International Journal of Application or Innovation in Engineering & Management (IJAIEM)
Web Site: www.ijaiem.org Email: editor@ijaiem.org
Volume 4, Issue 5, May 2015 **ISSN 2319 - 4847**

Mathematical models for movement bubble in the water based on the capture images using SONY camera

[1]Fatima I. Abbas , Ali Abid Dawood Al – Zuky [2] , Anwar H.Mahdy[3]

[1]Department of Physics, College of Education, AL-Mustansiriyah University, Iraq.

[2]Department of Physics, College of Science, AL-Mustansiriyah University, Baghdad, Iraq.

[3]Department of Computing, College of Science, AL-Mustansiriyah University, Baghdad, Iraq.

ABSTRACT

Bubbles and foams are important features of liquid surface phenomena, this research Provides, the study of the movement of air bubble in the water and rising through the channel (water hose) tight, with different diameters (0.4, 0.5,0.7,1, 1.2)cm, using image processing technology this by build a computer algorithms to analysis the motion in image planes for the successive frames for extract images of video clip using SONY camera, considering the number of snapshot represent function of the time, and determine the rising bubble location, speed and Acceleration . By using suitable processing for a software package, such as programs (Ulead Studio 11, Matlab 2012b and Table Curve(TC) version 5.01). After that using fitting processing to estimate the best fitting model for the motion parameters. Then comparing between the real motion parameter deter as a function of times and the estimated values from the estimated model. Where have been found high a agreement between them.
Keywords: Bubble, motion parameters, location, velocity, acceleration, rising, liquid.

1.1.INTTDUCTION

Bubbles Known, it is a gas, surrounded by a liquid membrane or solid, very thin. And playing surface tension, essential role in the formation of bubbles, surface tension, is a physical property, attract particles with each other, thus behaving liquid surface like a roped membrane, make surfaces has a smaller space, thus take bubble form spherical [1]. Bubbles consists, as a result of arent pressure or shaking out, and immediately remove the pressure or shaking out or arent, pushed, gas bubbles rising to the outside to the top surface of the water[1].

Air bubbles are used in chemical, biochemical, environmental, and food process for improving the heat and mass transfer. Bubbles play an important role in many applications such as; in the fermentation process, in the cooking processes, in determining the rates of heat and mass transfer and coalescence, in the pipeline transport applications, in polymer and sludge processes and others [2]. The bubbles find uses in many process industries such as in vacuum pan operation in sugar industries which is an important process for the production of raw sugar. A number of researches have been conducted to study the bubbles and its properties in different ways, in the following some of these studies as following:

Jeong-Mo Hong et. al.[3] in 2003They presented a new fluid animation technique in which liquid and gas interact with each other, using the example of bubbles rising in water. In addition to the flowing motion, the interactions between liquid and gas cause buoyancy, surface tension, deformation and movement of the bubbles. They combine the volume-of-fluid method and the front-tracking method developed in the field of computational fluid dynamics.

Hassan N.M.S.et. al.[4] in 2007 Presented the bubble rise phenomena in different low concentration polymer solutions for higher Reynolds number(R_e). The main characteristics, namely, the bubble velocity, the bubble trajectory and the drag relationship are investigated. They results show that the average bubble rise velocity increases with the increase in bubble volume for different low concentration polymer solutions and the bubble velocity is not dependant on the size of the test rig. In trajectory analysis, they seen that the smaller bubbles show helical or zigzag motion and larger bubbles follow spiral motion.

Jeong-Mo Hong et. al.[5] in 2008 they presented a hybrid of Eulerian grid-based simulation and Lagrangian smoothed particle hydrodynamics (SPH) for the realistic simulation of multiphase fluids, focusing on bubbles. Using there heuristic bubble model, they could generate natural looking computer generated bubbly water.

Also Ho-Young Lee et. al.[6] in 2009 they seeded Lagrangian bubble particles in air bubbles and caused them to move like molecules in order to create turbulence and consequently simulate realistic fluid in a grid based fluid simulation. In conclusion, the contributions of there paper is that it enables animators to control various fluid motions by creating turbulence with bubble particles and correcting the volume of air bubbles.

Markus Ihmsen et. al.[7] in 2011 They proposed a velocity based heuristic that generates air bubbles for inflows. Thereby, trapped air is animated efficiently, i.e. without explicitly simulating the air phase surrounding the liquid. Also employ a simple foam model in order to simulate floating air bubbles.

While N.M.S.Hassan et. al.[8] in 2012 They propose a comprehensive comparison of experimental results of the bubble trajectory and shapes are made for water, polymeric solutions and a crystal suspension.They were seen from there study that the trajectories of bubbles were significantly influenced by the bubble deformations and the surrounding liquid flow.

Our research about study of the rising air bubble in the water basin, using image processing technology to bubble tracker, to generate the trajectory of an bubble over the time by locating its position in every frame of video Typically, tracking over time consists of matching moving bubbles in successive frames, using FUJIFILM camera. Then estimated mathematical models for rising bubble motion ,location, velocity and Acceleration as function of the time.

1.2Fluid DYNAMIC DEFINITIONS

In fluid dynamics, the Morton number (M_o) is a dimensionless number used together with the Eötvös number(E_o) or Bond number(B_o) to characterize the shape of bubbles or drops moving in a liquid or continuous phase, The Morton number(M_o) and Eötvös number(E_o) are defined as,[9].

$$M_o = \frac{g \mu_l^4 \Delta\rho}{\rho_l^2 \sigma^3} \qquad (1)$$

$$E_o = B_o = \frac{g L^2 \Delta\rho}{\sigma} \qquad (2)$$

Where g is the acceleration of gravity(m/s^2), $\mu_l$is the viscosity of the surrounding fluid(Pascal.sec), ρ_l the density of the surrounding fluid (kg/m^3), $\Delta\rho$ the difference in density of the phase(kg/m^3), σ is the surface tension coefficient (N/m), and L is characteristic length(m). The Morton number(M_o) can also be expressed by using a combination of the Weber number(W_e), Froude number(F_r) and Reynolds number(R_e),[9].

$$M_o = \frac{W_e^3}{F_r R_e^4} \qquad (3)$$

The Froude number(F_r) in the above expression is defined as,[9]:

$$F_r = \frac{v_b^2}{g d_{eq}} \qquad (4)$$

Where v_b is the bubble rise velocity(m/sec) and d_{eq} is the equivalent diameter(m) of the drop or bubble. A high value of the Eötvös(E_o) or Bond number(B_o) indicates that the system is relatively unaffected by surface tension effects; a low value (typically less than one) indicates that surface tension dominates. Intermediate numbers indicate a non-trivial balance between the two effects, where weber number(W_e), and Reynolds number(R_e) are defined as,[9]:

$$W_e = \frac{\rho_l v_b^2 d_{eq}}{\sigma} \qquad (5)$$

$$R_e = \frac{\rho_l d_{eq} v_b}{\mu_l} \qquad (6)$$

1.4.BUBBLS MOTION AND TYPES

The characteristics of bubble motion in liquids are still not well understood because many parameters influence the terminal rise velocity, trajectory and shape of bubbles[10]. As the bubble motion is a complex problem, the degree of the complexity increases with bubble size[11]. When bubble rises through liquid, the most resistance will be imposed directly on top and the bubble first moves along a straight vertical path and then develops a zigzag motion which consequently can change into a spiraling motion, at the same incidence as the preceding zigzag[12]. In most reported studies, very small bubbles (less than 1 mm) rise through water maintaining their spherical shape due to surface tension. The trajectory of these bubbles follows a straight line until it completes its journey[13]. On the other hand, considerable deformations are observed for bubbles with diameters larger than 1 mm [14]. This deformation occurs due to the increase in the variations of hydrostatic and dynamic pressure over the bubble's surface[15]. Therefore, large bubbles cannot remain spherical and deform into oblate spheroids first and then become ellipsoidal, and with further increase in size they switch into a spherical or ellipsoidal cap. Bubble motion such as velocity and trajectory also change with the increase in bubble size [16].

The bubble is not always rising in straight path. When the bubble size increases, a straight path turns into zigzag or spiral in fluids of small Morton number(M_o). Then the path becomes nearly straight again for a spherical cap bubble. Only a straight path is observed in liquids of large Morton number(M_o) [17]. Aybers, N. M.et.al. reported different types of trajectories such as zigzag, helical or spiral and rocking motions[18]. Haberman, W. L.et.al. also observed rectilinear (R_e (Renold's number)< *300*), spiral and rocking motions[19]. They indicated that the spiral path could be either clockwise or counter-clockwise, depending on the conditions of bubble release. The major axis of the bubble is

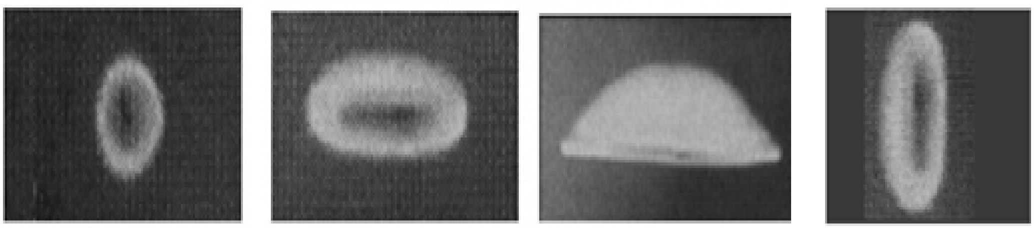
always directed perpendicular to the direction of motion. Saffman, P. G. observed only zigzag bubble rise motions as the bubble rises in water when the radius of the bubble was less than 1 mm, but bubbles of larger radius showed either zigzag or spiral motions[20]. Feng, Z. C. et.al. verified various possible trajectories for different shape regimes[21]. A single bubble can follow a zigzag path at Re ≈600, accompanied with vortex shedding behind the bubble. Under the same experimental conditions, Yoshida, S. et.at. reported that the bubbles can also follow a spiral trajectory without vortex shedding[22]. Tsuge, H. et.al. reported that the trajectories of rising spherical and ellipsoidal gas bubbles at higher Reynolds numbers are identical[23].
The trajectories of bubbles are strongly influenced by the bubble deformations and the surrounding fluid flow [24]. Bubble deformations and fluid flow could be explained with dimensionless groups such as the Reynolds Number (R_e), the Weber number (W_e), the Morton number (M_o) and bubble aspect ratio (E) [16,25]. In fluid mechanics, R_e gives a measure of the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions. On the other hand, W_e is often useful in analyzing fluid flows where there is an interface between two different fluids, especially for multiphase flows such as bubble rise in liquids. It can be thought of as a measure of the relative importance of the fluid's inertia compared to its surface tension. The quantity is useful in analyzing the formation of droplets and bubbles. The dimensionless number such as M_o, is also used together with the Eötvös number (E_o) to characterize the shape of bubbles or drops moving in a surrounding fluid or continuous phase. Eötvös number(E_o) is considered as proportional to buoyancy force divided by surface tension force,[8].

$$E = \frac{d_w}{d_h} \qquad (7)$$

Where d_w is represent semi major axis, and d_h represent semi minor axis. Usually, R_e controls the liquid flow regime around the bubble and W_e, M_o and E characterize the bubble deformations and bubble shapes. Therefore, the influences of R_e , W_e and M_o are seen as important for elucidation of the bubble trajectories[8].

1.5BUBBLE SHAPES

The shape of the bubbles greatly influences the bubble rise velocity and it has a significant role in determining the rates of heat and mass transfer and coalescence. Normally, a motionless bubble has a spherical shape because surface tension minimizes surface area for a given volume. When a bubble has motion, different forces exist such as drag caused by the liquid, viscosity of the liquid, pressure difference between the top and bottom of the bubble as well as the wall effects. Mainly, three types of shape such as spherical, ellipsoidal and spherical-cap or ellipsoidal cap in free motion under the influence of gravity are observed in Newtonian liquids[8].



(A) (B) (C) (D)

Figure 1 Different types of bubble shape in Newtonian fluid[8].

The shapes of bubble are related to the R_e, at low R_e, the bubble retains its shape as a sphere because interfacial forces and viscous forces are much more important than inertia forces. Most bubbles of small size fall into this category. The spherical shape of the bubble is shown in the Figure(1A). The next category of bubbles is termed "ellipsoidal"; these are oblate with a convex interface around the surface when viewed from the inside. The liquid viscosity may affect the bubble shape, stretching the bubble out laterally, so that actual shapes may differ considerably from true ellipsoids. However, the general shape is comparable to an ellipsoid which is shown in Figure(1B and 1D)[8].
Large bubbles have a flat base or a spherical wedge, which may look very similar to segments cut from a sphere. They are heavily distorted from the equilibrium shape of a sphere. In this case, the Reynolds number is high and these bubbles are termed as spherical-cap or ellipsoidal-cap. This type of bubble is shown in Figure(1C),[8].

2.1 DETERMINE THE BUBBLES MOTIONPARAMETERS MODELS

A digital waterproof SONY camera(DSC-TX10, Resolution 16.20 megapixels) inside the water has been used in this study. The work system show as in the block-diagram in figure(2), design to determine the location r(x,y), change in displacement (Δr), velocity(v) and acceleration(A) for rising bubble in the water, in which the work was divided into several steps, the practical work and detail of the uses, tools, devices, algorithms and software for each steps will be explain:

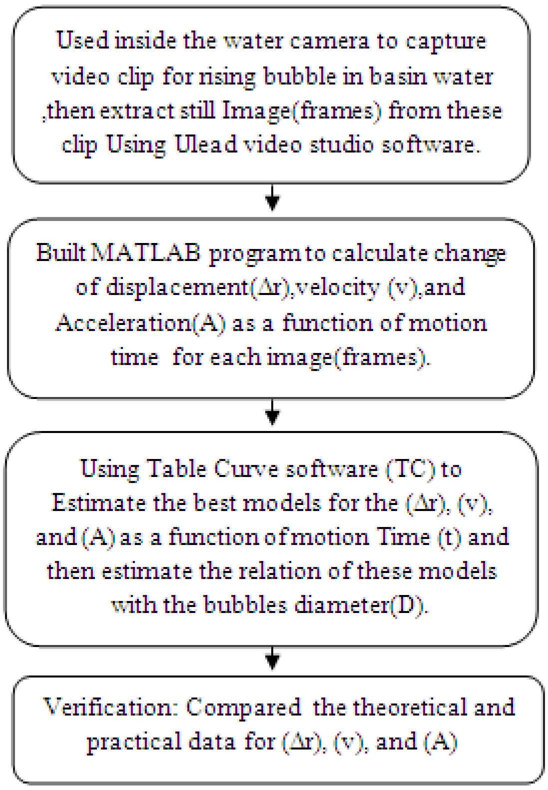


Figure 2 The Block diagram explain the steps of estimate the motion statistic for rising bubble in the water.

The following steps, explain phases of sequence work

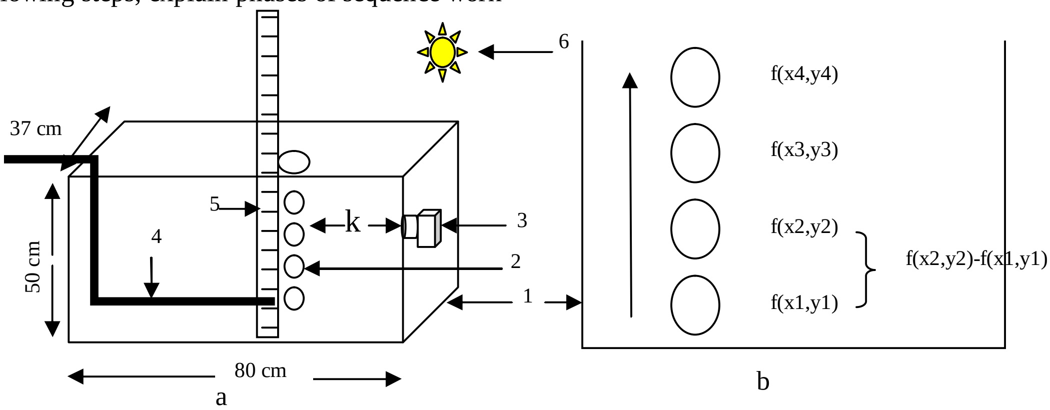


Figure 3 (a) the used basin: (1)basin water,(2)rise bubble,(3) waterproof camera(sony), (4)hose,(5)ruler,(6) source of light.(b) The sketch of the rising bubble in basin .

1. Extract Still Images (frames) From Video Clip: Rising bubbles produce by pumping a gas in hose then the gas will be reach the terminal end of the hose at the lowest depth of water in the basin . This gas after exit from the hose will be produce a bubble move up ward to the upper surface. waterproof digital camera (SONY -DSC-TX10, Resolution 16.20 megapixels) inside the water imaging was performed. Basin dimensions is (80,37,50) cm^3 filled with water as show in figure(3a), have been used five different diameters(D) of hoses (0.4,0.5,0.7,1,and 1.2)cm. For constant space (k=50 cm, distance between the camera and rising bubble), have been taken video clip. Using Ulead video studio 2011 plus software to convert video clip into still images (frames) this frames save in JPEG format, these video clip each second converted into 30 (fps). First frame $f(x_1,y_1)$ started from the bottom at initial time=0 sec, second frame $f(x_2,y_2)$ its time 1/30 sec, third frame $f(x_3,y_3)$ its time 2/30 sec…etc, these images extract as a frame of time $f(x_t,y_t)$. Thus determine the difference between two points from, $[f(x_{t+1},y_{t+1})-f(x_t,y_t)]$ as shown in the figure(3b), to reach the surface of the basin.

International Journal of Application or Innovation in Engineering & Management (IJAIEM)
Web Site: www.ijaiem.org Email: editor@ijaiem.org
Volume 4, Issue 5, May 2015 **ISSN 2319 - 4847**

2. Analysis Algorithms: MATLAB software has been used to build the introduced Algorithms; The first algorithm to compute the scale factor (Scf) for the more visible frame image, this is performed by measure the real value of bubble diameter(d_{cm}) by ruler putting in the basin through the itself image, and determining the bubble diameter(d_{pixel}) in picture plane by selected two points on the boundary in the image using computer mouse, see figure (4). Then used distance law as in the algorithm(1) to compute scale factor(Scf), this scale factor can used in another algorithm computation in this work.

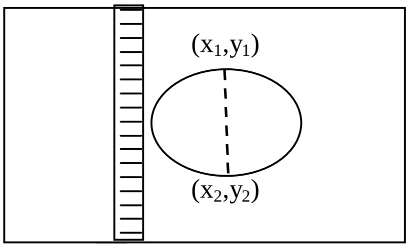


Figure 4 shows the Dotted line in picture plane to determine the bubble diameter once manually by mouse in pixel unit and another by ruler in cm unit.

Algorithm (1) Determine of Scale factor(Scf) for the frame image

Input:
1- The image of rising bubble in the basin (img).
2- Input known length (l_{cm}) on the ruler.

Out put:
Scale factor (Scf).

Start algorithm
1- load image(img) , to determine diameter of the bubble (x,y) .
2- manually selected two fixed points length information on the ruler to determine the Diameter (corresponding points) of bubble's in the image plane to compute bubble diameter in pixel .
this process can be explain in two step:
i. Used the property (mouse click) and determine the first point on the ruler(x_1, y_1) corresponding to point of the first end of known ruler length ($l$cm) in image plane.
ii. Move and used the property again (mouse click) and determine the second point on the ruler (x_2, y_2) to second end of the known ruler length ($l$cm) in the image plane, see figure (4).
3- Compute the length in pixels between point(1) (x_1, y_1) and point(2) (x_2, y_2) using:

$$l_{pixel} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

4- Compute scaling factor for this image frame using:

$$Scf = \frac{l_{cm}}{l_{pixel}}$$

5- Print Scf .
6- End algorithm.

The second Algorithm describe the process in detail how can be calculate the rising bubble location r(x,y), change of displacement (Δr), the velocity(v) and the Acceleration(A) for each frame image, by using following algorithm:

Algorithm (2) calculate the location r(x,y), change of displacement(Δr), the Velocity(v) and the Acceleration(A) of the moving bubble inside the water.

Input:

1- Scale factor (Scf).
2- The sequence n-frames $(img_{(1)}),(img_{(2)}),\ldots..(img_{(n)})$.

Out put:

i. The distance between two balls images both in pixels and centimeters, (Δr)pixel, Δr(cm).
ii. The velocity(v) in centimeters.
iii. The Acceleration(A) in centimeters .

Step1 : manually selected two bottom points in input images $(img_{(i)})$ as a fallow:

i. Used the property of (mouse click) and determine the bottom point (x_i,y_i) of ball image $(img_{(i)})$.
ii. Used the property again (mouse click) and determine the bottom point (x_{i+1},y_{i+1}) of ball image $(img_{(i+1)})$.

Step 2 : Compute the distance in pixels between the two points (x_i,y_i), (x_{i+1},y_{i+1}) using:

$$\Delta r_{pixel} = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}$$

Step 3 : Converted Δr from pixels to centimeters (cm) using:

$$\Delta r_{cm} = Scf * \Delta r_{pixel}$$

Step4 : Compute the velocity(v_i) in centimeter, depending on Δr_{cm} using the following equation :

$$\upsilon_i = \frac{\Delta r}{\Delta t}$$ Where $t = \frac{1}{n}$ sec, and Δt, represent the difference of two sequence frame.

Step 5 : input image $(img_{(i+2)})$ and repeat steps (from step1 to step 4) for $(img_{(i+1)})$ and $(img_{(i+2)})$sequence frame by frame to compute (v_{i+1}) .

Step 6 : Compute Δv_{cm} using :

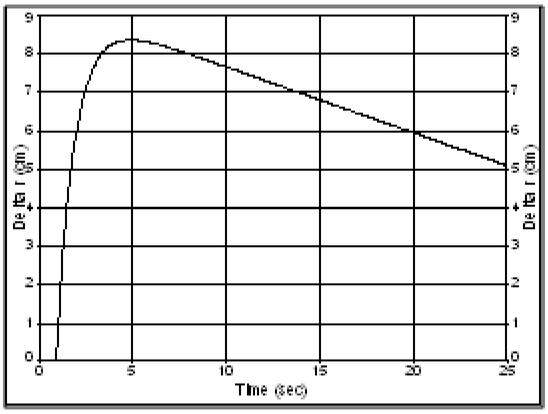
$$\Delta \upsilon_{cm} = v_{i+1} - v_i$$

Step 7 : Compute the Acceleration (A) in centimeter as following equation :

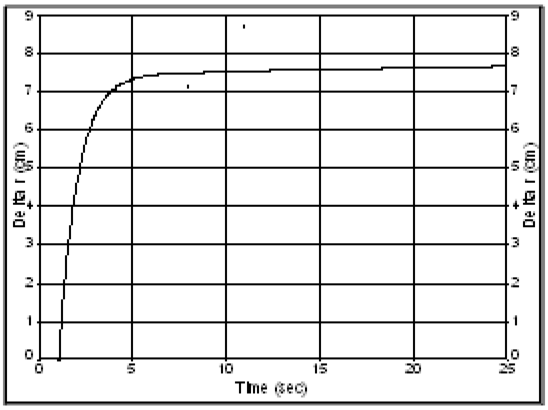
$$A = \frac{\Delta \upsilon}{\Delta t}$$

Step 8 : End algorithm.

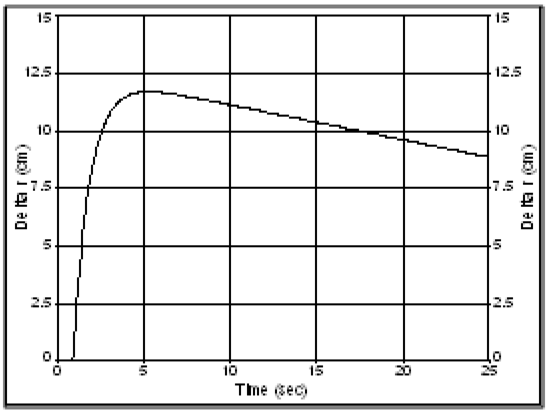
3. Motion Parameters Modeling : Used "Table Curve 2D Version 5.01" to fitting the practical Δr, υ and A data for different Diameter of hose(D) to introduce appropriate mathematical function for captured Δr, υ and A data for the frame images. This section contains the results of performing the suggested algorithms for the images frames Δr, υ and A data fitting of a rising bubble in the water at constant distance space k=50 cm(distance between the camera and the rising bubbles), and various Diameter of hose D=0.4,0.7,1, and 1.2cm, see figures(5,6,7).



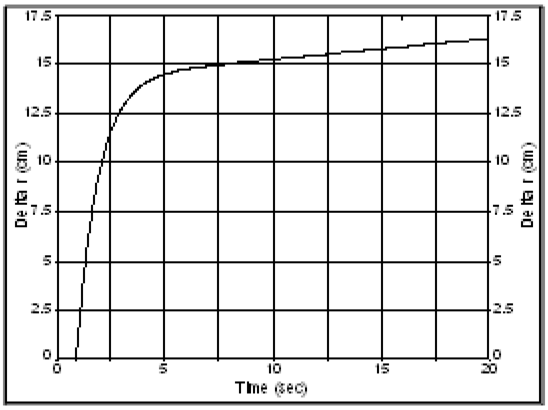
D=0.4 cm



D=0.7 cm

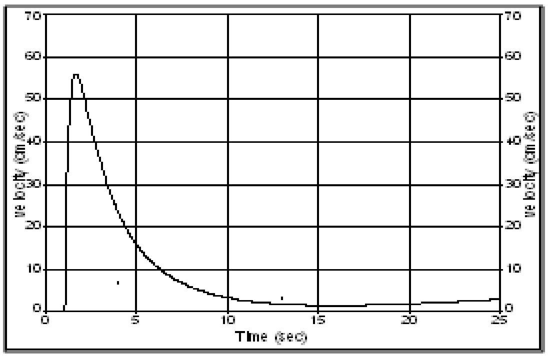


D=1 cm

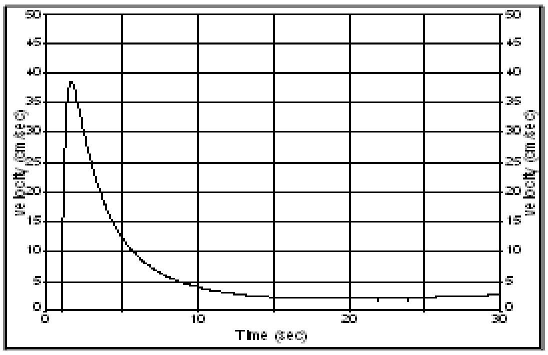


D=1.2 cm

Figure 5 shows change distance(Δr) cm as a function of the time(sec) for various diameters D(cm)) of the hoses in water

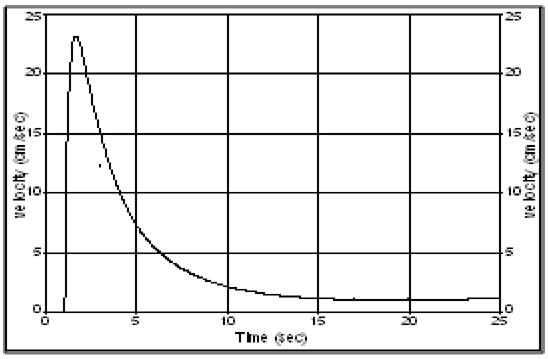


D=0.4 cm

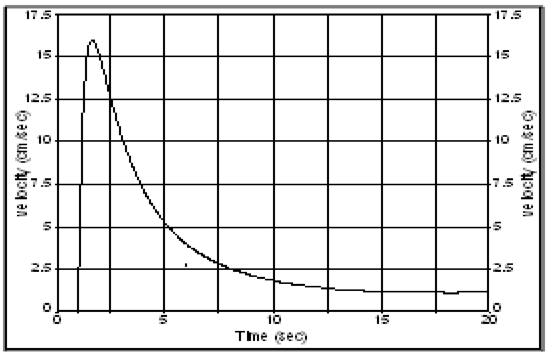


D=0.7 cm

Figure 6 shows velocity (v) cm/sec as a function of the time (sec) for various diameters D (cm)) of the hoses in water.

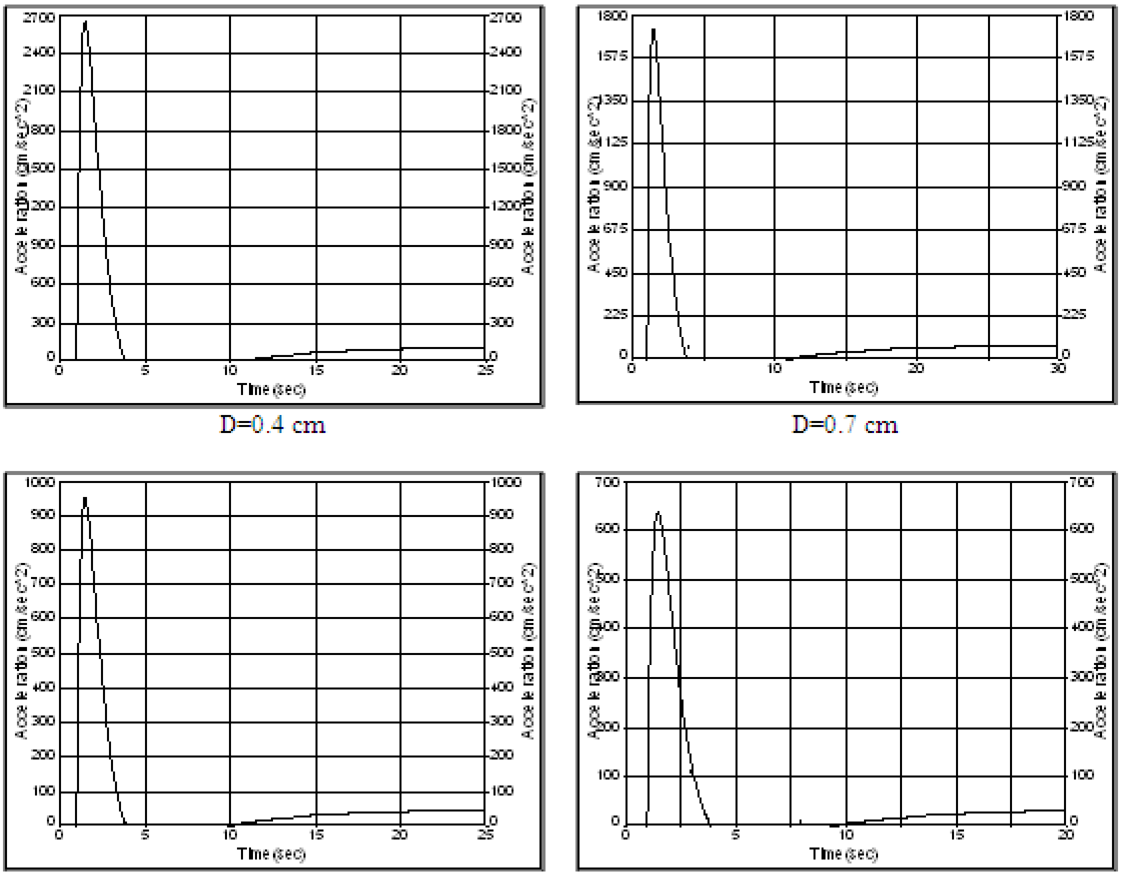


D=1 cm



D=1.2 cm

Continue figure6 shows velocity (v) cm/sec as a function of the time (sec) for various diameters D (cm)) of the hoses in water.



D=0.4 cm D=0.7 cm

D=1 cm D=1.2 cm

Figure 7 shows Acceleration (A) cm/sec^2 as a function of the time(sec) for various diameters D(cm)) of the hoses in water

The estimate best equations for the relationship between the three functions (Change of displacement(Δr),Velocity(υ) and Acceleration(A)) and the motion frame time as follows:

$$\Delta r = a_i + b_i t + c_i e^{-t} \qquad (8)$$

$$v = a_j + b_j t + \frac{c_j \ln(t)}{t^2} \qquad (9)$$

$$A = a_k + \frac{b_k}{t^2} + c_k e^{-t} \qquad (10)$$

Where the values of the constant $a_{i,j,k}$, $b_{i,j,k}$, $c_{i,j,k}$ and correlation(ρ^2) are reported in Table (1) for each Diameters (D).

Table 1: The parameters of the fitting equations for three functions

Δr,v,A data and real various Diameter(D).

Diameter (D) cm	Change of displacement(Δr)				Velocity(v)				Acceleration (A)			
	a_i	b_i	c_i	ρ^2	a_j	b_j	c_j	ρ^2	a_k	b_k	c_k	ρ^2
0.4	9.38	-0.17	-22.9	0.84	-8.49	0.39	346.7	0.84	135.8	-15748.5	42624.6	0.93
0.7	12.88	0.009	-32.2	0.82	-2.56	0.15	222.28	0.85	84.34	-10285.8	27845.6	0.92
1	12.64	-0.15	-31.4	0.81	-1.83	0.09	135.19	0.96	55.69	-5666.73	15303.0	0.94
1.2	14.19	0.106	-35.9	0.85	-0.98	0.07	91.58	0.96	40.95	-3788.03	10215.5	0.94

The relation between (a_i,b_i and c_i)-parameters as a function of the Diameter(D), using table curve TC software obtain the following equations:

$$a_i = 12.66 + 28.45(\ln(D))^2 + \frac{4.74\ln(D)}{D^2} \quad (11)$$

$$b_i = -6.94 - 16.69D\ln(D) + 6.78D^{2.5} \quad (12)$$

$$c_i = -31.45 - 83.64(\ln(D))^2 - \frac{13.74\ln(D)}{D^2} \quad (13)$$

Here the eqs.(11,12 and 13) representing the relation between (a_i,b_i and c_i)-parameters and Diameters (D) of hoses. By substituting these equations into eq. (8) get mathematical models that relates change displacement (Δr), Time (t) and Diameters (D):

$$\Delta r = \left(12.66 + 28.45(\ln(D))^2 + \frac{4.74\ln(D)}{D^2}\right) + (-6.94 - 16.69D\ln(D) + 6.78D^{2.5})t + \left(-31.45 - 83.64(\ln(D))^2 - \frac{13.74\ln(D)}{D^2}\right)e^{-t} \quad (14)$$

Here eq.(14) represented the mathematical motion model for Δr parameter of rising bubble in water.
While the mathematical model for velocity (v) parameter can be estimated in same way of estimated Δr, the estimated parameter is as follow:

$$a_j = -1.34 - 8.52(\ln(D))^2 \quad (15)$$

$$b_j = 0.083 + 0.368(\ln(D))^2 \quad (16)$$

$$c_j = 461.89 - 318.73D \quad (17)$$

Where the eqs.(15,16 and 17) representing the relation between (a_j,b_j and c_j)-parameters and Diameters (D). By substituting these equations into eq.(9) get mathematical model that relates velocity(v), Time (t) and Diameters (D) .

$$v = (-1.34 - 8.52(\ln(D))^2) + (0.083 + 0.368(\ln(D))^2)t + (461.89 - 318.73D)\frac{\ln(t)}{t^2} \quad (18)$$

Here equation(18) represent the mathematical model for v parameter of motion for the rising bubble in water at the constant space k=50 cm (distance between camera and rising bubbles).
Also can be found mathematical model for Acceleration (A)- parameters with the same way estimated in of Δr and v, the estimated parameter -is:

$$a_k = 176.06 + 117.42D \quad (19)$$

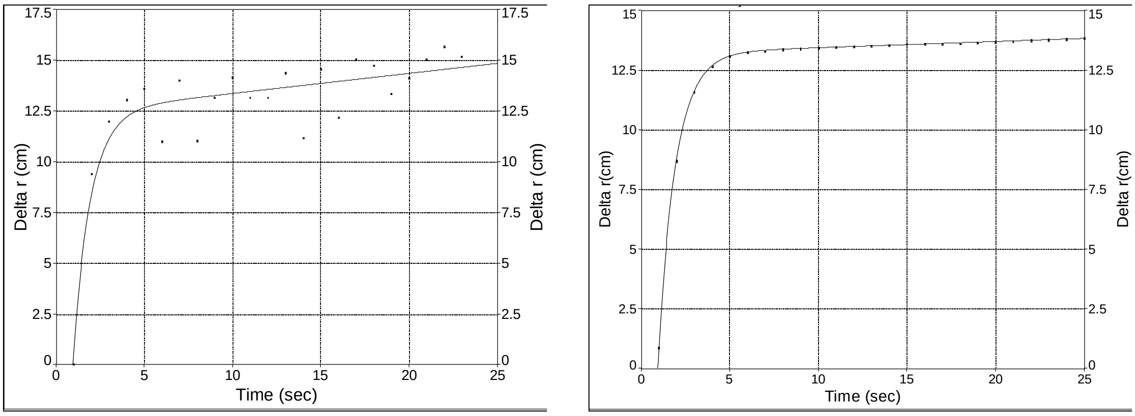
$$b_k = -21368.89 + 15147.41D \quad (20)$$

$$c_k = 57866.68 - 41053.87D \quad (21)$$

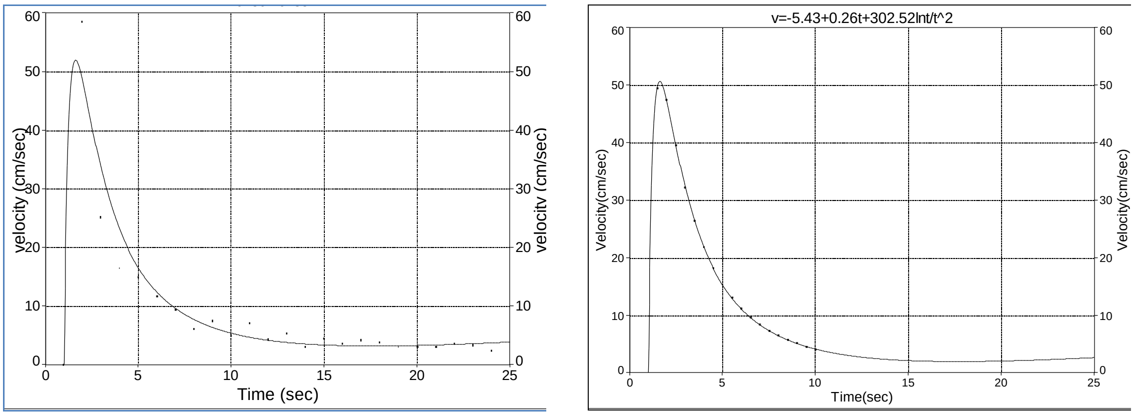
Where the eqs.(19,20 and 21) representing the relation between (a_k,b_k and c_k)-parameters and Diameters (D) for an acceleration of the rising bubbles. By substituting these equations into (10) get mathematical model that relates Acceleration (A), Time (t) and Diameters (D):

$$A = (176.06 + 117.42D) + \frac{(-21368.89 + 15147.41D)}{t^2} + (57866.68 - 41053.87D)e^{-t} \quad (22)$$

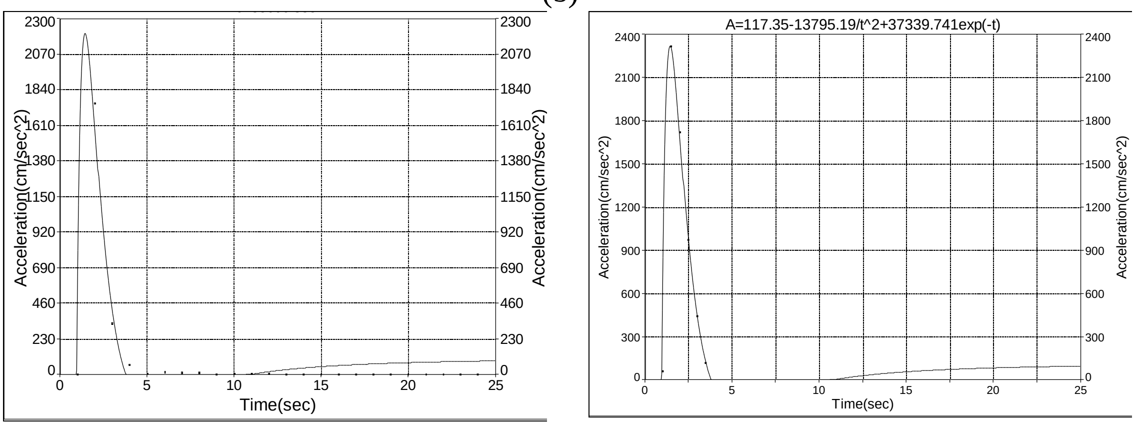
Here eq.(22) represent the mathematical model for the acceleration(A) of rising bubble motion in water.
4. Models Verification: The verification have been preformed by canceled the practical results for diameter (D=0.5 cm) from fitting operation in previous modeling process. In order to determine their values theoretically from the estimated mathematical model and make a comparison between the experimental and theoretical data. now will be reviewing the graphically details for the practical and theoretical for each functions (Δr,v and A) of rising bubble, see figure(8).



(a)



(b)



(c)

Figure 8 Matching the practical graphic at left side and theoretical graphic at the right side (a) of Δr,(b) Velocity(υ), and (c)Acceleration(A) for the rising bubble in water.

As in figure (8) notes that excellent match between the practical and theoretical curves, An error percentage resulting between the practical and theoretical for Diameter (D=0.5cm) tabulated in the table(2).

Table 2: Shows the error percentage resulting between the practical and theoretical data, D=0.5 cm

Functions	SONY camera		
	Theoretical	Practical	Error percentage %
Delta r (cm)	13.089	12.492	**4.56%**
Velocity	50.89	51.96	**2.05%**
Acceleration	2317.8	2209.2	**4.68%**

2.2 CONCLUSIONS

From the previous results of the present work, can be obtain a three new mathematical models of for the bubble motion parameters in the water based on using captured images for the moment bubbles using by (SONY DSC-TX10, Resolution 16.20 megapixels) camera;

First model: The change in displacement (Δr) as in Eq.(14) its accuracy 4.56%.

- Second model: The velocity (v) as in Eq.(18) its accuracy 2.05%.
- Third model: The Acceleration (A) as in Eqs.(22) its accuracy 4.68%.

It is clear that from practical work small bubbles followed a helical motion while larger bubbles followed a spiral motion. As the bubble size increases, initially, the bubble follows straight path, attains its terminal velocity and shape, then it switches to spiral path. Addition to from the data notes bubble rise velocity increases with the increase in bubble volume. This confirming the validity interpretations of previous research.

References

[1]. Ho-Young Lee, Jeong - Mo Hong ,Chang – Hun Kim, “Simulation of swirling bubbly water using bubble particles", The Visual Computer , Volume 25 , Number 5-7, 707-712 , 2009.
[2]. Shosho, C. and Ryan, Micheal, E.,"An experimental study of the motion of long bubbles in inclined tubes", Chemical engineering science, 56, 2191-2204, 2001.
[3]. Jeong-Mo Hong and Chang-Hun Kim ,“Animation of Bubbles in Liquid", Department of Computer Science, Korea University,2003
[4]. Hassan, N. M. S., KHAN, M. M. K. and Rasul, M. G.,"Characteristics of Air Bubble Rising in Low Concentration Polymer Solutions", AUSTRALIA,2007.
[5]. Jeong-Mo Hong, Ho-Young Lee, Jong-Chul Yoon, Chang-Hun Kim_, "Bubbles Alive",Korea University,2008.
[6]. Ho-Young Lee _ Jeong-Mo Hong _Chang-Hun Kim ,"Simulation of Swirling Bubbly Water using Bubble Particles", 2009.
[7]. Markus Ihmsen, Julian Bader, Gizem Akinci, Matthias Teschner ,"Animation of air bubbles with SPH", University of Freiburg,2011
[8]. N.M.S. Hassan, M.M.K. Khan and M.G. Rasul ,"Bubble Rise Phenomena in Non-Newtonian Crystal Suspensions", University AUSTRALIA, 2012 .
[9]. Clift, R.; Grace, J. R.; Weber, M. E. ," Bubbles Drops and Particles ", New York: Academic Press. ISBN 0-12-176950-X, 1978.
[10]. Frank, X., Li, H. Z., Funfschilling, D., Burdin, F. and Ma, Y.,"Bubble motion in non- Newtonian fluids and suspensions”, Can J Chem Eng., 81:483–490, 2003.
[11]. Kulkarni, A. A. and Joshi, J. B.,"Bubble formation and bubble rise velocity in gas-liquid systems ", 44, 5873-5931, Ind. Eng. Chem. Res., 2005.
[12]. Zenit R and Magnaudet J ,"Measurements of the stream wise vorticity in the wake of an oscillating bubble", Int. J. Multiph. Flow 35 195–203, 2009.
[13]. Duineveld, P. C., "The rise velocity and shape of bubbles in pure water at high Reynolds number", Journal of Fluid Mechanics, 292, 325-332, 1995.
[14]. Ellingsen K and Risso F "On the rise of an ellipsoidal bubble in water: oscillatory paths and liquid induced velocity ", J. Fluid Mech. 440 235–68, 2001.
[15]. Magnaudet J and Eames I, "The motion of high-reynolds-number bubbles in inhomogeneous flows", Annu. Rev.Fluid Mech. 32 659–708, 2000.
[16]. Hassan, N. M. S., Khan, M. M. K and Rasul, M. G.,"Modelling and experimental study of bubble trajectory in non-Newtonian crystal suspension", Fluid Dynamics Research, IOP Publishing, 42, 065502, 2010a.
[17]. Yang B," Numerical studies of single gas and vapor bubble flows ", PhD Thesis The John Hopkins University, Baltimore, MD, 2006.
[18]. Aybers, N. M., and Tapucu, A.," The motion of gas bubble rising through stagnant liquid ", Warme und Stoffubertraguang 2, 118-128, 1969.
[19]. Haberman, W. L. and Morton, R. K.," An experimental study of bubbles moving in liquids ", Trans ASCE 2799, 227-252, 1954.
[20]. Saffman, P. G.," On the rise of small air bubbles in water ", Journal of Fluid Mechanics, Digital Archive, 1: 249-275, 1956.
[21]. Feng, Z. C. and Leal, L. G.," Nonlinear bubble dynamics ", Ann. Rev. Fluid Mech. 29, 201-243, 1997.
[22]. Yoshida, S. and Manasseh, R.,"Trajectories of rising bubbles", 16th Japanese Multiphase Flow Symposium , Touha, Hokkaido, 1997.
[23]. Tsuge, H. and Hibino, S. I.," The onset conditions of oscillatory motion of single gas bubbles rising in various liquids" , Journal of Chemical Engineering of Japan 10, 66-68, 1997.
[24]. Ellingsen K and Risso F," On the rise of an ellipsoidal bubble in water: oscillatory paths and liquid induced velocity ", J. Fluid Mech. 440 235–68, 2001.
[25]. Hassan, N. M. S., Khan, M. M. K. and Rasul, M. G.," An investigation of bubble trajectory and drag co-efficient in water and non-Newtonian fluids", WSEAS Transactions on Fluid Mechanics, ISSN: 1790-5087, 3(3), 261 -270, 2008a.