for Phase 1 and 2 consists of students' written work (tasks and assignments) and recordings (video and audio) which capture all kinds of discourses that are taking place during the sessions. In Phase 2 a questionnaire will also be given to the students at the beginning and at the end of the project. Discussions between the students and with the lecturer will be recorded both in video and in audio form. In every classroom separate cameras and audio devices will record the focusing on the group and to the whole classroom. In addition, the selected groups will be provided with a special device (LiveScribe 3 Smartpen) for more accurate and secured data collection. The same equipment will be used in all interviews with the task designer.

Data Analysis

Data from RQ_1 and RQ_2 will allow me to address the four bodies of knowledge proposed by the ATD, the first two from a detailed task-design analysis, the third from the above mentioned recordings and the last from a general assessment (formal exams and general performance in the classroom during the sessions). A task-design analysis, for example, can provide what the existing literature (mathematical biology) provides on population dynamics and exponential growth (scholarly knowledge) but also which task was finally decided to be presented (knowledge to be taught) and this will happen for every different modeling block.

The multi-dimensional model functions in such a way that whenever one or more dimensions may display a change (progression) then the volume (student's competence profile) of the cuboid changes. At this point a better analysis of these three dimensions is necessary.

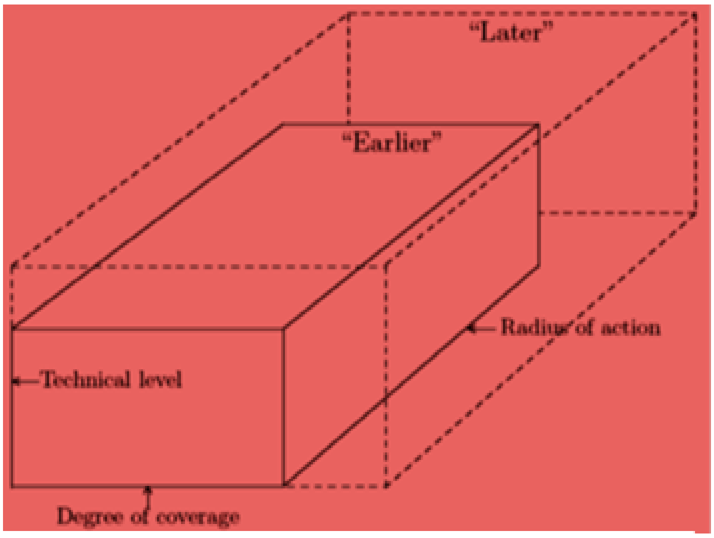

Figure 3.The three dimensional model (Niss & Højgaard, 2011)

- **Technical Level**: indicates how and to what degree (how advanced) a student may use his/her tools and mathematical entities which belong to his/her cognitive set of knowledge in order to activate a certain competence.
- **Radius of Action**: illustrates the range of action a student may take in terms of context and didactical situations. It shows where a student can activate a specific competence.
- **Degree of Coverage**: indicates to what extent a student is developing a competence in terms of its specific characteristics.

For the needs of this analysis I constructed a coding system which breaks down into smaller parts the verbal, mental and written actions of every single competence. This system, which is illustrated with data from Phase 1, functions as a decoding tool that assigns every student's discourse action to specific parts of a certain mathematical competence. In order to be successful in this attempt I need strong indicators that correspond to a specific competence and the frequency of appearance of these codes can be an indicator of progression (or stagnation) of a specific competence. It is in my intention to improve the reliability of this coding system by grading every code depending on the different tasks the students encountered during the modelling sessions.

Abstract from my coding system: the Reasoning Competence

- When a student is able to follow and assess a chain of arguments.
  Code: Flw. Arg.

- Knowing the difference between a formal mathematical proof and other kinds of mathematical reasoning. Code: Pr. ≠ Math. R.

- Separating main lines from details and ideas from technicalities during a line of arguments posed by anyone in the classroom. Code: Sep.

- When a student has the skill to devise formal and informal mathematical arguments. This may differ from a typical mathematical proof in our study therefore we could include the term: proving statements. Code: Pr. St.

Data from Phase 1

At the extract below we can see a discussion, between the members of Group2 in a university department of Biology about a modelling task. The students should come up with a solution in a time frame of 15 minutes and then present their possible solutions on a whiteboard in front of the other groups. The colored parts of the extract are based on the coding system above.

The task

Uncontrolled geometric growth of the bacteria Escherichia coli (E. coli) is the theme of the best-selling Michael Crichton's science fiction thriller, The Andromeda Strain. At some point the author claims that: ''**In a single day, one cell of E. coli could produce a super-colony equal in size and weight to the entire planet Earth.**'' If a single cell of the bacterium E. coli divides every 20 minutes, how many E. coli would be there in 24 hours? The mass of an E. coli bacterium is 1.7×10^{-12} g while the mass of the Earth is 6.0×10^{27} g.

- Is Crichton's claim accurate?
- If not, how much time should be allowed for this statement to be correct?

The students are trying to find a way to mathematize the assumption: if a single cell of the bacterium E. coli divides every 20 minutes. They should come up with this expression: 2^{3x}

1. A: what about 24x8 (checking calculator) and then I get…oh we have to compare it with, we can take 10 to the power of…isn't that very close to Earth's mass?
2. B: close is not enough for mathematics.
3. A: yes but we have something to compare it with. Is 8 our ground number (meaning base) or is it 24? I don't know what power I should put.
4. B: we have 2, 4, 8 … (almost silent)
5. C: so it's always double.
6. B: so it goes 16, 32, 64…
7. A: it may be 2^x ? Since it's always changing.
8. C: But our ground number?
9. A: Our ground number is 2, when we have 4 it is 2^2 then 2^n=8
10. B: No you have to put 3 to get 2^3=8
11. C: You have 8, 16, 3, 64… so is there …? How is it called?
12. A: (writes 2^3, 2^4…) Is this the first line with 2^1? The starting point? Oh we can take just 2^{25} and then we have (a huge number appears at the calculator)

The Reasoning Competence is not the only one that appears in the text but for the interests and page restrictions of this report I included only this specific type of mathematical competence. It is quite possible that episodes of overlapping competencies may occur but this is not an obstacle when it comes to identifying the progression of a specific competence.

Goals & Addressed Questions

My main goal is to create a dynamic competence profile for every student and it is in my intention to redefine the term good student by that of competent and try to find a way to identify students' learning skills, which in this study are considered as mathematical competencies. I therefore consider that the didactical environment of mathematical modelling is a suitable one for my research interests.

It would be more than helpful for my dissertation, if I could have some feedback on the following questions that are closely related with my data analysis:

1. What statistical tool would be ideal for my code analysis?
2. Is there a solid connection between ATD, mathematical competencies and mathematical modelling tasks?

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